Resonant atomic gases

Leo Radzihovsky

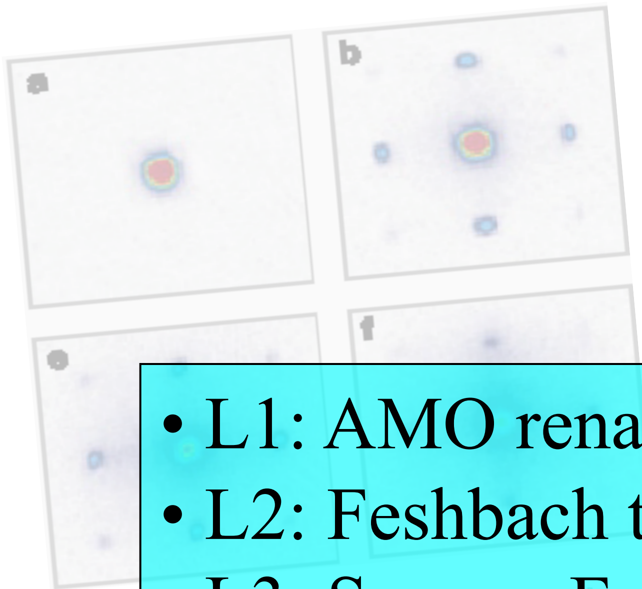
for details see: *Gurarie, L.R., Annals of Physics, 322, 2-119 (2007)*
Sheehy, L.R., Annals of Physics, 322, 1790 (2007)

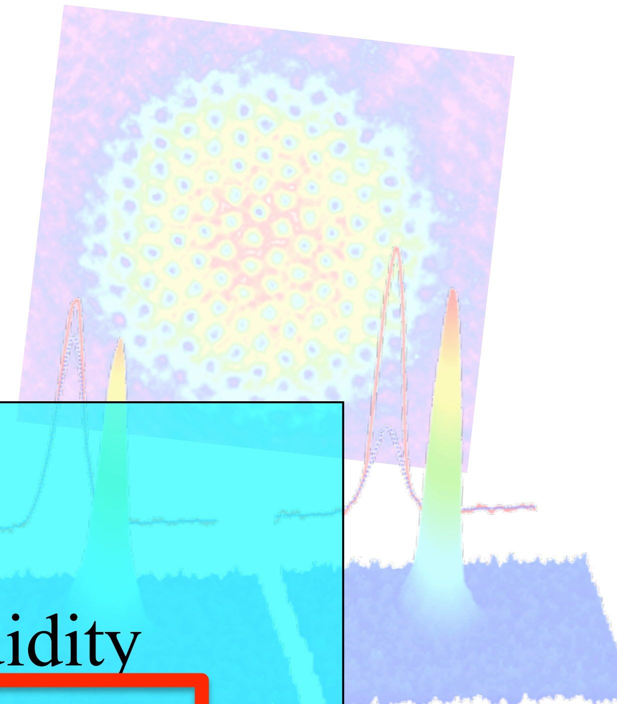
Giorgini, et al., RMP, 80, 885 (2008)
Ketterle and Zwierlein, Varenna lectures (2006)

$: NSF

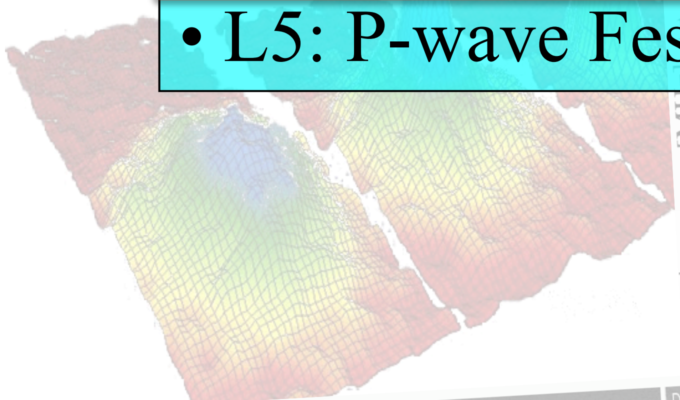
Mysore, India, Dec 2010

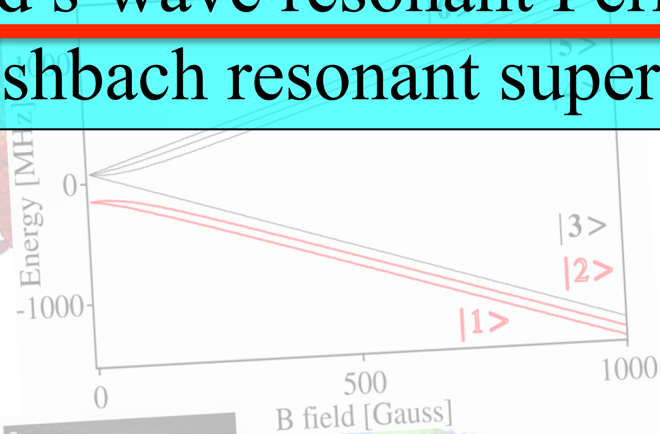
Course outline

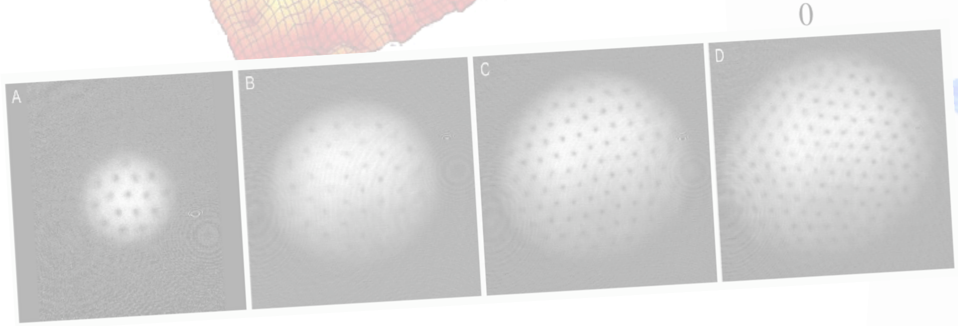


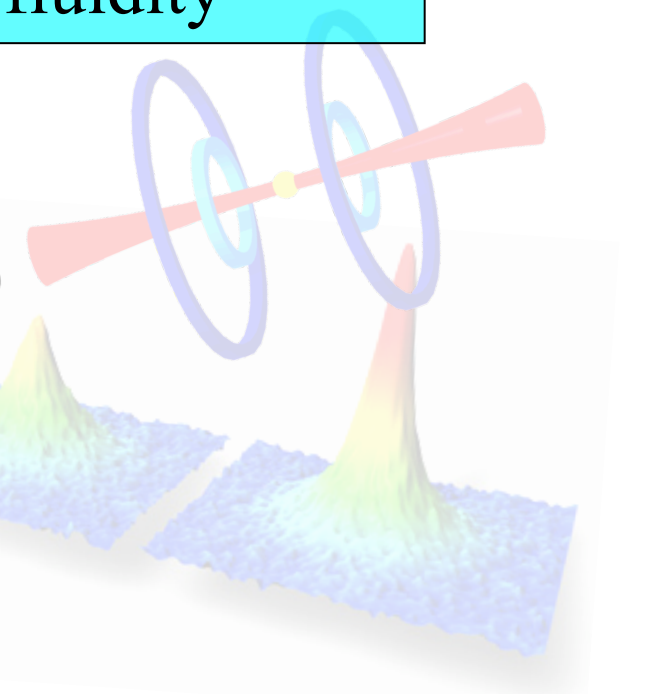


- L1: AMO renaissance overview
- L2: Feshbach two-atom scattering
- L3: S-wave Feshbach resonant superfluidity
- **L4: Imbalanced s-wave resonant Fermi gases**
- L5: P-wave Feshbach resonant superfluidity



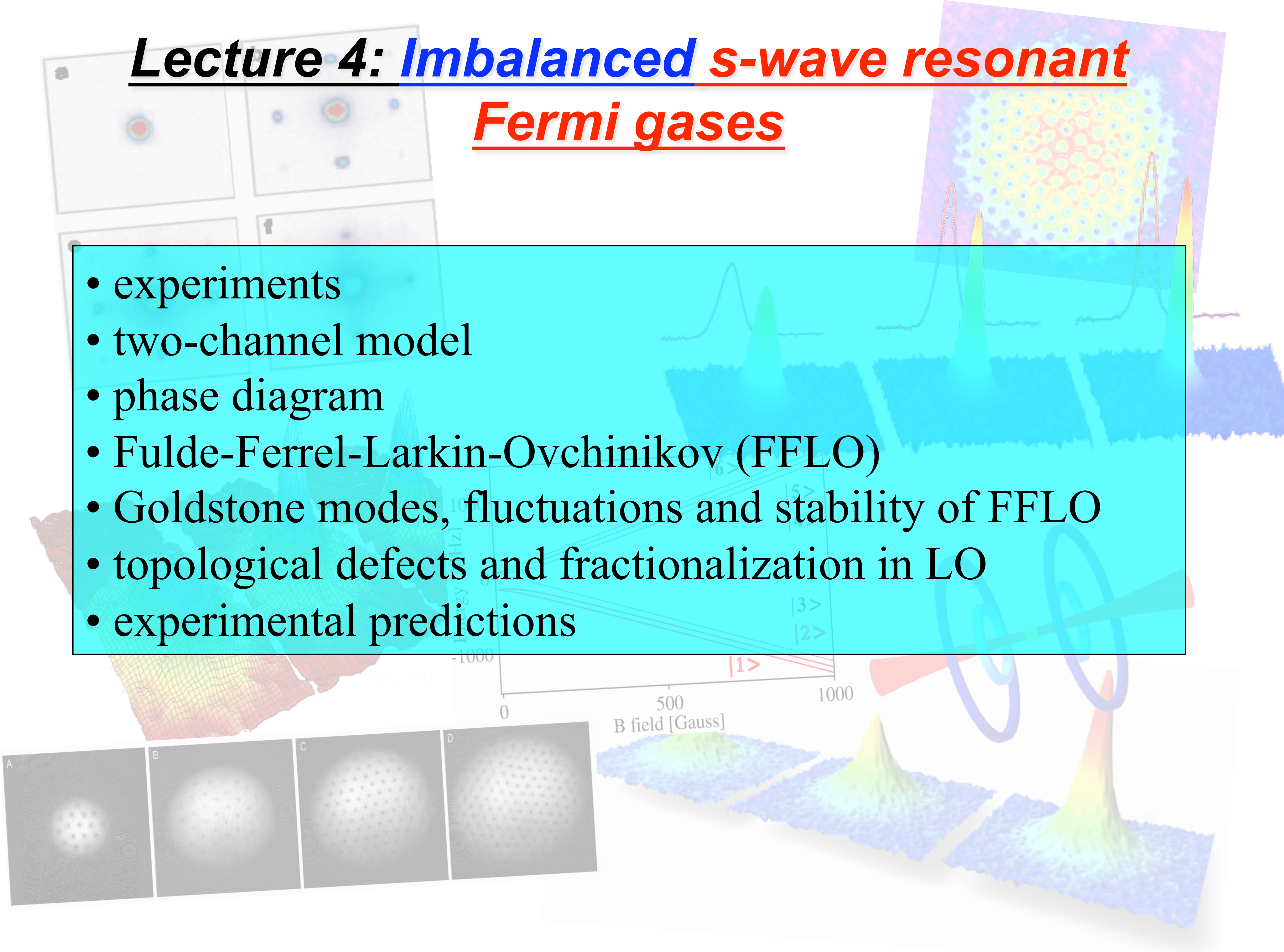






Lecture 4: Imbalanced s-wave resonant Fermi gases

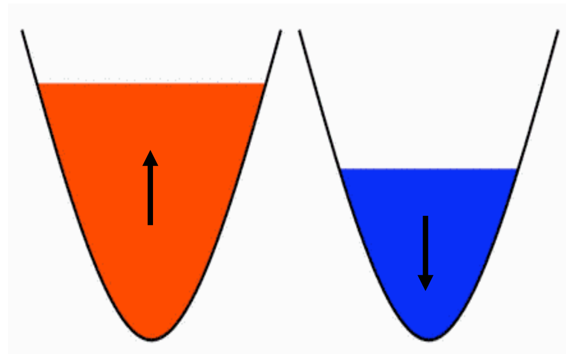
- experiments
- two-channel model
- phase diagram
- Fulde-Ferrel-Larkin-Ovchinikov (FFLO)
- Goldstone modes, fluctuations and stability of FFLO
- topological defects and fractionalization in LO
- experimental predictions



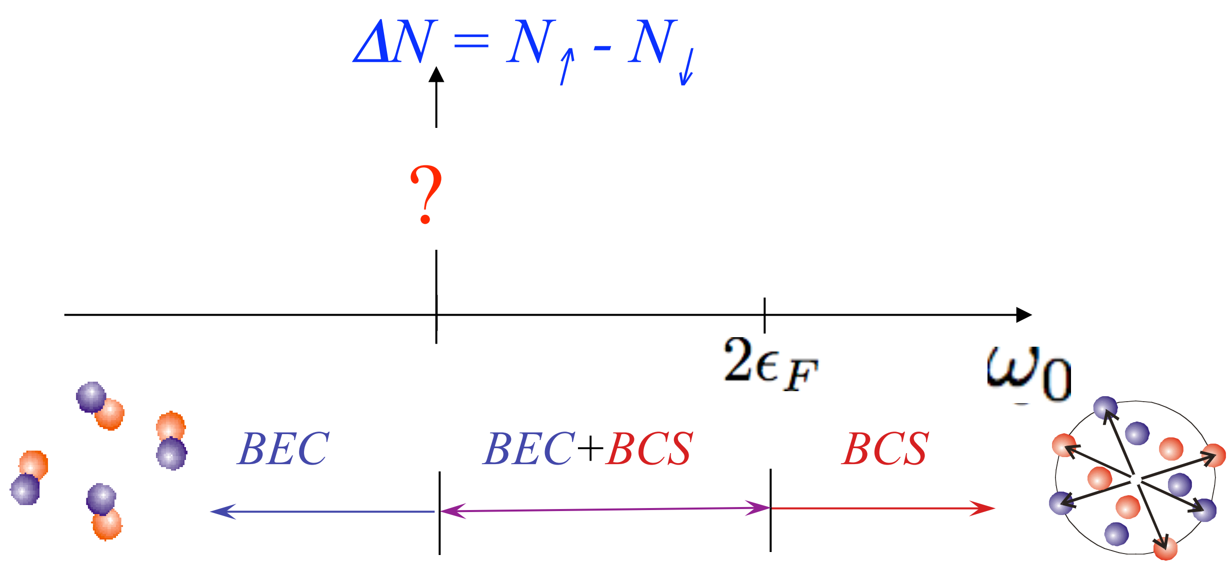
Imbalanced ("magnetized") BEC-BCS

- motivation: *superconductivity in B field, quarks-gluon plasma,…*
- natural realization in cold atoms: $H_h = H - h(N_\uparrow - N_\downarrow)$

$$\mathcal{H} = \psi_\sigma^\dagger\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_\sigma + \lambda\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$$



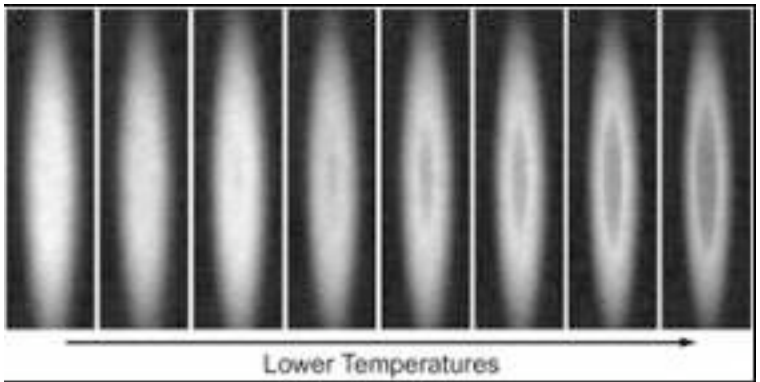
$$n = \psi_\uparrow^\dagger\psi_\uparrow + \psi_\downarrow^\dagger\psi_\downarrow, \quad \Delta n = \psi_\uparrow^\dagger\psi_\uparrow - \psi_\downarrow^\dagger\psi_\downarrow$$

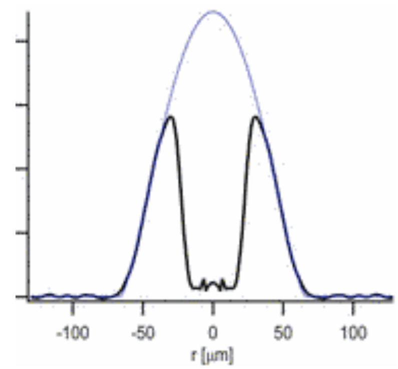


Imbalanced BEC-BCS experiments

• Ketterle's experiments (vortices, phase separation)

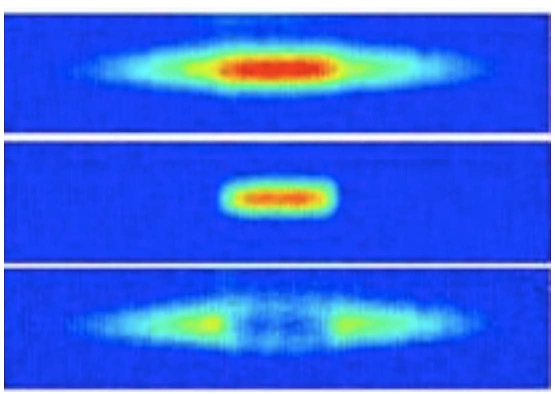
Science (2006)

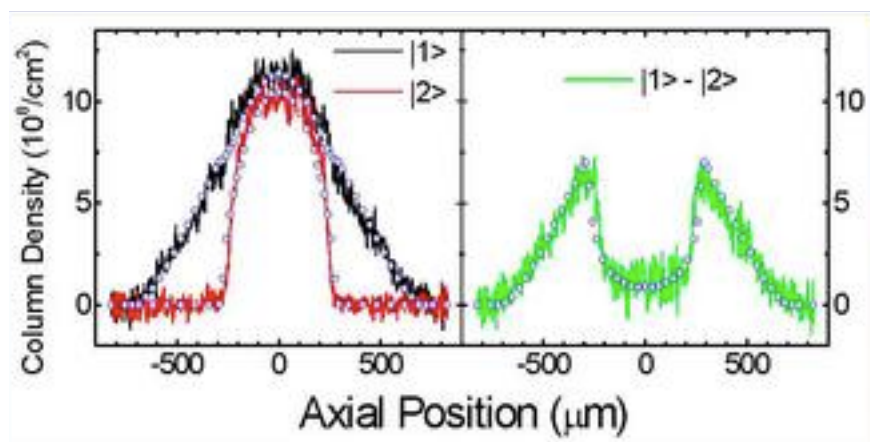




• Hulet's experiments (phase separation, surface tension)

Science (2006)

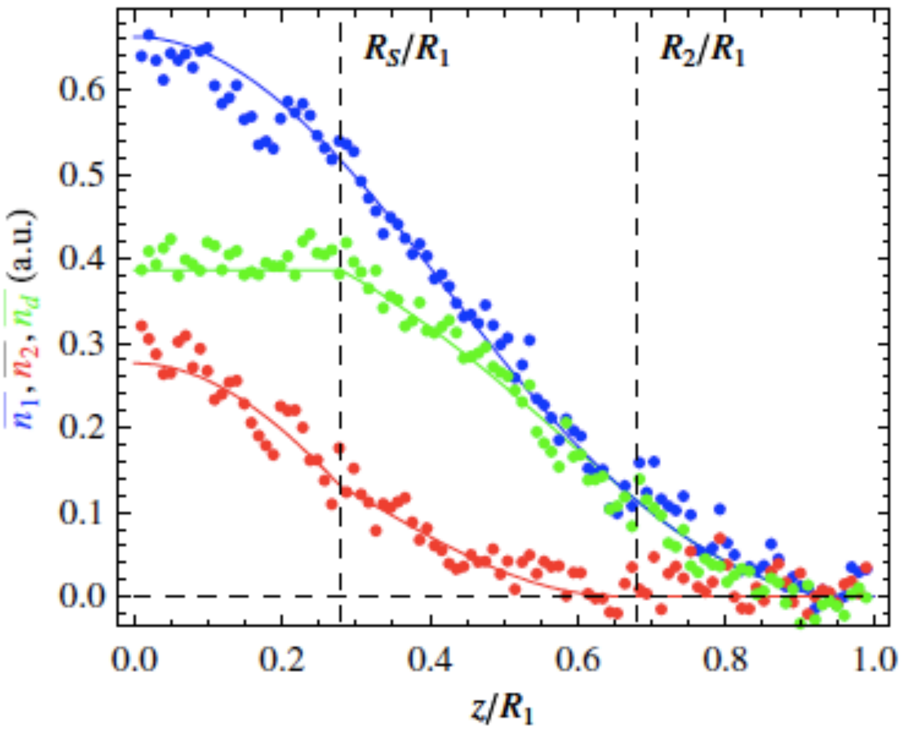


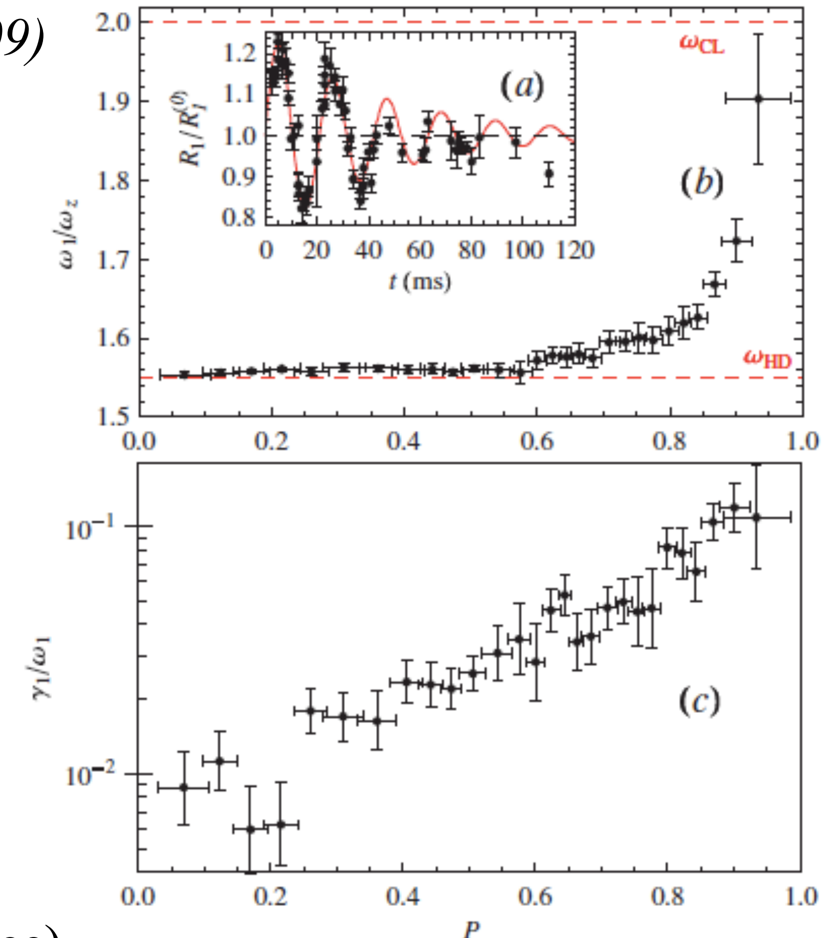


Imbalanced BEC-BCS experiments

- Salomon's experiments (phase separation, oscillations)

(PRL 2009)





- N = 10^4, axial trap with 20:1 anisotropy (cf Rice)
- superfluid core disappears at P_{c2}= 0.76 (cf MIT)
- LDA works (cf MIT)
- no visible surface tension effects (cf MIT)

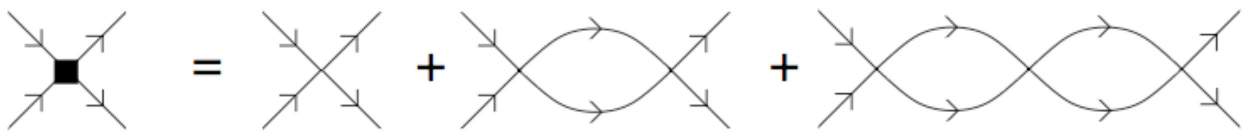
$\gamma >> 1$

Broad resonance scattering

$$\mathcal{H}_{2ch} \rightarrow \mathcal{H}_{1ch} = \psi_\sigma^\dagger\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_\sigma + \lambda\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$$

- scattering T-matrix relates λ to a:

$T_{k\,k'} =$

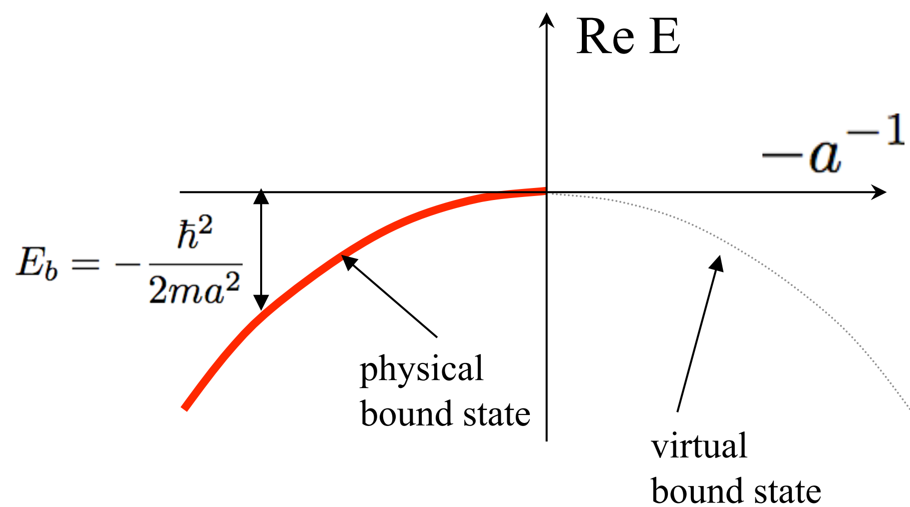


$$+ \ldots = \frac{\lambda}{1-\lambda\Pi}$$

$$= -\frac{4\pi\hbar^2}{m} f_{kk'} \approx \frac{4\pi\hbar^2}{m}\frac{1}{a^{-1}+ik}$$

$$\rightarrow \boxed{a = \frac{m}{4\pi\hbar^2}\frac{\lambda}{1+\lambda/\lambda_c}}$$

$(\lambda_c = \pi d/m)$



$b_q = B_Q \,\delta_{q,Q}$

Mean-field theory

(valid for $\gamma \sim g^2/\epsilon_F^{1/2} \ll 1$ *)*

$$H_{\mu,h} = H - \mu N - h\Delta N$$

$N = N_{a\uparrow} + N_{a\downarrow} + 2\,N_b$

$\Delta N = N_{a\uparrow} - N_{a\downarrow}$

- ***ground state:*** $|gs\rangle = \Pi'_{\mathbf{k}}\left(u_{\mathbf{k},\mathbf{Q}} + v_{\mathbf{k},\mathbf{Q}}\, a^{\dagger}_{-\mathbf{k}+\mathbf{Q}/2,\downarrow}\, a^{\dagger}_{\mathbf{k}+\mathbf{Q}/2,\uparrow}\right)|0\rangle$

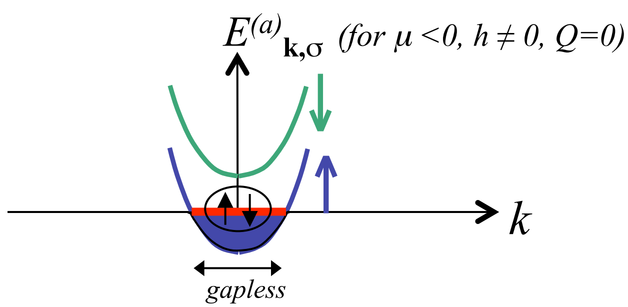
- ***ground state energy:*** $E_{gs} = \left(\frac{Q^2}{4m} + \delta - 2\mu\right)B_Q^2 - \sum_{\mathbf{k}}(E_k - \varepsilon_k) + \sum_{\mathbf{k}}\left[E_{\mathbf{k},\uparrow}\theta(-E_{\mathbf{k},\uparrow}) + E_{\mathbf{k},\downarrow}\theta(-E_{\mathbf{k},\downarrow})\right]$

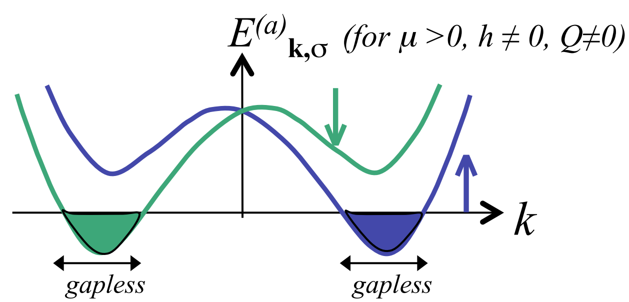
$$E_k = (\varepsilon_k^2 + g^2 B_Q^2)^{1/2}, \quad \varepsilon_k = \frac{k^2}{2m} - \mu + \frac{Q^2}{8m}$$

- ***excitation spectrum:*** $H_{ex} = \sum'_{\mathbf{k},\sigma} E^{(a)}_{\mathbf{k},\sigma}\,\alpha^{\dagger}_{\mathbf{k},\sigma}\alpha_{\mathbf{k},\sigma} + \sum'_{\mathbf{k},\sigma} E^{(b)}_{\mathbf{k},\sigma}\,\beta^{\dagger}_{\mathbf{k}}\beta_{\mathbf{k}}$

$$E^{(a)}_{\mathbf{k},\sigma} = E_k \mp (h + \mathbf{k}\cdot\mathbf{Q}/2m), \qquad E^{(b)}_k = \sqrt{\epsilon_k^2 + V_0\epsilon_k} \quad (\text{for } \; Q = 0)$$

(gapped and gapless k's) *(gapless k's collective; also phonons* $Q \neq 0$*))*





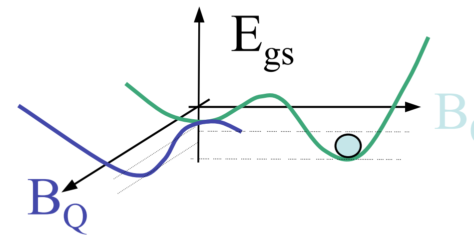
- ***determine*** $\boldsymbol{B_Q, N_b, N_{a\uparrow}, N_{a\downarrow}\ (\Delta N_a), Q}$ ***by:***

energy minimization ⇨ $\dfrac{\partial E_{gs}}{\partial B_Q} = 0$ *(gap equation),* $\dfrac{\partial E_{gs}}{\partial \mathbf{Q}} = 0$ *(*$P_{total}=0$*)*

μ, h fixed

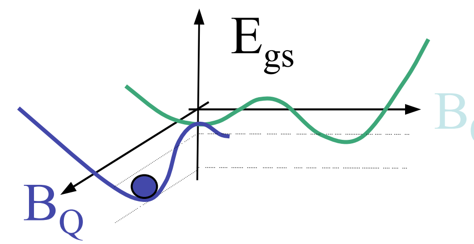
BCS and crossover regimes ($\delta > 0$)

- **BCS SF** $B_0 \neq 0$, $B_Q = 0$, $\Delta N = 0$: $0 < h < h_c = \frac{1}{\sqrt{2}}\, g\, B_0(\mu,\delta)$

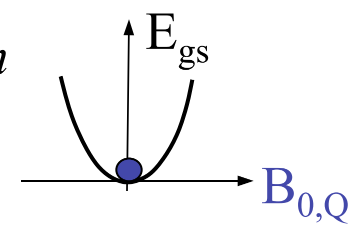


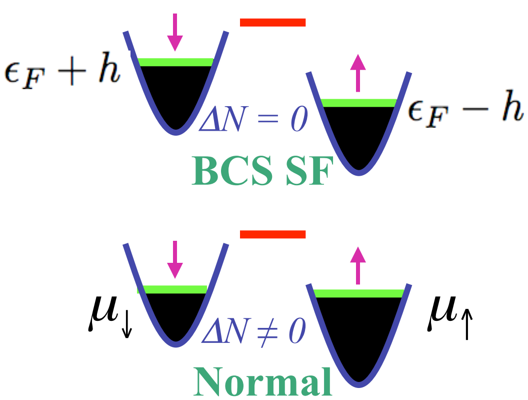
- **FFLO** $B_0 = 0$, $B_Q \neq 0$, $\Delta N \neq 0$: $h_c < h < h_{FFLO}(\delta) \xrightarrow{\delta \gg 2\epsilon_F} 1.1\, h_c$ *(Fulde-Ferrell)*; $\xrightarrow{\delta \approx 2\epsilon_F} h_c$

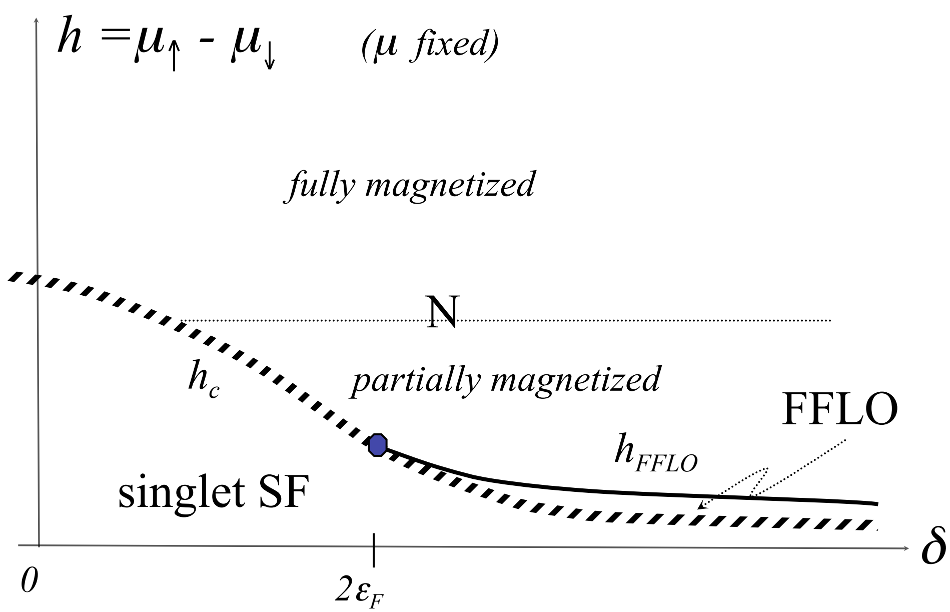
"supersolid": *broken rotational and translational symmetry*



- **Normal** $B_0 = 0$, $B_Q = 0$, $\Delta N \neq 0$ *(Pauli "paramagnet")*: $h_{FFLO}(\delta) < h$

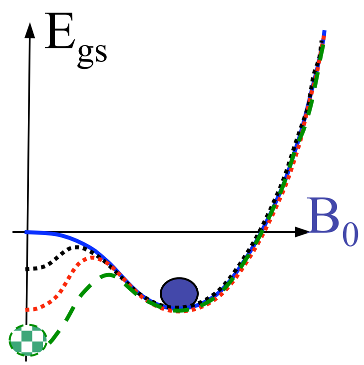






Phase separation $(\delta > 0)$

***N*, h fixed**

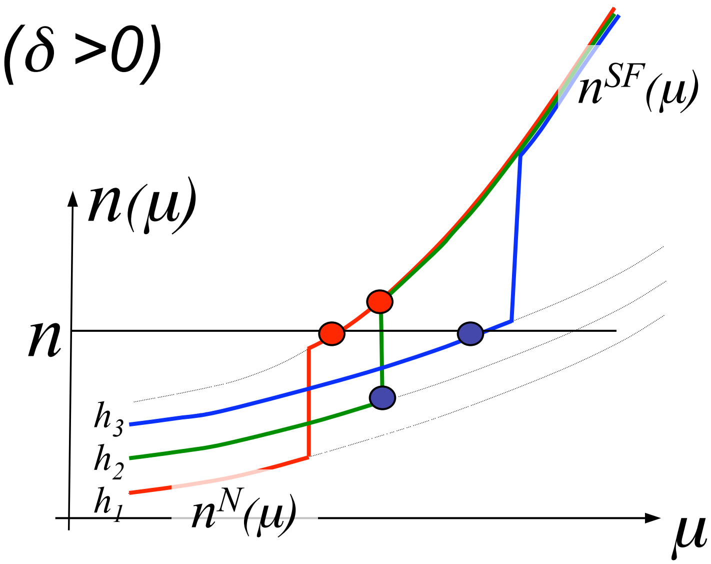


$$h_c$$

$$h_1 < h_{c1}(\delta) < h_2 < h_{c2}(\delta) < h_3$$

SF N

$$n = (1-x)n^{SF} + x\, n^N$$



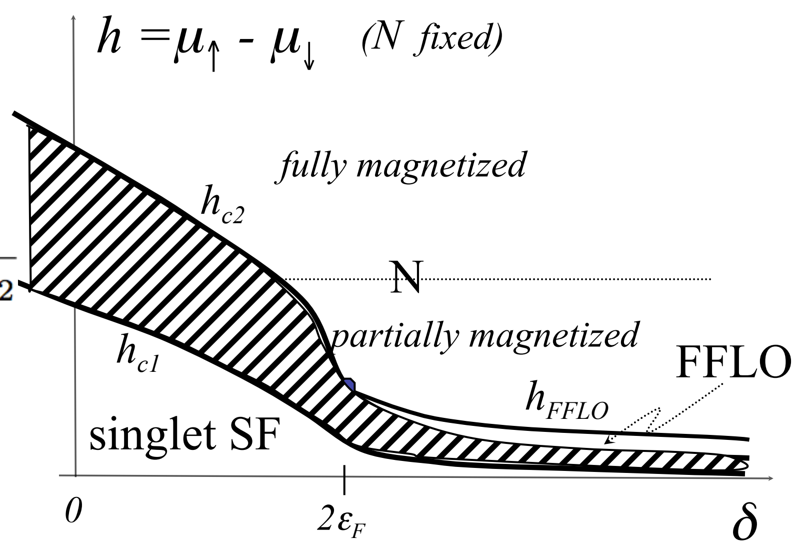
phase separation $x(h,\delta) = (n^{SF} - n)/(n^{SF} - n^N)$

$$h_{c2}(\delta) = h_c(\mu^{(FFLO)}(N,\delta),\delta) \qquad h_{c1}(\delta) = h_c(\mu^{(SF)}(N,\delta),\delta)$$

$0 < \delta \ll 2\epsilon_F$

$$h_{c2}(\delta) \approx 2^{3/2}\epsilon_F - \delta/2$$

$$h_{c1}(\delta) \approx \epsilon_F \sqrt{\frac{\gamma}{3}}\sqrt{1-(\delta/2\epsilon_F)^{3/2}}$$



$2\epsilon_F \ll \delta$

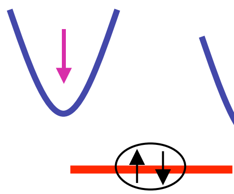
$$h_{FFLO}(\delta) \longrightarrow 1.1 h_c$$

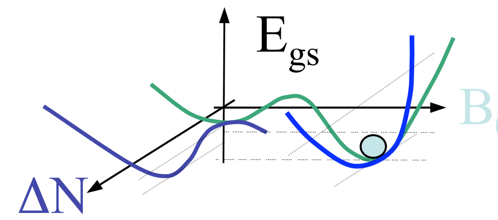
$$\left.\begin{array}{l} h_{c2}(\delta) \\ h_{c1}(\delta) \end{array}\right\} \rightarrow \frac{1}{\sqrt{2}}\Delta_{BCS}$$

BEC regime ($\delta < 0$)

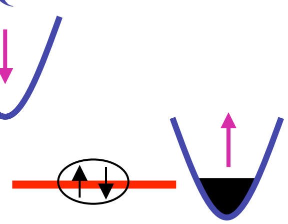
***N*, h fixed**

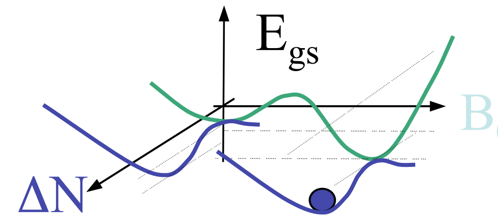
- ***BEC SF*** $B_0 \neq 0$, $B_Q = 0$, $\Delta N = 0$: $0 < h < h_m(\delta) \approx -\delta/2$



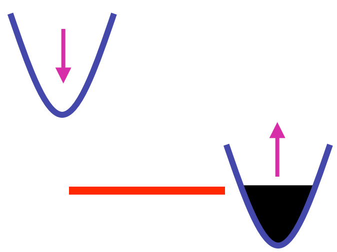


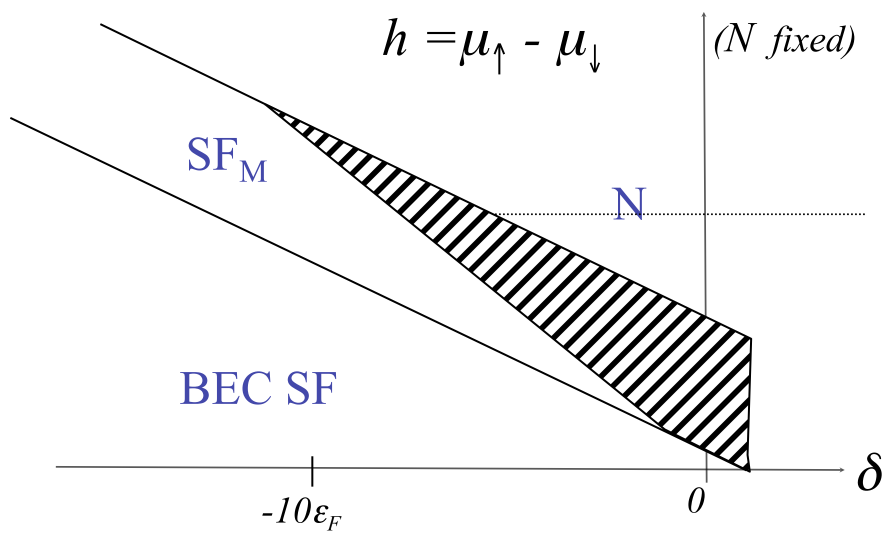
- ***SF_M*** $B_0 \neq 0$, $B_Q = 0$, $\Delta N \neq 0$: $h_m < h < h_{c1}(\delta) \approx -0.65\delta$

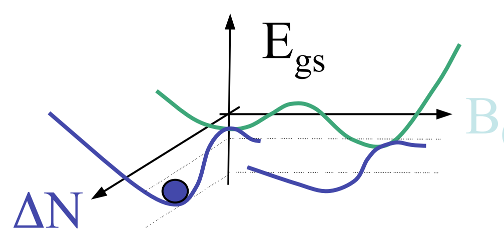




- ***Normal*** $B_0 = 0$, $B_Q = 0$, $\Delta N \neq 0$ *(Pauli "paramagnet")*: $h > h_{c2}(\delta) \approx 2^{3/2}\epsilon_F - \delta/2$

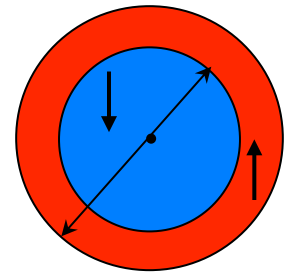




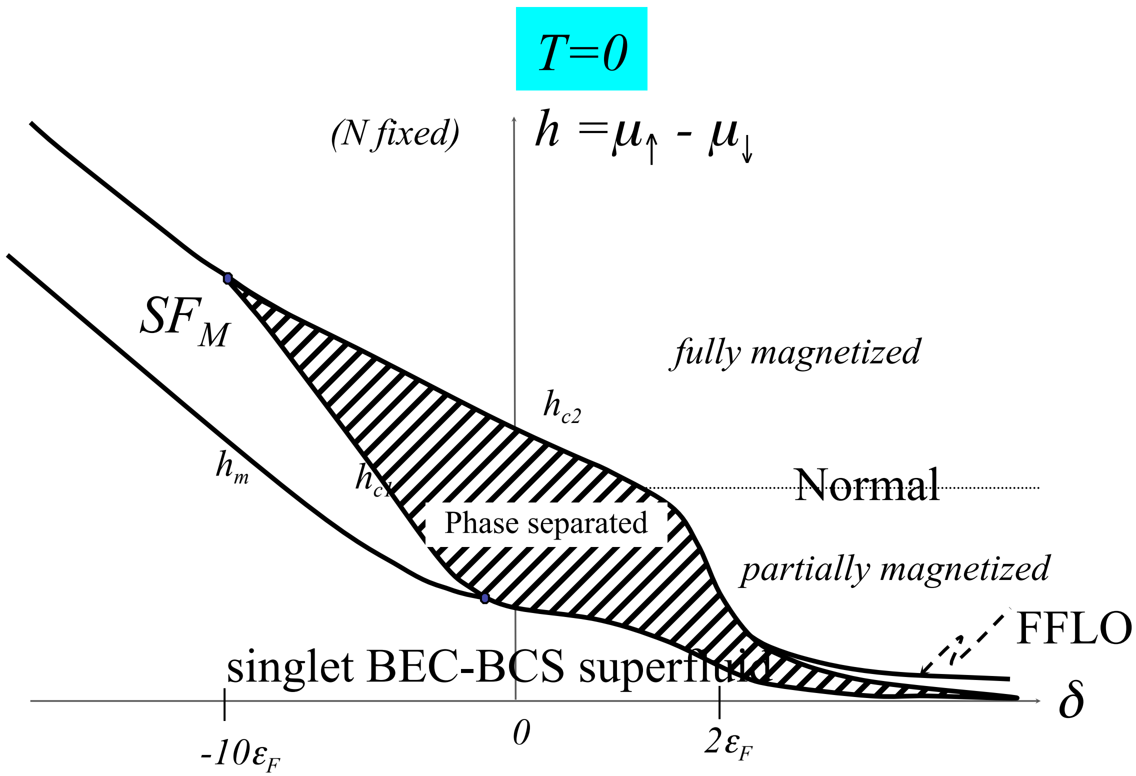


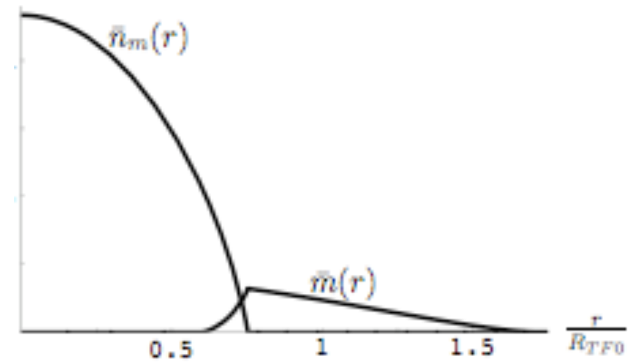
Imbalanced BEC-BCS

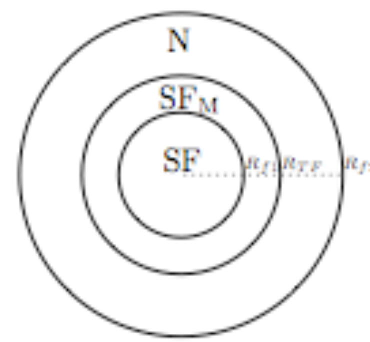
Sheehy, L.R. '05

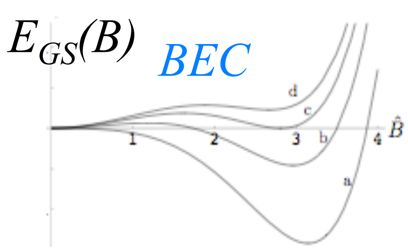


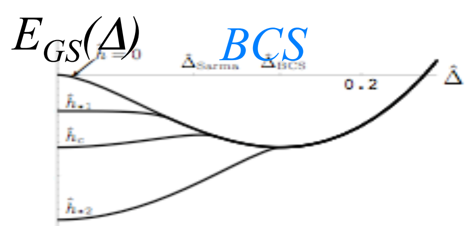
- 1st order transitions and phase separation





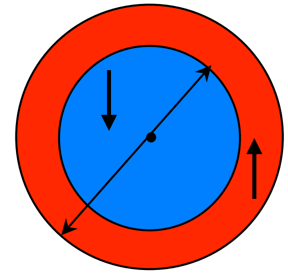




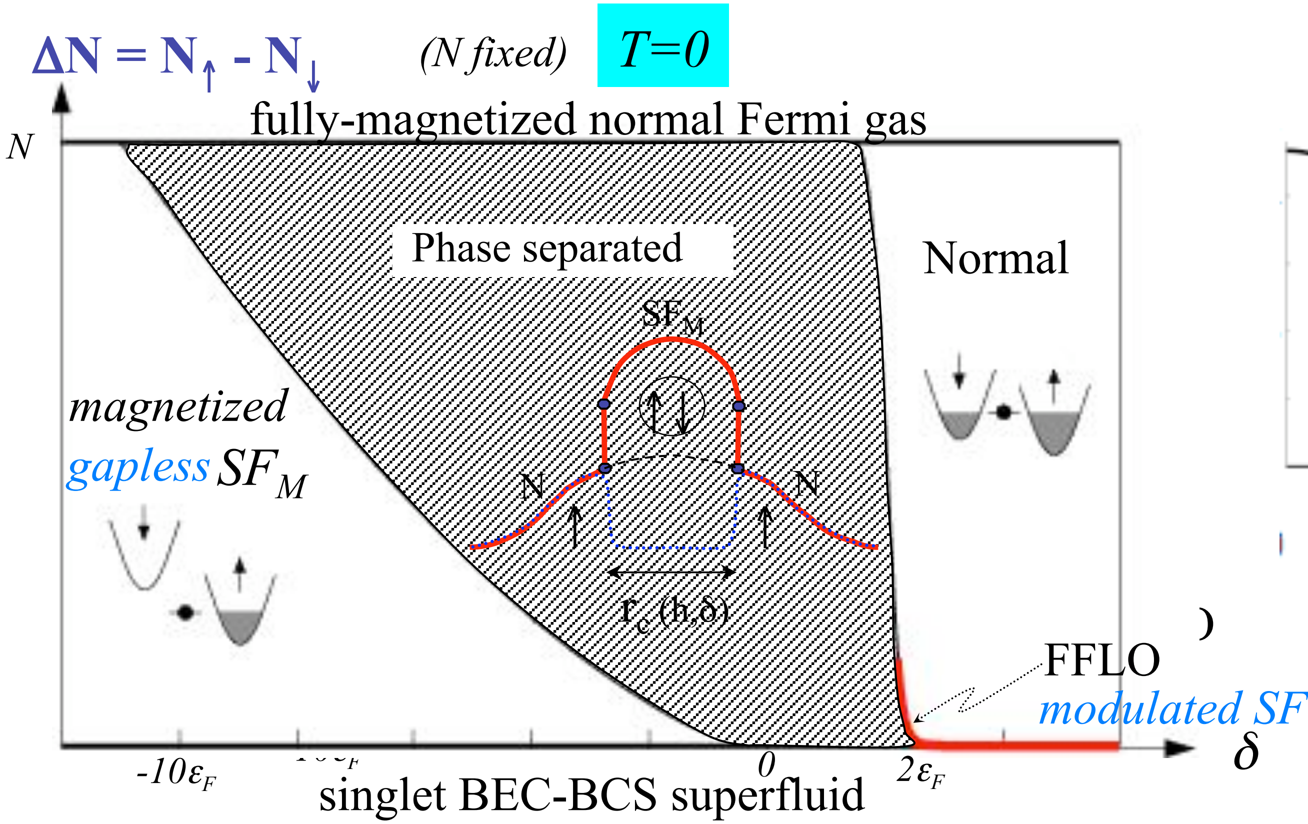


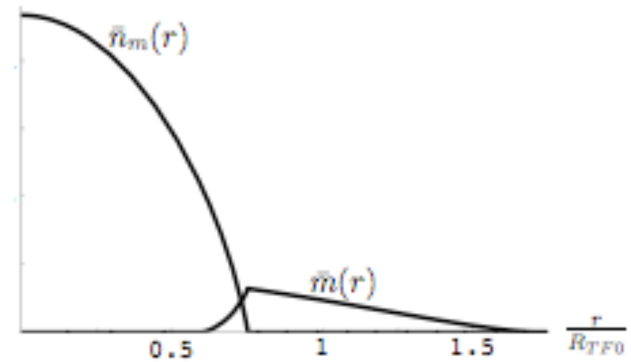
Imbalanced BEC-BCS

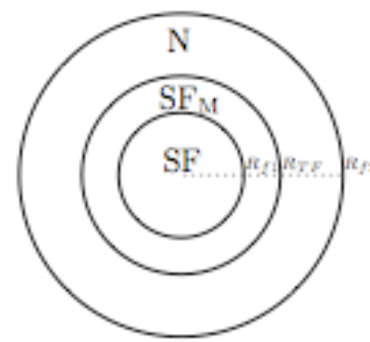
Sheehy, L.R. '05

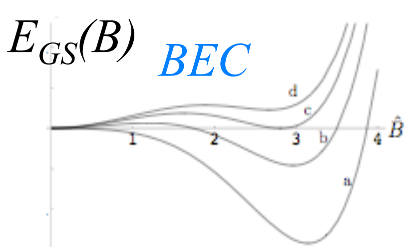


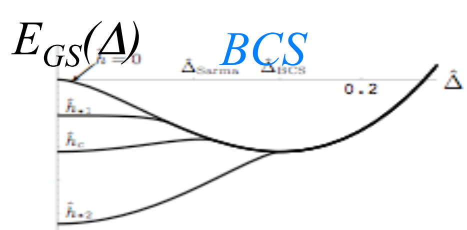
- 1st order transitions and phase separation





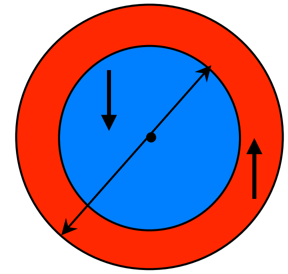






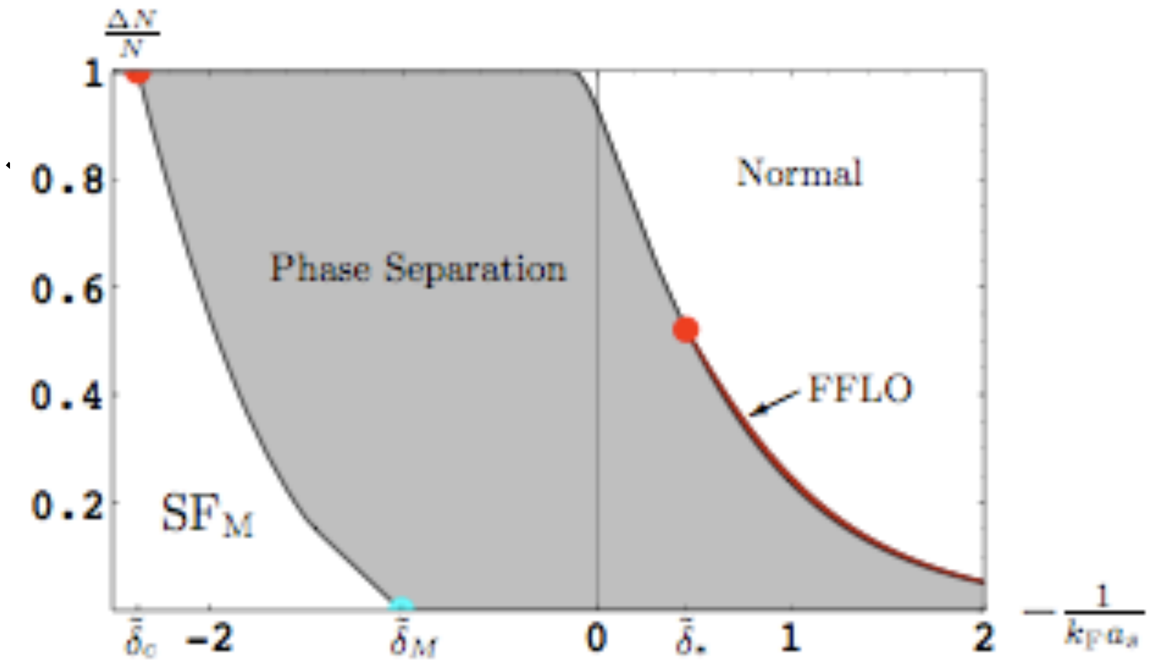
Imbalanced BEC-BCS

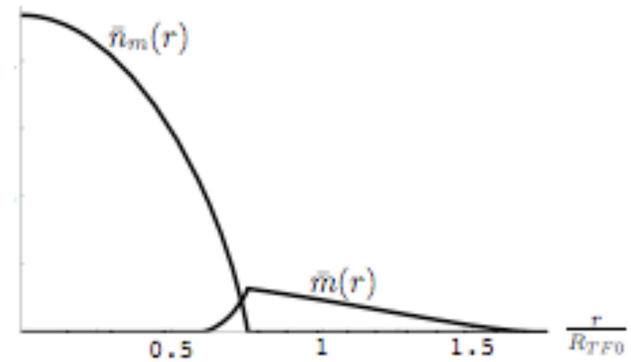
Sheehy, L.R. '05

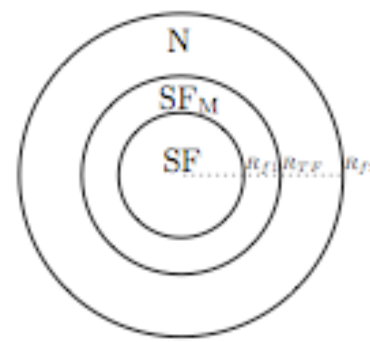


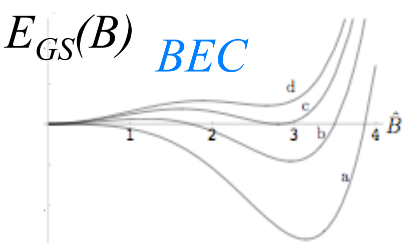
- 1st order transitions and phase separation

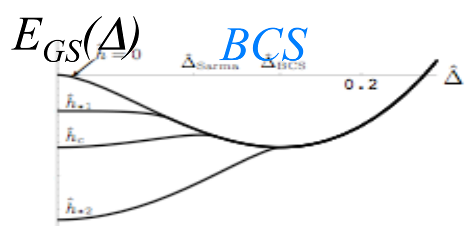
T=0





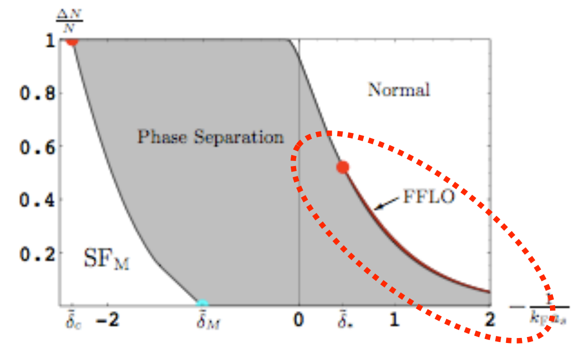




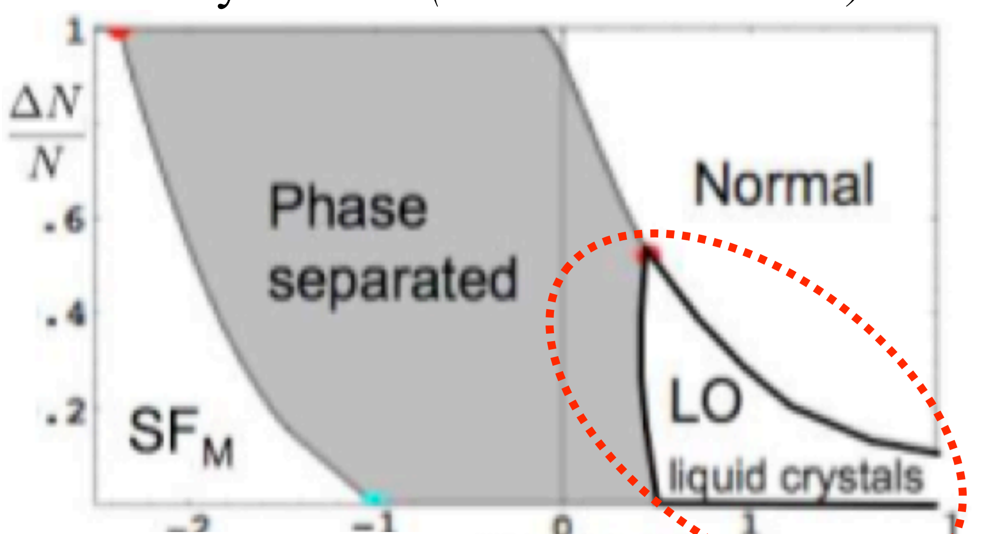


FFLO state

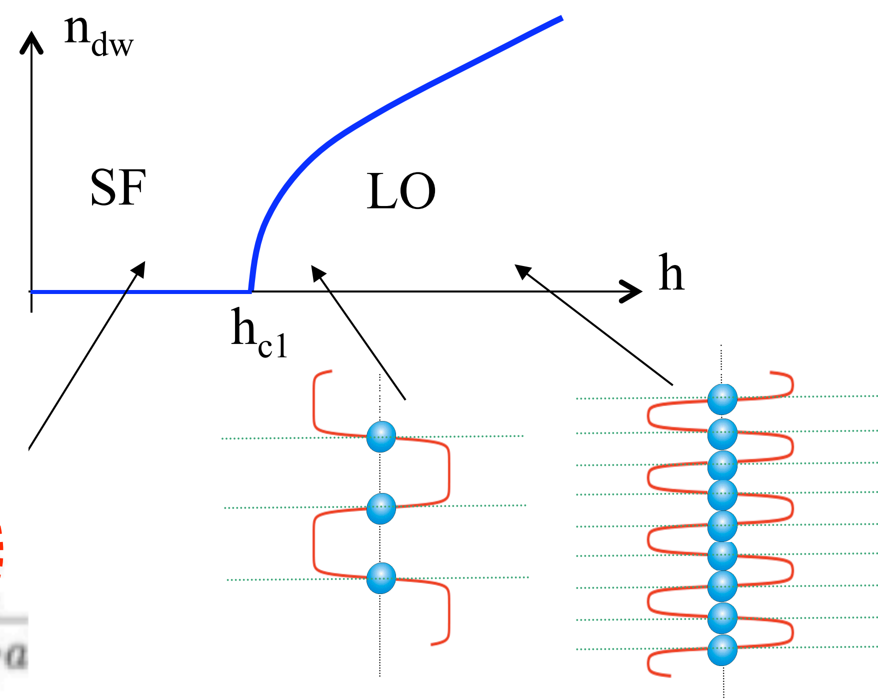
- pair "density" wave: $\Delta = \sum_{\mathbf{Q}} \Delta_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{x}}$
- motivation:
  - *stabilized in lower dimensions (Huse, et al)*
  - *negative surface tension for ± Δ domain wall (Matsuo,et al.;Yoshida+Yip)*
  - ⟹ *SF→ LO: C-I transition of domain-wall proliferation?*



- "yes" in 1d *(Machida-Nakanishi '84)*



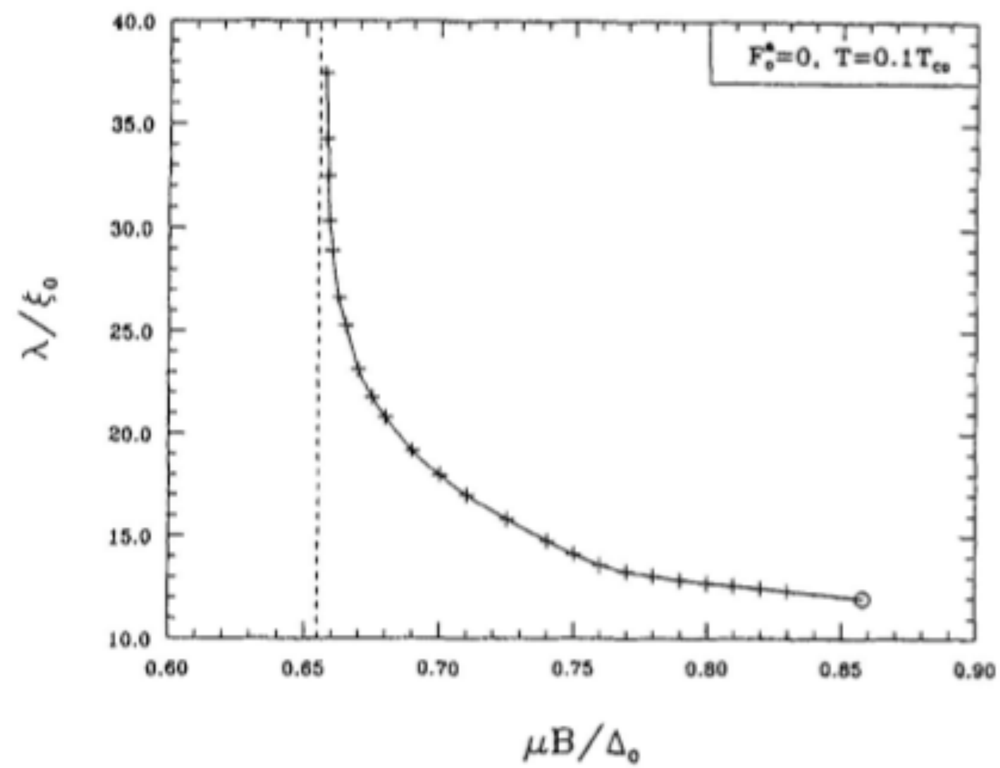
Burkhardt, Rainer '94
Machida, Nakanishi '84



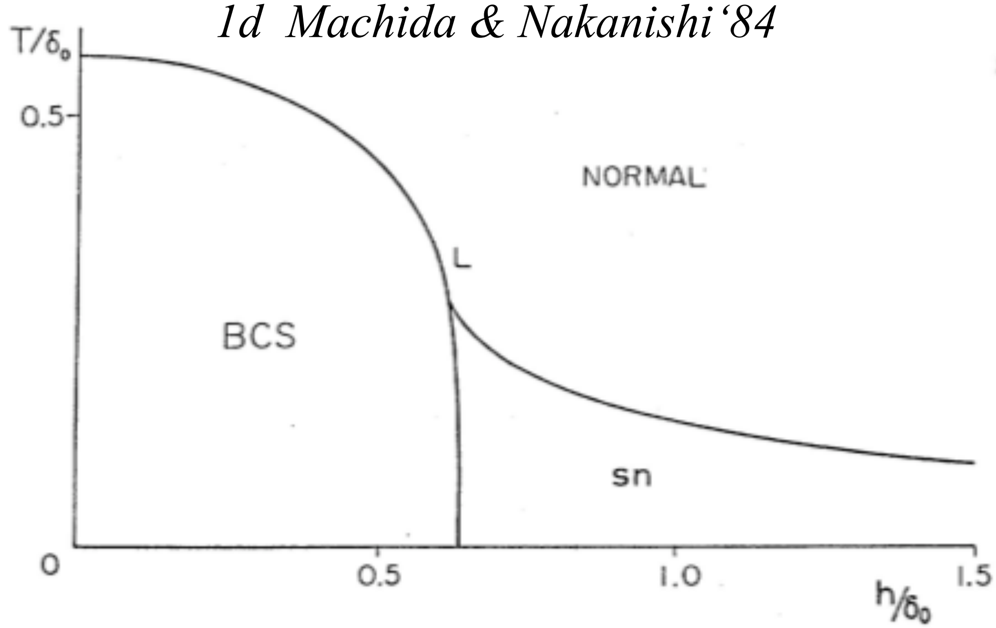
- excess fermions sit on domain walls *(cf. polyacetylene of Schrieffer, Su, Heeger)*
- microphase separation *(cf. H_{c1} transition to vortex state in type II sc's)*

Evidence in 1d and 2d

2d Burkhardt & Rainer '94



1d Machida & Nakanishi '84

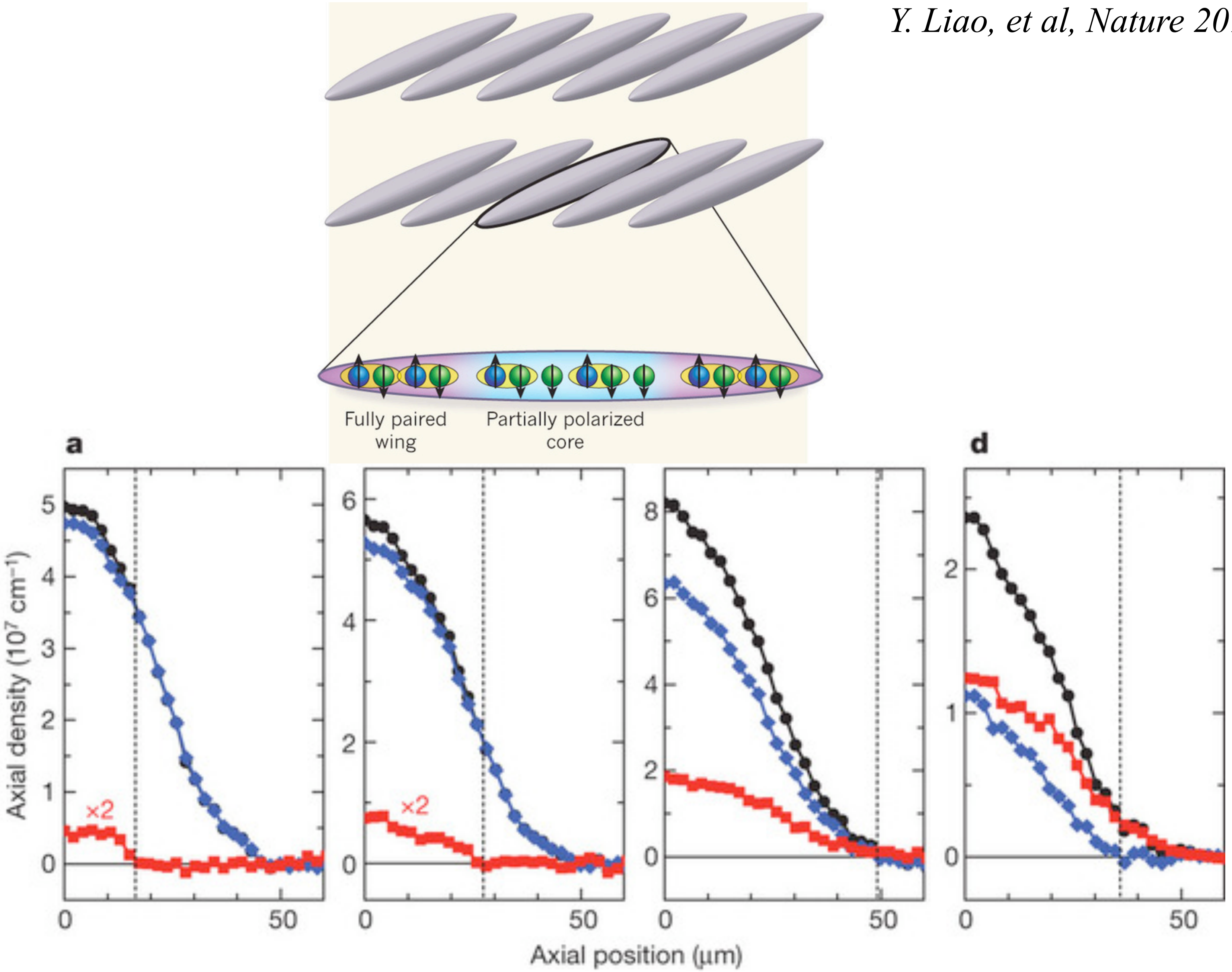


in 1d:

- Bethe ansatz exists
- bosonization spin gap closing, 1 ➔ 2 LL modes

Experimental realization in quasi-1d

Y. Liao, et al, Nature 2010



Experimental realization in quasi-1d

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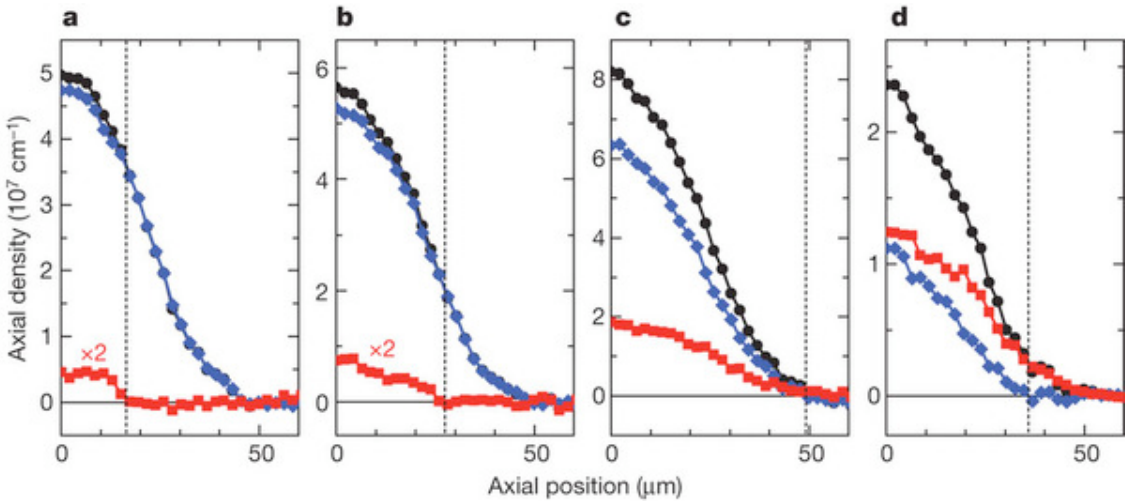
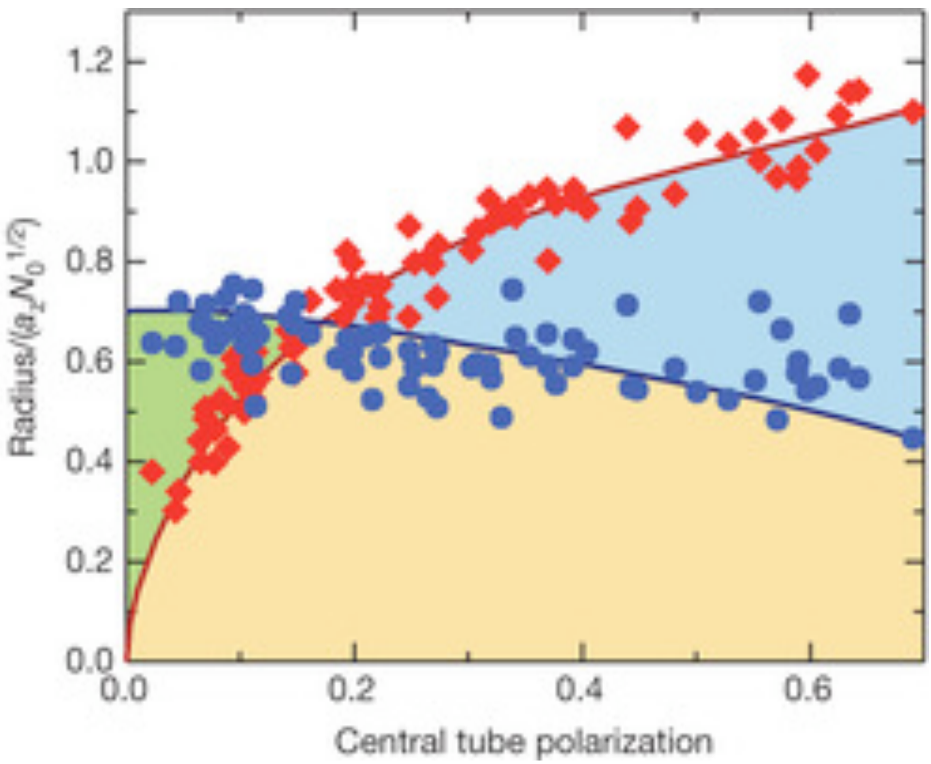
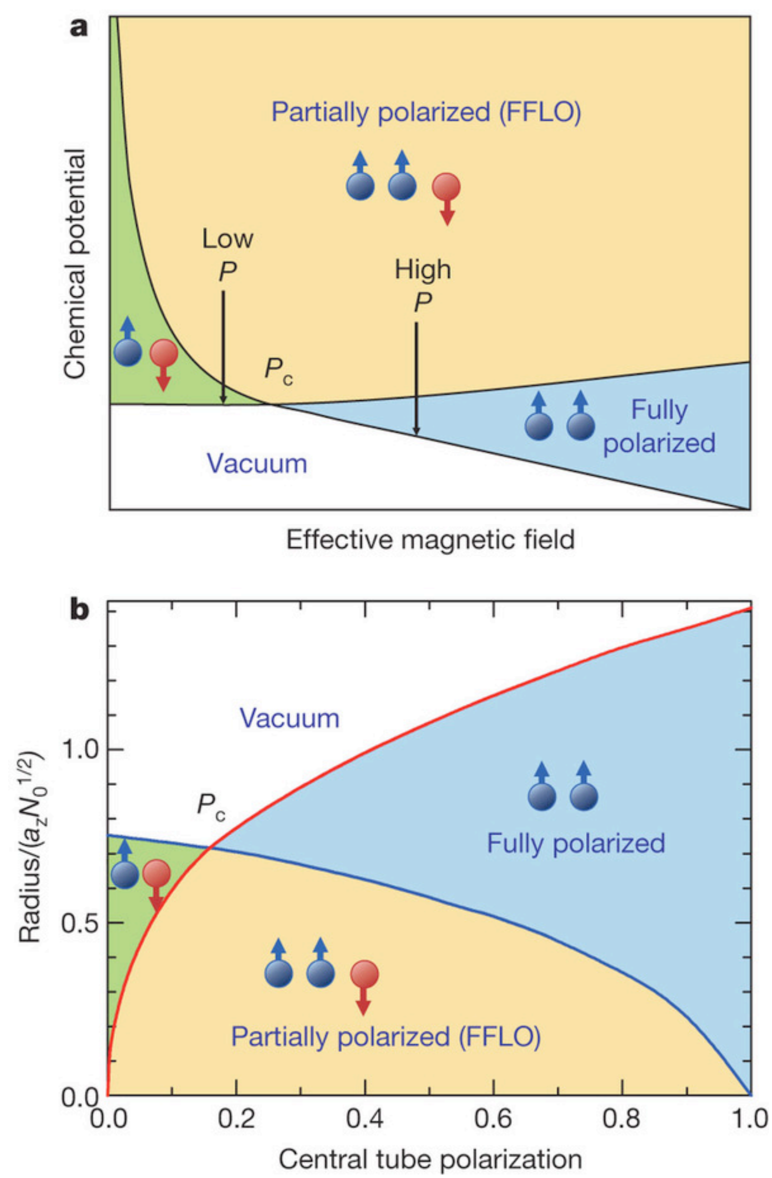






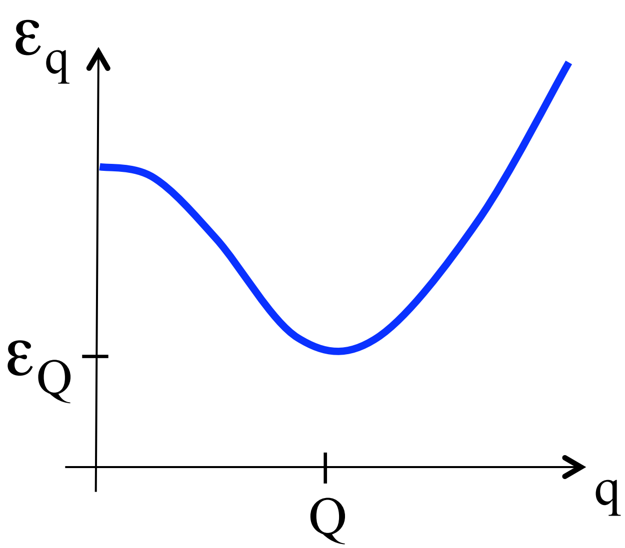
Microscopics to Ginzburg-Landau

LR, unpublished '07

$$H_{BCS}[c_\sigma, c_\sigma^\dagger] \xrightarrow{\text{near } h_{c2}} \quad \text{with } \Delta = V\langle c_\downarrow c_\uparrow \rangle$$

$$H_{GL}[\Delta] = \sum_{\mathbf{q}} \bar{\Delta}_{\mathbf{q}} \varepsilon_q \Delta_{\mathbf{q}} + \sum_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3} v_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\mathbf{q}_4} \bar{\Delta}_{\mathbf{q}_1} \Delta_{\mathbf{q}_2} \bar{\Delta}_{\mathbf{q}_3} \Delta_{\mathbf{q}_4} + \ldots$$

$$\approx J\bar{\Delta}\left(-\nabla^2 - Q^2\right)^2 \Delta + \varepsilon_Q |\Delta|^2 + \frac{v_1}{2}|\Delta|^4 + \frac{v_2}{2}\mathbf{j}^2 + \ldots$$



$$J \approx \frac{n}{\epsilon_F Q^4}$$

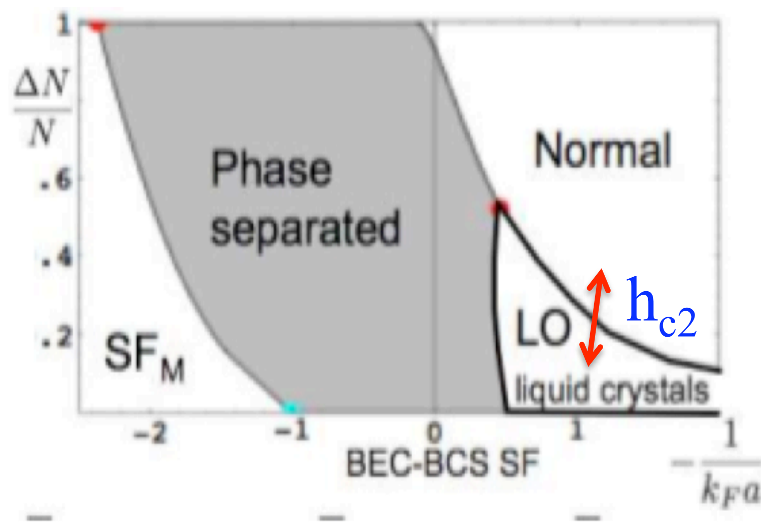
$$Q \approx \frac{\Delta_{BCS}}{\hbar v_F}$$

$$\varepsilon_Q \approx \frac{n}{\epsilon_F} \ln\left[\frac{h}{h_{c2}}\right]$$

$$h_{c2} \approx \frac{3}{4}\Delta_{BCS}$$

$$v_1 \approx \frac{n}{\epsilon_F \Delta_{BCS}^2}$$

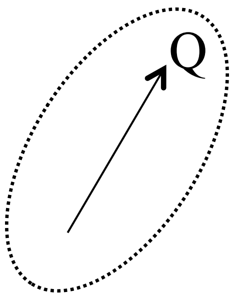
$$v_2 \approx \frac{nm^2}{\epsilon_F \Delta_{BCS}^2 Q_0^2}$$



Broken symmetries in LO/FF states

LR, Vishwanath
PRL, 2009

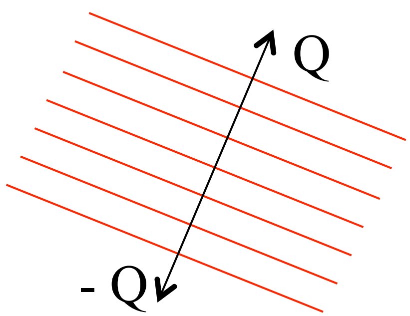
- Fulde-Ferrell: $\Delta_{FF}(\mathbf{x}) = \Delta_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{x}}$



- <u>broken</u>: *time reversal, orientational, off-diagonal*

orientationally-ordered superfluid

- Larkin-Ovchinnikov: $\Delta_{LO}(\mathbf{x}) = \Delta_{\mathbf{Q}} \cos \mathbf{Q}\cdot\mathbf{x}$



- <u>broken</u>: *orientational, translational, off-diagonal*

superconducting smectic

superfluid liquid crystals

Low-energy excitations in LO/FF states

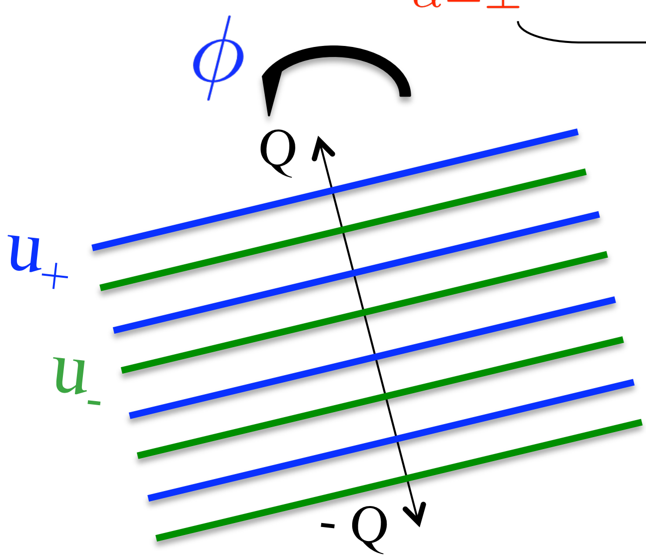
- order parameter: $\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta_+} e^{i\mathbf{Q}\cdot\mathbf{x}} + \Delta_0 e^{i\theta_-} e^{-i\mathbf{Q}\cdot\mathbf{x}}$

$$= 2\Delta_0 e^{i\theta} \cos[\mathbf{Q}\cdot\mathbf{x} - Qu]$$

- superfluid phase and phonon: $\theta = \frac{1}{2}(\theta_- + \theta_+)$ $u = \frac{1}{2Q}(\theta_- - \theta_+)$

- coupled incommensurate smectics u_+, u_- :

$$\mathcal{H}_{LO} = \sum_{\alpha=\pm} \left[\frac{K}{2} \boxed{(\nabla^2 u_\alpha)^2} + \frac{B}{2} (\partial_z u_\alpha)^2 \right]$$



rotational invariance of smectic liquid crystal

$$E[u^0_\pm(\mathbf{x})] = 0, \text{ for :}$$

$$u^0_\pm(\mathbf{x}) = \phi\, x$$

Low-energy excitations in LO/FF states

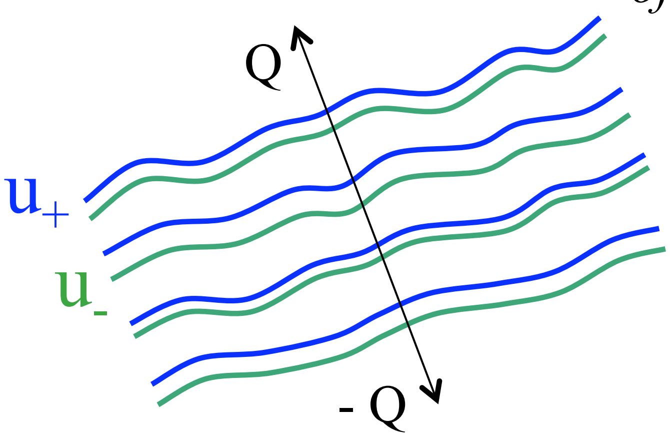
- order parameter: $\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta_+} e^{i\mathbf{Q}\cdot\mathbf{x}} + \Delta_0 e^{i\theta_-} e^{-i\mathbf{Q}\cdot\mathbf{x}}$
$= 2\Delta_0 e^{i\theta} \cos[\mathbf{Q}\cdot\mathbf{x} - Qu]$

- superfluid phase and phonon: $\theta = \frac{1}{2}(\theta_- + \theta_+)$ $u = \frac{1}{2Q}(\theta_- - \theta_+)$

- coupled incommensurate smectics u_+, u_- :

$$\mathcal{H}_{LO} = \sum_{\alpha=\pm} \left[\frac{K}{4}(\nabla^2 u_\alpha)^2 + \underbrace{\frac{B}{4}\left(\partial_z u_\alpha + \frac{1}{2}(\nabla u_\alpha)^2\right)^2}_{\text{rotational invariance of smectic liquid crystal}} \right]$$

rotational invariance
of smectic liquid crystal



$E[u^0_\pm(\mathbf{x})] = 0$ for $u^0_\pm(\mathbf{x}) = z(\cos\phi - 1) + x\sin\phi$

Low-energy excitations in LO/FF states

- order parameter: $\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta_+} e^{i\mathbf{Q}\cdot\mathbf{x}} + \Delta_0 e^{i\theta_-} e^{-i\mathbf{Q}\cdot\mathbf{x}}$
$$= 2\Delta_0 e^{i\theta} \cos[\mathbf{Q}\cdot\mathbf{x} - Qu]$$

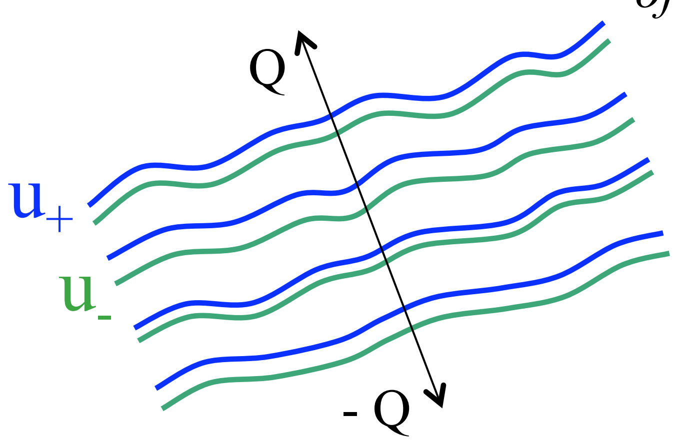
- superfluid phase and phonon: $\theta = \frac{1}{2}(\theta_- + \theta_+)$ $\quad u = \frac{1}{2Q}(\theta_- - \theta_+)$

- coupled incommensurate smectics u_+, u_- :

$$\mathcal{H}_{LO} = \sum_{\alpha=\pm}\left[\frac{K}{2}(\nabla^2 u_\alpha)^2 + \frac{B}{2}\left(\partial_z u_\alpha + \frac{1}{2}(\nabla u_\alpha)^2\right)^2\right] + \frac{\gamma}{2}(\nabla u_+ - \nabla u_-)^2$$

rotational invariance of smectic liquid crystal

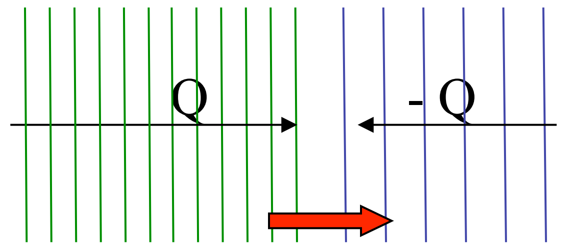
$j = j_+ + j_- = 0$



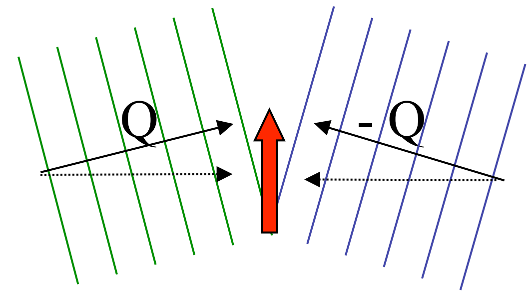
$$E[u^0_\pm(\mathbf{x})] = 0 \quad \text{for} \quad u^0_\pm(\mathbf{x}) = z(\cos\phi - 1) + x\sin\phi$$

“Infinitely” anisotropic superfluid

- supercurrents:



$j_{\|} \sim \partial_{\|} u_+ - \partial_{\|} u_-$



$j_{\perp} \sim \nabla_{\perp} u_+ - \nabla_{\perp} u_-$

- Goldstone modes “elastic” theory:

$$\mathcal{H}_{LO} = \sum_{\alpha=\pm} \left[\frac{K}{4} (\nabla^2 u_\alpha)^2 + \frac{B}{4} \left(\partial_z u_\alpha + \frac{1}{2} (\nabla u_\alpha)^2 \right)^2 \right] + \frac{\gamma}{2} (\nabla u_+ - \nabla u_-)^2$$

$$\approx \underbrace{\frac{K}{2} (\nabla_\perp^2 u)^2 + \frac{B}{2} (\partial_z u)^2}_{\text{smectic elasticity}} + \underbrace{\frac{\rho_s^i}{2} (\nabla_i \theta)^2}_{\text{superfluid stiffness}}$$

- superfluid stiffness *anisotropy*:

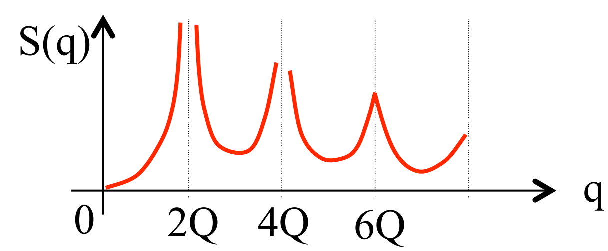
$$\frac{\rho_s^\perp}{\rho_s^\|} = \left(\frac{\Delta_Q}{\Delta_{BCS}} \right)^2 \approx \ln \left(\frac{h_{c2}}{h} \right) \ll 1$$

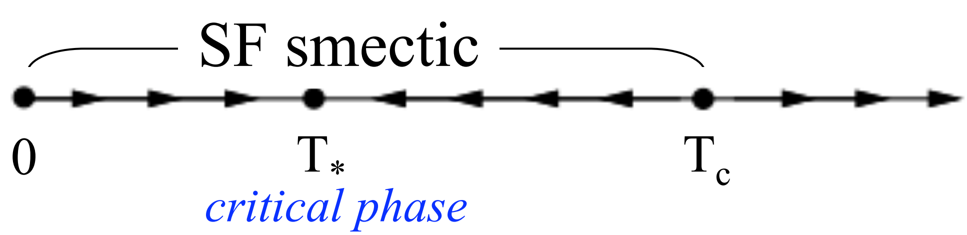
Fluctuations and stability of LO/FF states

- fluctuations at T=0: $$\mathcal{L}_{LO} = \frac{\chi}{2}(\partial_\mu\theta)^2 + \frac{\rho}{2}(\partial_t u)^2 + \frac{B}{2}(\partial_z u)^2 + \frac{K}{2}(\nabla^2 u)^2$$
  - $\langle\theta^2\rangle$, $\langle u^2\rangle \sim$ finite for $d > 1$ $\Rightarrow$ *LO stable to quantum fluctuations*
- fluctuations at T≠0:
  - $\langle\theta^2\rangle \sim$ finite for $d > 2$ $\Rightarrow$ *SF order stable to $k_B T$ fluctuations*
  - $\langle u^2\rangle \sim$ diverges for $d \leq 3$ $\Rightarrow$ *positional order unstable*

➡ LO = superfluid smectic (SF_{sm}) with:

- quasi-Bragg peaks (3d), Lorentzian (2d)
- anomalous elasticity *(Grinstein and Pelcovits)*
- transitions to superfluid nematic (SF_N)



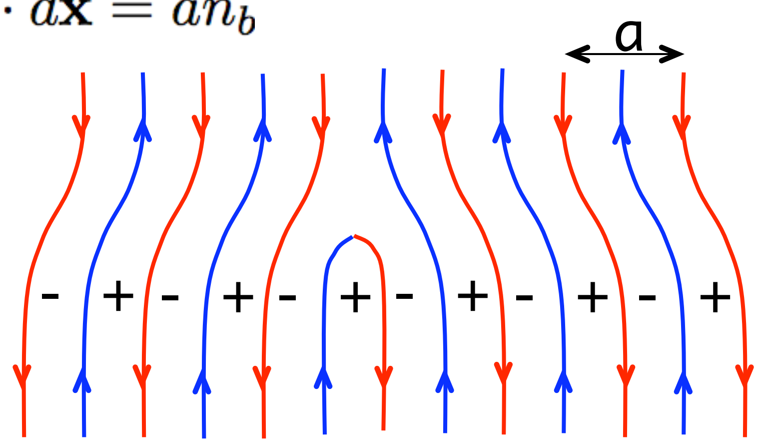


Topological defects

$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})}\cos[Q(z-u(\mathbf{x}))] \qquad \frac{U(1)\times U(1)}{Z_2}$$

- integer dislocations in u: $\oint \nabla u \cdot d\mathbf{x} = a n_b$

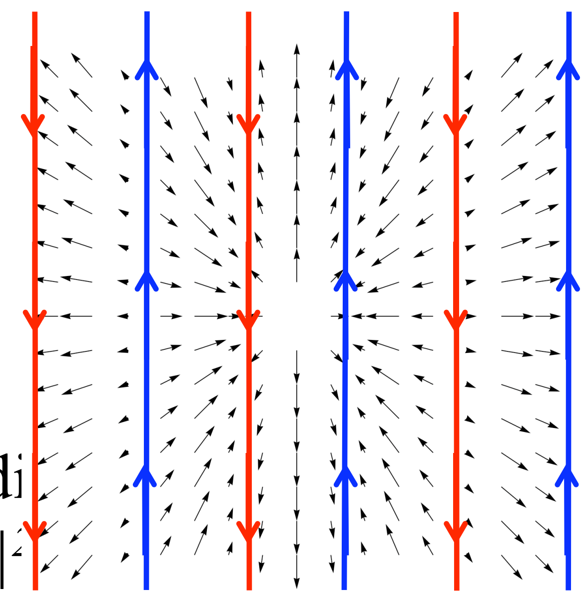
$(n_v, n_b) = (0,1)$



  - destroy LO order ("charge"-*2* SF *and* full smectic periodicity)
  - retain "charge" ≥ *4* homogeneous SF (Δ^2)

- integer vortices in θ: $\oint \nabla \theta \cdot d\mathbf{x} = 2\pi n_v$

$(n_v, n_b) = (1,0)$



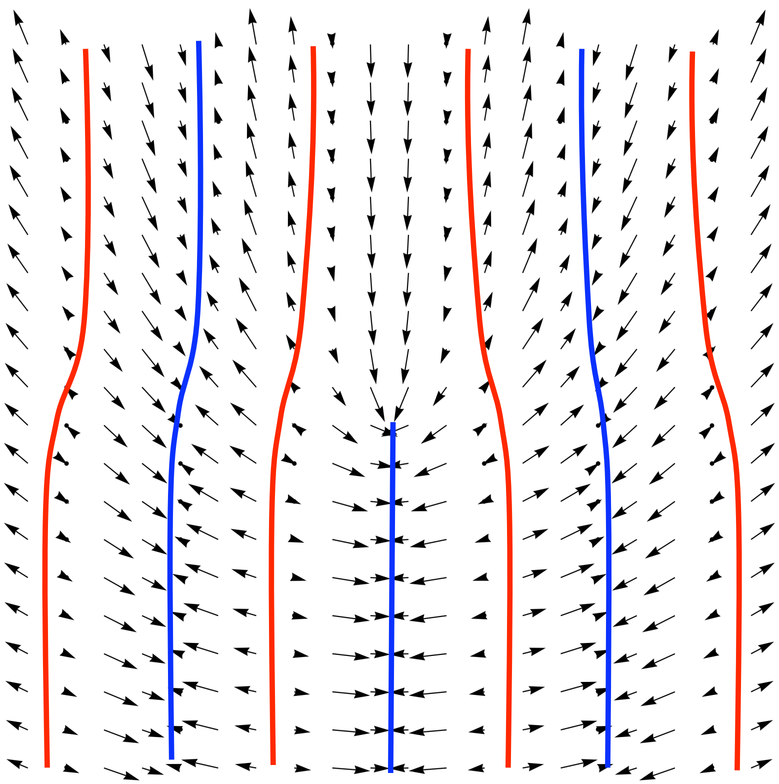
  - destroy LO order (full SF *and* *Q* smectic periodi
  - retain wavevector ≥ *2Q* smectic periodicity (|Δ|

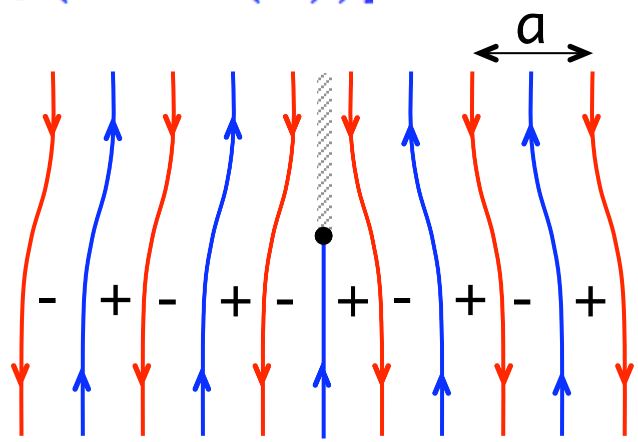
Fractional topological defects

$$\frac{U(1)\times U(1)}{Z_2}$$

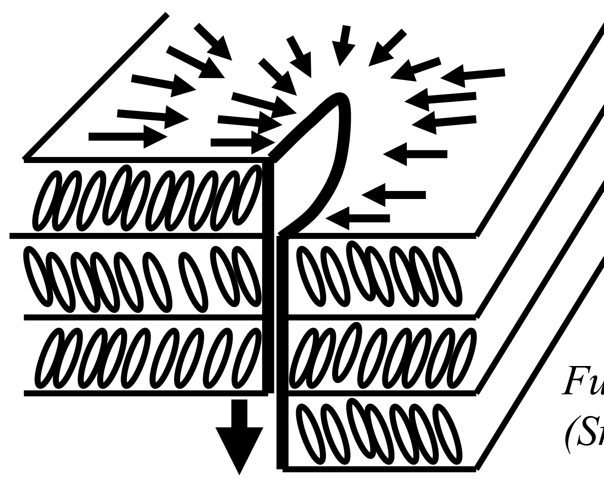
$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})}\cos[Q(z-u(\mathbf{x}))]$$

- π-vortex — a/2 dislocation pairs:





$(n_v,n_b) = (1/2,1/2)$ $(n_v,n_b) = (1/2,-1/2)$



dispiration in SmC_A

Fukuda, et al., PRB 45 '92 (Smalyukh)

- destroy LO order
- restore full translational invariance and atom "conservation"

Topological defects energetics

$$\frac{U(1)\times U(1)}{Z_2}$$

$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})}\cos[Q(z-u(\mathbf{x}))]$$

- integer a-dislocation in u (composite): $E_{(0,\,a)} \approx K\,L$

- integer 2π-vortex in θ (composite): $E_{(2\pi,\,0)} \approx \rho_s\,L \log L$

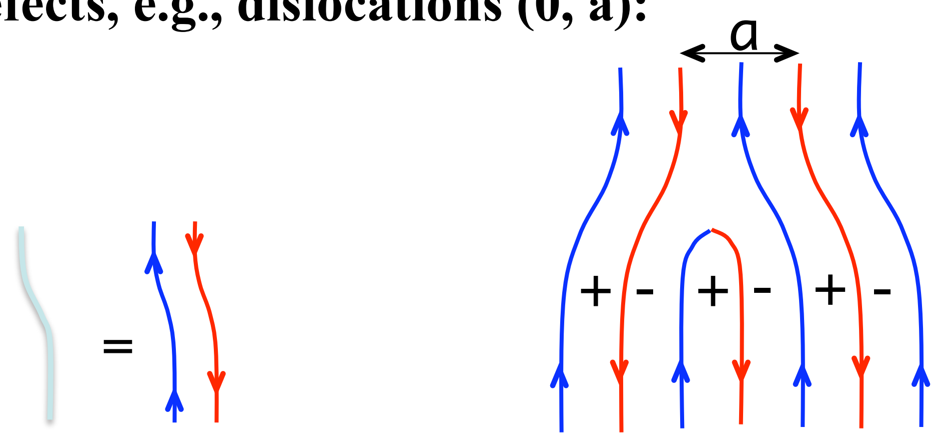
- π-vortex – a/2-dislocation (elementary): $E_{(\pm\pi,\,a)} \approx \frac{1}{4}\rho_s\,L\log L + \frac{1}{4} K\,L$

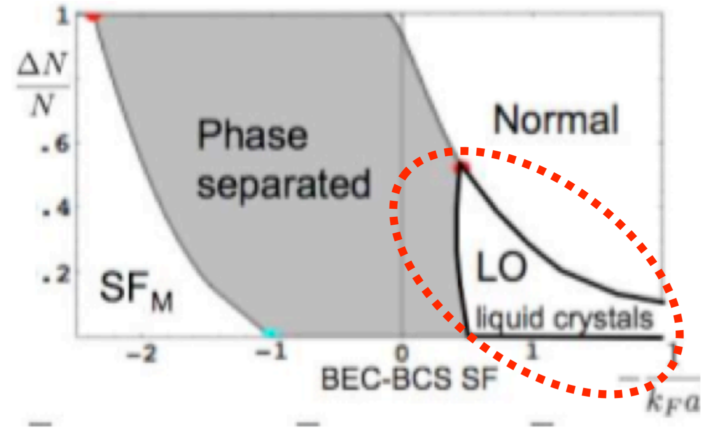
Composite defects (a-dislocation) unbind 1st → "fractionalized" phases

$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})} \cos[Q(z - u(\mathbf{x}))]$

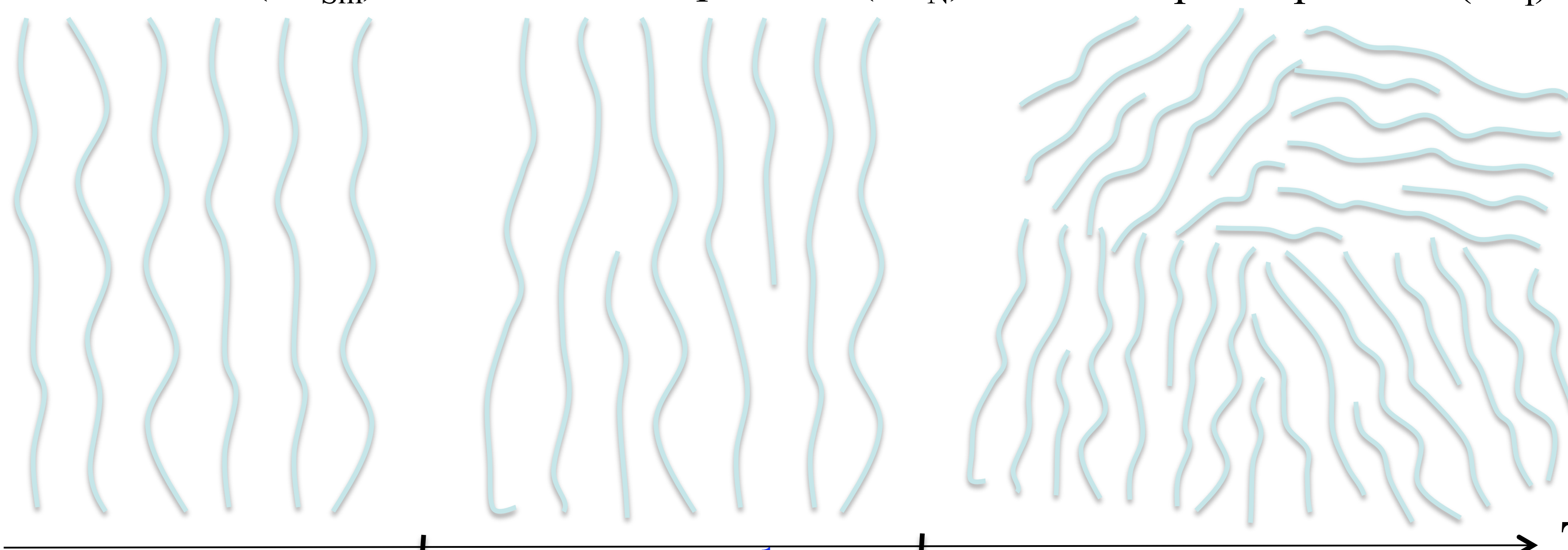
Phase transitions

unbind defects, e.g., dislocations (0, a):





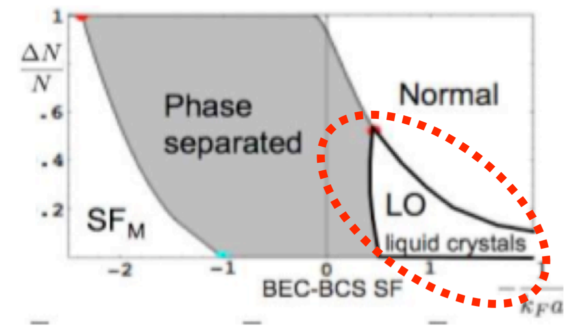
LO Smectic (SF_{Sm}) Nematic Superfluid (SF_N) Isotropic Superfluid (SF_I)

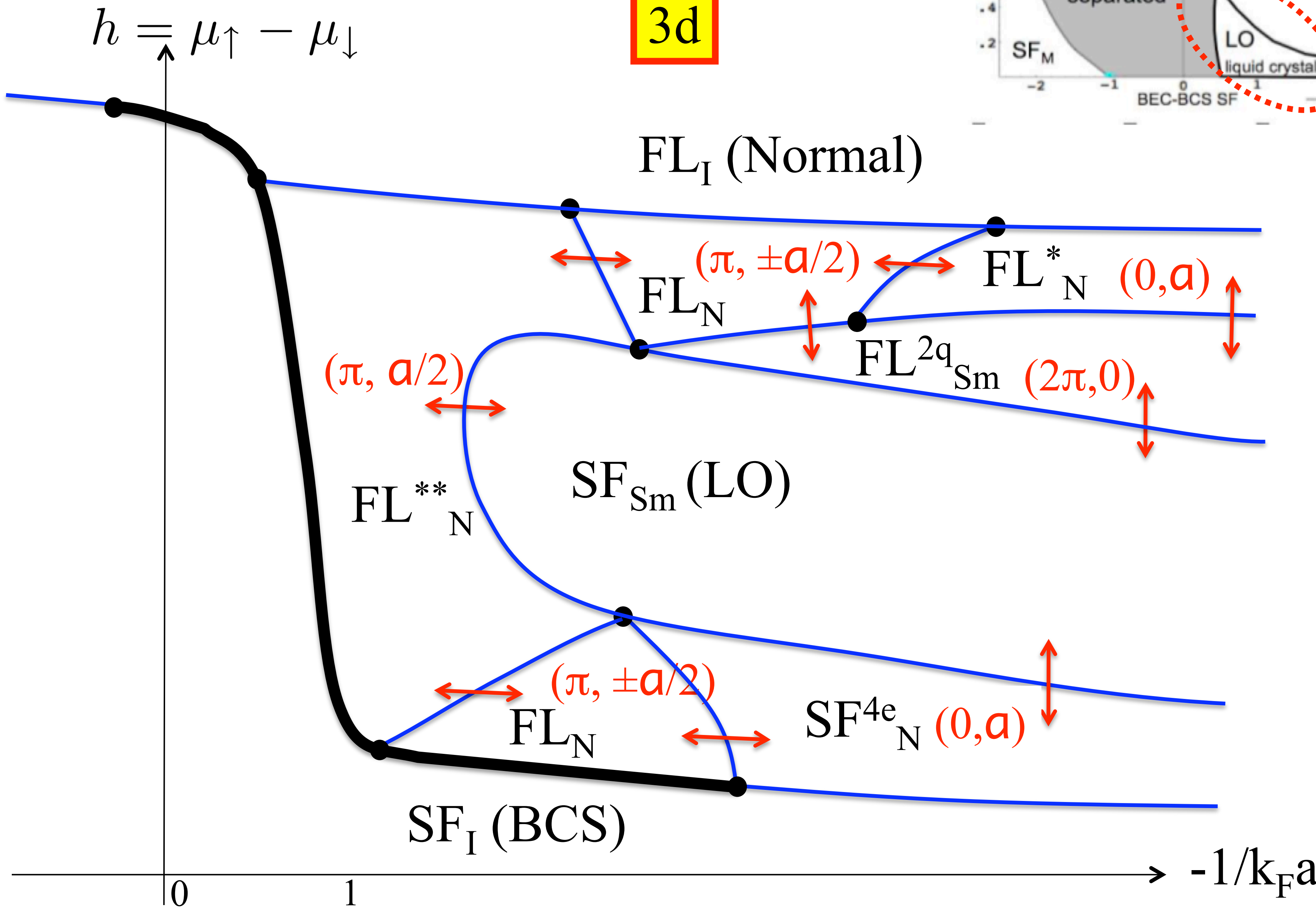


$e^{i\theta} \cos[Q(z-u)]$ T_{NSm} $e^{i2\theta}(\hat{n}_i \hat{n}_j - \frac{1}{3}\delta_{ij})$ T_{IN} $e^{i\theta}$ T

Phase transitions

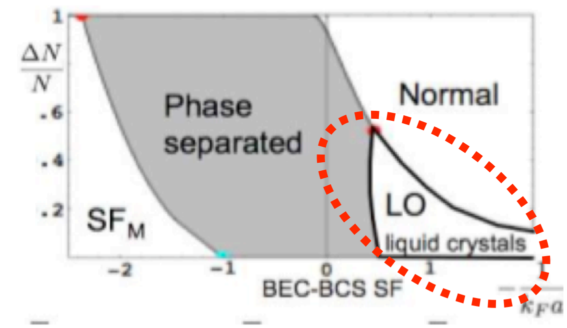
3d

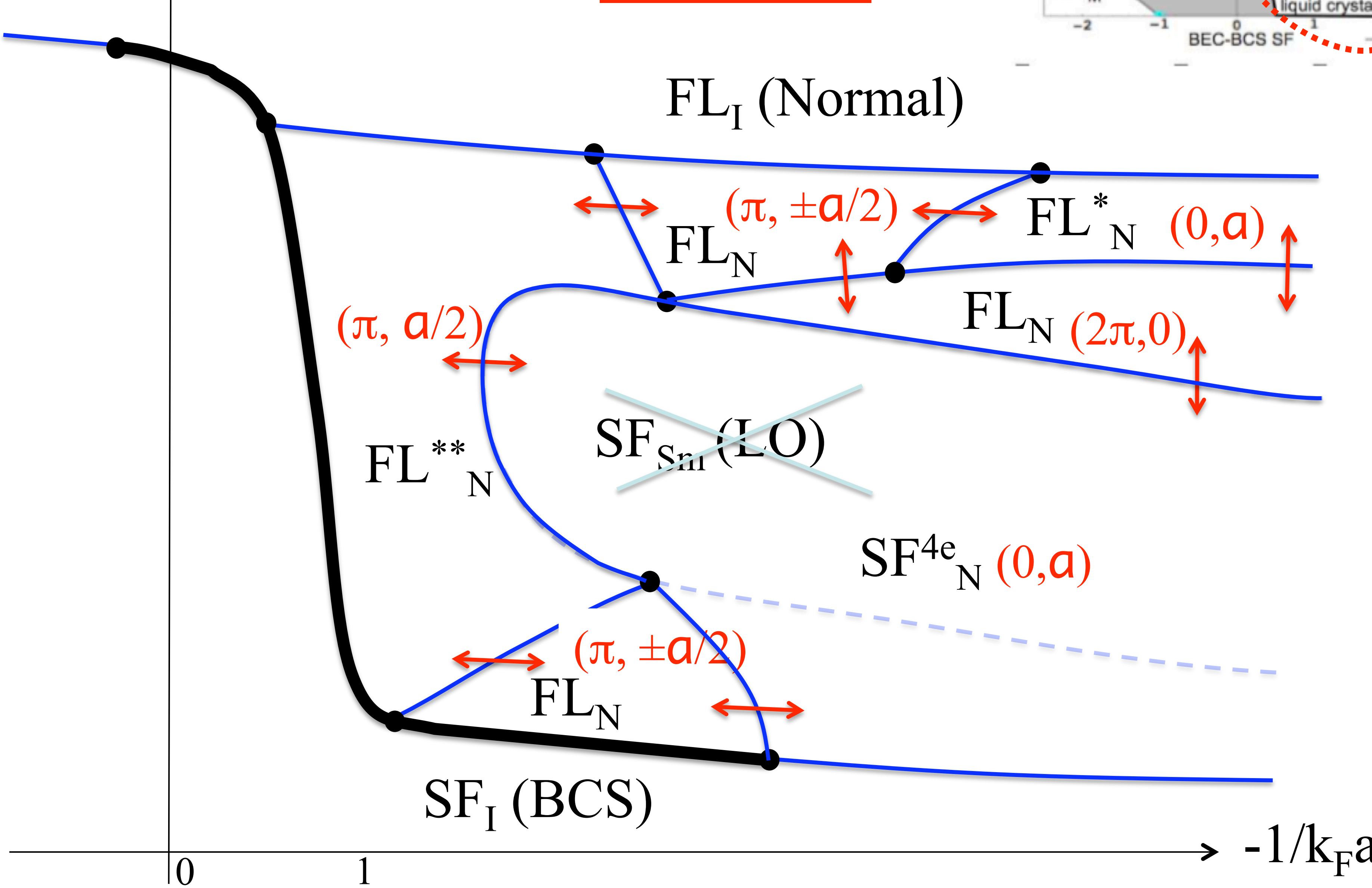




Phase transitions

2d finite T

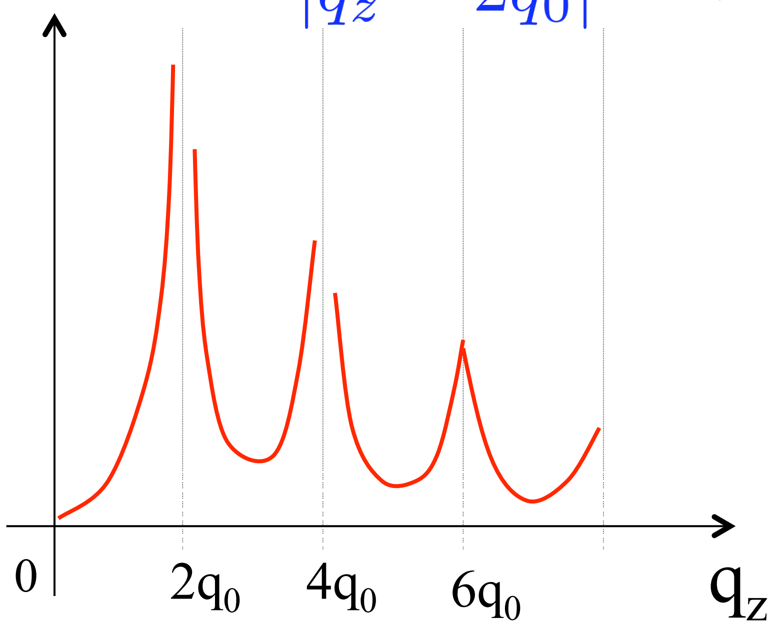




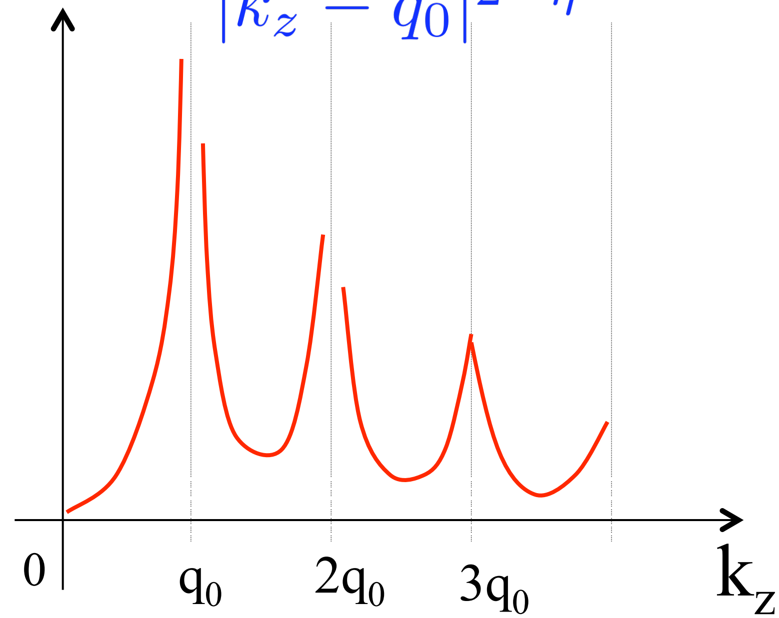
Structure function and time of flight

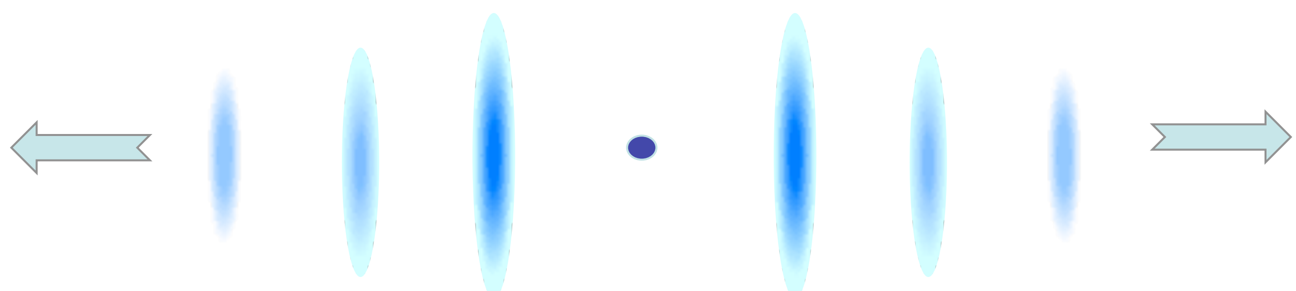
quasi-long-range order in 3d for T > 0

$$S(q_z) \sim \frac{1}{|q_z - 2q_0|^{2-4\eta}}$$



$$n_k \sim \frac{1}{|k_z - q_0|^{2-\eta}}$$



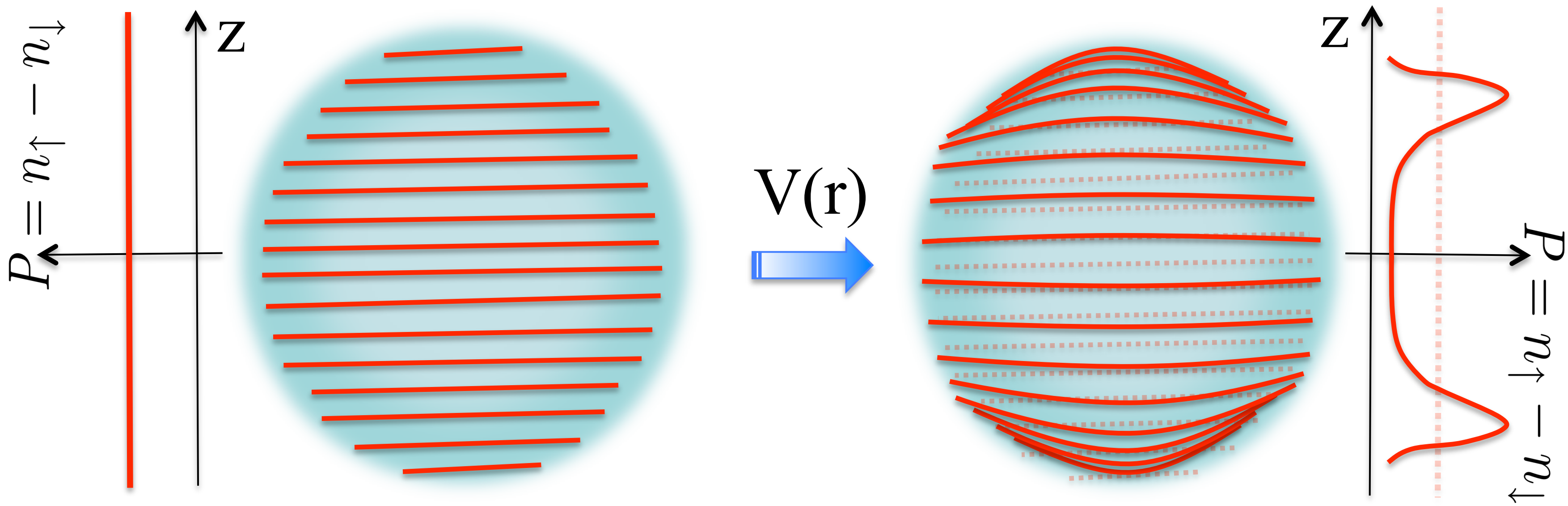


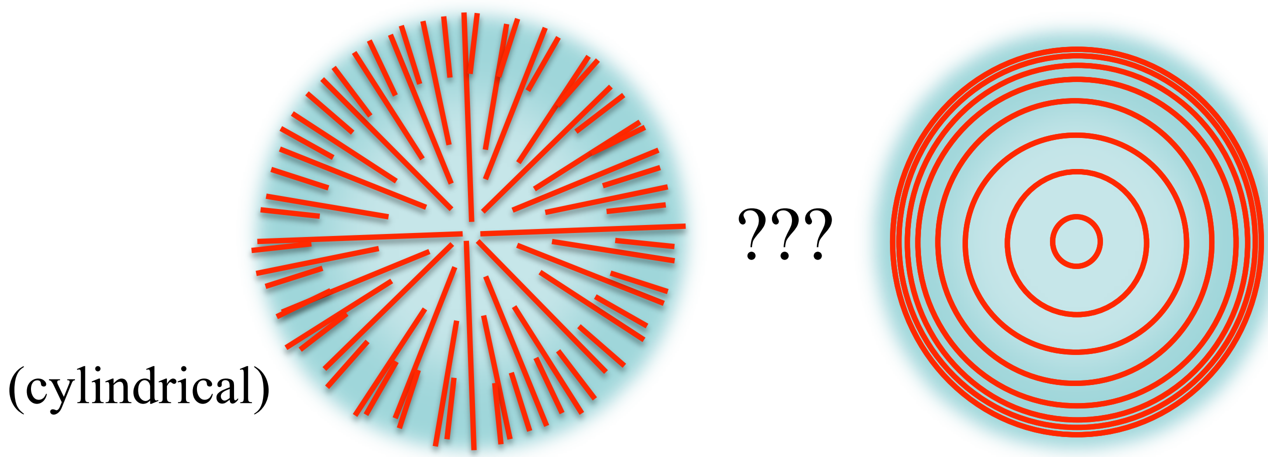
Finite trap geometry

$\mu_{\text{eff}}(r) = \mu(1 - r^2/R^2)$

$$\mathcal{H}_{LO} \approx \frac{\rho_s^i}{2}(\nabla_i \theta)^2 + \frac{K}{2}(\nabla_\perp^2 u)^2 + \frac{B}{2}(\partial_z u)^2 + n_0 V(\mathbf{r}) \partial_z u$$

$$\delta P(\mathbf{r}) \sim -\partial_\mathbf{z} \mathbf{u}(\mathbf{r}) = \partial_\mathbf{z} \int_{\mathbf{r}'} \frac{-1}{\mathbf{K}\nabla_\perp^4 - \frac{\mathbf{B}}{2}\partial_\mathbf{z}^2} \mathbf{n_0}(\mathbf{r}) \partial_\mathbf{z} \mathbf{V}(\mathbf{r}) \approx \frac{\mathbf{n_0}(\mathbf{r})}{\mathbf{B}} \mathbf{V}(\mathbf{r})$$

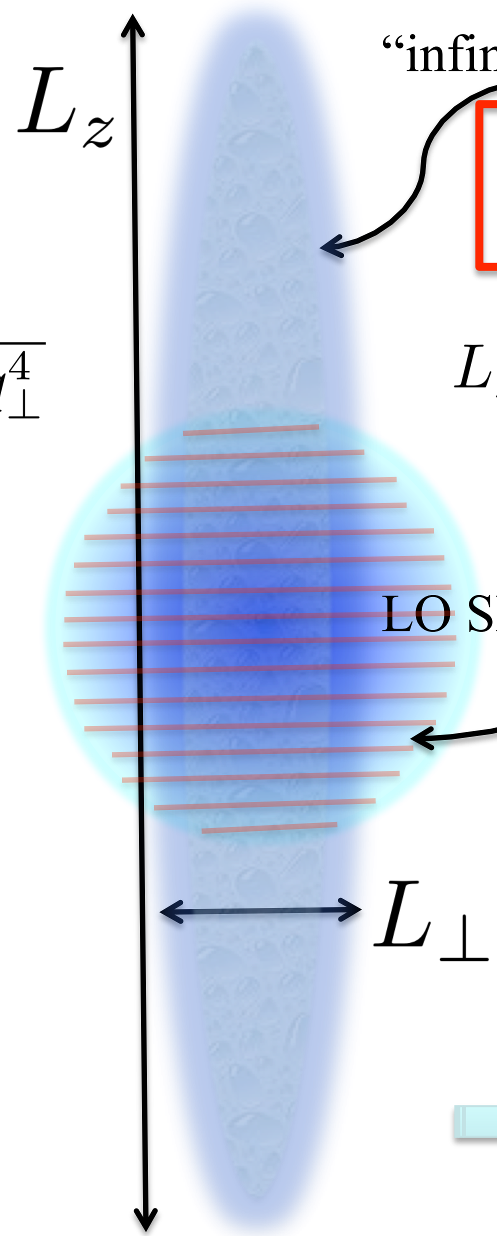




$u(\mathbf{r}) = \int \mathbf{G}(\mathbf{r} - \mathbf{r}') n_0(\mathbf{r}') \partial_z \mathbf{V}(\mathbf{r}')$ [illegible] $\quad G(\mathbf{r}) = FT\left[\frac{1}{\text{[illegible]}}\right] = \frac{a}{\text{[illegible]}} \mathrm{sgn}(z) \left[\mathrm{erf}\left(\frac{x}{\text{[illegible]}}\right) + 1\right]$

Fluctuations and stability in a trap

$$\langle u^2 \rangle \sim \frac{1}{\omega^2 + Bq_z^2 + Kq_\perp^4}$$



"infinitely" anisotropic fluctuations

$$q_z \approx \lambda q_\perp^2 \ll q_\perp \rightarrow 0$$

$$L_z \approx L_\perp^2/\lambda \gg L_\perp \rightarrow \infty$$

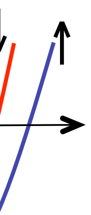
LO SF in isotropic trap

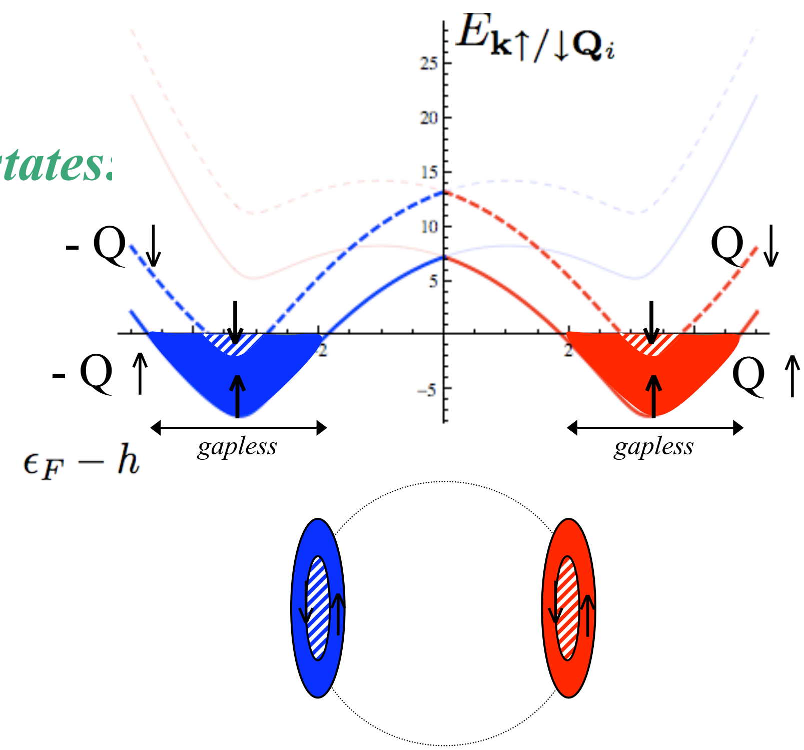
3d → 2d crossover in isotropic trap!

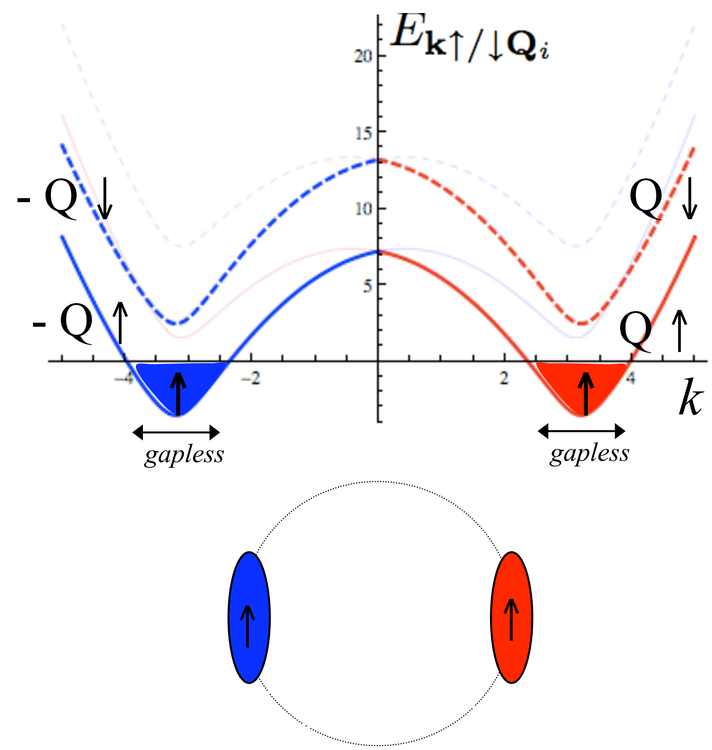
Fermionic sector of LO state

- ***excitation spectrum:*** $$E_{\mathbf{k}\uparrow/\downarrow\mathbf{Q}_i} = (\varepsilon_k^2 + \Delta_Q^2)^{1/2} \mp (h + \frac{\mathbf{k}\cdot\mathbf{Q}_i}{2m})$$
  (gapped and gapless k's)

- ***2 distinct LO states:***







- ***ground state:*** $$|LO_{\mathbf{Q}}\rangle = \prod_{\mathbf{k},\mathbf{Q}_i \in E_{\mathbf{k}\sigma\mathbf{Q}_i}<0} \alpha^\dagger_{\mathbf{k}\sigma\mathbf{Q}_i}|BCS_{\mathbf{Q}}\rangle,$$
$$= \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_3} c^\dagger_{\mathbf{k}+\frac{\mathbf{Q}_i}{2}\downarrow} \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_2} c^\dagger_{-\mathbf{k}+\frac{\mathbf{Q}_i}{2}\uparrow} \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_1} (u_{\mathbf{k}} + v_{\mathbf{k}} c^\dagger_{\mathbf{k}+\frac{\mathbf{Q}_i}{2}\downarrow} c^\dagger_{-\mathbf{k}+\frac{\mathbf{Q}_i}{2}\uparrow})|0\rangle$$

$$H_f^{ex} = \sum \left[E_{\mathbf{k}\sigma\mathbf{Q}_i}\Theta(E_{\mathbf{k}\sigma\mathbf{Q}_i})\alpha^\dagger_{\mathbf{k}\sigma\mathbf{Q}_i}\alpha_{\mathbf{k}\sigma\mathbf{Q}_i} - E_{\mathbf{k}\sigma\mathbf{Q}_i}\Theta(-E_{\mathbf{k}\sigma\mathbf{Q}_i})\alpha_{\mathbf{k}\sigma\mathbf{Q}_i}\alpha^\dagger_{\mathbf{k}\sigma\mathbf{Q}_i}\right]$$

Fermionic sector of LO state

- *ground state:*

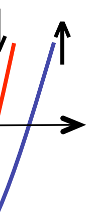
$$|LO_{\mathbf{Q}}\rangle = \prod_{\mathbf{k},\mathbf{Q}_i \in E_{\mathbf{k}\sigma\mathbf{Q}_i}<0} \alpha^\dagger_{\mathbf{k}\sigma\mathbf{Q}_i}|BCS_{\mathbf{Q}}\rangle,$$

$$= \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_3} c^\dagger_{\mathbf{k}+\frac{\mathbf{Q}_i}{2}\downarrow} \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_2} c^\dagger_{-\mathbf{k}+\frac{\mathbf{Q}_i}{2}\uparrow} \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_1} (u_{\mathbf{k}} + v_{\mathbf{k}} c^\dagger_{\mathbf{k}+\frac{\mathbf{Q}_i}{2}\downarrow} c^\dagger_{-\mathbf{k}+\frac{\mathbf{Q}_i}{2}\uparrow})|0\rangle$$

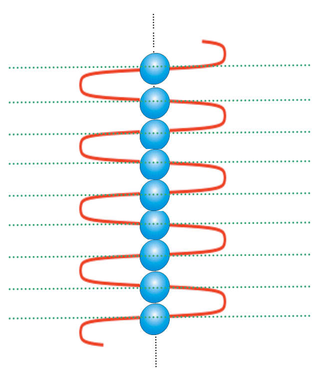
- *excitation spectrum:*

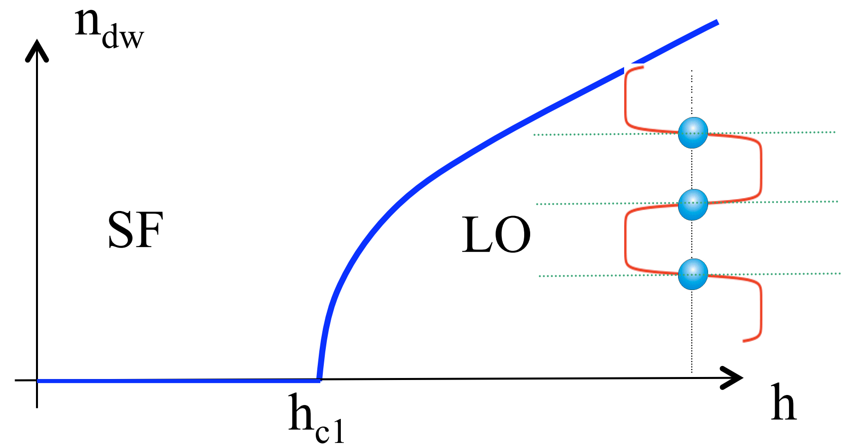
  (gapped and gapless k's)

$$E_{\mathbf{k}\uparrow/\downarrow\mathbf{Q}_i} = (\varepsilon_k^2 + \Delta_Q^2)^{1/2} \mp (h + \frac{\mathbf{k}\cdot\mathbf{Q}_i}{2m})$$



- *gapless fermionic excitations band of Andreev states:*



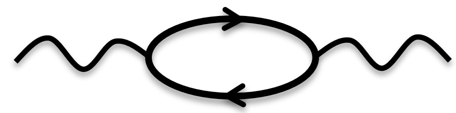


$$H_f^{ex} = \sum \left[E_{\mathbf{k}\sigma\mathbf{Q}_i}\Theta(E_{\mathbf{k}\sigma\mathbf{Q}_i})\alpha^\dagger_{\mathbf{k}\sigma\mathbf{Q}_i}\alpha_{\mathbf{k}\sigma\mathbf{Q}_i} - E_{\mathbf{k}\sigma\mathbf{Q}_i}\Theta(-E_{\mathbf{k}\sigma\mathbf{Q}_i})\alpha_{\mathbf{k}\sigma\mathbf{Q}_i}\alpha^\dagger_{\mathbf{k}\sigma\mathbf{Q}_i}\right]$$

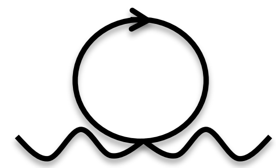
Fermion-Goldstone modes coupling in LO state

- ***supercurrent-current:*** $H_{j_s,j} \sim \nabla\theta \cdot \overline{\psi} i \nabla \psi + h.c.$

➡ $|\omega|\sigma_{ij}(\omega, \mathbf{q})\nabla_i\theta\nabla_j\theta$



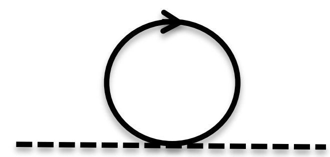
- ***supercurrent-density:*** $H_{j_s,n} \sim (\nabla\theta)^2\overline{\psi}\psi$



➡ $n_f(\nabla\theta)^2$

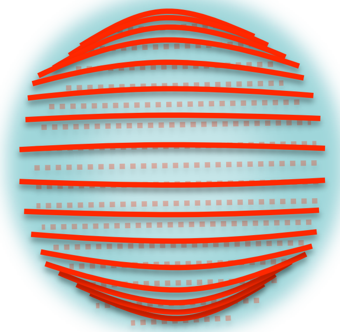
- ***atom-phonon:*** $H_{a-p} \sim \left(\partial_z u + \frac{1}{2}(\nabla u)^2\right)\overline{\psi}\psi + (\nabla u \cdot \overline{\psi} i \nabla \psi)^2 + h.c.$

➡ $n_f\left(\partial_z u + \frac{1}{2}(\nabla u)^2\right)$



- **How do these affect Goldstone modes and fermions?**
  - (*weak) Landau damping, finite corrections to* q_0, ρ_s, *K, B,…*
  - *fermions retain their anisotropic pocket Fermi surface*

Experiments

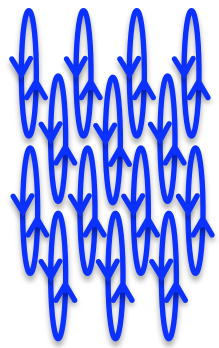


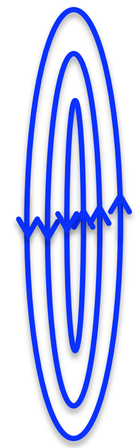
- *trap suppression of fluctuations:*

$$\Delta_{LO} \sim R^{-\eta(T)} \sim N^{-\frac{1}{5}\eta(T)} \sim \omega_{\mathrm{tr}}^{\eta(T)} \to 0$$

- *anisotropic vortices:*

$$\mathbf{v} = \sqrt{\rho_x^s \rho_y^s}\,\frac{(-y,x)}{\rho_y^s x^2 + \rho_x^s y^2}$$

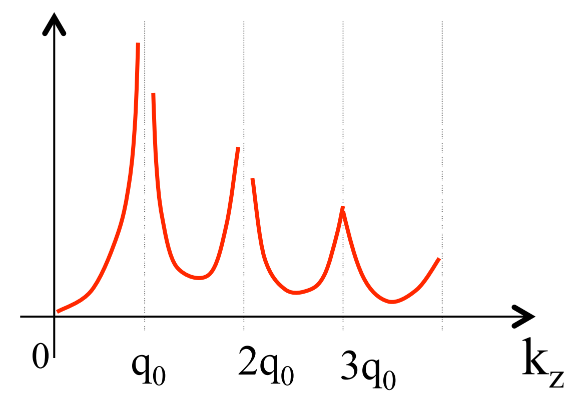




novel vortex phases?

- *π-vortices:*

$$n_v = 4\Omega_r \frac{m}{\hbar}$$

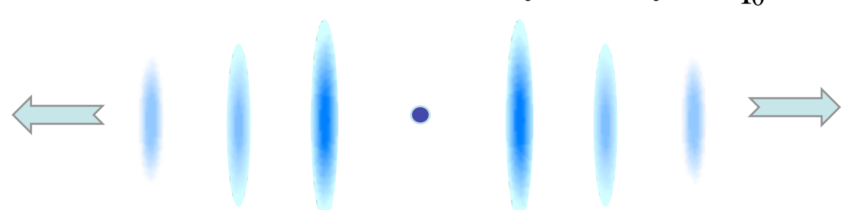


- *momentum distribution (time of flight):*

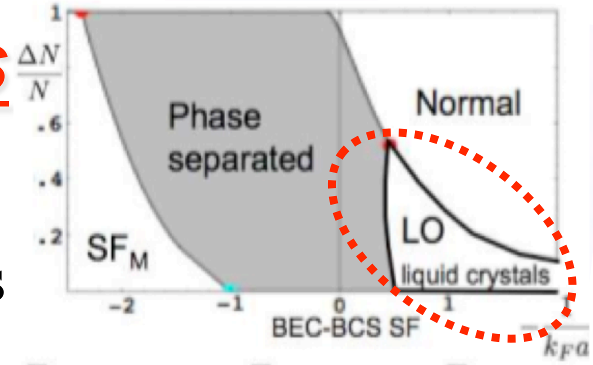
$$n_k \sim \frac{1}{|k_z - q_0|^{2-\eta}}$$

- *structure function:*

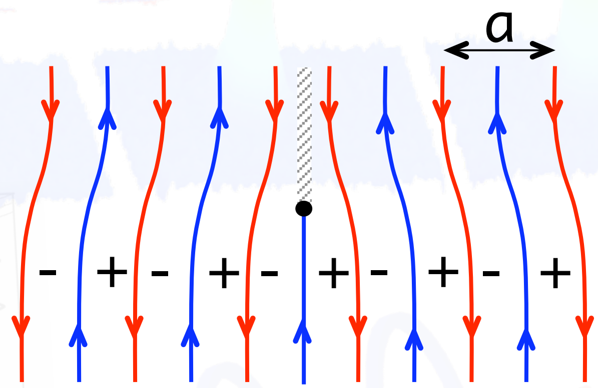
$$S(k_z) \sim \frac{1}{|k_z - 2q_0|^{2-4\eta}}$$



Summary and directions



- Larkin-Ovchinnikov state ⇔ superfluid smectic
- critical phase at finite T with universal properties
- half-integer vortex and dislocation defects
- transitions to N-Sm$_{2Q}$ and SF$_4$-Nm ("charge"-4 SF nematic) phases



...many remaining questions:

- effects of Fermi pockets - Goldstone modes interactions?
- better microscopic support for the energetics?
- connection to experimental knobs: detuning and imbalance?
- explore further experimental consequences, detection signals?
- charge-4e SC? …

Next time

- L1: AMO renaissance overview
- L2: Feshbach two-atom scattering
- L3: S-wave Feshbach resonant superfluidity
- L4: Imbalanced s-wave resonant Fermi gases
- L5: P-wave Feshbach resonant superfluidity