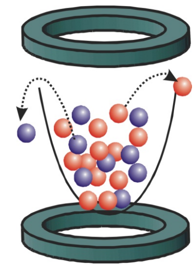
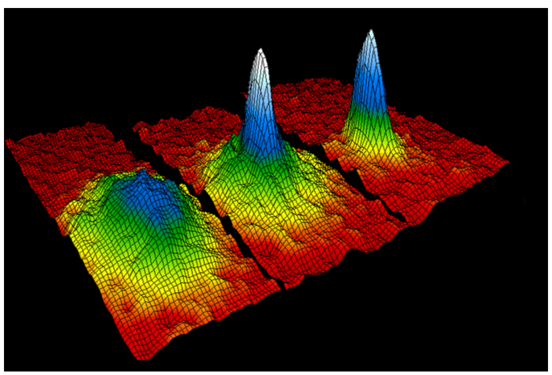
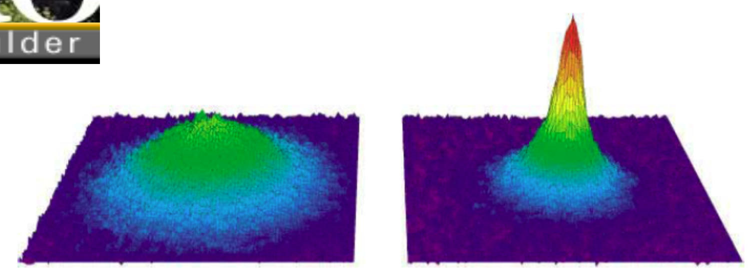
Finite-momentum superfluidity and phase transitions in p-wave resonant Bose gas







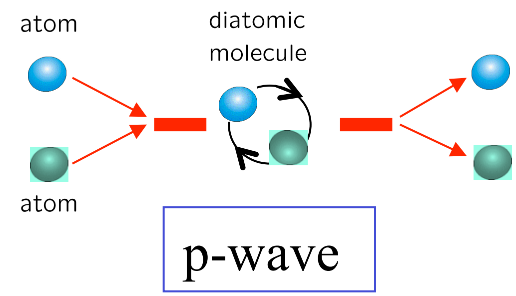
with *Sungsoo Choi*

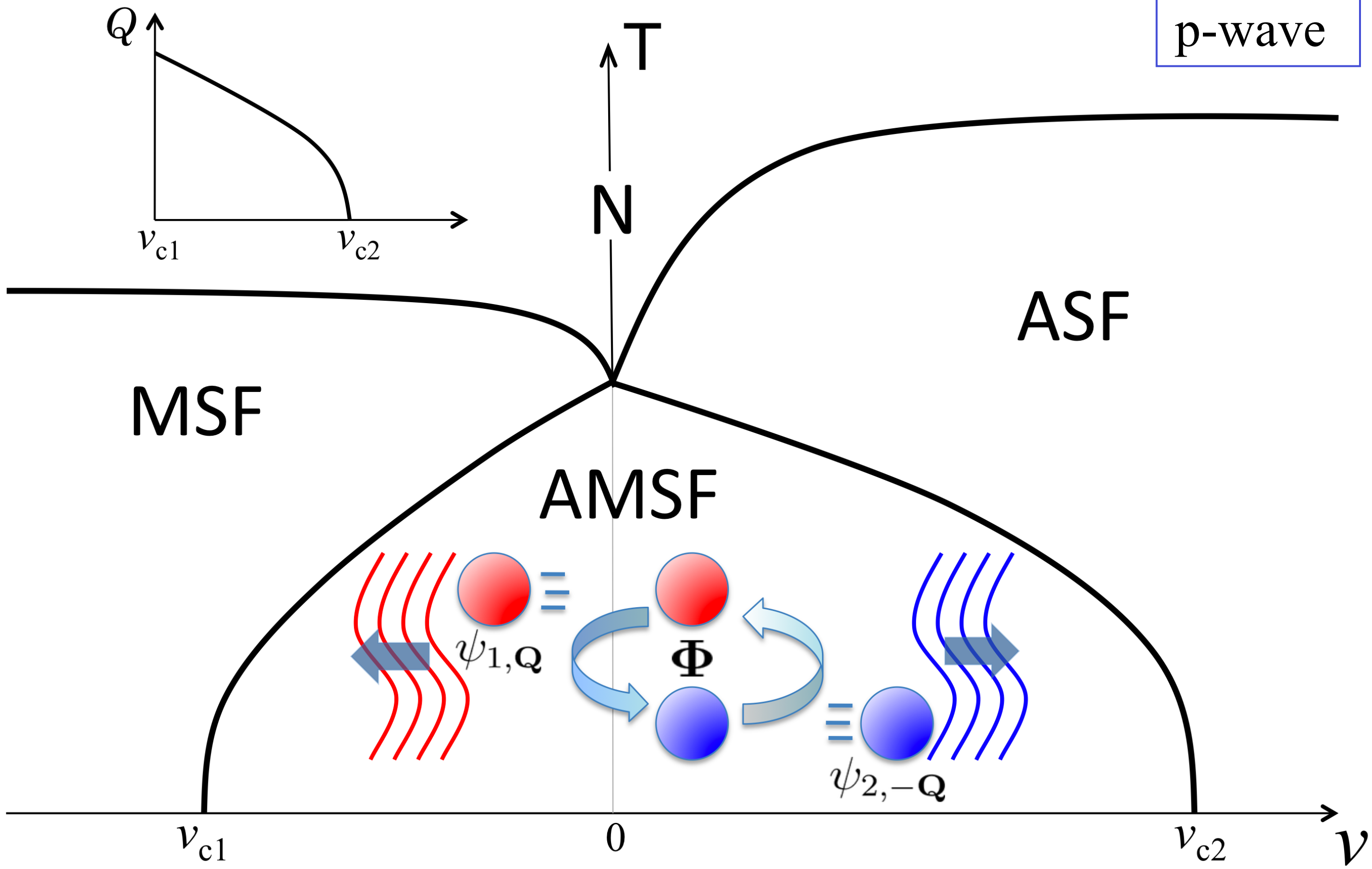
(for details see: ***arXiv:1106.5765*** and PRL '09
also see: Liu, Wu, and Kuklov '06)

support by: ***NSF Materials Theory***

ICTP Tieste, July 2011

Summary





- atomic (ASF) and spinor-molecular (MSF) superfluids
- atomic-molecular superfluid (AMSF) with ***finite momentum*** atomic BEC
- quantum and thermal phase transitions

Outline

- Motivation
- Feshbach resonances
- Model
- Phases and transitions
- Excitations
- Conclusions

Motivation

- **Bosonic Feshbach resonances exist in e.g., Rb87, Rb85,…**
  → ultracold coherent bosonic atom-molecule mixtures

- Resonant fermionic systems led to molecular superfluids, BEC-BCS crossover, strongly interacting superfluid, etc.

- Allow SF-SF quantum phase transitions (cf. just crossover for fermions)

Rb85 Feshbach resonances

Atom–molecule coherence in a Bose–Einstein condensate

Elizabeth A. Donley, Neil R. Claussen, Sarah T. Thompson & Carl E. Wieman

JILA, University of Colorado and National Institute of Standards and Technology, Boulder, Colorado 80309-0440, USA

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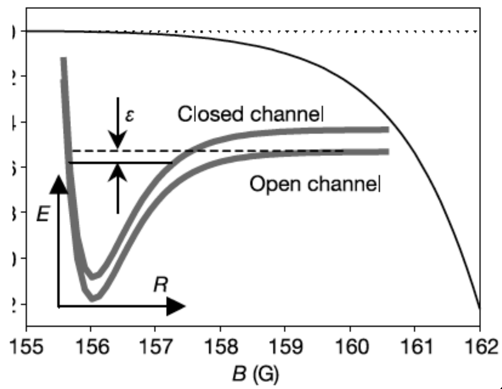
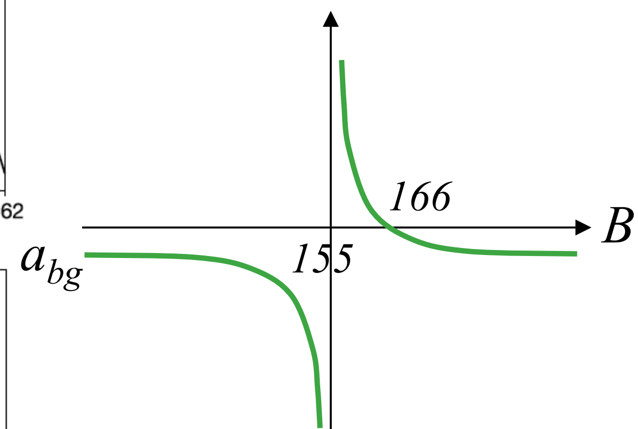
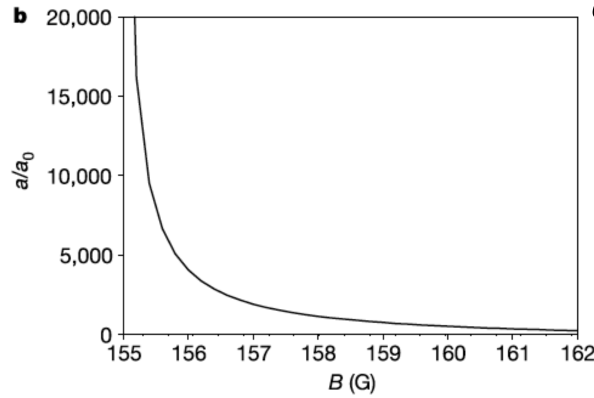
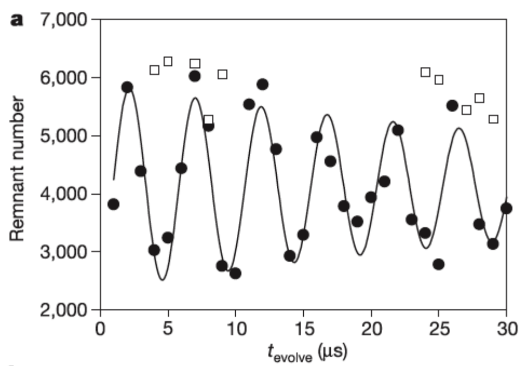


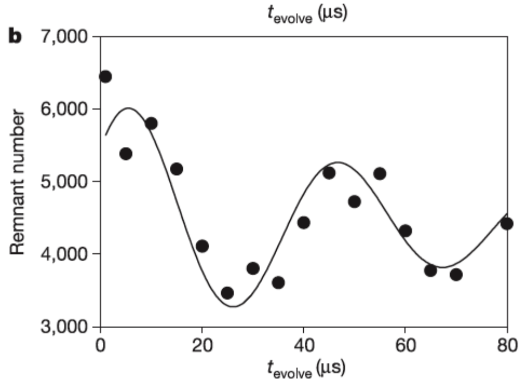
$$|F=2, m_F=-2\rangle$$

$$a = a_{bg}\left(1 - \frac{\Delta B}{B-B_0}\right)$$

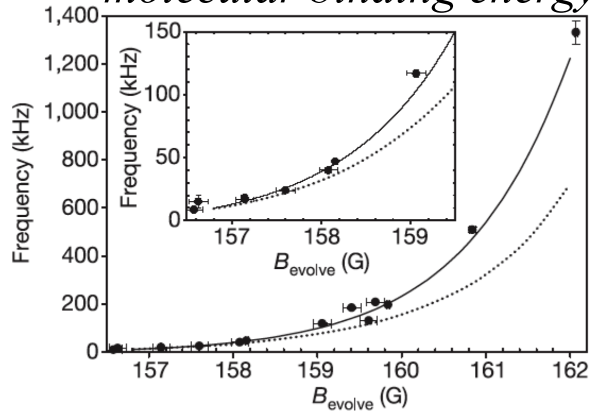




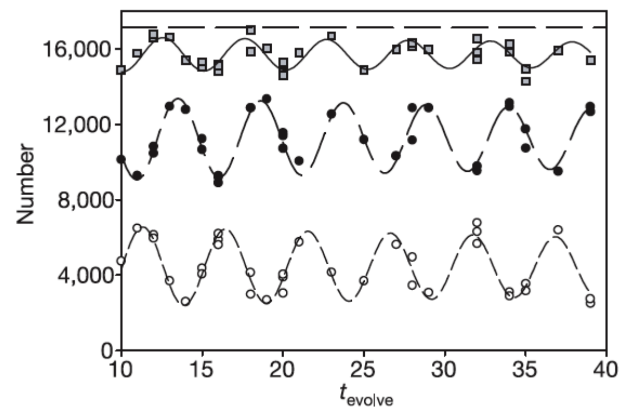




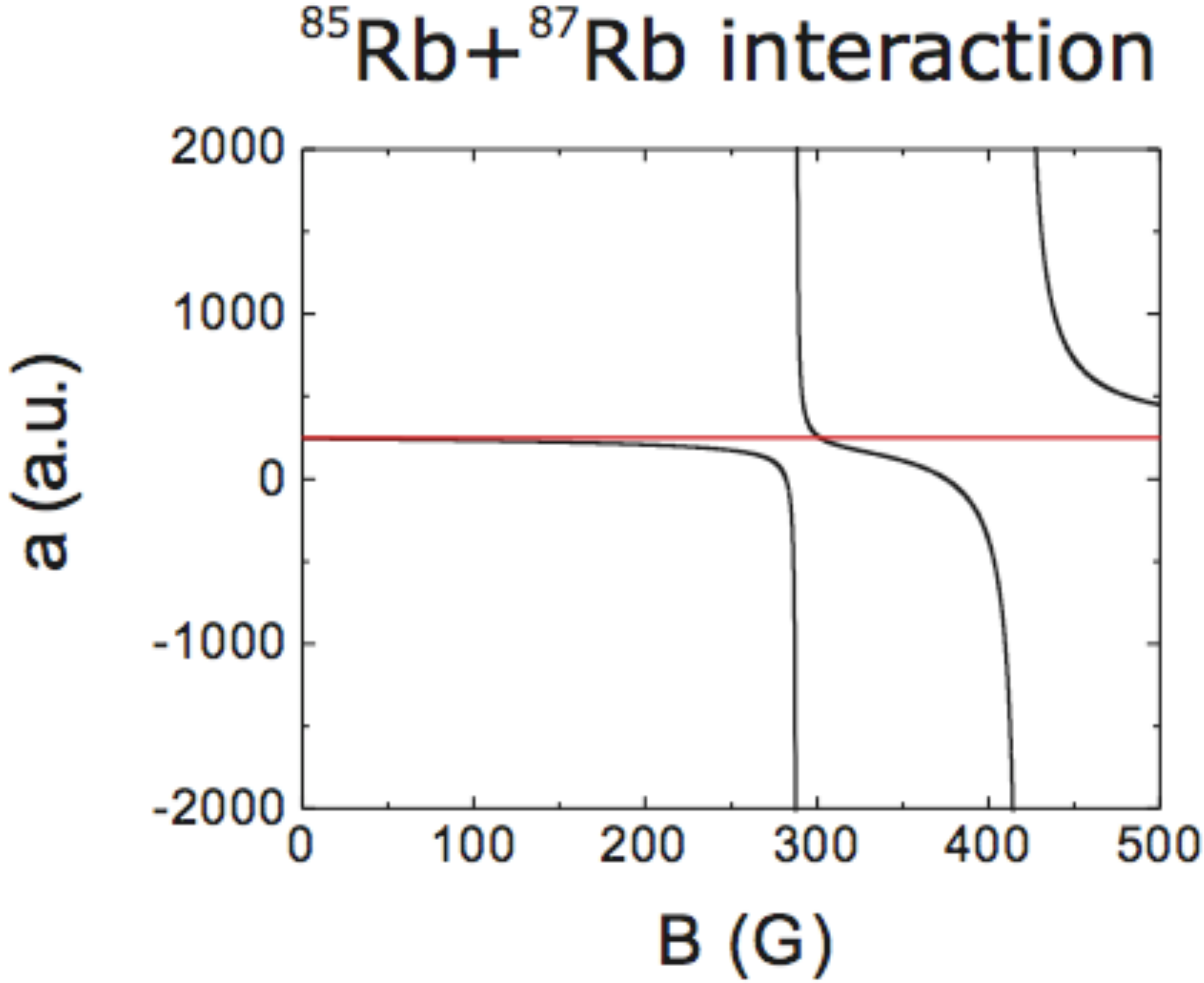
molecular binding energy



Rabi oscillations

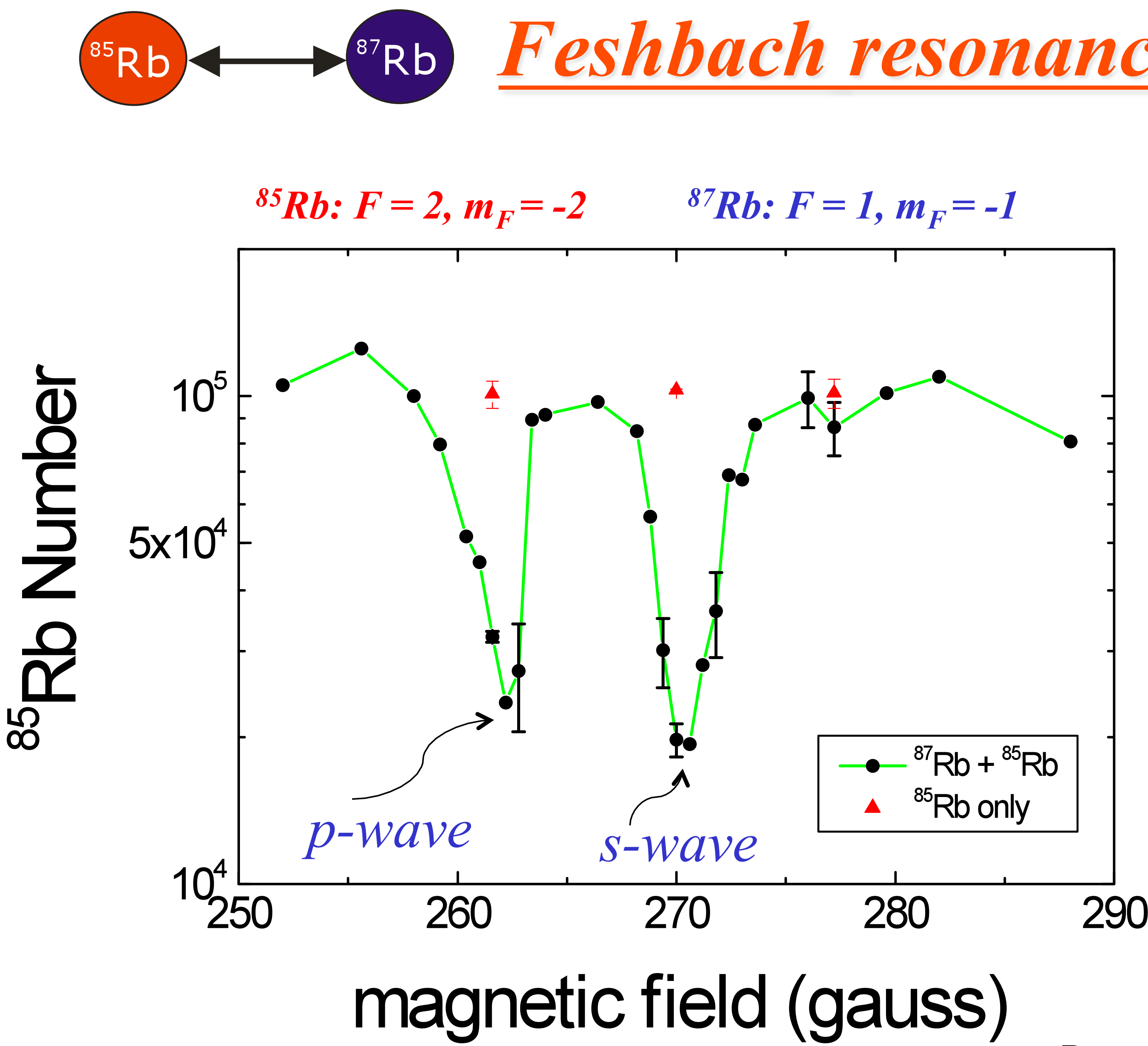


Rb85-Rb87 Feshbach resonances



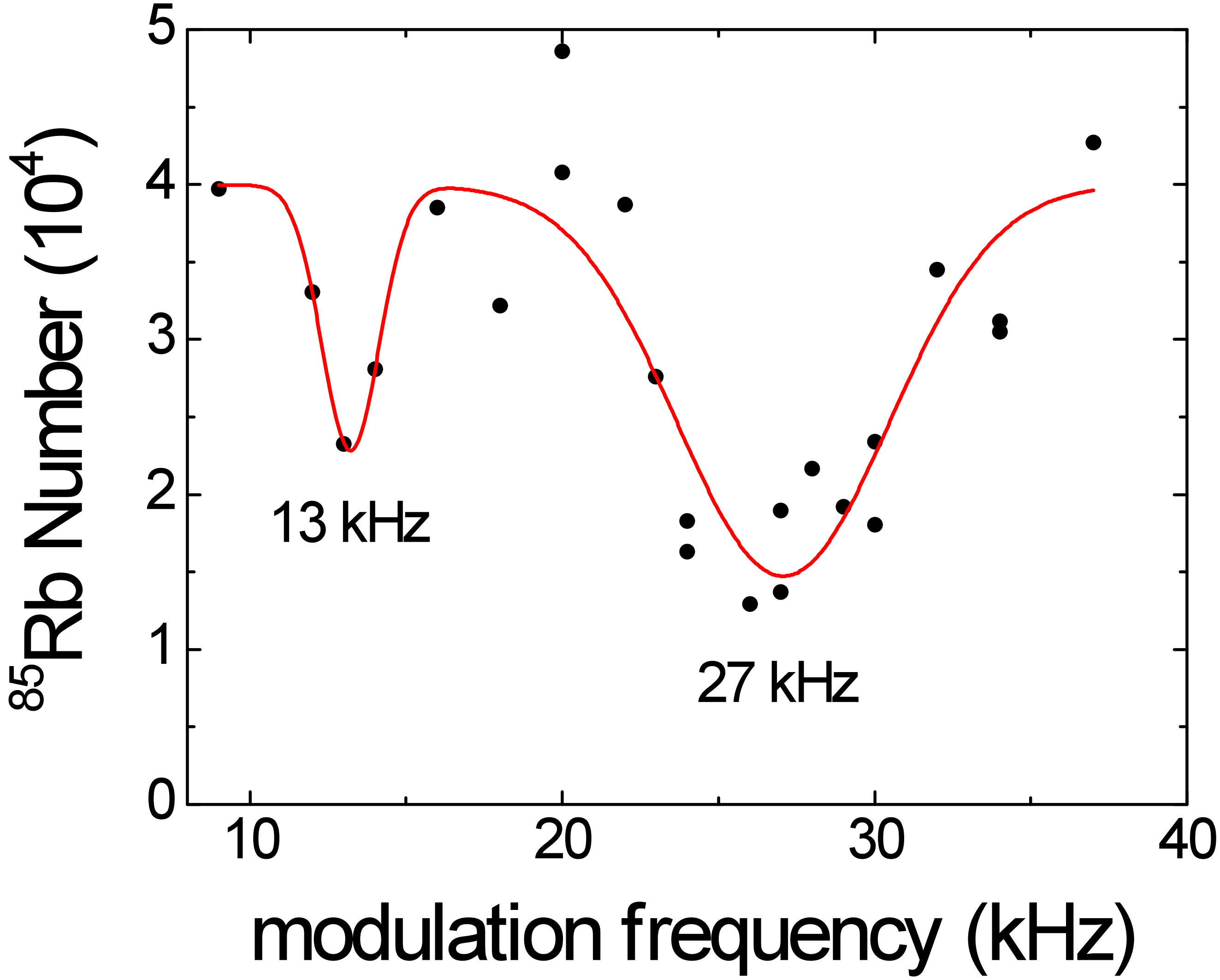
Papp, Pino, Wieman

Feshbach resonances



Papp, Pino, Wieman

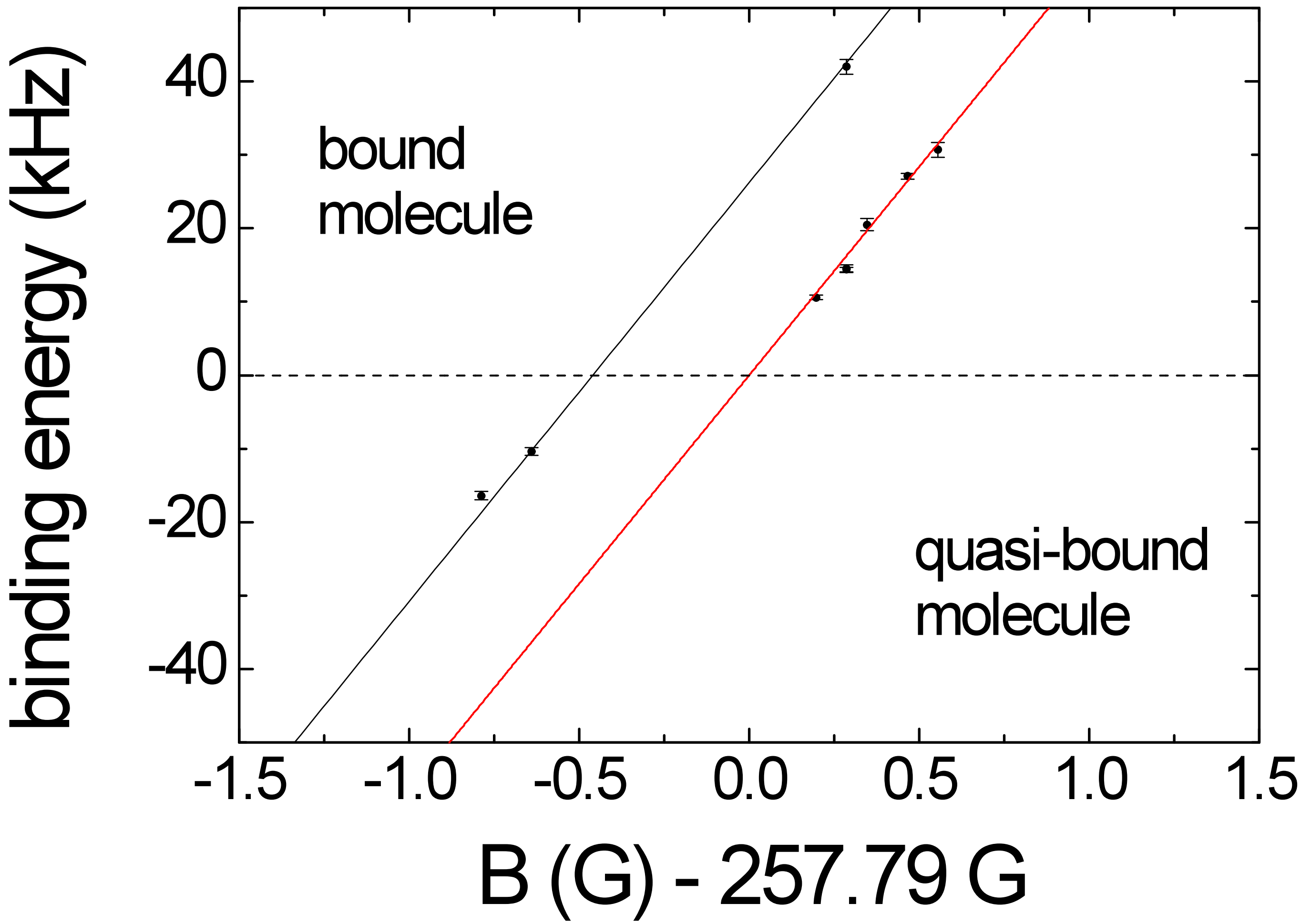
p-wave resonant modulation



Papp, Pino, Wieman

Rb85-Rb87 p-wave molecules

two m_l levels



Papp, Pino, Wieman

Motivation

- Bosonic Feshbach resonances exist in e.g., Rb87, Rb85,…
  → ultracold coherent bosonic atom-molecule mixtures

- **Resonant fermionic systems led to molecular superfluids, BEC-BCS crossover, strongly interacting superfluid, etc.**

- Allow SF-SF quantum phase transitions (cf. just crossover for fermions)

Feshbach resonance (Fermi)

Emergence of a molecular Bose–Einstein condensate from a Fermi gas

Markus Greiner[1], Cindy A. Regal[1] & Deborah S. Jin[2]

[1] *JILA, National Institute of Standards and Technology and Department of Physics, University of Colorado,* [2] *Quantum Physics Division, National Institute of Standards and Technology, Boulder, Colorado 80309-0440, USA*

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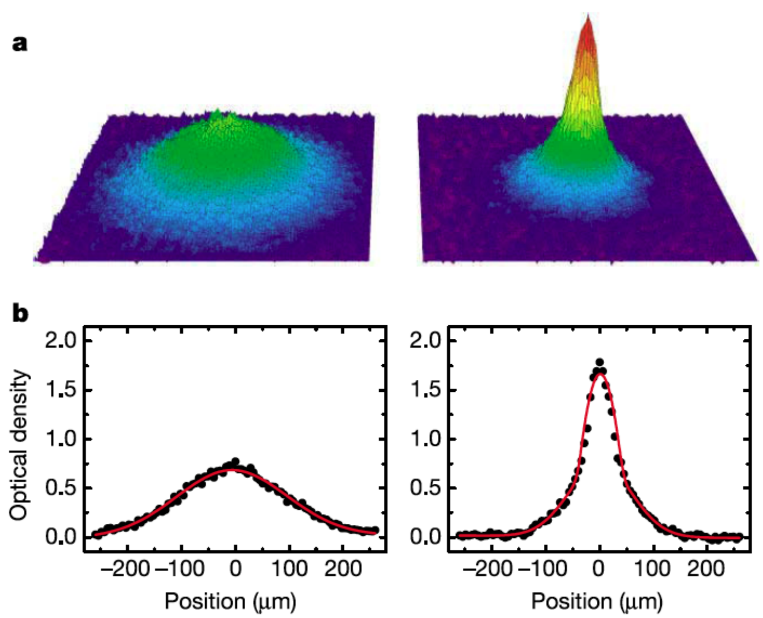
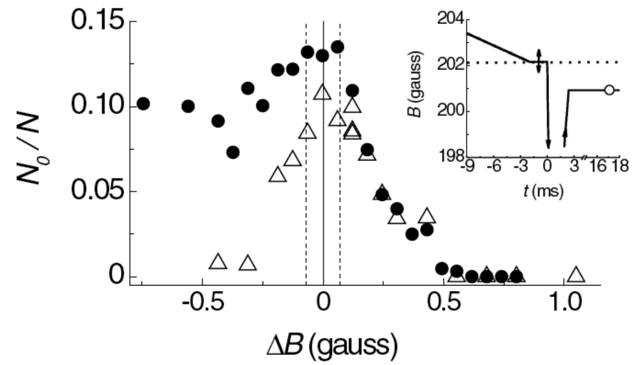
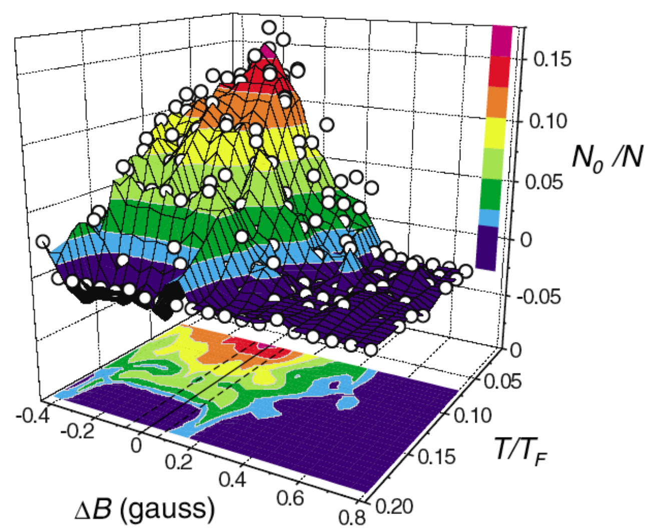


Observation of Resonance Condensation of Fermionic Atom Pairs

C. A. Regal, M. Greiner, and D. S. Jin*

Physical Review Letters 92, (2004)



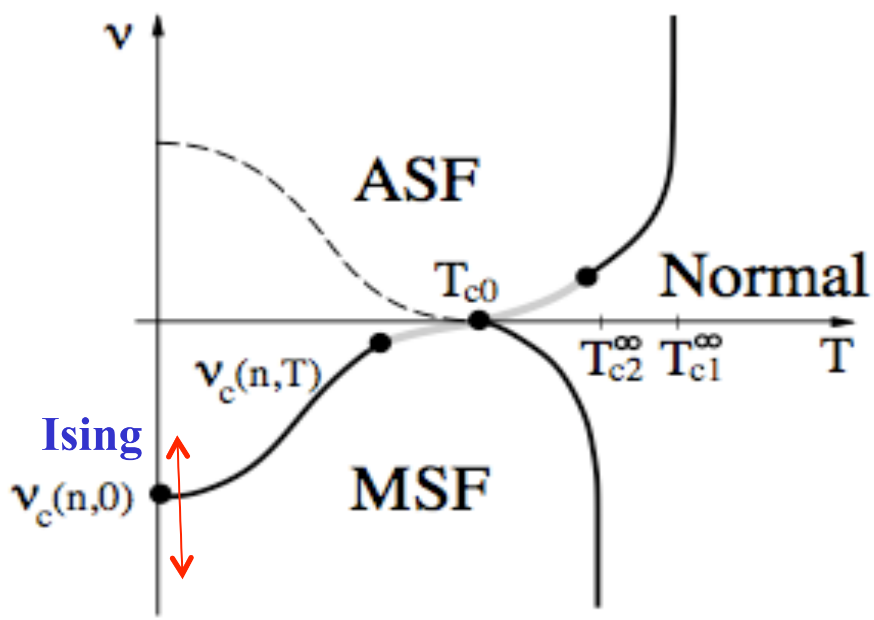


Motivation

- Bosonic Feshbach resonances exist in e.g., Rb87, Rb85,…
  → ultracold coherent bosonic atom-molecule mixtures

- Resonant fermionic systems led to molecular superfluids, BEC-BCS crossover, strongly interacting superfluid, etc.

- **Allow SF-SF quantum phase transitions (cf. just crossover for fermions) even for s-wave resonance** -

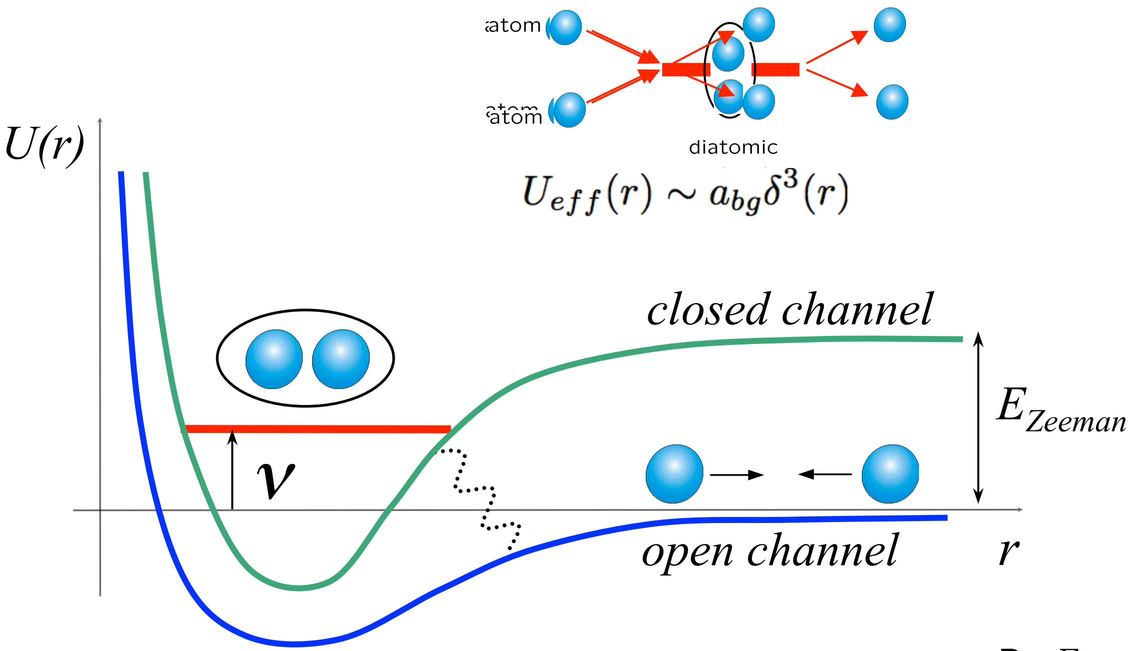


original proposal in continuum:
LR, Park, Weichman, PRL '04
Romans, et al, PRL'04

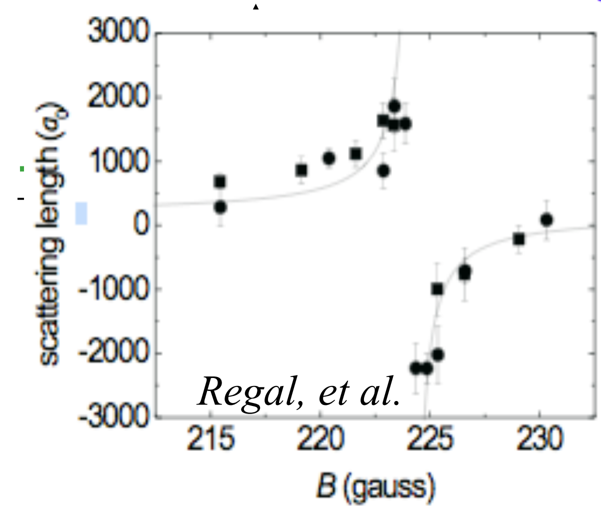
more recently on lattice:
- *Diehl, et al, 2010*
- *Ejima, et al, 2011 (DMRG)*
- *Bonnes, Wessel, 2011 (QMC)*

Feshbach resonance

- tunability (strength and sign) of interactions (sudden and adiabatic) via Feshbach resonances



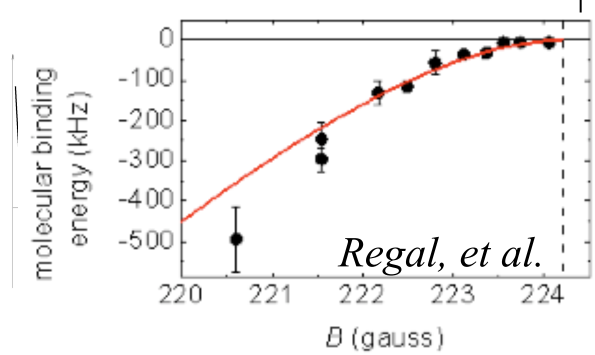
$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

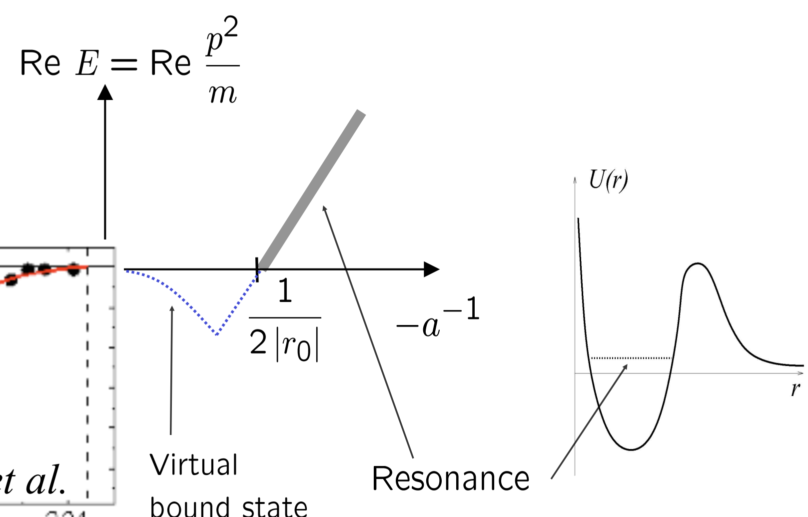


$$f_s(k) = \frac{1}{-a^{-1} + \frac{r_0}{2}k^2 - ik}$$

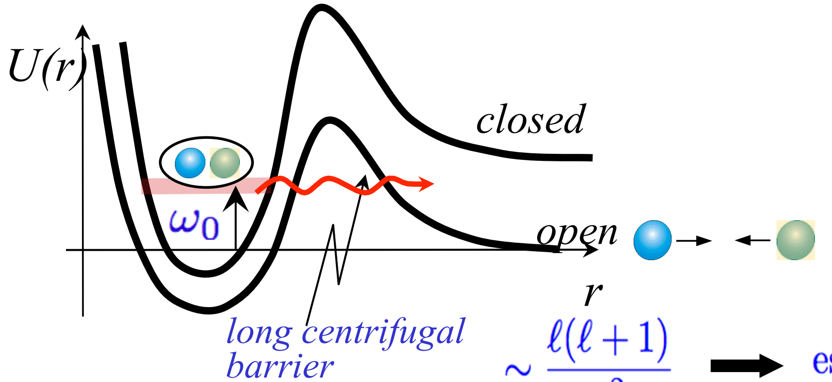
with $a \sim -\frac{g^2}{\omega_0} \sim a_{bg}\frac{\Delta B}{B_0 - B}$

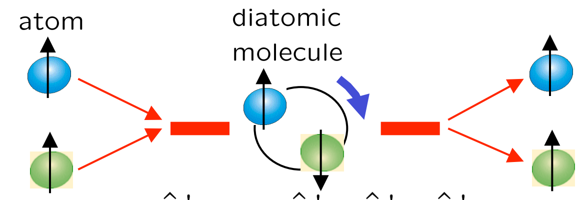
$r_0 \sim -\frac{1}{g^2}$





P-wave Feshbach resonant scattering





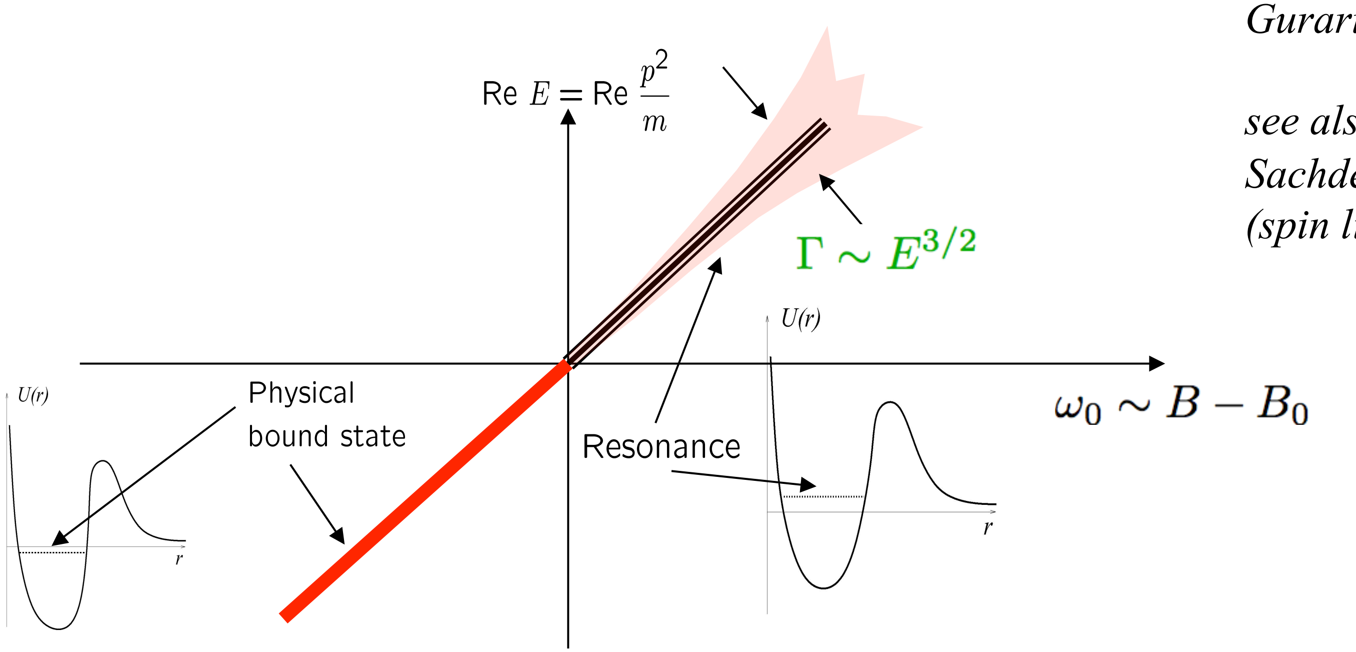
$\hat{\phi}^\dagger = (\hat{\phi}_x^\dagger, \hat{\phi}_y^\dagger, \hat{\phi}_z^\dagger)$

$\sim \frac{\ell(\ell+1)}{r^2}$ ➡ escape (molecular life) time $\tau \sim \Gamma^{-1} \sim E^{-\frac{3}{2}} \gg E^{-1}$, for $E \to 0$

$$H = \psi_\sigma^\dagger \frac{\hat{p}^2}{2m}\psi_\sigma + \vec{\phi}^\dagger \cdot \left(\frac{\hat{p}^2}{4m} + \nu_0\right)\vec{\phi} - i\alpha\vec{\phi}^\dagger \cdot \hat{\psi}_1 \vec{\nabla}\psi_2 + h.c.$$

Gurarie, L.R., AOP '09

see also:
Sachdev + Read '91
(spin liquids)



p-wave resonant Bose model

- **two distinguishable open-channel bosonic atoms:** $\hat{\psi}^\dagger_\sigma = \left(\hat{\psi}^\dagger_1, \hat{\psi}^\dagger_2\right)$
- **p-wave closed-channel molecule:** $\hat{\vec{\phi}}^\dagger = (\hat{\phi}^\dagger_x, \hat{\phi}^\dagger_y, \hat{\phi}^\dagger_z)$

- **model**: $$H = H_a + H_m + H_{am} + H_{FR}$$

two species BEC:
$$H_a = \sum_{\sigma=1,2} \left(\psi^\dagger_\sigma (-\frac{\nabla^2}{2m} - \mu_\sigma)\psi_\sigma + \frac{\lambda_\sigma}{2}\psi^{\dagger 2}_\sigma \psi^2_\sigma \right) + \lambda_{12}\psi^\dagger_1\psi^\dagger_2\psi_2\psi_1$$

spinor=1 condensates:
$$H_m = \vec{\phi}^\dagger(-\frac{\nabla^2}{4m} - \mu_m)\vec{\phi} + \frac{g_1}{2}|\vec{\phi}^\dagger \cdot \vec{\phi}|^2 + \frac{g_2}{2}|\vec{\phi} \cdot \vec{\phi}|^2$$

$\mu_m = \mu_1 + \mu_2 - \nu$

ν *- detuning*

nonresonant interaction:
$$H_{am} = g_{am}\psi^\dagger_\sigma\psi_\sigma\vec{\phi}^\dagger \cdot \vec{\phi}$$

Feshbach resonant interaction:

$$H_{FR} = -i\frac{\alpha}{2}\left[\vec{\phi}^\dagger \cdot (\psi_1\vec{\nabla}\psi_2 - \psi_2\vec{\nabla}\psi_1) + h.c.\right]$$

Landau theory

$$F \approx -\mu_m|\vec{\Phi}|^2 + \frac{g_1}{2}(\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2}|\vec{\Phi} \cdot \vec{\Phi}|^2$$

$$+(\frac{Q^2}{2m} - \mu_\sigma)|\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2}|\Psi_{Q,\sigma}|^4 + \dots$$

$$+\alpha(\vec{\Phi}^* \cdot \vec{Q})\Psi_{Q,1}\Psi_{-Q,2} + c.c.$$

conserved: $n_1 + n_m$, $n_2 + n_m$ $\qquad$ $\mu_m = \mu_1 + \mu_2 - \nu$

Landau theory

*large **negative** detuning* ⟶ $\mu_\sigma < 0, \quad \mu_m > 0$

$$F \approx -\mu_m|\vec{\Phi}|^2 + \frac{g_1}{2}(\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2}|\vec{\Phi} \cdot \vec{\Phi}|^2$$

$$+(\frac{Q^2}{2m} - \mu_\sigma)|\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2}|\Psi_{Q,\sigma}|^4 + \ldots$$

$$+\alpha(\vec{\Phi}^* \cdot \vec{Q})\Psi_{Q,1}\Psi_{-Q,2} + c.c.$$

conserved: $n_1 + n_m$

$n_2 + n_m$

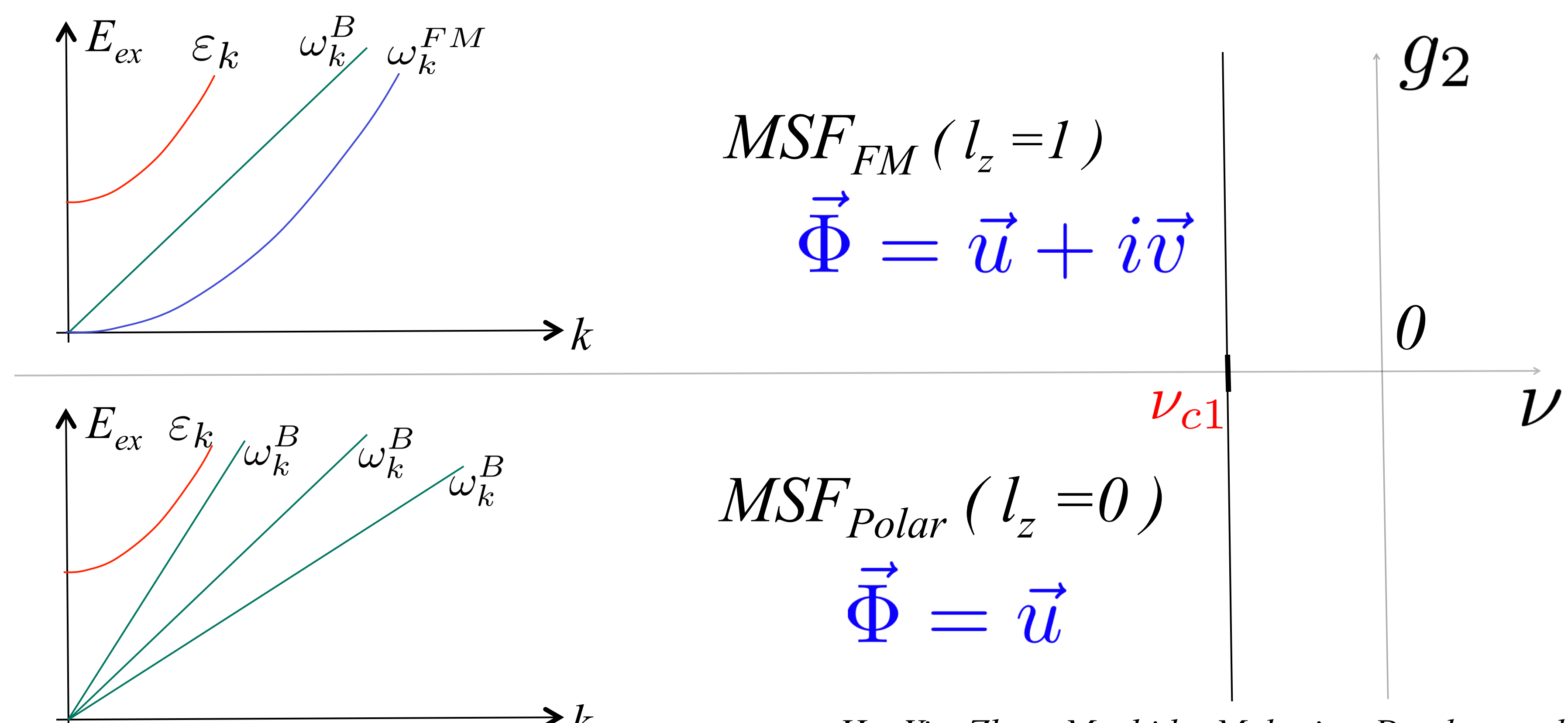
$$\mu_m = \mu_1 + \mu_2 - \nu$$

L=1 molecular superfluid (MSF)

$\vec{\Phi} \neq 0, \quad \Psi_\sigma = 0$

*large **negative** detuning* ⟶ $\mu_\sigma < 0, \quad \mu_m > 0$

$$F_{MSF} \approx -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2}(\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2}|\vec{\Phi} \cdot \vec{\Phi}|^2$$



Ho, Yip, Zhou, Machida, Mukerjee, Demler, et al.

Landau theory

large ***positive*** *detuning* ⟶ $\mu_\sigma > 0, \quad \mu_m < 0$

$$F \approx -\mu_m|\vec{\Phi}|^2 + \frac{g_1}{2}(\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2}|\vec{\Phi} \cdot \vec{\Phi}|^2$$

$$+(\frac{Q^2}{2m} - \mu_\sigma)|\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2}|\Psi_{Q,\sigma}|^4 + \dots$$

$$+\alpha(\vec{\Phi}^* \cdot \vec{Q})\Psi_{Q,1}\Psi_{-Q,2} + c.c.$$

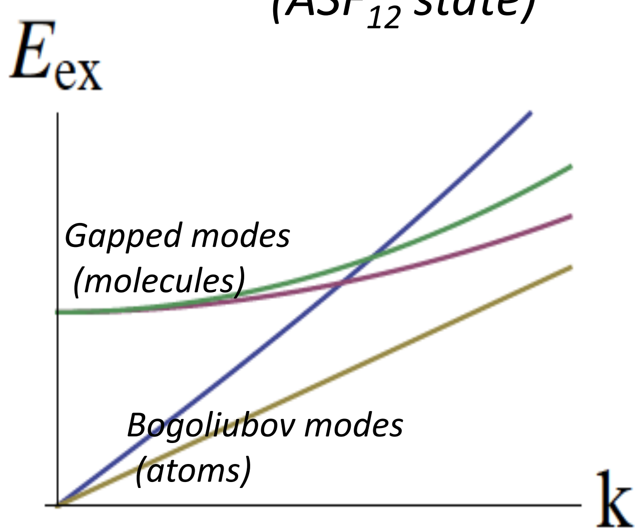
Atomic superfluid (ASF)

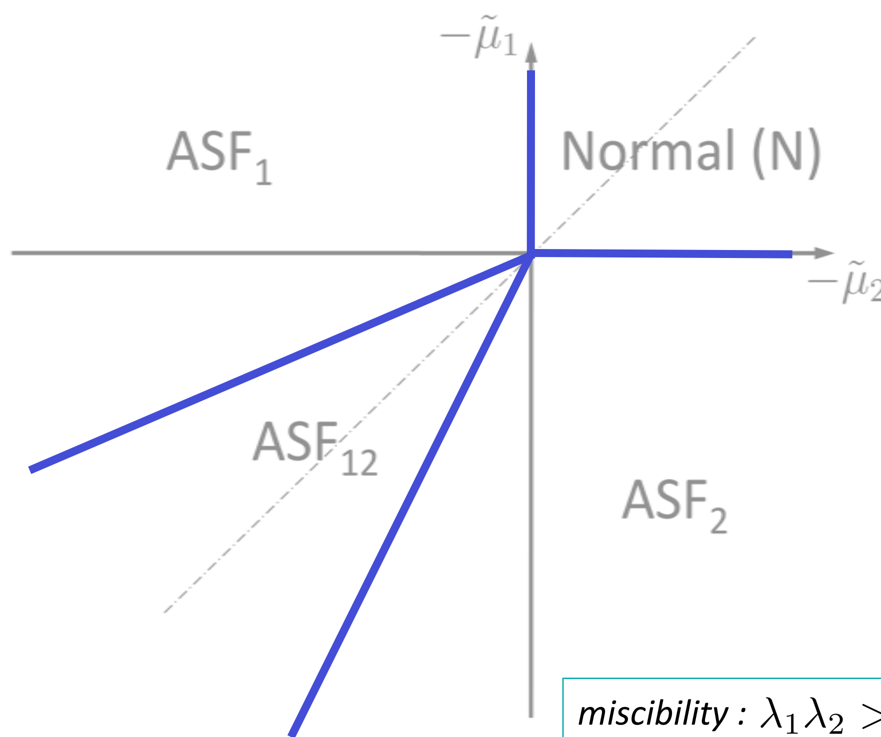
$\vec{\Phi} = 0\,, \quad \Psi_\sigma \neq 0$

large positive detuning ⟶ $\mu_\sigma > 0, \quad \mu_m < 0$

$$F_{ASF} \approx -\mu_\sigma |\Psi_\sigma|^2 + \frac{\lambda_\sigma}{2}|\Psi_\sigma|^4 + \frac{\lambda_{12}}{2}|\Psi_1|^2|\Psi_2|^2$$

(ASF_{12} state)





miscibility : $\lambda_1\lambda_2 > \lambda_{12}^2$

Esry, Greene '97

Landau theory

intermediate detuning $\nu_{c1} < \nu < \nu_{c2}$ ⟶ $\mu_\sigma < 0, \quad \mu_m > 0$

$$F \approx -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2}(\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2}|\vec{\Phi} \cdot \vec{\Phi}|^2$$

$$+(\frac{Q^2}{2m} - \mu_\sigma)|\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2}|\Psi_{Q,\sigma}|^4 + \ldots$$

$$\boxed{+\alpha(\vec{\Phi}^* \cdot \vec{Q})\Psi_{Q,1}\Psi_{-Q,2} + c.c.}$$

Atomic-molecular superfluid (AMSF) $\vec{\Phi} \neq 0, \quad \Psi_{Q,\sigma} \neq 0$

intermediate detuning $\nu_{c1} < \nu < \nu_{c2}$ ⟶ $\mu_\sigma < 0, \quad \mu_m > 0$

$$F_{AMSF} \approx (\frac{Q^2}{2m} - \mu_\sigma)|\Psi_{Q,\sigma}|^2 + \alpha(\vec{\Phi}^* \cdot \vec{Q})\Psi_{Q,1}\Psi_{-Q,2} + \ldots$$

$$\approx -\frac{\mu_+}{2}|\Psi_Q^{(+)}|^2 - \frac{\mu_-}{2}|\Psi_Q^{(-)}|^2 + \ldots$$

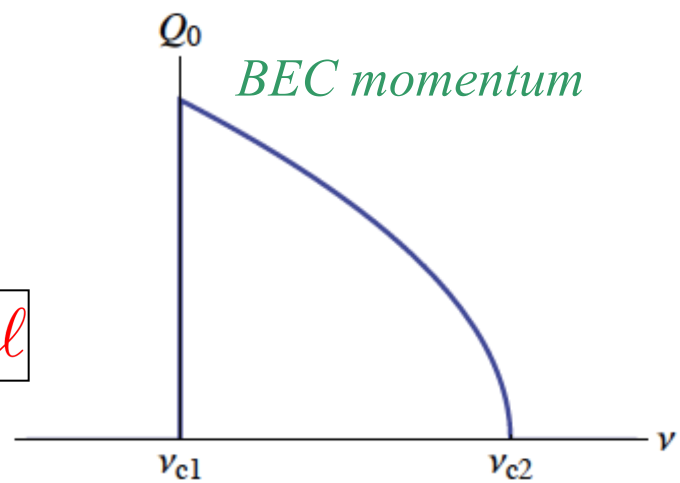
$$\mu_\pm = -(\frac{Q^2}{2m} - \mu) \pm \alpha|\vec{\Phi} \cdot \vec{Q}|$$

⟶ *transition to AMSF at ν_{c1} ($\mu_+ = 0 > \mu_-$)* $\quad \Psi_\pm = \Psi_{Q,1} \pm e^{i\varphi}\Psi^*_{-Q,2}$

physics of Q ≠ 0: $\quad \frac{Q^2}{2m} \sim \alpha\vec{Q} \cdot \vec{\Phi}$

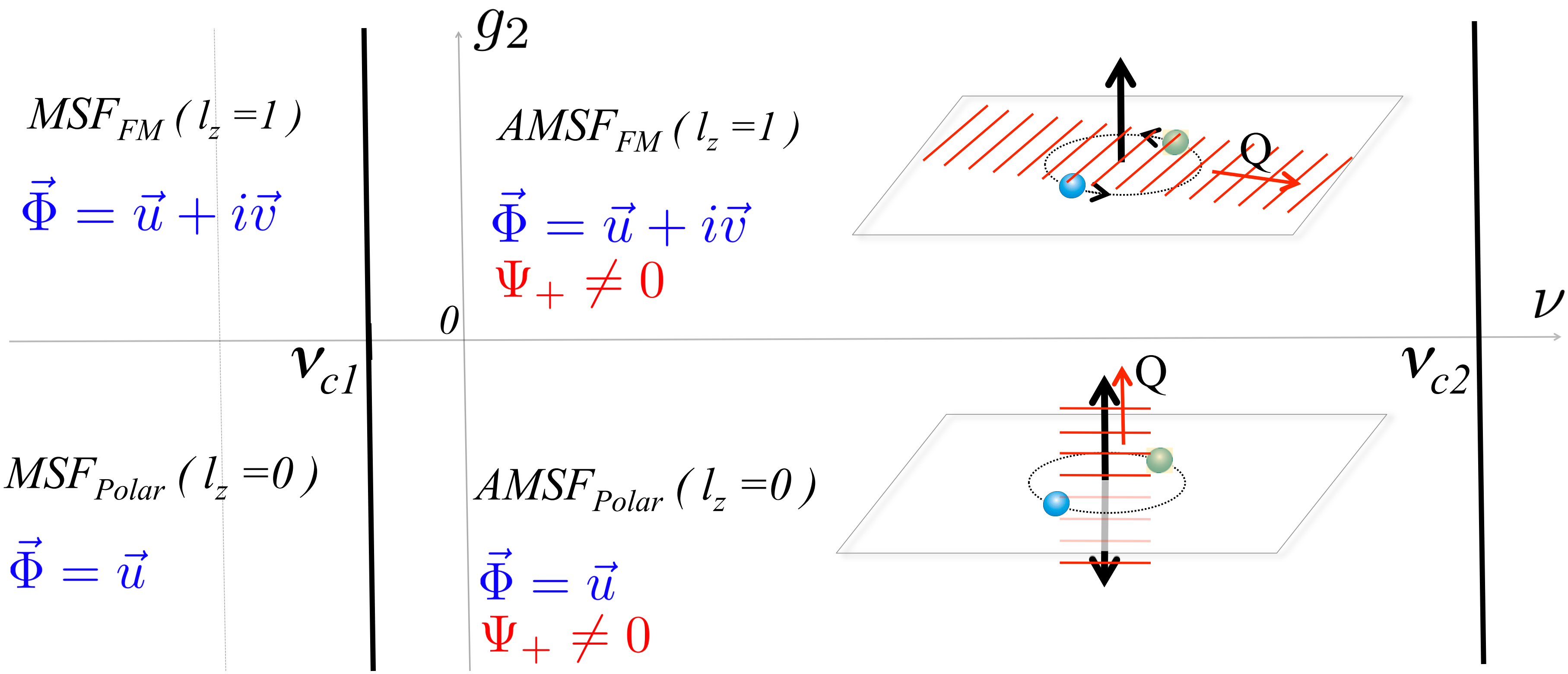
⟶ $Q \approx \alpha m\sqrt{n_m} \sim \sqrt{\gamma_p \ell n_m} \lesssim \sqrt{\gamma_p}/\ell$

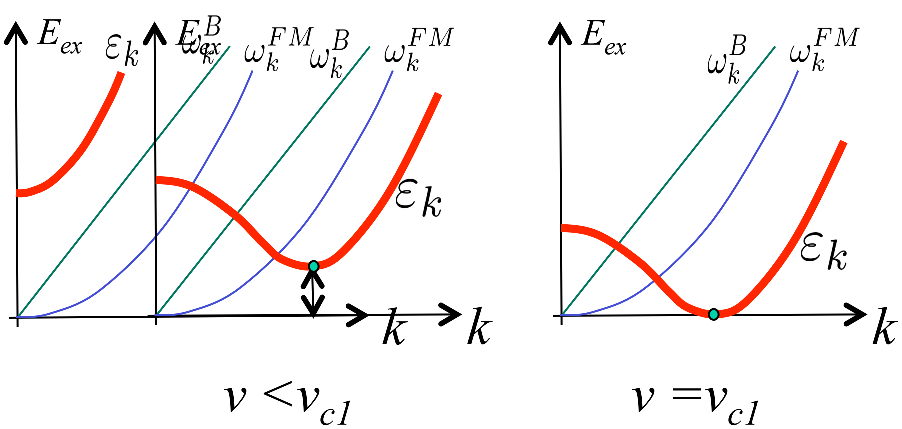
(tunable with ν)



$\nu_{c1} = -(m\alpha^2 + g)n_m \quad \nu_{c2} = (2\lambda - g_{am})n_a$

Near MSF-AMSF transition



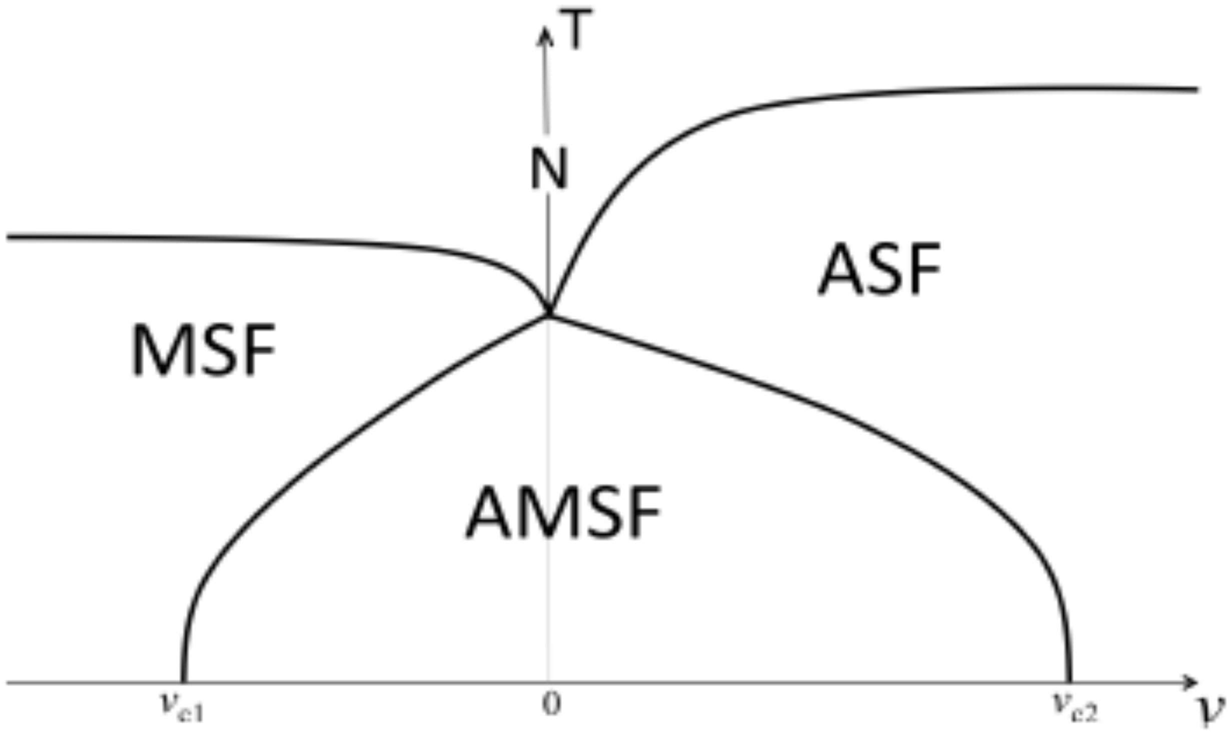


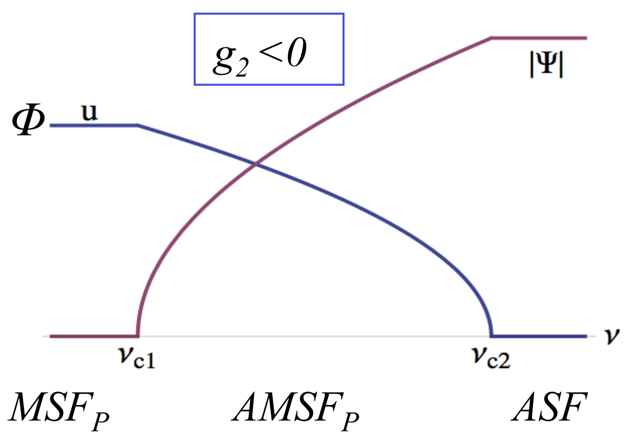
$$H_a^{MSF} = \begin{pmatrix} \frac{k^2}{2m} - \mu & \alpha\vec{\Phi}\cdot\vec{k} \\ \alpha\vec{\Phi}^*\cdot\vec{k} & \frac{k^2}{2m} - \mu \end{pmatrix}$$

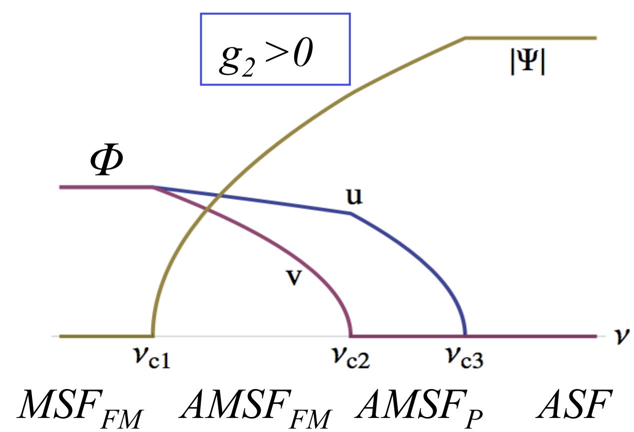
$$E_{ex} = \sqrt{\mu_+\mu_-} = \sqrt{\left(\frac{k^2}{2m} - \mu\right)^2 - |\alpha\vec{\Phi}\cdot\vec{k}|^2}$$

→ **co-linear (FF) for** MSF_{Polar}

Global phase diagram

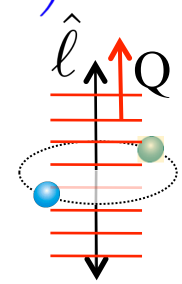






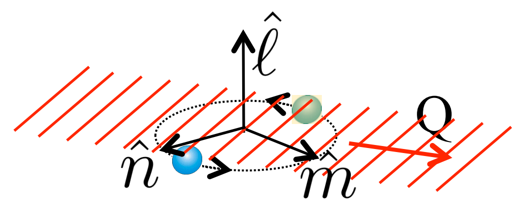
Symmetries, order parameters, Goldstone modes

- AMSF$_{\text{Polar}}$
  - *OP:* $\vec{\Phi} = e^{i\phi}\hat{\ell}, \quad \Psi = \sum_Q \Psi_Q e^{i\theta_Q + i\vec{Q}\cdot\vec{r}} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \approx e^{i\theta} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \cos(\vec{Q}\cdot\vec{r} + Qu)$
  - *breaks:* $U_N(1) \times U_{\Delta N}(1) \times SO(3) \times T_{\varepsilon\hat{\ell}}$
  - *GM:* θ_1, θ_2, φ, $\hat{\ell}$ → Higgs'ed: θ_c, θ_s



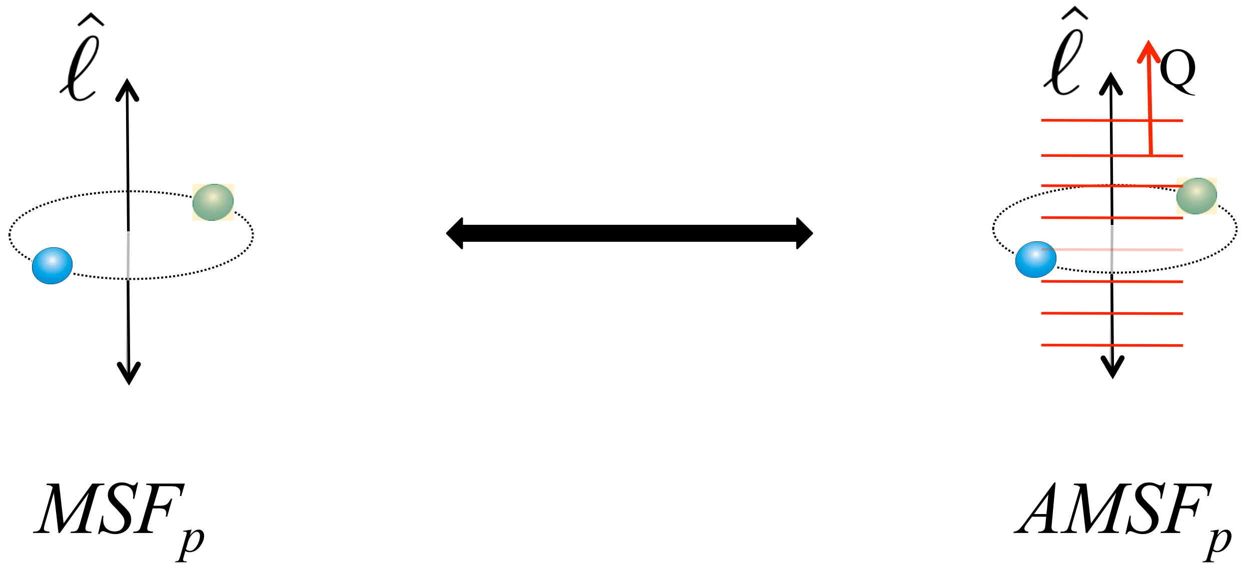
$$\mathcal{L}_p = \frac{n_c}{2}(\partial_\mu \theta_c)^2 + \frac{\chi_s}{2}(\partial_\tau \theta_s)^2 + \frac{n_s}{2}(\partial_{||}\theta_s)^2 + \frac{K}{2}(\nabla_\perp^2 \theta_s)^2$$

- AMSF$_{\text{FM}}$
  - *OP:* $\vec{\Phi} = \hat{n} + i\hat{m}, \quad \Psi = \sum_Q \Psi_Q e^{i\theta_Q + i\vec{Q}\cdot\vec{r}} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \approx e^{i\theta} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \cos(\vec{Q}\cdot\vec{r} + Qu)$
  - *breaks:* $U_N(1) \times U_{\Delta N}(1) \times SO(3) \times T_{\varepsilon\hat{\ell}} \times \Theta$
  - *GM:* θ_1, θ_2, φ, $\hat{n}$, $\hat{m}$ → Higgs'ed: θ_c, θ_s, γ



$$\mathcal{L}_{fm} = \frac{n_c}{2}(\partial_\mu \theta_c)^2 + \frac{\chi_s}{2}(\partial_\tau \theta_s)^2 + \frac{n_s}{2}(\partial_{||}\theta_s)^2 + \frac{K}{2}(\nabla_\perp^2 \theta_s)^2 + i\kappa \partial_y \theta_s \partial_\tau \gamma + \frac{J}{2}(\nabla\gamma)^2$$

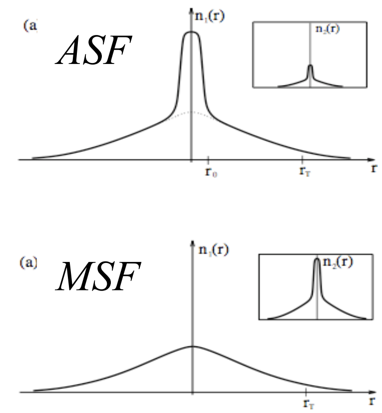
MSF – AMSF transition



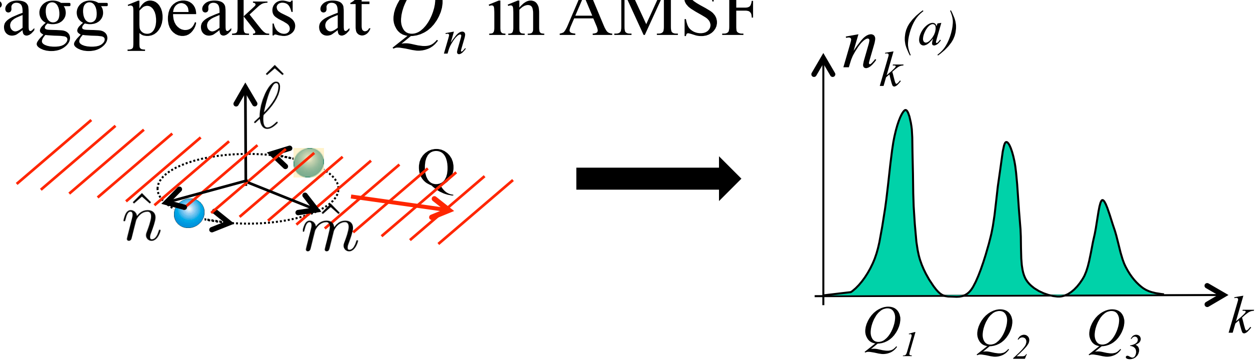
Abelian Higgs (quantum de Gennes) model:

$$\mathcal{L}_p = |\partial_\tau \psi|^2 + \frac{1}{2m}|\left(i\nabla - Q\delta\hat{\ell}\right)\psi|^2 + \epsilon_+|\psi|^2 + \frac{\lambda}{2}|\psi|^4 + \frac{1}{2g_\ell}(\partial_\mu\hat{\ell})^2 + \frac{1}{2g_\varphi}(\partial_\mu\varphi)^2$$

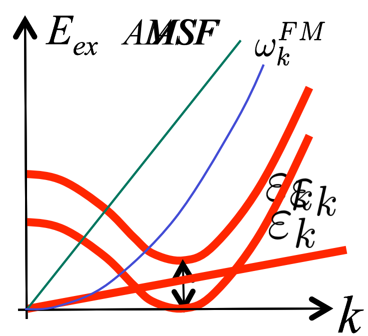
Experimental signatures



- momentum distributions $n_k^{(a)}$, $n_k^{(m)}$
- Bragg peaks at Q_n in AMSF

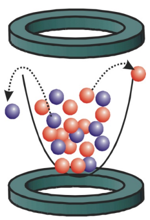


- thermodynamic singularities at transitions
- excitation spectra (phonons, Bogoluibov and spin-wave modes)
  via Bragg spectroscopy

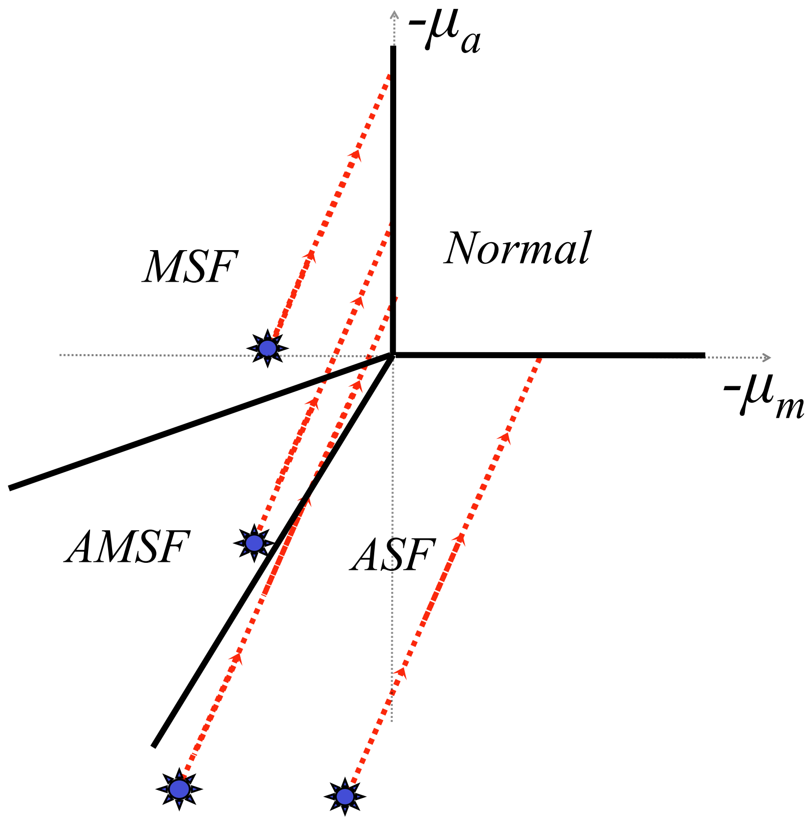


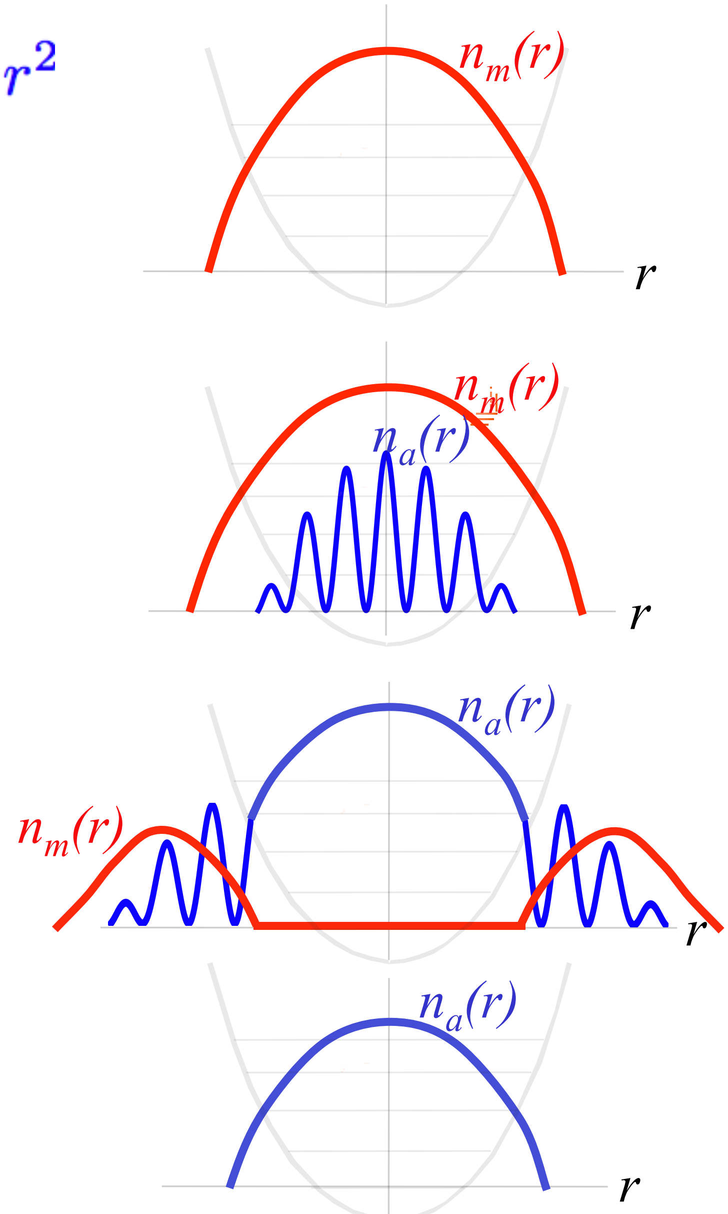
- novel vortices and dislocations

Trapped profiles via LDA



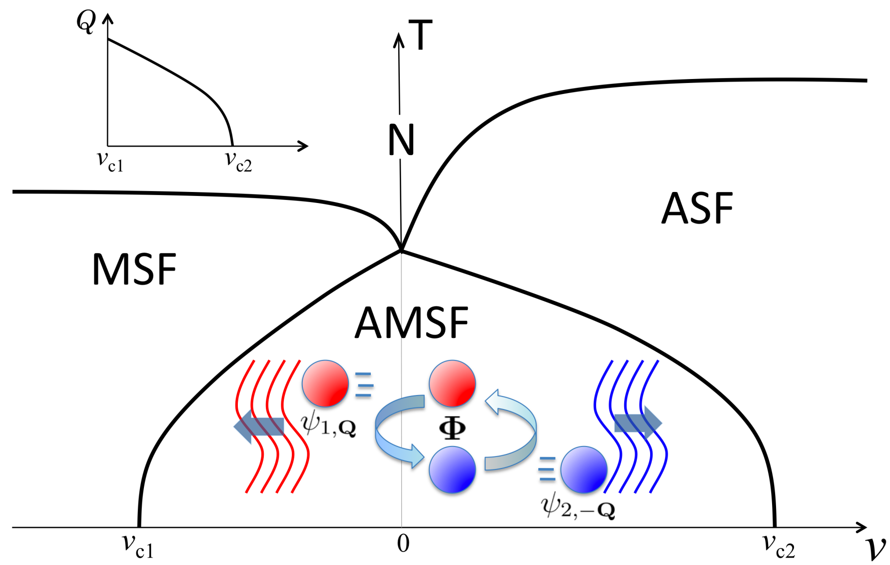
$$\mu \rightarrow \mu_{\text{eff}}(r) = \mu - \frac{1}{2} m \omega^2 r^2$$





Summary and conclusions

- resonantly interacting Bose gas:
  - *atomic and molecular superfluids*
  - *atomic supersolid, tunable Q(ν)*
  - *quantum, thermal transitions*
  - *topological defects…*



- questions:
  - *nature of the AMSF solidity: vortex lattice? 3d crystal?*
  - *stability? expect short lifetime due to 3-body instabilities*
  - …

- fixes:
  - *optical lattice?*
  - *avoid immediate vicinity of FBR?*