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ABSTRACT

We detail an experimentally simple approach for centering a beam of light to the axis of a rotating surface. This technique can be understood as a rotating analog to knife-edge profilometry, a common experimental technique wherein the intensity (or power) of various masked portions of a beam is used to ascertain the transverse intensity profile of the beam. Instead of collecting the light transmitted through a mask, we give the surface a variable reflectivity (such as with a strip of retro-reflective tape) and sample the light scattered from the surface as it rotates. We co-align the transverse position (not the tilt) of the axis of rotation and the beam centroid by minimizing the modulation amplitude of this scattered light. In a controlled experiment, we compare the centroid found using this approach to the centroid found using the canonical knife-edge approach in two directions. We find our results to be accurate to within the uncertainty of the benchmark measurement, ±0.03 mm (±2.9% of the beam waist). Using simulations that mimic the experiments, we estimate that the uncertainty of the technique is much smaller than that of the benchmark measurement, ±0.01 mm (±1% of the beam waist), limited here by the size of the components used in these experiments. We expect this centering technique to find applications in experimental and industrial fabrication and processing settings where alignment involving rotating surfaces is critical.

I. INTRODUCTION

In applications ranging from manufacturing to physics experiments, it can be necessary to center a beam of light to the axis of rotation of a rotating surface. For example, fabricating high precision components such as diffractive optical elements (DOEs) may require aligning a beam of light to the axis of rotation of a spindle.[1–4] Misalignment in these manufacturing processes limits the quality of the DOEs that can be produced.[4] One common alignment strategy used in these industrial applications involves focusing light onto a grating affixed to the rotating surface and imaging the interference fringes formed by the overlapping +first and zeroth orders and the overlapping −first and zeroth orders.[5] A beam not centered on the axis of rotation translates over many grating lines as the grating rotates, causing relative movement of the interference fringes. As the alignment between the beam and the axis of rotation improves, this fringe movement decreases, eventually ceasing with perfect alignment. Yet, aligning a beam to a surface in this way can introduce technically demanding limitations into the entire optical system. For example, wavefront aberrations of the light incident on the grating must be minimal, as these can give rise to both changing numbers of fringes and distorted shapes, which complicate the analysis.[5] An alternative alignment strategy designed with applications in microfabrication in mind can be used to position a beam perpendicular to a spinning surface at a fixed offset from the axis of rotation.[6] This technique entails mounting a specially designed mirrored beam splitter to the rotating surface and monitoring the reflected beams with quadrant photodetectors.[6] Yet, like the alignment technique from Ref. 5, this technique from Ref. 6 requires mounting bulk optics to the rotating surface, which may be difficult in fragile or compact systems. Doing so could strain the mechanical system, shifting the position of the axis of rotation found in the alignment process from that in the intended application.

In the research lab, a high degree of alignment to a rotating surface is required for studying the rotational Doppler effect (RDE). The prototypical RDE experiment involves measuring the angular frequency of a surface rotating at a prescribed frequency Ω using changes in the orbital angular momentum (OAM) between the incident and scattered light.[7,8] Typically, the illuminating light comprises two OAM modes, $\pm l\hbar$. In this case, the change in the OAM between the incident and scattered light is detected as an intensity modulation and is expected to occur with a frequency $f_{mod} = \Delta l\Omega/(2\pi)$, where $\Delta l = 2|l|$. In practice, however, intensity modulations may also be detected in the harmonics of $f = \Omega/(2\pi)$.[9] This can occur if the OAM of the light differs from the anticipated $\pm l$. Such discrepancies between the expected and the actual OAM modal spectra can be caused by misalignment between the axis of rotation of the spinning surface and the direction of propagation of the beam, angularly, translationally, or in both degrees of freedom.[10–13] For example, a Laguerre–Gauss (LG) beam comprising the $LG^{p=0}_{l=1}$ mode displaced laterally by a fraction of a beam waist, $0.8w$, retains less than 40% of its energy in the intended LG^0_1 mode, with energy distributed in the beam's surrounding l modes.[10] This is because the OAM modal spectrum can have an extrinsic contribution, meaning its modal spectrum depends on the frame of reference in which it is measured.[10,13] Therefore, if the beam is mispositioned, laterally displaced from the axis of rotation, the actual OAM of the light incident on the rotating surface might not necessarily be the intended $\pm l$. A broadened spectrum can lead to errant heterodyne beat notes in the acquired signal, which can complicate analysis and create ambiguities. Drawing fundamental conclusions from RDE experiments, then, requires that the incident OAM modal spectrum be well characterized. This, in turn, implies that the light must be well aligned to the spinning system.

While aligning the beam to the spinning surface angularly can be accomplished by retracing backreflections from the surface back through the system, ensuring the beam is translationally aligned to the axis of rotation of the surface is more challenging. One possible strategy for translational alignment is to use the RDE, the very effect the experiments of Refs. 7, 8, 11, and 12 seek to characterize, and to maximize the power at the expected frequency f_{mod}. If Ω is not known *a priori* but the spacing between the harmonics of f can be resolved, the spacing between these harmonics can be used to determine Ω. Then, the power in the harmonic at f_{mod} can be maximized. However, if the OAM modal spectrum decomposed about the axis of rotation of the rotating system differs from the intended $\pm l$ due to the compounded effects of poor angular and translational alignment[13] or imperfect OAM generation, this strategy can result in unintended misalignment. Therefore, an alignment strategy distinct from the RDE measurement is necessary.

Here, we present a simple, translational alignment technique, which positions the beam centroid on the axis of rotation of the rotating surface with an estimated error of ±1% of the beam waist. This technique provides a low-profile and simple alternative to other alignment techniques for applications that cannot tolerate the weight or space of bulk optics mounted on the rotor or for applications that involve incoherent light, which would preclude monitoring interference fringes. This alignment technique involves segmenting a rotating surface into two portions, one with a detectably higher reflectivity than the other, with their interface passing through or close to the axis of rotation. This can be accomplished by, for example, adhering a strip of retro-reflective tape to the surface, positioning its edge close to the axis of rotation of the surface. As this bireflective surface rotates, the light scattered from it is detected and analyzed. While the scattered intensity from a well-centered beam has little dependence on the orientation of the rotating surface, that from a misaligned beam peaks when the beam primarily illuminates the more reflective side and dips when it illuminates the less reflective side. Our alignment technique involves minimizing the variation in the intensity of light scattered from the rotating surface. We conduct experiments to benchmark the performance of this centering technique, using the beam centroid coordinates on a digital micromirror device (DMD) determined using a knife-edge test as the true position of the axis of rotation. We conduct experiments with a Gaussian beam, but we note that this technique could be feasible using any beam such as an LG beam. The structure of this paper is as follows. In Sec. II, we describe the technique. Next, in Sec. III, we experimentally demonstrate the technique. In Sec. IV, we detail the simulations used to estimate the uncertainty of the technique. Finally, in Sec. V, we conclude this paper.

II. DESCRIPTION

It is well known that the intensity of scattered light from a diffuse surface is a function of the surface reflectivity. If a surface is engineered to have a known spatially varying reflectivity, the relative intensities of light scattered from different regions of the surface can be used to determine the position of the beam incident on the surface.

Consider a target comprising two surfaces with different reflectivities that meet at a linear interface. When this target moves through the beam, the intensity-weighted fraction of the beam scattered by each surface changes at a rate given by the velocity of the interface and the intensity profile of the beam. For example, in the case of such a reflector translating through a Gaussian beam, the intensity of the light scattered from the leading surface diminishes as the intensity of the light scattered from the trailing surface grows. The total light scattered from this reflector is the sum of the intensity-weighted fraction of the beam scattered from each material. Encoded in the intensity of the scattered light is information about the size and position of the incident beam. In the same way that knife-edge profilometry uses intensity or power measurements of various masked portions of a beam to determine the transverse profile of the beam,[14–16] light scattered from these surfaces can also be used to construct a beam profile. Importantly, however, if we know or make assumptions about the beam profile *a priori*, we can also locate the beam relative to the interface of the two materials.

To begin with, we assume that the beam incident on the spinning surface is axisymmetric, noting that if the beam is not axisymmetric, this method will align its centroid, which may differ from its center, to the axis of rotation. We make one portion of the rotating surface detectably more reflective than the other by, for example, covering one side of the surface with a strip of reflective tape that scatters more light than the surface itself does. The positioning of the tape is not critical, as will be discussed below, though the shortest

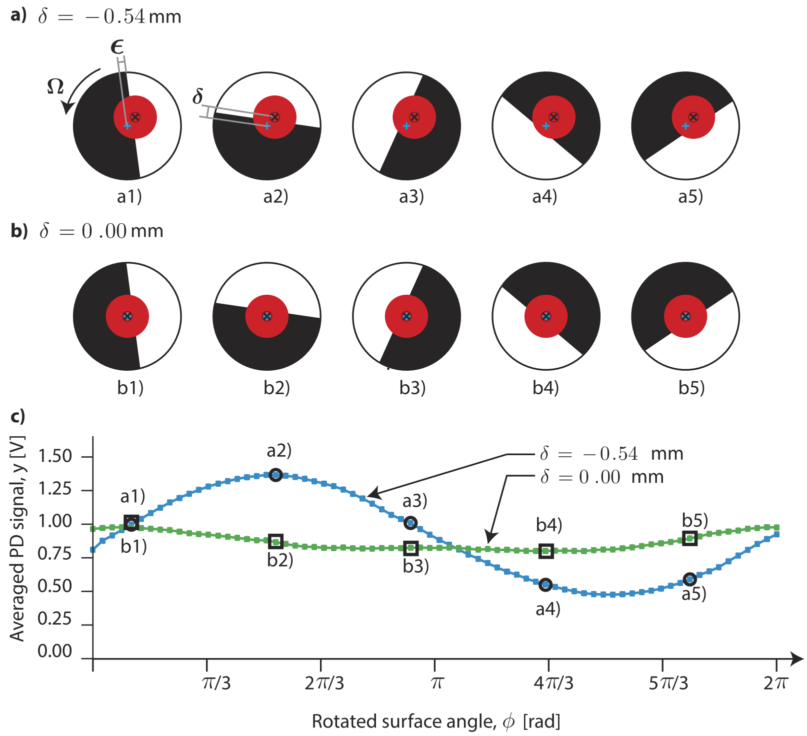

FIG. 1. A surface rotates with an angular frequency Ω. A Gaussian beam (red) is (a) misaligned and (b) aligned to the axis of rotation of the spinning surface, offset from it by a distance δ. More light scatters from the more reflective surface (white) than from the less reflective surface (black). The interface between these two surfaces lies at the shortest distance ϵ from the axis of rotation. (c) Experimental data corresponding to the configurations in (a) and (b). Fluctuations in the averaged photodetector signal arise due to variations in the amount of the Gaussian beam sampled by the reflective surface. These fluctuations depend on the relative orientation of the surface and the beam. ϵ = 0.51 mm, w = 1.05 ± 0.03 mm. δ, indicated in the figure, is referenced to the centroid of the beam found using the knife-edge test.

distance between its edge and the axis of rotation ϵ should be roughly less than a beam waist w. When the surface spins, variations in the scattered intensity are periodic with the angular frequency of the surface, Ω. This concept is illustrated in Fig. 1, where we show how the scattered intensity fluctuates when a beam is off-center from the axis of rotation by a distance δ but remains relatively constant when a beam is coaligned with the axis of rotation.

While a beam offset from the axis of rotation by a distance δ alternately scatters high and low intensities as the surface rotates, the intensity variations of the scattered light of a centered beam are small and are limited by the quality of the incident beam. This is because, in perfect alignment, the intensity-weighted fraction of the beam reflecting from each of the two surfaces is invariant under rotation. In contrast, if δ is nonzero (misalignment exists), the intensity-weighted fraction of the beam scattering from the two surfaces depends on the rotated surface angle ϕ. Translating the beam across the rotating surface toward the axis of rotation decreases the peak-to-peak variation of the intensity or the power of the scattered light as the surface rotates. Therefore, minimizing this peak-to-peak variation by adjusting the relative positions of the rotating surface and the incident beam serves as a viable strategy for centering the beam on the rotating surface.

III. EXPERIMENTAL DEMONSTRATION

To demonstrate our alignment technique, we experimentally replicate a spinning surface using a digital micromirror device (DMD, Texas Instruments DLP4500) so that we can verify the positioning of our beam relative to the location that will serve as the axis of rotation with a second, independent measurement: a knife-edge test. This allows us to perform highly controlled experiments in which we translate the spinning structure relative to the incident beam of light by playing a different video on the DMD, rather than physically moving it or the spinning surface. Our experimental setup, shown in Fig. 2, is based on those from Refs. 7, 8, 11, and 12, but we have replaced the spinning disk from these experiments with a DMD. Typically, RDE experiments display computer generated holograms (CGHs) on a spatial light modulator (SLM) to generate light with the OAM. Here, we use our SLM (Cambridge Correlators, SDK1024) to direct a Gaussian beam into the same angle OAM beams generated for RDE experiments would diffract. We note that the SLM is not necessary and is only included here for completeness, as it was used in our experiments. After the SLM, we spatially separate the diffracted orders and isolate the first diffracted order with a spatial filter. The light from this order is then directed to the DMD. A video playing on the DMD effectively samples the incident beam. The sampled portions of the beam impinge on a photodetector (Thorlabs, DET36A). In these experiments, we average 0.3 s of data collected at 100 kHz for each video frame. The uncertainty of each data point is calculated as the standard deviation of the signal over the averaging time.

Prior to determining the translational position of the beam to the surface of the DMD, we confirm the beam is normally incident on the flat mirrors of the DMD to within ~1.15° by ensuring its backreflection from the DMD is centered on the 400 μm diameter pinhole of the spatial filter after being focused by lens L4 (focal length 100 mm). If the reflected light traveled at an angle different

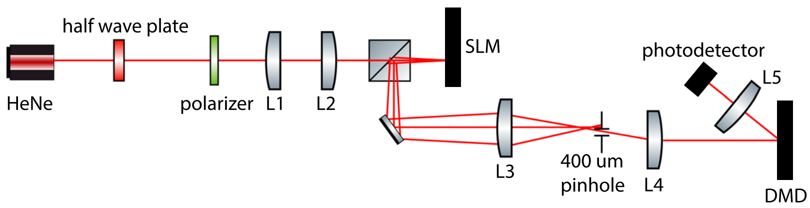

FIG. 2. Experimental setup. The size and polarization of a beam from HeNe are prepared and directed to a spatial light modulator (SLM), which displays a computer generated hologram (CGH). Light diffracts from the CGH into many orders, but we select only the first order with a spatial filter. This order is then sent to the digital micromirror device (DMD), which replicates a spinning surface. Light sampled by the DMD is collected on the photodetector. L#: lenses.

from that of the incident light, the reflected spot was visible on the housing of the pinhole. Iteratively rotating the DMD to narrow in on the angle that the spot appeared on opposite sides of the pinhole, we were able to set the angle so that the reflected beam passed through the hole. Accordingly, the perpendicularity is calculated as the ratio of twice the beam waist to the focal length of the lens. Next, we identify the position of the beam centroid as well as the beam size with a knife-edge test replicated on the DMD in two orthogonal directions. Note here that while it is known that skew can affect some results of the knife-edge test, skew has no impact on determining the position of the centroid of a Gaussian beam. Later, in Sec. IV, we will find that simulations that the beam waist extracted here agree quite well with our experimental data, suggesting the influence of any remaining skew on the beam waist is minimal. In our knife-edge test, we program the DMD to sample light from the incident beam in the same way that a knife moving through the beam does, sending samples of the beam to the photodetector. Fitting an error function to the averaged data plotted as a function of the position relative to the lower left corner of the DMD, we find that the beam is located at (6.39 ± 0.03 mm, 5.54 ± 0.03 mm) and has a $1/e^2$ beam waist $w = 1.05 \pm 0.03$ mm. The uncertainties in these measurements arise due to the discretized nature of this knife-edge test and correspond to the distance between successive samples.

Now that we know the location of the beam's centroid, we test the proposed alignment technique. We program the DMD to play a video replicating the spinning disk described above in Sec. III. We vary the center of rotation along (6.35 mm, 5.54 + δ mm), where $-0.44 \le \delta$ (mm) ≤ 0.49 for four values of ϵ, where $0.04 \le \epsilon$ (mm) ≤ 0.72. We show representative samples of the resulting averaged signal from the photodetector, y, as a function of the rotated surface angle ϕ for $\epsilon = 0.29$ in Fig. 3(a). From these and similar traces, we extract the peak-to-peak voltages V_{pp}, the difference between the maximum and minimum y of the trace. The uncertainty of V_{pp} is calculated as the uncertainties of the maximum and minimum of y added in the quadrature. V_{pp} for all experiments is shown in Fig. 3(b). Note that effects leading to a bias in the background signal on the photodetector (e.g., ambient light) are eliminated when we calculate V_{pp}.

As discussed previously, if the center of rotation was perfectly aligned with the beam, we would expect no modulation ($V_{pp} = 0$) in the signal as a function of the rotated surface angle ϕ because at every angle, the sampled intensity-weighted fraction of the beam would remain constant. For nonzero δ, however, this sampled portion of the beam varies as the reflective surface sweeps through changing fractions of the beam with ϕ, so $V_{pp} > 0$. This increasing dependence of y on ϕ as δ increases can be seen in the traces in Fig. 3(a).

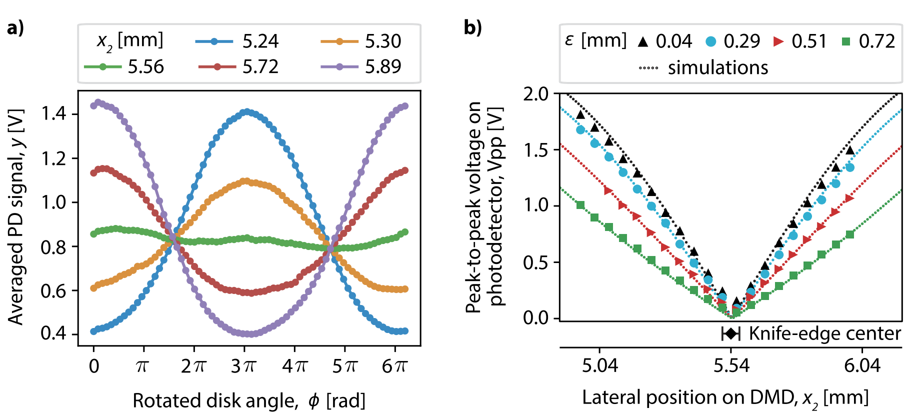

FIG. 3. (a) Representative averaged signals from the photodetector y as a function of the rotated disk angle ϕ for various lateral positions of the axis of rotation on the DMD, x_2. Recall that the centroid found by the knife-edge test is 5.54 ± 0.03 mm. Uncertainty bars, which correspond to the standard deviation of the averaged signal for each frame, are smaller than each point. $\epsilon = 0.29$ mm. (b) Peak-to-peak voltage on the photodetector, V_{pp}, as a function of the lateral position of the axis of rotation on the DMD, x_2, for various ϵ. Experimental and simulated data are indicated with the symbols and the dotted lines, respectively. Uncertainty bars corresponding to the uncertainties of the minima and maxima of y used in generating V_{pp} added in quadrature are smaller than the data points. Also indicated is the centroid as measured by the knife-edge test (black diamond). In contrast to the other data points, the uncertainty in the position of this knife-edge centroid is much larger than the data point and is indicated with the horizontal error bars.

Minimizing V_{pp} sets the x_2 position of the beam centroid. To completely center the beam, this procedure should be repeated in the x_1 direction. We note that this technique can be implemented in real time, monitoring V_{pp} with an oscilloscope.

IV. SIMULATIONS

In the experiments, the centroid position found using the knife-edge test has a relatively large uncertainty. We anticipate the uncertainty of the centering technique discussed in this paper is much smaller, and thus, the knife-edge test does not provide an accurate enough "truth" to assess the uncertainty. To estimate the uncertainty of our centering technique, we turn to simulations.

In simulating our centering technique, we calculate the spatial intensity of a Gaussian beam with the same beam waist as in the experiments, $w = 1.05$ mm. Then, for each ϵ, we calculate V_{pp} as the difference between the summed intensity of the beam with (a) $x_2 < \delta + \epsilon$ and (b) $x_2 \geq \delta - \epsilon$. These sampling positions capture the minimum and maximum expected intensities sampled by the spinning surface, respectively. To match these simulations to our experiments, we assume that the only light that scatters from the rotating surface is that incident on the more reflective of the two surfaces. We model this reflective surface as a perfect mirror (reflectance = 100%), and we assume that the reflected intensity corresponds linearly to the anticipated signal on the photodetector. To superimpose the simulated data on our experimental data, we calculate a scaling parameter by dividing the summed intensity of the entire simulated Gaussian beam by the maximum averaged signal on the photodetector, y, found in the knife-edge test when the entire beam is directed to the photodetector. The same value of this scaling parameter is used in each of our simulations for $\epsilon = \{0.04, 0.29, 0.51, 0.72\}$ mm.

As shown in Fig. 3(b), we see good qualitative agreement between our simulations and our experiments. These simulations suggest that the uncertainty in the technique is less than 0.01 mm (1% of the beam waist), the smallest possible uncertainty permissible in these experiments due to the size of the individual mirrors of the DMD. Note that these simulations assume the maximum difference in reflectivity between the two surfaces. We anticipate that outside of this limit, if the surfaces had more similar reflectivities, this could increase the magnitude of the uncertainty.

V. DISCUSSION

As misalignment between the beam centroid and the axis of rotation δ tends to 0, the variation of the intensity as measured by the photodetector output, V_{pp}, decreases. Therefore, translating the axis of rotation through the light, or vice versa, and finding the location that minimizes V_{pp} can be used to align the axis of rotation to the beam centroid. As shown in Fig. 3(b), the minima of V_{pp} for each transit of a spinning structure through the beam correspond well with the known centroid position to within the uncertainty of the centroid position, where $\delta = 0 \pm 0.03$. Our simulations, which show good qualitative agreement to the data, suggest that the uncertainty of this centering technique demonstrated in these experiments is on the same order as the size of the individual mirrors of the DMD, ± 0.01 mm. We expect that the uncertainty of our centering technique is limited by pixilation from the mirrors, and thus, better performance would be possible with a device with smaller mirrors or by making the entire rotating structure monolithic. Indeed, knife-edge profilometry techniques have been demonstrated in non-discretized systems to have a precision of ~$1/8\lambda$.[17]

We find that this alignment technique is insensitive to the position of the interface between the two reflective portions of the surface when ϵ, the shortest distance between the interface and the axis of rotation, is smaller than the beam waist w, $\epsilon < w$. As long as the scattering surface is segmented within approximately the beam waist of the axis of rotation, the strategy proposed here can be used. The implication of this is that with just a strip of reflective tape crudely placed such that its edge passes close to but not necessarily through the axis of rotation, quick and easy centering can be performed.

Here, we have experimentally demonstrated how minimizing V_{pp} can be used to radially align a beam to the rotation axis of a planar object. We identified the centroid of a beam using a standard knife-edge approach, and then we used our V_{pp} minimization technique to find the same centroid by scanning the axis of rotation of a spinning structure. While we only minimized V_{pp} along one direction, we note that this technique can be used to minimize δ along a different subsequent direction. We have demonstrated this alignment technique using a Gaussian beam, but we expect that the same strategy could be developed for any beam, such as an LG beam. While this alignment technique was developed with RDE experiments in mind, we anticipate that it could be useful in other applications where radial alignment is critical, as in precision manufacturing. The simplicity of this alignment technique makes it a good candidate for systems that cannot tolerate mounting additional optics to the rotor. Because variations in the intensity (or power) underlie the technique and imaging interference fringes is not necessary, this strategy can be implemented for beams that may have aberrations, and it can be used with incoherent light.

AUTHORS' CONTRIBUTIONS

A.Q.A. and E.F.S. identified the need for this work. E.F.S. developed this technique, conducted the demonstration experiments, and developed the simulations. E.F.S. wrote the manuscript. E.F.S., A.Q.A., J.T.G., and G.B.R. edited the manuscript.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

[1] Y. Xie, Z. Lu, F. Li, J. Zhao, and Z. Weng, Opt. Express **10**, 1043 (2002).
[2] L. Fengyou, L. Zhenwu, X. Yongjun, and Z. Dianwen, Acta Photonica Sin. **31**, 616 (2002).
[3] T. Wang, W. Yu, D. Zhang, C. Li, H. Zhang, W. Xu, Z. Xu, H. Liu, Q. Sun, and Z. Lu, Opt. Express **18**, 025102 (2010).
[4] A. G. Poleshchuk, V. P. Korolkov, V. V. Cherkashin, S. Reichelt, and J. H. Burge, "Polar-coordinate laser writing systems: error analysis of fabricated DOEs," in *Lithographic and Micromachining Techniques for Optical Component Fabrication*, Vol. 4440 (SPIE, 2001).
[5] T. D. Milster and C. L. Vernold, Opt. Eng. **34**, 2840 (1995).
[6] S.-J. Jeong, S.-K. Lee, and S.-H. Jeong, Opt. Laser Technol. **36**, 401 (2004).
[7] M. P. J. Lavery, F. C. Speirits, S. M. Barnett, and M. J. Padgett, Science **341**, 537 (2013).
[8] M. P. J. Lavery, S. M. Barnett, F. C. Speirits, and M. J. Padgett, Optica **1**, 1 (2014).
[9] D. B. Phillips, M. P. Lee, F. C. Speirits, S. M. Barnett, S. H. Simpson, M. P. J. Lavery, M. J. Padgett, and G. M. Gibson, Phys. Rev. A **90**, 011801 (2014).
[10] M. V. Vasnetsov, V. A. Pas'ko, and M. S. Soskin, New J. Phys. **7**, 46 (2005).
[11] S. Qiu, T. Liu, Z. Li, C. Wang, Y. Ren, Q. Shao, and C. Xing, Appl. Opt. **58**, 2650 (2019).
[12] S. Qiu, T. Liu, Y. Ren, Z. Li, C. Wang, and Q. Shao, Opt. Express **27**, 24781 (2019).
[13] I. V. Basistiy, V. V. Slyusar, M. S. Soskin, M. V. Vasnetsov, and A. Y. Bekshaev, Opt. Lett. **28**, 1185 (2003).
[14] J. M. Khosrofian and B. A. Garetz, Appl. Opt. **22**, 3406 (1983).
[15] J. A. Arnaud, W. M. Hubbard, G. D. Mandeville, B. de la Clavière, E. A. Franke, and J. M. Franke, Appl. Opt. **10**, 2775 (1971).
[16] D. R. Skinner and R. E. Whitcher, J. Phys. E: Sci. Instrum. **5**, 237 (1972).
[17] A. H. Firester, M. E. Heller, and P. Sheng, Appl. Opt. **16**, 1971 (1977).