Resonant atomic gases

Leo Radzihovsky

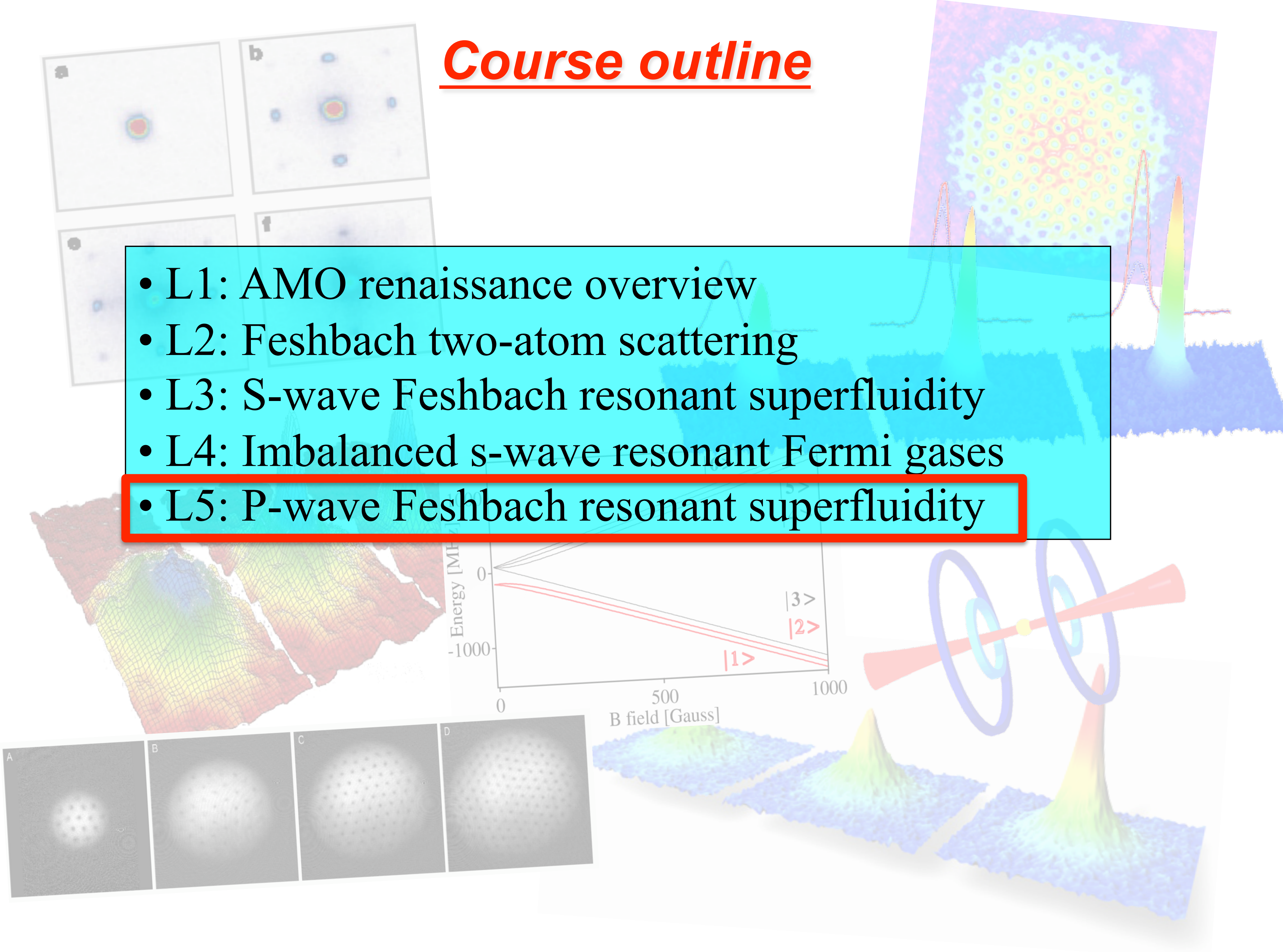
for details see: *Gurarie, L.R., Annals of Physics, 322, 2-119 (2007)*
Sheehy, L.R., Annals of Physics, 322, 1790 (2007)

Giorgini, et al., RMP, 80, 885 (2008)
Ketterle and Zwierlein, Varenna lectures (2006)

$: NSF

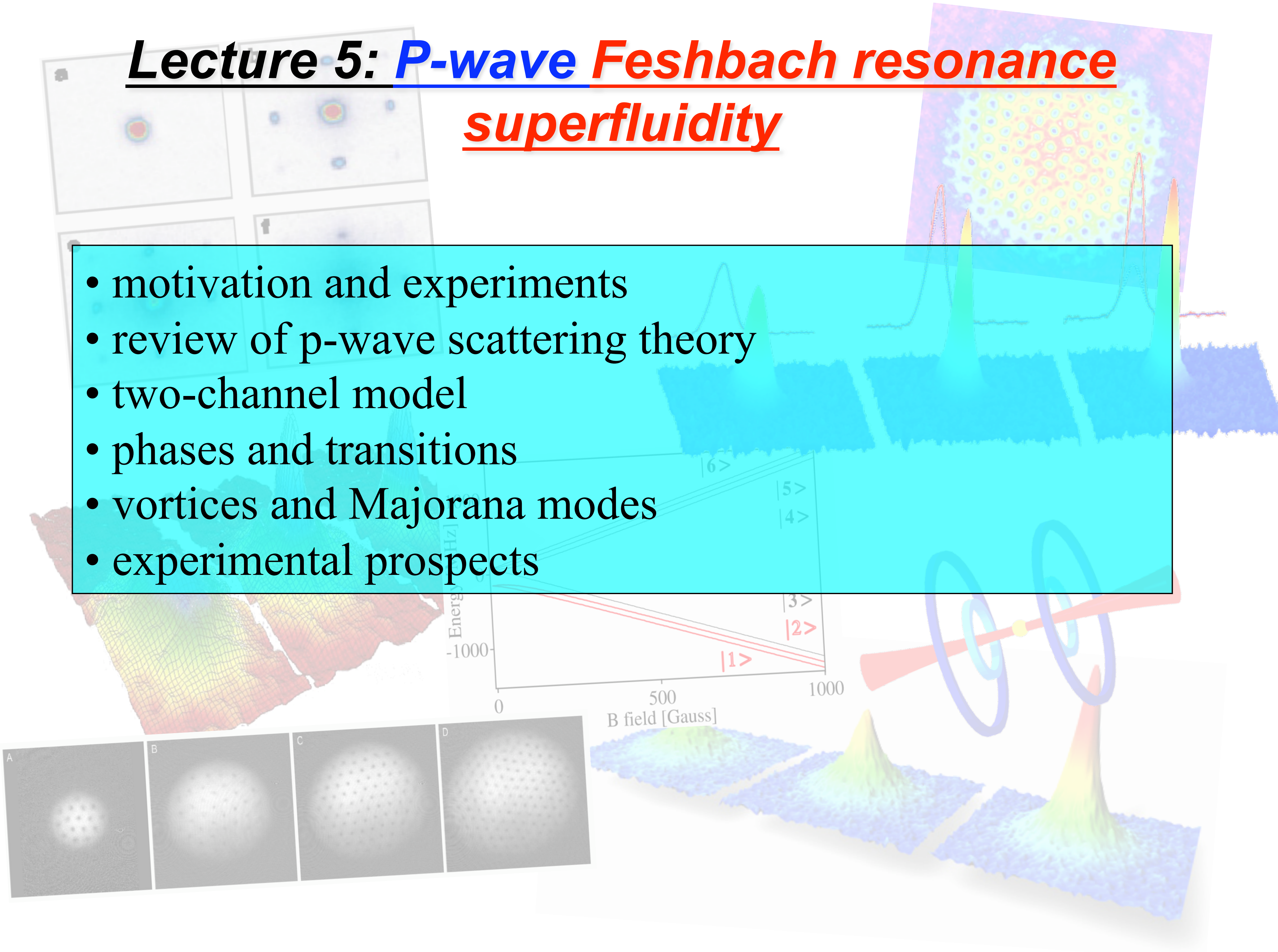
Mysore, India, Dec 2010

Course outline



- L1: AMO renaissance overview
- L2: Feshbach two-atom scattering
- L3: S-wave Feshbach resonant superfluidity
- L4: Imbalanced s-wave resonant Fermi gases
- L5: P-wave Feshbach resonant superfluidity

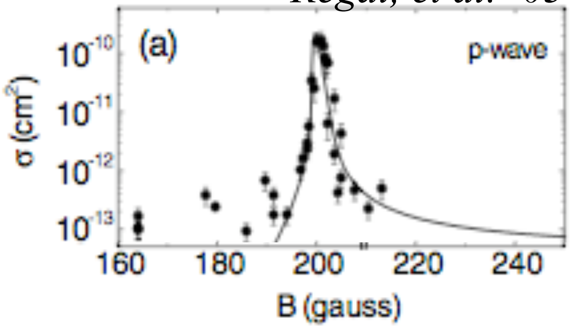
Lecture 5: P-wave Feshbach resonance superfluidity



- motivation and experiments
- review of p-wave scattering theory
- two-channel model
- phases and transitions
- vortices and Majorana modes
- experimental prospects

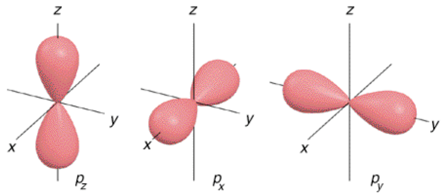
Finite angular momentum superfluidity

Regal, et al. ‘03

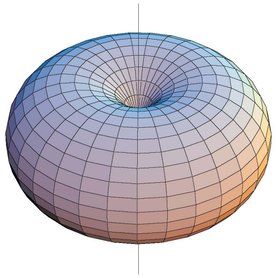


Motivation:

- *p-wave Feshbach resonances exist*
- *examples of ^{3}He and high-Tc superconductors*
- *multiple superfluids phases*

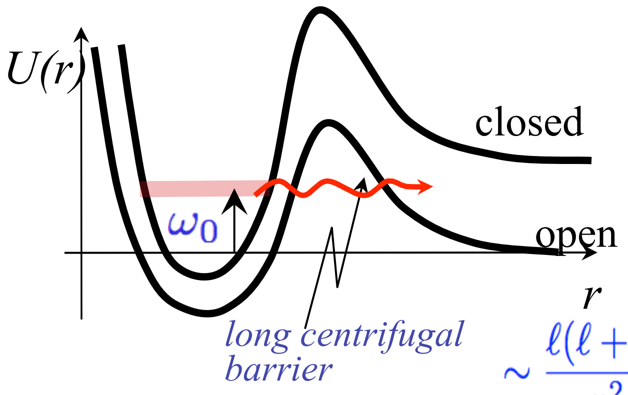


- *anisotropic gap with gapless excitations*

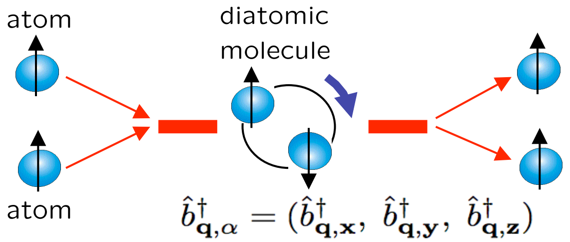


- *conventional (thermal and quantum) and topological phase transitions with detuning*
- *non-Abelian vortex excitations ⇒ topological QC?*

P-wave Feshbach resonant scattering



naturally ***narrow!***



$\hat{b}^\dagger_{\mathbf{q},\alpha} = (\hat{b}^\dagger_{\mathbf{q,x}}, \hat{b}^\dagger_{\mathbf{q,y}}, \hat{b}^\dagger_{\mathbf{q,z}})$

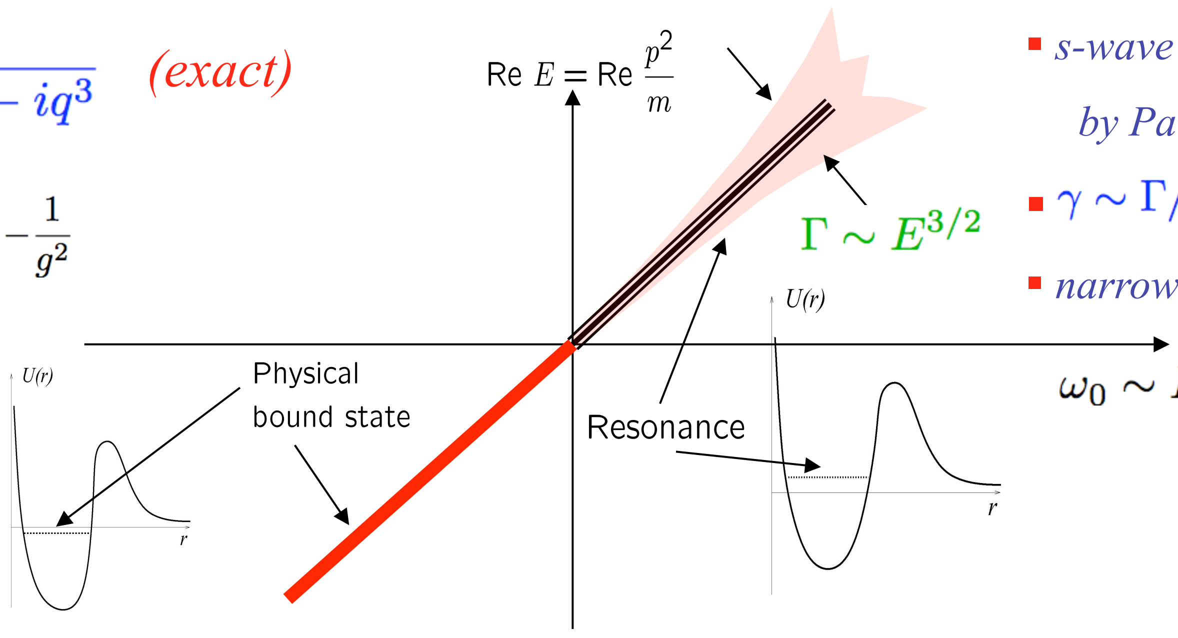
$\sim \frac{\ell(\ell+1)}{r^2}$ ➡ escape (molecular life) time $\tau \sim \Gamma^{-1} \sim E^{-\frac{3}{2}} \gg E^{-1}$, for $E \to 0$

$$\mathcal{H}_{2ch} = \psi^\dagger \frac{\hat{p}^2}{2m}\psi + \vec{\phi}^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\vec{\phi} - ig\vec{\phi}\cdot\psi^\dagger\nabla\psi^\dagger$$

$$f_p = \frac{q^2}{-v^{-1} + \frac{q_0}{2}q^2 - iq^3}$$ *(exact)*

with $v^{-1} \sim -\frac{g^2}{\omega_0}$, $q_0 \sim -\frac{1}{g^2}$

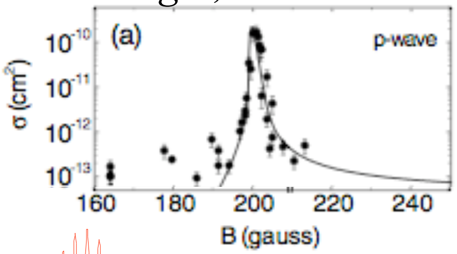
$$f_p(q) = \frac{q^2}{F(q^2) - iq^3}$$



- *s-wave suppressed by Pauli principle*
- $\gamma \sim \Gamma/E \sim E^{\frac{1}{2}} \ll 1$
- *narrows with* ϵ_F, n

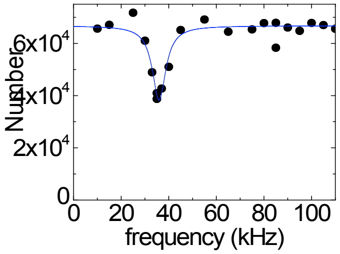
Experimental hopes for p-wave superfluidity

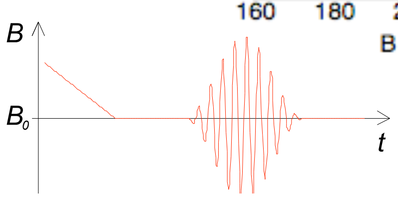
Regal, et al.'03



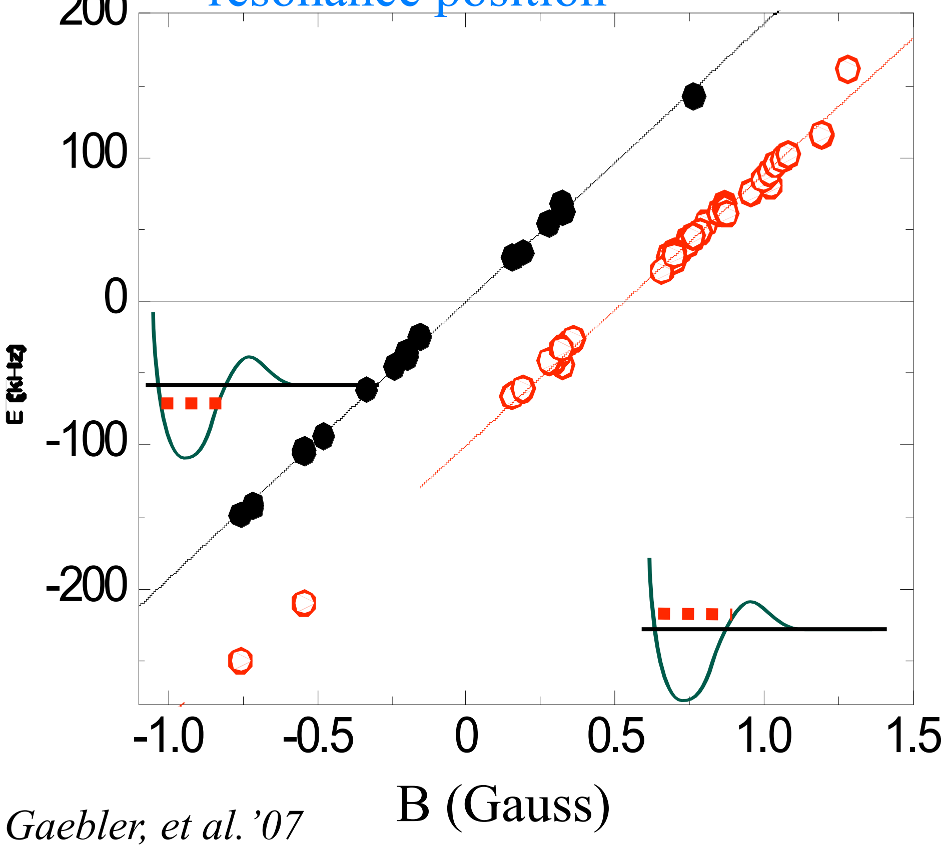
- p-wave Feshbach resonance in ^{40}K, ^{6}Li
- making p-wave molecules:

resonant disappearance of atoms with oscillating B(t)

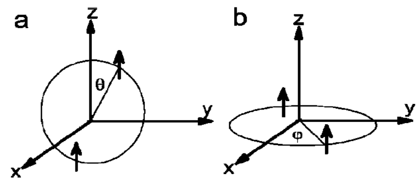




resonance position

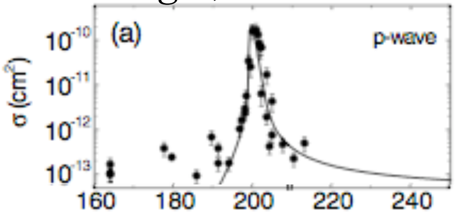


Gaebler, et al.'07



Experimental hopes for p-wave superfluidity

Regal, et al.'03



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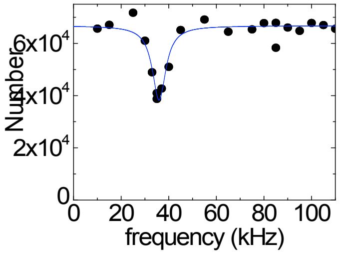
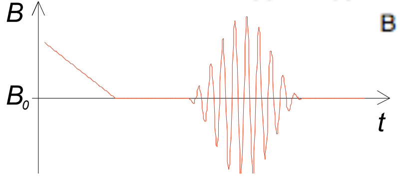
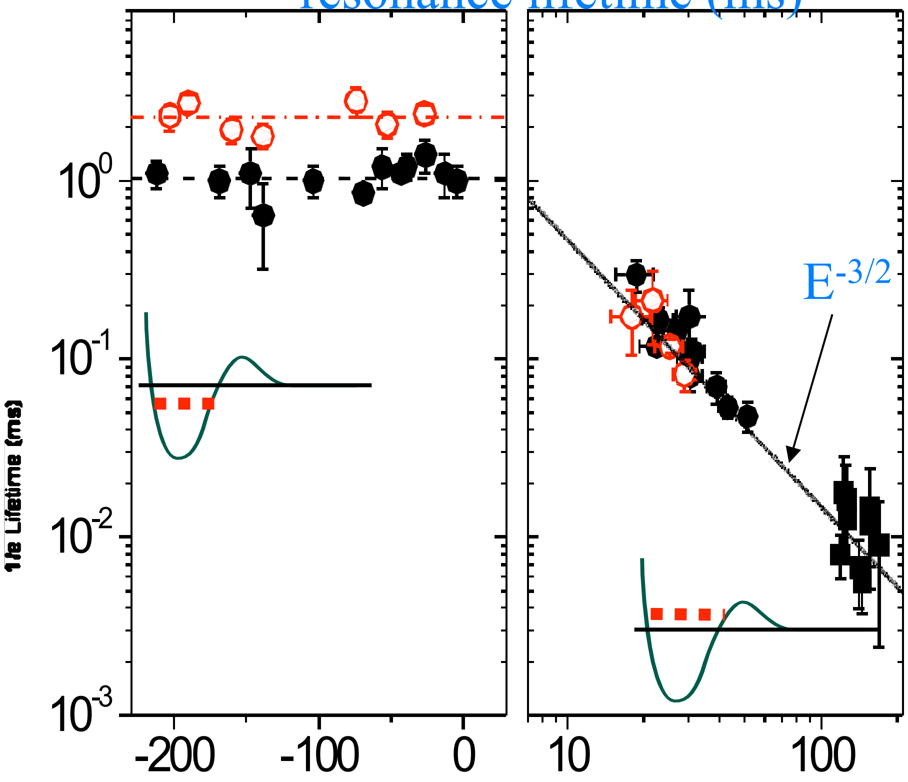
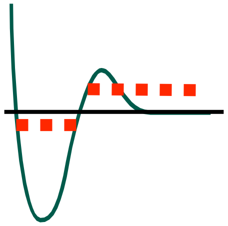




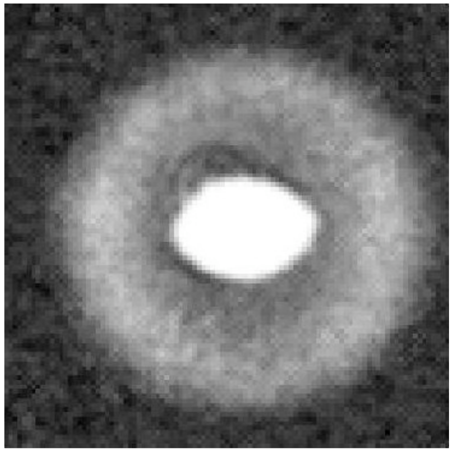
resonance lifetime (ms)



Gaebler, et al.'07



to see molecules:



look for energetic atoms

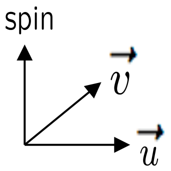
P-wave resonant superfluidity

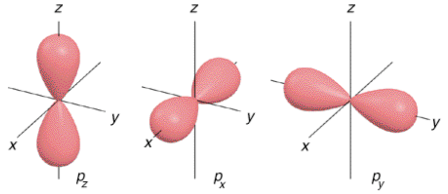
$$\mathcal{H}_{2ch} = \psi^\dagger \frac{\hat{p}^2}{2m}\psi + \vec{\phi}^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\vec{\phi} - ig\vec{\phi}\cdot\psi^\dagger\nabla\psi^\dagger$$

dimensionless coupling: $\gamma \sim \left(\frac{g\sqrt{n}k_F}{\epsilon_F}\right)^2 \sim g^2\epsilon_F^{1/2} \sim \frac{n^{1/3}}{q_0}$

- ***narrow*** resonance $\gamma \ll 1 \;\rightarrow\; \mathrm{MFT}: \vec{\phi}(x) = \vec{B}$

- ***complex vector*** order parameter:





$$\vec{B} = \vec{u} + i\,\vec{v} \;\Longleftrightarrow\; \psi_0 = B_z,\; \psi_\pm = \pm(B_x \pm iB_y)$$

$$\vec{B}\cdot\vec{k} = \sum_{m=0,\pm k}\psi_m Y_{1,m}(\hat{k})k$$

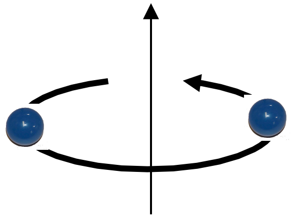
- sample states:

$v = 0 \;\Longleftrightarrow\; |m = 0\rangle$ along $\vec{u}$

$(k_x \quad \beta - state\ in\ {}^3He)$

$u = v \;\Longleftrightarrow\; |m = 1\rangle$ along $\vec{u}\times\vec{v}$

$(k_x + ik_y \quad "axial"\ Anderson - Morel\ state\ in\ {}^3He)$



$L_z = \pm 1, 0$

Mean-field theory $(\gamma \sim g^2 \epsilon_F^{1/2} \ll 1)$

$$H_p = \sum_{\mathbf{k}} \frac{k^2}{2m} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \sum_{\mathbf{q},\alpha} (\frac{q^2}{4m} + \epsilon_{0\alpha}) b^\dagger_{\mathbf{q},\alpha} b_{\mathbf{q},\alpha} + \sum_{\mathbf{k}} [\Delta_{\mathbf{k}}(\vec{B}) a^\dagger_{\mathbf{k}} a^\dagger_{-\mathbf{k}} + h.c.]$$

- *superfluid ground state:*

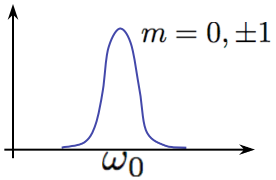
$$\text{molecular BEC } |\vec{B}\rangle \; + \; \text{Cooper pairing } |\text{BCS}_{\vec{\text{B}}}\rangle = \Pi_k (u_{\mathbf{k}} + v_{\mathbf{k}} a^\dagger_{-\mathbf{k}} a^\dagger_{\mathbf{k}})|0\rangle$$

(closed) *(open)*

- *excitation spectrum:* $H_{ex} = \sum_{\mathbf{k}} E^{(a)}_{\mathbf{k}} \alpha^\dagger_{\mathbf{k}} \alpha_{\mathbf{k}} + \sum_{\mathbf{k},\alpha} E^{(m)}_{\mathbf{k},\alpha} \beta^\dagger_{\mathbf{k},\alpha} \beta_{\mathbf{k},\alpha}$

$$E^{(a)}_{\mathbf{k}} = \sqrt{\varepsilon^2_{\mathbf{k}} + \Delta^2_{\mathbf{k}}}, \qquad E^{(m)}_{\mathbf{k},\alpha} = \sqrt{\epsilon^2_{\mathbf{k}} + \mu_\alpha \epsilon_{\mathbf{k}}} \qquad \textit{with gap}: \; \Delta_k = 2g|\vec{B}\cdot\vec{k}|$$

- $\vec{B}, \;\; n_b, \;\; , n_a, \;\; \mu$ *determined by :*
  - energy minimization (gap equation) $\rightarrow \dfrac{\partial E(\vec{B})}{\partial B_\alpha} = 0$
  - atom number equation $\rightarrow 2n_b + n_a = n$



Isotropic resonance at T=0

$(\omega_\alpha = \omega_0)$

$$E = (u^2+v^2)\,[\omega_0 - 2\mu + \underbrace{a_1 \ln\{a_0(u+v)\}] + a_1\frac{u^3+v^3}{u+v}}_{\text{BCS}} + \underbrace{a_2\left[\left(u^2+v^2\right)^2 + \frac{1}{2}\left(u^2-v^2\right)^2\right]}_{\text{BEC}:\ (\vec{B}^*\cdot\vec{B})^2 + \frac{1}{2}|\vec{B}\cdot\vec{B}|^2}$$

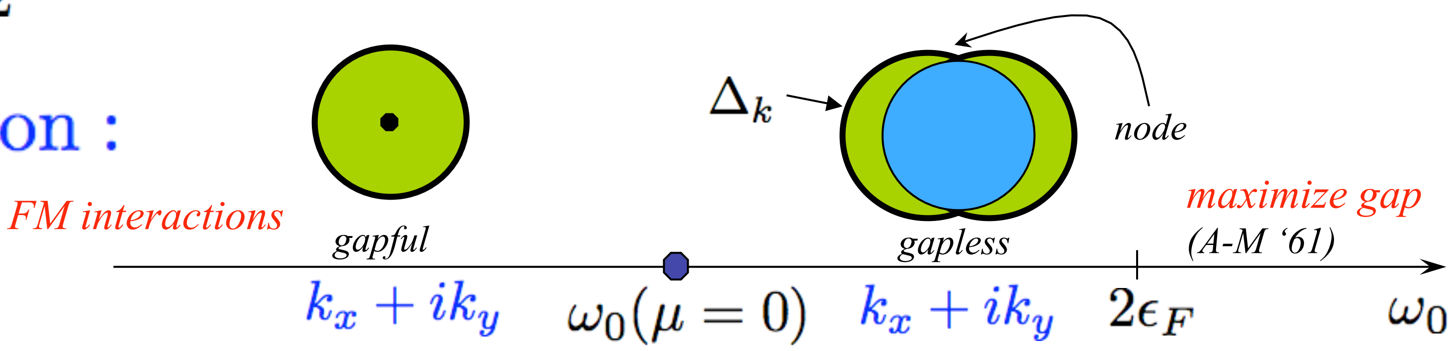
- BCS $(\omega_0 \gg 2\epsilon_F)$:
  - ➢ $\mu \approx \epsilon_F + \mathcal{O}(\gamma)$
  - ➢ $\dfrac{E_{k_x+ik_y}}{E_{k_x}} = \dfrac{1}{2}e > 1$

$\Longrightarrow$ $(Anderson - Morel\ A_1\ phase)$

$u = v \sim e^{-(\omega_0 - 2\epsilon_F)/\gamma\epsilon_F} \Longrightarrow k_x + ik_y\ (m = 1)$

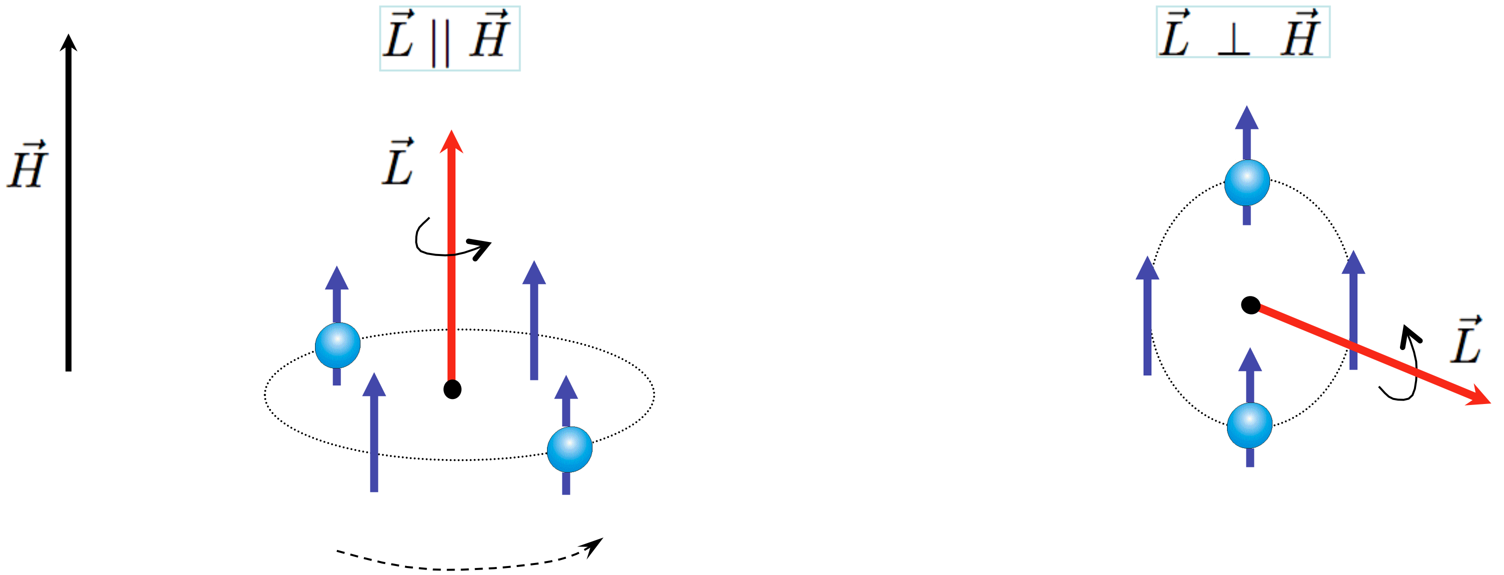
- BEC $(\omega \ll 2\epsilon_F)$:
  - ➢ $\mu \approx \dfrac{1}{2}\omega_0 + \mathcal{O}(\gamma)$ $\Longrightarrow$ $u = v \approx \sqrt{n - n(\omega_0/2)} \Longrightarrow k_x + ik_y\ (m = 1)$

- transition :

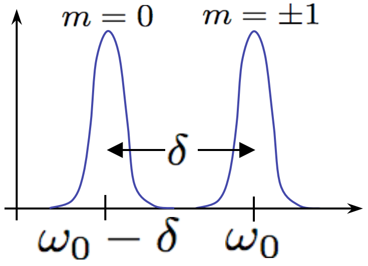


Dipolar-interaction FR splitting

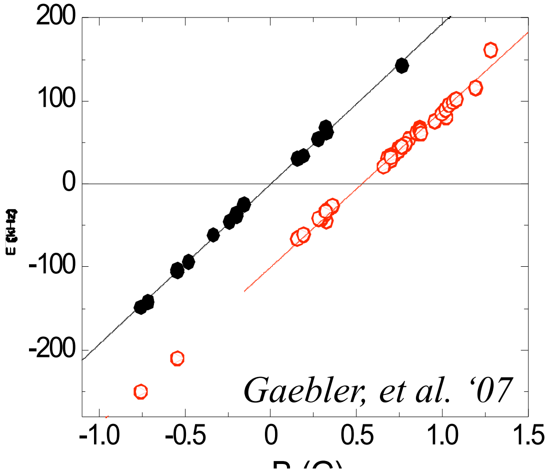
Ticknor, et al '03



$$H_{\text{molecule}} = \omega_{||} b^{\dagger}_{||} b_{||} + \omega_{\perp} \vec{b}^{\dagger}_{\perp} \cdot \vec{b}_{\perp} \qquad \omega_{||} < \omega_{\perp}$$



Ticknor, Regal, et al. '03



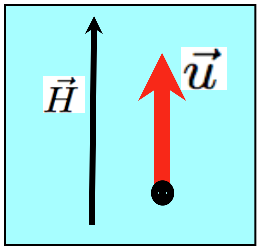
Gaebler, et al. '07

$$E = \sum_{\alpha} (u^2_{\alpha} + v^2_{\alpha})[\omega_{\alpha} - 2\mu + \ldots$$

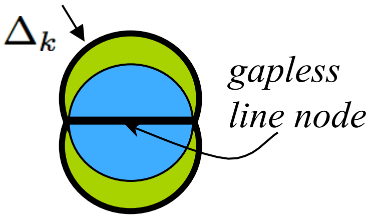
P-wave superfluid phases

$$E = \sum_\alpha (u_\alpha^2+v_\alpha^2)\left[\omega_{0\alpha} - 2\mu + a_1 \ln\{a_0(u+v)\}\right] + a_1 \frac{u^3+v^3}{u+v} + a_2\left[\left(u^2+v^2\right)^2 + \frac{1}{2}\left(u^2-v^2\right)^2\right]$$

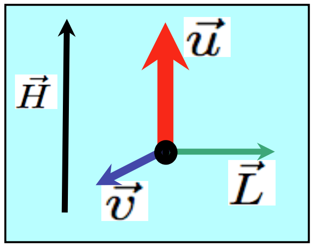
- k_x - *state:* *β-phase of* ^3He ($m_{||}=0$)



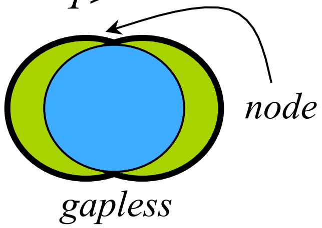
  - $u \approx u_0 e^{\delta}$, $v = 0$
  - equatorial node line for $\mu > 0 \Rightarrow C \sim T^{\alpha}$, …
  - fully gapped for $\mu < 0 \Rightarrow C \sim e^{-|B-B_0|^2/T}$, …
  - spontaneously broken symmetries: *U(1)*



- $k_x + i\,\sigma\, k_y$- *state:* *“deformed” A_1 -phase of* ^3He ($m_\perp=1$)



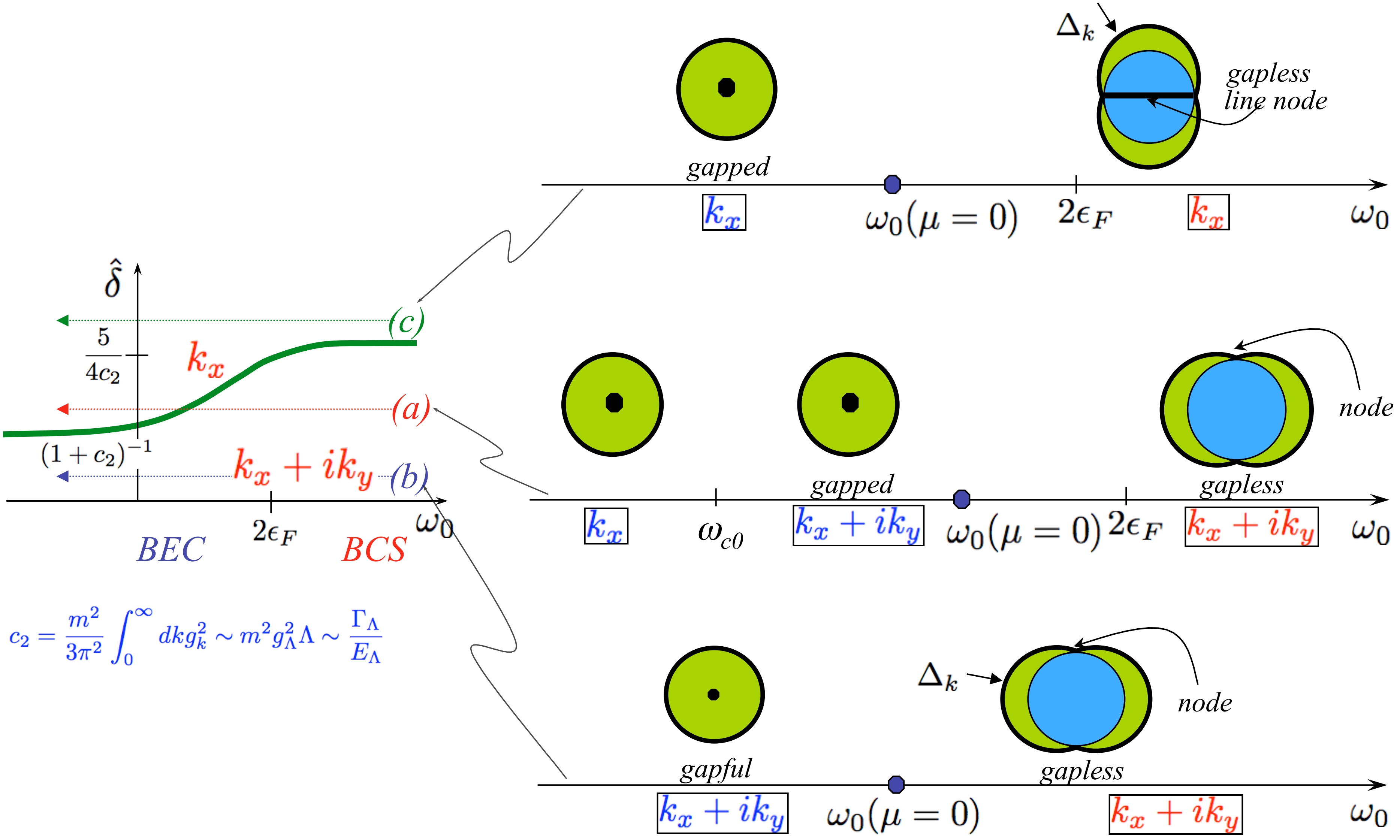
  - $u \approx u_0 (1+\delta)\, e^{\delta/2}$, $v \approx u_0 (1-\delta)\, e^{\delta/2} \Rightarrow |m_\perp = 1> + \delta\, |m_\perp = -1>$
  - polar point nodes for $\mu > 0 \Rightarrow C \sim T^{\alpha}$, …
  - fully gapped for $\mu < 0 \Rightarrow C \sim e^{-|B-B_0|^2/T}$, …
  - spontaneously broken symmetries: *U(1), O(2), T*



T=0 phase diagrams

also see Cheng, Yip '05

$$E=\sum_{\alpha}(u_{\alpha}^{2}+v_{\alpha}^{2})\left[\omega_{0\alpha}-2\mu+a_{1}\ln\{a_{0}\,(u+v)\}\right]+a_{1}\frac{u^{3}+v^{3}}{u+v}+a_{2}\left[\left(u^{2}+v^{2}\right)^{2}+\frac{1}{2}\left(u^{2}-v^{2}\right)^{2}\right]$$

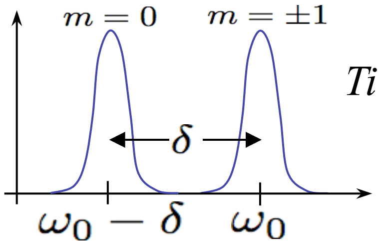


$$c_2=\frac{m^2}{3\pi^2}\int_0^\infty dk g_k^2\sim m^2g_\Lambda^2\Lambda\sim\frac{\Gamma_\Lambda}{E_\Lambda}$$

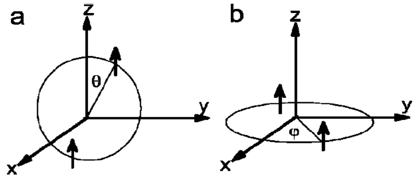
Anisotropic p-wave superfluidity

Gurarie, L.R., Andreev '05
Cheng and Yip '05

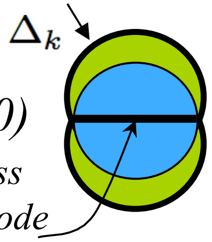
- *resonance splits:*

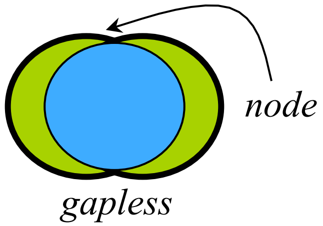


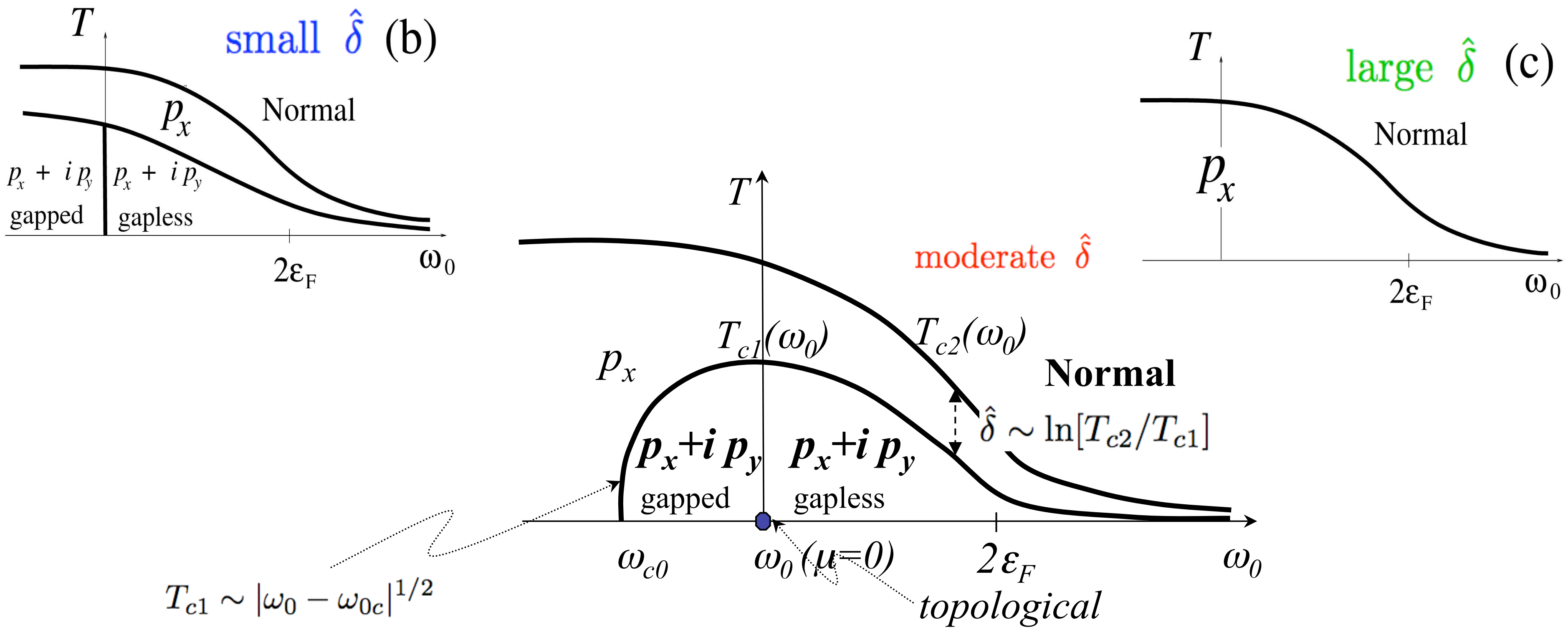
Ticknor, Regal, et al.



- *two competing states:* p_x $(m_z=0)$ and $p_x+i\,p_y$ $(m_z=\pm1)$

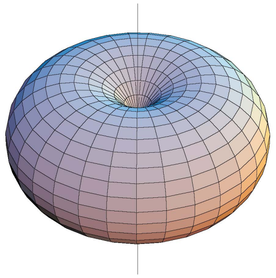


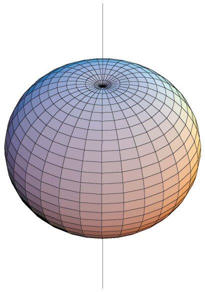




Gapless ⟷ gapped superfluid transitions

$p_x + i\,p_y$ $\xrightarrow{\mu \text{ changes sign}}$ $p_x + i\,p_y$





$$E_p = \sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + g^2\,(p_x^2 + p_y^2)\,|B|^2}$$

$$E_p = \sqrt{\left(\frac{p^2}{2m} + |\mu|\right)^2 + g^2\,(p_x^2 + p_y^2)\,|B|^2}$$

G. E. Volovik, JETP Lett. 80, 343 (2004)

p_x $\xrightarrow{\mu \text{ changes sign}}$ p_x

$$E_p = \sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + g^2\,p_x^2\,|B|^2}$$

$$E_p = \sqrt{\left(\frac{p^2}{2m} + |\mu|\right)^2 + g^2\,p_x^2\,|B|^2}$$

$p_x + i\, p_y$ superfluid in 2D

- Pfaffian (Moore-Read) state from FQH $$|p_x + ip_{y\,BCS}\rangle = \prod_p \left[u_p + v_p a^\dagger_{-p} a^\dagger_p\right]|0\rangle$$

$$\Psi\left(z_1, z_2, \ldots, z_{2N}\right) = \sum_P (-1)^P \frac{1}{z_{P_1} - z_{P_2}} \frac{1}{z_{P_3} - z_{P_4}} \cdots \frac{1}{z_{P_{N-1}} - z_{P_N}}$$

- topological classification in terms of u_p and v_p

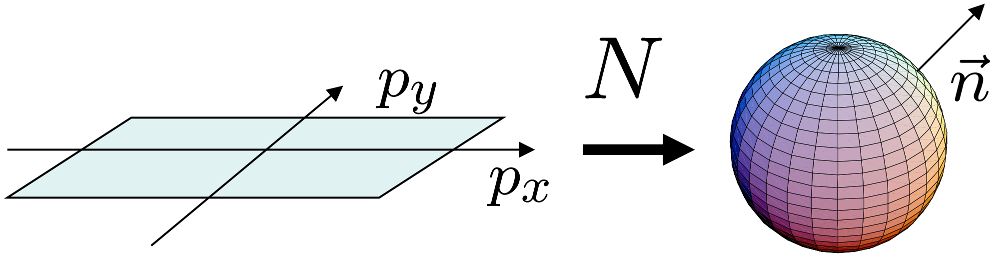
Anderson's pseudospin $$\begin{cases} n_x + i n_y = 2v^* u \\ n_z = |v|^2 - |u|^2 \end{cases}$$

$$\vec{n} = \frac{1}{\sqrt{\left(\frac{p^2}{2m_a} - \mu\right)^2 + 4B^2\left(p_x^2 + p_y^2\right)}} \begin{pmatrix} 2gBp_x \\ -2gBp_y \\ \frac{p^2}{2m_a} - \mu \end{pmatrix}$$

Explicit calculations show that

N=*0* if $\mu < 0$

N=*1* if $\mu > 0$



$$N = \frac{1}{8\pi} \int d^2p \;\left[\vec{n} \cdot \partial_\alpha \vec{n} \times \partial_\beta \vec{n}\;\, \epsilon_{\alpha\beta}\right]$$

topological invariant

$p_x + i\, p_y$ superfluid in 2D

- Pfaffian (Moore-Read) state from FQH

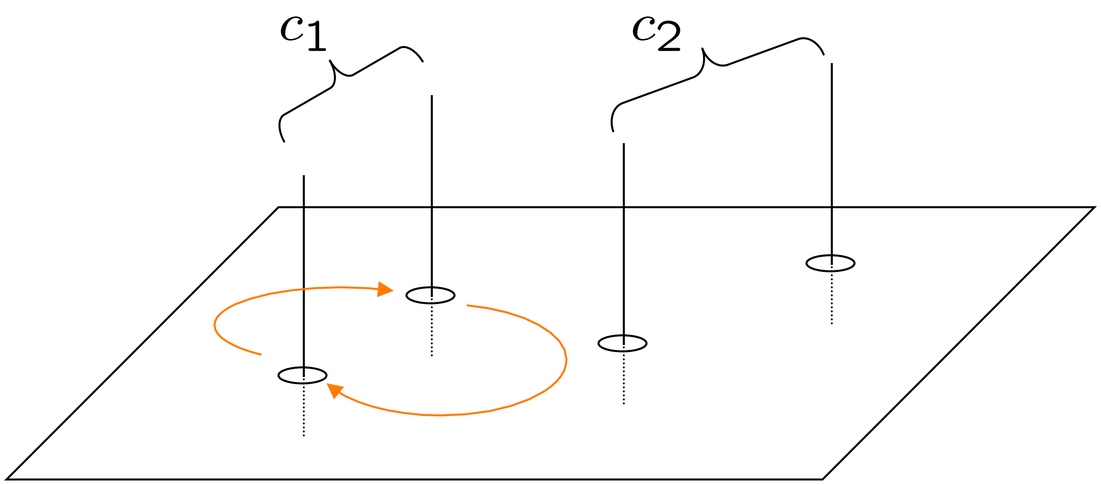
$$|p_x + ip_{y\,BCS}\rangle = \prod_p \left[u_p + v_p a^\dagger_{-p} a^\dagger_p\right] |0\rangle$$

$$\Psi\left(z_1, z_2, \ldots, z_{2N}\right) = \sum_P (-1)^P \frac{1}{z_{P_1} - z_{P_2}} \frac{1}{z_{P_3} - z_{P_4}} \cdots \frac{1}{z_{P_{N-1}} - z_{P_N}}$$

- topological classification in terms of u_p and v_p
- gapped (N=1, BCS) ⇒ gapped (N=0, BEC) superfluid transition at μ=0

Read and Green, PRB 61, 10267 (2000)

- vortex excitations with non-Abelian statistics *Ivanov, PRL (2001)*



one fermion (2 states –either empty or occupied fermion) per two vortices

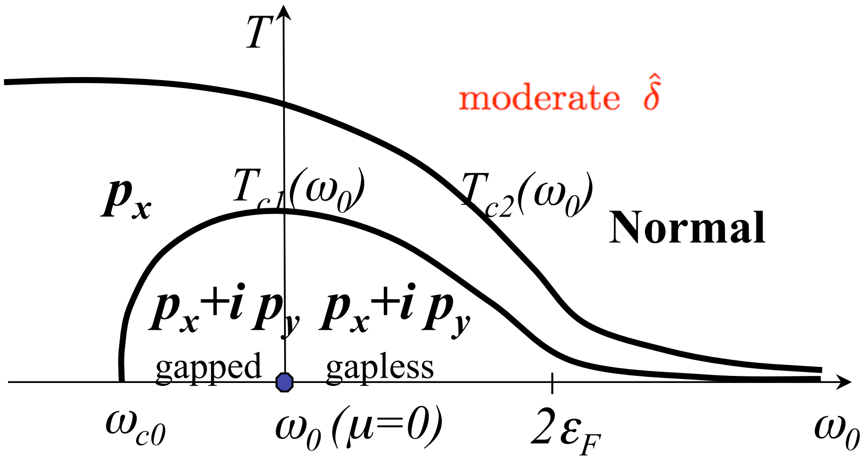
$2^{\frac{n}{2}}$ states per *n* vortices

- suggested to be used as qubits for quantum computers

Kitaev, Ann. Phys. 303, 2 (2003)

Summary of p-wave superfluidity

- mapped out T, $\omega_0 \propto B$, δ phase diagram for p-wave Feshbach resonant Fermi gas
    - *p_x and $p_x + i\,p_y$ superfluids*
    - *thermal, quantum and topological SF => SF transitions*



- quantitatively accurate description for small $\gamma=\Gamma/\varepsilon_F$ (low n)
- realization of topological states, majorana zero modes, and non-Abelian statistics of Pfaffian (Moore-Read) state
- p-wave Feshbach molecules observed in K^{40}
- …BUT
    - ❖ *short (msec) molecular lifetime (see Levinson, et al, PRL 2007)*
    - ❖ *what about Li^6*
    - ❖ *need better quantitative understanding of stability*