# A Comparison of GEO Satellites Using Chemical and Electric Propulsion

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**A telecommunication satellite in a geostationary orbit requires a significant amount of propellant for station-keeping maneuvers, in addition to the propellant needed for transfer from GTO to the geostationary orbit. The propellant mass significantly increases the launch mass of the spacecraft, requiring a larger launch vehicle and increasing the cost of launch. However, low-thrust electric propulsion can lower the necessary propellant mass and reduce the launch cost. This paper seeks to explore the mass savings of electric propulsion through simple numerical simulations. Analysis indicates that the launch mass of an electric satellite is only 45% of that for a satellite with conventional, chemical propulsion.**

## Nomenclature

| | | |
|---|---|---|
| A | = | cross-sectional area ($m^2$) |
| AU | = | astronomical unit |
| **a** | = | acceleration ($m/s^2$) |
| a | = | semi-major axis (km) |
| $C_R$ | = | coefficient of reflectivity |
| c | = | speed of light (299,792,458 m/s) |
| ECEF | = | Earth-centered, Earth-fixed (coordinate frame) |
| ECI | = | Earth-centered, inertial (coordinate frame) |
| F | = | thrust (N) |
| GEO | = | geostationary orbit |
| GTO | = | geostationary transfer orbit |
| $g_0$ | = | standard gravitational acceleration (9.81 $m/s^2$) |
| $J_2$, $J_3$ | = | Earth gravitational field parameters |
| $I_{sp}$ | = | specific impulse (s) |
| i | = | inclination (°) |
| K | = | control gain |
| LEO | = | low Earth orbit |
| MMH | = | monomethylhydrazine |
| MON | = | mixed oxides of nitrogen |
| $m_i$ | = | initial mass (kg) |
| $m_f$ | = | final mass (kg) |
| $R_0$ | = | one astronomical unit (149,597,870.7 km) |
| $R_E$ | = | Earth radius (6,378.1363 km) |
| **r** | = | position vector (km) |
| SRP | = | solar radiation pressure |
| r | = | orbital radius (km) |
| t | = | time (s) |
| V | = | speed (km/s) |
| XIPS | = | xenon ion propulsion system |

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| | | |
|---|---|---|
| $\Delta V$ | = | change in velocity (km/s) |
| $\lambda$ | = | longitude (°) |
| $\mu$ | = | gravitational parameter ($km^3/s^2$) |
| $\theta$ | = | angle (°) |
| $\phi$ | = | latitude (°) or average solar radiation flux (1357 $W/m^2$) |
| $\omega_E$ | = | Earth rotation rate (°) |

## I. Introduction and Spacecraft Parameters

Satellites in geostationary orbits (GEO) provide lucrative business opportunities for telecommunication companies. The lifetimes for these satellites can be as long as 15 years of operation. The mass of propellant onboard a satellite limits the lifetime, as the satellite must periodically perform station keeping maneuvers to retain its specified orbit. Once the satellite runs out of propellant, it can no longer perform maneuvers, and the satellite drifts from its orbit. Therefore, more propellant extends the lifetime of a satellite; however, this additional propellant increases the launch mass of the spacecraft and so increases the cost to launch the satellite. It is desirable, then, to reduce the amount of propellant aboard the spacecraft, while still meeting mission requirements for station keeping.

Traditionally, small thrusters using hypergolic propellants or a monopropellant have been used on GEO satellites for station keeping. Likewise, a chemical rocket has been used as an apogee kick motor to place the satellite in its orbit. However, the low specific impulse of chemical rockets necessitates a large amount of propellant, according the ideal rocket equation:

$$\Delta V = I_{sp} g_0 \ln\left(\frac{m_i}{m_f}\right) \tag{1}$$

The ratio of the final mass of the rocket to the initial mass has an exponential dependency on the specific impulse, so any change in the specific impulse drastically changes the required initial mass of the spacecraft. The limitation on the specific impulse for chemical rockets increases the amount of propellant requirement.

More recently, though, electric propulsion has been used onboard GEO satellites. Electric propulsion can achieve much higher specific impulses than chemical rockets, allowing for significant mass savings. The trade-off is that electric propulsion is limited by the amount of electrical power that the satellite can generate. For GEO satellites, power is generally provided by solar panels, so the power is limited by the size of the panels. Too large of panels increases the mass, complexity, and cost of the spacecraft. The limitation on power prevents electric propulsion from producing large thrusts; an electric thruster usually cannot produce more than 1 N of thrust.

Low thrust requires a change in mission design from chemical propulsion. Chemical rockets produce high thrust at low specific impulse, so they can be fired infrequently for brief periods to perform maneuvers. A short period of thrusting creates the necessary $\Delta V$ for a maneuver, and the burn time is short enough that the maneuver can be treated as an impulse, that is, an instantaneous change in velocity. Electric rockets, however, provide low thrust at high specific impulse, so they must burn for longer periods of time to achieve the same $\Delta V$ as a chemical rocket. The time of the burn is too long to treat as an impulse; rather, a nearly constant thrust acts on the spacecraft over a longer period of time.

One example of an electric geostationary satellite is Eutelsat 115 West B. It was launched aboard a Falcon 9 rocket on March 2, 2015, from the Kennedy Space Center. Its launch mass was 2,205 kg. After approximately eight months of orbit-raising maneuvers, the satellite entered service in a GEO located at a longitude of 114.9° West. The expected lifetime of the spacecraft is 15 years. Power is generated by two solar arrays that span a distance of 33 meters, and lithium-ion batteries store the power. The Eutelsat 115 West B satellite is unique in that it is one of the first GEO satellites to use electric propulsion not only for station-keeping maneuvers, but also for raising its orbit to GEO. Whereas previous satellites have used chemical apogee kick motors to enter GEO, and then used electric propulsion for station keeping, Eutelsat 115 West B relies solely on electric propulsion. [1,2,3]

The Eutelsat 115 West B satellite utilizes four XIPS-25 thrusters for propulsion. The XIPS-25 thruster is an electro-static thruster that uses xenon as a propellant. The designation of 25 indicates the diameter of the grid used to accelerate the xenon ions; the XIPS-25 thruster can provide up to nine times the thrust of the earlier XIPS-13 thruster. The thruster can operate in low- and high-power modes: the high-power mode is used for orbit-raising, and the low-power mode is employed for station-keeping maneuvers. Table 1 summarizes the performance characteristics of a XIPS-25 thruster.[1,4]

Table 1: Performance Characteristics for the XIPS-25 Thruster

| Parameter | Low-Power Mode | High-Power Mode |
|---|---|---|
| Input Power (W) | 2,067 | 4,215 |
| Thrust (mN) | 79 | 165 |
| Specific Impulse (s) | 3,400 | 3,500 |
| Electrical Efficiency (%) | 87 | 87 |

Figure 1 provides a concept image for the Eutelsat 115 West B satellite, and Figure 2 gives a schematic for a XIPS thruster.

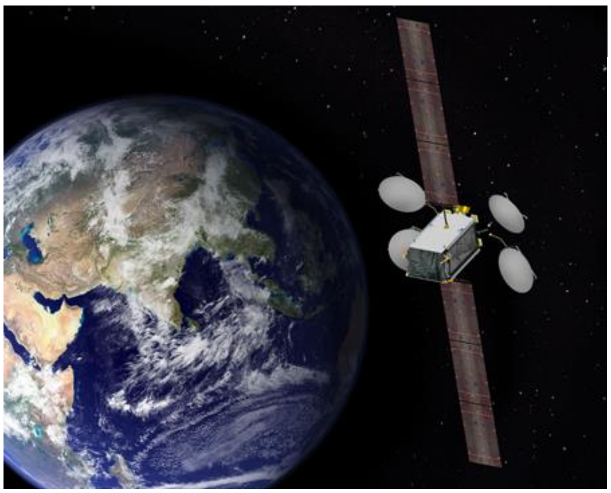

Figure 1. Concept Image of Eutelsat 115 West B.[1]

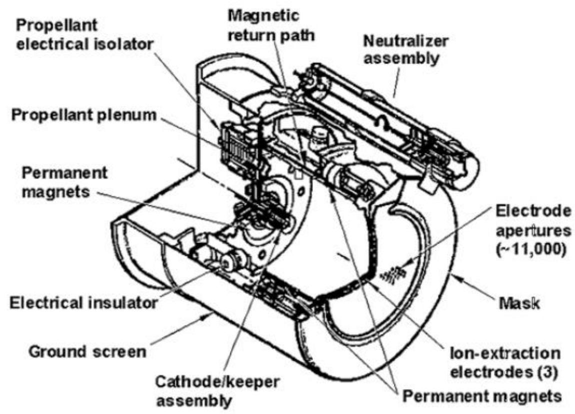


Figure 2. Schematic of XIPS thruster.[5]

In contrast to a satellite with electric propulsion, the Eutelsat W3C satellite used a liquid S400 engine to achieve GEO. The S400 is a bipropellant engine that uses MMH fuel with MON oxidizer at a mixture ratio of 1.65. It provides a thrust between 340 and 440 N at a specific impulse of 318 s. The engine can thrust for up to 8.3 hours and 100 cycles of operation. With fuel, the launch mass of Eutelsat W3C was 5,456 kg, almost 2.5 times the launch mass of Eutelsat 115 West B. While the thrusters used for station keeping on Eutelsat W3C are not known, it can be assumed that they use hydrazine (which has a specific impulse around 230 s), as has been used for previous Eutelsat missions.[6,7,8]

## II. Transfer to GEO

First, the transfer from LEO to GEO must be considered, as the propulsions system must be able to provide the necessary impulse to reach a GEO orbit. The required propellant mass for the transfer is considered for the cases of a chemical rocket and a low-thrust electric system.

### A. Chemical Rocket

For a chemical rocket, the equations for the required $\Delta V$ to reach GEO are well known. Two-body motion is considered for this case, and maneuvers are treated as short impulses. For a Hohmann transfer, two burns are required to raise the orbit from LEO to GEO: one burn at LEO to enter the transfer orbit, and another at the apogee of the GTO to circularize the orbit. To enter an equatorial orbit, the burns must also perform an inclination change. The following equations are used to calculate the $\Delta V$ necessary to reach GEO.[9]

The radius for a circular orbit at geostationary altitude is 35,786 km. Assuming that the spacecraft begins in a LEO of altitude 150 km (radius of 6528 km), the semi-major axis of the transfer orbit can be found by treating the LEO radius as perigee and the GEO radius as apogee, as given in Eq. (2).

$$a_{trans} = \frac{1}{2}(r_{LEO} + r_{GEO}) \tag{2}$$

Equation (3) gives the speed of a spacecraft in a circular orbit, and Eq. (4) provides the speed of a spacecraft in an elliptical orbit. These equations can be used to calculate the speed at LEO to be 7.814 km/s, the speed after the LEO burn at GTO perigee to be 10.028 km/s, the speed at GTO apogee to be 1.592 km/s, and the speed at GEO to be 3.075 km/s.

$$V_{circ} = \sqrt{\frac{\mu}{r}} \tag{3}$$

$$V_{ellp} = \sqrt{\mu\left(\frac{2}{r} - -\frac{1}{a}\right)} \tag{4}$$

A plane change must also lower the inclination of the orbit. A spacecraft cannot directly enter an orbit at an inclination lower than the latitude its launch site. For a launch from the Kennedy Space Center, this inclination is 28.5°. A geostationary orbit is necessarily equatorial, with an inclination of 0°, so the maneuvers must also decrease the inclination while raising the orbit.

If the entirety of the plane change is performed during the second burn (from GTO apogee to GEO), then Eq. (5) provides the ΔV for the second burn, while Eq. (6) gives the ΔV for the first burn. For this scenario to GEO, the first burn requires a ΔV of 2.469 km/s, and the second requires a ΔV of 1.840 km/s. The total ΔV for both maneuvers is 4.309 km/s.

$$\Delta V_1 = V_{GTO,p} - V_{LEO} \tag{5}$$

$$\Delta V_2 = \sqrt{V_{GTO,a}^2 + V_{GEO}^2 - 2V_{GTO,a}V_{GEO}\cos \Delta i} \tag{6}$$

However, a small savings of ΔV can be achieved by performing a portion of the inclination change at the first burn, and performing the rest at the second burn. Equations (7) and (8) calculate each ΔV for this case. The change in inclination achieved by the first burn is designated as $\Delta i_1$. Summing these two equations gives the total ΔV, and differentiating this total with respect to $\Delta i_1$ allows for a computation of the change in inclination that minimizes the total ΔV required. For this case of LEO to GEO, the optimal inclination changes are 2.154° at the first burn (for a ΔV of 2.492 km/s) and 26.346° at the second burn (for a ΔV of 1.793 km/s). The total ΔV for both maneuvers is 4.285 km/s, saving 24 m/s from the previous case.

$$\Delta V_1 = \sqrt{V_{LEO}^2 + V_{GTO,p}^2 - 2V_{LEO}V_{GTO,p}\cos \Delta i_1} \tag{7}$$

$$\Delta V_2 = \sqrt{V_{GTO,a}^2 + V_{GEO}^2 - 2V_{GTO,a}V_{GEO}\cos(\Delta i - \Delta i_1)} \tag{8}$$

If the chemical rocket were required to provide the thrust for both maneuvers, the propellant mas required would be prohibitively large. For the Eutelsat W3C satellite specified in Section 1, the propellant mass needed would be 4,075 kg out of the initial mass of 5,456 kg. This would take 75% of the spacecraft's mass for propellant just to reach GEO, without considering later station-keeping maneuvers. However, it can be assumed that the launch vehicle can insert the payload into GTO, and the spacecraft's motor only needs to burn at GTO apogee to enter GEO. In this case, for a burn of only 1.793 km/s, the required propellant mass is lowered to 2,386 kg, taking only 44% of the spacecraft's initial mass.

Despite the high propellant requirements, chemical propulsion to GEO does possess the advantage of a shorter transfer time than low-thrust, electrical propulsion. For a Hohmann transfer, the total time is simply half of the transfer orbit's period, as given in Eq. (9). For a LEO to GEO transfer, this time is only 5.25 hours. The satellite can reach GEO in a few hours and can more quickly enter into operation.

$$t_{trans} = \pi\sqrt{\frac{a_{trans}^3}{\mu}} \tag{9}$$

### B. Electric Propulsion

For low-thrust, electrical propulsion, the thrust can no longer be treated as an instantaneous impulse. Rather, the thruster must burn continually over a long period of time to produce the same ΔV as a chemical rocket. Furthermore, the slow change in orbital radius during a burn lead to gravity losses and a lower efficiency than a short chemical burn. While these losses are less significant than the propellant saving due to the higher specific impulse, they do affect the propellant mass calculations.

Equations for electric propulsion use an assumption of vanishingly small thrust that is tangent to the velocity vector of the spacecraft. The total ΔV needed is given in Eq. (10).[9] For a transfer from LEO to GEO, this amounts to a ΔV of 4.739 km/s. For the XIPS-25 thruster on high-power mode and Eutelsat 115 West B satellite discussed in Section 1, a propellant mass of 284.3 kg is required. This propellant takes 13% of the spacecraft's initial mass of 2,205 kg. Even though the ΔV is higher than for the chemical rocket, the higher specific impulse causes the propellant mass to be lower.

$$\Delta V = V_{LEO} - V_{GEO} \tag{10}$$

One disadvantage of using chemical propellant is the larger transfer time necessary to reach GEO. Equation (11) gives the transfer time, assuming that the thruster burns continuously for the entire duration of the transfer.[9] For the Eutelsat 115 West B satellite, the thrust is taken to be that of two XIPS-25 thrusters. The satellite possesses four thrusters, but in a configuration with two on each side, so only two of the thrusters can ever thrust in the same direction. This gives the satellite a constant thrust of 330 mN. The total transfer time is 342.4 days, which is significantly longer than that of the chemical rocket case. The savings on propellant mass are offset by the fact that the satellite will not be in position for operation for a longer time.

$$t_b = \frac{I_{sp} m_i g_0}{F}\left(1 - \exp\left(\frac{V_{GEO} - V_{LEO}}{I_{sp} g_0}\right)\right) \tag{11}$$

However, this analysis does not account for the plane change that must lower the inclination of the orbit. It is assumed that rather than thrusting tangentially to the satellite's velocity, the thrusters burn in a direction out of the orbital plane by an angle θ. In this case, the tangential thrust used in Eq. (11) is depends on the burn angle, as in Eq. (12). The rest of the thrust occurs in the direction normal to the orbital plane and so can change the inclination. This thrust can be directed to be below the orbital plane when the satellite is above the equator and to be above the orbital plane when the satellite is below the equator, so that the burn continually lowers the inclination.

$$F_{tan} = F \cos\theta \tag{12}$$

An angle of 40° was chosen for the direction of the electric thrust. According to Eq. (11) and (12), the total burn time is 446.9 days. The required propellant mass can be calculated from the burn time and the mass flow rate, as given in Eq. (13). This propellant mass is 371.1 kg, or 17% of the initial spacecraft mass.

$$m_{prop} = \dot{m} t_b = \frac{T t_b}{I_{sp} g_0} \tag{13}$$

A numerical simulation is implemented to verify these results. The equations of motion for the satellite are given in Eq. (14), which depend on the two-body acceleration due to the Earth's gravity and the thrust vector of the spacecraft. Note that in this simulation, the thrust is not constant in the inertial frame; it is constant in the orbital frame of the spacecraft, but must then be rotated into the inertial frame.

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{|\boldsymbol{r}|^3}\boldsymbol{r} + \frac{\boldsymbol{F}}{m} \tag{14}$$

The numerical simulation integrates Eq. (14) over time for a constant thrust angle of 40°. The results are shown in Figures 3 and 4. Figure 3 plots the trajectory of the spacecraft in inertial space over the entire transfer. The vertical axis for Figure 3 is not to scale with the horizontal axes; this is necessary for visual clarity in the plot. Figure 3 shows how the orbit decreases in inclination as its semi-major axis increases, eventually ending at a final inclination of 0.02°, which is essentially equatorial. The semi-major axis increases very slowly, such that the early orbits appear to overlap. The green dot highlights the initial position, and the red dot gives the final position of the spacecraft. Figure 4 shows the semi-major axis and inclination as functions of time.

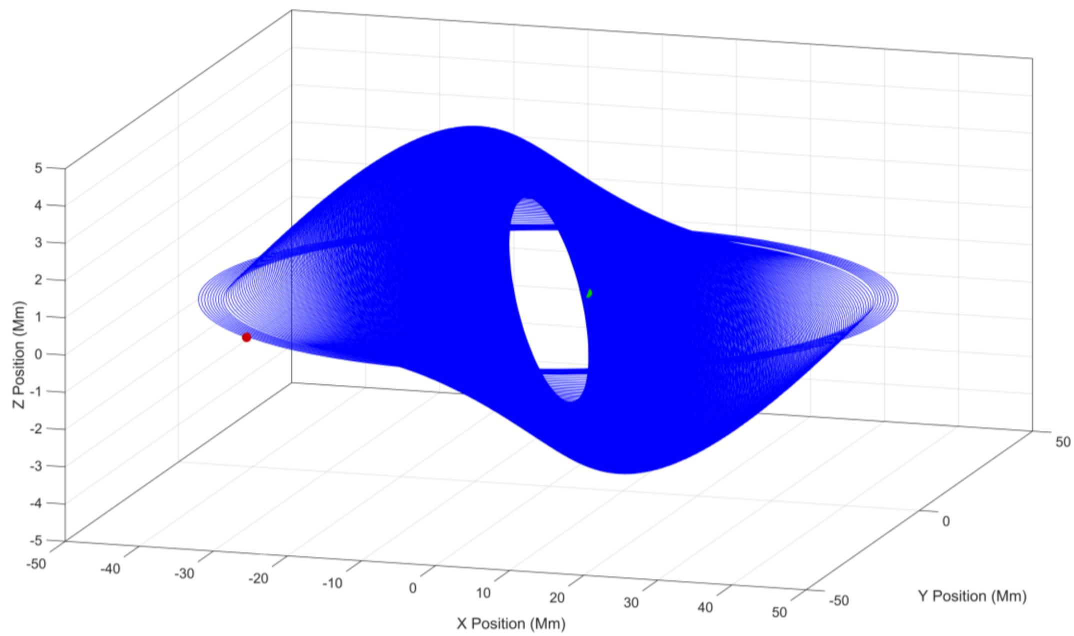


Figure 3. Trajectory for Low-Thrust Transfer to GEO.

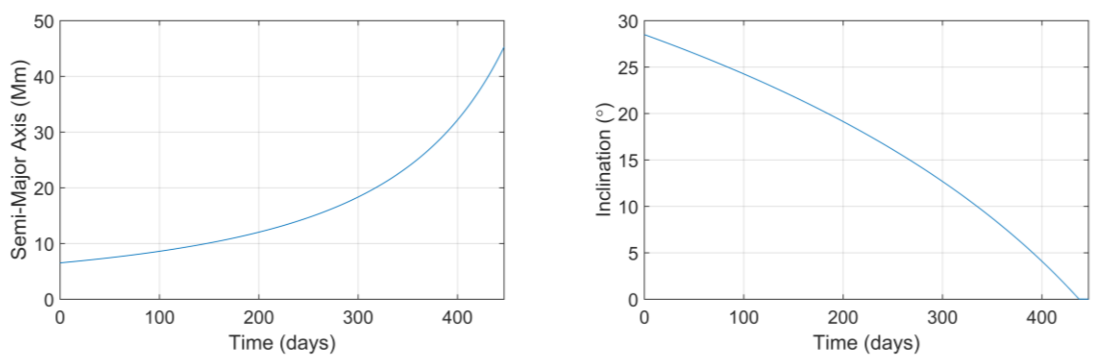


Figure 4. Semi-Major Axis and Inclination for Low-Thrust Transfer to GEO.

## III. Station Keeping at GEO

After achieving GEO, the satellite's propulsion system must maintain the desired orbit in the presence of different perturbing forces. A numerical simulation is implemented to investigate the magnitudes of these perturbations and the required maneuvers that must be performed to maintain GEO.

### A. Orbital Perturbations

Besides the dominant two-body acceleration from the gravity of the Earth, numerous other accelerations affect the motion of a satellite in GEO. These include third-body gravitational effects from the Sun and the Moon, higher-order gravity terms in the Earth's gravitational field, and solar radiation pressure. Due to the high altitude of the GEO orbit, atmospheric drag is quite small, so it will be ignored in this analysis.

The masses of the Sun and the Moon not only exert a gravitational acceleration on the satellite, but they also affect the Earth. A third-body acceleration can be expressed through Eq. (15), where all positions are relative to the center of the Earth. The subscript B indicates the third body.[10]

$$\boldsymbol{a}_B = \mu_B \left( \frac{\boldsymbol{r}_B - \boldsymbol{r}}{|\boldsymbol{r}_B - \boldsymbol{r}|^3} - \frac{\boldsymbol{r}_B}{|\boldsymbol{r}_B|^3} \right) \quad (15)$$

Solar radiation pressure can be expressed using Eq. (16).[10] In this equation, c is the speed of light, A is the cross-sectional area of the spacecraft, $C_R$ is the coefficient of reflectivity (known to be approximately 0.25 for solar panels[9], which comprise most of the are of a GEO satellite), $R_0$ is the average distance from the Earth to the Sun (1 AU), and ϕ is the average solar radiation flux of 1357 W/m$^2$ at a distance of 1 AU.

$$\boldsymbol{a}_{SRP} = -\frac{\phi}{c}\left(\frac{R_0}{|\boldsymbol{r}_{Sun} - \boldsymbol{r}|}\right)^2 \left(\frac{A}{m}\right) C_R \left(\frac{\boldsymbol{r}_{Sun}}{|\boldsymbol{r}_{Sun}|}\right) \tag{15}$$

The gravitational field of the Earth can be represented through a series of harmonic terms, though most of these terms are small enough to ignore for a high-altitude orbit. The two largest terms are the $J_2$ term, which represents the Earth's oblateness, and the $J_3$ term. The gravitational potential functions for these terms are given in Eq. (16) and (17), respectively, with $R_E$ representing the radius of the Earth and z representing the component of position not in the equatorial plane.[11] Taking the gradients of these potential functions with respect to the position components gives the acceleration vectors; these acceleration equations are omitted for brevity.

$$U_{J_2} = -\frac{J_2}{2}\left(\frac{R_E}{r}\right)^2 \left(3\left(\frac{z}{r}\right)^2 - 1\right) \tag{16}$$

$$U_{J_3} = -\frac{J_3}{2}\left(\frac{R_E}{r}\right)^3 \left(5\left(\frac{z}{r}\right)^3 - 3\left(\frac{z}{r}\right)\right) \tag{17}$$

Figure 5 plots the magnitudes of each of the perturbing accelerations as functions of times over a period of one month. The dominant accelerations come from Sun and Moon third-body effects and from the $J_2$ oblateness term. Solar radiation pressure creates a much smaller effect, and the $J_3$ acceleration is even smaller. Since $J_3$ is so small, it demonstrates that other Earth gravity terms (which are smaller than the $J_3$ effect) can safely be ignored. The magnitude of the Moon third-body perturbation varies over the course of a month depending on the Moon's distance from the Earth, in addition to the shorter periodic changes due to the satellite's orbit.

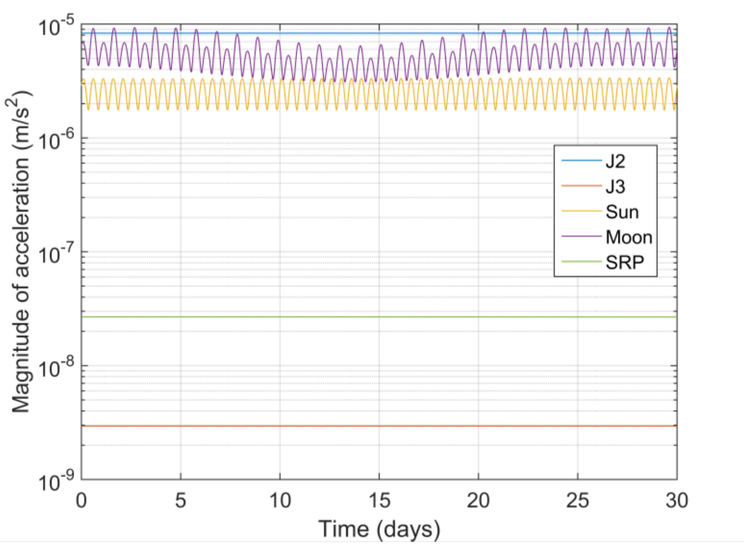


Figure 5: Magnitudes of Perturbing Accelerations

Figure 6 shows the changes in the semi-major axis and inclination of the orbit due to these perturbations. The semi-major axis experiences large oscillations, as well as a secular decrease. These changes cause the satellite to drift from its geostationary position. In addition, the inclination slowly begins to increase, and the satellite leaves the equatorial plane. Maneuvers must account for both of these types of drifts to keep the satellite in its geostationary orbit.

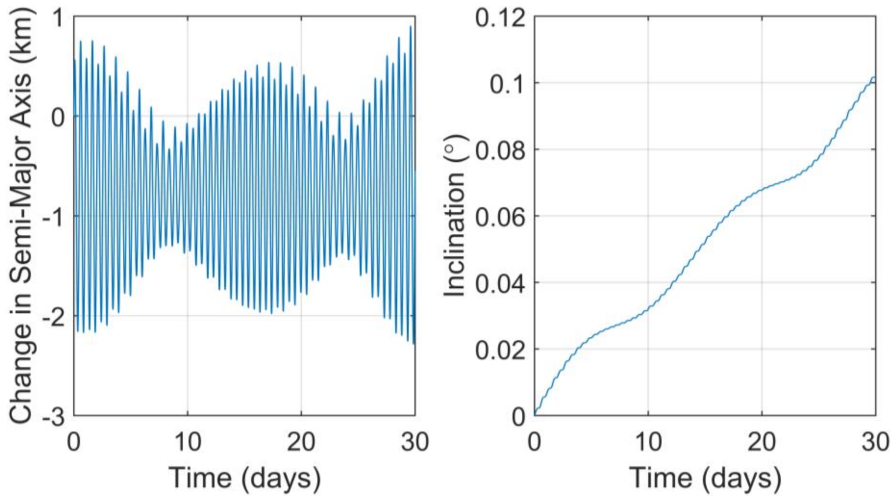

Figure 6: Changes in Orbital Elements Due to Perturbing Accelerations.

### B. Chemical Propulsion

A chemical rocket uses impulse burns to correct for drifts from the orbit. While much research has gone into developing algorithms that optimize the maneuvers to save on propellant mass, a simple model will suffice for this simulation. At each instant, the position of the satellite is calculated in an Earth-fixed frame, using a rotation matrix based on the Earth's rotational rate to convert from the inertial position to the Earth-fixed position, as in Eq. (18). Then, the longitude, latitude, and radius of the satellite can be computed, as in Eq. (19).[12]

$$\boldsymbol{r}_{ECEF} = \begin{bmatrix} \cos\omega_E t & \sin\omega_E t & 0 \\ -\sin\omega_E t & \cos\omega_E t & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{r}_{ECI} \tag{18}$$

$$\lambda = \tan^{-1}\left(\frac{r_{ECEF,y}}{r_{ECEF,x}}\right), \qquad \phi = \sin^{-1}\left(\frac{r_{ECEF,z}}{|\boldsymbol{r}|}\right), \qquad r = |\boldsymbol{r}| \tag{19}$$

The latitude and longitude are then compared to the reference latitude and longitude of the geostationary orbit. Following the Eutelsat 115 West B mission, the desired longitude is 114.9° West, and the desired latitude of any geostationary orbit is 0°. The simulation is set so that the spacecraft thrusts whenever the latitude or longitude drifts by more than a specified threshold from the reference value. To calculate the direction and magnitude of the change in velocity, Lambert's problem is solved to determine the orbit needed to connect the current location of the satellite to the desired position over a set transfer time. Solving Lambert's problem gives the required velocity of the spacecraft at its current location, and subtracting the current velocity from this required velocity gives the $\Delta V$ of the maneuver. The explanation of how to solve Lambert's problem and compute the required velocity is more detailed than can be adequately covered here; Ref. 11 contains more information.

Figure 7 displays the longitude and latitude of the spacecraft over a period of ten days. A strict requirement of 0.02° is imposed on both the latitude and longitude, and to maintain this requirement, a burn is executed whenever the drift exceeds 0.01°. (It takes time for the satellite to move back toward the reference location after a burn, so the maneuvers are always executed well before the requirement of 0.02° is exceeded). The green dot indicates the desired position of the satellite, and the red box gives the limits on the latitude and longitude. The blue curve shows the trajectory of the satellite.

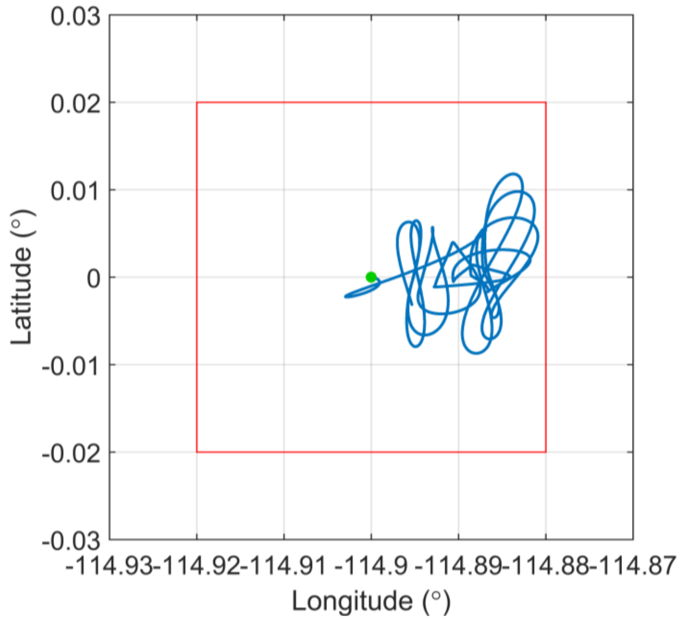

Figure 7: Longitude and Latitude of Satellite with Chemical Thruster

In the ten-day simulation, the satellite executes ten burns, averaging one burn every day. However, each burn is quite small, and the total ΔV over all of these burns is 2.12 m/s. Assuming hydrazine propellant, this corresponds to the expulsion of 2.89 kg of propulsion. Extrapolating these results over a fifteen-year mission duration, the total ΔV is 1,162 m/s, requiring 1,580 kg of propellant. Given that Section 2 calculates a spacecraft mass of 3,071 kg upon arrival at GEO, the mass of the spacecraft after these station-keeping maneuvers is 1,490 kg.

At the end of the satellite's life, the satellite must be boosted to a higher orbit to clear its geostationary orbit for use by future satellites. A Hohmann transfer to a final orbit that is 300 km above GEO requires an additional ΔV of 10.88 m/s. If the apogee kick motor provides this boost, the propellant needed is 5.82 kg.

After the final boost, the final mass of the spacecraft is 1,485 kg. This final mass is the mass available (out of the launch mass of 5,456 kg) that is actually usable for the structure and payload of the satellite. In other words, 73% of the initial mass of the payload is propellant, and only 27% is usable for the actual mission of the satellite.

### C. Electric Propulsion

Next, station keeping is considered for a spacecraft with electric propulsion. Whereas the chemical thrusters burn for short periods of time to provide large impulses, electric thrusters must burn continuously over longer periods of time to provide the necessary thrust. This changes the analysis needed for station keeping maneuvers.

A simple control law is set up to determine the necessary thrusts for station keeping. The satellite's current position and velocity are compared to those of the desired GEO, and the errors are multiplied by a gain, according to Eq. (20). The output from this control law is the desired acceleration of the spacecraft to maintain its position. This control law assumes that perturbations from the reference orbit are small enough that nonlinear orbital dynamics are negligible, so that a simple, linear control law is valid. A gain of $10^{-5}$ is found to be reasonable for this case.

$$\boldsymbol{a}_{control} = K\big((\boldsymbol{r} - \boldsymbol{r}_{GEO}) + (\boldsymbol{v} - \boldsymbol{v}_{GEO})\big) \tag{20}$$

However, the control law in Eq. (20) assumes a variable thrust, so as to provide the exact acceleration requested. The XIPS-25 thruster, though, only operates at two possible thrusts, 79 mN and 165 mN. Therefore, the control law is modified so that only constant thrusts are applied. If the necessary thrust calculated from Eq. (20) exceeds 100 times the acceleration that the thruster could provide in its low-thrust mode, the thruster fires at full thrust in the direction specified by Eq. (20). Otherwise, the thruster does not fire, and the satellite moves according to its natural, uncontrolled dynamics.

Figure 8 plots the longitude and latitude of the satellite over a period of ten days. As with the chemical rocket, the longitude and latitude are required to be within 0.02° of the reference orbit. However, Figure 8 indicates that the spacecraft never gets near this limit; the thruster keeps the satellite within 0.001° of the reference. In fact, the blue trajectory is barely visible near the green dot of the desired GEO. Over such small deviations from the reference orbit, the linear control law of Eq. (20) holds, and nonlinear orbital dynamics do not need to be considered.

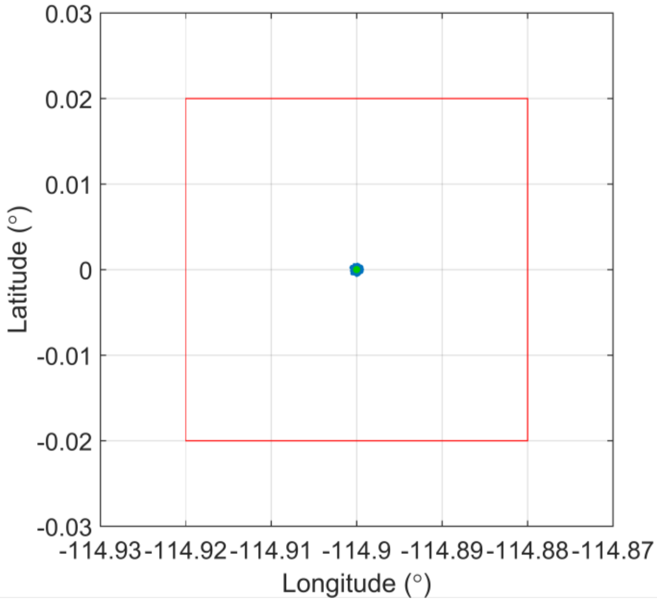

Figure 8: Longitude and Latitude of Satellite with Electric Thruster

Over the course of the ten-day simulation, the thruster is firing for 44% of the time. It expels 0.895 kg of propellant to produce a ΔV of 16.3 m/s. While the ΔV required is larger than that for the chemical rocket case, the mass of propellant needed is lower. With this average rate of propellant use, the spacecraft expel a total of 490 kg of propellant over fifteen years. According to Section 2, the initial mass of the satellite once it reaches GEO is 1,834 kg. After fifteen years, this mass drops to 1,344 kg.

Once the satellite has reached the end of its mission, it is boosted to an orbit that is 300 km above GEO. According to the corresponding of Eq. (10), the ΔV required to raise the orbit is 10.88 m/s, and the propellant mass needed is 0.44 kg. As such, the final mass of the spacecraft is still (after rounding) 1,344 kg. Out of the initial mass of 2,005 kg, 39% of the mass is propellant, and 61% is usable for the satellite's structure and mission payload.

## IV. Results and Conclusions

Table 2 summarizes the masses of the two satellites throughout the simulation. The payload of the satellite with chemical propulsion is only 141 kg larger than that for the satellite with electric propulsion, but the initial mass is 3,251 kg larger. The chemical rockets require much more propellant, such that 73% of the spacecraft mass is propellant. In contrast, the case with electric thrusters results in only 39% of the total mass being propellant.

Table 2: Satellite Masses throughout the Simulated Mission

| | Satellite with Chemical Propulsion | | Satellite with Electric Propulsion | |
|---|---|---|---|---|
| Mission Phase | Mass (kg) | Fraction of initial mass (%) | Mass (kg) | Fraction of initial mass (%) |
| Launch | 5,456 | 100 | 2,205 | 100 |
| GEO | 3,071 | 56 | 1,834 | 83 |
| End of mission | 1,490 | 27 | 1,344 | 61 |
| End of life | 1,485 | 27 | 1,344 | 61 |

Using the ratios in Table 2, the initial mass of the satellite with electric propulsion can be calculated if it were given the payload of the chemical propulsion satellite. For a payload mass of 1,485 kg, the required propellant mass would be 952 kg, and the initial mass would be 2,437 kg. This initial mass is 45% of the initial mass of the satellite with chemical propulsion. In other words, for the same payload mass, the required initial mass using electric propulsion is only 45% of the initial mass using chemical propulsion. The initial mass of the satellite can be decreased by 55% using electric propulsion, so a less powerful launch vehicle is needed. This drastically decreases the launch cost for the satellite. Alternatively, for a 55% decrease in mass, the same launch vehicle can be used to launch two spacecraft that use electric propulsion, as was the case for the actual launch of the Eutelsat 115 West B satellite.

Numerical simulations have indicated that while electric propulsion requires a larger $\Delta V$, the higher specific impulse ultimately lowers the mass of the propellant needed. One disadvantage of using electric propulsion is the significantly longer time needed for the transfer from LEO to GEO, but this delay may be acceptable if it means reducing the launch mass by 55%. The cost benefits that arise from saving mass demonstrate the utility of using electric propulsion, and demonstrate why recent satellites sent to GEO rely solely on electric propulsion.

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