Analysis of Space Elevator Technology

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The concept of an elevator which could lift payloads into space is a very intriguing and potentially economically advantageous concept. The space elevator would eliminate the need to use large launch vehicles to escape the Earth's gravity well and, therefore, would allow large payloads to be launched for much less money. The purpose of this paper is to examine the state of space elevator technology, analyze the propulsion and economic benefits of a space elevator, and describe the challenges in space elevator construction. In the end, it will be shown that a space elevator could reduce the cost of launching payloads into orbit to $250 per kilogram, representing a 99% savings over current launch technologies.

Nomenclature

a	=	Orbit semi-major axis
a_{cent}	=	Local acceleration due to centripetal force
a_{grav}	=	Local acceleration due to gravity
e	=	Orbit eccentricity
F	=	Local force on the cable section
G	=	Gravitational constant
i	=	Orbit inclination
M_{Earth}	=	Earth's mass
S_0	=	Ground cable thickness
S_1	=	Geostationary cable thickness
T_{GEO}	=	Geostationary orbit period
T_{trans}	=	Transfer orbit period
V	=	Tangential velocity
α	=	Angle between current and target geostationary longitude
μ_{Earth}	=	Gravitational parameter of Earth
ω_{Earth}	=	Rotation rate of Earth
LEO	=	Low Earth Orbit
GEO	=	Geostationary Earth Orbit

I. Introduction and Theory

The concept of a space elevator was first proposed in 1895 by Konstanine Tsiolkovsky. Tsiolkovsky proposed a tower structure which would extend from the Earth's surface to the altitude of geostationary orbit and support its weight from below. The entire tower would rotate at the same rate as the Earth, and therefore, objects at the top of the tower would have geosynchronous orbital velocity. Objects could simply be released from the top of the tower and would stay in geosynchronous orbit.

A. Modern Elevator Concept

The primary problem with Tsiolkovsky's tower concept is that the base of the tower would not have the strength to support the weight of the rest of the structure using any known materials. A more feasible concept is to build a structure which is supported by tension. The tension in the cable can be calculated by determining the apparent gravitational field at each point along the cable. This gravitational field is the sum of the force of the gravity on the cable due to the Earth and the centrifugal force due to the cable's rotation as shown in Equation 1 through Equation 3 below.

$$a_{grav} = -\frac{GM_{Earth}}{r^2} \quad \text{Equation 1}$$

$$a_{cent} = \omega_{Earth}^2 r$$ *Equation 2*

$$F = m\left(a_{grav} + a_{cent}\right)$$ *Equation 3*

Using the above equations, one can calculate that the net force on a section of cable will be toward the Earth if the section is below the altitude of geostationary orbit and upward if the section is above geostationary orbit. Therefore, if the center of mass is above the altitude of geostationary orbit, the structure will be in tension. The center of mass of the system can be raised above geostationary altitude by attaching the free end to a counterweight, as shown in Figure 1, or by extending the cable well passed geostationary orbit. The center of mass must be located high enough above geostationary orbit to provide enough tension to support the weight of the structure and the weight of any vehicles climbing the cable.

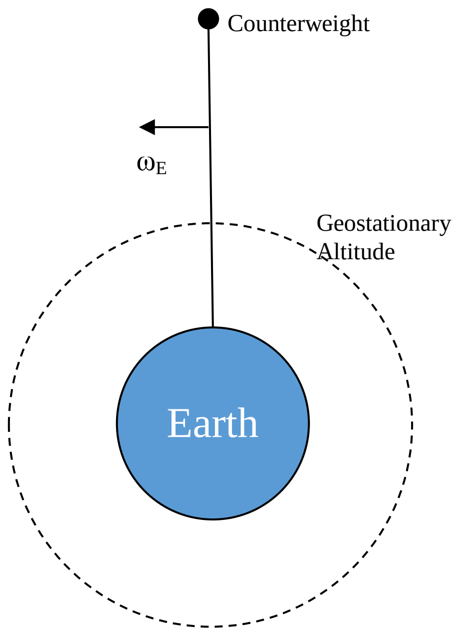

Figure 1. Space Elevator Concept

With the center of mass appropriately placed, the next design consideration is to select a cable material and cross section that will support the weight of the structure. Because the apparent gravitational field varies with height, the cable cross section must be wider at certain heights than others must. The maximum tension occurs at the height of geostationary orbit, while the minimum occurs at the Earth's surface (Aravind, 2006). The ratio of the cross section area at the surface of the Earth to the cross section at geostationary orbit can be found using Equation 4 below.

$$\frac{S_0}{S_1} = e^{\frac{\rho}{\sigma} g_0 r_0 \left(1 + \frac{x}{2} - \frac{3}{2} x^{\frac{1}{3}}\right)}$$ *Equation 4*

One can see that the ratio in Equation 4 is heavily dependent on the material strength to density ratio. Specifically, the higher the ratio, the smaller the taper ratio needs to be. Obviously, the need to launch the initial cable places a constraint on how large the taper ratio can be. In fact, only carbon nanotubes, with a yield strength around 100 GPa and a low density, provide a realistic taper ratio of 1.9 (Pugno, 2006). For this reason, carbon nanotubes are the material of choice for modern space elevator concepts.

B. Climber Concept

With the cable in place, elevator vehicles, or climbers, would need to be designed that could carry payloads up the cable. In order to minimize the required cable length, the elevator would not consists of moving cables which raise the payload, as in a traditional building elevator, but instead would have one fixed cable and the climbers would supply the power to climb up and down the cable. The amount of energy that the climber would need to provide can be calculated by calculating the potential energy the climber must acquire using Equation 5 (assuming a loss-free system).

$$E = mgh$$ *Equation 5*

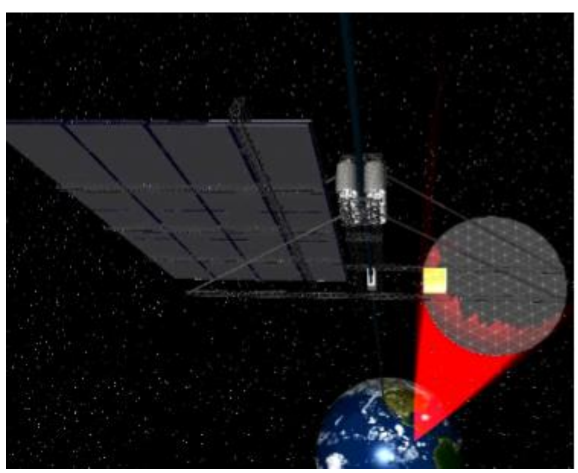

Figure 2. Climber Concept. (Bradley, 2003)

Considering the vast distance that the climber may have to travel in order to the release height, carrying on board propellant would require a prohibitively large mass. Using gasoline, acquiring the required potential energy would require 6720 kg of gasoline for a 500 kg climber. A proposed solution to this problem is to use an electric motor and wirelessly transfer power to the climber using lasers. Phasing of a geostationary satellite shows a climber concept with the power receiving panel on the far left. The power that the laser must provide is dictated by the speed of the climber. Current concepts call for a laser, which can provide around 200 kW, allowing the climber to reach geostationary orbit in around four days.

The capacity the climber can lift depends on the engine design used as well as the material properties of the cable-wheel interface. Since the climber holds to the cable using friction, the carrying capacity is limited by the force of friction between the wheels and the cable. Practical estimates suggest that climbers would be able to carry around 13 tons of cargo and weigh around 7 tons.

C. Base Station Concept

Astrodynamics requires that the base of any space elevator be anchored at the equator so that the cable can be stationary with respect to the Earth's surface. In addition, it may be desirable to move the anchor point slightly in order to avoid orbital collisions. Finally, the base station would need to be quite large in order to accommodate payload delivery, climber staging, and power beaming stations. For these reasons, most concepts feature a floating base station as shown in Figure 3. This concept would have the additional benefit of allowing any nation to construct a space elevator, not just nations along the equator.

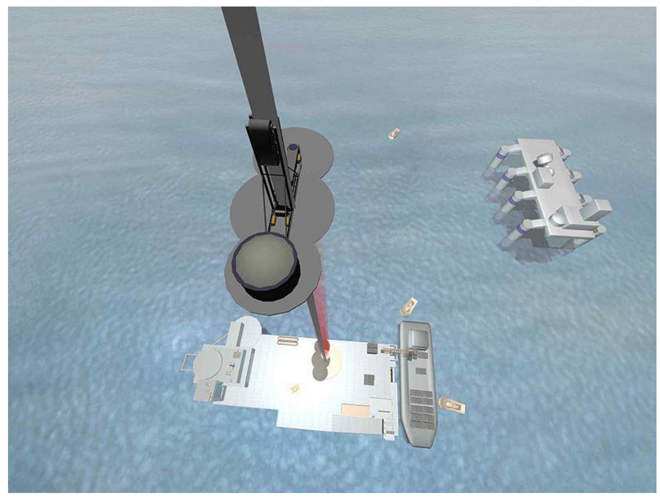

Figure 3. Elevator Base Station Concept (Bradley, 2003)

The precise base station location would need to be selected based on a variety of factors. Weather patterns in the area would need to be as mild as possible in order to minimize the risk of damage to the cable. In addition, easy access for shipping would be a key parameter. Careful analysis is still needed in order to precisely place the elevator.

D. Space Elevator Astrodynamics

Once the climber has reached its target height, the payload can be deployed into orbit. Recall that the entire space elevator rotates at the same rate as the Earth. Therefore, the velocity of a payload and climber at a given height while attached to the cable can be found using the tangential velocity relation shown in Equation 6 below.

$$V = \omega_{Earth} r \quad \text{Equation 6}$$

The velocity required to achieve circular orbit at a given height can be calculated using Equation 7 below.

$$V = \sqrt{\frac{\mu_{Earth}}{r}} \quad \text{Equation 7}$$

Given the two equations above, one can find that payloads released below geostationary altitude will have less than circular orbit velocity and will therefore enter into an elliptic orbit with the apogee height at the release height. The semi-major axis length of the orbit can be calculated by solving the Vis-Viva equation as shown in Equation 8 below.

$$V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} \quad \text{Equation 8}$$

The above equations allow the orbit shape to be calculated for payloads that are simply released from the elevator. Many missions can then be accomplished by using onboard engines to adjust the orbit after release in order to achieve more useful results.

II. Application to Propulsion

Any mission which could be accomplished with a traditional rocket launch could be accomplished with a space elevator. For the purpose of this paper, three common mission types will be examined: Delivering payloads to geostationary orbit, ISS rendezvous, and interplanetary transit.

A. Delivering payload to geostationary orbit

The simplest mission that could be accomplished with a working space elevator is the delivery of cargo to geostationary orbit. Using Equation 6 above, one can see that the tangential velocity that the payload has while attached to the cable at geostationary orbit is already equal to the circular orbit velocity at this altitude. Thus, any payload released at geostationary altitude will orbit along with the top of the cable.

In most geostationary applications, however, it is desirable for the satellite to be stationary over a specific target on the Earth which likely means that the satellite would need to be phased away from the cable. Phasing the satellite to the correct position in the geostationary belt is typically accomplished by performing a burn to enter a larger elliptical orbit which will have a longer period than a geostationary orbit as shown in Figure 4.

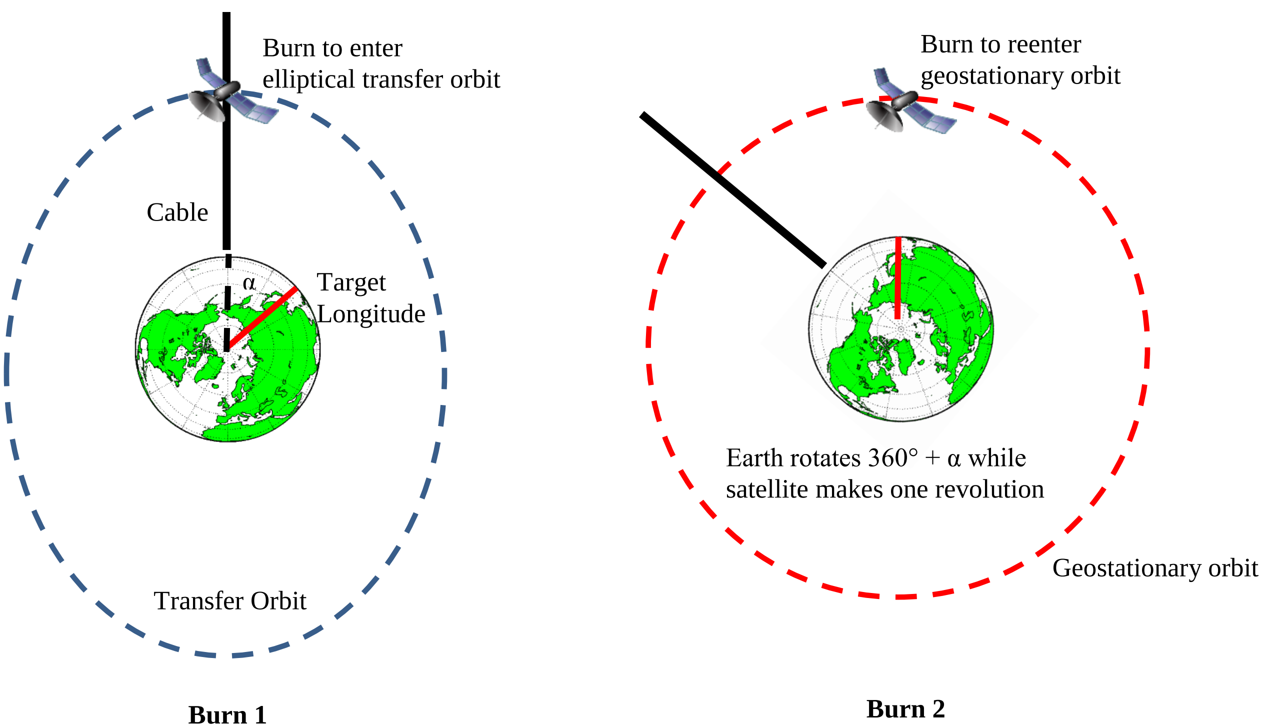

Figure 4. Phasing of a geostationary satellite

This will allow the Earth to rotate more than one revolution in the same time it takes the satellite to complete on revolution. Once the satellite returns to the geostationary altitude after completing one revolution, another burn is performed to recircularize the orbit. If the size of the elliptical transfer orbit is carefully chosen, the satellite will recircularize over the target longitude. Equation 9 through Equation 11 provide an algorithm to calculate the ΔV necessary to phase the satellite α degrees away from the cable.

$$T_{trans} = T_{Earth}\left(1 + \frac{\alpha}{360}\right) \qquad \textit{Equation 9}$$

$$a_{trans} = \left(\mu\left(\frac{T_{trans}}{2\pi}\right)^2\right)^{\frac{1}{3}}$$ *Equation 10*

$$\Delta V = 2\left(\sqrt{\mu\left(\frac{2}{r_{GEO}} - \frac{1}{a_{trans}}\right)} - \sqrt{\frac{\mu}{r_{GEO}}}\right)$$ *Equation 11*

Using the above algorithm, one can calculate that the maximum ΔV necessary occurs when the satellite is required to phase 240 degrees away from the cable with a value of 860 m/s. For comparison purposes, consider this mission being carried out using a Star 48-B engine. This engine is used on the third stage of the Delta IV rocket, commonly used for geostationary transfer missions. The relevant performance specifications for this transfer are shown in Table 1 below.

Table 1. Star 48-B Performance Specifications

Parameter	Value	Units
Specific Impulse	286	Sec
Stage empty mass	126	kg
Stage total mass	2137	kg

Using the Star 48-B engine and the maximum ΔV of 860 m/s, one can calculate that the maximum payload mass that can be delivered is 5,475 kg. This is roughly equivalent to the payload mass that the Delta IV medium can deliver to geostationary transfer orbit. Careful placement of the elevator station on the Earth's surface could serve to limit the required phasing and allow larger payloads to be launched.

B. ISS rendezvous

Next, consider a mission to deliver payload to the International Space Station. The orbital parameters for the ISS are shown in Table 2 below.

Table 2. ISS Orbital Parameters

Parameter	Value
Apogee Height	426 km
Perigee Height	421 km
Eccentricity	3e-4
Inclination	51.65°

One can see that the ISS is in a nearly circular orbit at a high inclination. Payloads being released from the space elevator at a height equal to the ISS apogee height would not have enough velocity to maintain orbit and would also be at 0^{o} inclination. Therefore, propulsion must be provided to boost the perigee height of the satellite and change the orbital inclination. Using Equation 8, the ISS orbital velocity at apogee can be found to be 7.65 km/s. Using Equation 6, the velocity of the satellite while attached to the cable at 426 km height is 0.49 km/s, yielding a ΔV of 7.15 km/s. In addition, the ΔV required to change the orbital inclination can be found using Equation 12 below.

$$\Delta V = \frac{2\sin\frac{\Delta i}{2}\sqrt{1 - e\hat{}2}\cos(w + f)na}{1 + e\cos f}$$ *Equation 12*

For a 51.65^{o} inclination change, the required ΔV is 6.67 km/s; adding this value to the ΔV required to boost the perigee height gives a total required ΔV for the mission of 13.82 km/s. One can see that this ΔV value is quite large, primarily due to the large plane change maneuver. In fact, it takes only 10.1 km/s of ΔV to reach the ISS when launching from Kennedy Space Center (assuming total ΔV losses of 1 km/s). Clearly, releasing LEO satellites at the intended orbital height is not a viable option.

In order for the space elevator to provide ΔV savings in LEO, satellites must be released from a much higher altitude than that of the intended final orbit. Releasing from a higher altitude provides two advantages for LEO satellite deployment: First, the satellite has a higher velocity while attached to the cable due to the tangential velocity relation in Equation 6. This means that less ΔV is required to match the desired orbital shape. Second, since the satellite enters an elliptical orbit when released, the plane change can be done before the circularization burn, resulting in a lower orbital velocity at the time of the plane change, and thus a lower ΔV required for plane change.

Returning to the ISS rendezvous case, the total ΔV required to enter the orbit of the ISS for various release heights was calculated using Equation 7 and Equation 8 and the results are shown in Figure 5 below.

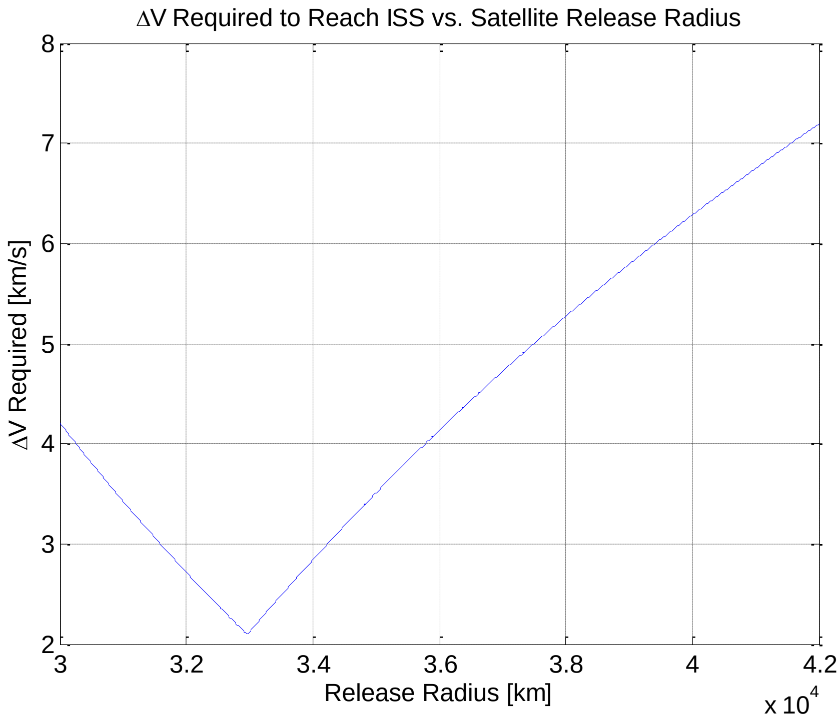

Figure 5. Delta V required to reach ISS for various satellite release radii

The minimum ΔV required to reach the ISS occurs at a release radius of 32,970 km with a required ΔV of 2.09 km/s. This represents a savings of 8.01 km/s versus launching from Kennedy Space Center. Similar savings can be calculated for other LEO missions, making the space elevator an excellent propulsion option at this altitude.

C. Interplanetary transit

One of the most surprising applications of the space elevator system is the potential ability to use the elevator to deploy interplanetary spacecraft. In order to leave the Earth and travel to another planet, a satellite must achieve escape velocity, which can be calculated for a given orbital radius using Equation 13 below.

$$V_{esc} = \sqrt{\frac{2\mu}{r}}$$ *Equation 13*

Using Equation 13 above, the escape velocity was calculated for various radii along the elevator. Figure 6 shows this velocity along with the velocity a satellite attached to the cable would have.

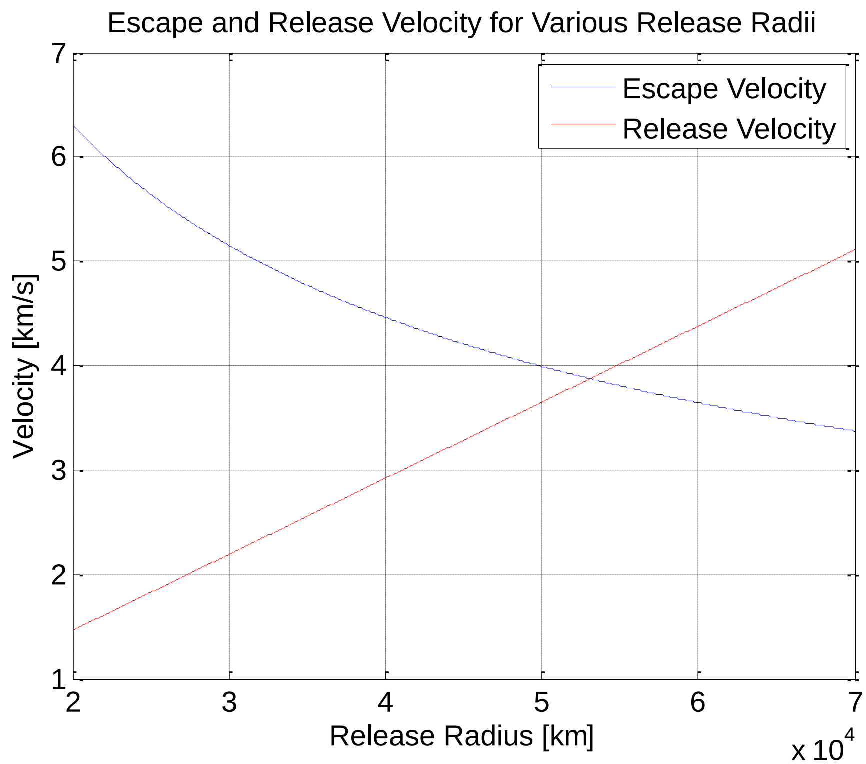

Figure 6. Escape and Release Velocity for Various Release Radii

One can see that at a radius of 52,140 km, a satellite attached to the cable will have a velocity equal to the escape velocity and will enter a hyperbolic orbit. However, in order for a satellite to travel to another planet, the satellite's velocity must exceed the escape velocity. This excess velocity, known as the hyperbolic excess velocity, can be calculated using Equation 14 below.

Equation 14

$$V_\infty = \sqrt{V_{release}^2 - V_{esc}^2}$$

The hyperbolic excess velocity that a released satellite would have is shown in Figure 7 for various release radii.

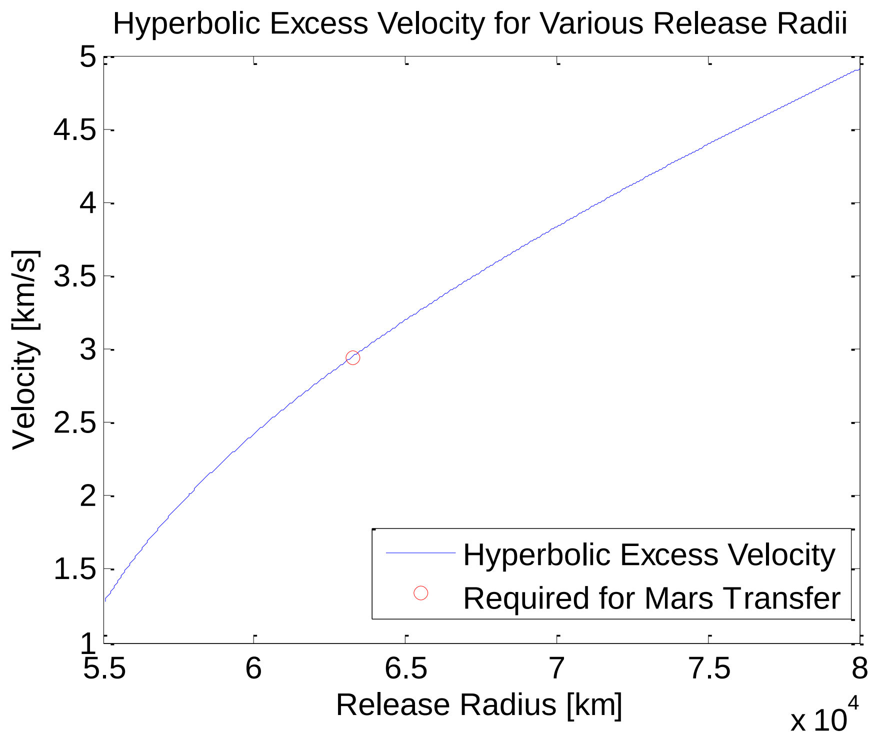

Figure 7. Hyperbolic Excess Velocity for Various Release Radii

Using a Hohmann transfer, the hyperbolic excess velocity required to reach Mars is 2.94 km/s. This velocity is achieved at a release height of 63,250 km with no additional propulsion required. This capability would allow any payload that can be carried up the elevator to be injected into Mars transfer orbit, greatly increasing the mass that can be sent to Mars with a single launch. Using a cable of 100,000 km, which has been proposed in several case studies, the velocity at the end of the cable would be sufficient to send a satellite as far as Jupiter, from which gravitational assists could be used to reach any other planet in the solar system. The space elevator could be a system which could truly open up the possibility of regular interplanetary travel.

III. Elevator Construction

The above section demonstrated that the space elevator system has many very useful applications; however, significant challenges exist in the construction of such a system. Because the structure is a tensile one, it could not be built from the ground up and must instead be lowered from space. Most concepts for elevator construction involve first launching a spool of relatively thin cable into geostationary orbit and then unraveling the cable as shown in Figure 8. In order to keep the spool over a fixed location on the Earth, the center of mass would need to initially remain in geostationary orbit, implying that the cable would need to be unravelled both toward the Earth and away simultaneously. Once the cable was fully unravelled, the spool could be elevated to the top of the cable to form part of the counterweight. Additional counterweight could then be added to raise the center of mass and increase the tension on the cable. Finally, additional climbers could then be sent up the cable, dragging additional cables, which would be attached to the original cable to provide increased strength.

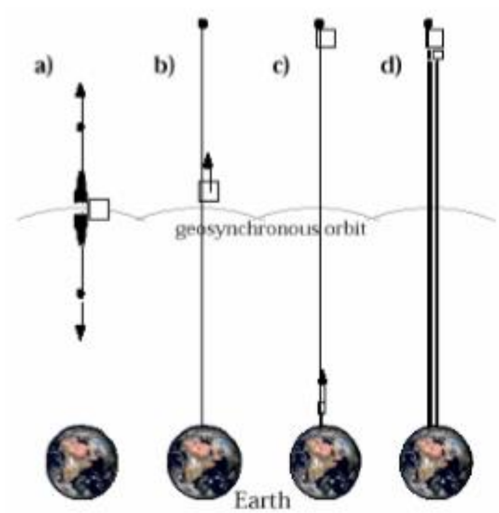

Figure 8. Space Elevator Construction Steps

There would be a large amount of technical risk in the construction plan detailed above, however. First, an in-space deployment on such a scale has never been attempted. While it has been shown that launch of a spool of sufficient length would be possible, the mechanisms and control systems necessary to guide the cables to Earth would need to be developed with a large amount of rigor before such a risky deployment could be attempted. Next, significant improvements would need to be made in the manufacture and availability of carbon nanotube material. Studies have shown that carbon nanotube fiber can be produced with the requisite strength to build a space elevator; however, batches of fibers have been tested and it has been found that the strength of the fibers currently produced have is inconsistent. Given the large quantity of fiber necessary for the completion, it would be impractical to strength test every batch, and therefore material quality would need to be improved greatly before construction. Finally, maintenance for the space elevator would need to be considered in the initial construction plan. Because the climbers would climb using friction, the cable would be subjected to wear and fatigue. The cable would need to be constructed in such a way that outer layers could be replaced after they fatigue without the need to bring down the entire cable. These are but a few of the technical risks, which would face the space elevator builders. Additional questions about funding, material supply, and orbital operations would need to be answered as well. For these reasons, even the most optimistic estimates place the first space elevator construction 20-30 years in the future (Swan, 2006), if the funding and political will can be mustered.

IV. Economic Benefits

The space elevator would provide a truly reusable form of space access which would carry with it great economic benefits. Estimates from experts on the subject have suggested that the space elevator could reduce the cost of launching to $250 per kilogram (Bradley, 2000). This is a massive savings over today's launch costs, approximately $10,000 per kilogram. In addition, the space elevator cost would apply to any mission, regardless of the final destination. For interplanetary missions, this could mean such a drastic reduction in cost that regular interplanetary transit and even colonization could become a reality.

However, the initial construction costs could be prohibitive. Experts estimate that the first elevator could be conceivably be constructed for a total cost of ten billion dollars (Bradley, 2000). In terms of today's launch costs, this is equal to 1,000 launches of 1,000 kg payloads at the average cost. At an average of 122 launches per year, the space elevator would not be cost effecive for over 9 years, as shown in Figure 9.

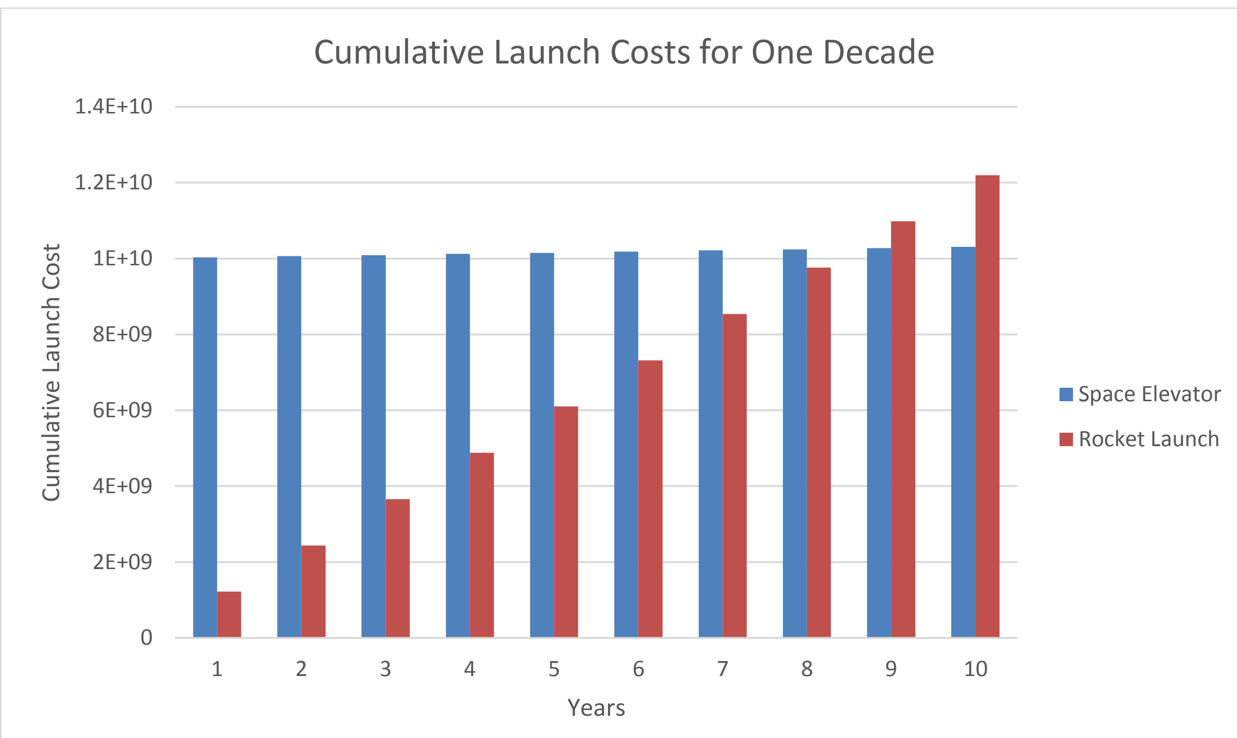

Figure 9. Cumulative launch cost for space elevator and traditional rocket launch

This is likely too slow to attract the attention of private investors, and the project likely caries too much risk to attract government support. The space industry will need to continue to grow before funding for the space elevator will likely be possible.

V. Conclusion

In conclusion, it has been shown that a space elevator would provide many benefits over conventional launch systems used today. Missions to GEO and LEO would benefit from massive reductions in the amount of propellant needed, and thus more massive payloads could be deployed. Even interplanetary missions could be launched using the space elevator, allowing for potential interplanetary colonization. All of this would be provided at an estimated cost of only $250 per kilogram, a 99% savings over today's launch methods. However, significant technical and financial risk would need to be taken in order for the project to begin. In all likelihood, the space elevator will remain a concept for the near future.

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