Introduction

The principal behind all propulsion systems is Newton's Third Law, for every action, there is an equal and opposite reaction. Traditional space propulsion systems have utilized an onboard reaction mass that is accelerated to high velocities via an exothermic reaction or electromagnetic forces. The concept of solar sailing eliminates the need for carrying an on board propellant by harnessing the solar pressure.

Since the forces generated from an individual photon is incredibly small, large numbers of photons must be intercepted which requires a large surface area. The momentum transferred to the spacecraft can be nearly double that of the photons due to reflection. In the best situation, only 9 Newtons of force are available for every square kilometer located at 1 AU. By controlling the angle of the sail relative to the sun, the spacecraft can lose momentum and spiral towards the sun, or gain momentum and spiral away from the sun.

History

The concept of light pressure was demonstrated by James Clerk Maxwell in 1873. It wasn't until Peter Lebedew in 1900 that light pressure was experimentally measured. However, science fiction writings describing a spacecraft powered by large mirrors date as early as 1889. The first technical paper written on the topic of solar sailing was by Fridrickh Tsander in 1924.

Solar sailing essentially died with Tsander in 1933 and wasn't resurrected again until Carl Wiley, under the pseudonym of Russell Sanders published a detailed technical discussion of the physics of solar sailing in no other journal than *Astounding Science Fiction* in 1951.

The first technical paper published in the western world was written by Richard Garwin in *Jet Propulsion* in 1958. Garwin discussed the elegancy of solar sailing and recognized the considerable difficulties are of getting the sail into space rather than the design of the sail itself.

Popularity of solar sailing grew by several orders of magnitude in 1963 when Arthur C. Clarke published his short story *The Wind from the Sun*. The story described in detail a sailing race in Earth orbit similar to current yacht races.

The First Sail Missions

In the early 1970's, NASA began low-level studies of solar sail architecture. During these studies, Jerome Wright discovered a trajectory that would allow for a four year transit time to rendezvous with comet Halley at its perihelion. The mission called for launch in late 1981 or early 1982. The initial design called for an 800 x 800 m square sail shown in Figure 1. This design was dropped due to the risk of deployment failure. A heliogyro configuration was investigated next. This design utilized 12-7.5 km long blades that was spin-stabilized. This configuration is shown in Figure 2.



Figure 1: Halley Comet Square Sail



Figure 2: Halley Comet Heliogyro

Difficulties in design and escalating cost estimates put a stopper on using a sail for rendezvous. An advanced electric propulsion system was also considered, but was also scrapped. Comet Halley was never investigated by NASA, but Soviet, Japanese and European spacecrafts all made intercepts.

Even though a true solar sail has not yet flown, several preliminary tests have been made. The Russian Space Regatta Consortium successfully deployed a 20 m spinning reflector from a Progress supply vehicle in February of 1993. The deployment was observed from MIR and shown in Figure 3.



Figure 3: Soviet Reflector Deployment

Solar Sail Configurations

The basic goal for any solar sail design is to provide a large, flat, reflective surface with the minimum amount of mass. Other considerations involve manufacturing ease and deployment reliability. To keep a sail flat and to prevent billowing, tensile forces must be applied to the film. This is accomplished via several methods. Booms, similar to the mast on a sailing ship, can be used to pull outwards on the film. The spacecraft may also be spun, providing a centripetal force on the film.

Several sail configurations have been designed. The square sail, considered for the Halley rendezvous, has four deployable spars from which, the sails are unfurled. The spars and sail are deployed from a central hub that can be discarded after deployment to decrease the overall spacecraft mass. The easiest, but slowest method of attitude control for a square sail is to have articulating reflecting vanes at the boom tips to increase or decrease the reflecting area on a side. Two other, more complicated methods involve either displacing the center of pressure or the center of mass of the spacecraft. To displace the center of pressure, the sail is furled or unfurled on one side of the spacecraft. To displace the center of mass, the payload needs to be mounted on boom normal to the sail. This boom is then displaced to shift the center of mass. The primary challenge to this concept is packing and deployment. A large number of consecutive and concurrent operations are needed during deployment and must be designed to compensate. Thus, the spars can comprise a large mass fraction of the sail.

Another concept is the heliogyro configuration. The sail area is divided into several long blades attached at a central hub. Deployment is achieved by slowly rotating the spacecraft and allowing the blades to unfurl themselves. The spin puts tension on the blades and eliminates the need for heavy spars. Attitude control is achieved via spin stabilizing the spacecraft and by varying the blade pitch over the rotation, similar to a helicopter. Since the blade pitch is being altered, edge stiffeners are needed to prevent blade twist. This configuration has easy packing and deployment.

The last concept is a hybrid of the previous two. A disc sail consists of a single film of sail that is spun from a central hub. Radial spars provide some stiffness and help aid in deployment via the elastic energy stored in the spars when they are wrapped around the hub for packing.

Sail Performance

The solar radiation pressure at 1 AU is $4.56 \times 10^{-6} \text{ N/m}^2$. Giving a sail a finite efficiency η , the characteristic acceleration of a sail can be written as shown in Equation 1.

Equation 1
$$a_o = \frac{2\eta P}{\sigma}$$

Where σ is called the sail loading and is the average mass per unit area of the spacecraft.

The traditional efficiency rating for a propulsion system is the specific impulse (I_{sp}). Tsiolkovsky's equation states:

Equation 2
$$\Delta v = I_{sp} g_o \ln\left(\frac{m_1}{m_2}\right)$$

It is often claimed that solar sails have an infinite specific impulse. However, infinite specific impulse is only achieved with infinite mission duration. If a payload is being delivered to a designated orbit, the solar sail with which the payload is delivered becomes redundant. Thus, if the mass of the sail is greater than the mass of a chemical or electric system that would accomplish the same mission, the sail is not the best option. The effective specific impulse of a solar sail can be calculated by calling the initial mass m_1 , the sum of the sail and payload and the final mass, m_2 , only the payload. It can also be assumed that the total Δv given to the spacecraft is the characteristic acceleration multiplied by the mission duration, T. Thus, the specific impulse of a solar sail can be calculated as

Equation 3

$$I_{sp} \sim \frac{a_o T}{g_o} \ln \left(\frac{m_1}{m_2}\right)^{-1}$$

However, the sail will not always be pointed directly at the sun and the mission will not completely take place at the earth's orbit, thus, Equation 3 is only an approximation. Figure 4 shows a plot of specific impulse vs. mission duration for several solar sail accelerations. This is calculated assuming a payload fraction of 1/3. Common propulsion systems are also displayed.



Figure 4: Isp vs. Mission Duration

Solar Radiation Pressure

Two methods can be used to derive the effects of solar radiation pressure; the quantum description, and the electromagnetic description. The quantum description will be discussed here.

The energy of any wave can be described via Planck's Law

Equation 4

$$\hat{E} = hv$$

Special relativity also allows the energy to be described as the sum of rest energy and kinetic energy where m_o is the rest mass of the particle and p is the particle's momentum.

Equation 5
$$E^2 = m_o^2 c^4 + p^2 c^2$$

Since photons have zero rest mass, the energy may be written as

Equation 6
$$E = pc$$

Equating Equation 4 and Equation 6 yields

Equation 7 $p = \frac{hv}{c}$

The energy flux, W_E at Earth may be written as a function of the solar luminosity, L_s and the sun-Earth distance, R_E .

Equation 8 $W_E = \frac{L_s}{4\pi R_E^2}$

This can be scaled to any heliocentric distance by

Equation 9
$$W = W_E \left(\frac{R_E}{r}\right)^2$$

where r is the heliocentric distance. The total energy across a finite area for a given time can be written as

Equation 10
$$E = WA\Delta t$$

This corresponds to a momentum transfer per unit area per unit time or, more simply a pressure of



Forces on a Perfect Sail



Figure 5: Sail Coordinate System

There are two fundamental forces acting on a solar sail: solar pressure and gravity The solar forces on a sail can be scaled into an acceleration shown in

Equation 12
$$\vec{a} = 2\frac{W_E}{c}\frac{1}{\sigma}\left(\frac{R_E}{r}\right)^2 \cos^2 \alpha \vec{n}$$

Vectorially adding the solar pressure acceleration with the common gravitational acceleration yields

Equation 13
$$\vec{a} = \beta \frac{GM_s}{r^2} (\hat{r} \cdot \vec{n})^2 \vec{n}$$

where β can be defined as the ratio of solar radiation pressure to gravitational acceleration.

Equation 14
$$\beta = \frac{\sigma^*}{\sigma}$$

Since both photon flux and gravity decrease by the inverse square of the heliocentric distance, the ratio of the two yields what has been deemed the critical solar sail loading parameter, σ^* .

Equation 15
$$\sigma^* = \frac{L_s}{2\pi GM_s c}$$

The critical sail loading is 1.53 g/m^2 corresponding to a characteristic acceleration of 6 mm/s².

Solar Sail Orbital Mechanics

Since a solar sail can provide a continuous force, it can be utilized to create extremely exotic orbits. The most basic orbits achieved by solar sails are modified Keplerian orbits. These orbits are achieved by keeping the sail directly normal to the sun line. The fundamental orbital equation of motion must be modified to take into account the addition of the ever present solar pressure.

Equation 16
$$\frac{d^2r}{dt^2} + (1-\beta)\frac{\mu}{r^2}\hat{r} = 0$$

From Equation 16, one can observe that there is now an effective gravitational constant defined as:

Equation 17
$$\tilde{\mu} = \mu (1 - \beta)$$

As one can observe in Equation 17, matters get interesting as the value of β changes. Values of β from 0 to $\frac{1}{2}$, will allow a spacecraft to achieved modified elliptic orbits. A β of $\frac{1}{2}$ will allow for a parabolic escape orbit. A β between $\frac{1}{2}$ and 1 will create a hyperbolic escape trajectory. A solar sail with a critical sail loading, β of 1, will achieve a straight since the solar radiation pressure will be balanced equally with the gravitational force. And finally, a β greater than 1 will allow a spacecraft to travel on a hyperbolic orbit away from the sun. These orbits are displayed in Figure 6.



Figure 6: Keplerian Orbits with Solar Sails

Another orbit of interest is a logarithmic spiral. This orbit can be achieved with any propulsion system capable of delivering a constant low thrust. When sailing, these orbits can easily be achieved by a simple steering law; rotating the entire spacecraft one rotation per orbit. The sail angle and β will affect the angle of the spiral relative to a circular orbit. Figure 7 displays various affects of these parameters on the sail angle.



The spiral orbit is attractive for interplanetary trajectories. There is no limitation on launch windows when using a spiral trajectory. The sail angle need only needs to be altered in order to vary the transfer time for planetary rendezvous. Missions to Venus and Mercury are extremely popular. A straight Hohmann transfer requires a ΔV of 13 km/s to travel to Mercury. This ΔV can be reduced with gravitational assists, however at a large expense of transfer time. Using a sail with a characteristic acceleration of 0.25 mm/s² will be able to achieve Mercury orbit in 3.5 years. The time to capturing and spiral into a scientific orbit around Mercury is nearly negligible since solar sail acceleration at Mercury is nearly ten times that at Earth.

An interplanetary transfer to Mars can also be performed by a solar sail. However transfer time of a 1 mm/s^2 sail takes around 400 days with an additional 100 days of

capture and orbit adjustments. However, the use of a solar sail for a two way trip from Earth to Mars and back again for a sample return mission is exceedingly attractive. The primary limitation of a sample return mission is the propellant mass needed to be launched to Mars for the return transfer. A solar sail does not need this extra propellant mass and can accomplish the mission with a significantly reduced launch mass.

The orbits investigated thus far have been in-plane orbits. By angling the sail such that the force is normal to the orbit, one can crank the inclination of the orbit about the line of nodes. To achieve this, the sail angle need only be rotated once every half orbit. The close the spacecraft is to the sun, the quicker the inclination changes. However, thermal constraints on the sail film limit how close the spacecraft can get to the sun. Figure 8 displays the variation of the basic spacecraft parameters over several orbits.



A similar maneuver can be used to vary the ascending node, shown in Figure 9.



Figure 9: Varying Ascending Node

Another non-Keplerian orbit is an out of plane orbit that can allow constant observation of a portion of the sun shown in Figure 10. The orbital mechanics for these orbits are complex and active control of the orbit is necessary to prevent the orbit from degrading and spiraling into the sun.



Figure 10: Out of Plane Orbit

These orbits can be patched together to form a pseudo-cubic heliocentric orbit.



Figure 11: Pseudo-Cubic Orbits

Non-Ideal Solar Sails

Several factors affect a solar sail's performance that prevents the sail from ideal performance. The first cause of a sail's deviation from ideal is its reflectivity. A sail may not be flat and may billow of have wrinkles. In addition, a sail may not be able to reflect every photon striking its surface. Some photons may be absorbed by the sail film heating it up. This leads to another aspect of non-ideal sail performance. The only method of cooling in space is radiation. When the sail heats up from absorbed photons, more photons are emitted from the sail on both sides. The momentum of the emitted photons subtracts from the momentum of the spacecraft.

Other factors to consider are the pressure due to the solar wind and atmospheric drag. The solar wind pressure at the Earth's orbit is only $3 \times 10^{-9} \text{ N/m}^2$, 10^{-4} times smaller than the solar photon pressure. The atmosphere also restricts solar sails from deploying from an altitude less than 600-900 km.

Conclusions

Although several mechanical and structural issues remain to be solved before a true solar sail is ready to be implemented, the prospects of their use are astounding. Solar sail powered spacecraft allow for orbits that cannot be achieved otherwise. They can decrease interplanetary transfer times to the inner planets and decrease launch mass for missions to the outer planets. The possible science that can be accomplished from the use of solar sailed powered spacecraft is endless. The only hurdle left is for someone to take a financial risk to develop and implement one.