Competitive Differential Pricing

Yongmin Chen* Jianpei Li† Marius Schwartz‡

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Abstract. This paper analyzes welfare under differential versus uniform pricing across oligopoly markets that differ in costs of service. We establish general demand conditions for differential pricing by symmetric firms to increase consumer surplus, profit, and total welfare. The analysis reveals why competitive differential pricing is generally beneficial—more than price discrimination—but not always, including why profit may fall, unlike for monopoly. The presence of more competitors tends to enlarge consumers' share of the gain from differential pricing, though profits often still rise. When firms have asymmetric costs, however, profit or consumer surplus can fall even with 'simple' linear demands.

Keywords: differential pricing, price discrimination, demand curvature, cross-price elasticity, pass-through, oligopoly

*University of Colorado Boulder; yongmin.chen@colorado.edu

†University of International Business and Economics (UIBE); lijianpei2007@gmail.com

‡Georgetown University; mariusschwartz@mac.com

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1. Introduction

Distinct consumer groups—or 'markets'—for a product frequently differ in their costs of service or demands. A large literature studies the welfare effects, relative to uniform pricing, of differential pricing across markets that entail equal costs of service and differ solely in demand elasticities—classic third-degree price discrimination—under monopoly or oligopoly.[1] Very little work has compared uniform pricing (UP) and differential pricing (DP) when, instead, markets vary in costs of service. Yet DP motivated (at least partly) by cost differences is controversial and frequently subject to various constraints in monopoly or oligopoly markets, such as gender-neutral requirements in insurance or pensions, universal-service mandates on utilities, antidumping rules in international trade, and consumer resistance to add-on pricing such as airline bag fees.[2]

This paper analyzes the welfare effects of (cost-based) differential pricing by oligopoly firms. If markets were perfectly competitive, DP obviously would be desirable, as prices would equal marginal costs in each market hence maximize welfare, whereas UP would distort the output allocation. However, the ranking is no longer clear when prices exceed marginal costs. For monopoly, Chen and Schwartz (2015) show that DP increases consumer surplus (aggregated across markets) under fairly general conditions, though it is possible for consumer surplus and total welfare to fall, albeit under rather stringent demand conditions. Under oligopoly competition, the welfare analysis of DP is even richer for at least two reasons: (1) when firms supply differentiated products, the pricing equilibria will depend additionally on cross-price elasticities[3]; and (2) if firms differ in cost within each market, the pricing equilibria will depend on the pattern of cost asymmetry even with homogeneous products.

We consider an arbitrary number (n) of competing firms, each selling its product in $M \geq 2$ distinct markets that have equal demand elasticities at any common price but different marginal

[1] See, for example, Aguirre, Cowan and Vickers (2010) and references therein for monopoly third-degree price discrimination; and Holmes (1989) and Stole (2007) for the oligopoly case.

[2] The constraints on cost-based DP can stem from various sources: government policy, contractual restrictions, consumer perceptions of the likely effects, or transaction costs. For further discussion and examples, see Chen and Schwartz (2015), Edelman and Wright (2015), and Nassauer (2017). On add-on pricing generally see Ellison (2005) and Brueckner et al. (2015).

[3] Mrázová and Neary (2017) show that any well-behaved demand function for a single product can be represented by its elasticity and curvature. With differentiated products, cross-price elasticity is additionally needed.

costs of service (the polar opposite case from classic price discrimination). Firms sell symmetrically differentiated or homogeneous products and compete in prices under the alternative UP or DP regimes.[4] Our main model has firms with symmetric costs—the same cost within a market—selling (symmetrically) differentiated products. This environment serves two purposes. It reveals how the welfare properties of DP under monopoly (Chen and Schwartz, 2015) may differ in oligopoly solely due to the cross-elasticity/substitution effect; and it permits a natural comparison to price discrimination in symmetric oligopoly, analyzed by Holmes (1989). An extension of the model introduces asymmetric costs between firms, while retaining symmetric demands.

In our main model, a major factor is the pass-through rate from firms' common marginal cost to their symmetric equilibrium price.[5] Under UP each firm sets a single price based on the average of its marginal costs across markets, whereas under DP it sets prices based on each market's specific marginal cost. Thus, moving to DP effectively lowers firms' marginal cost in some markets and raises cost in the others, with the equilibrium price adjustments determined by the pass-through rate. By analyzing when each welfare measure is convex or concave in marginal cost, we obtain necessary and sufficient conditions for DP to raise consumer surplus, profit, or total welfare (Propositions 1-3). The conditions involve general properties of the demand system (which also determine the pass-through rate): curvature and own- and cross-price elasticities, and how they possibly vary as firms change price equally or as the number of firms changes. These conditions reduce to their counterparts for monopoly DP—and hence neatly nest the latter—when cross-price effects vanish. We trace the welfare changes to familiar forces, such as the change in average price and in total output across markets.

Consumer surplus is subject to the same forces as under monopoly when moving to DP. Consumers in the aggregate benefit from the price dispersion. But average price can rise with DP, as occurs if the pass-through rate is greater at higher than at lower prices along the common demand function of the different market segments. Potentially, consumers might lose on balance,

[4]Hereafter, unless stated otherwise, DP refers to 'cost-based' differential pricing. These alternative pricing regimes can be attained through interventions that do not require knowledge of costs: laissez faire yields DP, whereas prohibiting any price differences yields UP. Cowan (2018) analyzes regulatory schemes that constrain a monopolist's price-cost margins, schemes that improve welfare but require the regulator to know costs.

[5]For a general analysis of pass-through in various applications see Weyl and Fabinger (2013).

though we have not found such an example.[6]

Unlike for monopoly, DP can reduce profits in oligopoly.[7] This may occur in two ways. First, DP induces excessive output reallocation between markets when pass-through exceeds one, as the price difference 'overshoots' the cost difference so the profit margins become smaller in the (low-cost) markets that gain output. Under monopoly, DP nevertheless raises profit for any pass-through rate, because a rate above one requires demand to be highly convex, in which case the price dispersion chosen by a monopolist yields a large output expansion (Chen and Schwartz, 2015). In oligopoly, however, pass-through can be high not only due to demand curvature but also due to cross-price effects between firms. Consequently, DP may reallocate output excessively without expanding total output enough to outweigh the misallocation. Second, DP can reduce average price, while also lowering output. If the products are closer substitutes at lower prices than at higher prices, moving to DP can reduce price by more in low-cost markets than it raises price in the high-cost markets even if demand curvature is smaller at lower prices—which explains why output can decrease. In some such cases, total welfare also declines.

Our general symmetric oligopoly setting also lets us examine how more intense competition, represented by a larger number of firms (n) that drives prices closer to marginal costs, affects the distribution of gains from DP between consumers and firms. For a linear demand system, we show that DP always benefits consumer surplus and profits, but the *share* of the gains that accrues to consumers increases with n (Proposition 4). The same pattern is found through simulations for the CES demand system and multinomial logit. Since both consumers and firms tend to benefit from DP in oligopoly, a move all the way to perfect competition—where profits are zero and welfare is maximized—necessarily increases consumers' share of the welfare gains. Our findings suggest that this pattern holds also for incremental changes in competition intensity within oligopoly, for some familiar classes of demands.

Overall, our analysis suggests that—while there are exceptions—cost-based DP in symmetric oligopoly is broadly beneficial. Furthermore, DP is more beneficial for consumers and total welfare than classic third-degree price discrimination. In both cases, consumers gain from the

[6]As detailed in Subsection 3.4, we analyze as examples three classes of demand systems under symmetric oligopoly: linear, CES, and multinomial logit. DP raises consumer surplus in these cases.

[7]In such cases, firms would jointly gain from committing to uniform pricing, but such a commitment would not be unilaterally optimal.

price dispersion by adjusting quantities. But price discrimination has a bias to raise average price, which harms consumers, whereas cost-based DP does not, for a broad class of demand functions. Regarding total welfare, classic price discrimination misallocates output while UP does not, whereas when markets differ in costs of service, UP misallocates output and DP can improve the allocation. We provide an example where markets differ both in costs of service and in demand elasticities and, as expected, differential pricing is beneficial for consumers as well as firms if the cost differences are large relative to the demand differences.

Do the generally favorable effects of DP persist when firms have asymmetric costs in a given market? For tractability, we consider two firms and two market segments, with a focus on homogeneous products.[8] In one scenario, the same firm has a cost advantage in both markets. Consumer surplus rises under DP due to price dispersion, but profit can readily fall if the cost difference across markets for the lower-cost firm is smaller than for the rival (Proposition 5). In an alternative scenario, each firm has a cost advantage in one market (but under UP each firm must serve both markets). Average price across markets under DP then exceeds the uniform price, because cost dispersion—which determines equilibrium markups—is higher under DP. If cost heterogeneity between firms is large relative to that across markets, the price-increasing effect of DP dominates the beneficial price dispersion effect hence consumer surplus falls (Proposition 6). As a robustness check, we extend these findings to differentiated products with symmetric linear demands, showing that DP can reduce either profit or consumer surplus if products are sufficiently close substitutes and firms have asymmetric costs (Proposition 7).

To our knowledge, the only other analysis of cost-based DP in oligopoly is by Adachi and Fabinger (2019). Our contributions are complementary. Adachi and Fabinger add cost differences between markets to Holmes' (1989) symmetric oligopoly setting. They provide sufficient conditions for DP to lower or raise total welfare, using similar techniques as Aguirre, Cowan and Vickers (2010; 'ACV'), who study monopoly price discrimination with no cost differences. Their conditions resemble ACV's, in comparing weighted markups between markets at the equilibrium price(s), but are more complex because the weights depend additionally on cross-price effects. Our analysis of symmetric oligopoly assumes equally-elastic demands across markets in order

[8] Since DP by symmetric firms is always beneficial with homogeneous products (Bertrand competition then yields the same first-best outcome as perfect competition), this case sharply highlights the role of cost asymmetries.

to focus sharply on the role of cost differences. By analyzing when each welfare measure is a convex function of marginal cost, we provide transparent necessary and sufficient conditions on the demand system for DP to raise or lower consumer surplus, profit, or total welfare, and decompose the underlying forces. Our analysis also offers new insight on how the effects of DP on consumers and firms may vary with competition intensity. In addition, we highlight the effects of cost asymmetries between firms.

The next section presents the main model and some preliminary results. Section 3 analyses the effects of DP with symmetric firms. Section 4 considers the case where firms have asymmetric costs. We conclude in Section 5, and gather all proofs in the Appendix.

2. A Model With Symmetric Firms

There are $n \geq 2$ symmetric firms, each producing a horizontally differentiated product. The demand function for firm $i \in \{1, \cdots, n\}$ is $\widetilde{D}(p_i, p_{-i})$, where $p_{-i} \equiv (p_1, \cdots, p_{i-1}, p_{i+1}, \cdots, p_n)$. The continuous and differentiable function $\widetilde{D}(p_i, p_{-i})$ is decreasing in p_i and increasing in every element of p_{-i}.[9] There are $M \geq 2$ distinct groups of consumers or market segments. Each firm's constant marginal cost is c_m to serve group $m \in \{1, \cdots, M\}$, with $c_1 < c_2 < ... < c_M$. Group m's demand for firm i's product is $\lambda_m \widetilde{D}(p_i, p_{-i})$ with $\lambda_m \in (0,1)$ and $\sum_{m=1}^{M} \lambda_m = 1$. Since the demand functions for the M market segments differ only in scale, they have equal price elasticities at any common prices.

Firms compete by simultaneously choosing prices, in two alternative pricing regimes. Under uniform pricing (UP) each firm sets a single price for all consumers, whereas under differential pricing (DP) each firm can charge different prices for the distinct consumer groups.

For convenience, we use $D(p, \hat{p}) \equiv \widetilde{D}(p, \hat{p}, ..., \hat{p})$ to denote firm i's demand when it charges price p and all its competitors charge price $\hat{p}$. At equal prices $\hat{p} = p$, the demand for any firm i is then $D(p, p)$, and we define the industry demand as

$$Y(p) \equiv nD(p, p), \quad \text{with} \quad Y'(p) < 0. \tag{1}$$

[9] We shall also briefly address the case where the n symmetric firms produce a homogeneous product so that $\widetilde{D}(p_i, p_{-i})$ is not continuous in p_i at $p_1 = \cdots = p_n$. The analysis is then straightforward.

Given firms' symmetry, we will analyze the symmetric equilibria in which prices are the same for all firms under UP or under DP. Under DP, given rivals' prices, each firm charges a market-specific price p_m for each group m that maximizes profit for marginal cost c_m. Under UP, each firm draws customers from group m in proportion to the relative mass λ_m, hence its virtual marginal cost will be a weighted average of c_m:

$$\bar{c} \equiv \sum_{m=1}^{M} \lambda_m c_m, \tag{2}$$

and the symmetric uniform price p_U maximizes a firm's profit for marginal cost $\bar{c}$ given that all its competitors set p_U.[10] We refer to the market segments with $c_m < \bar{c}$ as low-cost markets and the segments with $c_m > \bar{c}$ as high-cost markets. The above setting lets us study the welfare effects of (cost-based) differential vs. uniform pricing in a general symmetric oligopoly, including how the effects may depend on the number of competitors.

We assume standard demand conditions such that DP raises equilibrium prices in the high-cost segments and reduces equilibrium prices in the low-cost segments: $p_m < p_U$ if $c_m < \bar{c}$ and $p_m > p_U$ if $c_m > \bar{c}$. Let $q_U = Y(p_U)$, and $q_m = Y(p_m)$. Then $q_U > q_m$ if $c_m > \bar{c}$ and $q_U < q_m$ if $c_m < \bar{c}$. Denote $\Delta q_m \equiv q_m - q_U$, then $\Delta q_m > 0$ if $c_m < \bar{c}$ and $\Delta q_m < 0$ if $c_m > \bar{c}$. Define

$$p_D \equiv \sum_{m=1}^{M} \lambda_m p_m \tag{3}$$

as the *average* (equilibrium) price under DP weighted by the relative sizes of the M market segments, which equal their relative consumption quantities under UP.

To use consumer surplus as a welfare measure, let the demand functions be derived from optimization by a representative consumer with quasi-linear utility function $V(q_1, ..., q_n) + q_0$, where V is increasing and concave, and q_0 is consumption of the numeraire good. When all

[10] At a common uniform price p_U, firm i's profit is $\sum_{m=1}^{M}(p_U - c_m)\lambda_m D(p_U, p_U) = (p_U - \bar{c})D(p_U, p_U)$.

firms set equal price p, the aggregate consumer surplus can be written as[11]

$$S(p) \equiv \int_p^\infty nD(x,x)dx = \int_p^\infty Y(x)dx. \tag{4}$$

Notice that $S(p)$ is convex, and the consumer surplus under UP and DP are respectively $S_U = S(p_U)$ and $S_D = \sum_{m=1}^M \lambda_m S(p_m)$. We thus have:

Remark 1 *DP increases consumer surplus if average price does not rise ($p_D \le p_U$).*

Intuitively, when the pricing regime moves from UP to DP, if $p_D \le p_U$ the representative consumer can still afford the old consumption bundle under UP, $(\lambda_1 q_U, \cdots, \lambda_M q_U)$, but will exploit the price dispersion by increasing quantity where price fell and decreasing quantity where price rose (Waugh, 1944).

Next, consider profit. Denote the industry output under DP as $q_D \equiv \sum_{m=1}^M \lambda_m q_m$. Moving from UP to DP changes industry profit by

$$\Delta\Pi \equiv \Pi_D - \Pi_U = \sum_{m=1}^M \lambda_m (p_m - c_m) q_m - (p_U - \bar{c})\, q_U.$$

The difference in industry profit, $\Delta\Pi$, can be decomposed as follows:

$$\Delta\Pi = \underbrace{(p_D - p_U)\, q_U}_{\text{Average-P Effect}} + \underbrace{\sum_{m=1}^M \lambda_m (q_m - q_D)(p_m - c_m)}_{\text{Reallocation Effect}} + \underbrace{(q_D - q_U)(p_D - \bar{c})}_{\text{Output Effect}}, \tag{5}$$

in which $p_m - c_m$ is the price-cost margin in market m under DP and $p_D - \bar{c}$ is the (weighted) average price-cost margin under DP.

DP reallocates output from high-cost markets ($c_m > \bar{c}$) to low-cost markets ($c_m < \bar{c}$). The reallocation effect will be positive if the margins, $(p_m - c_m)$, which were higher in low-cost markets than in high-cost markets under UP, remain so under DP. Under classic third-degree price discrimination, i.e., markets face different prices but have the same cost of service, the reallocation effect is necessarily *negative*: output shifts to the market where price falls and,

[11] Under our quasi-linear utility assumption, the indirect utility as a function of the prices of the n goods and income I can be written as $v(p_1, ..., p_n) + I$, with the demand for good i being $\widetilde{D}(p_i, p_{-i}) = -\partial v(\cdot)/\partial p_i$. Hence, $S(p) = v(p, ..., p) = \int_\infty^p \frac{dv(x, ...x)}{dx} dx = \int_p^\infty nD(x,x)\, dx$.

hence, where the margin is lower. Thus, profitable price discrimination requires an increase in output or in the average price. In contrast, (cost-based) differential pricing can be profitable even when output and average price fall, because the reallocation effect often is positive. This distinction will prove useful in Subsection 3.6.

3. Welfare Analysis

If the firms supply a homogeneous product, then under uniform pricing (UP) each firm charges $p_U = \bar{c} = \sum_{m=1}^{M} \lambda_m c_m$, from (2); whereas under differential pricing (DP) each firm charges $p_m = c_m$, yielding the same average price $p_D = \sum_{m=1}^{M} \lambda_m p_m = p_U$. Thus, DP is beneficial for consumers (Remark 1) due to the price dispersion, while profit is zero under both regimes.

However, the results are no longer obvious when products are (symmetrically) differentiated. The remainder of this section addresses that case. Subsections 3.1-3.3 analyze the effects of DP compared to UP on consumer surplus, profits, and total welfare, respectively, using general properties of the demand system. Subsection 3.4 provides illustrative examples using specific demand functions. Subsection 3.5 studies how changes in competition may influence the welfare effects of DP. Subsection 3.6 compares the welfare effects of our cost-based differential pricing to classic third-degree price discrimination.

3.1 Equilibrium Prices and Consumer Surplus

Since market demands are proportional, markets essentially differ only in marginal cost c, assumed symmetric among firms. Therefore, we can analyze the properties of all relevant variables as functions of c. Furthermore, since we focus on symmetric equilibria in this section, firm i's decision problem can be formulated as choosing p to maximize

$$\pi(p, \hat{p}) = (p - c) D(p, \hat{p}),$$

given all competitors choosing price $\hat{p}$, with $c = \bar{c}$ under UP and $c = c_m$ in market m under DP.

Define

$$D_1(p, \hat{p}) \equiv \frac{\partial D(p, \hat{p})}{\partial p} < 0, \qquad D_2(p,\hat{p}) \equiv \frac{\partial D(p, \hat{p})}{\partial \hat{p}} > 0, \tag{6}$$

in which $\frac{\partial D(p,\hat{p})}{\partial \hat{p}} = (n-1)\frac{\partial \widetilde{D}(p_i,\cdot)}{\partial p_j}$ for $j \neq i$ and $p_j = \hat{p}$. $D_2(p,\hat{p})$ summarises the cross-price effect from a common price rise by all $n-1$ competitors.

At a symmetric equilibrium, $(p,\hat{p}) = (p^*, p^*)$ where $p^* \equiv p^*(c)$ satisfies the first-order condition

$$\frac{\partial \pi(p^*, p^*)}{\partial p} = D(p^*, p^*) + (p^* - c) D_1(p^*, p^*) = 0. \tag{7}$$

A sufficient condition for the existence of a unique equilibrium, which we shall maintain, is

$$\frac{\partial^2 \pi(p,\hat{p})}{\partial p^2} < 0 \quad \text{and} \quad -\frac{\partial^2 \pi(p,p)}{\partial p^2} > \frac{\partial^2 \pi(p,p)}{\partial p \partial \hat{p}} > 0. \tag{8}$$

The inequality $\frac{\partial^2 \pi(p,p)}{\partial p \partial \hat{p}} > 0$ implies that firms' prices are strategic complements.

Substituting the relevant value of c in (7) yields the equilibrium prices in the two regimes as

$$\text{UP:} \quad p_U = p^*(\bar{c}), \qquad\qquad \text{DP:} \quad p_m = p^*(c_m). \tag{9}$$

Moving from UP to DP therefore can be analyzed as if marginal cost fell in low-cost markets (those with $c_m < \bar{c}$) from the virtual level $\bar{c}$ to c_m and rose in high-cost markets ($c_m > \bar{c}$) from $\bar{c}$ to c_m. Profits per firm under UP and DP are

$$\pi_U \equiv \pi(p_U, p_U), \qquad\qquad \pi_D \equiv \sum_{m=1}^{M} \lambda_m \pi(p_m, p_m). \tag{10}$$

Define:

$$\text{own-price elasticity:} \quad \eta_{11}(p) \equiv -D_1(p,p)\frac{p}{D(p,p)} > 0. \tag{11}$$

$$\text{(aggregate) cross-price elasticity:} \quad \eta_{12}(p) \equiv D_2(p,p)\frac{p}{D(p,p)} > 0. \tag{12}$$

$$\text{(aggregate) elasticity ratio:} \quad R(p) \equiv \frac{\eta_{12}(p)}{\eta_{11}(p)} > 0. \tag{13}$$

$$\text{elasticity of industry demand:} \quad \eta(p) \equiv -Y'(p)\frac{p}{Y(p)} > 0. \tag{14}$$

A larger $R(p)$ reflects greater substitutability between the products: in symmetric equilibrium, $R(p)$ equals the aggregate *diversion ratio,* a measure of product substitutability commonly used in antitrust (e.g., Chen and Schwartz, 2016).[12] Since $Y'(p) < 0$ from (1), we have $-D_1(p,p) > D_2(p,p)$, and hence $R(p) \in (0,1)$. Furthermore, with $Y(p) = nD(p,p)$, we have $\eta(p) = -\frac{p}{D(p,p)}[D_1(p,p) + D_2(p,p)] = \eta_{11}(p) - \eta_{12}(p)$, the difference between firm i's own- and (aggregate) cross-price elasticity when all firms set equal price p.[13]

Define the (adjusted) curvature of any firm's demand at equal prices p as

$$\Phi(p) \equiv \frac{D(p,p)}{[D_1(p,p)]^2}\frac{d}{dp}D_1(p,p) = \underbrace{\left(-\frac{D(p,p)}{pD_1(p,p)}\right)}_{1/\eta_{11}(p)}\underbrace{\left[-\frac{p}{D_1(p,p)}\frac{d}{dp}D_1(p,p)\right]}_{\text{Elasticity of the slope of } i\text{'s demand}}. \tag{15}$$

The square-bracketed term above is the elasticity of the slope of firm i's demand with respect to an equal change in p for all firms, which can be considered as the curvature of the demand for one product. Thus, $\Phi(p) = 0$ if D is linear, and $\Phi(p) > (<)\ 0$ if D is strictly convex (concave) in symmetric price p.

Using (7), the pass-through rate from marginal cost to equilibrium price is

$$\begin{aligned}
p^{*\prime}(c) &= -\frac{-D_1(p^*,p^*)}{2D_1(p^*,p^*) + D_2(p^*,p^*) + (p^*-c)\frac{d}{dp^*}D_1(p^*,p^*)} \\
&= \frac{1}{2 + \frac{D_2(p^*,p^*)}{D_1(p^*,p^*)} + \frac{p^*-c}{p^*}\cdot\frac{p^*}{D_1(p^*,p^*)}\cdot\frac{d}{dp^*}D_1(p^*,p^*)} \\
&= \frac{1}{2 - [R(p^*) + \Phi(p^*)]} > 0,
\end{aligned} \tag{16}$$

where the inequality follows from assumption (8). The third equality above uses the familiar inverse elasticity rule $\frac{p^*-c}{p^*} = \frac{1}{\eta_{11}(p^*)}$ to obtain $\Phi(p^*)$. The term $R(p^*)$ summarizes the relative importance of cross-price to own-price effects when each firm has multiple competitors charging the same equilibrium price p^*.

It is useful to compare the (adjusted) curvature term $\Phi(p)$ in (15) and the pass-through rate $p^{*\prime}(c)$ in (16) for symmetric oligopoly with their counterparts for a single-product monopolist. For a monopolist with demand function $q = D(p)$, $\alpha \equiv -\frac{p}{D'(p)}D''(p)$ is the curvature of demand at price p, while $\sigma \equiv -\frac{q}{P'(q)}P''(q)$ is the curvature of the inverse demand $P(q) \equiv D^{-1}(q)$ at

[12] With $q_i = \widetilde{D}(p_i, p_{-i})$, the aggregate diversion ratio from product i to rival products at equal prices is $-\frac{(n-1)(\partial q_j/\partial p_i)}{(\partial q_i/\partial p_i)} = \frac{\eta_{12}q_j}{\eta_{11}q_i}$, and $q_i = q_j$ under symmetry.

[13] Notice that if $n = 2$ our η_{11}, η_{12}, and η correspond to ε_i^F, ε_i^C, and ε^I, respectively, in Holmes (1989), with all elasticities again defined as positive. In his notation, our elasticity ratio $R(p)$, instead, is $e_i^C\ /\ e_i^F$.

output q; and these two notions of demand curvatures are connected by the price elasticity of demand $\eta \equiv -\frac{pD'(p)}{D(p)}$ through the relation: $\sigma = \frac{\alpha}{\eta}$ (e.g., Chen and Schwartz, 2015). Thus, analogous to the monopoly case, $\Phi(p)$ in (15) can be considered as the demand curvature for a product adjusted by its own price elasticity, and corresponds to σ under monopoly. In expression (16) for the pass-through rate, the monopoly case would have $R(p^*) = 0$, $\Phi(p^*) = \sigma$, with p^* now being the monopoly price, and the pass-through rate would reduce to $p^{*'}(c) = \frac{1}{2-\sigma}$.

Equation (16) shows that a marginal increase in c affects equilibrium price in oligopoly through two channels. One is the curvature of a firm's demand: $p^{*'}(c)$ is larger when the firm's demand is convex ($\Phi(p^*) > 0$) rather than concave ($\Phi(p^*) < 0$), as under monopoly (Bulow and Pfleiderer, 1983). The second channel, specific to oligopoly, is the degree of product substitutability: *ceteris paribus,* a higher $R(p^*)$ raises $p^{*'}(c)$. Intuitively, a common increase in marginal cost c will raise also the rivals' price, which magnifies a firm's own profit-maximizing price increase when prices are strategic complements, and this cross-effect increases with $R(p^*)$.

We now can compare the (group-weighted) average price under differential pricing (p_D) to the uniform price (p_U). Using (2), (3) and (9): $p_D = \sum_{m=1}^{M} \lambda_m p^*(c_m)$ while $p_U = p^*(\bar{c}) = p^*\left(\sum_{m=1}^{M} \lambda_m c_m\right)$. Therefore, $p_D > p_U$ if $p^*(c)$ is convex and $p_D < p_U$ if $p^*(c)$ is concave. From (16),

$$p^{*''}(c) = \left[\Phi'(p^*) + R'(p^*)\right]\left[p^{*'}(c)\right]^3. \tag{17}$$

Thus, $p^{*''}(c)$ has the same sign as $[\Phi'(p^*) + R'(p^*)]$, implying the following conditions for the average price under differential pricing to be higher or lower than the uniform price:

Remark 2 *(i) If $\Phi'(p) + R'(p) > 0$ for all $p \in [p^*(c_1), \ p^*(c_M)]$, then $p_D > p_U$; and (ii) if $\Phi'(p) + R'(p) \leq 0$ for all $p \in [p^*(c_1), \ p^*(c_M)]$, then $p_D \leq p_U$.*

Note that the conditions in (i) and (ii) are sufficient, but not necessary; for example, we can have $p_D > p_U$ if $\Phi'(p) + R'(p) > 0$ for 'most' but not all of prices in the relevant range. However, if $[\Phi'(p) + R'(p)]$ has a consistent sign over $p \in [p^*(c_1), \ p^*(c_M)]$, then the condition $\Phi'(p) + R'(p) > 0$ is sufficient and *also necessary* for DP to raise average price: $p_D > p_U$, and similarly the condition in (ii) is necessary for $p_D \leq p_U$. This observation also applies to the subsequent Propositions 1 through 3.

To obtain some intuition for Remark 2, consider case (*ii*). If $\Phi'(p) + R'(p) \leq 0$, the pass-through rate weakly decreases with price for $c \in [c_1, c_M]$, because one or both of the following hold: at higher prices the demand curvature term is lower ($\Phi'(p) \leq 0$) and/or product substitutability is weaker ($R'(p) \leq 0$). Pass-through then will be smaller in high-cost markets, where a move to DP raises marginal cost (from the virtual level $\bar{c}$) and, hence, price, than where marginal cost and price fall, explaining why the average price (weakly) falls. Both $\Phi(p)$ and $R(p)$ are constant for some familiar classes of demand functions—including linear and CES demands (see Subsection 3.4 below). For these classes of demands, the average price under cost-based differential pricing is equal to the uniform price.

The condition $\Phi'(p) + R'(p) \leq 0$, implying $p_D \leq p_U$, is sufficient for DP to raise consumer surplus since consumers gain from price dispersion, but consumer surplus can increase even if average price rises somewhat. Thus, we can derive a tighter sufficient condition for consumers to benefit from DP. Rewrite consumer surplus (4) as a function of c

$$s(c) \equiv S(p^*(c)) = \int_{p^*(c)}^{\infty} Y(x)dx. \tag{18}$$

Since $S_D = \sum_{m=1}^{M} \lambda_m s(c_m)$ and $S_U = s(\bar{c}) = s(\sum_{m=1}^{M} \lambda_m c_m)$, DP raises or lowers consumer surplus when $s(c)$ is convex or concave. Analyzing the sign of $s''(c)$ yields a tighter condition than $\Phi'(p) + R'(p) \leq 0$ for $S_D > S_U$:

Proposition 1 *Consumer surplus is higher under differential pricing than under uniform pricing if inequality (19) holds for all* $p \in [p^*(c_1), \ p^*(c_M)]$, *and is lower if the inequality in (19) is reversed for all* $p \in [p^*(c_1), \ p^*(c_M)]$.

$$\frac{-[\Phi'(p) + R'(p)]}{2 - [\Phi(p) + R(p)]} + \frac{\eta(p)}{p} > 0. \tag{19}$$

The first term in (19) is the average price effect when moving from UP to DP. Since $\frac{1}{2-[\Phi(p)+R(p)]} = p^{*'}(c) > 0$ from (16), the first term takes the sign of $-[\Phi'(p) + R'(p)]$. When $\Phi'(p) + R'(p) \leq 0$, DP weakly lowers the average price, which benefits consumers.[14] The second

[14] The term $-[\Phi'(p) + R'(p)]$ will also appear in condition (23) for total welfare. It will appear with the opposite sign in condition (21) for profit, since a fall in the average price benefits consumers and also total welfare (via the output expansion), but harms profit.

term, $\frac{\eta(p)}{p} > 0$, reflects the price-dispersion effect: when the price elasticity of market demand $\eta(p)$ is higher, consumers are more capable of making quantity adjustments and thus benefit more from the price dispersion. On balance, DP raises consumer surplus if it does not raise average price too much, i.e., if $\Phi(p) + R(p)$ does not increase too fast with the common price p.

Under monopoly, the corresponding condition for differential pricing to raise consumer surplus given in Chen and Schwartz (2015) is

$$\frac{\sigma'(q)}{2-\sigma(q)} + \frac{1}{q} > 0 \Longleftrightarrow \frac{-\sigma'(q)\, q'(p)}{2-\sigma(q)} + \frac{\eta(p)}{p} > 0.$$

As explained, $\sigma \equiv -\frac{q}{P'(q)} P''(q)$, the curvature of inverse demand, corresponds to $\Phi(p)$, hence $-\sigma'(q(p))\, q'(p)$ corresponds to $-\Phi'(p)$. Thus, condition (19) in oligopoly reduces to its monopoly counterpart for $R'(p) = R(p) = 0$ and $\Phi(p) = \sigma(q(p))$. Condition (19) holds for a broad class of demand functions under oligopoly or monopoly (see examples in Subsection 3.4 and in Chen and Schwartz, 2015) and, hence, differential pricing tends to benefit consumers.

3.2 Profit

Equilibrium profit for firm i under marginal cost c is

$$\pi(c) \equiv \pi(p^*(c), p^*(c)) = [p^*(c) - c]\, D(p^*(c), p^*(c)).$$

Thus, using the envelope theorem:

$$\pi'(c) = -\underbrace{D(p^*(c), p^*(c))}_{\text{direct effect}} + \underbrace{[p^*(c) - c]\, D_2(p^*(c), p^*(c)) p^{*\prime}(c)}_{\text{rivals' effect}},$$

where the second term, the rivals' effect, can be rewritten as

$$D(p^*, p^*) \left[D_2(p^*, p^*) \frac{p^*}{D(p^*, p^*)} \right] \left[\frac{p^* - c}{p^*} \right] p^{*\prime}(c) = D(p^*, p^*) \frac{\eta_{12}(p^*)}{\eta_{11}(p^*)} p^{*\prime}(c),$$

using the inverse-elasticity rule $(p^* - c)/p^* = 1/\eta_{11}(p^*)$. Since $\eta_{12}(p)/\eta_{11}(p) = R(p)$, we can express $\pi'(c)$ as

$$\pi'(c) = -D(p^*(c), p^*(c))\left[1 - p^{*'}(c)R(p^*(c))\right]. \tag{20}$$

For a monopolist, an increase in c lowers profit because the profit margin is reduced. Under competition, this margin reduction is alleviated by the rise in the rivals' price due to the common increase in c, as reflected in the additional term $-p^{*'}(c)R(p^*)$ in (20). Thus, $\pi'(c) < 0$ if and only if $p^{*'}(c)R(p^*) < 1$. Condition (8) for a unique equilibrium does not ensure $p^{*'}(c)R(p^*) < 1$, and we will consider also $p^{*'}(c)R(p^*) \geq 1$.

The result below is derived by analyzing when $\pi(c)$ is convex or concave.[15]

Proposition 2 *Profit is higher under differential pricing than under uniform pricing if inequality (21) holds for all $p \in [p^*(c_1), \ p^*(c_M)]$, and is lower if the inequality in (21) is reversed for all $p \in [p^*(c_1), \ p^*(c_M)]$.*

$$\frac{R(p)\left[\Phi'(p) + R'(p)\right]}{2 - \Phi(p) - R(p)} + R'(p) + \frac{\eta(p)}{p}\left[2 - \Phi(p) - 2R(p)\right] > 0. \tag{21}$$

For a monopolist, $R(p) = R'(p) = 0$ and $\Phi(p) = \sigma(q(p))$, so (21) reduces to $\frac{\eta(p)}{p}\left[2 - \sigma(q(p))\right] > 0$. It represents a monopolist's gain from adjusting outputs across markets in response to mean-preserving cost dispersion[16] and is proportional to demand elasticity, akin to the flexibility gain for consumer surplus. As with consumer surplus, the condition for cost-based differential pricing to raise profit in oligopoly embeds and generalizes the condition under monopoly. Next, consider the three terms in (21) for our oligopoly case.

The first term in (21), $\frac{R(p)[\Phi'(p)+R'(p)]}{2-\Phi(p)-R(p)}$, takes the sign of $[\Phi'(p) + R'(p)]$ which determines the direction of change in average price when moving to DP (Remark 2). It affects the firm's profit via the *rivals'* price response to the common cost shocks, $1/\left[2 - \Phi(p) - R(p)\right] = p^{*'}(c)$, in proportion to the substitutability term, $R(p)$. An increase in average price boosts industry profit because competition under uniform pricing forces price too low from the industry's standpoint.

[15] We are greatly indebted to an anonymous referee and the editor, David Myatt, for pointing out an error in the ensuing condition (21) in an earlier draft of this paper.

[16] Recall that moving from UP to DP can be analyzed as a virtual decrease in marginal cost from $\bar{c} = \sum_{m=1}^{M} \lambda_m c_m$ to c_m in low-cost markets ($c_m < \bar{c}$) and an increase from $\bar{c}$ to c_m in high-cost markets ($c_m > \bar{c}$), with weight λ_m in each market m.

The middle term, $R'(p)$, reflects an output externality. When moving from UP to DP, each firm sets its (market-specific) price based on the firm elasticity of demand, $\eta_{11}(p)$, but total output is determined by the (lower) market elasticity, $\eta(p) = \eta_{11}(p) - \eta_{12}(p)$. Each firm ignores that (a) its price increase in high-cost markets expands the competitors' output and (b) its price decrease in low-cost markets reduces their output. The size of each externality is proportional to the aggregate diversion ratio $R(p) \equiv \eta_{12}(p)/\eta_{11}(p)$. If $R'(p) > 0$, the diversion ratio is larger at higher prices than at lower prices, hence the positive externality (a) exceeds the negative externality (b). In the move from uniform pricing to cost-based differential pricing, each firm therefore induces a positive net externality on competitors' output, boosting their profit.

The last term, $\frac{\eta(p)}{p}\left[2 - \Phi(p) - 2R(p)\right]$, can be written (using (16) that defines $p^{*'}(c)$) as $\frac{\eta(p)}{p}\frac{1}{p^{*'}(c)}\left[1 - p^{*'}(c)R(p)\right]$: a monopolist's gain from adjusting prices and outputs across markets in response to cost dispersion, modified in oligopoly by the impact of the rivals' symmetric price responses ($p^{*'}(c)R(p) > 0$). Suppose $p^{*'}(c)R(p) \in (0, 1)$. Since the rivals' response in each market does not outweigh the own-cost effect, cost dispersion still benefits a firm (though less than if it were a monopolist), in proportion to the elasticity of market demand.

However, it is possible to have $p^{*'}(c)R(p) \geq 1$ and for DP to (weakly) decrease profit, unlike for monopoly. The condition $p^{*'}(c)R(p) \geq 1$ requires $p^{*'}(c) > 1$ (since $R(p) < 1$), hence the virtual cost decrease in low-cost markets reduces profit margins; while the reverse pattern occurs in high-cost markets. Consequently, the profit margins under DP are *lower* in low-cost markets than in high-cost markets,[17] so the output reallocation to low-cost markets harms profit, from (5). For a monopolist, DP nevertheless raises profit because $p^{*'}(c) > 1$ requires demand to be sufficiently convex that the monopolist's optimal pricing expands output enough to outweigh the harmful misallocation (Chen and Schwartz, 2015).[18] In oligopoly, however, $p^{*'}(c) > 1$ can arise from a high cross-elasticity term, $R(p)$, so the price reductions in low-cost markets are driven sufficiently by product substitutability rather than demand curvature that output may not increase sufficiently. Example 2 in Subsection 3.4 illustrates a case with $p^{*'}(c)R(p^*) \to 1$ in the limit, hence DP fails to raise profits.

[17] For any two markets with $c_L < \bar{c} < c_H$, the difference in margins is $p_L - c_L - (p_H - c_H) = (c_H - c_L) - \int_{c_L}^{c_H} p^{*'}(c)\, dc < 0$ if $p^{*'}(c) > 1$.

[18] By revealed preference, monopoly profits must be no lower under DP than under the constraint of UP. The text explained why this holds even when $p^{*'}(c) > 1$.

A second way differential pricing can reduce profit in oligopoly arises when $R'(p) < 0$. DP then can lower both average price *and total output*, and reduce profit via the first two terms in (21) (tracking the first and third terms in decomposition (5)). See Example 3 in Subsection 3.4, and the further discussion in Subsection 3.6.

Summarizing, cost-based differential pricing by symmetric oligopolists may reduce profit, but the required demand conditions seem rather special. In the 'normal' case $(p^{*'}(c)R(p) < 1)$, there is a systematic force pushing towards greater profit: the beneficial output reallocation effect captured by the last term in (21). Profit and consumer surplus both benefit from greater scope for output reallocation, reflected in a larger elasticity of market demand $\eta(p)$.

3.3 Total Welfare

Given marginal cost c and the associated equilibrium price $p^*(c)$, the equilibrium total welfare in a market with marginal cost c can be written as

$$\begin{aligned} w(c) &\equiv W\left(p^*\left(c\right)\right) = s(c) + n\left[p^*(c) - c\right] D(p^*(c), p^*(c)) \\ &= \int_{p^*(c)}^{\infty} Y(x)dx + \left[p^*(c) - c\right] Y(p^*(c)). \end{aligned} \tag{22}$$

Analyzing when $w\left(c\right)$ is convex, we obtain the following condition for DP to raise or lower total welfare.

Proposition 3 *Total welfare is higher under differential pricing than under uniform pricing if inequality (23) holds for all $p \in \left[p^*\left(c_1\right),\ p^*\left(c_M\right)\right]$, and is lower if the inequality in (23) is reversed for all $p \in \left[p^*\left(c_1\right),\ p^*\left(c_M\right)\right]$.*

$$\frac{-\left[\Phi'(p) + R'(p)\right]\left[1 - R(p)\right]}{2 - \Phi(p) - R(p))} + R'(p) + \frac{\eta(p)}{p}\left[3 - \Phi(p) - 2R(p)\right] > 0. \tag{23}$$

The first term corresponds to the first terms in (19) and (21), reflecting the net effect of change in average price on consumer surplus plus profit: when $\left[\Phi'(p) + R'(p)\right] \leq 0$, average price weakly falls, hence the net effect is weakly positive due to industry output expansion given $1 - R(p) > 0$. The second term, $R'(p)$, is the same as the middle term in (21) and captures the net output externality under competition across all market segments. Together, the first and second terms in (23) reflect how DP affects total welfare through the change in total output. The last term, $\frac{\eta(p)}{p}\left[3-\Phi(p)-2R(p)\right]$, is positive (since $2-\Phi(p)-R(p)>0$ from (16) and $R(p)<1$). It reflects the output reallocation: the price variation caused by moving from UP to DP induces an output reallocation across markets that is beneficial for total welfare, and the size of the output reallocation increases with $\eta(p)$. Differential pricing can raise total welfare by improving the output allocation and/or expanding output, but neither of them alone is necessary for welfare to rise.[19] And like its counterparts for consumer surplus and profit, (19) and (21), condition (23) is met for a broad class of demands, such as those with constant $\Phi(p)$ and $R(p)$, but not always; see Example 3 below.

The counterpart of condition (23) under monopoly with demand $D(p)$ (Chen and Schwartz, 2015, condition (A1B)) can be written as

$$\frac{-\sigma' D'(p)}{(2-\sigma)}+\frac{\eta}{p}\left[(2-\sigma)+1\right]>0.$$

As with consumer surplus and profit, the condition for DP to raise total welfare in oligopoly, (23), embeds and generalizes its monopoly counterpart: $R(p)=R'(p)=0$ for the single-product monopolist, while $\Phi(p)$ in oligopoly corresponds to σ under monopoly and $\Phi'(p)$ corresponds to $\sigma' D'(p)$.

Finally, observe that under monopoly, DP always increases profit, hence total welfare rises under broader conditions than does consumer surplus (compare Propositions 1 and 2 of Chen and Schwartz, 2015). In oligopoly, DP can reduce profit, hence consumer surplus may rise yet total welfare fall, as in Example 3. Thus, the conditions for differential pricing to raise consumer surplus or total welfare, (19) and (23), are no longer nested.

3.4 Examples

The ensuing examples show that the conditions in Propositions 1-3 for differential pricing (DP) to benefit consumers, profits, and overall welfare relative to uniform pricing (UP) are met by

[19] This is a key difference between cost-based DP and price discrimination, since the latter can increase welfare only if output expands.

familiar demand functions—though not always.[20] The examples also illustrate the underlying economic forces. For this subsection, we focus on the case of two market segments: $m \in \{L, H\}$ with $c_L < c_H$ and the sizes of consumer groups denoted $\lambda_L = \lambda$, $\lambda_H = 1 - \lambda$.

Example 1 *Linear demand (DP increases consumer surplus and profit)*[21]*:*

$$\widetilde{D}(p_i, p_{-i}) = \frac{1}{n}\left(a - p_i - \gamma\left(p_i - \frac{\sum_{j=1}^{n} p_j}{n}\right)\right), \quad a > c_H,\ \gamma > 0. \tag{24}$$

Then, $R(p) = \frac{(n-1)\gamma}{n(1+\gamma)-\gamma}$, $Y(p) = a - p$, $\eta(p) = \frac{p}{a-p}$, *and* $\Phi(p) = 0$.

It follows that both (19) and (21) hold, hence DP increases consumer surplus and profit. Average price and total output are the same under UP and DP, so the gains come solely from reallocating output between markets. The gains can be significant. For instance, let $n = 2$ and $\{a, \gamma, c_L, c_H, \lambda\} = \{8, 1, 2, 2 + t, 0.5\}$. Then, if $t = c_H - c_L = 0$, DP and UP yield the same level of welfare. As the cost difference t increases for $t \in (0, 3.429)$, the incremental benefits from DP over UP increase. Profit and consumer surplus both rise by 4% at $t = 2$, by 11.11% at $t = 3$, and by 16% at $t = 3.429$. The gain from DP exhibits an inverted U-shape in the relative size of the two markets, $\lambda :$ the gain is zero for $\lambda = 0$ or 1 (since in both cases there is only a single market), and is maximized at an intermediate λ.

Example 2 *CES demand (DP increases consumer surplus and profit)*[22]*:*

$$\widetilde{D}(p_i, p_{-i}) = \theta^{\frac{1}{1-\theta}} p_i^{-\rho}\left(p_1^{1-\rho} + \cdots + p_n^{1-\rho}\right)^{\frac{\frac{1}{1-\theta}-\rho}{\rho-1}}, \quad \rho > 1, \quad 0 < \theta < \frac{\rho - 1}{\rho}. \tag{25}$$

[20] Armstrong and Vickers (2018) characterize an important class of demand systems in which consumer surplus is a homothetic function of quantities. The demand functions in our examples below are all part of this class.

[21] This demand function is adopted from Shubik and Levitan (1971) and can be derived by utility maximization of a representative consumer with utility function:

$$u(q_0, q_1, \cdots, q_n) = q_0 + a\sum_{i=1}^{n} q_i - \frac{1}{2}(\sum_{i=1}^{n} q_i)^2 - \frac{n}{2(1+\gamma)}\left[\sum_{i=1}^{n} q_i^2 - \frac{\left(\sum_{i=1}^{n} q_i\right)^2}{n}\right].$$

A slightly different form of this demand system is used by Myatt and Wallace (2018) in a different context.

[22] This demand can be derived from the utility maximization of a representative consumer with the following utility

$$u(q_0, q_1, \cdots, q_n) = q_0 + \left(q_1^{\frac{\rho-1}{\rho}} + \cdots + q_n^{\frac{\rho-1}{\rho}}\right)^{\frac{\theta\rho}{\rho-1}}.$$

Then, $p^*(c) = \frac{(1+\rho(1-\theta)(n-1))c}{\theta\rho+(\rho-1)(n-\theta n-1)}$, $p^{*\prime}(c) = \frac{1+\rho(1-\theta)(n-1)}{\theta\rho+(\rho-1)(n-\theta n-1)} > 1$, $R(p) = \frac{(n-1)((1-\theta)\rho-1)}{1+(n-1)(1-\theta)\rho}$, $Y(p) = \frac{\theta^{\frac{1}{1-\theta}} n^{\frac{\theta}{(1-\theta)(\rho-1)}}}{p^{\frac{1}{1-\theta}}}$, $\eta(p) = \frac{1}{1-\theta} > 0$, $\Phi(p) = \frac{(2-\theta)n}{1+(1-\theta)(n-1)\rho} > 0$.

Once again, both (19) and (21) hold, so that differential pricing increases consumer surplus and profits. Since $\Phi(p)$ and $R(p)$ are independent of price p, DP does not change the average price (by Remark 2). Consumer surplus increases due to the output reallocation, as well as output expansion since demand is convex ($Y''(p) > 0$). Although $p^{*\prime}(c) > 1$, hence the output reallocation is excessive for profit, the condition $p^{*\prime}(c)R(p) < 1$ holds for all feasible parameters so profit still increases due to the output expansion. However,

$$p^{*\prime}(c)R(p) = \frac{(n-1)\left((1-\theta)\rho-1\right)}{\theta\rho+(\rho-1)(n-\theta n-1)} \to 1 \text{ as } \theta \to 0 \text{ or as } \rho \to \infty.$$

Therefore, in this example DP can fail to raise profit in the limit as $\theta \to 0$ or as $\rho \to \infty$.

Example 3 *Multinomial Logit demand with outside option (DP increases consumer surplus but can reduce total output, profit, and total welfare):*

$$\widetilde{D}(p_i, p_{-i}) = \frac{e^{\frac{a-p_i}{\mu}}}{\sum_{j=1}^n e^{\frac{a-p_j}{\mu}} + A}, \qquad A > 0, \quad \mu > 0. \tag{26}$$

Then, $p^* = c+\mu+\frac{\mu}{Ae^{\frac{-a+p^*}{\mu}}+n-1}$, $p^{*\prime}(c) = \frac{\left((n-1)e^{\frac{a}{\mu}}+Ae^{\frac{p^*}{\mu}}\right)^2}{(n-1)^2e^{\frac{2a}{\mu}}+A^2e^{\frac{2p^*}{\mu}}+(2n-1)Ae^{\frac{a+p^*}{\mu}}}$; $R(p) = \frac{n-1}{n-1+Ae^{\frac{-a+p}{\mu}}}$, $R'(p) < 0$; $\Phi(p) = \frac{Ae^{\frac{p}{\mu}}\left(Ae^{\frac{p}{\mu}}+(n-2)e^{\frac{a}{\mu}}\right)}{\left((n-1)e^{\frac{a}{\mu}}+Ae^{\frac{p}{\mu}}\right)^2} > 0$, $\Phi'(p) > 0$; $\eta(p) = \frac{Ae^{\frac{p}{\mu}}p}{ne^{\frac{a}{\mu}}\mu+Ae^{\frac{p}{\mu}}\mu}$.

In Example 3, condition (19) always holds, hence DP raises consumer surplus. However, conditions (21) and (23) can be reversed, so profit and total welfare can fall. For instance, let $n = 2$, $A = 0.01$, $c_L = 0$, $c_H = 2$, $\mu = 1$, $a = 0$, and $\lambda = 0.5$. Then: $p_L - c_L = 1.94 > p_H - c_H = 1.71$, hence the output reallocation benefits profit and total welfare. But DP lowers the average price, from $p_U = 2.85$ to $p_D = 2.82$, *and* lowers total output, from $q_U = 0.92$ to $q_D = 0.90$, causing profit to fall: $\pi_D = 1.65 < \pi_U = 1.71$. The output reduction also reduces welfare, albeit slightly (by 0.067%). Instead, if $\mu = 0.3$, $a = 0.5$, then DP lowers profit but raises consumer surplus by more, hence total welfare increases.

The price and output changes can be understood as follows. Demand is more convex at higher prices ($\Phi'(p) > 0$), which pushes the pass-through $p^{*'}(c)$ to be increasing; however, the products' substitutability is smaller at higher prices ($R'(p) < 0$), which pushes $p^{*'}(c)$ to be decreasing. On balance, $\Phi'(p)+ R'(p)$ can be positive or negative, depending on parameter values, hence the average price may rise or fall with DP. When $R'(p) < 0$ it is possible not only for average price to fall but for total output to fall as well. In some such cases, profit decreases. But consumer surplus always increases under DP, primarily due to the positive output reallocation effect.

Summarizing our findings, cost-based differential pricing raises both consumer surplus and profit when the demand curvature ($\Phi(p)$) and the elasticity ratio ($R(p)$) do not change too fast with price relative to the size of market-demand elasticity ($\eta(p)$). Under those conditions, DP will not change average price by 'much' but will yield sufficient output adjustments by consumers and firms for both groups to benefit. In particular, DP raises both consumer surplus and profit when $\Phi(p)$ and $R(p)$ are constant, such as for linear and CES demand. There are demands for which, under some parameter values, profit or total welfare can be lower under differential pricing than under uniform pricing, but such cases seem unusual.[23]

3.5 Effects of Competition Intensity

In this subsection, we analyze how the distribution of gains from differential pricing (DP) relative to uniform pricing (UP) is affected by the number of competitors. Consider first the class of linear demand functions given by (24). There, a larger number of firms (n) drives price closer to marginal cost and thereby represents more intense competition.[24]

Recall from Example 1 that for the demand system (24), DP always raises both consumer surplus and profit. Solving explicitly for the symmetric equilibrium allows us to obtain comparative statics with respect to n. Let $\Delta S \equiv S_D - S_U$, $\Delta\Pi \equiv \Pi_D - \Pi_U$, and $\Delta W \equiv W_D - W_U$ denote the gains from DP to consumers, firms, and total welfare, respectively.

[23] Under our symmetric setting, we have not found an example in which DP lowers consumer surplus, even though it is conceivably possible. However, as we show in Section 4, DP can lower consumer surplus if costs differ between firms. Thus, our broad message is that DP is often, but not always, beneficial to consumers.

[24] A second parameter in this demand system is the degree of product substitutability (γ). The products are highly differentiated as $\gamma \to 0$ and highly substitutable as $\gamma \to \infty$. Increasing γ has the same qualitative effects as increasing n, and Proposition 4 below applies also to varying γ.

Proposition 4 *Under the linear demand (24), stronger competition magnifies the gains from differential pricing to consumers and total welfare, but reduces the gain in industry profit: (i)* $\Delta S > 0$ *and* $\Delta W > 0$ *both increase with* n*; (ii)* $\Delta\Pi > 0$ *but decreases with* n*. Thus, consumers' relative gain from DP, the ratio* $\Delta S/\Delta\Pi$*, increases in* n*.*

To understand result (i), recall that moving to DP leaves average price and total quantity unchanged under linear demand, so the gains to consumers and total welfare come solely from the output reallocation induced by the price dispersion. The pass-through rate is

$$p^{*'}(c) = \frac{n + \gamma(n-1)}{2n + \gamma(n-1)},$$

which increases in n. Thus, an increase in n magnifies the price dispersion under DP. Since market demand has constant slope (invariant to n), the beneficial output reallocation increases with the price dispersion.

To understand result (ii), why $\Delta\Pi$ decreases in n, suppose there are two market segments: $M = 2, m \in \{L, H\}$, and $\lambda_L = \lambda$, $\lambda_H = 1 - \lambda$. With linear demand, DP leaves average price and total output unchanged, hence industry profit changes solely due to the reallocation effect in (5). Since $q_D = \lambda q_L + (1-\lambda)q_H$, we have:

$$\Delta\Pi = \underbrace{\lambda(1-\lambda)(q_L - q_H)\left[(p_L - c_L) - (p_H - c_H)\right]}_{\text{Reallocation Effect, two markets}}. \tag{27}$$

As n rises, $p^{*'}(c)$ increases, which lowers the margin difference between the two markets, $[(p_L - c_L) - (p_H - c_H)]$. The output reallocation $(q_L - q_H)$ rises with n, due to the greater price dispersion, but at a slower rate than the margin difference falls, explaining why $\Delta\Pi$ decreases.[25] Since ΔS increases in n while $\Delta\Pi$ decreases in n, consumers' share of the gain from DP obviously rises as competition intensifies.

Consider next the CES demand in (25) and the multinomial logit demand in (26), where larger n again represents stronger competition, insofar as it drives prices closer to marginal costs. The welfare effects are more complex, however, since the slope of market demand varies with

[25] Specifically, $[(p_L - c_L) - (p_H - c_H)] = \frac{(c_H - c_L)n}{2n+\gamma(n-1)}$ and $q_L - q_H = \frac{(c_H - c_L)[n+\gamma(n-1)]}{2n+\gamma(n-1)}$. Thus, $\partial(q_L - q_H)/\partial n = -\partial\left[(p_L - c_L) - (p_H - c_H)\right]/\partial n$.

the price level and other parameters. We have found numerically, in the case of two markets ($M = 2$), that ΔS increases in n for a broad set of parameter values, but $\Delta\Pi$, while still positive, can decrease or increase in n. For both demand systems, however, the ratio $\Delta S/\Delta\Pi$ still increases in n for the various sets of parameter values that we checked, so that stronger competition again delivers to consumers a larger share of the gains from DP.

With CES demand, the underlying forces are as follows. Example 2 showed that average price does not change with DP ($p_D = p_U$), and that $\Delta S > 0$ due to the output reallocation and—since demand is convex—also the output expansion ($q_D - q_U > 0$); whereas the profit gain ($\Delta\Pi > 0$) comes solely from the output effect in (5), since the reallocation effect is negative because $p^{*'}(c) > 1$. The output expansion rises with n. The reason is subtle: increasing n lowers the uniform price p_U, hence the slope of demand flattens due to convexity, which magnifies the output expansion from price dispersion under DP.[26] The flatter slope of demand magnifies also the output reallocation.[27] Both forces, larger output expansion and larger output reallocation, cause ΔS to increase with n. For profit, the driving force causing $\Delta\Pi > 0$ is the output effect in (5), $(q_D - q_U)(p_D - \bar{c}) > 0$. Its magnitude is subject to opposing effects as n increases: $(q_D - q_U)$ rises, but the margin $(p_D - \bar{c})$ falls.[28] Depending on the parameters, either effect can dominate, and $\Delta\Pi$ may decrease or increase with n.

The specific reasons why stronger competition shifts a larger share of the gains from DP to consumers are therefore subtle. However, a common force is that stronger competition drives prices closer to marginal cost. With linear demand, the price dispersion and output reallocation from DP increase with n, which benefits consumers. The gain in profits, however, decreases with n despite the greater output reallocation, since the difference in margins between markets decreases as prices fall towards marginal cost. With CES demand, the output reallocation again increases with n (though for a different reason—convex demand), and this time also the output expansion, benefitting consumers. For profits, however, the potential gains from larger

[26] Although the price dispersion $(p_H - p_L)$ decreases in n (because, unlike with linear demand, the pass-through rate $p^{*'}(c)$ now decreases in n), this effect is outweighed by the flatter slope of demand.

[27] Moreover, as n increases the slope of demand flattens even at the initial p_U, since market output $Y(p)$ increases in n while market elasticity $\eta(p)$ is invariant to n. This further magnifies the output reallocation.

[28] The (negative) reallocation effect on $\Delta\Pi$, in (27), also is subject to opposing effects as n increases: $q_L - q_H$ rises but the margin difference $(p_L - c_L) - (p_H - c_H)$ falls in absolute value (since $p^{*'}(c)$ decreases in n). In the examples we considered, the behavior of $\Delta\Pi$ tracks the output effect, which dominates the reallocation effect.

output expansion are dampened by the reduced margin. Stepping beyond these examples, to the extent that stronger competition drives prices towards marginal costs, it raises welfare by more under DP than under UP while constraining profits, hence the efficiency gains from DP accrue increasingly to consumers.

3.6 Comparison to Oligopoly Price Discrimination

Holmes (1989) analyzed symmetric duopolists competing in two markets that differ only in demand elasticities, and compared uniform pricing to classic third-degree price discrimination instead of cost-based differential pricing. Our results exhibit similarities to his findings, but also differences.

In both settings, differential pricing—whether cost-based or demand-based—may reduce profit relative to uniform pricing, unlike for monopoly. Holmes shows this can occur if the market with the smaller elasticity of market demand has the larger cross-price elasticity between firms. Price discrimination then lowers price in the 'wrong market' and can reduce total output. In our setting, markets differ only in costs, but DP still can reduce output and profit if cross-price elasticity relative to a firm's own-price elasticity, for the common demand function across markets, is greater at lower prices than at higher prices (Example 3). Then price can fall by more in the low-cost markets than it rises in the other markets, while still reducing total output.[29]

For consumers, cost-based DP is more likely to be favorable than price discrimination. From decomposition (5), profitable price discrimination requires an increase in total output or in average price; indeed, price discrimination tends to raise average price, which of itself harms consumers.[30] By contrast, DP can raise profit even if average price does not rise (Examples 1-2), which ensures that consumers also benefit. The cost savings achieved by reallocating output to the lower-cost market provide firms an incentive to adopt DP also under demand conditions where average price does not rise, and consumers benefit from the price dispersion.

Total welfare also is more likely to rise with cost-based DP than with price discrimination.

[29] Additionally, cost-based DP can potentially fail to increase profit due to a second force: excessive output reallocation between markets when $p^{*\prime}(c) > 1$, as in Example 2 when $\theta \to 0$ or when $\rho \to \infty$.

[30] Holmes (1989, p. 248) notes: "There is a sense in which discrimination increases 'average' price; the increase in price in the strong market above the uniform price is 'large' relative to the decrease in the weak-market price." Chen and Schwartz (2015, pp. 449-451) discuss this issue further in the case of monopoly.

Discrimination misallocates output between markets, hence an increase in total output is necessary for total welfare to rise. In contrast, cost-based DP can increase welfare when output remains constant, as with linear demand (Example 1),[31] or even when output falls (as in Example 3 for some parameter values), by improving the output allocation.

Below we provide an example that nests the polar cases: markets may differ both in their costs of service and in their demand elasticities.

Example 4 *Markets differ in costs and demands (differential pricing is beneficial if the difference in costs is large relative to that in demands):*
Firm i faces the following demand system (equation (24) from Example 1 with $n = 2$) in market $m \in \{L, H\}$,

$$\widetilde{D}(p_{im}, p_{jm}) = \frac{1}{2}\left(a + b_m - p_{im} - \frac{\gamma}{2}(p_{im} - p_{jm})\right). \tag{28}$$

Let $b_L = 0$ and $b_H = b > 0$ so that H is the 'strong' market.[32] *Consumers are distributed between markets L and H in proportions λ and $1 - \lambda$, and the firms are symmetric in each market with marginal costs c_L and c_H. In the Appendix we show the following when moving from UP to DP: average price weighted by the consumption quantities under UP increases; total output does not change; and profit increases. Furthermore, there exist critical values b_1 and b_2 such that: i) consumer surplus increases if $b \leq b_1$; ii) total welfare increases if $b \leq b_2$.*

The weighted average price is analogous to p_D defined in (3), but the weights now reflect both the relative sizes of markets L and H (respectively λ and $1 - \lambda$) *and* that per capita quantity under UP is higher in the 'strong' market H. This weighted average price exceeds p_U, unlike in the base model where only costs differ, because now price discrimination is present. Profit rises for two reasons: the rise in average price, and cost savings from reallocating output. Consumer surplus rises (due to price dispersion) as does total welfare (due to cost savings) if the demand difference between markets is small relative to the cost difference, so that differential pricing is driven predominantly by cost differences. This generalizes the results from our main model where DP is driven solely by cost differences: there, DP always raises total welfare and

[31] In Holmes' setting, price discrimination can increase or decrease output even with linear demand, depending on his elasticity-ratio condition, which compares relative market-demand elasticities to relative cross-price elasticities. When output falls, total welfare also must fall (Holmes, 1989, fn.2).

[32] Cowan (2007) also uses a demand-shifting parameter to yield different price elasticities across markets.

its components under linear demand. Notice that a similar result also holds under monopoly in Chen and Schwartz (2015).

4. Firms With Asymmetric Costs

Do asymmetries between firms introduce new forces, beyond demand-side factors such as pass-through, that alter the welfare properties of differential pricing (DP) relative to uniform pricing (UP)? We address this issue in this section by assuming two firms and two markets, $n = 2$ and $m \in \{L, H\}$, but extending our model to allow cost asymmetries between firms for a given market, in addition to cost differences across markets as assumed until now. Thus, firm i has costs (c_{iL}, c_{iH}), where $c_{iH} > c_{iL}$, for $i = 1, 2$. The firms are still assumed symmetric in demand: they produce either a homogeneous product with industry demand $Y(p)$ (Subsection 4.1 below) or symmetrically differentiated products (Subsection 4.2 below).

4.1 Homogeneous Products

Consider two scenarios of cost asymmetries:

(1) *Global Cost Advantage*: the same firm, say firm 1, has a cost advantage in serving both markets: $c_{1L} < c_{2L}$ and $c_{1H} < c_{2H}$;

(2) *Local Cost Advantage*: each firm has a cost advantage in serving a different market. Without loss of generality, let $c_{1L} < c_{2L}$ and $c_{1H} > c_{2H}$, with $\bar{c}_1 \equiv \lambda c_{1L} + (1-\lambda)c_{1H} \leq \bar{c}_2 \equiv \lambda c_{2L} + (1-\lambda)c_{2H}$.

Global Cost Advantage

Assume that firms' costs are not too far apart, so the lower-cost firm must set price below its monopoly level. We adopt the standard assumption for Bertrand competition with asymmetric costs: the lower-cost firm captures the market by pricing at the rival's marginal cost. Under DP, competition occurs market-by-market and the equilibrium prices are therefore:

$$p_L = \max\{c_{1L}, c_{2L}\} = c_{2L}; \qquad p_H = \max\{c_{1H}, c_{2H}\} = c_{2H}. \tag{29}$$

Under UP, we assume that the lower-cost firm can capture both markets by pricing at the other firm's *average cost.* Therefore, the equilibrium uniform price is given by

$$p_U = \max\{\bar{c}_1, \bar{c}_2\} = \bar{c}_2. \tag{30}$$

The next result shows that, while DP benefits consumers, profits can readily fall. The profit comparison depends on the difference in marginal costs of serving the two markets for firm 1 ($\Delta c_1 \equiv c_{1H} - c_{1L} > 0$) relative to firm 2 ($\Delta c_2 \equiv c_{2H} - c_{2L} > 0$).

Proposition 5 *For any given pair of costs $\{(c_{1L}, c_{1H}), (c_{2L}, c_{2H})\}$ with $c_{1L} < c_{2L}$, $c_{1H} < c_{2H}$, and $\Delta c_i > 0$, $i = 1, 2$:*
(i) $p_D = p_U$, and hence $S_D > S_U$;
(ii) with linear demand, $\Pi_D > \Pi_U$ if $\Delta c_1 > \Delta c_2$ and $\Pi_D < \Pi_U$ if $\Delta c_1 < \Delta c_2$;
(iii) relative to linear demand, $\Pi_D - \Pi_U$ and $W_D - W_U$ are higher if demand is strictly convex and lower if demand is strictly concave.

Part (*i*) is straightforward. Consider part (*ii*). With linear demand, total output as well as average price are the same under DP and UP, hence the change in firm 1's profit (which equals industry profit) is determined entirely by the reallocation effect, given in (27) for the case of two markets. From (29), firm 1's prices are: $p_L = c_{2L}$ and $p_H = c_{2H}$. Therefore, the difference in firm 1's profit margins under DP between markets L and H is $(c_{2L} - c_{1L}) - (c_{2H} - c_{1H}) = \Delta c_1 - \Delta c_2$. The output reallocation under DP raises profit if the margin is higher in market L, which occurs if $\Delta c_1 > \Delta c_2$, and lowers profit if $\Delta c_1 < \Delta c_2$.[33] Intuitively, firm 1 is harmed by being constrained to adopt a price differential that exceeds the difference in its costs.

Turning to part (*iii*), in market H where price rises under DP, output decreases by less if demand is strictly convex instead of linear, while in market L where price falls under DP, output increases by more if demand is strictly convex instead of linear. Relative to linear demand, therefore, $\Pi_D - \Pi_U$ and $W_D - W_U$ are both higher if demand is strictly convex, and the conclusion is reversed if demand is strictly concave.

[33] With linear demand, total welfare rises if $\Delta c_1 \geq \Delta c_2$, but can fall if $\Delta c_1 < \Delta c_2$. As $\Delta c_1 \to 0$, the output allocation under uniform pricing converges to the first-best, but is inefficient under differential pricing, since $p_H - p_L = \Delta c_2 > 0$, hence $W_D < W_U$.

Local Cost Advantage

Under DP, firm 1 now serves market L at price $p_L = c_{2L} < \bar{c}_2$ and firm 2 serves market H at price $p_H = c_{1H} > c_{2H} > \bar{c}_2$. Under UP, we assume that each firm cannot refuse to serve its higher-cost market; it must be willing to sell in both markets or none. Suppose also that at equal prices ($p_1 = p_2$), if $\bar{c}_1 < \bar{c}_2$, firm 1 can capture both markets at price $\bar{c}_2$, while if $\bar{c}_1 = \bar{c}_2$, the firms split both markets equally. In both cases, the equilibrium uniform price is $p_U = \bar{c}_2$, and moving to DP lowers price in market L and raises price in market H, but raises average price.[34] The next result shows that while profit necessarily rises, consumer surplus can fall without requiring unusual demand conditions.

Proposition 6 *For any given pair of costs* $\{(c_{1L}, c_{1H}), (c_{2L}, c_{2H})\}$ *with* $c_{1L} < c_{2L} < c_{2H} < c_{1H}$, *and* $\bar{c}_1 \leq \bar{c}_2$:
(i) average price is higher under DP than under UP: $p_D > p_U$;
(ii) consumer surplus is higher under DP ($S_D > S_U$) *if the cost differences within markets,* $c_{2L} - c_{1L}$ *and* $c_{1H} - c_{2H}$, *are small, but* $S_D < S_U$ *if* $c_{2H} - c_{2L}$ *is small;*
(iii) profits are always higher under DP: $\Pi_D > \Pi_U$.

Consider first the average price. Under UP, price is determined by the firm with the higher *average* of the marginal costs across the markets. Under DP, each market's price is set by the higher of the two firms' marginal costs for that market. Since cost heterogeneity is greater market-by-market than on average, the average price is higher under DP. The driving force is similar to one noted by Dana (2012).

Industry profits always rise with DP, for three reasons: as with monopoly, output is reallocated to the lower-cost market; in addition, DP now leads to each market being served by the efficient firm in that market (firm 2 replaces 1 in market H) and, furthermore, DP raises average price by relaxing the competitive constraint.

Consumer surplus is subject to opposing effects: the price dispersion is beneficial but the increase in average price is harmful. When the cost difference between firms within each market ($c_{2m} - c_{1m}$, $m = L, H$) is sufficiently small, the average price under DP converges to the uniform

[34] We have considered a variant of this scenario, where firm 1 has lower cost to serve market A than market B and the reverse holds for firm 2. Differential pricing then raises price in both markets.

price, hence the price dispersion effect dominates and DP raises consumer surplus. The opposite arises if the cost difference between markets for firm 2 ($c_{2H} - c_{2L}$) is small: then $p_U = \bar{c}_2$ is close to c_{2L}, so that moving to DP lowers price in market L only slightly but raises price in market H substantially. However, DP can lower consumer surplus even when $c_{2H} - c_{2L}$ is not 'small'. For example, suppose $\lambda = 1/2$, $Y(p) = 10 - p$, $c_{1L} = 3, c_{2L} = 4$, $c_{2H} = 6, c_{1H} = 7$. Then $\bar{c}_1 = \bar{c}_2 = 5 = p_U$, $p_L = 4$, $p_H = 7$, $p_D = 5.5 > p_U$, and $S_D = 11.25 < S_U = 12.5$. Summarizing, DP raises consumer surplus if the cost difference across markets is large relative to the cost difference between firms in a given market, and lowers consumer surplus in the reverse case.

4.2 Differentiated Products With Linear Demands

Propositions 5 and 6 showed that DP can reduce profit or consumer surplus even with 'simple' demand functions—such as linear demand—when firms have asymmetric costs and produce homogeneous products. To check whether these findings may extend to imperfect substitutes, we consider differentiated products with the following linear demand system (equation (24) from Example 1 with $n = 2$):

$$\widetilde{D}(p_i, p_j) = \frac{1}{2}\left(a - p_i - \frac{\gamma}{2}(p_i - p_j)\right) \quad \text{for } i,\, j \in \{1, 2\}\ (j \neq i), \tag{31}$$

in which $\gamma \in (0, +\infty)$ measures the degree of product substitutability. As in Subsection 4.1, the two firms may have different costs of serving the same market; c_{1m} may differ from c_{2m}, for $m = L, H$, in arbitrary ways—including the cases of global or local cost advantage.

Proposition 7 *For the demand system in (31), there exist critical values γ_1 and γ_2 such that:*
(i) when $\gamma < \gamma_1$, consumer surplus and industry profit are both higher under differential pricing than under uniform pricing regardless of the cost asymmetry between firms;
(ii) when $\gamma > \gamma_2$, the following results in Propositions 5 and 6 hold: with global cost advantage ($c_{1L} < c_{2L}$ and $c_{1H} < c_{2H}$), DP reduces profit if $c_{1H} - c_{1L} < c_{2H} - c_{2L}$, whereas with local cost advantage ($c_{1L} < c_{2L}$, $c_{1H} > c_{2H}$ and $\bar{c}_1 \leq \bar{c}_2$), DP reduces consumer surplus if $c_{2H} - c_{2L}$ is sufficiently small.

When the products are sufficiently differentiated ($\gamma < \gamma_1$), under both UP and DP the equilibrium is *interior,* with both firms producing positive outputs and the prices determined by the standard first-order conditions. The average prices under DP and under UP are then equal, as in the case of symmetric costs and linear demands (Example 1). Hence, consumer surplus is higher under DP, and industry profit also is higher because it is a convex function of (c_1, c_2).

When products are sufficiently close substitutes ($\gamma > \gamma_2$), under both regimes we have a *corner equilibrium.* Under UP, firm 2 (the higher-cost firm here) sets price at marginal cost $\bar{c}_2$ while firm 1 captures the market by setting a limit price below $\bar{c}_2$ that induces zero demand for firm 2, and this limit price $\to \bar{c}_2$ as $\gamma \to \infty$; under DP, the lower-cost firm in each market, L or H, sets a limit price to capture that market. Thus, as the products converge to perfect substitutes, the outcome converges to the homogeneous-products case, described in Proposition 5 for global cost advantage and in Proposition 6 for local cost advantage.

5. Conclusion

The welfare properties of uniform versus differential pricing in oligopoly when markets differ in costs of service have gone largely unexplored, despite the prevalence of industries where firms face constraints on cost-based pricing. In a standard setting where firms face symmetric demands, we showed that the effects of cost-based differential pricing depend on whether products are homogeneous or differentiated and whether firms have symmetric or asymmetric costs.

With symmetric costs and homogeneous products, differential pricing obviously maximizes consumer welfare whereas uniform pricing does not, while profits are zero in both regimes. With differentiated products, differential pricing benefits consumers and firms under conditions met by many standard demand functions. The systematic force driving higher profit is cost savings from reallocating output between markets by adjusting prices; consumers benefit from this price dispersion provided average price does not rise too much. Stronger competition tends to shift more of the gains from differential pricing away from the firms towards consumers.

Although profit can fall with differential pricing—unlike for monopoly—and potentially consumer surplus too, such outcomes require demand conditions that seem rather stringent when firms have symmetric costs. When firms have asymmetric costs, however, differential pricing can reduce profit or, under an alternative cost configuration, reduce consumer surplus even for standard demand functions such as linear demands.

Thus, cost-based differential pricing in oligopoly can have subtle welfare effects. By elucidating these effects and the underlying economic forces, this paper advances our understanding of a significant issue in economics—in parallel to the extensive studies on third degree price discrimination—and helps evaluate prevalent constraints on a common business practice.

Appendix

Proof of Proposition 1. Using (18), we have

$$s'(c) = S'(p^*)\, p^{*'}(c) = -Y(p^*)\, p^{*'}(c),$$

$$\begin{aligned} s''(c) &= -Y'(p^*)\left[p^{*'}(c)\right]^2 - Y(p^*)\, p^{*''}(c) \\ &= -Y'(p^*)\left[p^{*'}(c)\right]^2 - Y(p^*)\left[R'(p^*) + \Phi'(p^*)\right]\left[p^{*'}(c)\right]^3 \\ &= \left[\frac{-Y'(p^*)}{Y(p^*)} - \left[R'(p^*) + \Phi'(p^*)\right] p^{*'}(c)\right] Y(p^*)\left[p^{*\prime}(c)\right]^2 \\ &= \left[\frac{\eta(p^*)}{p^*} - \frac{R'(p^*) + \Phi'(p^*)}{2 - \Phi(p^*) - R(p^*)}\right] Y(p^*)\left[p^{*\prime}(c)\right]^2. \end{aligned}$$

Thus,

$$\text{Sign } s''(c) = \text{Sign } \left[\frac{\eta(p^*)}{p^*} - \frac{R'(p^*) + \Phi'(p^*)}{2 - \Phi(p^*) - R(p^*)}\right].$$

Therefore, if (19) holds for all $p \in [p^*(c_1),\ p^*(c_M)]$, $s(c)$ is convex for all $c \in [c_1, c_M]$ and

$$S_U \equiv s(\bar{c}) = s\left(\sum_{m=1}^{M} \lambda_m c_m\right) < \sum_{m=1}^{M} \lambda_m s(c_m) \equiv S_D.$$

Similarly, if (19) is reversed, then $s(c)$ is concave and DP lowers consumer surplus. ■

Proof of Proposition 2. We derive the condition for equilibrium firm profit $\pi(c)$ to be convex or concave as follows:

$$\pi''(c) = \frac{dD(p^*, p^*)}{dp^*} p^{*'}(c)\left[p^{*'}(c)R(p^*) - 1\right] + D(p^*, p^*)\left[p^{*''}(c)R(p^*) + \left[p^{*'}(c)\right]^2 R'(p^*)\right]$$

$$= \frac{D(p^*, p^*)p^{*\prime}(c)}{p^*}\left\{\underbrace{\frac{-Y'(p^*)p^*}{Y(p^*)}}_{\eta(p^*)}\left[1 - p^{*\prime}(c)R(p^*)\right] + p^*\left[\left(R'(p^*) + \Phi'(p^*)\right)\left[p^{*'}(c)\right]^2 R(p^*) + p^{*\prime}(c)R'(p^*)\right]\right\}$$

$$= \frac{D(p^*,p^*)p^{*\prime}(c)}{p^*}\left\{\eta(p^*)\left[1-\frac{R(p^*)}{2-\Phi(p^*)-R(p^*)}\right]+p^*\left[\frac{\left(R'(p^*)+\Phi'(p^*)\right)R(p^*)}{\left(2-\Phi(p^*)-R(p^*)\right)^2}+\frac{R'(p^*)}{2-\Phi(p^*)-R(p^*)}\right]\right\}$$
$$= \frac{D(p^*,p^*)\left[p^{*\prime}(c)\right]^2}{p^*}\left\{\eta(p^*)\left[\left(2-\Phi(p^*)-R(p^*)\right)-R(p^*)\right]+p^*\left[\frac{\left(R'(p^*)+\Phi'(p^*)\right)R(p^*)}{\left(2-\Phi(p^*)-R(p^*)\right)}+R'(p^*)\right]\right\}.$$

Therefore, we have

$$\text{Sign } \pi''(c) = \text{Sign } \left\{\frac{\eta(p^*)}{p^*}\left[\left(2-\Phi(p^*)-R(p^*)\right)-R(p^*)\right]+\frac{\left(R'(p^*)+\Phi'(p^*)\right)R(p^*)}{2-\Phi(p^*)-R(p^*)}+R'(p^*)\right\}.$$

Then, when $\pi''(c) \gtreqless 0$,

$$\Pi_U \equiv n\pi\left(\bar{c}\right) = n\pi\left(\sum_{m=1}^{M}\lambda_m c_m\right) \lesseqgtr n\sum_{m=1}^{M}\lambda_m \pi(c_m) \equiv \Pi_D.$$

It follows that DP raises profit if (21) holds for all $p \in \left[p^*\left(c_1\right),\ p^*\left(c_M\right)\right]$, and DP lowers profit if (21) is reversed for all $p \in \left[p^*\left(c_1\right),\ p^*\left(c_M\right)\right]$. ■

Proof of Proposition 3. From (22),

$$w'(c) = -Y\left(p^*\right)p^{*'}(c) + \left[p^{*'}(c)-1\right]Y(p^*) + \left[p^*(c)-c\right]Y'(p^*)p^{*'}(c)$$
$$= p^{*\prime}(c)Y'(p^*)(p^*(c)-c) - Y(p^*).$$

Since $p^*\frac{Y'(p^*)}{Y(p^*)} = -\eta_{11}(p^*)+\eta_{12}(p^*)$ and $\frac{p^*(c)-c}{p^*} = \frac{1}{\eta_{11}(p^*)}$, we have

$$w'(c) = -Y(p^*)\left[p^{*\prime}(c)\left(1-R(p^*)\right)+1\right].$$

It follows that

$$w''(c) = -Y'(p^*)p^{*'}(c)\left[1+p^{*\prime}(c)\left(1-R(p^*)\right)\right] - Y(p^*)\left[p^{*''}(c)\left(1-R(p^*)\right)-\left[p^{*\prime}(c)\right]^2R'(p^*)\right]$$
$$= -Y'(p^*)p^{*'}(c)\left[1+p^{*\prime}(c)\left(1-R(p^*)\right)\right] - Y(p^*)\left[\left(R'(p^*)+\Phi'(p^*)\right)\left[p^{*\prime}(c)\right]^3\left(1-R(p^*)\right)-\left[p^{*\prime}(c)\right]^2R'(p^*)\right]$$

$$= Y(p^*)p^{*\prime}(c)\left\{\frac{-Y'(p^*)}{Y(p^*)}\left[1+p^{*\prime}(c)\left(1-R(p^*)\right)\right]-\left[\left(R'(p^*)+\Phi'(p^*)\right)\left[p^{*\prime}(c)\right]^2\left(1-R(p^*)\right)-p^{*\prime}(c)R'(p^*)\right]\right\}$$
$$= \frac{Y(p^*)p^{*\prime}(c)}{p^*}\left\{\eta(p^*)\left[1+\frac{1-R(p^*)}{2-R(p^*)-\Phi(p^*)}\right]+p^*\left[\frac{-\left[\Phi'(p^*)+R'(p^*)\right]\left(1-R(p^*)\right)}{\left[2-R(p^*)-\Phi(p^*)\right]^2}+\frac{R'(p^*)}{2-R(p^*)-\Phi(p^*)}\right]\right\}$$
$$= \frac{Y(p^*)p^{*\prime}(c)}{\left(2-R(p^*)-\Phi(p^*)\right)}\left\{\frac{-\left(\Phi'(p^*)+R'(p^*)\right)\left(1-R(p^*)\right)}{\left(2-R(p^*)-\Phi(p^*)\right)}+R'(p^*)+\frac{\eta(p^*)}{p^*}\left(3-\Phi(p^*)-2R(p^*)\right)\right\}.$$

Thus, we have

$$\text{Sign } w''(c) = \text{Sign }\left\{\frac{-\left(\Phi'(p^*)+R'(p^*)\right)\left(1-R(p^*)\right)}{\left(2-R(p^*)-\Phi(p^*)\right)}+R'(p^*)+\frac{\eta(p^*)}{p^*}\left(3-\Phi(p^*)-2R(p^*)\right)\right\}.$$

When $w''(c) \gtreqless 0$, we have

$$W_U \equiv w(\bar{c}) = w(\sum_{m=1}^{M}\lambda_m c_m) \lesseqgtr \sum_{m=1}^{M}\lambda_m w(c_m) \equiv W_D.$$

It follows that DP raises welfare if (23) holds for all $p \in [p^*(c_1)\,,\ p^*(c_M)]$, and DP lowers welfare if (23) is reversed for all $p \in [p^*(c_1)\,,\ p^*(c_M)]$. ■

Proof of Proposition 4. The equilibrium price, quantity, profit of each firm as functions of marginal cost c is

$$p^*(c) = \frac{c\gamma(n-1)+(a+c)n}{2n+\gamma(n-1)};\quad q^*(c) = \frac{(a-c)(n+\gamma(n-1))}{n\left(2n+\gamma(n-1)\right)};\quad \pi(c) = \frac{(a-c)^2\left(n+\gamma(n-1)\right)}{\left(2n+\gamma(n-1)\right)^2}.$$

Consumer surplus and total welfare as functions of c are

$$s(c) = \frac{(a-c)^2\left(n+\gamma(n-1)\right)^2}{2\left(2n+\gamma(n-1)\right)^2},\quad w(c) = \frac{(a-c)^2\left(n+\gamma(n-1)\right)\left(3n+\gamma(n-1)\right)}{2\left(2n+\gamma(n-1)\right)^2}.$$

We thus have:

$$\begin{aligned}\Delta S = S_D - S_U &= \sum_{m=1}^{M}\lambda_m s(c_m) - s(\bar{c})\\ &= \frac{\left(n+\gamma(n-1)\right)^2}{2\left(2n+\gamma(n-1)\right)^2}\left[\sum_{m=1}^{M}\lambda_m(a-c_m)^2-\left(a-\sum_{m=1}^{M}\lambda_m c_m\right)^2\right],\end{aligned}$$

in which the term in the square brackets is positive because $(a-c)^2$ is convex in c. Thus,

$\Delta S > 0$. Similarly,

$$\Delta\Pi = \Pi_D - \Pi_U = n\sum_{m=1}^{M}\lambda_m \pi(c_m) - n\pi(\bar{c})$$
$$= \frac{n\left(n+\gamma(n-1)\right)}{\left(2n+\gamma(n-1)\right)^2}\left[\sum_{m=1}^{M}\lambda_m(a-c_m)^2 - \left(a-\sum_{m=1}^{M}\lambda_m c_m\right)^2\right] > 0;$$

$$\Delta W = \Delta\Pi + \Delta S = \frac{\left(n+\gamma(n-1)\right)\left(3n+\gamma(n-1)\right)}{2\left(2n+\gamma(n-1)\right)^2}\left[\sum_{m=1}^{M}\lambda_m(a-c_m)^2 - \left(a-\sum_{m=1}^{M}\lambda_m c_m\right)^2\right] > 0.$$

Therefore:[35]

$$\frac{\partial\Delta S}{\partial n} = \frac{\gamma(n+\gamma(n-1))}{\left(2n+\gamma(n-1)\right)^3}\left[\sum_{m=1}^{M}\lambda_m(a-c_m)^2 - \left(a-\sum_{m=1}^{M}\lambda_m c_m\right)^2\right] > 0,$$

$$\frac{\partial\Delta\Pi}{\partial n} = -\frac{\gamma^2(n-1)}{\left(2n+\gamma(n-1)\right)^3}\left[\sum_{m=1}^{M}\lambda_m(a-c_m)^2 - \left(a-\sum_{m=1}^{M}\lambda_m c_m\right)^2\right] < 0,$$

$$\frac{\partial\Delta W}{\partial n} = \frac{\gamma n}{\left(2n+\gamma(n-1)\right)^3}\left[\sum_{m=1}^{M}\lambda_m(a-c_m)^2 - \left(a-\sum_{m=1}^{M}\lambda_m c_m\right)^2\right] > 0.$$

■

Proof of Example 4. Under DP, the symmetric equilibrium price and each firm's output and profit in market $m \in \{L, H\}$ can be written as:

$$p_m = \frac{2(a+b_m)+c_m(2+\gamma)}{4+\gamma}, \quad \tilde{q}_m = \frac{\lambda_m(a+b_m-c_m)(2+\gamma)}{2(4+\gamma)}, \quad \pi_m = \frac{\lambda_m(a+b_m-c_m)^2(2+\gamma)}{(4+\gamma)^2}$$

in which $\lambda_m = \lambda$ for $m = L$ and $\lambda_m = 1-\lambda$ for $m = H$.

Consumer surplus in market m is $s_m = \frac{\lambda_m(a+b_m-c_m)^2(2+\gamma)^2}{2(4+\gamma)^2}$.

Under UP, at symmetric price (p, p), each firm's demand in market m is proportional to $\frac{a+b_m-p}{2}$. Therefore, a firm's demand across both markets is $\lambda\frac{(a-p)}{2} + (1-\lambda)\frac{(a+b-p)}{2}$ if $p \leq a$. If $a+b > p > a$, market L is not served and each firm's demand is $(1-\lambda)\frac{(a+b-p)}{2}$. Thus, both markets will be served if and only if the equilibrium symmetric price satisfies $p \leq a$. We shall

[35] It is also straightforward to verify that $\frac{\partial\Delta S}{\partial\gamma} > 0$, $\frac{\partial\Delta\Pi}{\partial\gamma} < 0$, and $\frac{\partial\Delta W}{\partial\gamma} > 0$.

analyze this case, which holds when $b \leq \frac{(a-\bar{c})(2+\gamma)}{2(1-\lambda)}$, where $\bar{c} = \lambda c_L + (1-\lambda)c_H$.

With both markets served, the symmetric equilibrium price and each firm's output are:

$$p_U = \frac{2a + 2b(1-\lambda) + \bar{c}(2+\gamma)}{4+\gamma}, \qquad \tilde{q}_U = \frac{(a + b(1-\lambda) - \bar{c})(2+\gamma)}{2(4+\gamma)};$$

and each firm's equilibrium profit is

$$\begin{aligned}\pi_U &= \lambda\frac{(a-p_U)}{2}(p_U - c_L) + (1-\lambda)\frac{(a+b-p_U)}{2}(p_U - c_H)\\ &= \frac{((a-\bar{c})(2+\gamma) - 2b(1-\lambda))\,(a + b(1-\lambda) - \bar{c})}{(4+\gamma)^2} + \frac{1}{2}b(1-\lambda)\left(-c_H + \frac{2a + 2b(1-\lambda) + \bar{c}(2+\gamma)}{4+\gamma}\right).\end{aligned}$$

Total consumer surplus under UP is

$$S_U = \frac{(a+b-\bar{c})^2\,(2+\gamma)^2 + b\lambda\left(-2a\,(2+\gamma)^2 + 2(2+\gamma)^2\bar{c} + b(8-\gamma^2)\right) - 4b^2(3+\gamma)\lambda^2}{2\,(4+\gamma)^2}.$$

Note that the per-capita equilibrium quantity in market m under UP is given by

$$\widehat{q}_L \equiv \widetilde{D}(p_{iL}, p_{jL})\mid_{(p_U,p_U)} = \frac{a-p_U}{2}, \qquad \widehat{q}_H \equiv \widetilde{D}(p_{iH}, p_{jH})\mid_{(p_U,p_U)} = \frac{a+b-p_U}{2}.$$

Then $\frac{\lambda_m \widehat{q}_m}{q_U} \equiv \widehat{\lambda}_m$ is the share of total quantity demanded in market $m \in \{L, H\}$ under UP in equilibrium. Using these shares as weights, the weighted average price under DP exceeds the uniform price:

$$\left[\widehat{\lambda}_L p_L + \widehat{\lambda}_H p_H\right] - p_U = \frac{b\,(2b + (c_H - c_L)(2+\gamma))\,(1-\lambda)\lambda}{(2+\gamma)(a + b(1-\lambda) - \bar{c})} > 0.$$

Observing that $\tilde{q}_U = \tilde{q}_L + \tilde{q}_H$: total output of each firm remains unchanged when moving from UP to DP.

Industry profit is higher under DP:

$$\Pi_D - \Pi_U = 2\,(\pi_L + \pi_H - \pi_U) = \frac{(b(2+\gamma) + 2\,(c_H - c_L))\,(2b + (c_H - c_L)(2+\gamma))\,(1-\lambda)\lambda}{(4+\gamma)^2} > 0.$$

The change in consumer surplus is

$$S_D - S_U = (s_L + s_H) - S_U = \frac{\left((c_H - c_L)^2(2+\gamma)^2 - 2b(c_H - c_L)(2+\gamma)^2 - 4b^2(3+\gamma)\right)(1-\lambda)\lambda}{2(4+\gamma)^2}.$$

Let $b_1 = \min\{\frac{(a-\bar{c})(2+\gamma)}{2(1-\lambda)}, \frac{(c_H - c_L)(2+\gamma)}{2(3+\gamma)}\}$. Therefore, $S_D \geq S_U$ if $b \leq b_1$.

Comparing total welfare gives

$$W_D - W_U = \frac{\lambda(1-\lambda)\left(2b + (c_H - c_L)(2+\gamma)\right)\left((6+\gamma)(c_H - c_L) - 2b\right)}{2(4+\gamma)^2}.$$

Let $b_2 = \min\{\frac{(a-\bar{c})(2+\gamma)}{2(1-\lambda)}, \frac{1}{2}(c_H - c_L)(6+\gamma)\} > b_1$. Therefore, $W_D \geq W_U$ if $b \leq b_2$. ■

Proof of Proposition 5. (i) From (29) and (30):

$$p_D = \lambda p_L + (1-\lambda)p_H = \lambda c_{2L} + (1-\lambda)c_{2H} = p_U.$$

Then $S_D > S_U$ holds, from Remark 1.

(ii) For profit we only need to consider firm 1, since the higher-cost rival earns no profit under either pricing regime. Given $p_D = p_U$, linear demand implies that total output also remains unchanged:

$$q_D = \lambda Y(p_L) + (1-\lambda)Y(p_H) = Y(\lambda p_L + (1-\lambda)p_H) = Y(p_U) = q_U$$

With $p_D = p_U$ and $q_D = q_U$, it is straightforward that $\text{sign}(\Pi_D - \Pi_U) = \text{sign}\left[(p_L - c_{1L}) - (p_H - c_{1H})\right]$. Since $p_L = c_{2L}$ and $p_H = c_{2H}$, we have

$$(c_{2L} - c_{1L}) - (c_{2H} - c_{1H}) = c_{1H} - c_{1L} - (c_{2H} - c_{2L}) \equiv \Delta c_1 - \Delta c_2.$$

(iii) From (29) and (30), the prices p_L, p_H and p_U are determined by firm 2's marginal costs independent of the curvature of $Y(p)$. Suppose $Y(p)$ is strictly convex. Consider the linear demand $L(p)$ that is tangent to $Y(p)$ at p_U.[36] Uniform pricing yields the same price and output with $L(p)$ or $Y(p)$, hence the same profit and welfare. But under differential pricing, since

[36] The ensuing argument is inspired by Malueg (1993).

$p_D = p_U$, outputs in both markets will be greater with $Y(p)$ than with $L(p)$. Since firm 1's margins in both markets are positive ($p_L = c_{2L} > c_{1L}$, $p_H = c_{2H} > c_{1H}$), profit and total welfare will be higher with $Y(p)$ than with $L(p)$. The reverse holds if $Y(p)$ is strictly concave. ■

Proof of Proposition 6. (i) Price: $p_D = \lambda c_{2L} + (1-\lambda)\, c_{1H} > \lambda c_{2L} + (1-\lambda)\, c_{2H} = \bar{c}_2 = p_U$.
(ii) Consumer Surplus:

$$S_U = s(\bar{c}_2) = \int_{\bar{c}_2}^{\infty} Y(p)dp, \qquad S_D = \lambda \int_{c_{2L}}^{\infty} Y(p)dp + (1-\lambda) \int_{c_{1H}}^{\infty} Y(p)dp.$$

When $c_{2L} - c_{1L} \to 0$ and $c_{1H} - c_{2H} \to 0$, $\lambda c_{2L} + (1-\lambda)\, c_{1H} \to \bar{c}_2$, and hence, because $s(p)$ is strictly convex,

$$S_U = s(\bar{c}_2) \to s(\lambda c_{2L} + (1-\lambda)\, c_{1H}) < \lambda s(c_{2L}) + (1-\lambda)\, s(c_{1H}) = S_D.$$

On the other hand, when $c_{2L} \to c_{2H}$ so that $c_{2L} \to \bar{c}_2$,

$$S_D - S_U = \lambda \int_{c_{2L}}^{\bar{c}_2} Y(p)dp - (1-\lambda) \int_{\bar{c}_2}^{c_{1H}} Y(p)dp < 0.$$

(iii) Profits**:** Under UP, firm 2's profit is zero, but under DP, each firm earns positive profit. Total profits under the two regimes are

$$\Pi_U = (\bar{c}_2 - \bar{c}_1)Y(\bar{c}_2), \quad \Pi_D = \lambda(c_{2L} - c_{1L})Y(c_{2L}) + (1-\lambda)(c_{1H} - c_{2H})Y(c_{1H}).$$

Thus,

$$\begin{aligned}
\Pi_D - \Pi_U &= \lambda(c_{2L} - c_{1L})Y(c_{2L}) - (\bar{c}_2 - \bar{c}_1)Y(\bar{c}_2) + (1-\lambda)(c_{1H} - c_{2H})Y(c_{1H}) \\
&> \lambda(c_{2L} - c_{1L})Y(c_{2L}) - \lambda(c_{2L} - c_{1L})Y(\bar{c}_2) - (1-\lambda)(c_{2H} - c_{1H})Y(\bar{c}_2) \\
&= \lambda(c_{2L} - c_{1L})\,[Y(c_{2L}) - Y(\bar{c}_2)] + (1-\lambda)(c_{1H} - c_{2H})Y(\bar{c}_2) > 0.
\end{aligned}$$

■

Proof of Proposition 7. Following Section 4.1, suppose $\bar{c}_1 < \bar{c}_2$. Under UP, firm $i's$ profit function is:

$$\pi_i = \frac{1}{2}\left(a - p_i + \frac{\gamma}{2}(p_j - p_i)\right)(p_i - \bar{c}_i).$$

Suppose $\gamma \leq \gamma_U \equiv \frac{3a+\bar{c}_1-4\bar{c}_2+\sqrt{9a^2-2a\bar{c}_1+\bar{c}_1^2-16a\bar{c}_2+8\bar{c}_2^2}}{(\bar{c}_2-\bar{c}_1)}$. Using the first order conditions, the equilibrium prices and outputs of firms $i \neq j = 1, 2$ are:

$$\begin{aligned} p_{iU} &= \frac{a(8+6\gamma)+(2+\gamma)\left(\bar{c}_j\gamma+2\bar{c}_i(2+\gamma)\right)}{16+16\gamma+3\gamma^2}, \\ q_{iU} &= \frac{(2+\gamma)\left(\bar{c}_j\gamma(2+\gamma)+a(8+6\gamma)-\bar{c}_i(8+8\gamma+\gamma^2)\right)}{4(4+\gamma)(4+3\gamma)} \geq 0. \end{aligned}$$

Note that if $\gamma > \gamma_U$, then $q_{2U} < 0$ and the above p_{iU} and q_{iU} no longer form an equilibrium. Instead, the equilibrium will be a corner solution, described shortly.

Similarly, under DP, for $m = L, H$, there exists γ_m such that if $\gamma \leq \gamma_m$, the equilibrium prices and quantities of firms $i \neq j = 1, 2$ are:

$$\begin{aligned} p_{im} &= \frac{a(8+6\gamma)+(2+\gamma)\left(c_{jm}\gamma+2c_{im}(2+\gamma)\right)}{16+16\gamma+3\gamma^2}, \\ q_{im} &= \frac{(2+\gamma)\left(c_{jm}\gamma(2+\gamma)+a(8+6\gamma)-c_{im}(8+8\gamma+\gamma^2)\right)}{4(4+\gamma)(4+3\gamma)} \geq 0, \end{aligned}$$

while if $\gamma > \gamma_m$, and $c_{im} > c_{jm}$, then $q_{im} < 0$ and the equilibrium instead will be a corner solution.

Hence, when $\gamma \leq \gamma_1 \equiv \min\{\gamma_U, \gamma_L, \gamma_H\}$, the average prices under DP and UP are equal:

$$p_{1D} = \lambda p_{1L} + (1-\lambda)\, p_{1H} = p_{1U}; \qquad p_{2D} = \lambda p_{2L} + (1-\lambda)\, p_{2H} = p_{2U}.$$

Furthermore, it is straightforward to verify that when $\gamma \leq \gamma_1$, in equilibrium consumer surplus as a function of (q_1, q_2) and industry profit as a function of (c_1, c_2) are convex, implying that consumer surplus, industry profit and total welfare are all higher under DP than under UP.

Now turn to the case where $\gamma > \gamma_2 \equiv \max\{\gamma_U, \gamma_L, \gamma_H\}$. Suppose $\bar{c}_i < \bar{c}_j$. Under UP, the equilibrium is a corner solution in which $p_{jU} = \bar{c}_j$ and firm i captures all consumers in both markets by setting a limit price p_{iU} that induces zero demand from firm j:

$$q_{jU} = \frac{1}{2}\left(a - p_{jU} + \frac{\gamma}{2}(p_{iU} - p_{jU})\right) = 0,$$

where $p_{iU} = \frac{(\gamma+2)\bar{c}_j - 2a}{\gamma}$ and $q_{iU} = \frac{(a-\bar{c}_j)(1+\gamma)}{\gamma}$.

Similarly, under DP, the equilibrium is a corner solution in which the higher cost firm sets price $p_{jm} = c_{jm}$ and the lower cost firm chooses price $p_{im} = \frac{(\gamma+2)c_{jm}-2a}{\gamma}$ that induces zero demand from firm j in market m.

Suppose firm 1 has global cost advantage with $c_{1m} < c_{2m}$ as in Proposition 4. Then, $p_{2U} = \bar{c}_2$ and $p_{1U} = \frac{(\gamma+2)\bar{c}_2-2a}{\gamma}$ under UP, and $p_{2m} = c_{2m}$ and $p_{1m} = \frac{(\gamma+2)c_{2m}-2a}{\gamma}$ in market m under DP. Firm 2 receives zero profit under both DP and UP. For firm 1, we have $p_{1D} - p_{1U} = 0$. Note that $\Delta\Pi = \Pi_D - \Pi_U = \pi_{1D} - \pi_{1U}$ has the same sign as $(p_{1L} - c_{1L}) - (p_{1H} - c_{1H})$. Since

$$\begin{aligned}(p_{1L} - c_{1L}) - (p_{1H} - c_{1H}) &= \left[\frac{(\gamma+2)c_{2L} - 2a}{\gamma} - c_{1L}\right] - \left[\frac{(\gamma+2)c_{2H} - 2a}{\gamma} - c_{1H}\right] \\ &= (c_{1H} - c_{1L}) - \frac{\gamma+2}{\gamma}(c_{2H} - c_{2L}),\end{aligned}$$

it follows that $\Pi_D > \Pi_U$ if $\Delta c_1 > \frac{\gamma+2}{\gamma}\Delta c_2$ and $\Pi_D < \Pi_U$ if $\Delta c_1 < \frac{\gamma+2}{\gamma}\Delta c_2$.

Next consider local cost advantage with $c_{1L} < c_{2L}$, $c_{1H} > c_{2H}$, and $\bar{c}_1 \leq \bar{c}_2$. Under UP, $q_{2U} = 0$ and $q_{1U} = \frac{(a-\bar{c}_2)(1+\gamma)}{\gamma}$. Consumer surplus can be computed as

$$S_U = u(q_{1U}, q_{2U}) - p_{1U}q_{1U} - p_{2U}q_{2U} = \frac{(a-\bar{c}_2)^2(1+\gamma)(2+\gamma)}{2\gamma^2}.$$

Similarly, consumer surplus under DP can be computed as

$$s_L = \frac{(a-c_{2L})^2(1+\gamma)(2+\gamma)}{2\gamma^2}, \qquad s_H = \frac{(a-c_{1H})^2(1+\gamma)(2+\gamma)}{2\gamma^2}; \qquad S_D = \lambda s_L + (1-\lambda)s_H.$$

Therefore,

$$\Delta S = S_D - S_U = \frac{(1+\gamma)(2+\gamma)(1-\lambda)\left[(c_{2H} - c_{2L})^2\lambda - (2a - c_{1H} - c_{2H})(c_{1H} - c_{2H})\right]}{2\gamma^2},$$

with $\Delta S < 0$ if $c_{2H} - c_{2L}$ is sufficiently small, and $\Delta S > 0$ if $c_{1H} - c_{2H}$ is sufficiently small. ■

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