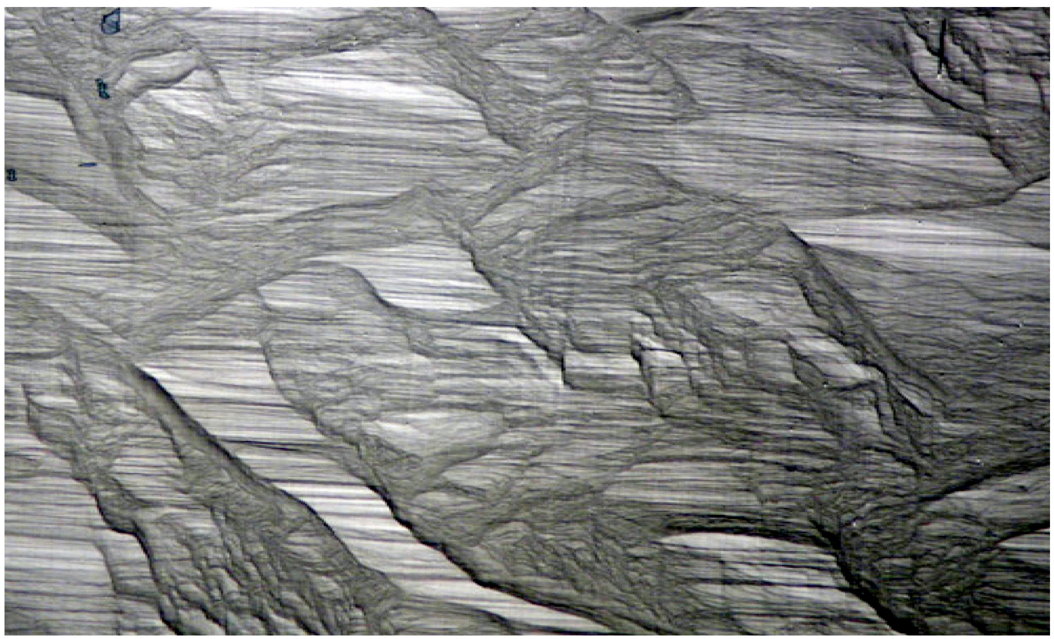
Liquid crystal cells with a "dirty" substrate

with Quan Zhang



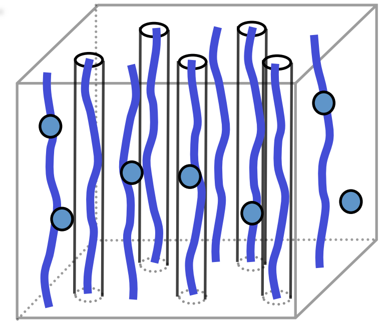
Jones and Clark

support by: NSF MRSEC, NSF Materials Theory

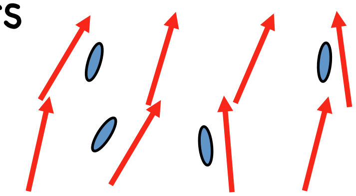
Outline

- Motivation for random pinning problems
- Realization of surface pinning: liquid crystal cells
- Questions of interest
- Models: nematic and smectic cells
- Analysis
- Challenges
- Conclusions

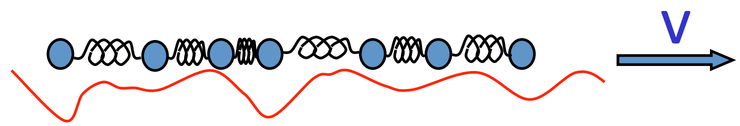
Randomly-pinned systems



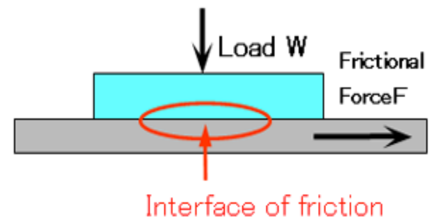
- Bulk-pinned systems (well explored)
    - Vortex lattices in type II superconductors



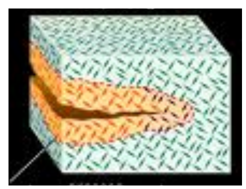
    - Magnets with impurities
    - Charge density waves in metals



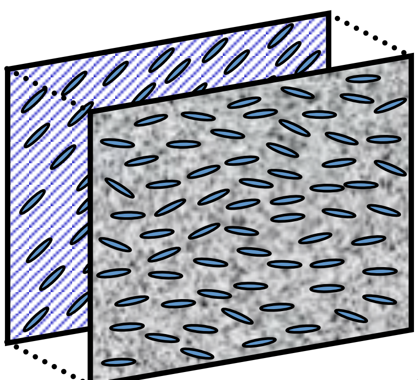
- Surface-pinned systems (nearly unexplored)



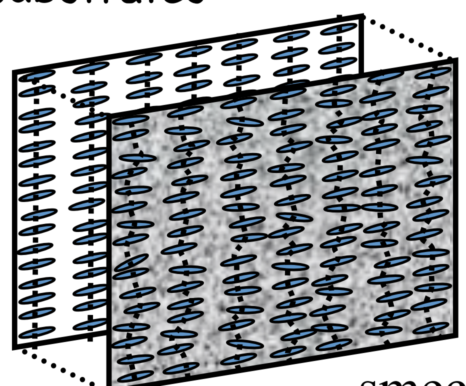
    - Friction and earthquakes



    - Cracks
    - Liquid crystal cells with patterned or "dirty" substrates



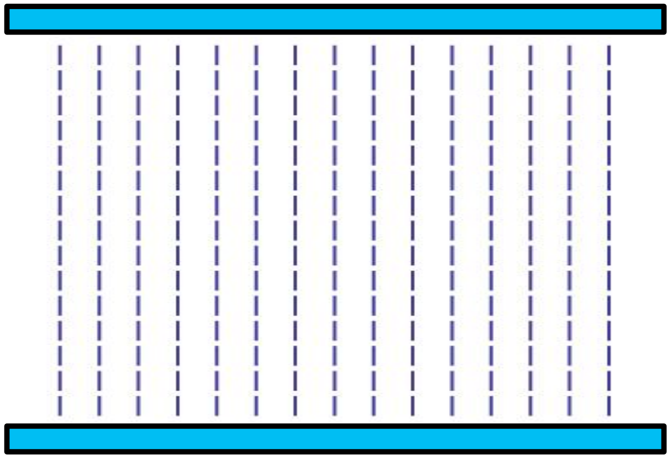
nematic cell



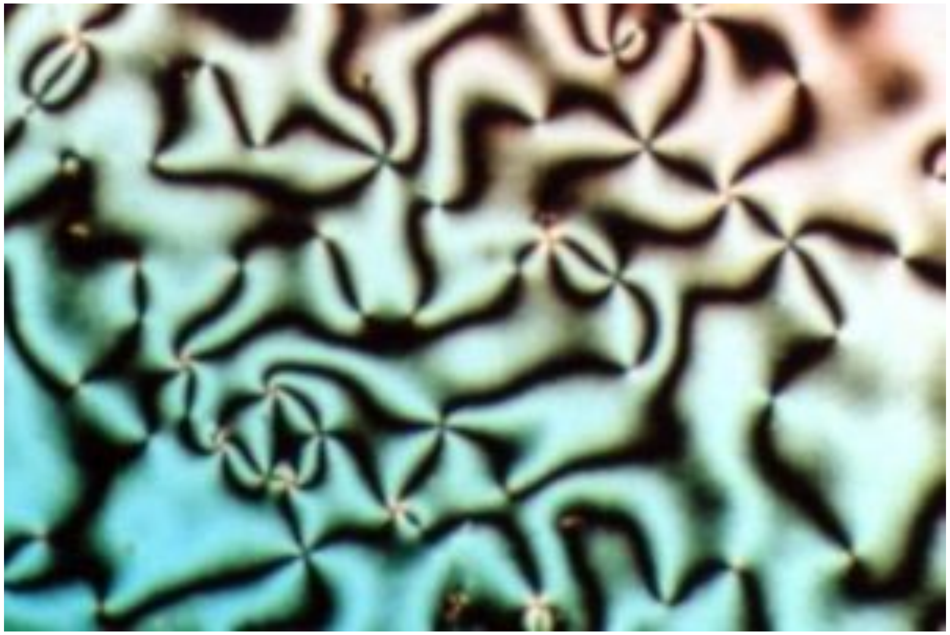
smectic cell

Motivation

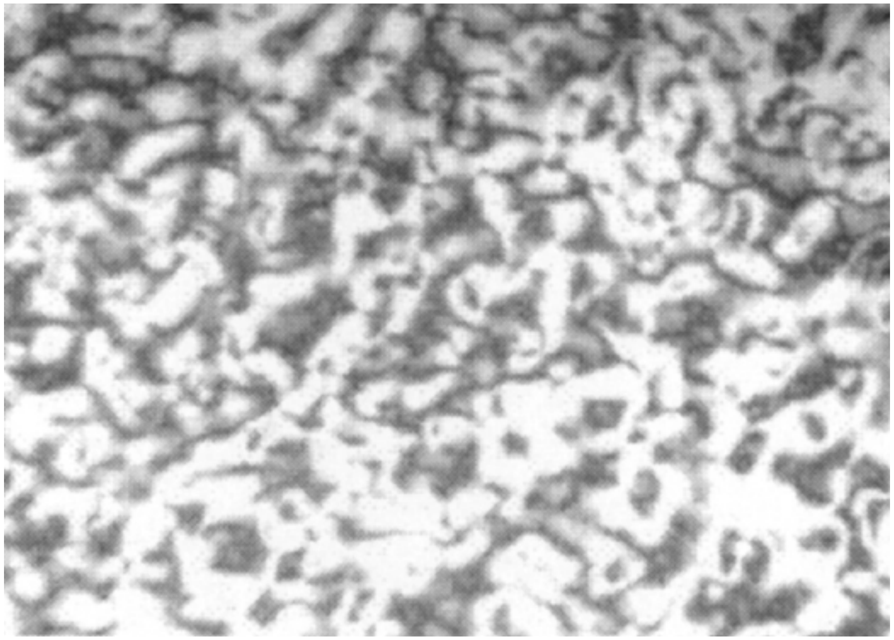
- basic scientific interest
- cell geometry in all applications
- surface pinning is essential
- random heterogeneous pinning if not rubbed:



differentially heated nematic cell



Schlieren texture in a Nematic cell

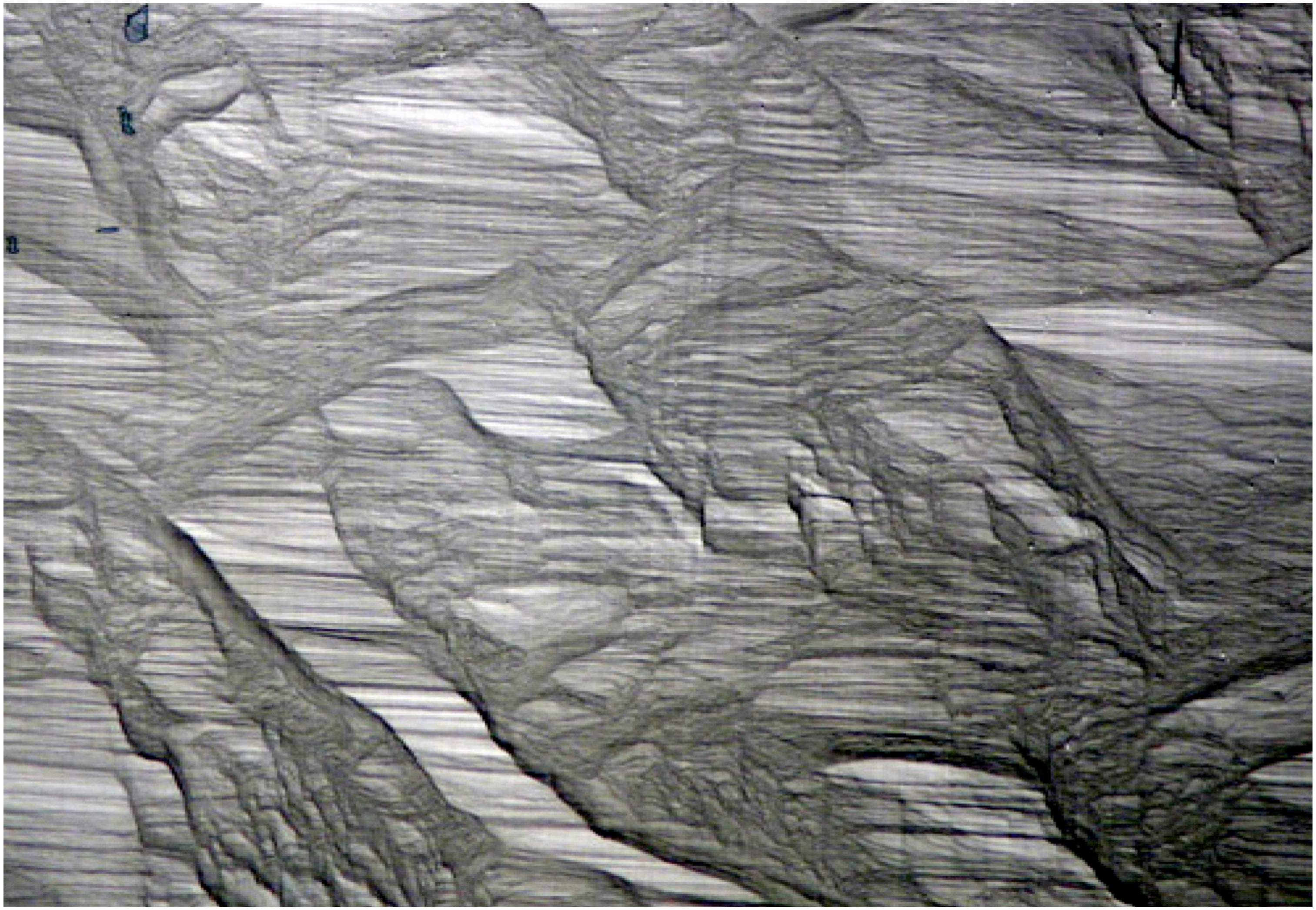


N. Aryasova et. al., Mol. Cryst. Liq. Cryst. (2004)

Riverbottom texture

smectic-C cell imaging stresses due to layers surface pinning

Jones and Clark

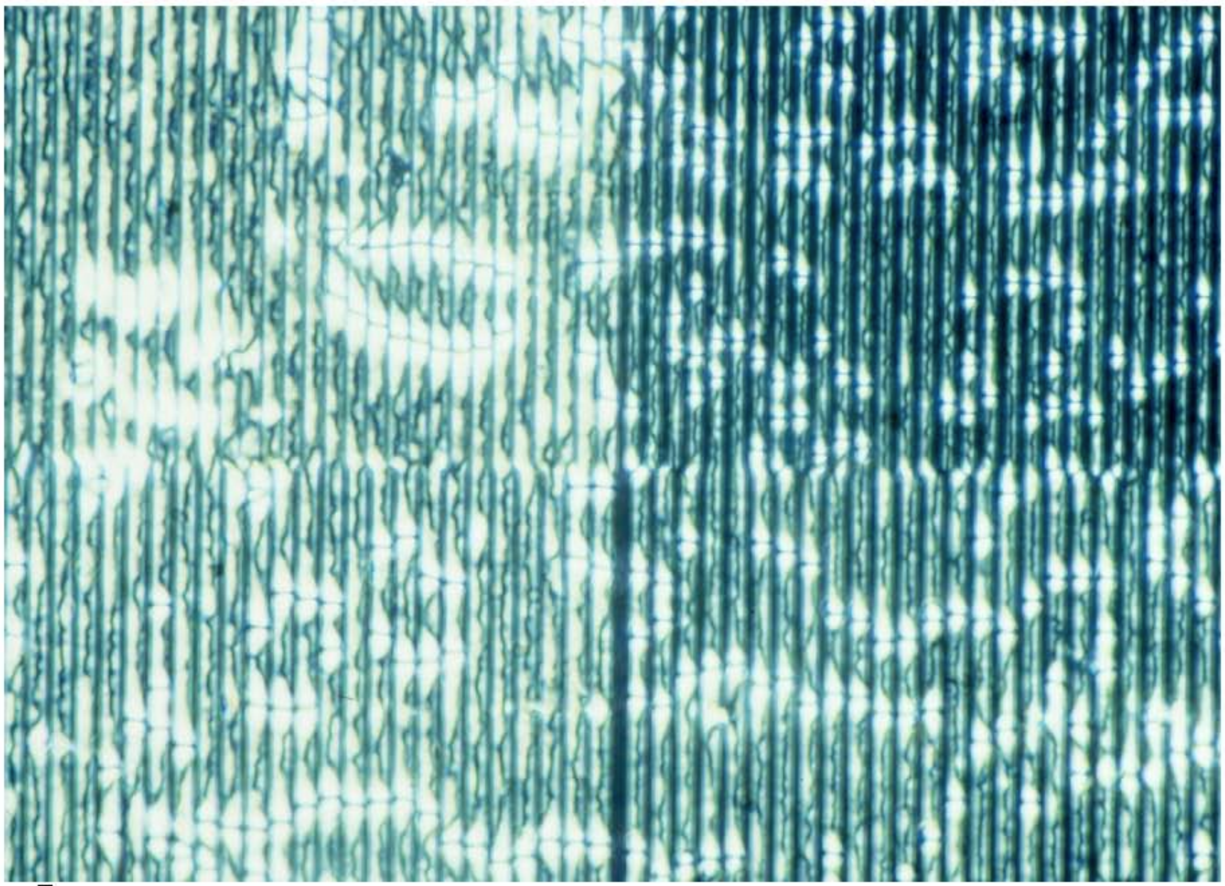


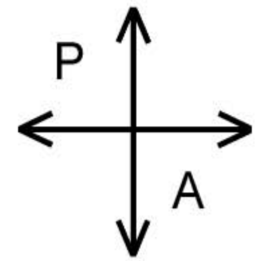
Experimental realizations

5μm cell with 5CB

Pattern:
stripes/spaces
in [μm]

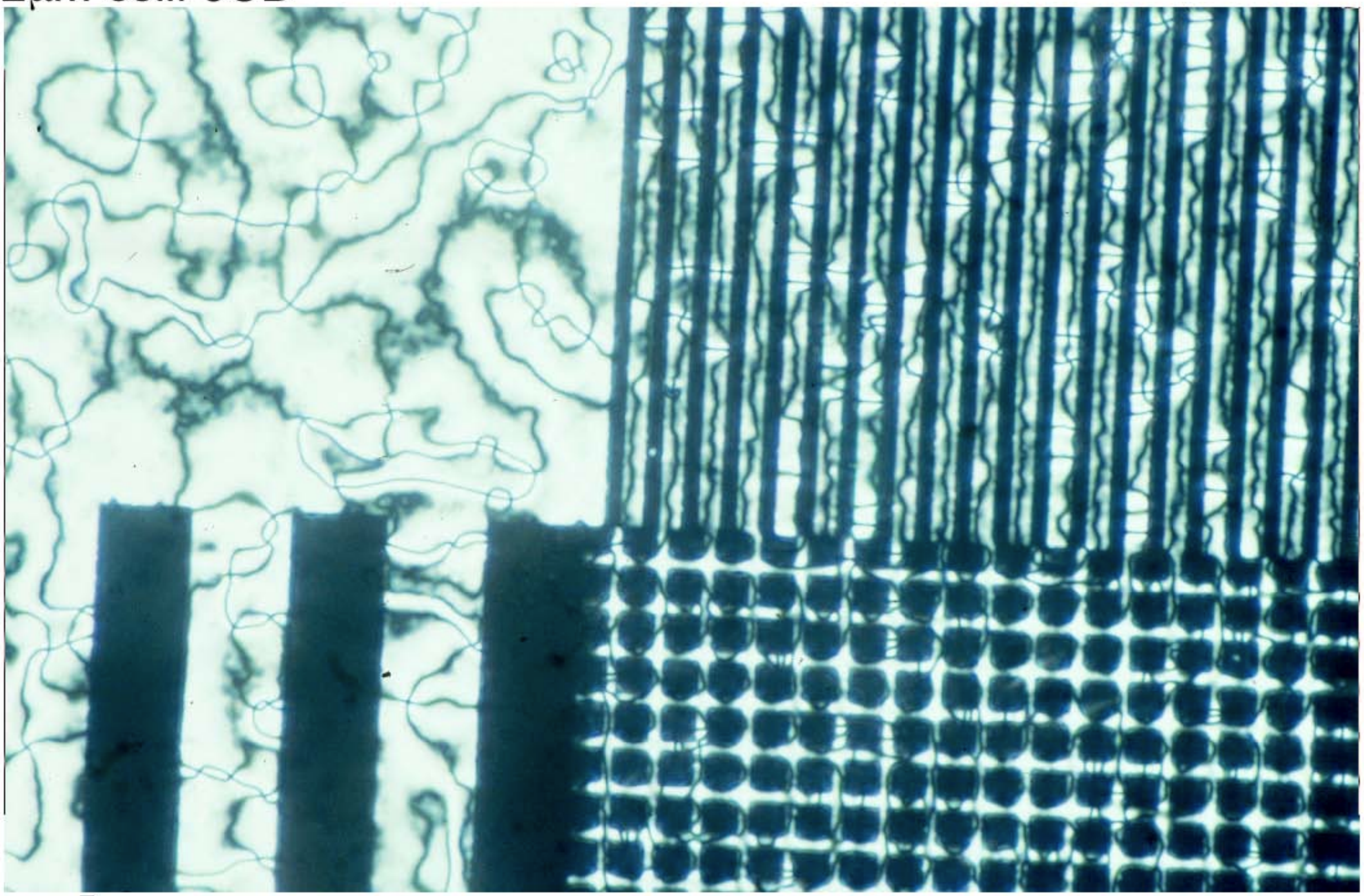
2/4	4/2
3/4	4/3





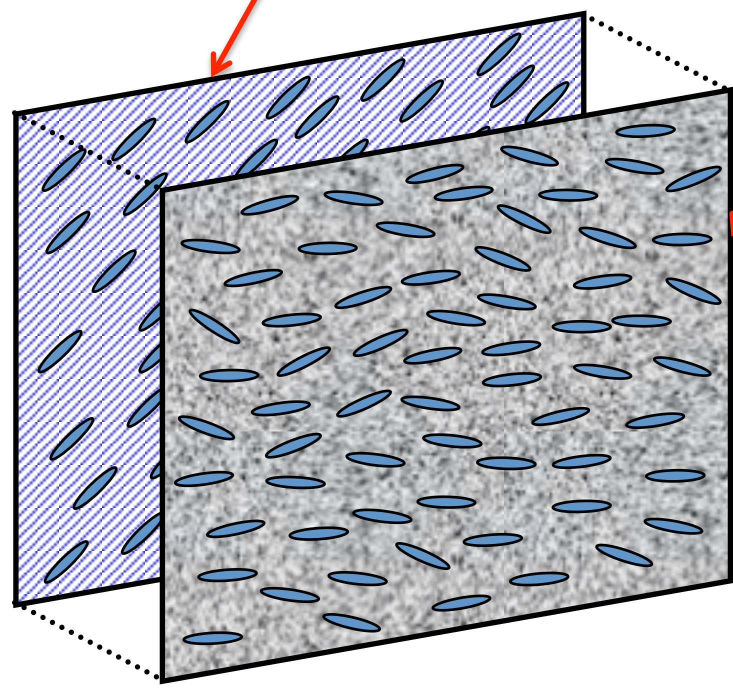
Experimental realizations

2μm cell: 5CB



Nematic cell

nematic cell with a rubbed and heterogenous substrates



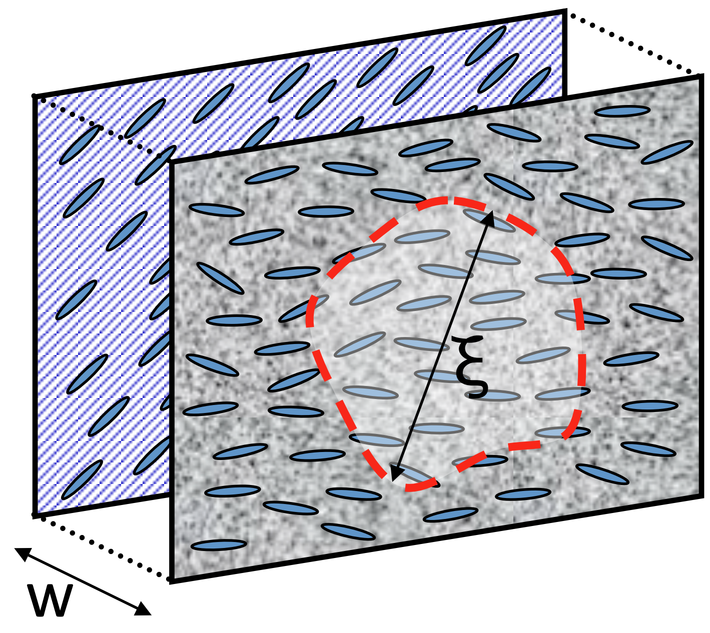
Competition between bulk interactions → ordering

and

substrate random pinning → disordering

Questions of interest

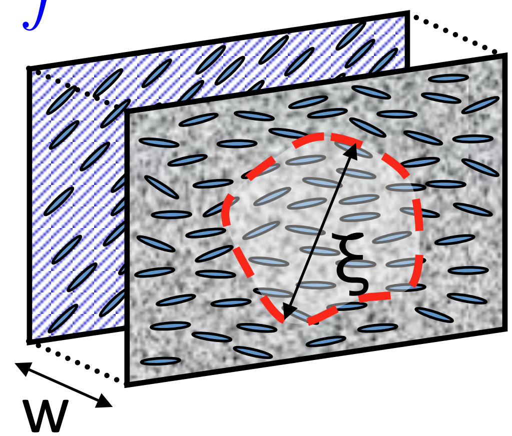
- stability of the ordered state to surface pinning?
- nature of the random state?
- domain size of ordered regions?
- statistics of random distortions
- effect of homogeneous substrate?
- elastic distortions vs topological defects?
- glassy dynamics?
- …



Imry-Ma energy balance

- Elastic energy of domain size ξ: $H_F = \frac{K}{2}\int d^2x dz (\nabla \hat{n})^2$

$$E_{elastic} \sim K L^{d-2}$$



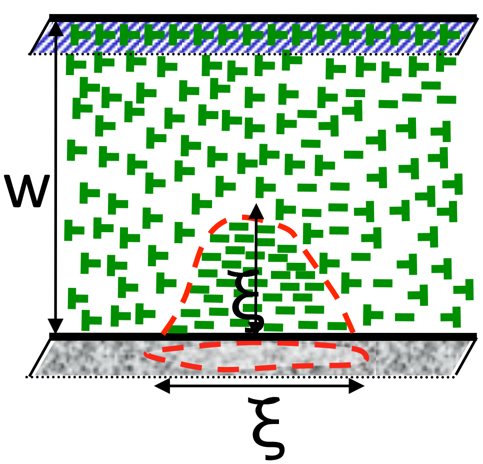
- Pinning energy of domain size ξ: $H_{pin} = -\int d^2x dz \delta(z)\Big(g(\mathbf{r})\cdot\hat{n}\Big)^2$

$$E_{pin} \sim V_p\sqrt{N_p} \sim V_p(L/\xi_0)^{(d-1)/2}$$

- Finite domain size for d ≤ d_c=3 (for thick cell):

  → pinning always wins, *no LR nematic order*

$$\xi_L \approx a e^{K^2/\Delta_p}$$ (in 3d)



Nematic cell model

- bulk Hamiltonian:

$$H = \int_{-\infty}^{\infty} d^2x \int_0^w dz \left[\frac{K}{2} (\nabla \phi(\mathbf{r}))^2 - V[\phi(\mathbf{r}), \mathbf{x}]\delta(z) \right].$$

- bulk vs surface modes: $\phi(\mathbf{q}, z) = \tau(\mathbf{q}) \frac{Kq\cosh[q(w-z)] + W_p \sinh[q(w-z)]}{KqW_p\cosh[qw] + K^2q^2\sinh[qw]}$

$$\phi^{(\infty)}(q, z) = \phi_0(q)e^{-qz}, \quad w \longrightarrow \infty$$

$$\phi^{(D)}(v, z) = \phi_0(q)\frac{\sinh[q(w-z)]}{\sinh(qw)}, \quad \text{rubbed}$$

$$\phi^{(B)}(v, z) = \phi_0(q)\frac{\cosh[q(w-z)]}{\cosh(qw)}, \quad \text{smooth}$$

- reduce to a substrate Hamiltonian: $\Gamma_q^{(\infty)} = Kq, \ \ w \to \infty$

$$H_s = \int_{q_\perp} \Gamma_q^{(a)} |\phi_0(q)|^2 - \int d^2x_\perp V[\phi_0(x_\perp), x_\perp]$$

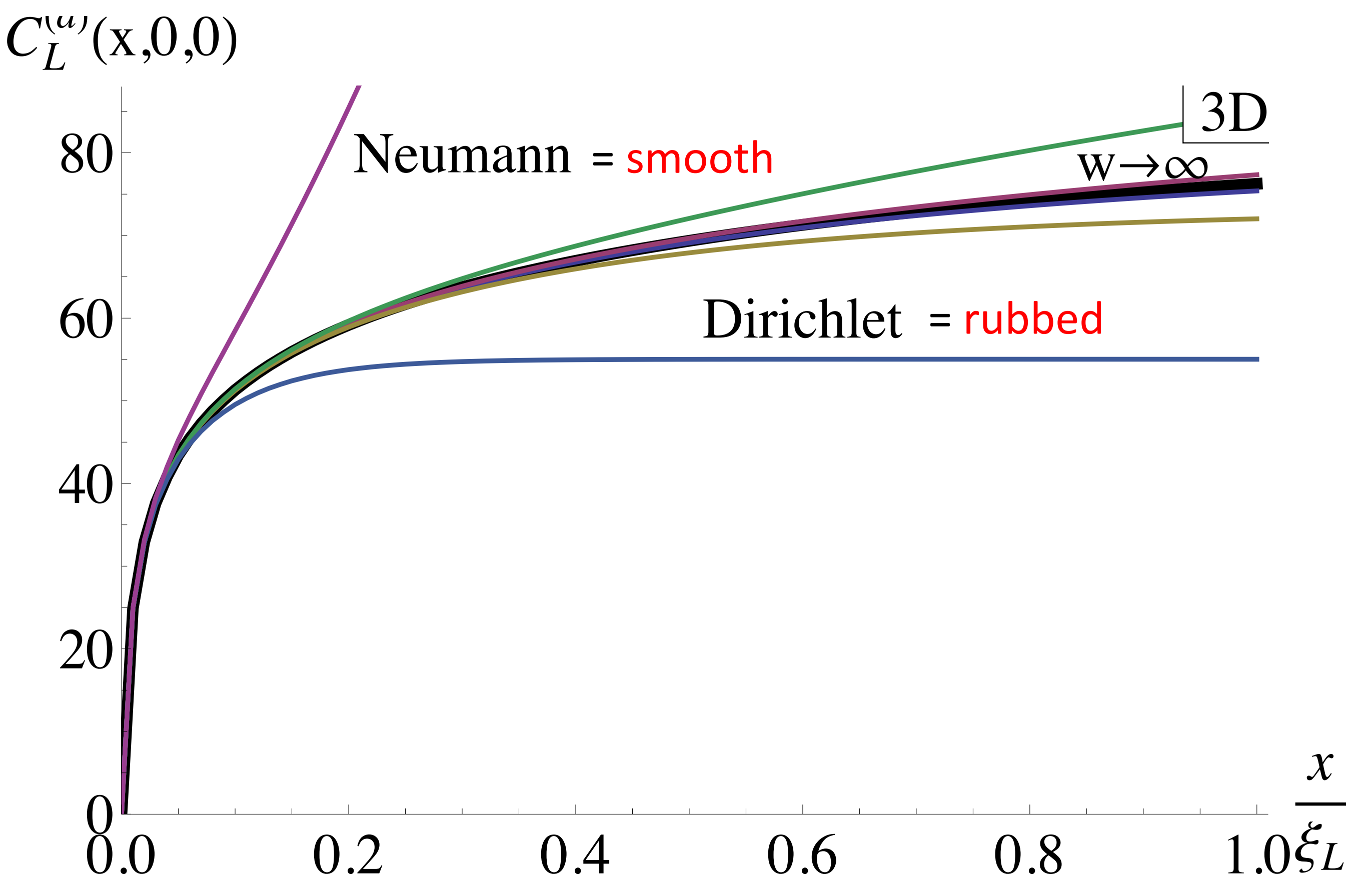
$$\Gamma_q^{(D)} = Kq\coth(qw), \ \text{rubbed}$$

$$\Gamma_q^{(N)} = Kq\tanh(qw), \ \text{smooth}$$

Director correlations

- *short-scales* director distortions on substrate, z=0:

$$C(\mathbf{x}, z, z) = \overline{\langle(\phi(\mathbf{x}, z) - \phi(0, z))^2\rangle}$$

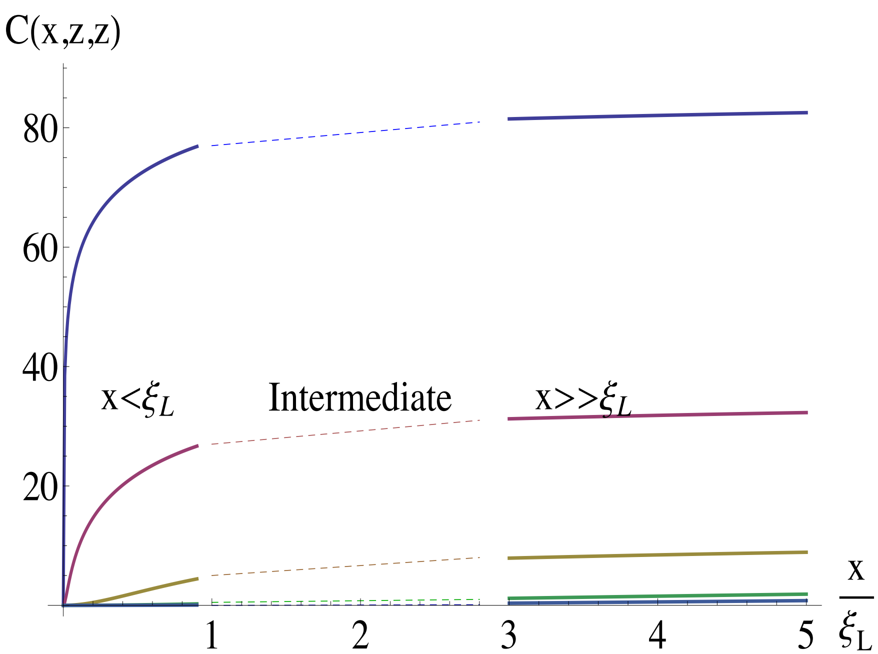


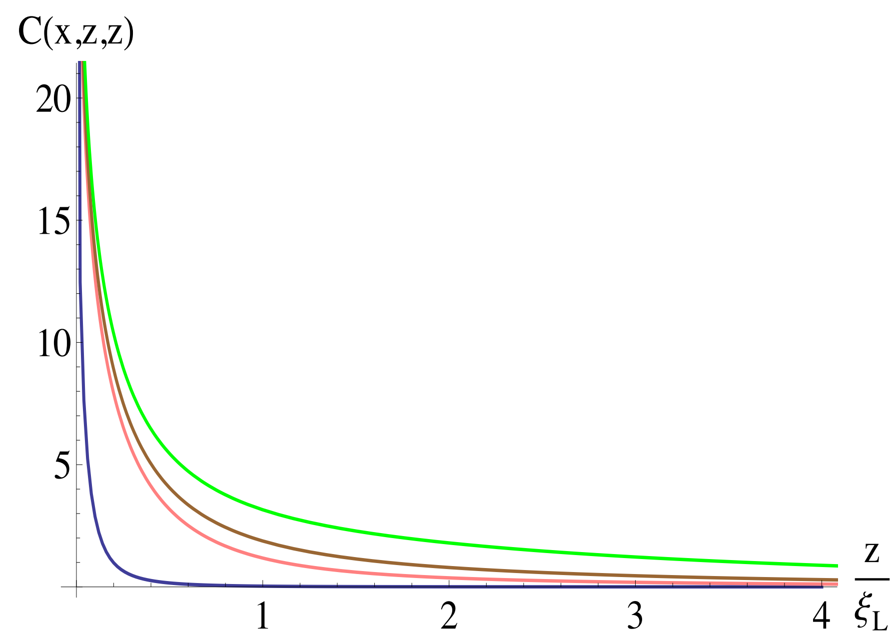
Director correlations

- director distortions at z: $C(\mathbf{x},z,z) = \overline{\langle(\phi(\mathbf{x},z)-\phi(0,z))^2\rangle}$

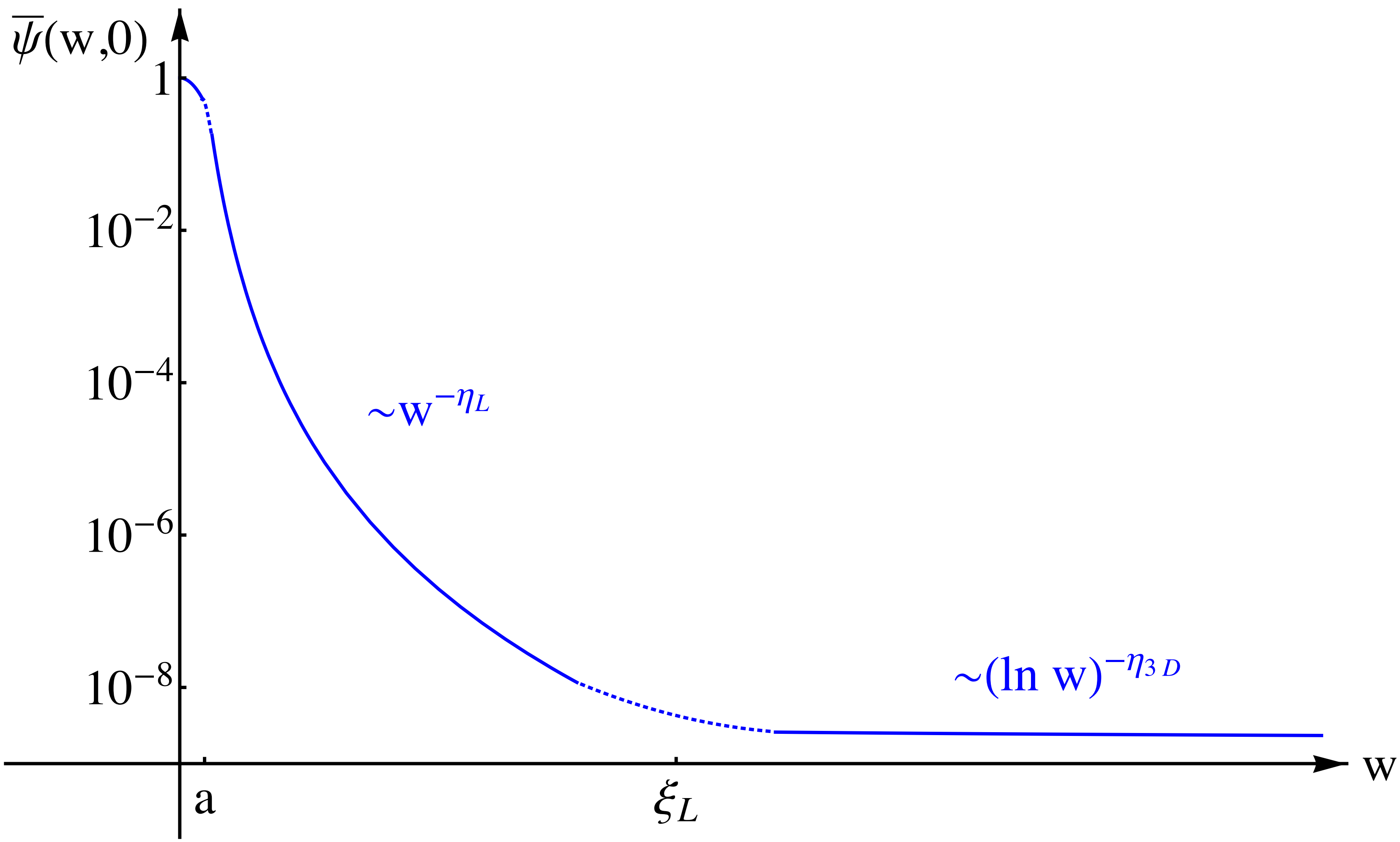
$$C^{(\infty)}(x,0,0) \approx \begin{cases} \frac{8\pi^2}{\ln\xi}\ln x, & x \ll \xi, \\ 8\pi^2 + \frac{2\pi^2}{9}\ln(\ln x), & x \gg \xi_L. \end{cases}$$

$$\boxed{C^{(\infty)}(x,z,z) \approx b e^{-2z/\xi_L} + \frac{2\pi^2}{9}\ln\left[\frac{\ln x}{\ln(\xi(z))}\right]}$$



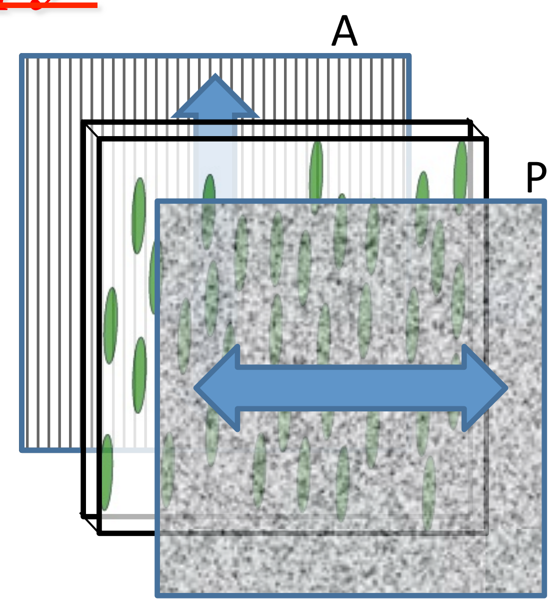


Surface nematic order parameter

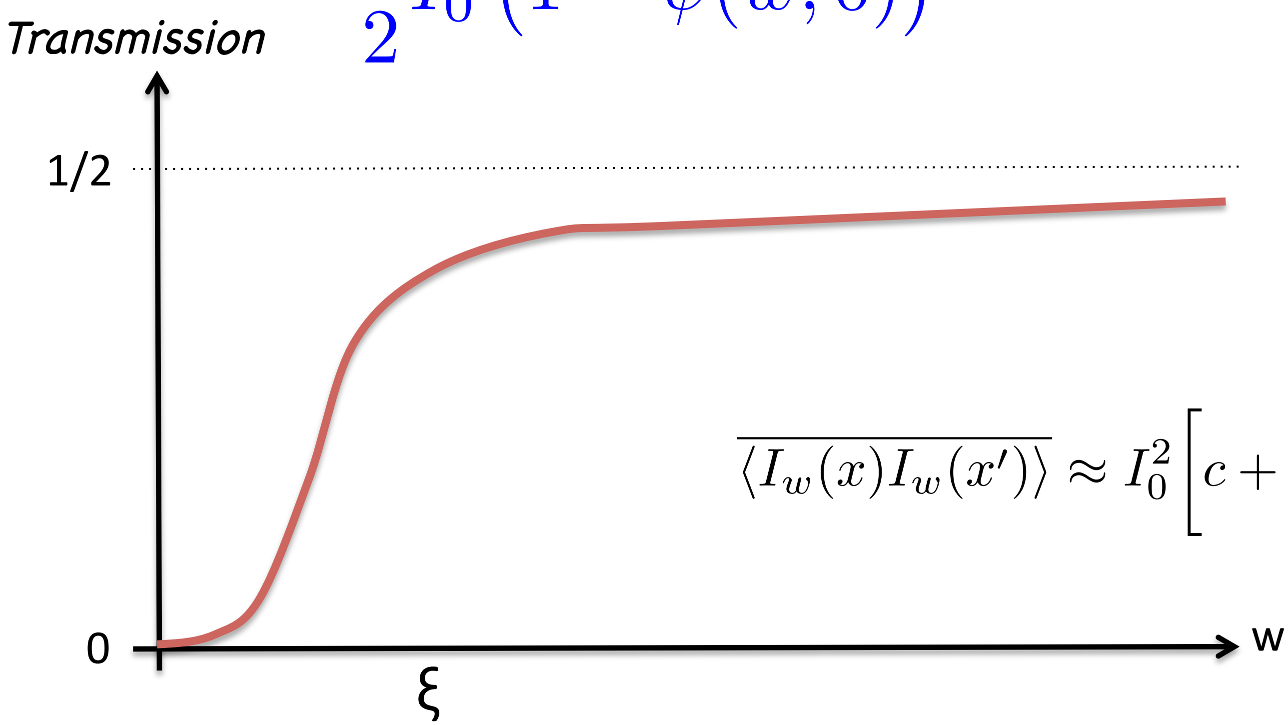


Polarized light microscopy

- rubbed back substrate along A
- Mauguin limit
- transmission through cell w:



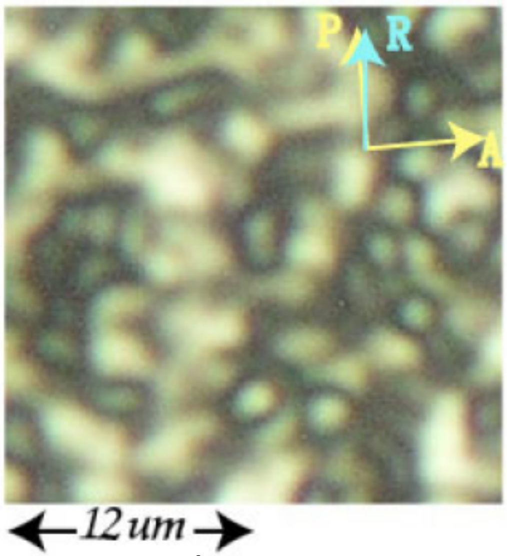
$$I_w = \overline{\langle I_0 \sin^2[\varphi(x,0)] \rangle} = \frac{1}{2} I_0 \left(1 - \overline{\psi}(w,0)\right)$$



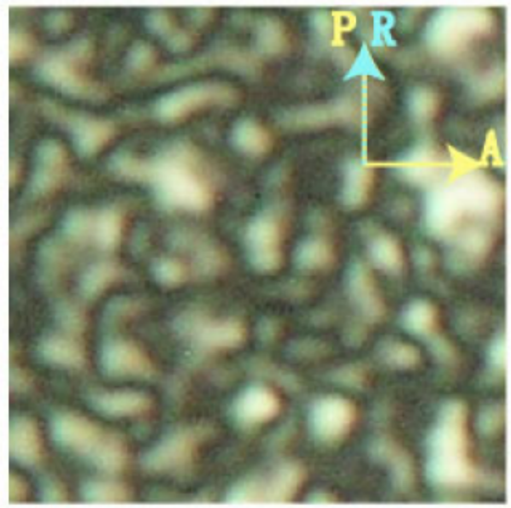
$$\overline{\langle I_w(x) I_w(x') \rangle} \approx I_0^2 \left[c + \frac{1}{8} e^{-\overline{\langle (\phi_0(x) - \phi_0(x'))^2 \rangle}/2} \right]$$

Recent experiments on nematic cells

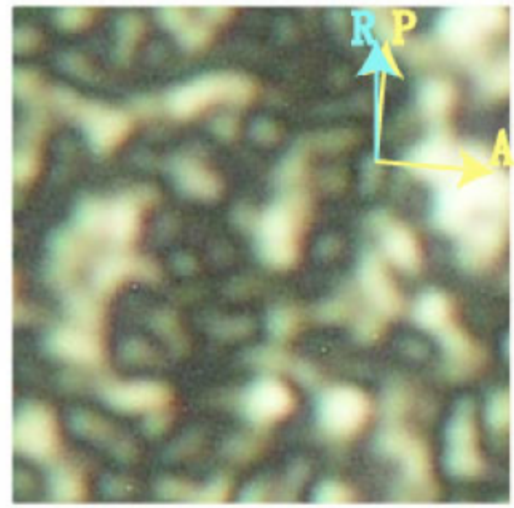
collaboration with Yue Shi and Noel Clark:



-5 degrees

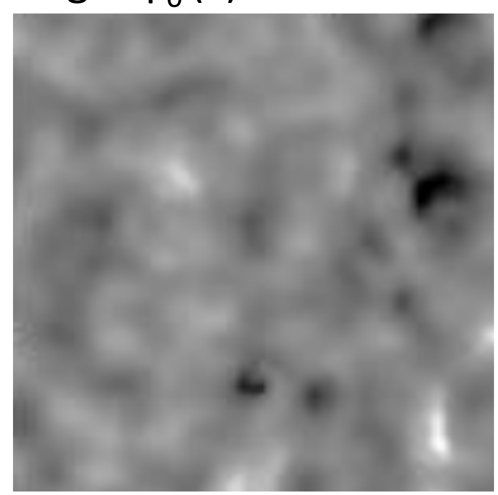


0 degrees



5 degrees

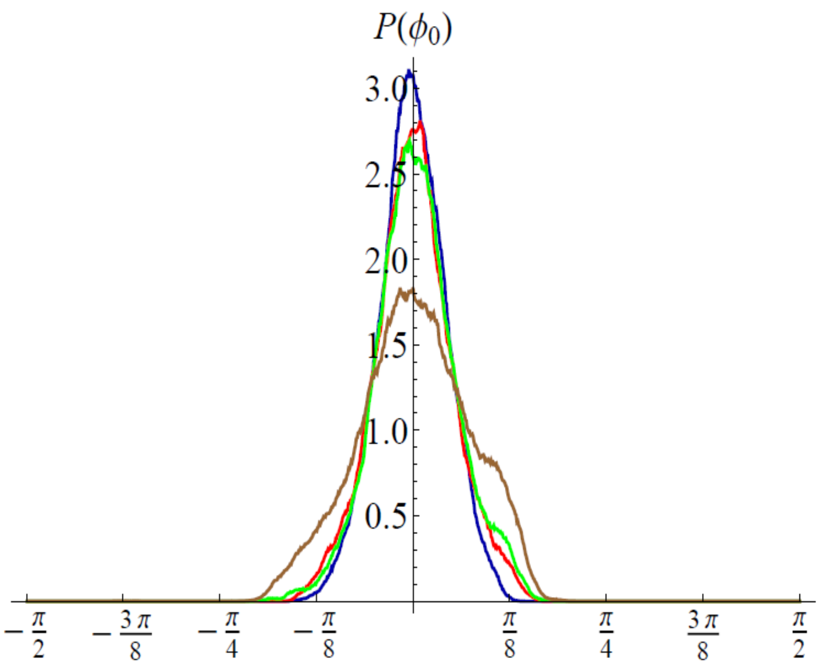
angle $\varphi_0(x)$ distribution

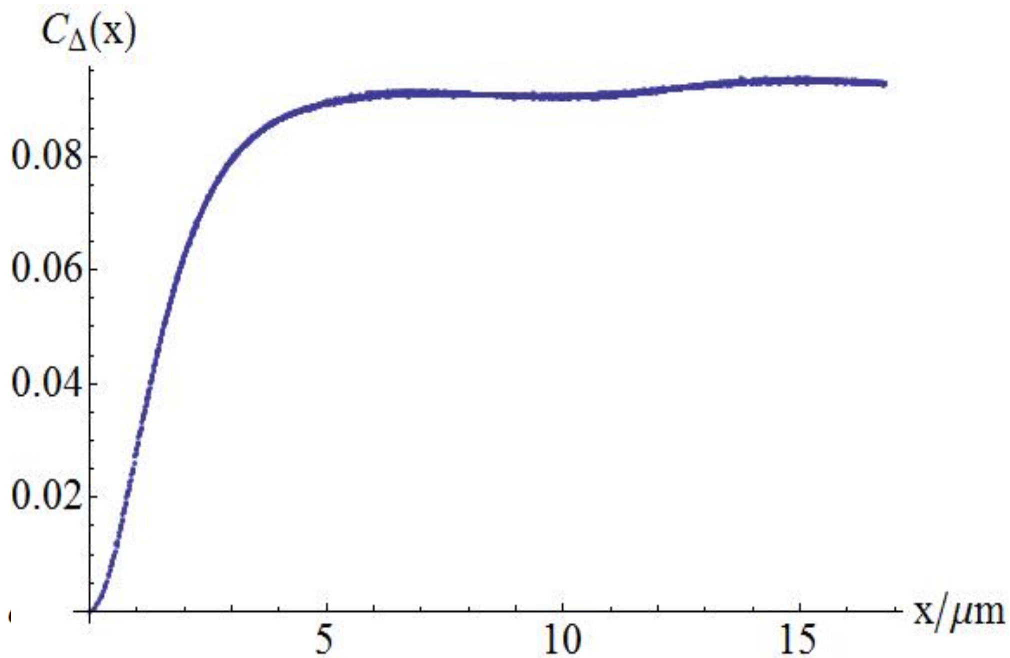


$$I(\phi(x), x) = I_0 + I_1 \sin^2[2\phi - \phi_0(x)]$$

Recent measurements on nematic cells

Yue Shi, Quan Zhang



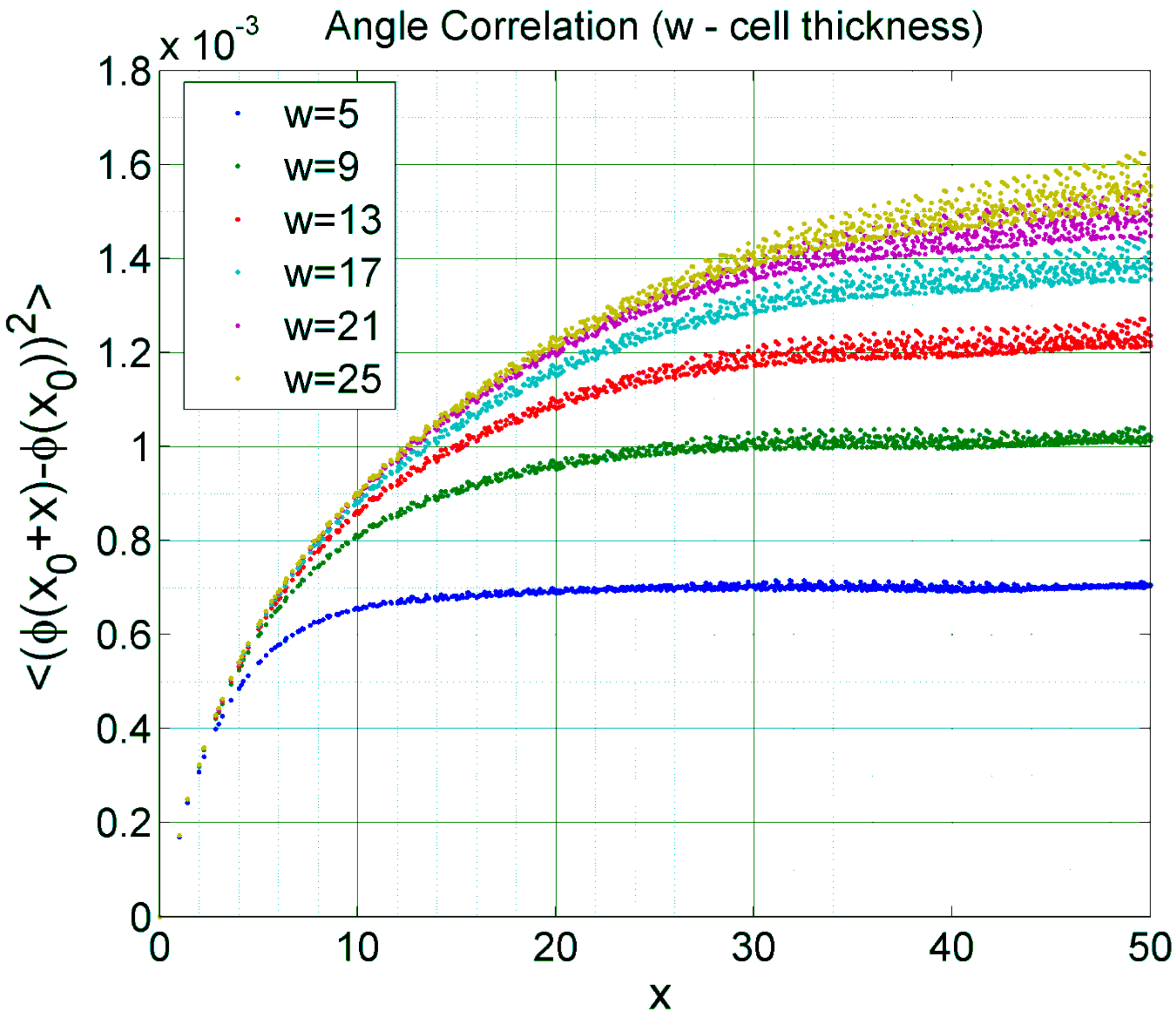


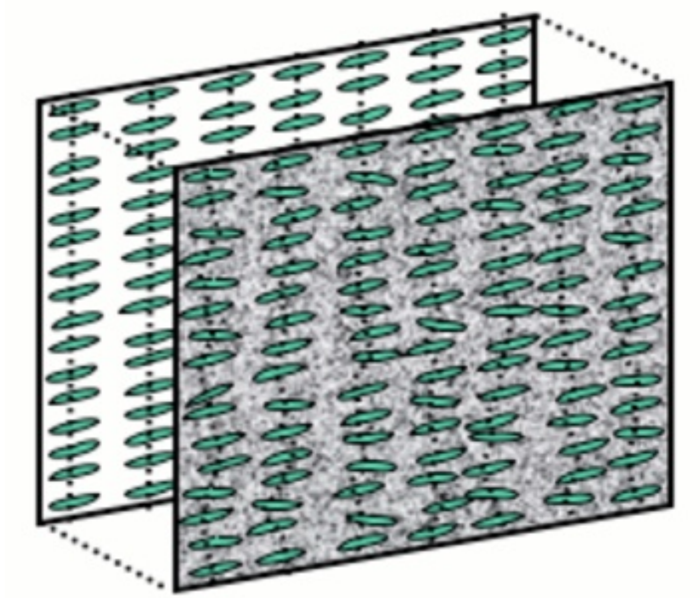
Distribution of azimuthal angle on the disordered substrate.

$$C_\Delta(x) = \overline{\langle [\phi_0(x'+x) - \phi_0(x')]^2 \rangle}$$

Monte Carlo simulations

Collaboration with Robin Selinger and Mikhail Pevnyi (KSU)

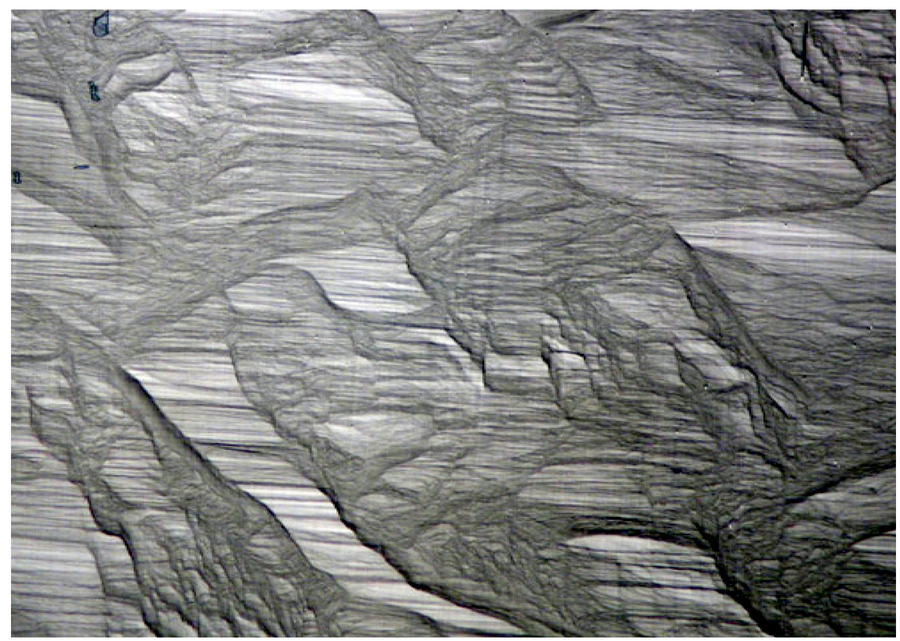




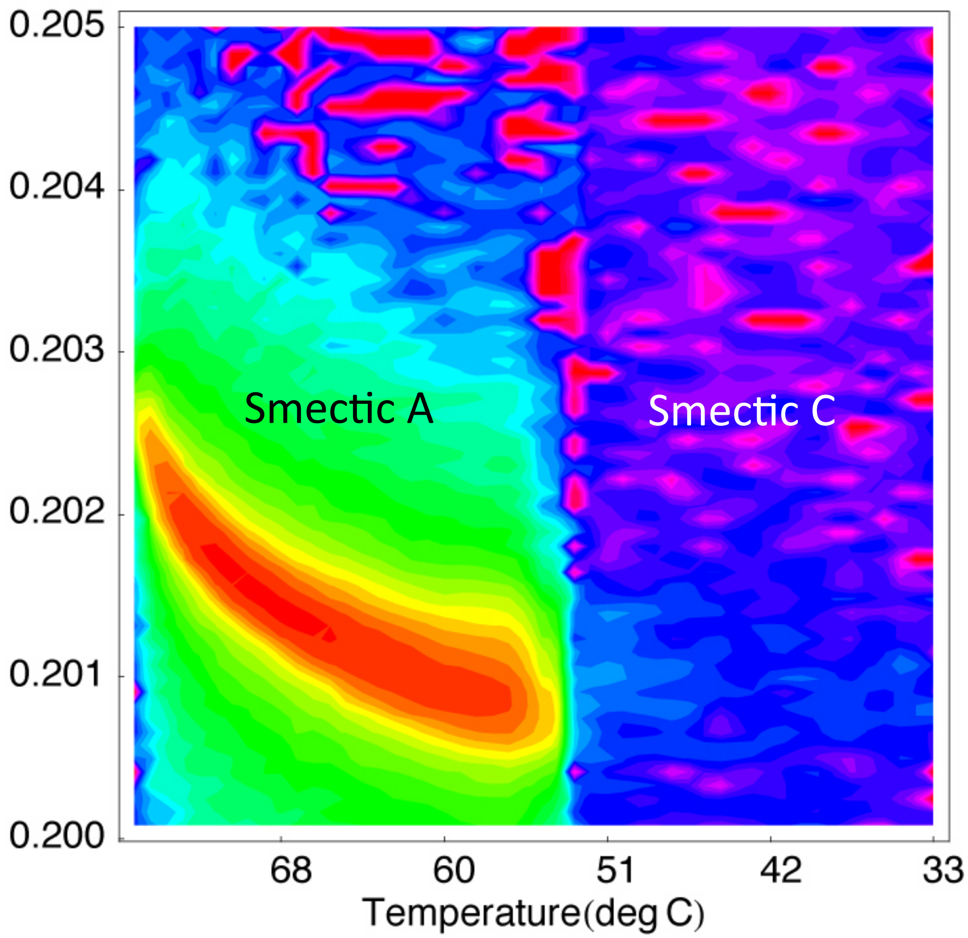
Smectic cell

What are the effects of substrate heterogeneity (disorder) on smectic order?

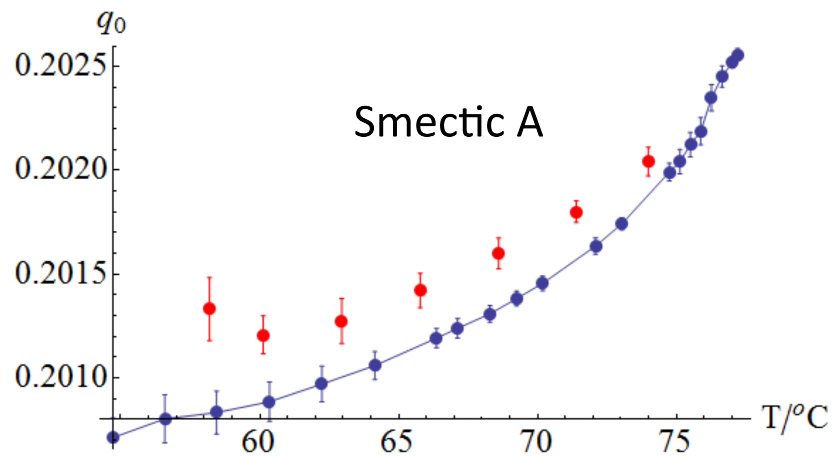
Jones and Clark



Broadening of X-ray peak at low T



C. D. Jones and N. A. Clark, Bull. Am. Phys. Soc. 49, 307 (2004).

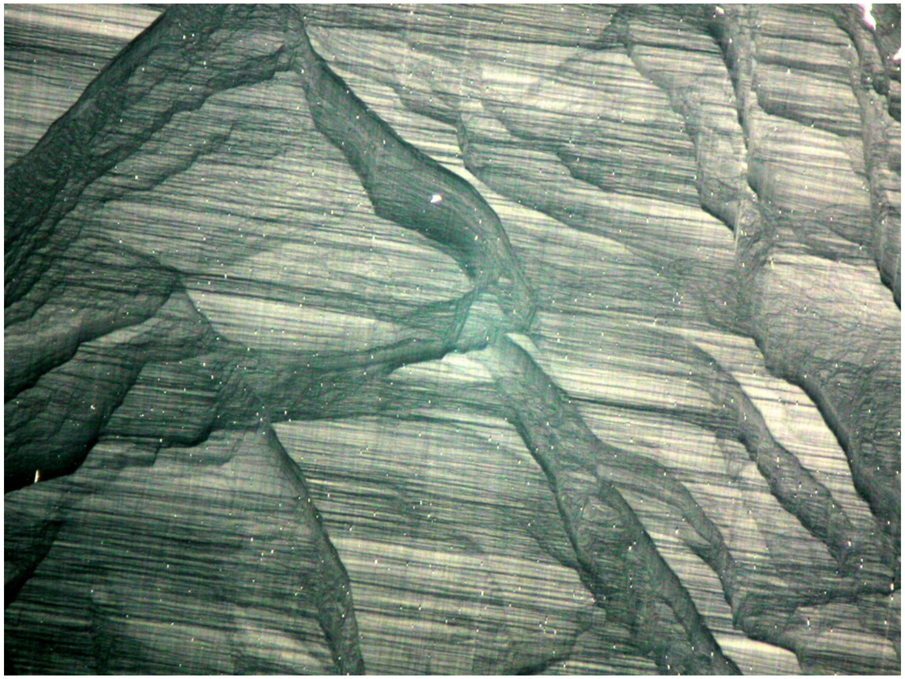


Red dots: data from bulk sample (in capillary).

Blue dots and line: data of surface disordered cell.

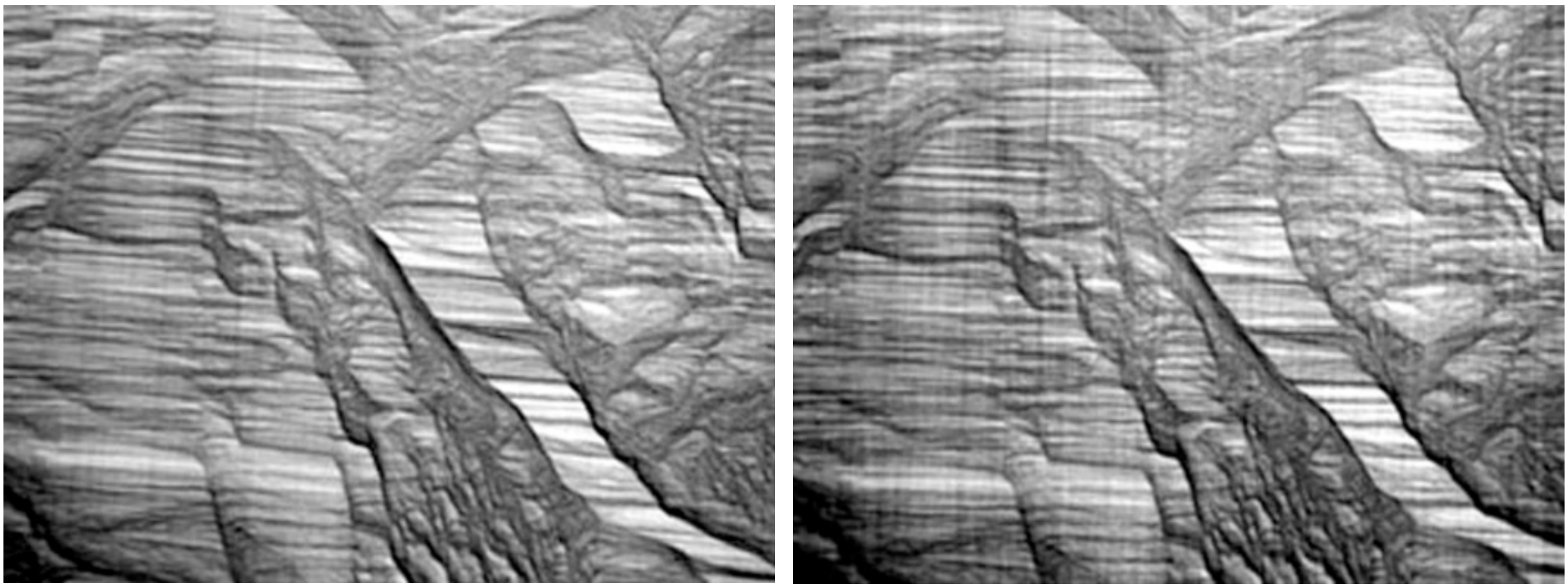
Polarized light microscopy

Sm A

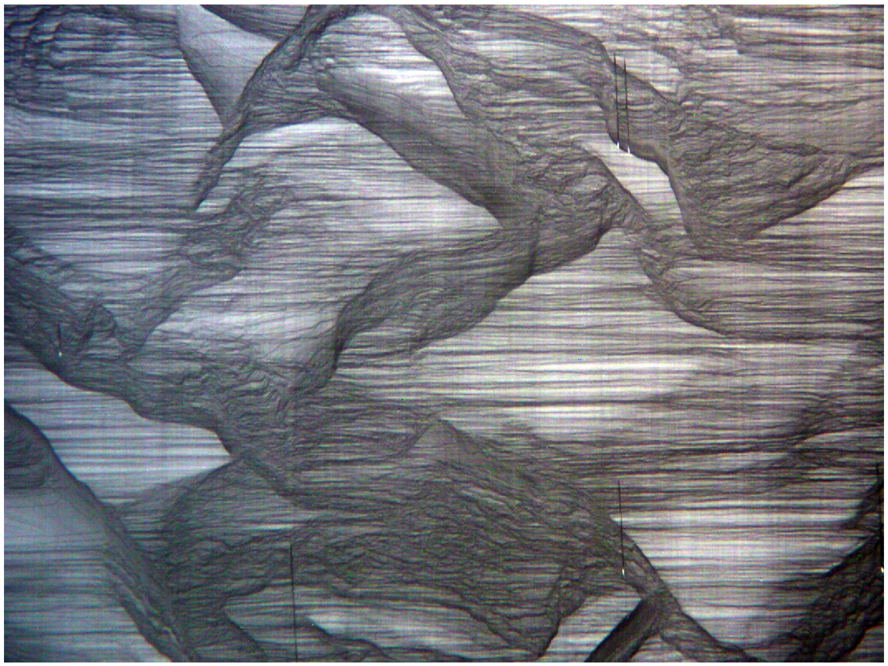


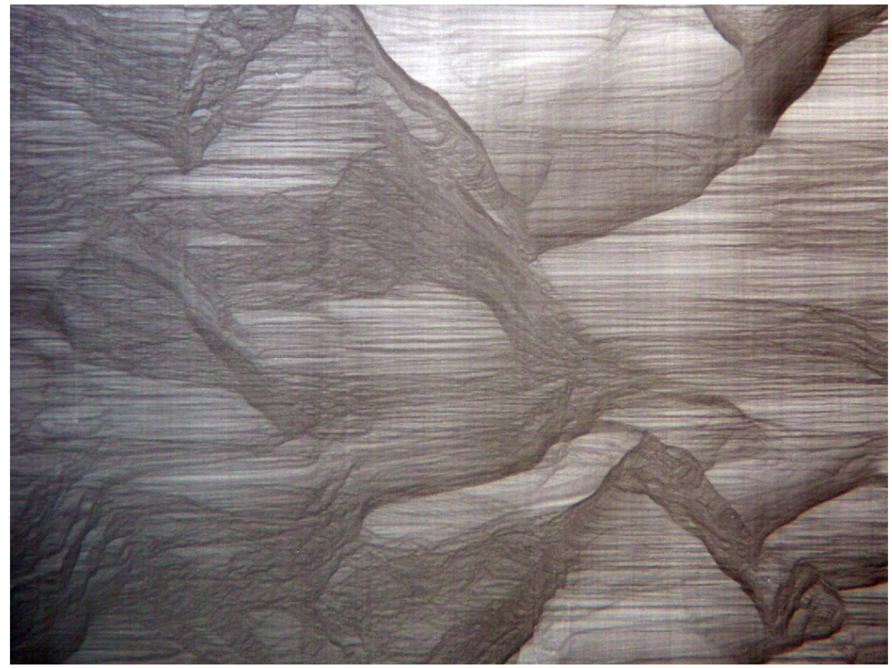
Sm C

Temperature cycle into Sm-A and back to Sm-C



Temperature cycle into Nematic

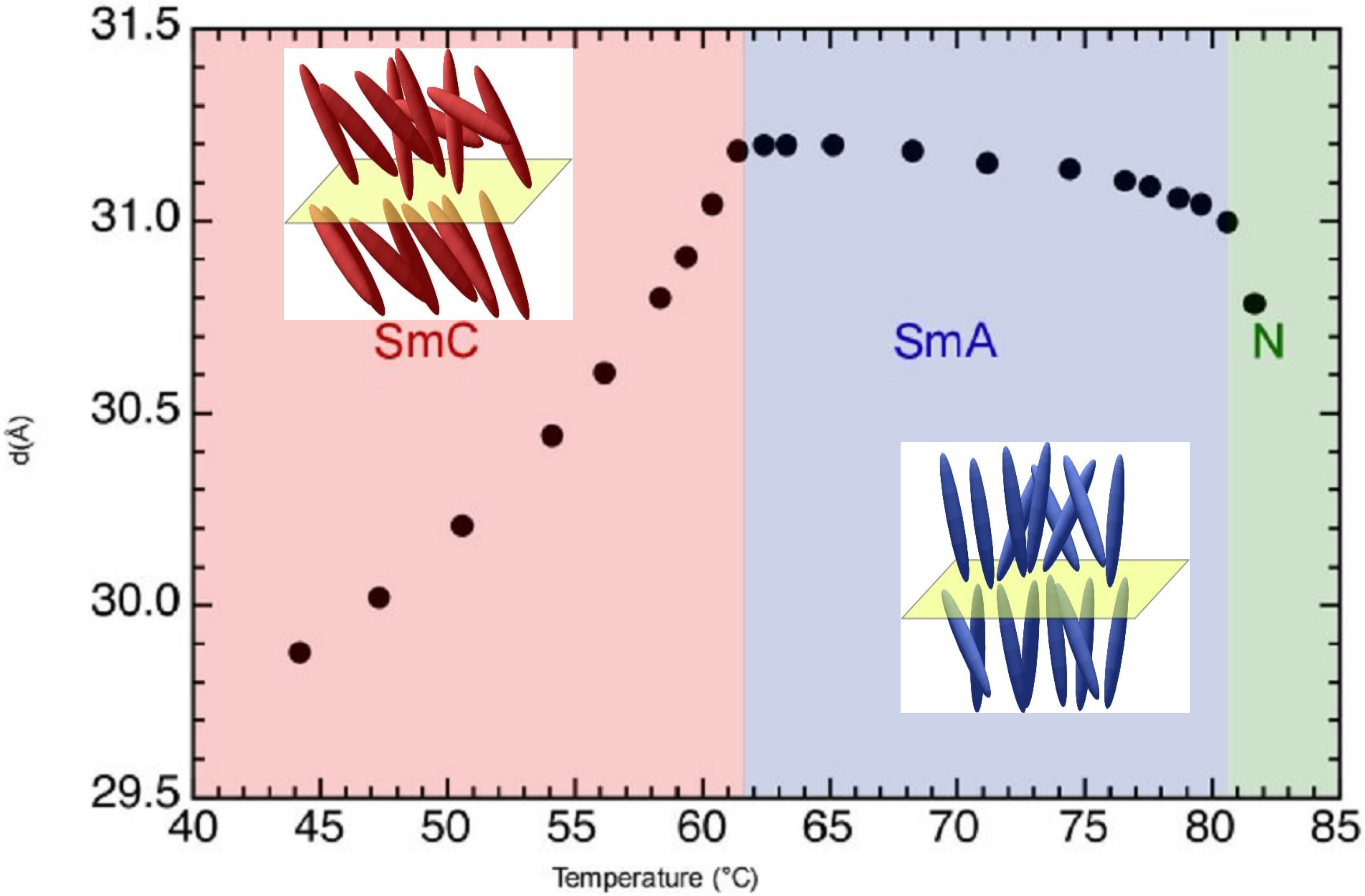




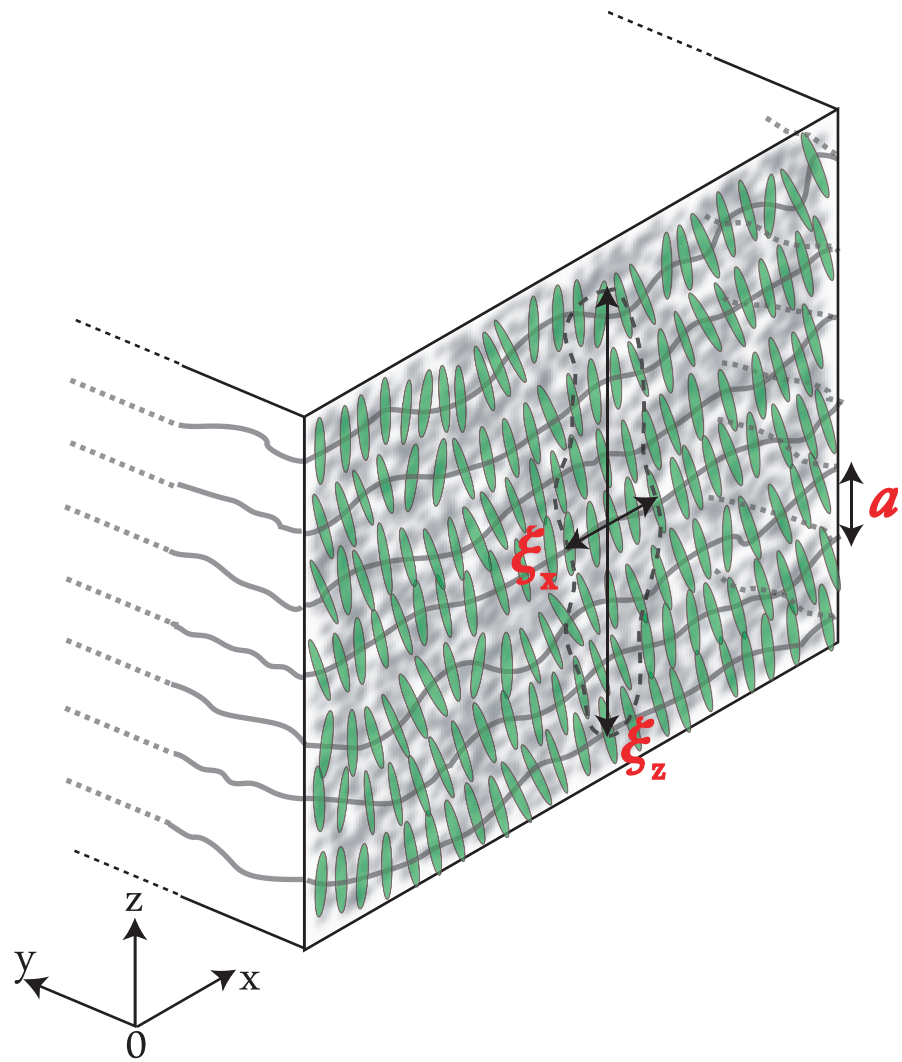
Both images are exactly the same area of the cell

Smectic layer spacing

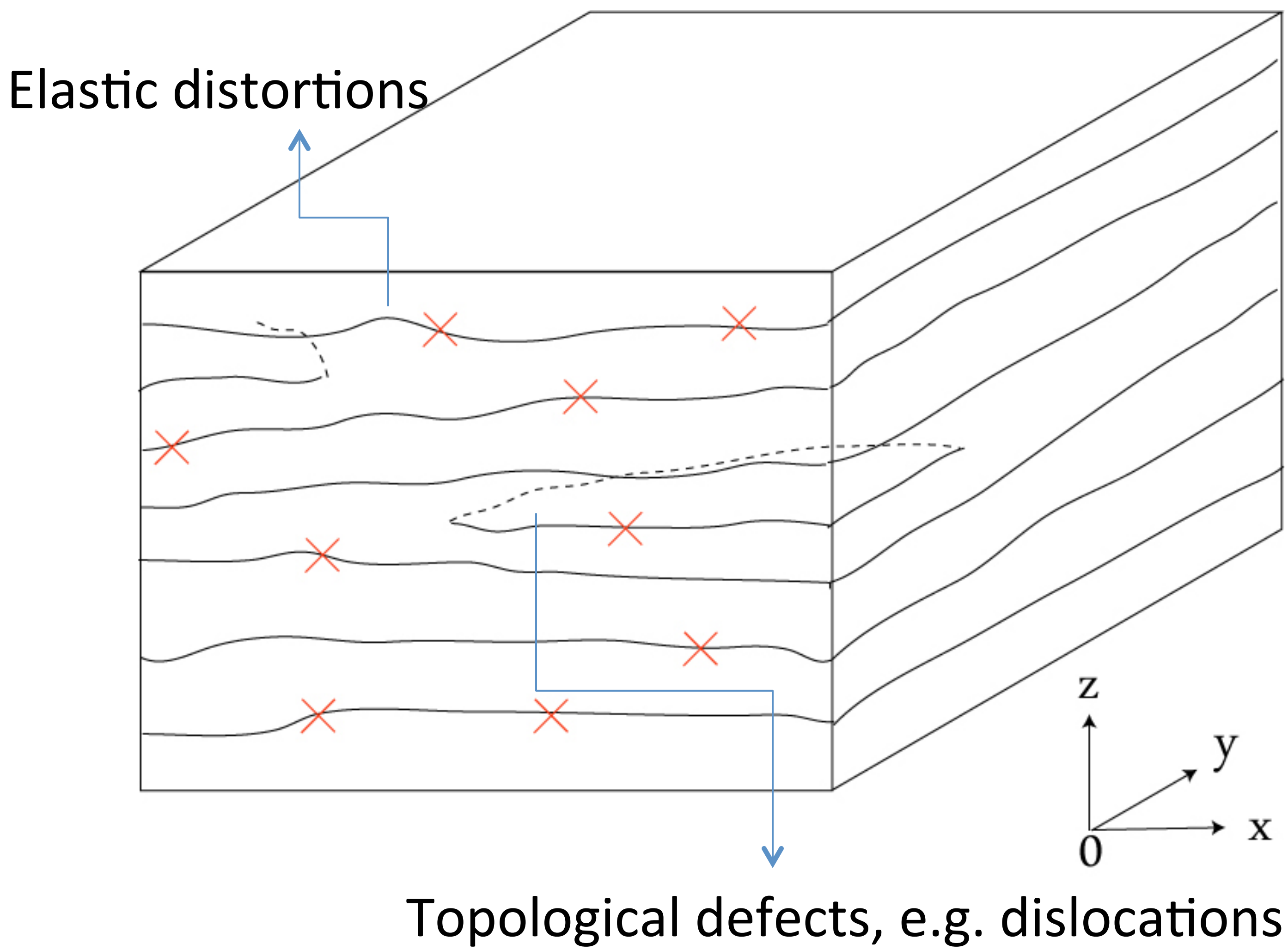
MX 8068: isotropic 85ºC nematic 81ºC SmA 62ºC SmC



Smectic cell



Substrate-induced distortions



Smectic cell bulk model

(bookshelf geometry)

- smectic elasticity:

$$H_{el} = \int d^2x_\perp \int_0^\infty dy \left[\frac{K}{2} (\nabla_\perp^2 u)^2 + \frac{B}{2} (\partial_z u)^2 \right]$$

- substrate pinning:

$$H_{pin} = \int d^2x_\perp [h(\mathbf{x}) \partial_x u_0 + V(u_0, \mathbf{x})]$$

Smectic cell substrate model

- smectic surface elasticity:

$$u(q_x, q_z, y) = u_0(q_x, q_z) e^{-\frac{y}{\xi_+(\mathbf{q})}} \left[\frac{\xi_-(\mathbf{q})}{\xi_+(\mathbf{q})} \sin\left(\frac{y}{\xi_-(\mathbf{q})}\right) + \cos\left(\frac{y}{\xi_-(\mathbf{q})}\right) \right]$$

$$\sqrt{2}/\xi_{q\pm} = \sqrt{\sqrt{q_x^4 + q_z^2/\lambda^2} + q_x^2}$$

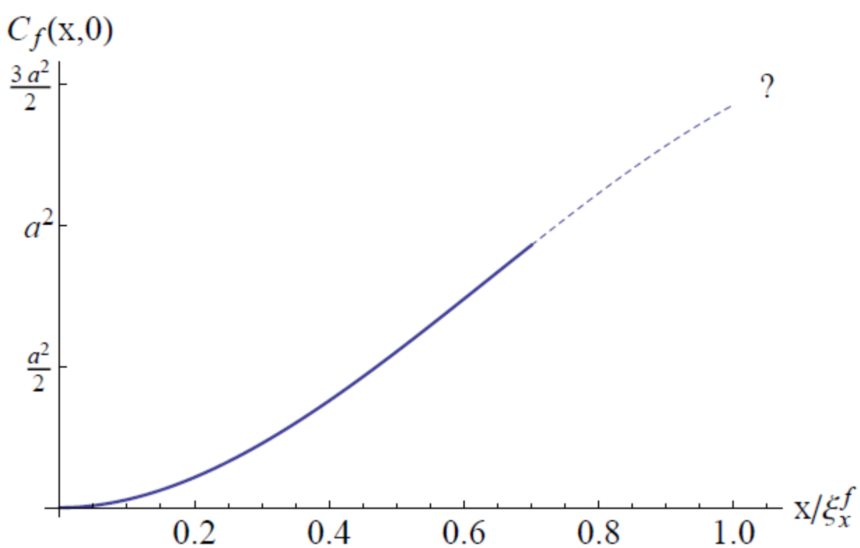
- substrate pinning:

$$H_{surface}[u_0] = \int_{q_\perp} \Gamma_{\mathbf{q}} |u_0(q_\perp)|^2 - \int d^2 x_\perp \left[h(\mathbf{x}_\perp) \partial_x u_0 + V(u_0, \mathbf{x}_\perp) \right]$$

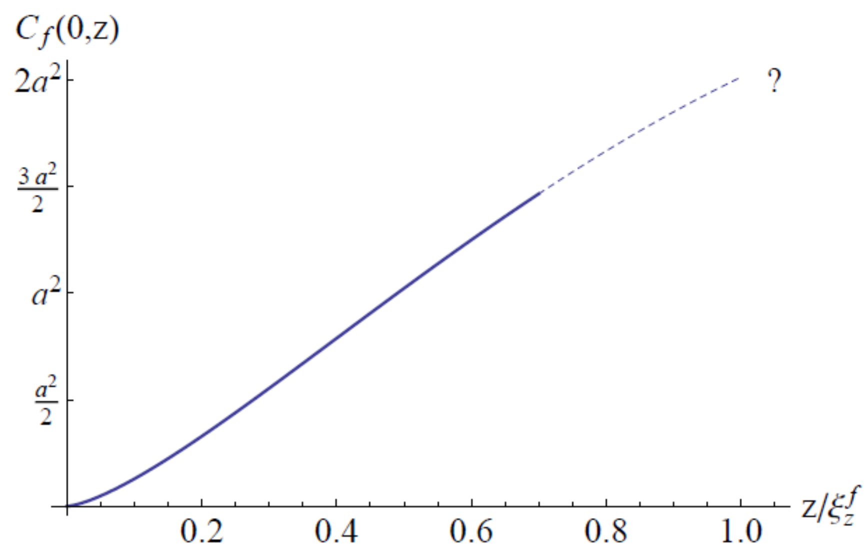
$$\Gamma_{\mathbf{q}} = \frac{2B\lambda}{\xi_{q+}} \sqrt{\lambda^2 q_x^4 + q_z^2}$$

Positional pinning correlations

- layer distortions:



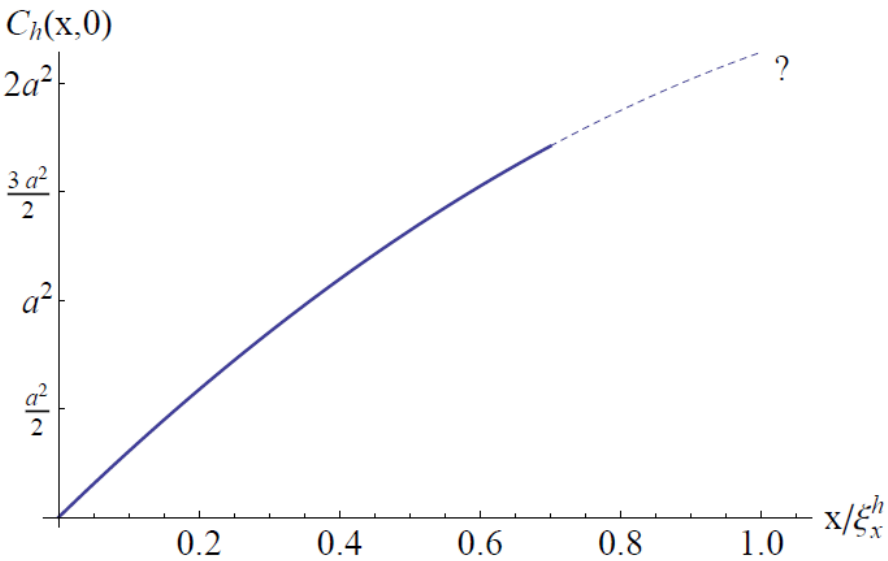
$$C_v(x,0) \sim a^2\left(\frac{x}{\xi_x}\right)^2$$

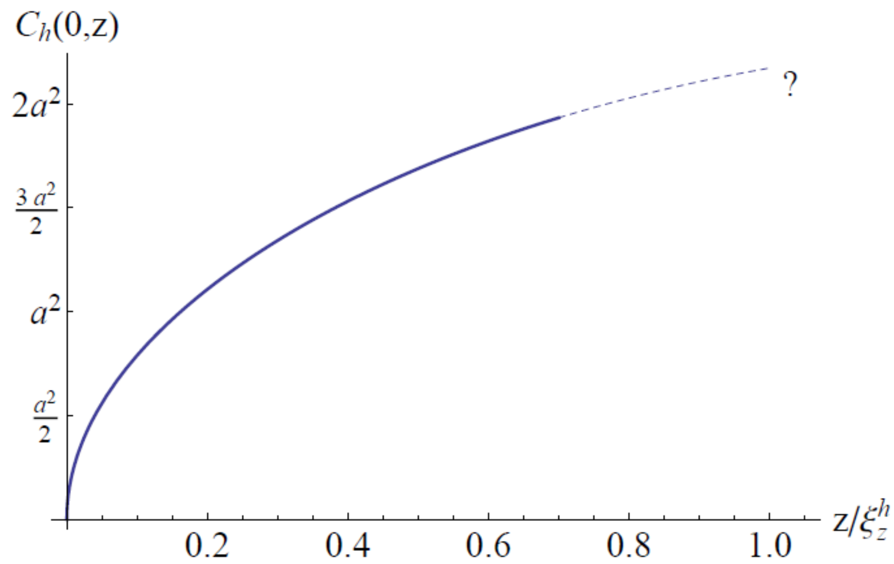


$$C_v(0,z) \sim a^2\left(\frac{z}{\xi_z}\right)^{3/2}$$

Orientational pinning correlations

- layer distortions:



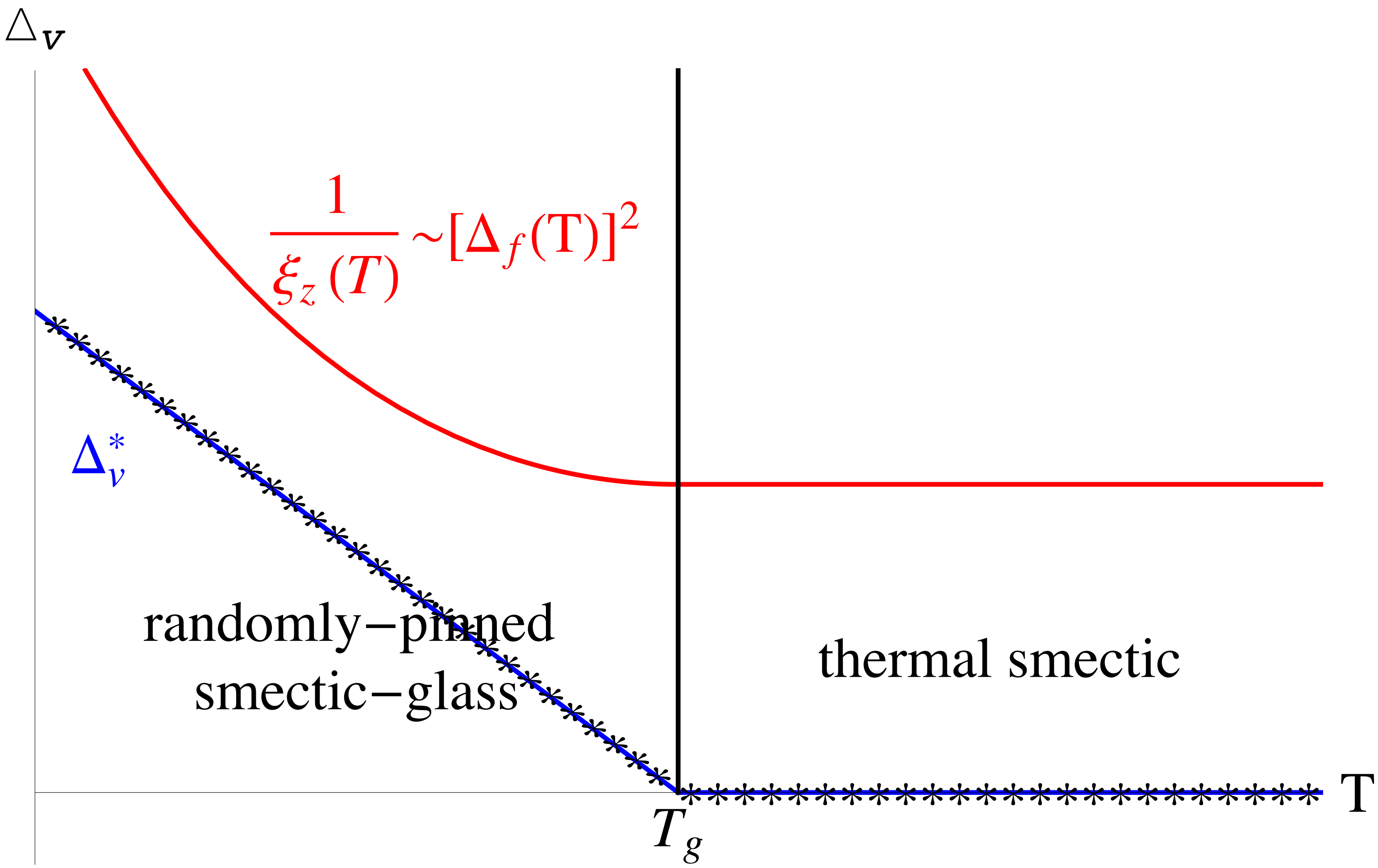


$$C_f(x,0) \sim a^2 \frac{x}{\xi_x}$$

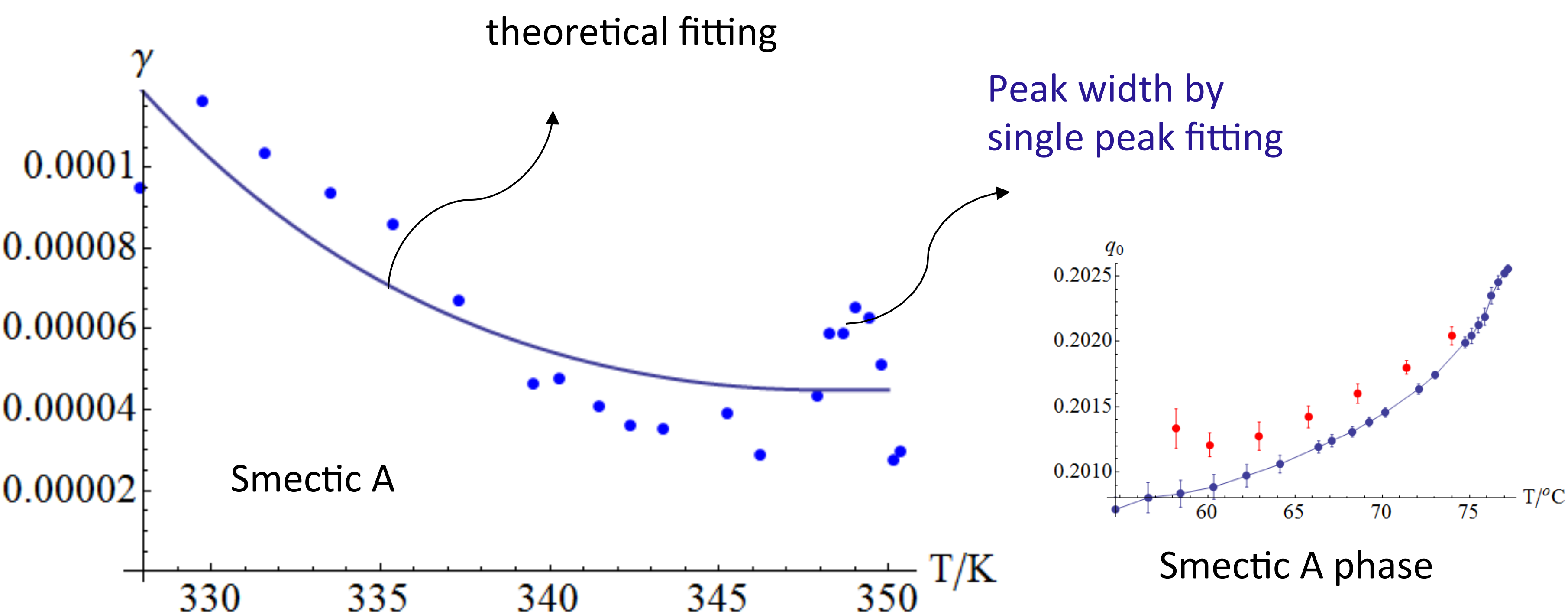
$$C_f(0,z) \approx a^2 \sqrt{\frac{z}{\xi_z}}$$

Smectic glass transition

Analog of Cardy-Ostlund transition



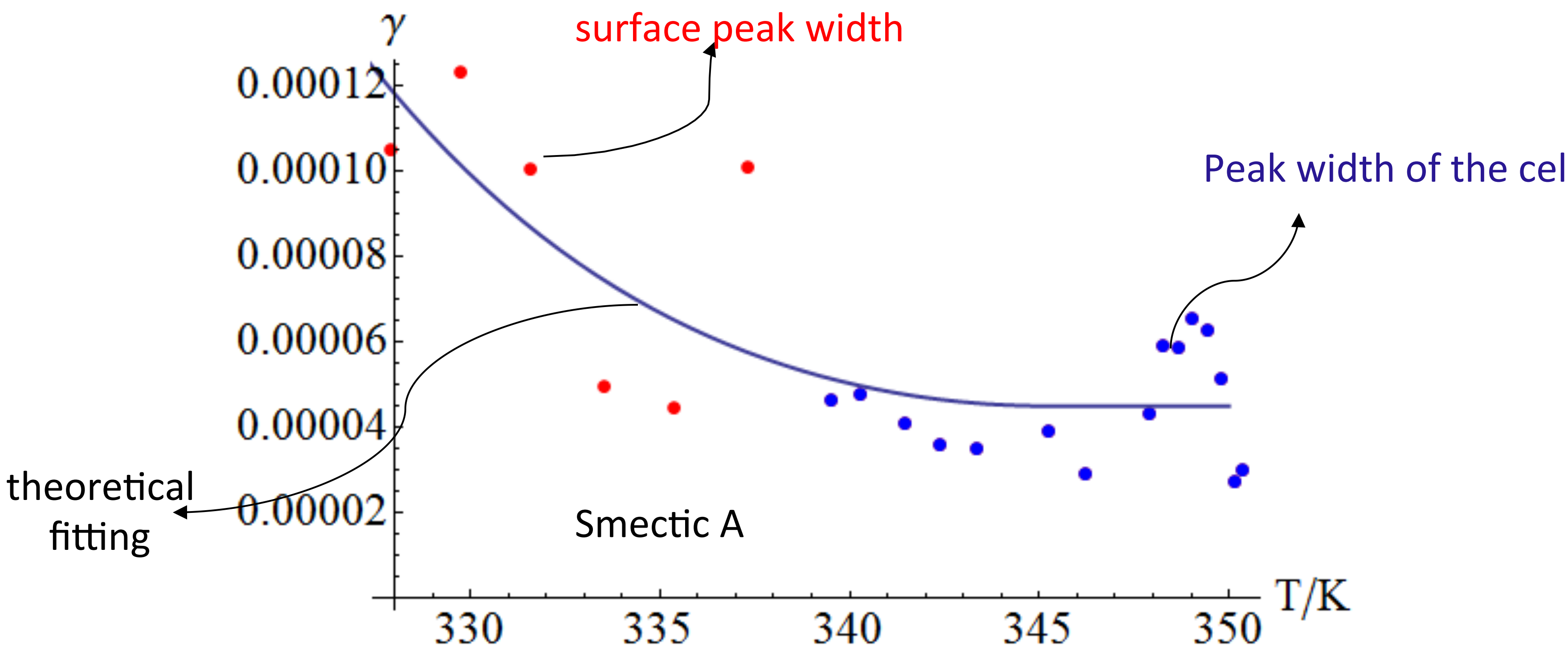
River-bottom experiments vs theory



Estimated domain size ξ_x ~1600 a ~ several microns
This provides a plausible explanation to the peak broadening at lower temperatures.

Model

Two peaks fitting of the exp.



Noticing that the lower temperature peaks broadens at high q side, we fit the x-ray peaks to surface and bulk peaks for lower temperatures. Estimated domain size ξ_x ~2200 a ~ several microns, which is the similar to the one peak fitting.

Nonlinear elasticity

Heterogenous substrate induces stress via nonlinear elasticity

Elastic energy of smectic system:

$$H = \int \left[\frac{K}{2}(\nabla_\perp^2 u)^2 + \frac{B}{2}(\partial_z u)^2 - \underbrace{\frac{B}{2}(\partial_z u)(\nabla_\perp u)^2}_{\sim (\partial_z u)\sigma} + \frac{B}{8}(\nabla_\perp u)^4 \right] dxdydz$$

$$\sigma \equiv \frac{B}{2}\langle|\nabla_\perp u|^2\rangle$$

For the two types of substrate pinning:

$$\langle|\partial_x u_0(x,z)|^2\rangle \sim \frac{\Delta_f}{B^2\lambda^3\sqrt{\lambda a}}$$

$$\langle|\partial_x u_0(x,z)|^2\rangle \sim \frac{\Delta_v}{4\pi B^2\lambda^3}\sqrt{\lambda\xi_z}$$

Smectic cell with surface stress

$$H = \int \left[\frac{K}{2} (\partial_y^2 u)^2 + \frac{B}{2} (\partial_z u)^2 \right] dydz - \int \sigma_0 (\partial_z u) \delta(y) dydz$$

Distortions away from the surface:

$$K\partial_y^4 u - B\partial_z^2 u = 0$$

solved by

$$u(y,z) = \sum_{n=0}^{\infty} a_n \sin{(q_n z)} e^{-\frac{y\sqrt{q_n}}{\sqrt{2\lambda}}} \times \left[\alpha_n \sin\left(\frac{y\sqrt{q_n}}{\sqrt{2\lambda}} \right) + \cos\left(\frac{y\sqrt{q_n}}{\sqrt{2\lambda}} \right) \right]$$

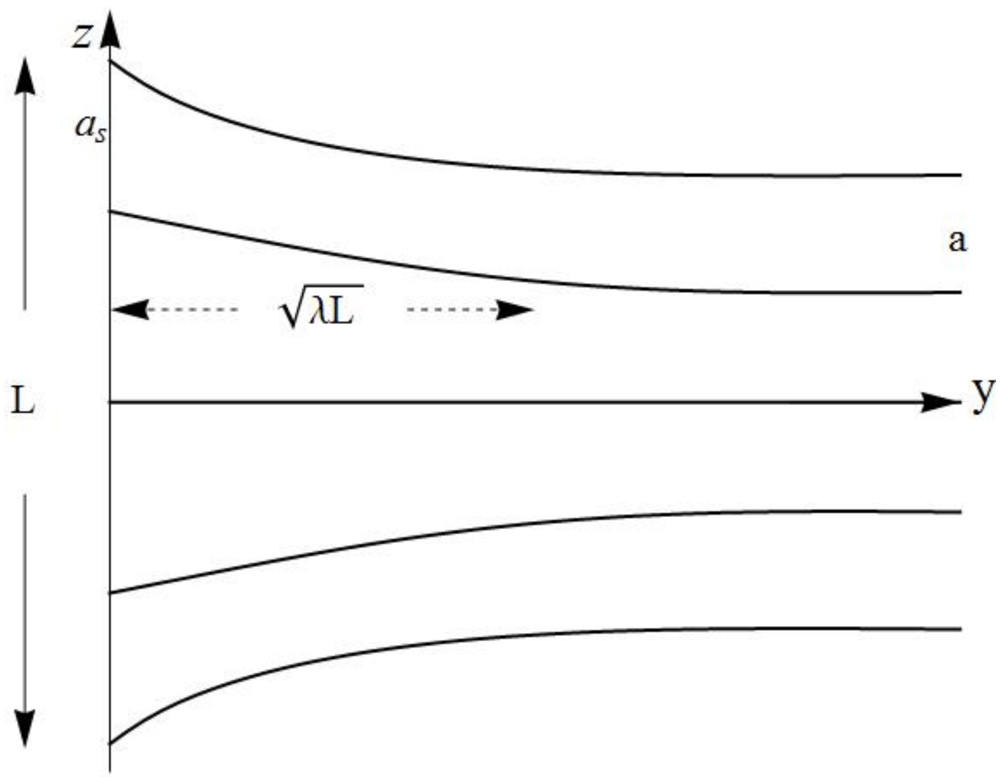
$$q_n = (2n+1)\pi/L$$

Leading to elastic energy:

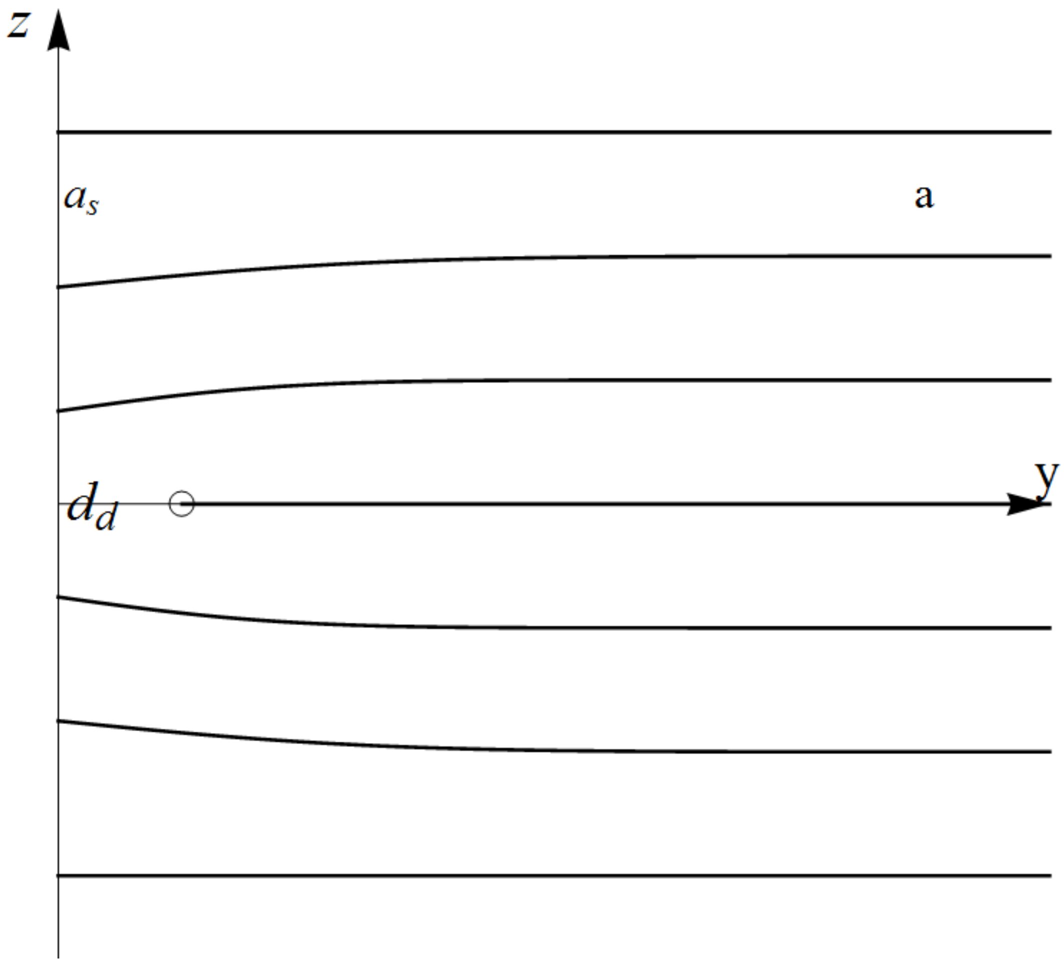
$$\boxed{E \sim \sqrt{L}\sigma_0^2}$$

Elastic distortion

$$\boxed{E \sim \sqrt{L}\sigma_0^2}$$



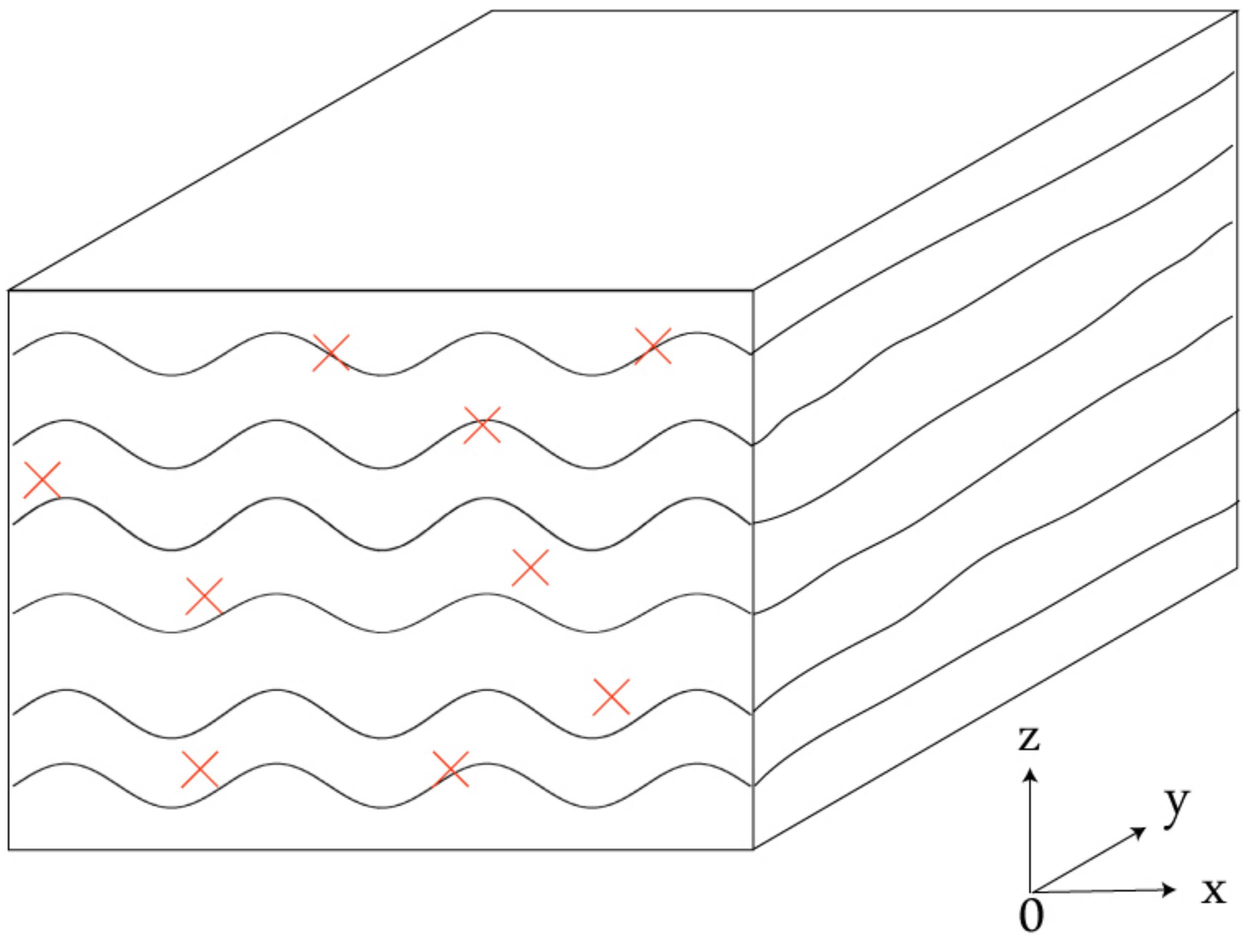
Dislocation nucleation



d_d = ?, n_d = ? …

Surface-pinning induced layer undulations

cf Clark – Meyer '73; Helfrich

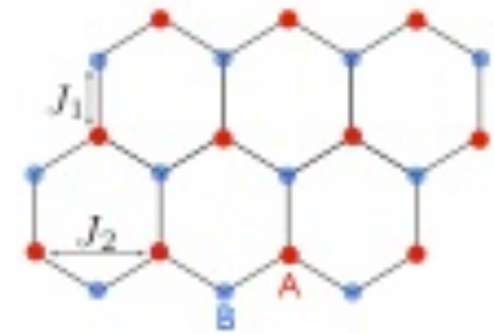


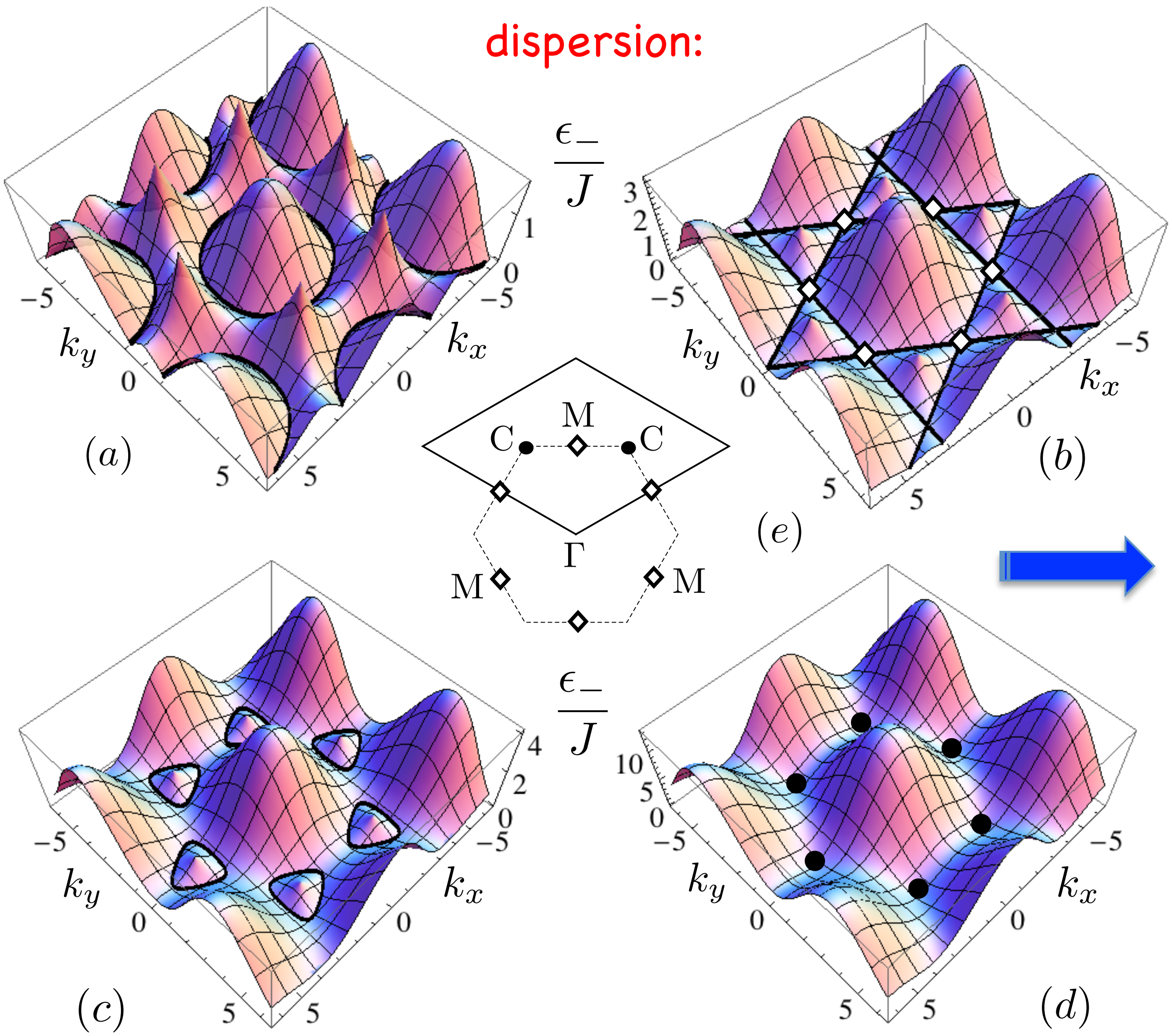
In progress and future

…Crazy out there in the left field…

Quantum liquid crystals

- spins and bosons on frustrate lattices *e.g., honeycomb lattice*

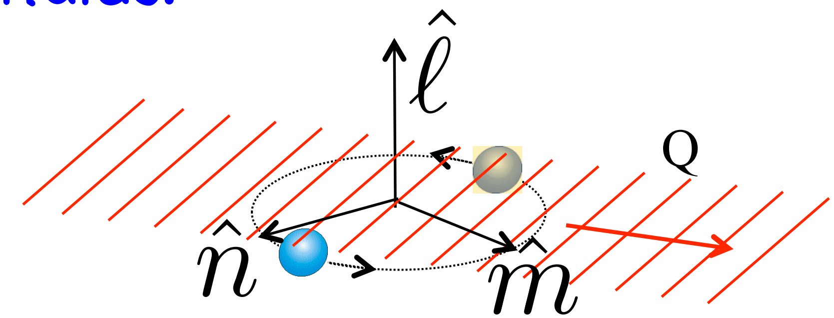


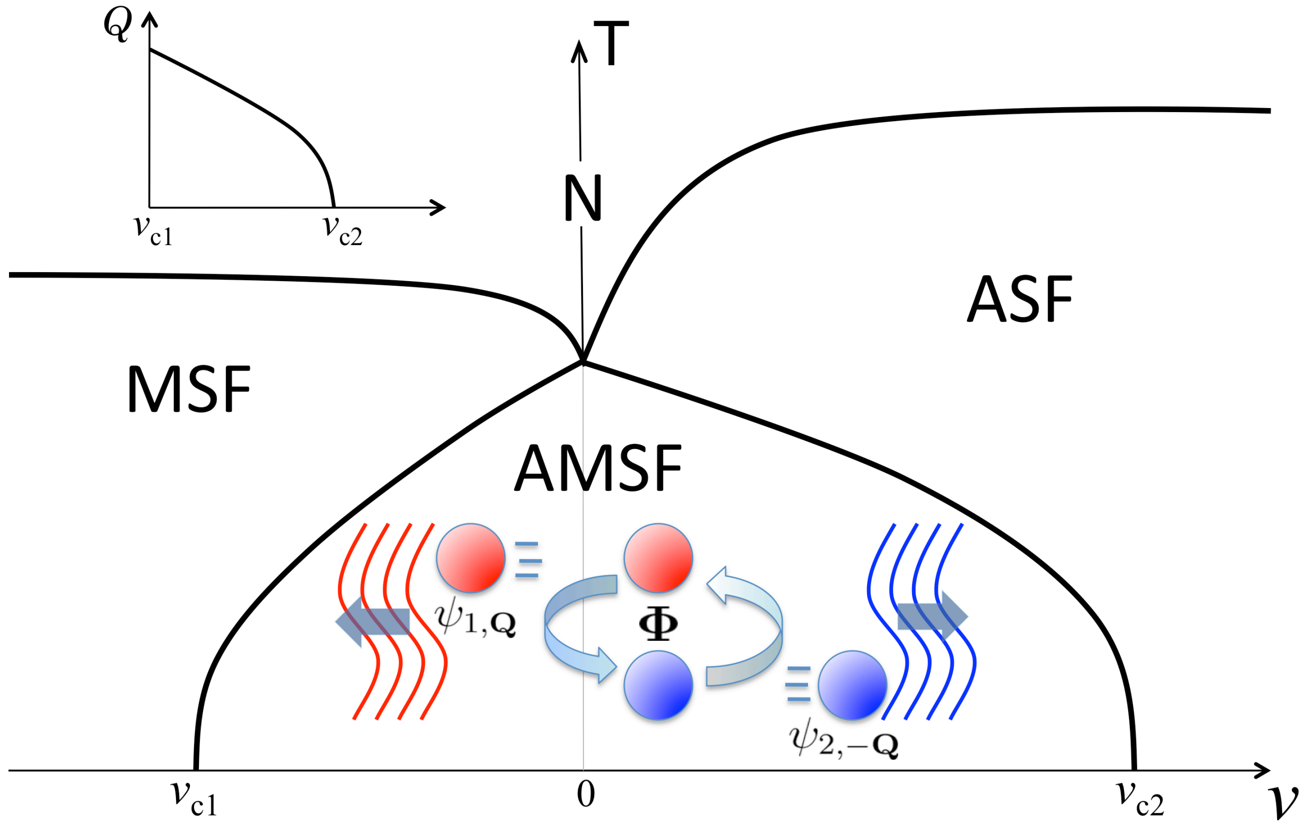


helical magnets
superfluid liquid crystals

Quantum liquid crystals

- p-wave resonant atomic superfluids:

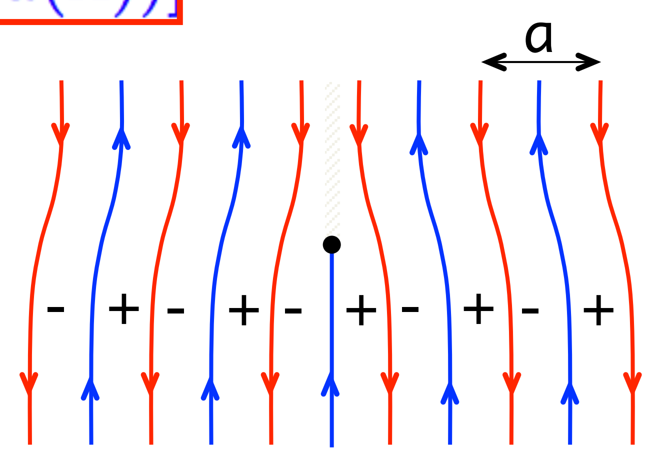


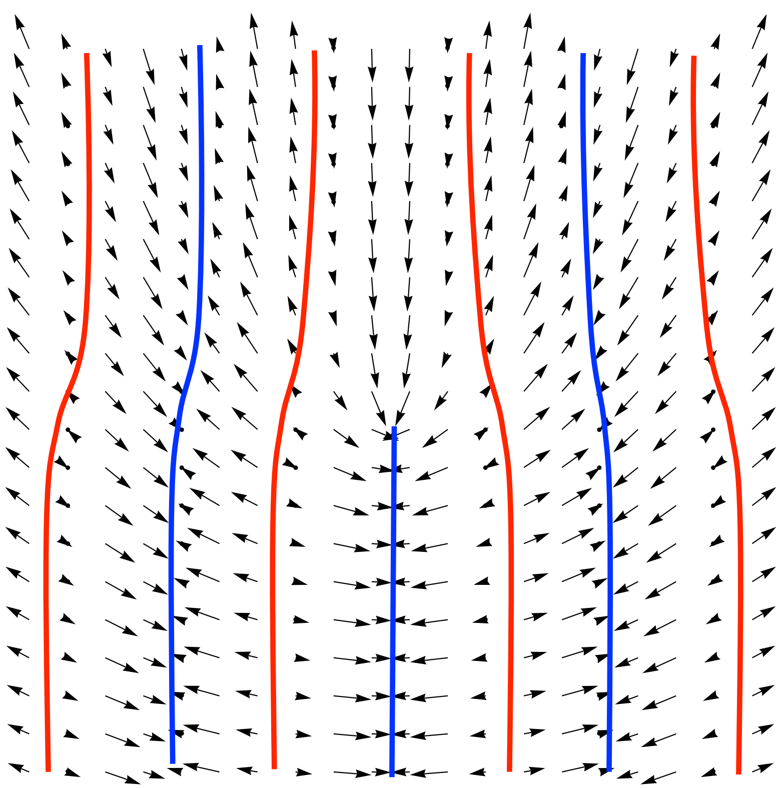


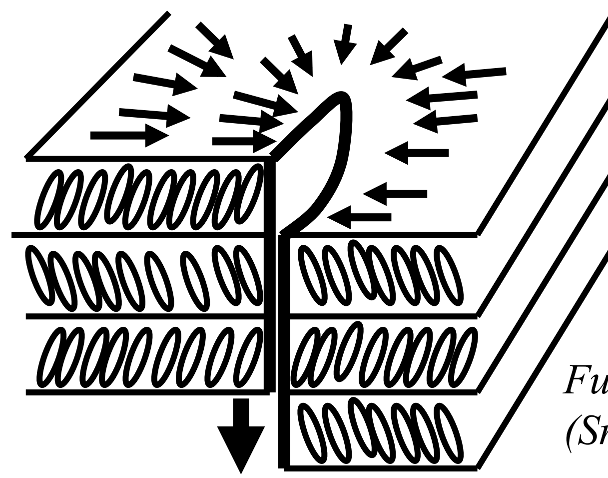
Quantum liquid crystals

- superconducting smectic in imbalanced fermionic atoms:

$$\boxed{\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})} \cos[Q(z - u(\mathbf{x}))]}$$







dispiration in SmC_A

Fukuda, et al., PRB 45 '92
(Smalyukh)

Summary

- nematic cell with "dirty" substrate
  - nematic order weakly unstable for thick cell
  - correlations of director distortions
  - transmission through cell
- smectic cell with "dirty" substrate
  - smectic order strongly unstable for thick cell
  - correlations of smectic layer distortions
  - smectic-glass phase transition at T_g
  - dislocations and nonlinear elasticity?
- quantum liquid crystals
  - helical order in frustrated magnets, $Ba_3NiSb_2O_6$, FeTe
  - superfluid (BEC) at finite momentum
  - superconductor at finite momentum (FFLO)