Fluctuations and stability of LO/FF states: quantum liquid crystals

with : ***Ashvin Vishwanath***

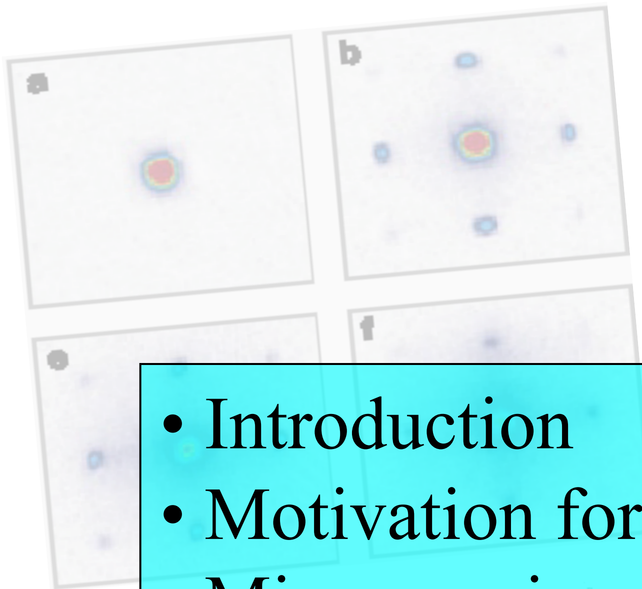
see: PRL 2009, detailed preprint

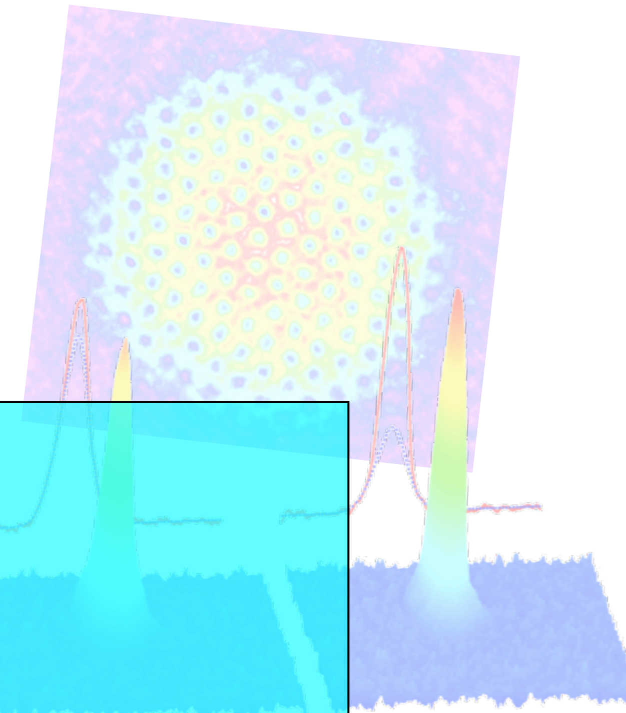
also see: Agterberg, Tsunetsugu, Nature (2008)
Berg, Fradkin, Kivelson, Nature (2009)

$: NSF, Packard, Miller

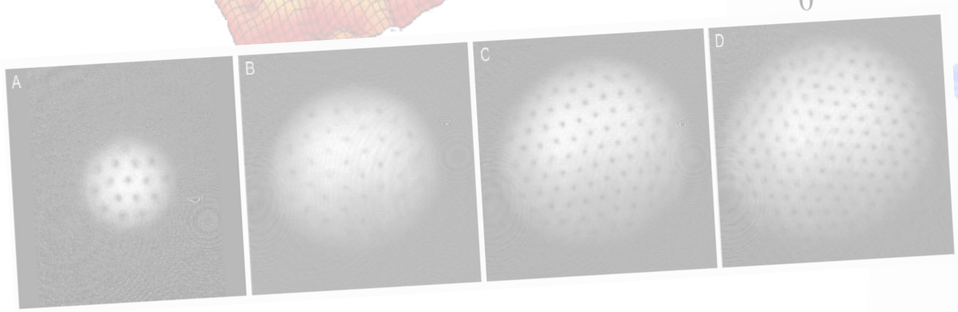
Nordita, Aug, 2010

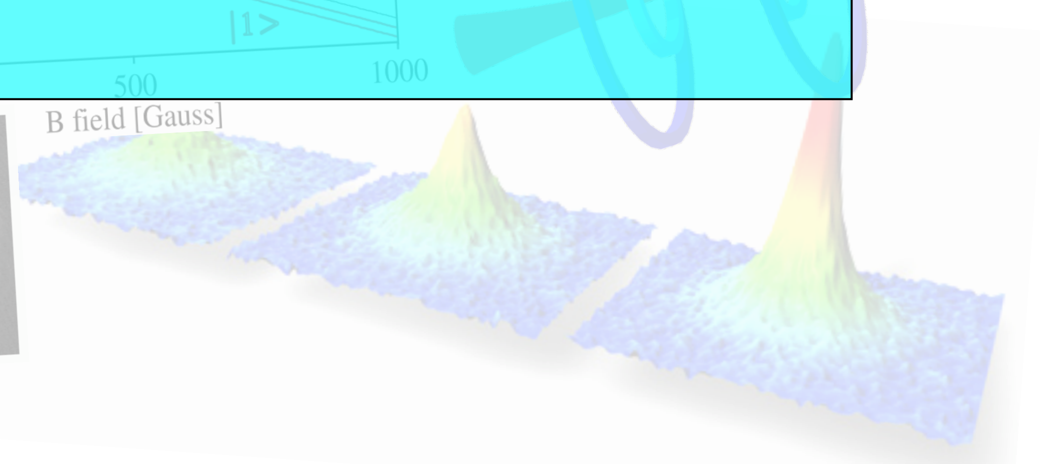
Outline





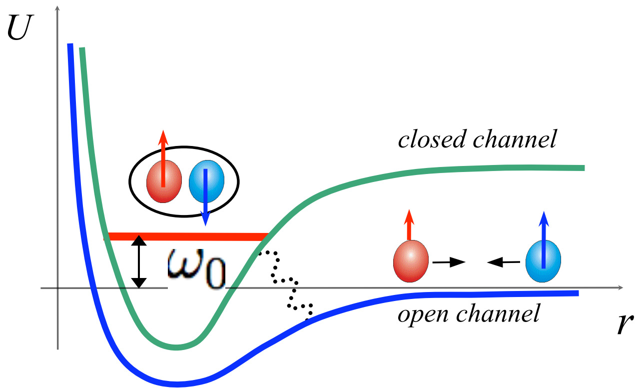
- Introduction
- Motivation for LO/FF state
- Microscopics
- Fluctuations and stability of LO/FF
- Topological defects
- Phase transitions
- Fermions
- Conclusions

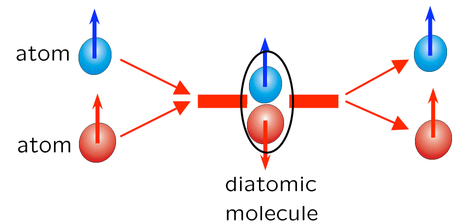


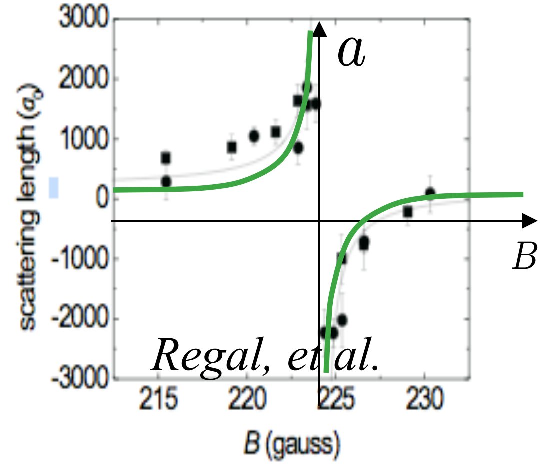


Strong correlations via Feshbach resonance

- tunability (strength and sign) of interactions (sudden and adiabatic)







⇒

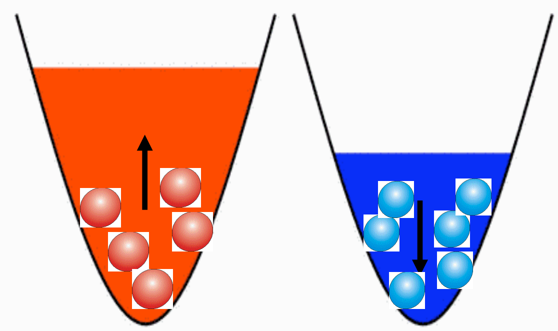
- s-wave BCS-BEC superfluidity
- p-wave superfluidity *(see e.g., Gurarie and LR, AOP 2007)*
- polarized superfluidity *(see e.g., Sheehy and LR, AOP 2007)*

…quite well understood:

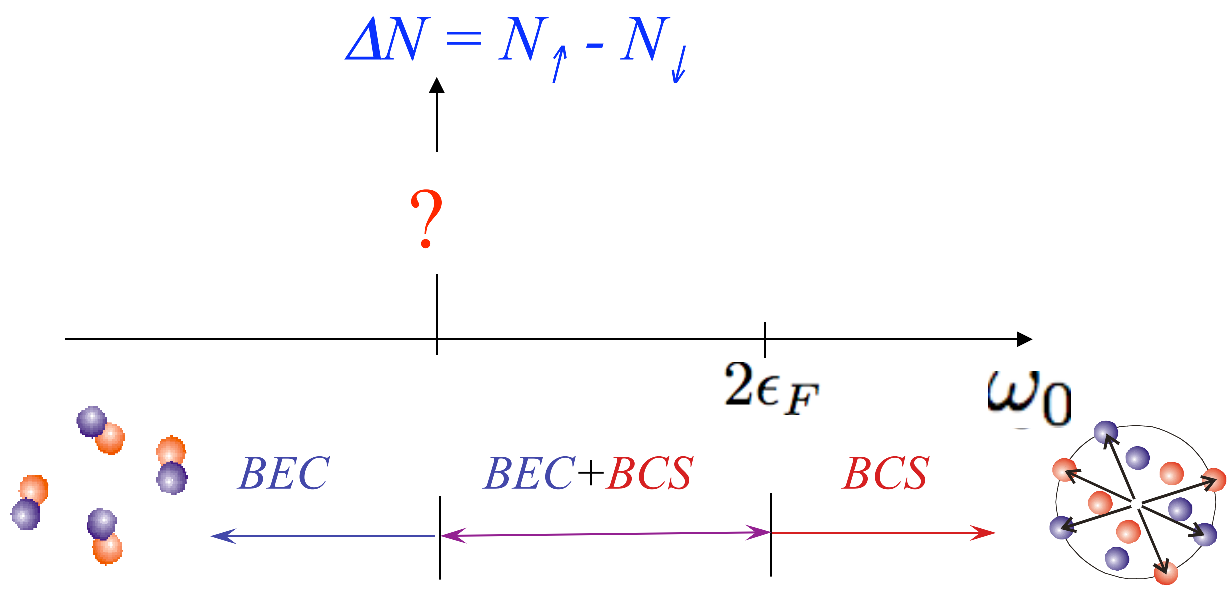
- *quantitatively for narrow (Γ/ε_F <<1) resonance*
- *qualitatively for broad (Γ/ε_F >>1) resonance*
  - mft, 1/N, ε-expansions ⟶ universality

(Veillette, Sheehy, LR '07; Nikolic, Sachdev '07; Nishida, Son '06)

Imbalanced ("polarized") BEC-BCS

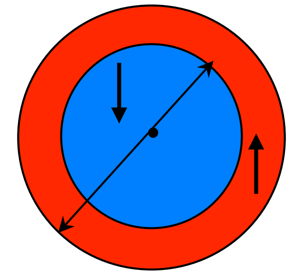


- motivation: *superconductivity in B field, quarks-gluon plasma,…*
- natural realization in cold atoms: $H_h = H - h(N_\uparrow - N_\downarrow)$



Imbalanced BEC-BCS

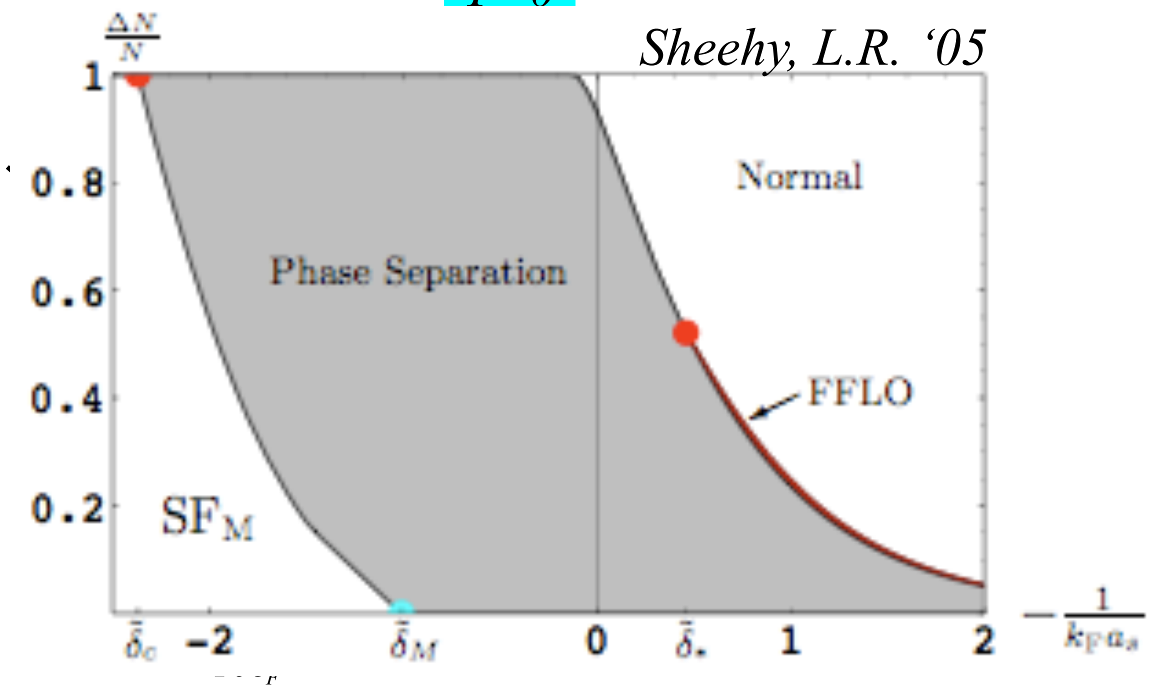
Sheehy, L.R. '05

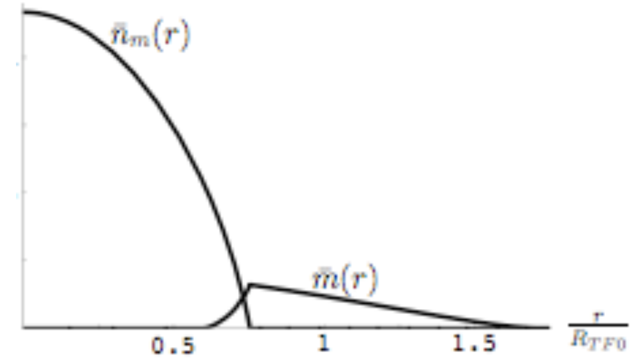


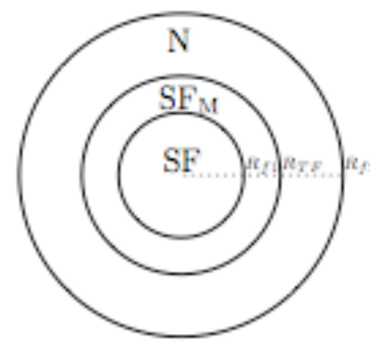
- 1[st]-order transitions and phase separation

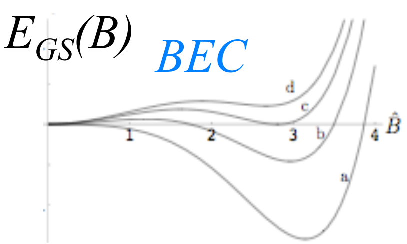
T=0

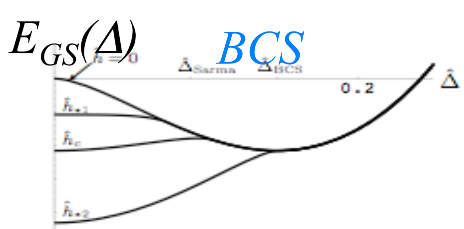
Sheehy, L.R. '05









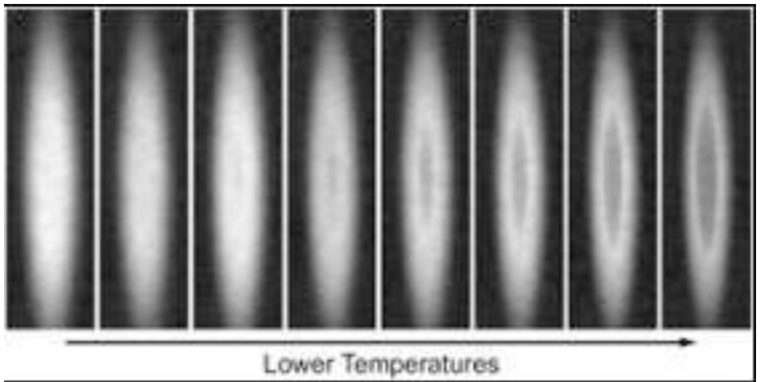


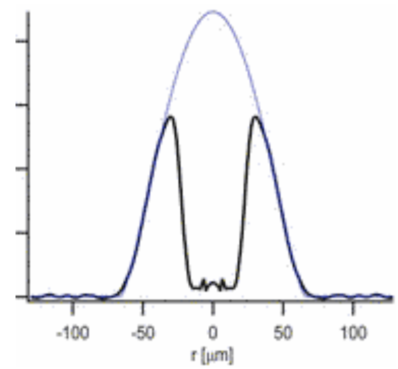
extended to finite T by Parish, et al. '07

Imbalanced BEC-BCS experiments

• Ketterle's experiments (vortices, phase separation)

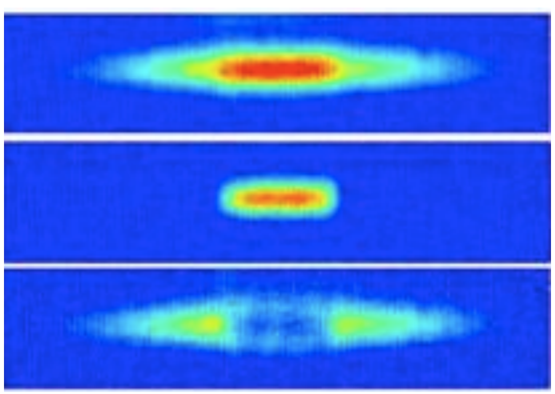
Science (2006)

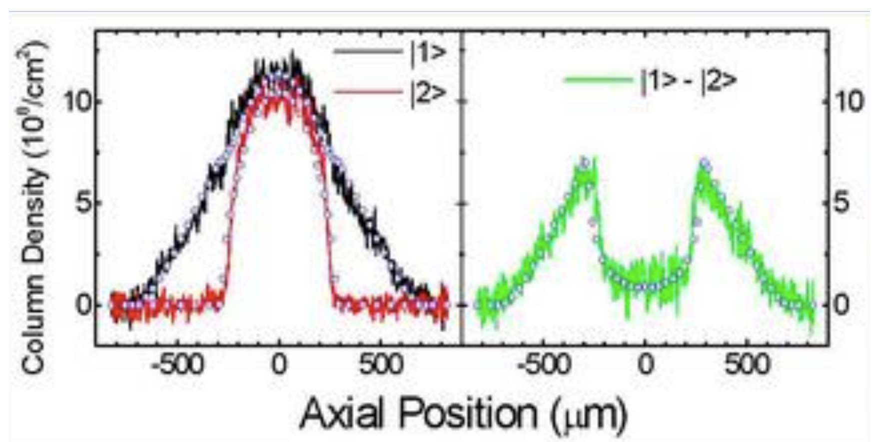




• Hulet's experiments (phase separation, surface tension)

Science (2006)

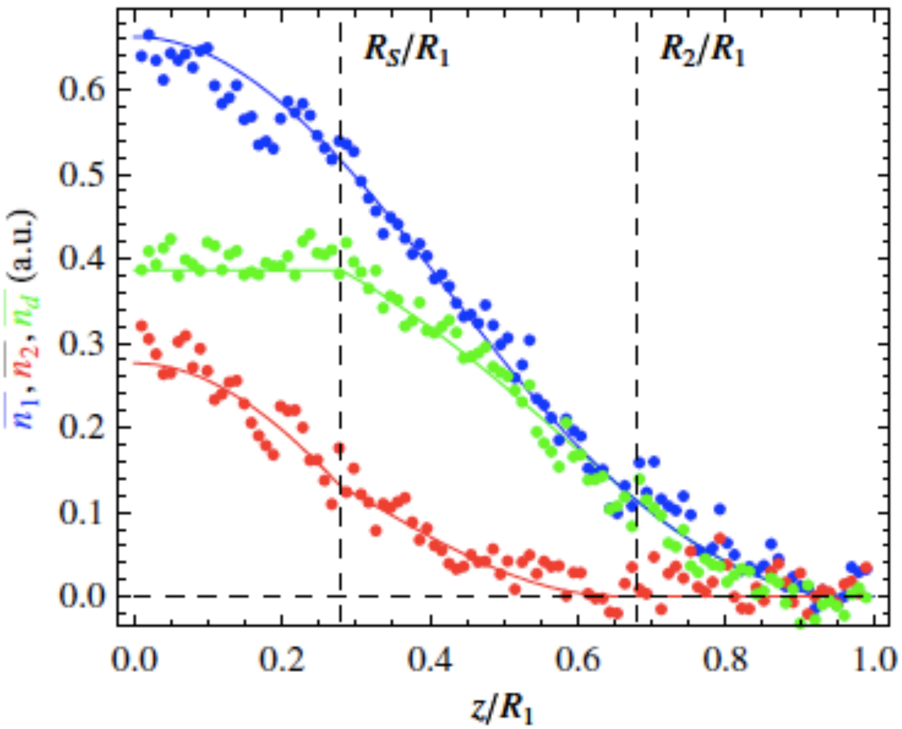


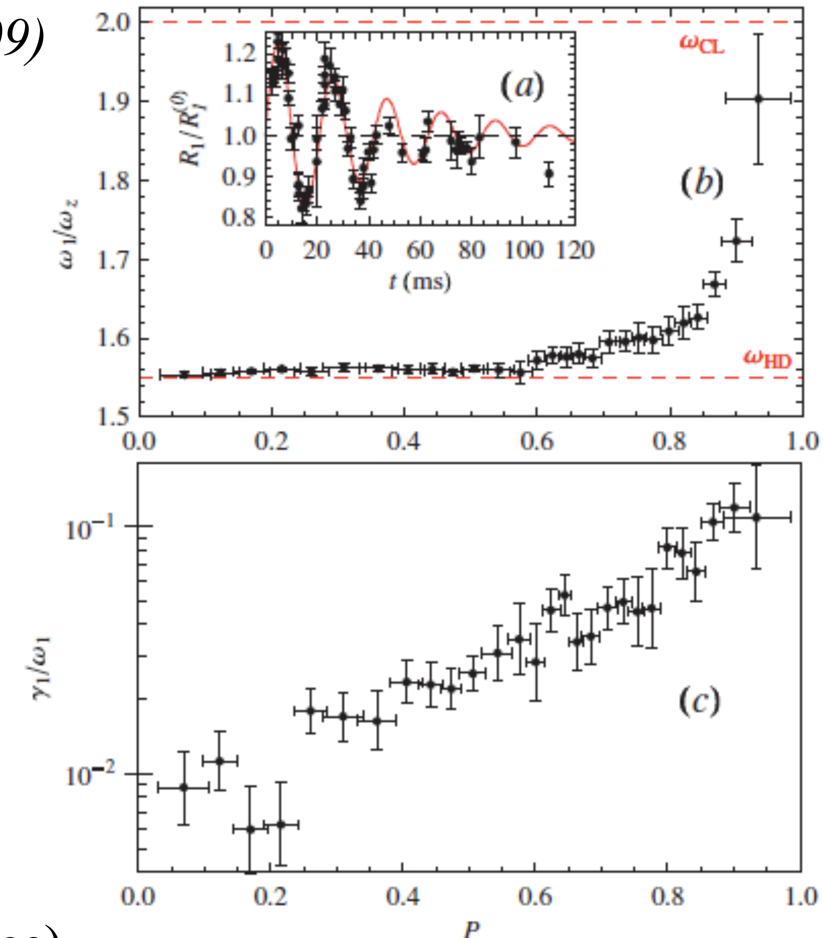


Imbalanced BEC-BCS experiments

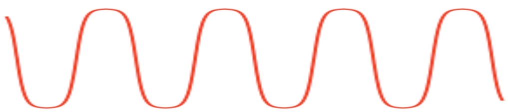
- Salomon's experiments (phase separation, oscillations)

(PRL 2009)

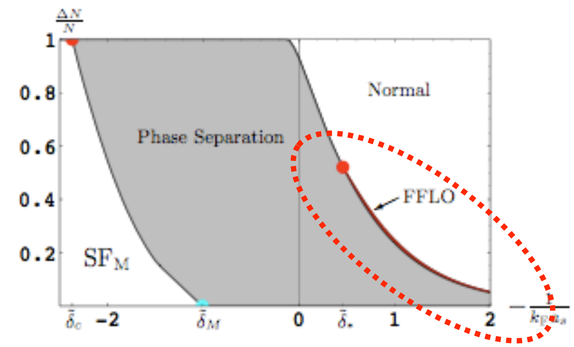




- N = 10^4, axial trap with 20:1 anisotropy (cf Rice)
- superfluid core disappears at P_{c2}= 0.76 (cf MIT)
- LDA works (cf MIT)
- no visible surface tension effects (cf MIT)

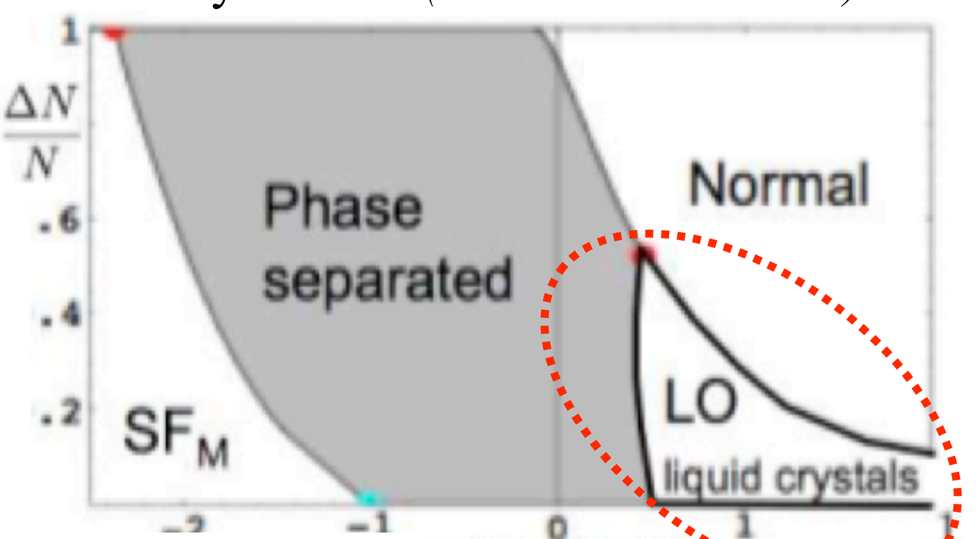


FFLO state

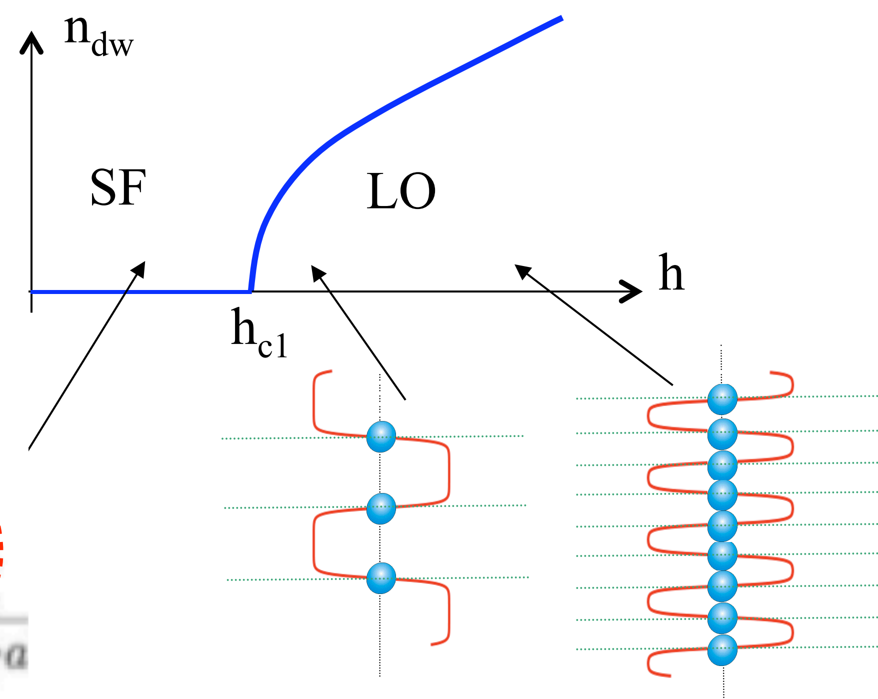


- pair "density" wave: $\Delta = \sum_{\mathbf{Q}} \Delta_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{x}}$
- motivation:
  - *stabilized in lower dimensions (Huse, et al)*
  - *negative surface tension for ± Δ domain wall (Matsuo,et al.;Yoshida+Yip)*
  - ⟹ *SF → LO: a PT transition of domain-wall proliferation?*

▪ "yes" in 1d *(Machida-Nakanishi '84)*



Burkhardt, Rainer '94
Machida, Nakanishi '84

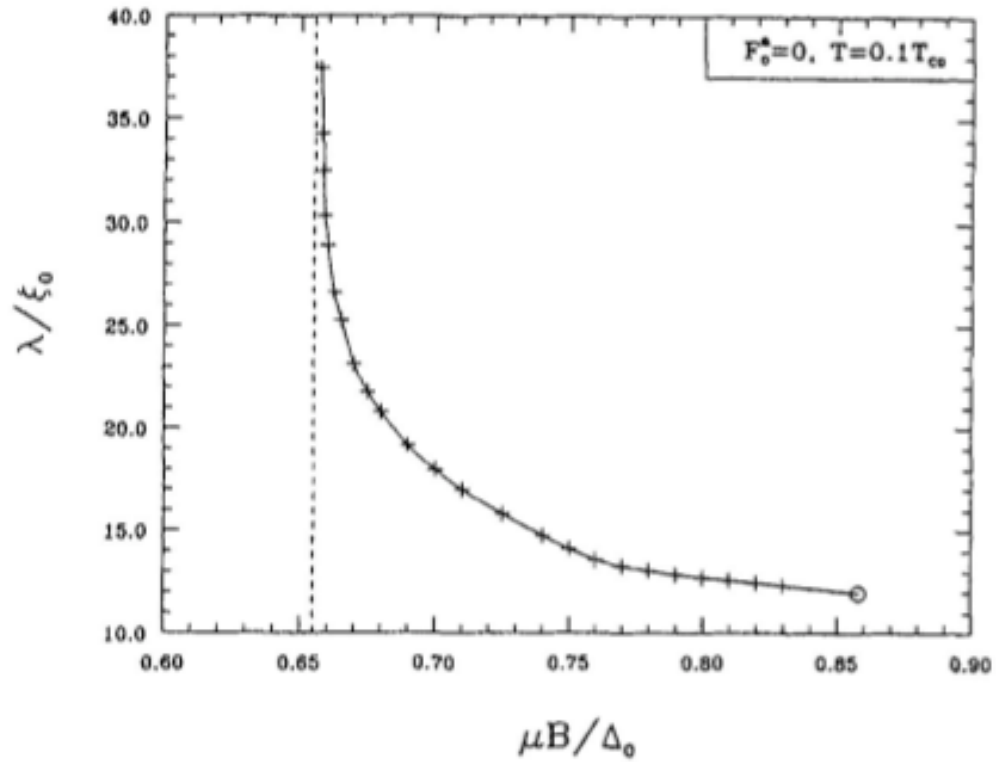


▪ excess fermions sit on domain walls *(cf. polyacetylene of Schrieffer, Su, Heeger)*

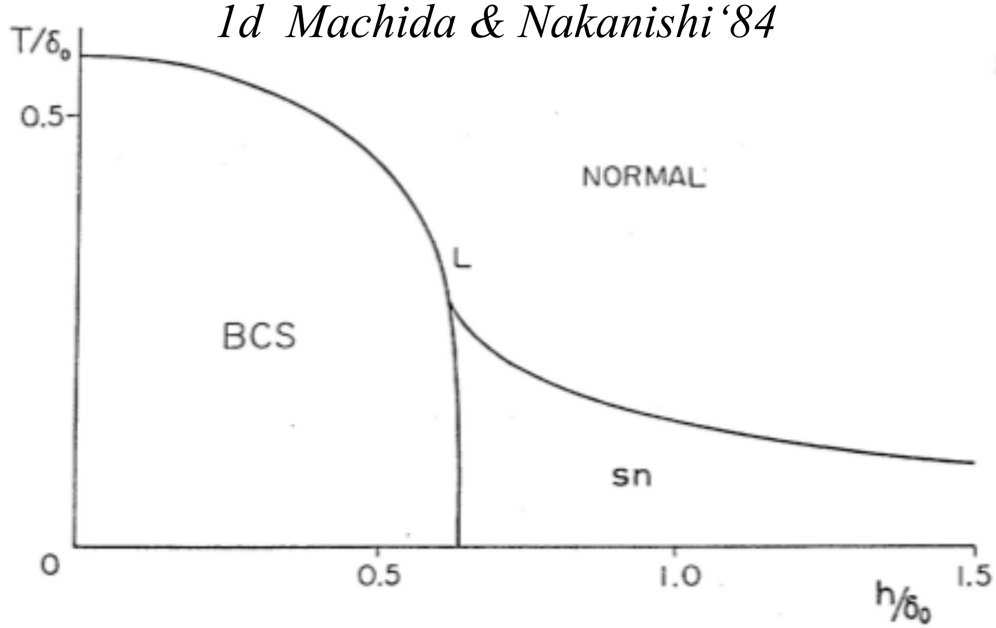
▪ microphase separation *(cf. H_{c1} transition to vortex state in type II sc's)*

Evidence in 1d and 2d

2d Burkhardt & Rainer '94



1d Machida & Nakanishi '84



in 1d:

- Bethe ansatz exists
- bosonization spin gap closing,
  1 ➔ 2 LL modes

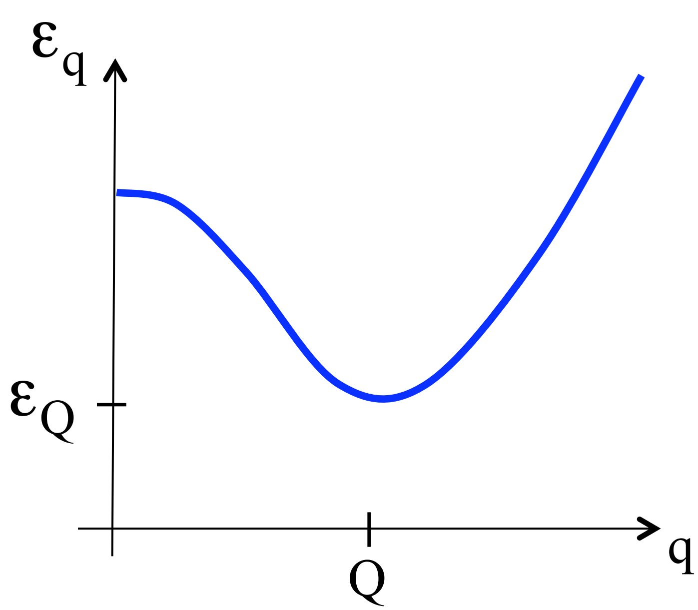
Microscopics to Ginzburg-Landau

LR, Vishwanath '08

$H_{BCS}[c_\sigma, c_\sigma^\dagger]$ $\xrightarrow{\text{near } h_{c2}}$ *with* $\Delta = V\langle c_\downarrow c_\uparrow \rangle$

$$H_{GL}[\Delta] = \sum_{\mathbf{q}} \bar{\Delta}_{\mathbf{q}} \varepsilon_q \Delta_{\mathbf{q}} + \sum_{\mathbf{q_1},\mathbf{q_2},\mathbf{q_3}} v_{\mathbf{q_1},\mathbf{q_2},\mathbf{q_3},\mathbf{q_4}} \bar{\Delta}_{\mathbf{q_1}} \Delta_{\mathbf{q_2}} \bar{\Delta}_{\mathbf{q_3}} \Delta_{\mathbf{q_4}} + \dots$$

$$\approx J\bar{\Delta}\left(-\nabla^2 - Q^2\right)^2 \Delta + \varepsilon_Q |\Delta|^2 + \frac{v_1}{2}|\Delta|^4 + \frac{v_2}{2}\mathbf{j}^2 + \dots$$



$$J \approx \frac{n}{\epsilon_F Q^4}$$

$$Q \approx \frac{\Delta_{BCS}}{\hbar v_F}$$

$$\varepsilon_Q \approx \frac{n}{\epsilon_F} \ln\left[\frac{h}{h_{c2}}\right]$$

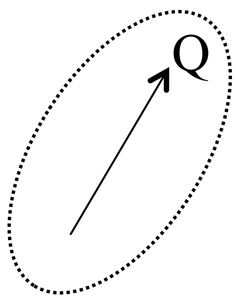
$$h_{c2} \approx \frac{3}{4}\Delta_{BCS}$$

$$v_1 \approx \frac{n}{\epsilon_F \Delta_{BCS}^2}$$

$$v_2 \approx \frac{nm^2}{\epsilon_F \Delta_{BCS}^2 Q_0^2}$$

Broken symmetries in LO/FF states

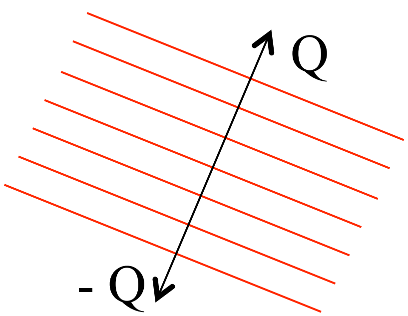
- Fulde-Ferrell: $\Delta_{FF}(\mathbf{x}) = \Delta_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{x}}$



- broken: *time reversal, orientational, off-diagonal*

orientationally-ordered superfluid

- Larkin-Ovchinnikov: $\Delta_{LO}(\mathbf{x}) = \Delta_{\mathbf{Q}} \cos \mathbf{Q}\cdot\mathbf{x}$



- broken: *orientational, translational, off-diagonal*

superconducting smectic

superfluid liquid crystals

Low-energy excitations in LO/FF states

- order parameter: $\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta_+} e^{i\mathbf{Q}\cdot\mathbf{x}} + \Delta_0 e^{i\theta_-} e^{-i\mathbf{Q}\cdot\mathbf{x}}$
$$= 2\Delta_0 e^{i\theta} \cos[\mathbf{Q}\cdot\mathbf{x} - Qu]$$

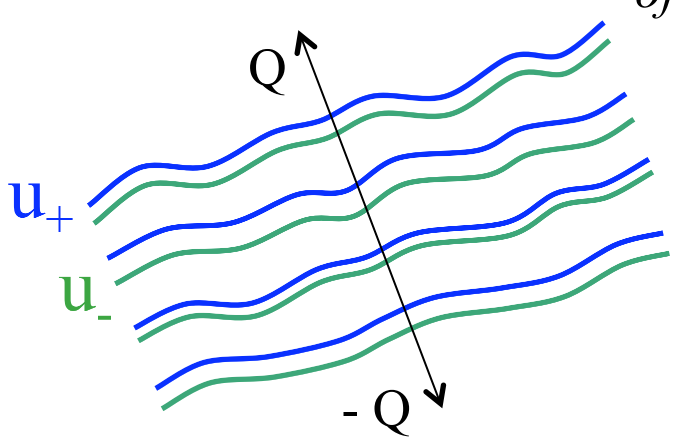
- superfluid phase and phonon: $\theta = \frac{1}{2}(\theta_- + \theta_+)$ $u = \frac{1}{2Q}(\theta_- - \theta_+)$

- coupled smectics u_+, u_- :

$$\mathcal{H}_{LO} = \underbrace{\sum_{\alpha=\pm}\left[\frac{K}{2}(\nabla^2 u_\alpha)^2 + \frac{B}{2}(\partial_z u_\alpha)^2\right]}_{\text{rotational invariance of smectic liquid crystal}} + \frac{\gamma}{2}\underbrace{(\nabla u_+ - \nabla u_-)^2}_{j = j_+ + j_- = 0}$$

rotational invariance
of smectic liquid crystal

$j = j_+ + j_- = 0$



$$E[u^0_\pm(\mathbf{x})] = 0 \;\; \text{for} \;\; u^0_\pm(\mathbf{x}) = z(\cos\phi - 1) + x\sin\phi$$

Low-energy excitations in LO/FF states

- order parameter: $\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta_+} e^{i\mathbf{Q}\cdot\mathbf{x}} + \Delta_0 e^{i\theta_-} e^{-i\mathbf{Q}\cdot\mathbf{x}}$
  $= 2\Delta_0 e^{i\theta} \cos[\mathbf{Q}\cdot\mathbf{x} - Qu]$

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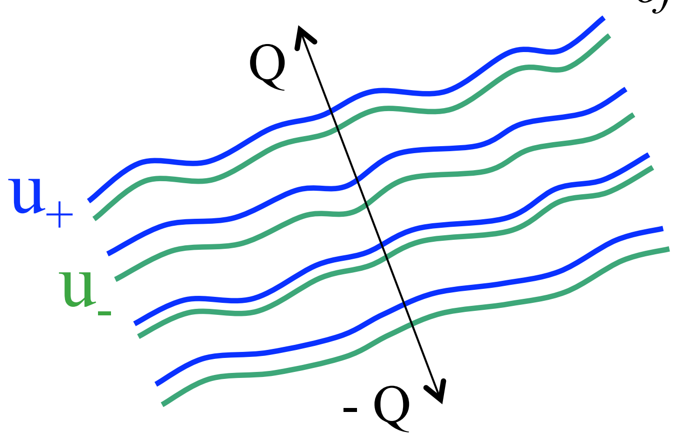
rotational invariance
of smectic liquid crystal

$j = j_+ + j_- = 0$



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Low-energy excitations in LO/FF states

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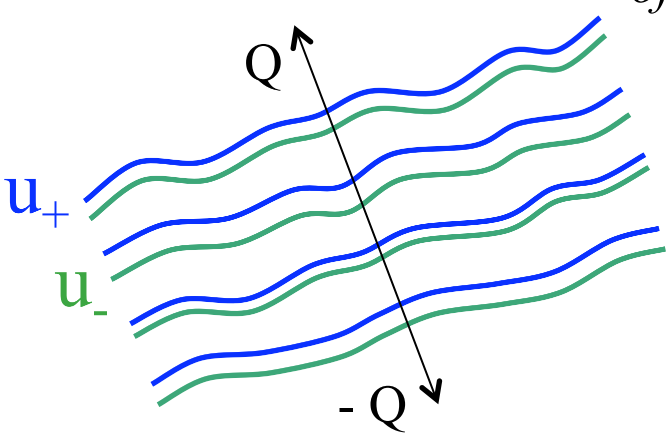
rotational invariance
of smectic liquid crystal

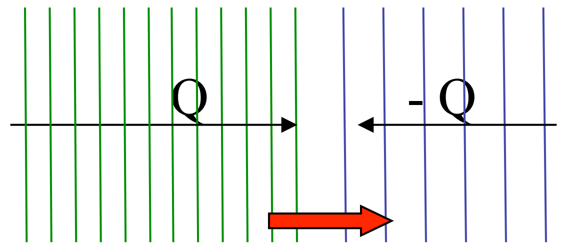
$j = j_+ + j_- = 0$



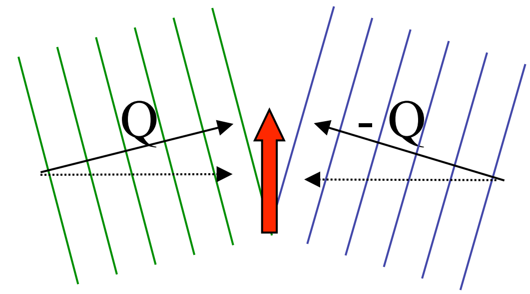
$$E[u^0_\pm(\mathbf{x})] = 0 \quad \text{for} \quad u^0_\pm(\mathbf{x}) = z(\cos\phi - 1) + x\sin\phi$$

“Infinitely” anisotropic superfluid

- supercurrents:



$j_{\|} \sim \partial_{\|} u_{+} - \partial_{\|} u_{-}$



$j_{\perp} \sim \nabla_{\perp} u_{+} - \nabla_{\perp} u_{-}$

- Goldstone modes “elastic” theory:

$$\mathcal{H}_{LO} = \sum_{\alpha=\pm}\left[\frac{K}{4}(\nabla^2 u_\alpha)^2 + \frac{B}{4}\left(\partial_z u_\alpha + \frac{1}{2}(\nabla u_\alpha)^2\right)^2\right] + \frac{\gamma}{2}(\nabla u_+ - \nabla u_-)^2$$

$$\approx \underbrace{\frac{K}{2}(\nabla_\perp^2 u)^2 + \frac{B}{2}(\partial_z u)^2}_{\text{smectic elasticity}} + \underbrace{\frac{\rho_s^i}{2}(\nabla_i \theta)^2}_{\text{superfluid stiffness}}$$

- superfluid stiffness *anisotropy*:

$$\frac{\rho_s^\perp}{\rho_s^\|} = \left(\frac{\Delta_Q}{\Delta_{BCS}}\right)^2 \approx \ln\left(\frac{h_{c2}}{h}\right) \ll 1$$

Fluctuations and stability of LO/FF states

- fluctuations at T=0: $\mathcal{L}_{LO} = \frac{\chi}{2}(\partial_\mu \theta)^2 + \frac{\rho}{2}(\partial_t u)^2 + \frac{B}{2}(\partial_z u)^2 + \frac{K}{2}(\nabla^2 u)^2$

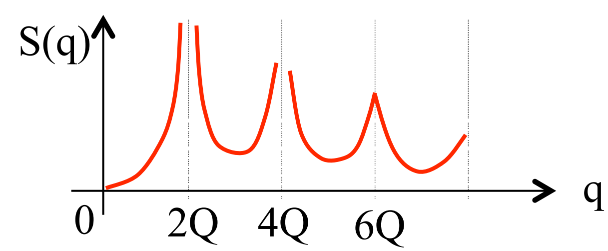
  - $\langle \theta^2 \rangle$, $\langle u^2 \rangle$ ~ finite for d > 1 ⇒ *LO stable to quantum fluctuations*

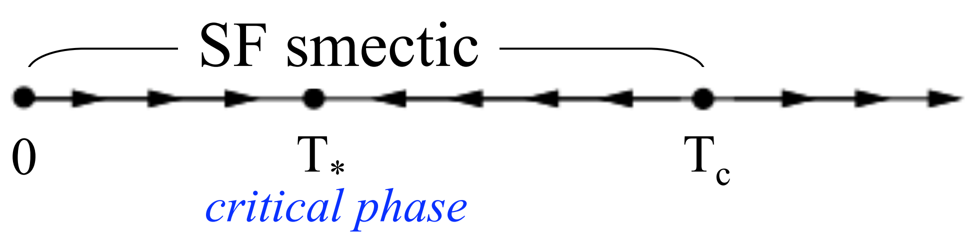
- fluctuations at T≠0:

  - $\langle \theta^2 \rangle$ ~ finite for d > 2 ⇒ *SF order stable to $k_B T$ fluctuations*
  - $\langle u^2 \rangle$ ~ diverges for d ≤ 3 ⇒ *positional order unstable*

➡ LO = superfluid smectic (SF_{sm}) with:

- quasi-Bragg peaks (3d), Lorentzian (2d)
- anomalous elasticity *(Grinstein and Pelcovits)*
- transitions to superfluid nematic (SF_N)



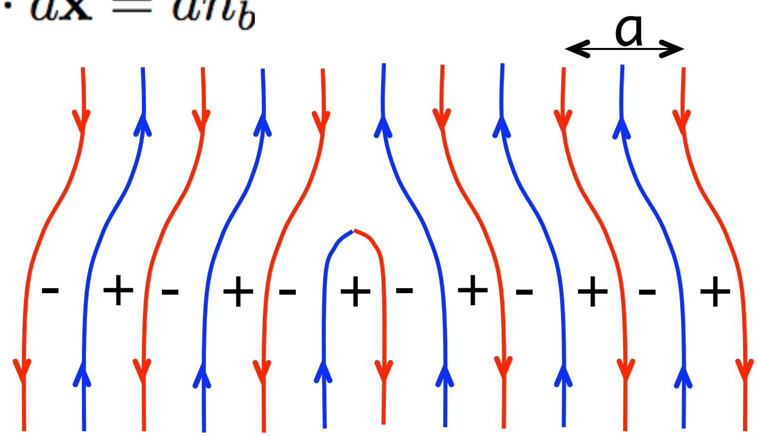


Topological defects

$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})} \cos[Q(z - u(\mathbf{x}))]$$

$$\frac{U(1) \times U(1)}{Z_2}$$

- integer dislocations in u: $\oint \nabla u \cdot d\mathbf{x} = a n_b$

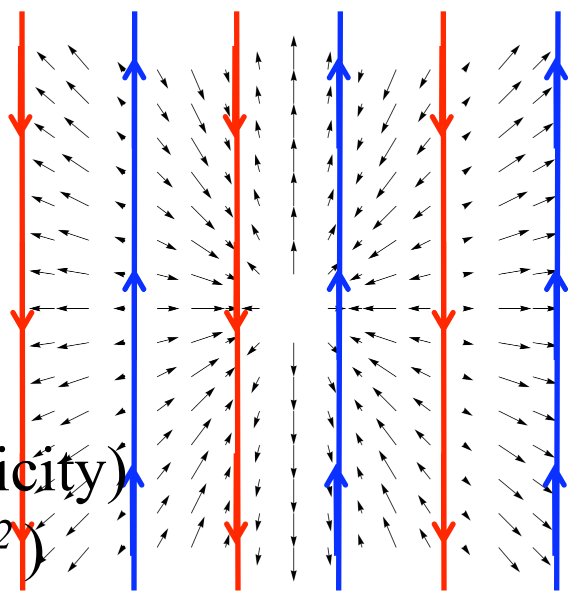


$(n_v, n_b) = (0,1)$

  - destroy LO order ("charge"-*2* SF *and* full smectic periodicity)
  - retain "charge" ≥ *4* homogeneous SF (Δ^2)

- integer vortices in θ: $\oint \nabla \theta \cdot d\mathbf{x} = 2\pi n_v$

$(n_v, n_b) = (1,0)$



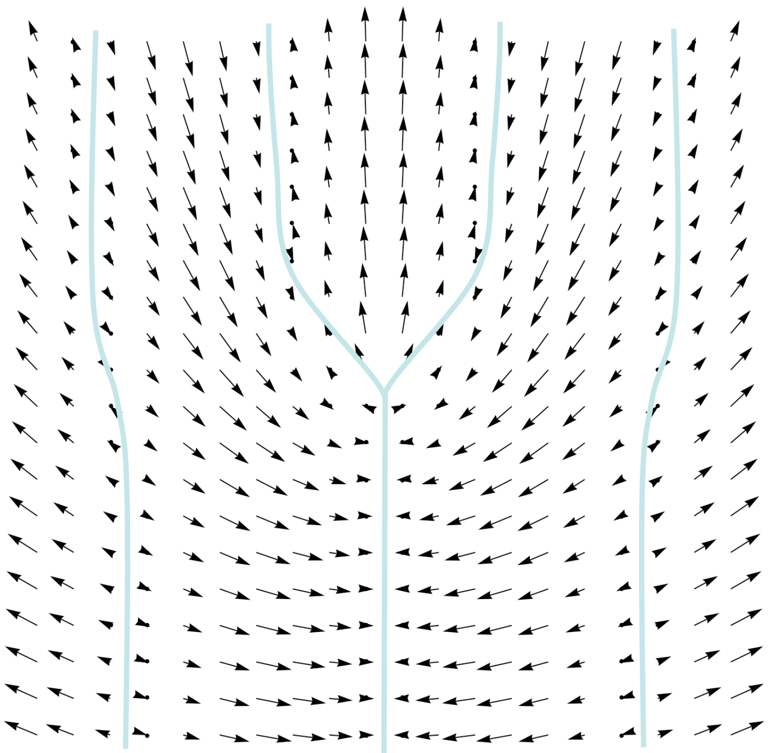
  - destroy LO order (full SF *and* *Q* smectic periodicity)
  - retain wavevector ≥ *2Q* smectic periodicity ($|\Delta|^2$)

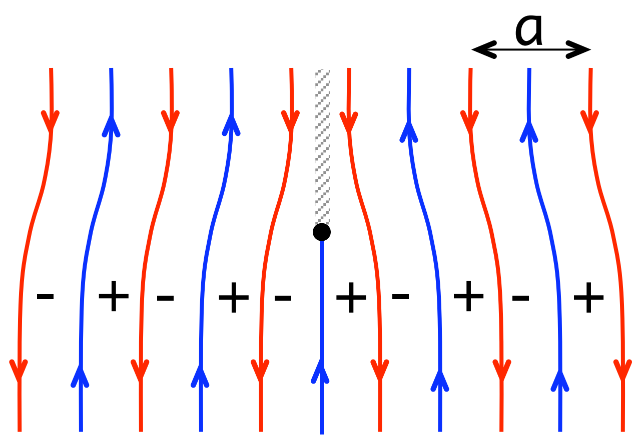
Fractional topological defects

$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})} \cos[Q(z - u(\mathbf{x}))]$$

$$\frac{U(1) \times U(1)}{Z_2}$$

- π-vortex — a/2 dislocation pairs:





$(n_v, n_b) = (1/2, 1/2)$

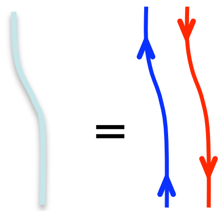
$(n_v, n_b) = (1/2, -1/2)$

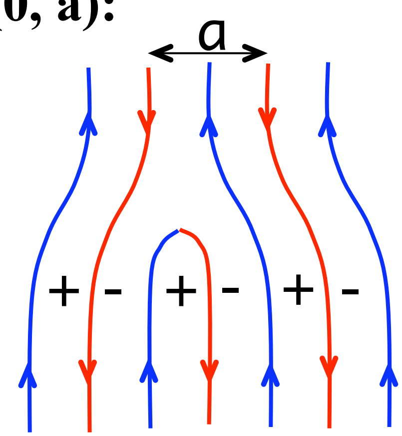
- destroy LO order
- restore full translational invariance and atom "conservation"

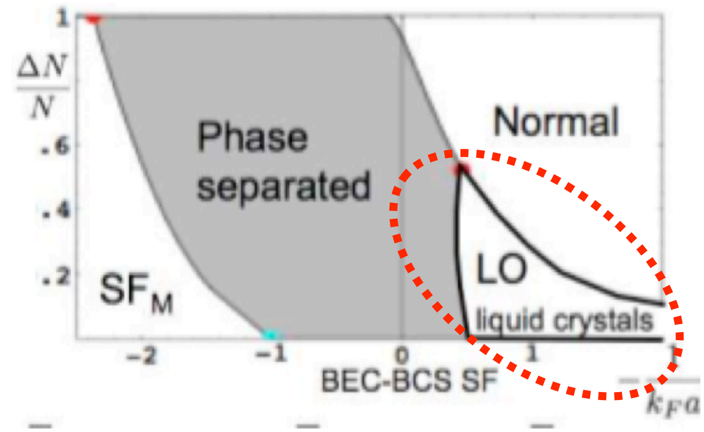
$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})} \cos[Q(z - u(\mathbf{x}))]$$

Phase transitions

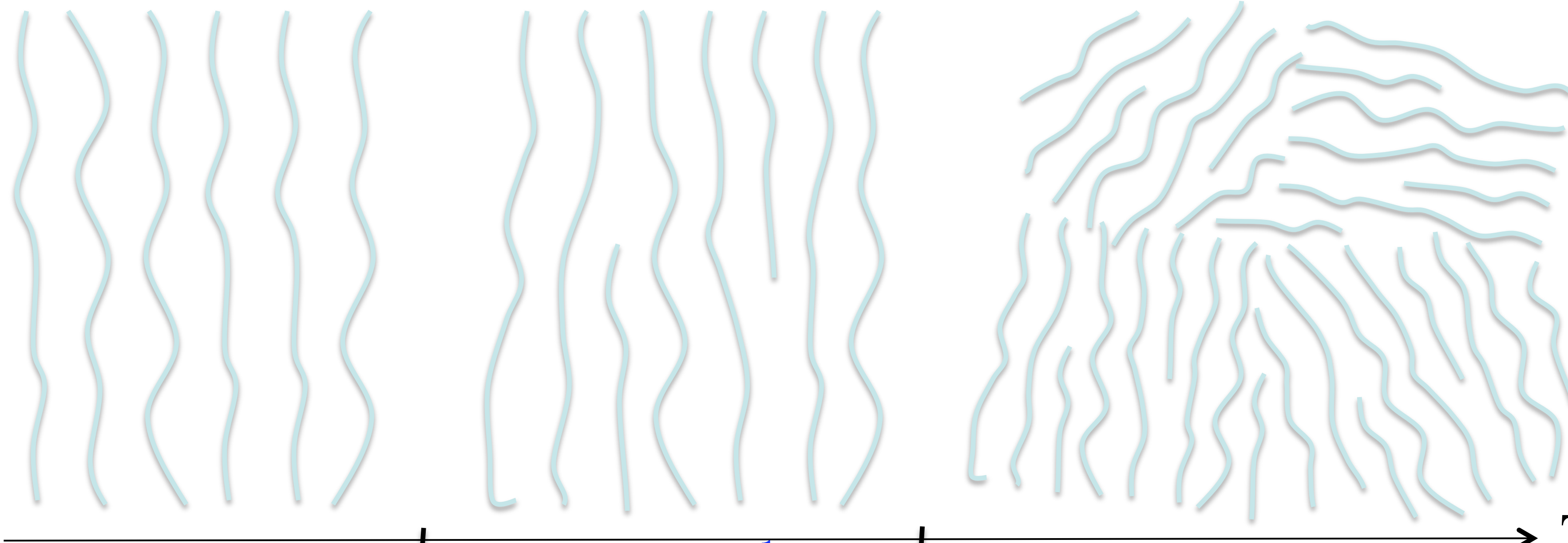
unbind defects, e.g., dislocations (0, a):







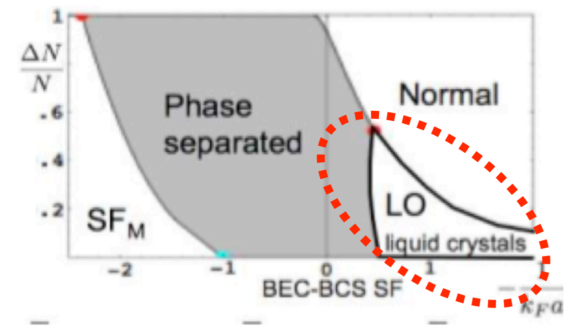
LO Smectic (SF_{Sm}) Nematic Superfluid (SF_N) Isotropic Superfluid (SF_I)

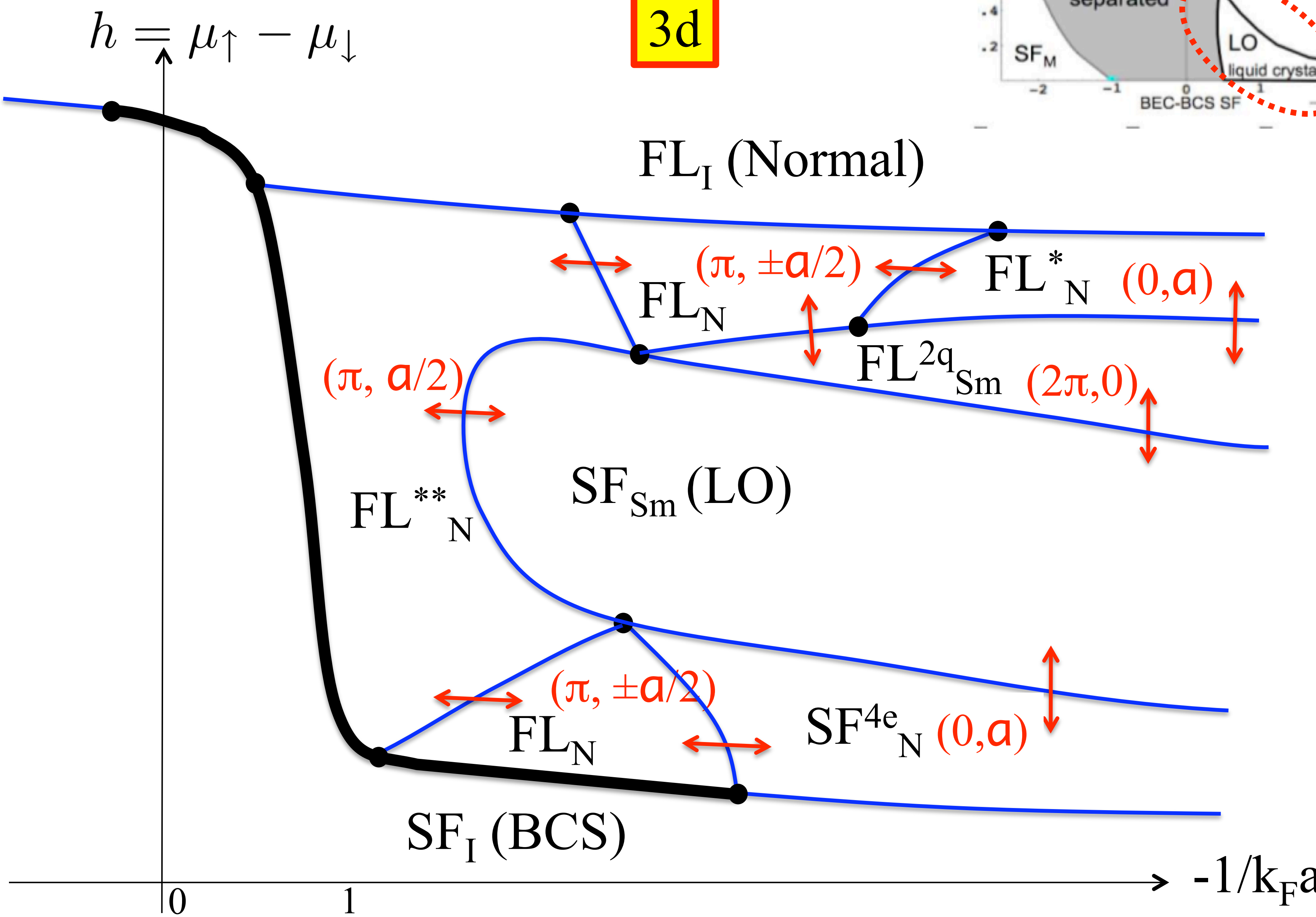


$e^{i\theta}\cos[Q(z-u)]$ T_{NSm} $e^{i2\theta}(\hat{n}_i\hat{n}_j - \frac{1}{3}\delta_{ij})$ T_{IN} $e^{i\theta}$ T

Phase transitions

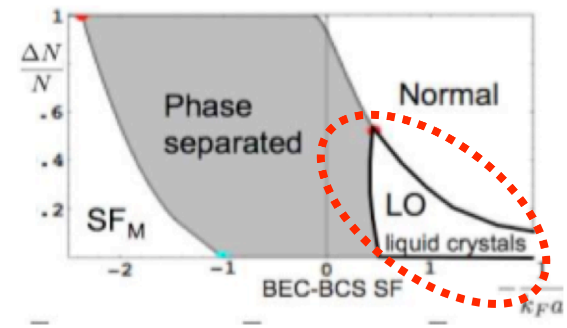
3d

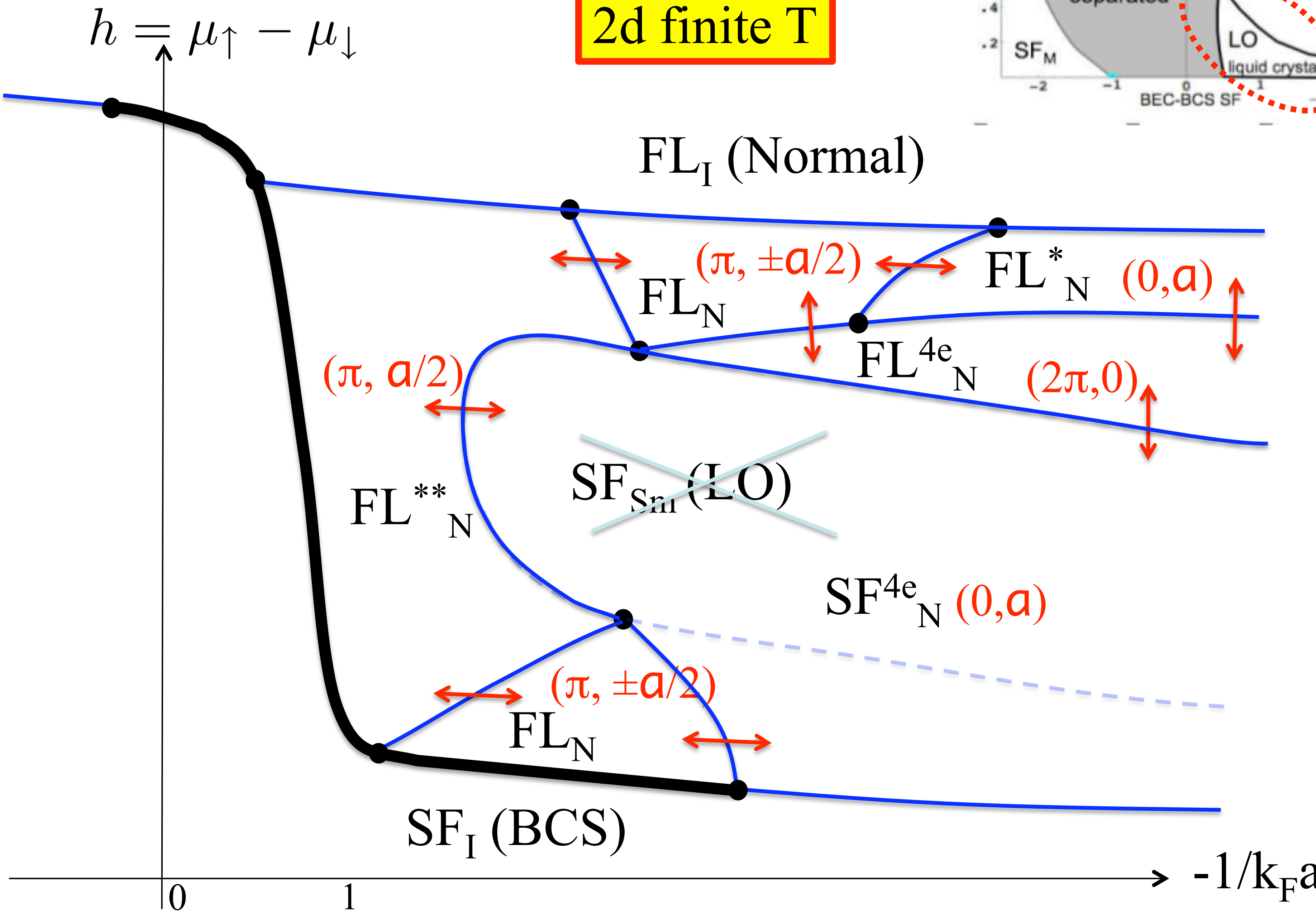




Phase transitions

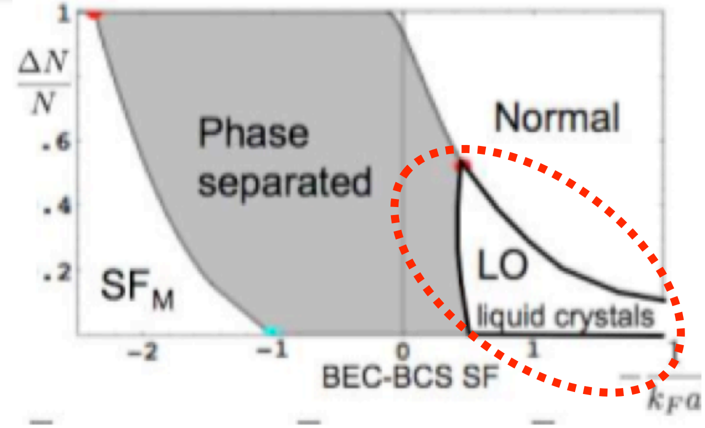
2d finite T

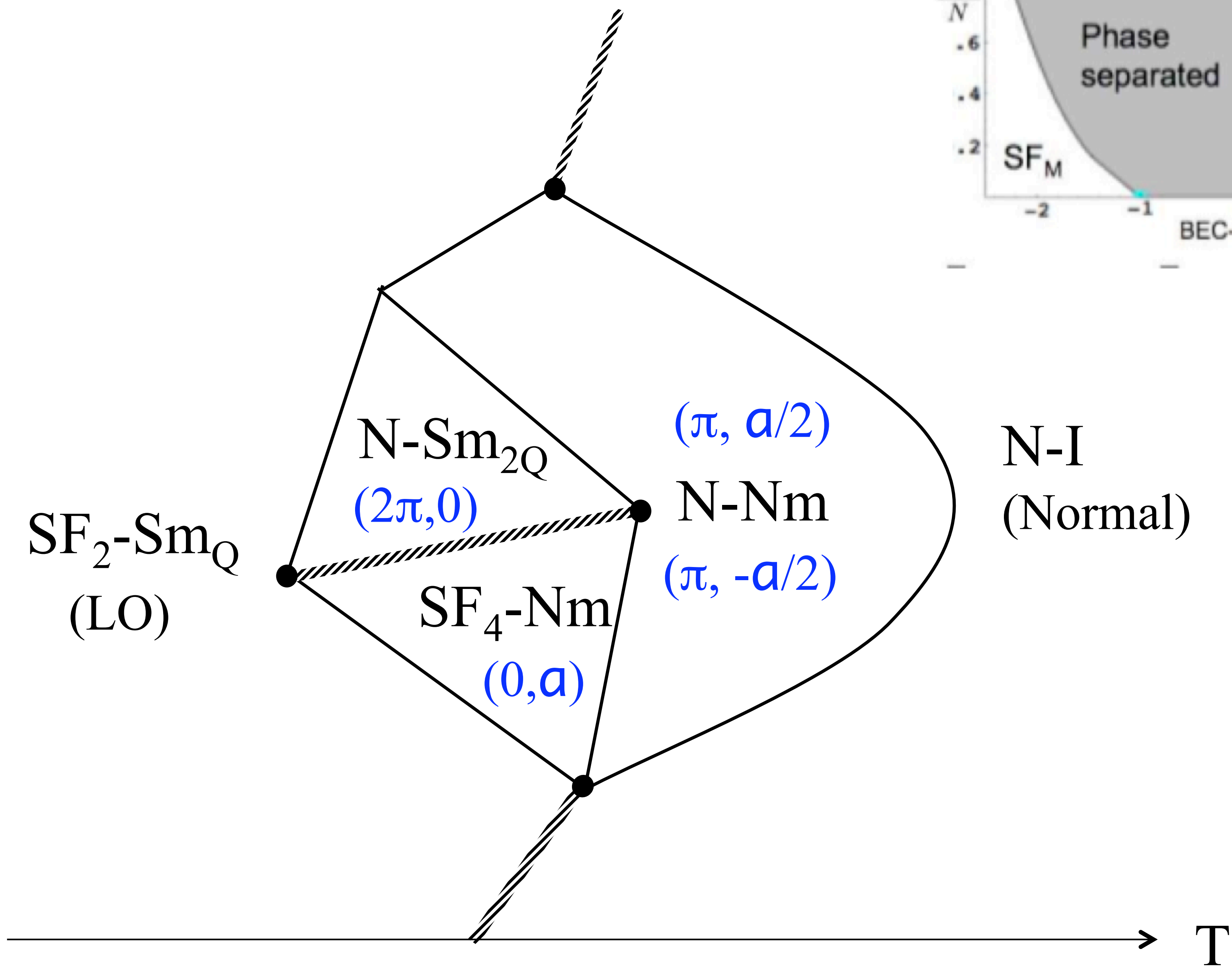




$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})} \cos[Q(z - u(\mathbf{x}))]$$

Phase transitions

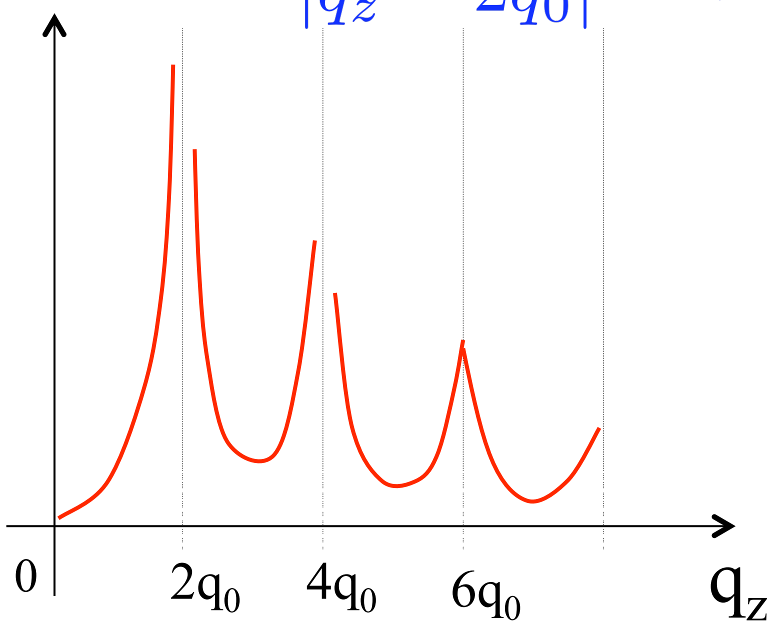




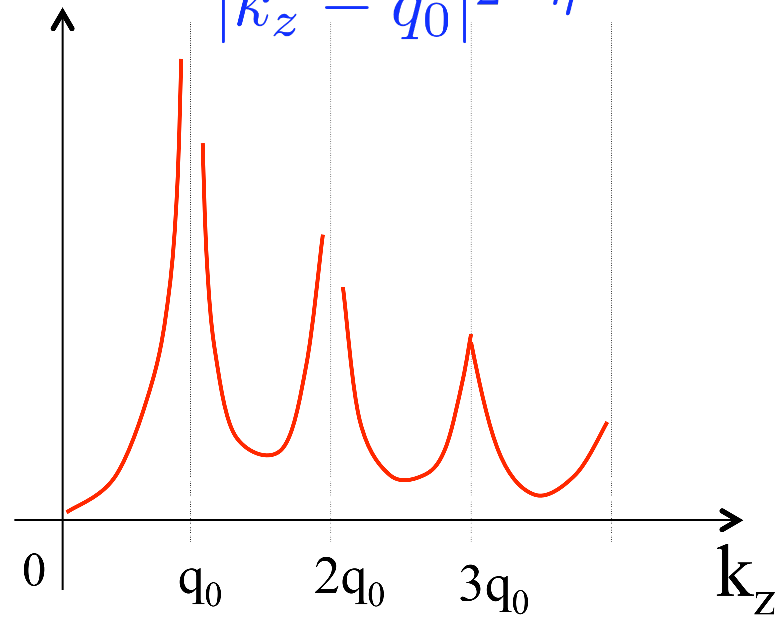
Structure function and time of flight

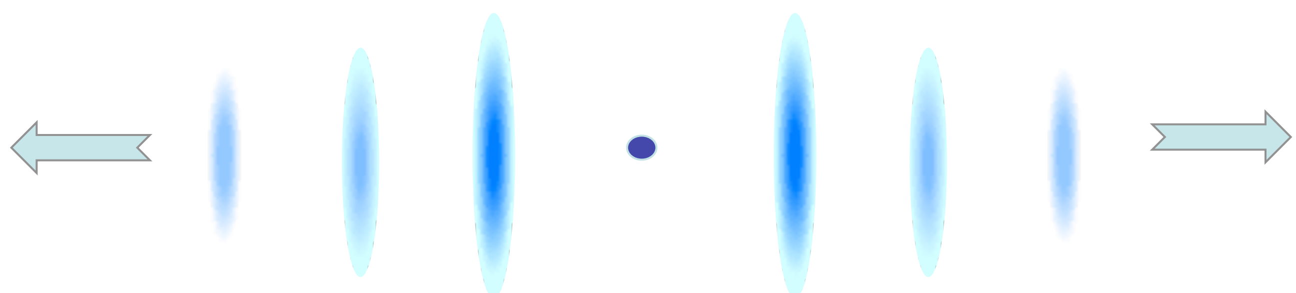
quasi-long-range order in 3d for T > 0

$$S(q_z) \sim \frac{1}{|q_z - 2q_0|^{2-4\eta}}$$



$$n_k \sim \frac{1}{|k_z - q_0|^{2-\eta}}$$



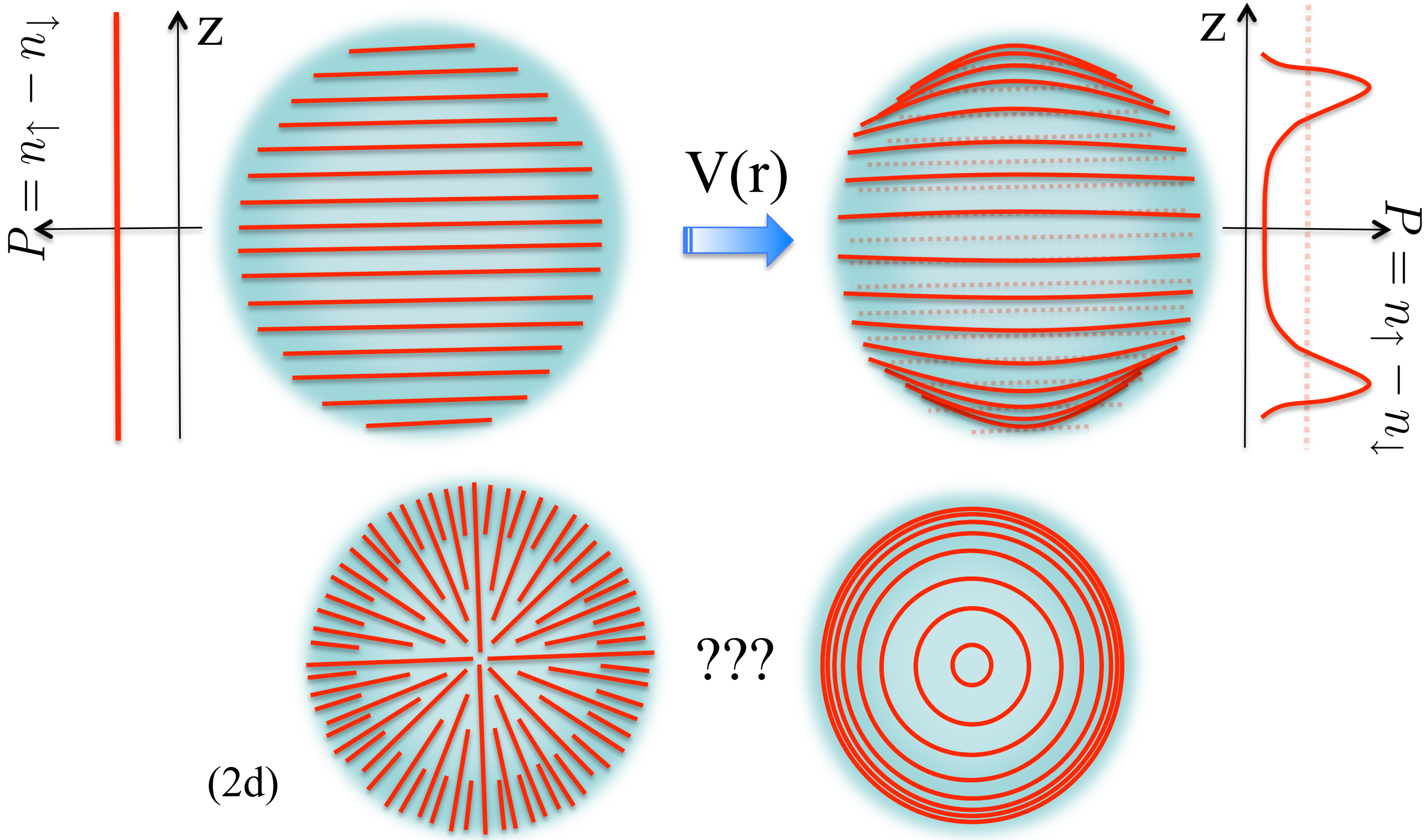


Finite trap geometry

$\mu_{\text{eff}}(r) = \mu(1 - r^2/R^2)$

$$\mathcal{H}_{LO} \approx \frac{\rho_s^i}{2}(\nabla_i\theta)^2 + \frac{K}{2}(\nabla_\perp^2 u)^2 + \frac{B}{2}(\partial_z u)^2 + n_0 V(\mathbf{r})\partial_z u$$

$$\delta P(\mathbf{r}) \sim -\partial_{\mathbf{z}}\mathbf{u}(\mathbf{r}) = \partial_{\mathbf{z}} \int_{\mathbf{r}'} \frac{-\mathbf{1}}{\mathbf{K}\nabla_\perp^{\mathbf{4}} - \frac{\mathbf{B}}{\mathbf{2}}\partial_{\mathbf{z}}^{\mathbf{2}}} \mathbf{n_0}(\mathbf{r})\partial_{\mathbf{z}}\mathbf{V}(\mathbf{r}) \approx \frac{\mathbf{n_0}(\mathbf{r})}{\mathbf{B}}\mathbf{V}(\mathbf{r})$$

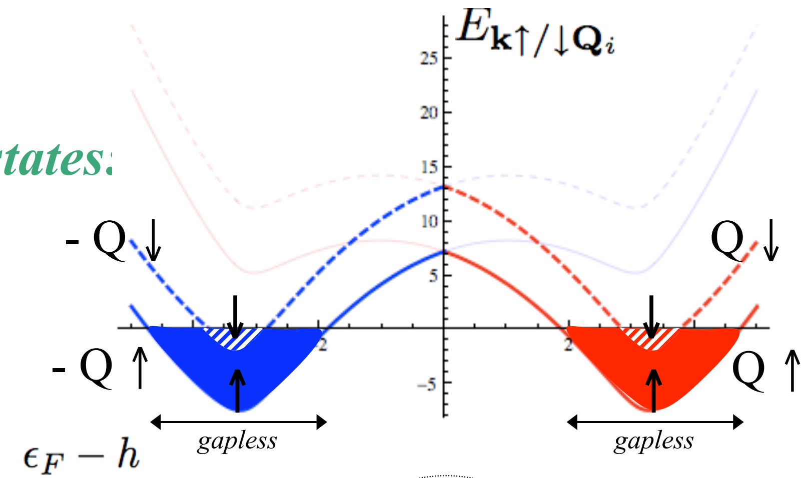


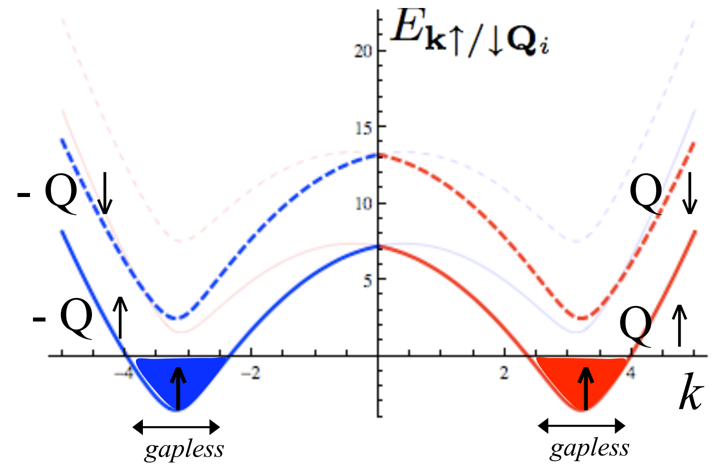
Fermionic sector of LO state

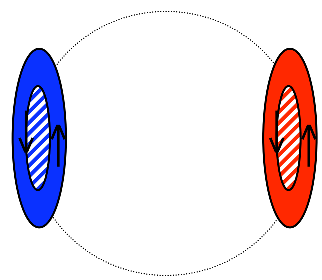
- ***excitation spectrum:*** $E_{\mathbf{k}\uparrow/\downarrow\mathbf{Q}_i} = (\varepsilon_k^2 + \Delta_Q^2)^{1/2} \mp (h + \frac{\mathbf{k}\cdot\mathbf{Q}_i}{2m})$

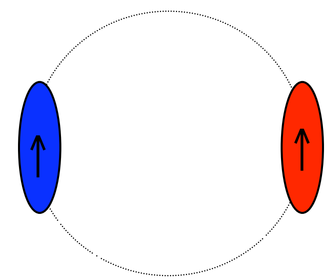
  (gapped and gapless k's)

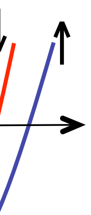
- ***2 distinct LO states:***











- ***ground state:***

$$|LO_{\mathbf{Q}}\rangle = \prod_{\mathbf{k},\mathbf{Q}_i \in E_{\mathbf{k}\sigma\mathbf{Q}_i}<0} \alpha^\dagger_{\mathbf{k}\sigma\mathbf{Q}_i}|BCS_{\mathbf{Q}}\rangle,$$

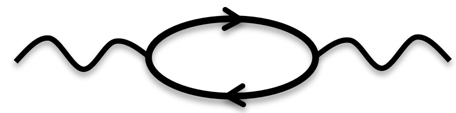
$$= \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_3} c^\dagger_{\mathbf{k}+\frac{\mathbf{Q}_i}{2}\downarrow} \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_2} c^\dagger_{-\mathbf{k}+\frac{\mathbf{Q}_i}{2}\uparrow} \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_1} (u_{\mathbf{k}} + v_{\mathbf{k}} c^\dagger_{\mathbf{k}+\frac{\mathbf{Q}_i}{2}\downarrow} c^\dagger_{-\mathbf{k}+\frac{\mathbf{Q}_i}{2}\uparrow})|0\rangle$$

$$H_f^{ex} = \sum \left[E_{\mathbf{k}\sigma\mathbf{Q}_i}\Theta(E_{\mathbf{k}\sigma\mathbf{Q}_i})\alpha^\dagger_{\mathbf{k}\sigma\mathbf{Q}_i}\alpha_{\mathbf{k}\sigma\mathbf{Q}_i} - E_{\mathbf{k}\sigma\mathbf{Q}_i}\Theta(-E_{\mathbf{k}\sigma\mathbf{Q}_i})\alpha_{\mathbf{k}\sigma\mathbf{Q}_i}\alpha^\dagger_{\mathbf{k}\sigma\mathbf{Q}_i}\right]$$

Fermion-Goldstone modes coupling in LO state

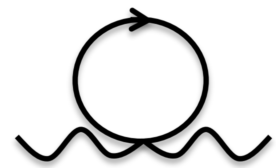
- ***supercurrent-current:*** $H_{j_s,j} \sim \nabla\theta \cdot \overline{\psi} i \nabla \psi + h.c.$

➡ $|\omega|\sigma_{ij}(\omega, \mathbf{q})\nabla_i\theta\nabla_j\theta$



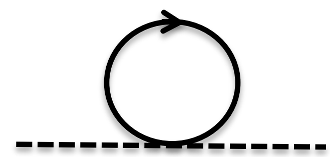
- ***supercurrent-density:*** $H_{j_s,n} \sim (\nabla\theta)^2\overline{\psi}\psi$

➡ $n_f(\nabla\theta)^2$



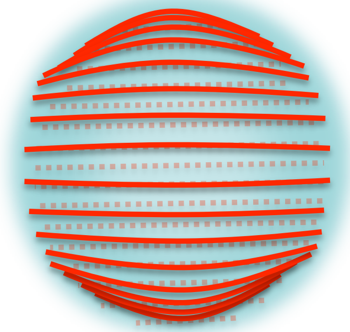
- ***atom-phonon:*** $H_{a-p} \sim \left(\partial_z u + \frac{1}{2}(\nabla u)^2\right)\overline{\psi}\psi + (\nabla u \cdot \overline{\psi} i \nabla \psi)^2 + h.c.$

➡ $n_f\left(\partial_z u + \frac{1}{2}(\nabla u)^2\right)$



- **How do these affect Goldstone modes and fermions?**
  - (*weak) Landau damping, finite corrections to q_0, ρ_s, K, B,…*
  - *fermions retain their anisotropic pocket Fermi surface*

Experiments

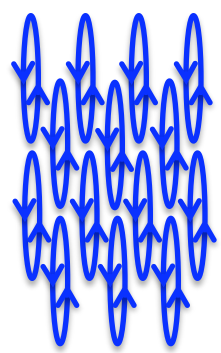


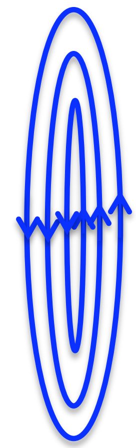
- ***trap suppression of fluctuations:***

$$\Delta_{LO} \sim R^{-\eta(T)} \sim N^{-\frac{1}{5}\eta(T)} \sim \omega_{\rm tr}^{\eta(T)} \to 0$$

- ***anisotropic vortices:***

$$\mathbf{v} = \sqrt{\rho_x^s \rho_y^s}\frac{(-y,x)}{\rho_y^s x^2 + \rho_x^s y^2}$$



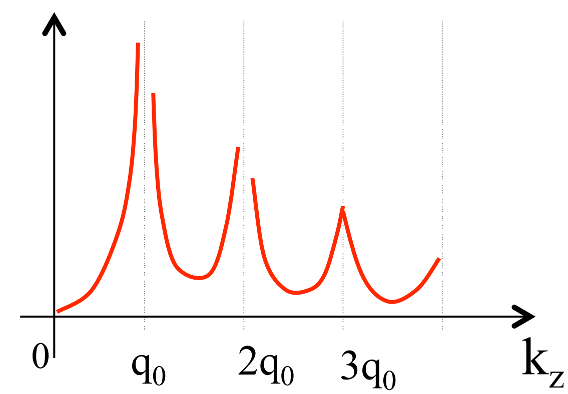


- ***π-vortices:***

$$n_v = 4\Omega_r \frac{m}{\hbar}$$

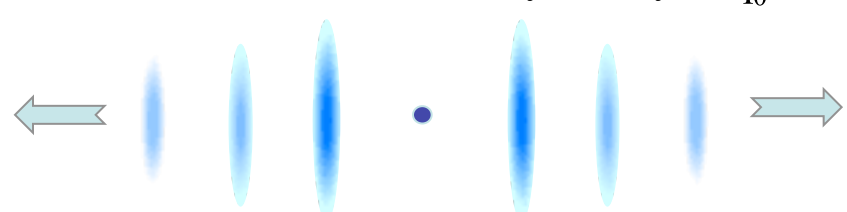
- ***momentum distribution (time of flight):***

$$n_k \sim \frac{1}{|k_z - q_0|^{2-\eta}}$$



- ***structure function:***

$$S(k_z) \sim \frac{1}{|k_z - 2q_0|^{2-4\eta}}$$



<u>*Summary and directions*</u>

- Larkin-Ovchinnikov state ⇔ superfluid smectic liquid crystal
- critical phase at finite T with universal properties
- half-integer vortex and dislocation defects
- transitions to N-Sm_{2Q} and SF_4-Nm ("charge"-4 SF nematic) phases

…many remaining questions:

- effects of Fermi pockets - Goldstone modes interactions?
- better microscopic support for the energetics?
- connection to experimental knobs: detuning and imbalance?
- explore further experimental consequences, detection signals?
- …