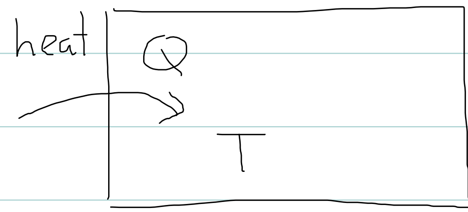
Thermodynamics 4230 Oct. 1, 2011

Lecture Set 4 (Leo Radzihovsky)

Heat Engines

- heat, entropy, temperature
- heat engine
- Carnot engine
- refrigerator
- real engines

S, Q, T



defn of T: $\frac{1}{T} = \frac{\partial S}{\partial E}\Big|_{N,V} \Rightarrow \Delta S = \frac{\Delta E}{T}\Big|_{V,N}$

V - const $\Rightarrow \Delta W = 0 \Rightarrow \Delta E \equiv \Delta U = Q$

$\Delta S = \frac{Q}{T}$ (reversible)

more generally:

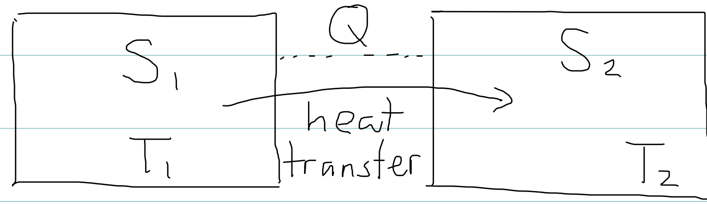
$dU = Q + W$ 1^{st} law

$$dU = T\,dS - p\,dV \Leftrightarrow dS = \underbrace{\frac{1}{T}}_{\frac{\partial S}{\partial U}\big|_{V,N}} dU + \underbrace{\frac{p}{T}}_{\frac{\partial S}{\partial V}\big|_{U,N}} dV$$

$\Rightarrow dS \geq \frac{Q}{T}$ ← energy conserved

can be spontaneously created, irreversible

$\Delta S = k_B N \ln \frac{V_f}{V_i} > Q = 0$
(fast)



heat is conserved: $Q_1 = -Q$, $Q_2 = Q$

$$\Delta S_1 = -\frac{Q}{T_1} \searrow \; ; \quad \Delta S_2 = \frac{Q}{T_2} \nearrow$$

$$\Delta S_{tot} = \Delta S_1 + \Delta S_2 = \underbrace{\left(\frac{1}{T_2} - \frac{1}{T_1}\right)}_{>0,\ T_2 < T_1} Q > 0 \nearrow$$

To tranfer heat need $T_1 > T_2$ (hot $\to$ cold)

... otherwise will not flow spontaneously (2^{nd} law)

recall Fourier heat conduction law:

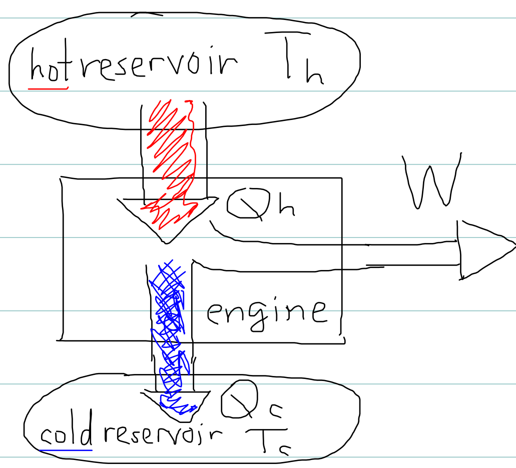
$$\frac{Q}{\Delta t} \equiv I_Q = -A\, k_Q \underbrace{\frac{dT}{dx}}_{\propto (T_1 - T_2)} \quad \left(\text{cf. Ohm's law: } I_e = G\left(-\frac{dV}{dx}\right)\right)$$

$$k_Q = \frac{1}{2} c_v \underbrace{\ell\, v_{rms}}_{D}$$

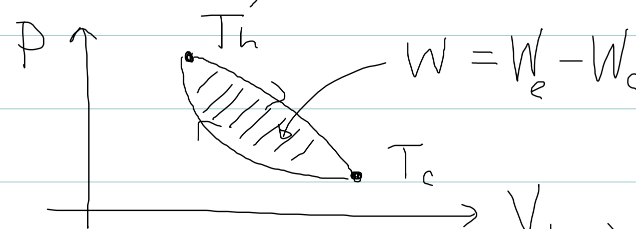
heat engine

heat engine: device that absorbs heat, & converts part of it into work

Ex's: steam turbine, internal combustion (car) engine
– almost all our electricity comes from heat engines
both coal & nuclear → steam turbines



must be cyclic:
take in Q_h, do W work
exhaust Q_c; $W = Q_h - Q_c$



(change sign convention)

efficiency: $e \equiv \dfrac{W}{Q_h} = \dfrac{Q_h - Q_c}{Q_h} = 1 - \dfrac{Q_c}{Q_h} < 1$

per cycle, $\Delta S_{engine} = 0$, since S is a state fnc.

$\Delta S_h = -\dfrac{Q_h}{T_h}$, $\Delta S_c = \dfrac{Q_c}{T_c}$ $\Rightarrow \Delta S_{universe} = \dfrac{Q_c}{T_c} - \dfrac{Q_h}{T_h} \geq 0$

2nd law

$\Rightarrow \dfrac{Q_c}{Q_h} \geq \dfrac{T_c}{T_h}$

$\Rightarrow e = 1 - \dfrac{Q_c}{Q_h} \leq 1 - \dfrac{T_c}{T_h}$ (high $e \Rightarrow$ high $T_h - T_c$)

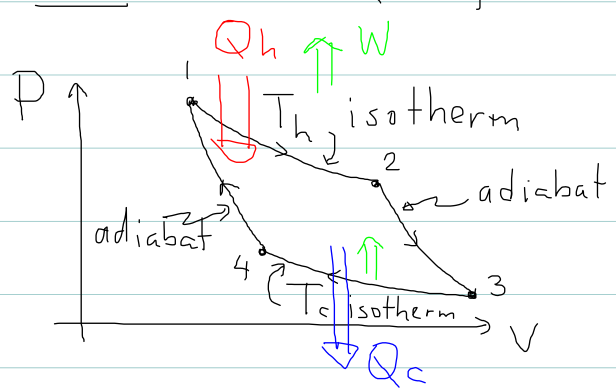
Carnot Cycle Engine

1^{st} law : "you cannot win" (conservation of energy)

2^{nd} law: "you cannot even break even" (cannot convert all of Q into W)

$e < 1$, but what's the best one can do?

most efficient (not practical) engine: Carnot engine :



$\Delta S = 0$, reversible

$$W = W_{12} + W_{23} + W_{34} + W_{41} = Q_h - Q_c$$

$Q_h = W_{12}$ (since $T_1 = T_2 \Rightarrow \Delta U_{12} = 0$) ; $Q_c = -W_{34}$ (since $T_3 = T_4, \Rightarrow \Delta U_{34} = 0$)

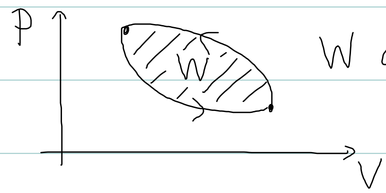
$$W_{23} + W_{41} = \Delta U_{23} + \Delta U_{41} = \cancel{U_3} - \cancel{U_2} + \cancel{U_1} - \cancel{U_4} = 0$$

use ideal gas & adiabat curves $\Rightarrow$ $\boxed{e_{Carnot} = 1 - \frac{T_c}{T_h}}$

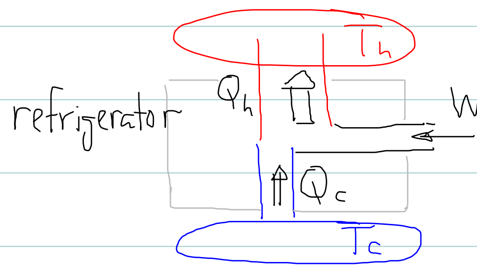
(hint: $\frac{V_3}{V_4} = \frac{V_2}{V_1}$; see hw 8)

Refrigerator

refrigerator: heat engine run in reverse



W done on "engine" to suck heat out from cold to hot



coefficient of performance $\equiv$ COP $\equiv \dfrac{Q_c}{W}$

1st law: $Q_h = Q_c + W \Rightarrow \text{COP} = \dfrac{Q_c}{Q_h - Q_c} = \dfrac{1}{\frac{Q_h}{Q_c} - 1}$

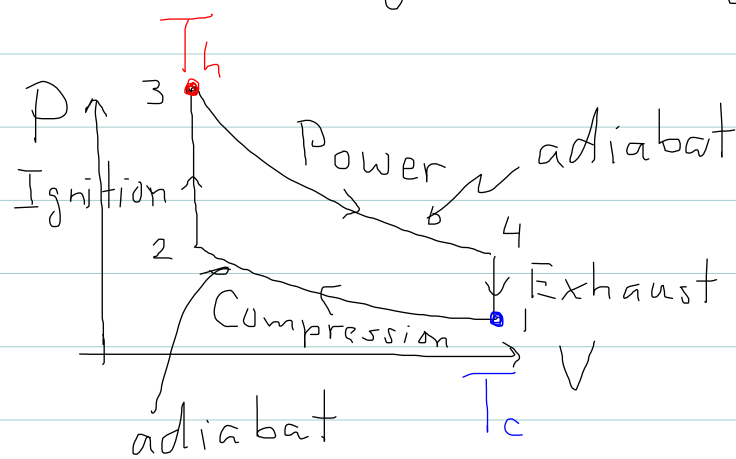
2nd law: $\dfrac{Q_h}{T_h} \geq \dfrac{Q_c}{T_c} \Rightarrow \text{COP} \leq \dfrac{1}{T_h/T_c - 1} = \dfrac{T_c}{T_h - T_c}$

kitchen fridge: $\left.\begin{array}{l} T_h = 293\,K \\ T_c = 263\,K \end{array}\right\} \Rightarrow \text{COP} = \dfrac{1}{\frac{293}{263} - 1} \lesssim 9$

$\Rightarrow$ 1 Joule of electrical energy (W) in
9 J of heat pumped out from interior (Q_c)
9+1 = 10 J of heat pumped into room (Q_h)

Real Engines

1. Otto cycle – gasoline engine



$$e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} < 1 - \frac{T_1}{T_3} \equiv e_{Carn}$$

compression ratio

2. Stirling engine (see Schroeder)

3. Steam engine (see Schroeder)

Read Schroeder, take thermo course in Engineering Dept.