Measurement of the M^2 beam propagation factor using a focus-tunable liquid lens

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We demonstrate motion-free beam quality M^2 measurements of stigmatic, simple astigmatic, and general astigmatic (twisted) beams using only a focus-tunable liquid lens and a CCD camera. We extend the variable-focus technique to the characterization of general astigmatic beams by measuring the 10 second-order moments of the power density distribution for the twisted beam produced by passage through multimode optical fiber. Our method measures the same M^2 values as the traditional variable-distance method for a wide range of laser beam sources, including nearly TEM_{00} ($M^2 \approx 1$) and general astigmatic multimode beams ($M^2 \approx 8$). The method is simple and compact, with no moving parts or complex apparatus and measurement precision comparable to the standard variable-distance method. © 2013 Optical Society of America

OCIS codes: 140.3295, 120.4800, 000.2170.

1. Introduction

Lasers are useful in many areas of modern society, ranging from communications to precision manufacturing, to medical diagnostics and surgery. Many of these applications rely on the tight focus possible with coherent light from high-quality laser beams. Of the many measures of beam quality, the International Organization for Standardization (ISO) has selected the M^2 beam propagation parameter as the standard metric of beam quality [1]. M^2 is the ratio of the beam parameter product (waist size times far-field divergence angle) compared to that of an ideal Gaussian beam. An ideal Gaussian beam is described by $M^2 = 1$. The M^2 parameter accurately describes the propagation of higher-order modes, based on the embedded Gaussian framework [2].

While useful for designing optical systems and comparing laser resonators, it must be noted that M^2 is not a complete description of the laser beam quality. Other proposed measures of beam quality include the Strehl ratio and power in the bucket. Strehl ratio quantifies the peak on-axis light intensity in the far field, while power in the bucket quantifies the amount of power within a given aperture size. Each of these provides a useful metric for specific applications, but no single value can completely characterize the quality of a beam. Nevertheless, the M^2 parameter is applicable for general use in beam characterization [3–8].

The standard method for determining M^2 involves measuring the beam size at a range of positions near its waist. Beam size measurements are typically performed with a sweeping razor blade or CCD camera translated along the optical axis, and a glass lens is used to form a beam waist [9–12]. The M^2 parameter is calculated by curve fitting multiple measurements of the beam size as a function of lens–camera separation (hereafter referred to as the *variable-distance method*). Recently, a variable-focus M^2 measurement technique was proposed [13] and demonstrated [14],

using a liquid lens with controllable focal length and a digital micromirror device (DMD). Combined with two photodetectors, the DMD was used as either a knife-edge or pinhole to measure the beam profile with the ability to measure high power beams. The tunable lens was used to move the beam waist location relative to the stationary beam profiler, sweeping out a beam width caustic similar to that measured using the variable-distance method.

Others have accomplished variable-focus measurements of the M^2 parameter using spatial light modulators (SLMs) to form a programmable lens [7,8]. Artifacts due to the quantized and pixilated nature of SLMs can affect the measured beam quality, however [7]. In addition, SLMs are sensitive to the polarization of the incident light, while lenses have no influence or dependence on polarization.

Another class of methods to measure M^2 allows single-measurement determination of the beam quality [3–6]. One method simultaneously images many cross sections of the test beam onto an array detector using a distorted Fresnel zone plate [3] or angled Fabry–Perot filter [4], but is limited to high power lasers. Modal decomposition is a robust technique for characterizing the output of laser resonators and multimode fiber [5,6] but relies on prior knowledge of the spatial modes and is not flexible enough for general laser beam characterization.

In this paper, we demonstrate and characterize a variable-focus method for measuring the beam propagation parameter of stigmatic, simple astigmatic, and general astigmatic laser beams using only a tunable lens and a CCD camera. Using only two components is an improvement over the standard variable-distance method, which relies on precise alignment of a lens, translation stage, and camera; using a CCD camera instead of DMD and photodetectors also simplifies alignment by reducing the number of components. A tunable lens mounted to a CCD camera forms a compact M^2 measurement system with no moving parts. These advantages of the variable-focus method carry over as we extend it to general astigmatic beams by demonstrating measurements of the 10 second-order moments of the power density distribution (with the addition of a glass cylindrical lens).

We will describe the theoretical background, practical implementation, and verification of the variable-focus method using a simple electrically controllable lens and CCD camera. After analyzing the method for stigmatic and simple astigmatic beams, we will extend the variable-focus method to general astigmatic beams. Further, we will discuss the advantages of the variable-focus method over the standard variable-distance method, particularly compactness and flexibility in analyzing noncollimated input beams.

2. Theory of Variable-Focus and Variable-Distance Methods

In order to illustrate the similarities and differences between the variable-focus and variable-distance beam characterization techniques, we follow a similar path to past work [7,13,15] and use paraxial ray-tracing methods to determine the spot size dependence on the following parameters: lens focal length f, lens-to-CCD distance d, and beam quality factor M^2. The expression we find will be used to fit experimental measurements of the beam spot size to determine the M^2 value characterizing a particular laser beam.

A Gaussian beam incident on the lens may be characterized by its beam radius ω_i, and radius of curvature (RoC) at the lens R_i, as well as its wavelength λ. This can be compactly expressed as the complex beam parameter, given by

$$\frac{1}{q_i} = \frac{1}{R_i} - i\frac{\lambda}{\pi\omega_i^2}. \tag{1}$$

The *ABCD* matrix for paraxial ray propagation through a lens of focal length f and a free-space distance d can be used to determine the q-parameter for the output beam, as follows:

$$\frac{1}{q_f} = \frac{Cq_i + D}{Aq_i + B}$$
$$= \frac{dfR_i\lambda + i\pi[d(f - R_i) + fR_i]\omega_i^2}{fR_i\lambda + i\pi(f - R_i)\omega_i^2}. \tag{2}$$

From Eq. (1), we see that the spot size ω can be extracted from the q parameter by

$$\omega^2 = -\frac{\lambda}{\pi\,\mathrm{Im}\left[\frac{1}{q}\right]}, \tag{3}$$

where Im[x] denotes the imaginary part of x. We can then determine ω_f in terms of the optical parameters of the system by plugging Eq. (2) into Eq. (3) to find

$$\omega_f^2 = \frac{d^2\lambda^2}{\pi^2\omega_i^2} + \left[\omega_i\frac{d(f - R_i) + fR_i}{fR_i}\right]^2. \tag{4}$$

The above analysis is valid for fundamental Gaussian beams, but it has been shown that multimode beams follow the same equations, with the difference that the multimode beam diameter is everywhere M times larger than the fundamental Gaussian [9,16]. That is,

$$W = \sqrt{M^2}\omega, \tag{5}$$

where W is the multimode beam radius, ω is the radius of the fundamental Gaussian beam, and M^2 is the parameter characterizing the beam quality. We can solve Eq. (5) for $\omega = W/\sqrt{M^2}$, plug this into Eq. (4) for both ω_i and ω_f, and rearrange slightly to find the final multimode beam radius:

$$W_f = \sqrt{\left(M^2\frac{d\lambda}{\pi W_i}\right)^2 + W_i^2\left(1 + \frac{d}{R_i} - \frac{d}{f}\right)^2}. \tag{6}$$

This equation can be used to fit either variable-focus (by holding d constant) or variable-distance (by holding f constant) beam size data, although the expression is much simpler in the variable-f case because the first term in Eq. (6) is constant. We also note that the expression for beam width is simpler if we consider the inverse RoC and inverse focal length (optical power).

In theory, both variable-focus and variable-distance methods can measure noncollimated beams because $1/R_i$ can be used as a fitting parameter. Typical commercial translation stage M^2 apparatus and software allow for small divergence angles (large $|R_i|$, $\theta < 5$ mrad [11]), but they are limited by the range of the translation stage. A longer translation stage would allow a larger range of angles but would also lead to a prohibitively large device. In contrast, a tunable lens M^2 measurement system can account for the incoming RoC simply by changing the focal length without requiring a larger apparatus.

3. Comparing Variable-Distance and Variable-Focus Methods

A variable-focus beam analysis method can be performed using only a tunable lens and a CCD camera (Fig. 1). Tuning the focal length allows measurement of the beam radius for multiple focal lengths. Fitting Eq. (6) to these beam radius data yields the M^2 value as well as other incident beam parameters, W_i and R_i. Focus-tunable lenses are now commercially available from a number of vendors. We have chosen to use the Optotune EL-10-30 lens (LD material) for its large aperture (10 mm) and adequately low wavefront error [17]. A driving current controls the lens focal length and is supplied by a precision laser diode current source (ILX LDX-3207). The capabilities of this liquid lens are discussed in Section 5; Table 1 summarizes the lens properties.

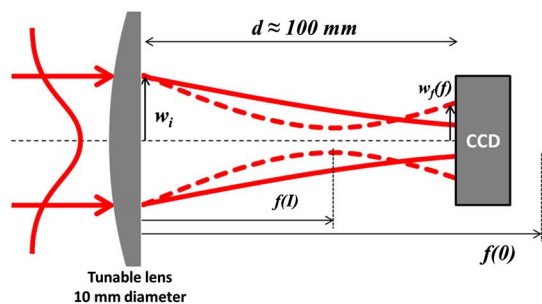The crux of any beam propagation measurement is the ability to accurately measure the laser beam

Fig. 1. (Color online) Schematic describing the variable-focus method of measuring the laser beam propagation factor, M^2. The only two components required are a CCD camera and a focus-tunable lens. A beam waist is formed at a position after the lens given by the resting focal length, $f(0)$; applying current I changes the lens focal length, $f(I)$, moving the waist location through the plane of the camera. Fitting Eq. (6) to beam size data, $W_f(f)$, yields the M^2 value as well as the initial beam size and RoC, W_i, and R_i. The lens was oriented with its optical axis vertical to avoid wavefront errors.

Table 1. Specifications: EL-10-30, LD Material (Values from [17] unless otherwise Indicated)

Parameter	Value
Diameter	10 mm
Focal tuning range[a]	~45–160 mm
Optical power range[a]	~6–20 m^{-1}
RMS wavefront error with vertical optical axis[a]	
Entire aperture	105 nm
2 mm aperture	<3 nm
Temperature influence	~0.2 m^{-1}/10°C
Settling time	<15 ms
Damage threshold (1064 nm)	10 kW/cm^2
Polarization dependence	None

[a]This work

width. Beam profiling CCD cameras are available for this purpose, and we have chosen to use the WinCamD-UCD12 (DataRay). DataRay software calculates the 4σ beam width (as per the ISO standard [1]) along the horizontal, vertical, major, and minor axes of the beam. The minor axis is the smallest dimension of the beam passing through the centroid; the major axis is perpendicular to the minor axis.

The proposed variable-focus method for measuring laser beam quality was tested by comparing with the traditional variable-distance method using a translation stage. A homebuilt variable-distance caustic measurement setup was used, based on a 150 mm travel motorized translation stage with 4 μm bidirectional repeatability (Newport UTS150CC). The liquid lens resting focal length (no current) was used as a fixed lens to create a beam waist in the variable-distance method. For both methods, the beam diameter was measured with a CCD camera and DataRay software, and curve fitting was performed using Eq. (6). The camera background was automatically subtracted in the software to improve the second-moment width calculations. Recorded beam diameters were averaged over 10 measurements, and all beam diameters used were larger than 10× (pixel pitch) (with 4.65 μm pixels) to ensure accurate beam width calculation. Approximately 20 beam width measurements were made along each beam caustic, exceeding the ISO recommended minimum.

A 632.8 nm helium–neon laser with $M^2 < 1.1$ (He–Ne, Thorlabs HRP050) was used as a high mode-quality laser source for testing. A lower-quality beam was obtained by passing the same laser beam through 3 m of optical fiber, which is slightly multimode for 632.8 nm (8.2 μm core diameter, $V = 5.7$, SMF28e). Based on the measured divergence angle from the fiber, $M^2 \approx 3$ is expected. Further beam quality reduction was obtained by passing the He–Ne laser beam through multimode fiber (25 μm core diameter, 125 μm cladding diameter, 0.10 NA, $V = 12.4$), resulting in $M^2 \approx 8$. To demonstrate the applicability of the variable-focus method, we also applied it to measure the beam propagation parameter of a multi-Watt wavelength-beam-combined (WBC) array of broad area diode lasers at 980 nm wavelength similar to that described in [18]. Testing

with a wide range of laser beam qualities shows the broad applicability of the variable-focus method.

First, we test the variable-focus method with a good quality He–Ne laser. Beam width measurements with both the variable-distance and variable-focus methods are shown side by side in Fig. 2 for comparison. Equation (6) was fit to the data with M^2, W_i, and R_i used as free parameters. Note the different units on the horizontal axes: distance in Fig. 2(a) and optical power in Fig. 2(b). The tunable lens and translation stage methods measure the same M^2 values, $M^2 = 1.05$ in the vertical dimension. This beam quality parameter is very close to 1, as expected for a TEM_{00} He–Ne laser. Reported uncertainties are twice the standard deviation of the fit (95% confidence interval).

The two methods also agree when applied to laser beams of lower quality as described above. Figure 3 shows the data with fitted M^2 factors using a beam coupled through multimode fiber. The data show a good match between the variable-focus and variable-distance methods for a wide range of M^2 values. The measured M^2 values for each method are summarized in Table 2. The variable-focus method agrees with the variable-distance results within the measurement uncertainty for all four laser sources tested.

The precision of the variable-focus method is comparable to that of the variable-distance method, evidenced by the uncertainties listed in Table 2. These data show the variable-focus method for measuring the beam propagation factor is valid and applicable over a broad range of laser beam qualities.

The He–Ne beam displays near-circular symmetry and is referred to as a stigmatic beam. The WBC diode laser array has two constant axes of symmetry, and so is a simple astigmatic laser beam. The coherent addition of the excited spatial modes in multimode optical fibers causes the two final test beams to twist slightly as they propagate: they are not circular and also do not have constant axes of symmetry. More involved methods are required to characterize these general astigmatic beams, as described in Section 4.

4. General Astigmatic Beams

General astigmatism describes the case of a laser beam with no assumptions about symmetry. An arbitrary beam can be described using the 10 second-order moments of the power density distribution [19]. These moments describe beam widths, divergence angles, radii of curvature, and the twist parameter.
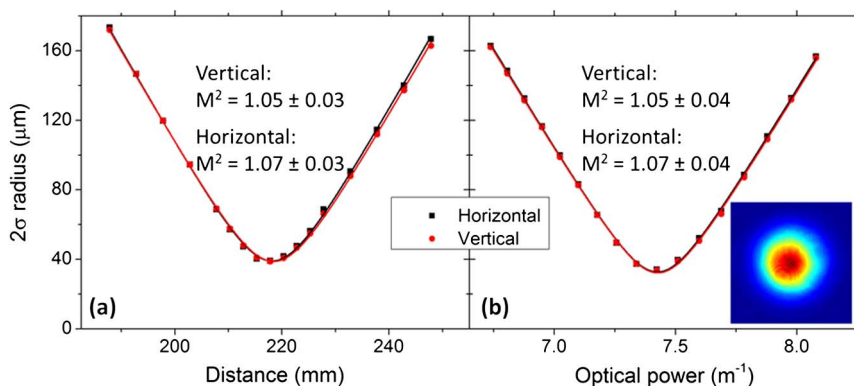

Fig. 2. (Color online) Comparison of data obtained using (a) variable-distance and (b) variable-focus methods. Beam widths are measured using a CCD based on the 4σ definition. Uncertainty in the beam width is smaller than the symbol size. Data is fitted using Eq. (6). Reported uncertainties are twice the standard deviation of the fit (95% confidence interval).
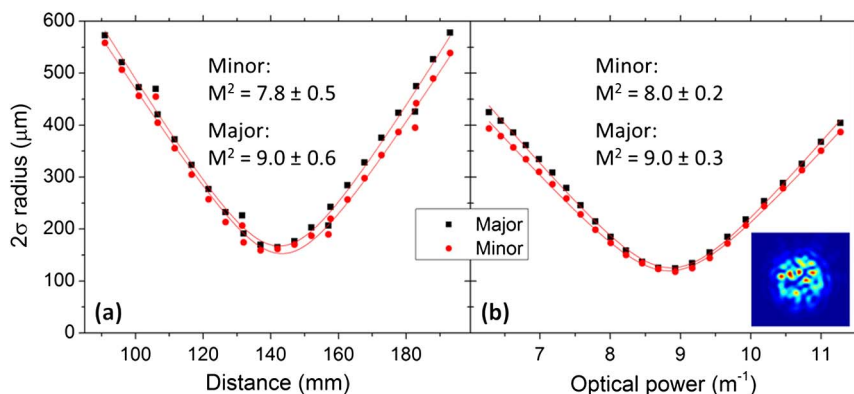

Fig. 3. (Color online) Comparison of (a) variable-distance and (b) variable-focus methods for a multimode beam: a He–Ne laser laser beam passed through 2 m long, 25 μm core fiber. Equation (6) was fit to data. Beam widths were measured along major and minor axes because the beam twists under propagation, as discussed in Section 4. Reported uncertainties are twice the standard deviation of the fit (95% confidence interval). The inset shows the near-field irradiance profile.

Table 2. Comparison of M^2 Values using the Variable-Distance and Variable-Focus Methods

	Variable-Distance	Variable-Focus
He–Ne laser beam		
Horizontal	1.07 ± 0.03	1.07 ± 0.04
Vertical	1.05 ± 0.03	1.05 ± 0.04
WBC diode array		
Horizontal	3.23 ± 0.13	3.12 ± 0.11
Vertical	1.32 ± 0.14	1.1 ± 0.4
8.2 µm core fiber		
Major	3.66 ± 0.07	3.61 ± 0.05
Minor	2.21 ± 0.05	2.20 ± 0.03
25 µm core fiber		
Major	9.0 ± 0.6	9.0 ± 0.3
Minor	7.8 ± 0.5	8.0 ± 0.2

Commonly the 10 moments are collected in the symmetric beam matrix:

$$P = \begin{pmatrix} \langle x^2 \rangle & \langle xy \rangle & \langle x\theta_x \rangle & \langle x\theta_y \rangle \\ \langle xy \rangle & \langle y^2 \rangle & \langle y\theta_x \rangle & \langle y\theta_y \rangle \\ \langle x\theta_x \rangle & \langle y\theta_x \rangle & \langle \theta_x^2 \rangle & \langle \theta_x\theta_y \rangle \\ \langle x\theta_y \rangle & \langle y\theta_y \rangle & \langle \theta_x\theta_y \rangle & \langle \theta_y^2 \rangle \end{pmatrix}. \qquad (7)$$

An effective beam quality factor that is invariant under free propagation can be defined based on the determinant of the beam matrix,

$$M^2_{\text{eff}} = \frac{4\pi}{\lambda}[\text{Det}(P)]^{1/4}. \qquad (8)$$

Variable-distance methods can be used to measure these 10 beam parameters [20–22]. In contrast, we propose a variable-focus method for measuring the parameters of general astigmatic beams and show this method matches the results of the standard variable-distance method.

The spatial moments, $\langle x^2 \rangle$, $\langle y^2 \rangle$, and $\langle xy \rangle$, are directly obtainable from irradiance measurements, such as those of a CCD camera. A total of nine beam parameters can be found by fitting caustic-type equations: eight of the second-order moments and the sum of the final two, $s = \langle x\theta_y \rangle + \langle y\theta_x \rangle$. The ISO standard describes the fit functions for a variable-distance method [19]. Using paraxial ray tracing through a lens of focal length f and free space distance d, we find equivalent fit functions for the variable-focus method:

$$\langle x^2 \rangle = \langle x^2 \rangle_i \left(1 - \frac{d}{f}\right)^2 + 2d\langle x\theta_x \rangle_i \left(1 - \frac{d}{f}\right) + d^2\langle \theta_x^2 \rangle_i$$

$$\langle y^2 \rangle = \langle y^2 \rangle_i \left(1 - \frac{d}{f}\right)^2 + 2d\langle y\theta_y \rangle_i \left(1 - \frac{d}{f}\right) + d^2\langle \theta_y^2 \rangle_i$$

$$\langle xy \rangle = \langle xy \rangle_i \left(1 - \frac{d}{f}\right)^2 + ds_i \left(1 - \frac{d}{f}\right) + d^2\langle \theta_x\theta_y \rangle_i. \qquad (9)$$

We choose to find the parameters $\langle ab \rangle_i$ in the plane just before the tunable lens. The beam matrix at any plane can be obtained via $ABCD$-type matrices [19].

The M^2 beam quality factor does not depend on this choice of measurement plane, because it is invariant under free propagation.

The twist parameter, $t = \langle x\theta_y \rangle - \langle y\theta_x \rangle$, completes the beam matrix, but is not accessible using only spherical optics. The second-order moments that enter into the beam matrix depend on combinations of s and t: $\langle x\theta_y \rangle = (s + t)/2$ and $\langle y\theta_x \rangle = (s - t)/2$. The twist parameter does not change under propagation in free space and through spherical lenses [19], so we are free to measure it in any plane.

In both variable-distance and variable-focus methods, finding the final parameter requires additional measurements with an astigmatic (cylindrical) lens. Only two beam measurements are necessary, but fitting measurements over a range of distances or focal lengths improves the twist determination [22]. The beam is measured with a cylindrical lens in both vertical- and horizontal-focusing orientations a distance d_c from the camera. Ray tracing again provides the fitting function to find the twist parameter from measurements of the spatial second-order moments:

$$\{\langle xy \rangle_v - \langle xy \rangle_h\}(f_c, d_c) = t\frac{d_c^2}{f_c}, \qquad (10)$$

where f_c is the cylindrical lens focal length.

Practically, we carry out measurements of beam widths in the same way as for the previously described variable-focus method. In addition, we record the values of $\langle xy \rangle$ at each focal length. Fitting Eq. (9) to these data allows finding 9 of the 10 parameters. Then we add a glass cylindrical lens to measure the twist parameter in the same manner as is typical for the variable-distance method: measuring $\langle xy \rangle$ with the CCD camera at a fixed distance from a cylindrical lens in both vertical- and horizontal-focusing orientations. The twist parameter could be found more accurately with a variable-focus cylindrical lens and fitting to Eq. (10), but it is sufficient to measure in only one plane with a glass cylindrical lens [19].

We have tested the variable-focus method for the general astigmatic beam formed by a He–Ne laser beam passed through 2 m long, 25 µm core fiber (the same beam as shown in Fig. 3). The measured spatial moments were fitted by Eq. (9), shown for both variable-distance and variable-focus methods in Fig. 4. We measured the twist parameter to be 0.055 µm · rad. The twist parameter and the nine fit parameters provide the entire beam matrix, P, with moments listed in Table 3. Then the effective M^2 value was calculated to be 8.1 ± 0.2 and 8.5 ± 0.2 from Eq. (8) with the variable-distance and variable-focus methods, respectively. Uncertainty in M^2 is calculated based on error analysis following that presented in [22].

The variable-focus method agrees with the variable-distance method for this test beam, validating the technique. The new method does not require a translation stage, and so simplifies alignment and
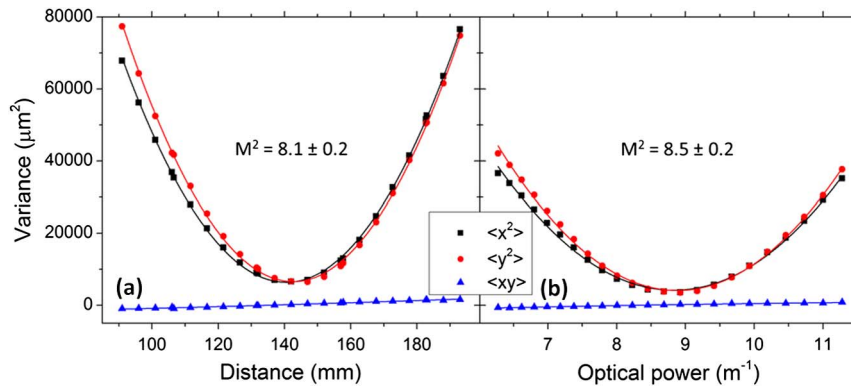
Fig. 4. (Color online) Comparison of (a) variable-distance and (b) variable-focus methods for measuring the 10 second-order moments of a general astigmatic. The test beam was identical to that of Fig. 3, a He–Ne laser laser beam passed through 2 m long, 25 μm core fiber. Equation (9) was fit to data. The nine fit parameters and the separately measured twist parameter completely characterize the beam and allow calculation of an effective M^2 value.

potentially improves measurement speed compared with the standard variable-distance method. Avoiding the need to add or switch optical components (e.g., cylindrical lenses) would further improve the variable-focus method. A device that can operate as both a tunable spherical lens and tunable cylindrical lens (such as a SLM or electrowetting lens with multiple electrodes) would allow a compact system for characterizing general astigmatic beams without need for switching lens components.

5. Discussion

Both variable-distance and variable-focus methods rely on a lens to create a beam waist. The variable-distance method can use glass lenses that may be made with very high optical quality. Tunable lenses used for the variable-focus method have only recently reached maturity and are not yet widely used or fully optimized. While aberrations and ohmic heating complicate its use, the Optotune EL-10-30 flexible-membrane liquid lens works well for measuring beam quality.

The lens is formed by a thin flexible membrane enclosing a liquid, with refraction occurring primarily at the air/membrane and membrane/liquid boundaries. Elastic membrane tension defines a spherical boundary, with curvature determined by the volume of liquid enclosed. Force from an electromagnetic coil compresses a liquid reservoir, controlling the volume of liquid in the lens and thus the curvature of the membrane and lens focal length. Accurate M^2 measurements require knowledge of the lens focal length tuning as a function of current, $f(I)$; incorrect expressions for $f(I)$ result in noticeably poor fits to the measured data (e.g., $R < 0.9$). To measure $f(I)$, a CCD camera was placed a known distance from the lens and current was applied to achieve the minimum spot size; given a collimated input beam, the lens-to-camera distance is the focal length for this applied current. Repeating with varying separation provides a focal length calibration curve for the lens used, shown in Fig. 5. The focal length uncertainty could be included in the fitted M^2 values, but this contribution is small compared to the uncertainty of the caustic fit itself.

The measured aberration of our tunable lens with no applied current is less than 3 nm across the 2 mm diameter region at the center (measured with a Shack–Hartmann wavefront sensor as in [23]) and allows measurement of high-quality beams, as evidenced by measurement of the He–Ne laser beam

Table 3. Second-Order Moments Measured with the Variable-Focus Method

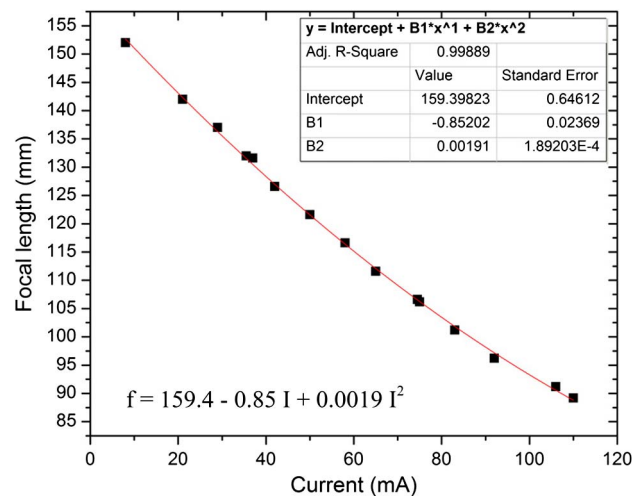| Moment	Value	
$\langle x^2 \rangle$	$(4.68 \pm 0.13) \times 10^5$	μm · rad
$\langle x\theta_x \rangle$	-0.28 ± 0.01	μm · rad
$\langle \theta_x^2 \rangle$	$(5.4 \pm 0.4) \times 10^{-7}$	rad^2
$\langle y^2 \rangle$	$(5.26 \pm 0.14) \times 10^5$	μm^2
$\langle y\theta_y \rangle$	-0.29 ± 0.01	μm · rad
$\langle \theta_y^2 \rangle$	$(5.3 \pm 0.4) \times 10^{-7}$	rad^2
$\langle xy \rangle$	$(-1.8 \pm 0.7) \times 10^3$	μm^2
$\langle x\theta_y \rangle$	$(1.56 \pm 0.06) \times 10^{-2}$	μm · rad
$\langle y\theta_x \rangle$	$(-3.96 \pm 0.06) \times 10^{-2}$	μm · rad
$\langle \theta_x\theta_y \rangle$	$(2.75 \pm 0.19) \times 10^{-8}$	rad^2

Fig. 5. (Color online) Measured focal length calibration curve for the Optotune EL-10-30 (with LD material), oriented with vertical optical axis.

(Fig. [2]). The effects of spherical aberration are negligible for small beams. For aberration introduced by a spherical lens,

$$M^2 \approx M_0^2 + \frac{1}{2M_0^2}\left(\frac{\omega_i}{\omega_q}\right)^8, \qquad (11)$$

where M_0^2 is the beam propagation parameter before the spherical lens and ω_q is the critical beam width for spherical aberration (approximated for small ω_i/ω_q) [24]. $\omega_q \approx 4$ mm for the tunable lens used in this work, assuming a plano–convex lens oriented optimally [24]. From this expression, we expect M^2 to change by only 0.01 for a 5 mm beam diameter. In addition to aberrations from the spherical lens surface, use of liquid in the lens means that gravity can distort the lens shape. Gravity causes 130 nm of coma when the lens is oriented with the optical axis horizontal [17]. To avoid distortion due to gravity, we oriented the lens with the optical axis vertical throughout this work.

During use, the temperature of the lens increases due to resistive heating of the actuating coil. As the liquid heats up, both the refractive index and volume of the liquid change, resulting in a change in optical power of about 0.2 m^{-1} per 10°C [17]. Ohmic heating increases with the square of the current, so this thermal effect increases rapidly at high currents. This temperature-dependent focusing can be avoided by allowing time for the lens to cool after tuning with high current, using small currents ($<$50 mA) when possible, and actively monitoring the wire temperature and adjusting the applied current to compensate. The two former methods were used in this work to achieve repeatable lens tuning, and the lens focal length was remeasured after periods of tuning to check repeatability.

As expected from Eq. ([6]), the M^2 value does not depend on the incident beam's RoC. R_i can be used as a fit parameter in addition to W_i and M^2. Both variable-distance and variable-focus methods share this quality, but the variable-focus method can more easily adapt to measure widely divergent or convergent beams. For the variable-distance and variable-focus methods, adaptation depends on accessing a large position range of the translation stage or large focal tuning range of the liquid lens, respectively. The tuning range is independent of liquid lens size, and large tuning ranges of more than 900 diopters have been achieved using small lenses with less than 1 mm diameter and thickness [25]. In contrast, a longer translation stage is needed to achieve a larger tuning range in variable-distance measurements. Additionally, the variable-distance method cannot focus a beam with incident RoC smaller than the focal length of the lens used to create the beam waist (no waist is formed for $R_i \leq f$). The variable-focus method is only limited by the minimum focal length of the liquid lens.

Therefore, the variable-focus method has greater potential for adaptation based on improvements in liquid lens designs rather than on scaling up the size of the apparatus. With fixed lens-to-camera distance and incident beam size, the optical system presented in this paper can account for incident RoC of 100 mm (divergence angles up to 15 mrad). For example, note that data in Fig. [2] represents $R_i \approx 500$ mm and $\theta \approx 2.5$ mrad. In contrast, with the 150 mm long translation stage used for these experiments, the variable-distance method could only account for divergence angles up to 5 mrad. Commercial systems can typically account for incident divergence angles of about 5 mrad for visible light [11]. The variable-focus method is more flexible while requiring a smaller apparatus.

The speed of the two methods is comparable. Lens tuning and translation stage movement times are similar, on the order of 0.1–1 s [17]. Faster tuning is a current topic in the development of tunable lenses and shows promise for improvement, but measurement time is also limited by the speed of acquisition with the beam profiling camera. Based on typical ∼10 Hz camera frame rate, the time to measure the beam size at each location is also on the order of 1 s (averaging over at least 5 images as per [1]), and complete M^2 measurements require on the order of 10 s. Current devices typically require at least 100 s for a complete M^2 measurement [10]. With a 1 kHz frame rate camera, 1 ms lens tuning time, and a sufficiently fast curve-fitting algorithm, M^2 measurements could be completed in approximately 100 ms. Simultaneous increases in camera frame rate and lens tuning speeds could lead to near-realtime M^2 beam analysis systems.

6. Conclusion

A variable-focus method for measuring the M^2 beam propagation parameter has been tested and verified for stigmatic, simple astigmatic, and general astigmatic beams. The new technique reproduces the M^2 values measured using the standard variable-distance method for a wide range of laser beam sources. In addition, the demonstrated variable-focus method of measuring M^2 provides advantages over the traditional translation stage method. No moving parts are required, enabling a robust measurement system. Lens tuning can account for large incident divergence angles that would require a prohibitively long translation stage with the standard method. These advantages show the potential for the variable-focus method to improve techniques and apparatus for beam propagation measurements.

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