TRAJECTORY DESIGN AND STATION-KEEPING ANALYSIS FOR THE WIDE FIELD INFRARED SURVEY TELESCOPE MISSION

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The Wide Field Infrared Survey Telescope (WFIRST) is an upcoming NASA-led observatory that will complete wide-field imaging and near-infrared sky surveys from a spacecraft in the Sun-Earth L_2 region. To identify a feasible mission trajectory, subject to geometric and maneuver constraints, an interactive trajectory design procedure, supported by dynamical systems techniques, is developed. This rapid and well-informed approach is implemented as a module of Purdue University's Adaptive Trajectory Design tool. In this paper, a feasible mission trajectory is constructed and output to a higher-fidelity modeling environment. Furthermore, station-keeping maneuvers are computed using a mode analysis strategy.

INTRODUCTION

The WFIRST spacecraft is the focus of a NASA-led observatory mission currently in the preliminary design development stage and involves a partnership between NASA Goddard Space Flight Center (GSFC) and the NASA Jet Propulsion Laboratory. The WFIRST mission concept is designed as a five-year mission, intended for launch in 2026 for collection of observations to study dark energy, to explore the origin and evolution of the universe and to seek exoplanets that may harbor life, while also supporting a guest observer program.¹ In support of these scientific objectives, WFIRST is designed to leverage an existing 2.4 meter telescope, a wide field instrument with a field of view 100 times wider than the Hubble Space Telescope, and a coronagraph instrument.² To achieve the primary objectives of the WFIRST mission concept, a baseline scientific orbit in the Sun-Earth (SE) L_2 region is currently of interest.

While meeting the thermal and dynamical requirements for the science instruments, the mission orbit and the transfer trajectory must satisfy additional geometrical constraints that create challenges during the trajectory design process. In particular, the mission orbit must avoid the Earth and lunar shadow regions. Additionally, the mission orbit must maintain an angle between the Earth-to-vehicle line and the SE L_2 line, labeled the *SEL2V* angle, that is less than 36 degrees.³ Violation of this constraint impedes clear communications with ground stations due to the spacecraft appearing too far below or above the horizon during the summer and winter seasons. A similar constraint on the transfer path requires that the spacecraft maintain a *SEL2V* angle that is less than 33 degrees along the path from low Earth orbit (LEO) to the mission orbit. In this investigation, the initial condition corresponding to the transfer path is assumed to correspond to a state that is supplied by the launch vehicle provider in the form of Earth-fixed position and velocity coordinates. This state corresponds to a 185 km altitude and 28.5 degree inclination LEO. To depart this LEO, the spacecraft implements a maneuver, labeled ΔV_{LEO} . This maneuver is operationally constrained to occur between 10 and 60 minutes beyond the initial state on the LEO. Furthermore, the maneuver must not

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possess a large out-of-plane component. Once the spacecraft is inserted onto the transfer path, a mid-course correction of up to 30 m/s in magnitude may be implemented to correct any state errors following the LEO departure maneuver. Then, an orbit insertion maneuver, labeled ΔV_{L2OI} , of magnitude 15 m/s or less, is leveraged to link the transfer path to the mission orbit. To ensure clear communications and reduce the time to reach orbit, ΔV_{L2OI} should occur near the closest intersection of the mission orbit with the Earth- L_2 line. Together, these constraints comprise the key challenges in identifying a feasible transfer path and mission orbit for the WFIRST spacecraft, offering an opportunity to develop a trajectory design strategy that employs dynamical systems techniques in an interactive environment.

While incorporating the mission constraints into the trajectory design procedure, a methodology that also leverages knowledge of natural dynamical structures in the Sun-Earth system is developed. Specifically, the Circular Restricted Three-Body Problem (CR3BP) offers an autonomous approximation to the dynamics in the Sun-Earth system. In this dynamical model, periodic orbits and their associated manifolds are identified and analyzed for their potential to satisfy the mission constraints. Candidate dynamical structures are then leveraged in combination with Poincaré mapping strategies to assemble an initial guess for a transfer trajectory. The resulting discontinuous initial guess, constructed using insight into the natural pathways within the Sun-Earth system, is corrected in the CR3BP and then in a point mass ephemeris model incorporating the gravitational influence of the Sun, Earth and Moon and augmented by the perturbative effect of solar radiation pressure. This systematic approach enables early incorporation of the mission constraints, while also supporting rapid trajectory design as the mission constraints and launch information are updated.

A feasible trajectory, designed using dynamical systems techniques, is exported to a higher-fidelity modeling software package for station-keeping analysis. In fact, station-keeping maneuvers are required to overcome the perturbations introduced by regular momentum unloads from the WFIRST spacecraft. To compute these maneuvers, a dynamical systems approach is employed, with the direction determined from the unstable modes associated with the perturbed path of the spacecraft.^{4,5} Then, the magnitude is calculated via a constrained optimization problem. The recovered station-keeping maneuvers are computed over a ten-year time interval for a baseline trajectory to a SE L_2 quasi-halo orbit. This strategy, informed by dynamical systems techniques, successfully recovers low-cost station-keeping maneuvers that are computationally efficient to identify in the nonlinear and chaotic gravitational environment of the Sun-Earth system.

In this investigation, the procedure is implemented via a graphical user interface within MATLAB as an additional module to the Adaptive Trajectory Design tool developed by Purdue University and NASA GSFC.⁶ Using this module, a trajectory is designed for the WFIRST mission and exported to a high-fidelity modeling environment for station-keeping analysis.⁷ Furthermore, the constructed methodology reveals the value of dynamical systems techniques in a supporting rapid and well-informed trajectory design procedure.

DYNAMICAL MODEL

To identify feasible transfer trajectories, the underlying structures associated with a representative, autonomous dynamical model are used to recover a solution in a higher-fidelity model. First, the equations of motion in the CR3BP are introduced along with the definition of the rotating frame. In this model, particular solutions are leveraged to construct an initial guess for a mission trajectory. Once corrected, these solutions are transitioned to a point mass ephemeris model.⁸ Accordingly, the equations of motion for a multi-body point mass ephemeris model are presented, along with an approximate model of solar radiation pressure. Following the definitions of these dynamical models, Poincaré mapping strategies are presented as a means for clear visualization of trajectories within the chaotic regime of the Sun-Earth-Moon system. This technique is used to identify and select arcs to construct an initial guess trajectory. Given an initial guess, a corrections scheme for recovering continuous trajectories is described. Together, these dynamical representations and associated computational techniques are useful in designing a feasible trajectory for the WFIRST spacecraft.

Circular Restricted Three-Body Problem

The dynamics in the Sun-Earth system are well-approximated via the CR3BP. This autonomous dynamical system models the motion of a spacecraft with negligible mass under the influence of the gravitational attraction of two point masses: the Sun and the Earth. These two primaries are assumed to follow circular orbits about their mutual barycenter. To enable the identification of natural structures within this dynamical environment, a rotating frame, $\hat{x}\hat{y}\hat{z}$, is commonly employed and rotates with the two primaries. In this coordinate frame, the \hat{x} direction is aligned with the Sun to Earth line, while the \hat{z} unit vector is parallel to the orbital angular momentum vector of the two primaries. Furthermore, position, velocity and time variables are nondimensionalized such that both the Sun-Earth distance and the mean motion of the primaries are equal to unity. In addition, mass quantities are normalized by the total mass of the system, such that the mass of the Earth is equal to the mass ratio, μ . In the CR3BP, the equations of motion for a spacecraft, described by the nondimensional state coordinates $\bar{x} = [x, y, z, \dot{x}, \dot{y}, \dot{z}]$ in the rotating frame, are written as:

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x}, \qquad \ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y}, \qquad \ddot{z} = \frac{\partial U}{\partial z}$$
 (1)

where U is the pseudo-potential function, $U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r} + \frac{\mu}{d}$, and $d = \sqrt{(x+\mu)^2 + y^2 + z^2}$, $r = \sqrt{(x-1+\mu)^2 + y^2 + z^2}$. Under the assumptions of the CR3BP, and when formulated in a rotating frame, the natural motion of a spacecraft is described by a constant energy-like integral labeled the Jacobi constant, $C_J = 2U - \dot{x}^2 - \dot{y}^2 - \dot{z}^2$. The motion of a spacecraft within this autonomous dynamical system is governed by a system of underlying dynamical structures, comprised of equilibrium points (three collinear points L_1, L_2, L_3 and two triangular points L_4, L_5), periodic and quasi-periodic orbits as well as their associated manifolds.⁸ Each of these types of particular solutions are identified and sampled to construct an initial guess for a trajectory within the Sun-Earth system.

For a given trajectory, the state transition matrix (STM) offers a measure of the sensitivity of a solution at a time downstream to perturbations in an initial state. In fact, for a given initial condition $\bar{x}_0 = [x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0]$ expressed in the rotating frame at a time t_0 , the STM evaluated at a time t is equivalent to the matrix of partial derivatives, $\Phi(t, t_0) = \frac{\partial \bar{x}}{\partial \bar{x}_0}$. This STM is evaluated either numerically via finite differencing or analytically by augmenting the state with the STM elements during the numerical integration process, with an initial value equal to the identity matrix. Since the STM is essentially a linear mapping from t_0 to a time t, it offers insight into the stability of a solution, proving useful during corrections and station-keeping analyses.

Ephemeris Model

To identify an end-to-end trajectory for a spacecraft within a higher-fidelity representation of the Sun-Earth system, a point mass ephemeris model is employed. In particular, three bodies – the Sun, Earth and Moon – each with a nondimensional mass, M_i , are modeled as point masses. Each point mass, body *i*, is located in an inertial frame, $\hat{X}\hat{Y}\hat{Z}$, at the nondimensional coordinates $\bar{R}_i = (X_i, Y_i, Z_i)$ relative to an inertially-fixed origin, *O*. Body *i* is then located relative to another point mass, body *j*, via the relative position vector $\bar{R}_{ji} = (X_i - X_j)\hat{X} + (Y_i - Y_j)\hat{Y} + (Z_i - Z_j)\hat{Z}$ expressed in nondimensional units. These definitions are leveraged to formulate the dynamics describing the natural motion of a spacecraft within a point mass ephemeris model incorporating the gravitational influence of the Sun, Earth and Moon. Specifically, the equations of motion for a spacecraft located at the coordinates $\bar{R}_{E,s/c} = (X_{E,s/c}, Y_{E,s/c}, Z_{E,s/c})$ relative to the Earth in an Earth-centered J2000 coordinate system are written in vector form as:

$$\bar{R}_{E,s/c}^{\prime\prime} = -\frac{GM_E}{R_{E,s/c}^3} \bar{R}_{E,s/c} + G \sum_{j=S,L} M_j \left(\frac{\bar{R}_{s/c,j}}{R_{s/c,j}^3} - \frac{\bar{R}_{E,j}}{R_{E,j}^3} \right)$$
(2)

where the subscript E identifies the Earth, s/c identifies the spacecraft, L corresponds to the Moon and S indicates the Sun, while capital letters indicate configuration space variables expressed in inertial coordinates.^{8,9} The variable G is the universal gravitational constant normalized by characteristic mass, length and time quantities defined to be consistent with the Sun-Earth CR3BP. These equations of motion are numerically integrated, with position coordinates for each planetary body accessed using the Jet Propulsion Laboratory DE421 ephemerides via the SPICE toolkit.^{10,11,12}

The additional acceleration due to solar radiation pressure (SRP) is incorporated into the differential equations governing the motion of the WFIRST spacecraft in the point mass ephemeris model. In fact, SRP contributes an additional force exerted on the surface of a spacecraft due to photons emitted from the Sun.¹³ To formulate an approximate and autonomous representation of SRP, the spacecraft is assumed to possess a spherical shape, thereby eliminating the need for time-dependent information on the spacecraft attitude. Rather, the spacecraft is modeled as a sphere of mass, $\tilde{M}_{s/c}$, with a constant cross-sectional area, A, and a constant reflectivity coefficient, k. Under these assumptions, the additional acceleration, \bar{a}_{SRP} , acting on the spacecraft due to SRP, located at a distance of $R_{S,s/c}$ from the Sun, is written in vector form as:

$$\bar{a}_{SRP} = \frac{kAS_0}{c\tilde{M}_{s/c}} \frac{R_0^2}{R_{E,s/c}^3} \bar{R}_{S,s/c}$$
(3)

where S_0 is the solar flux at the nominal Sun-Earth distance, R_0 , and c is the speed of light.¹⁴ For the WFIRST sample scenario, the spacecraft is assumed to possess a dry mass of 6877 kg with a fuel mass of 851 kg, and a cross-sectional area of 49.6 m². Once nondimensionalized using characteristic quantities consistent with the CR3BP, this additional acceleration is added to the equations of motion for the point mass ephemeris model described by Eq. (2). The resulting equations of motion offer an approximation to the dynamics in the vicinity of the Earth, Sun and Moon, capturing the dominant perturbations acting on the WFIRST spacecraft.

Corrections Procedure

To construct a continuous trajectory for the WFIRST spacecraft, a variable-time multiple shooting algorithm is employed. The general procedure for a multiple shooting scheme involves intermediate nodes, described by a set of free variables, that serve as initial states and are simultaneously integrated to produce a sequence of arcs. These nodes are then iteratively corrected to recover a continuous path that satisfies a set of constraints.⁸ This corrections algorithm is implemented first in the CR3BP and, then, the point mass ephemeris model. To further explore this procedure, consider a single trajectory discretized into a set of N arcs. Connectivity between two neighboring arcs requires both state and time continuity constraints. In particular, for two naturally-connected arcs, the position and velocity at the end of the *i*-th arc, integrated forward for a time $T_{int,i}$, must equal the position and velocity at the (i + 1)-th node to within an acceptable tolerance. For two neighboring arcs linked via an impulsive maneuver, only position continuity constraints are enforced. Note that in the point mass ephemeris model, the position and velocities of the Sun, Earth and Moon are time-dependent and the epoch, $T_{node,i} + T_{int,i}$, at the end of the *i*-th arc must equal the epoch, $T_{node,i+1}$, at the (i + 1)-th node; this condition is labeled a time continuity constraint.⁹ Together, the relevant constraints are simultaneously employed at each step in an iterative corrections algorithm, leveraging a Newton's method, to recover a continuous trajectory for the WFIRST spacecraft. This procedure is first implemented for a spacecraft in the CR3BP and, then, under the influence of a point mass ephemeris model incorporating the gravitational influence of the Earth, Moon and Sun with SRP. Additional constraints may also be placed on the state and/or time variables at a given node by augmenting additional equality constraints; such constraints are detailed in subsequent sections.

Poincaré Mapping

To reduce the complexity of visualizing a large variety of solutions within a chaotic dynamical system, a Poincaré mapping strategy is useful, enabling the identification of various arcs of interest in support of the construction of an initial guess trajectory.⁸ Poincaré mapping first involves the definition of a smooth surface of section, also labeled a hyperplane.¹⁵ Then, the flow in a desired region of the phase space is propagated forward or backwards in time, with each intersection of the hyperplane recorded. These crossings of the hyperplane are then plotted on a lower-dimensional map, thereby enabling clearer visualization of a variety of solutions. Definition of an appropriate surface of section depends predominantly upon the desired application. For instance, planes defined in configuration space have been explored extensively in the CR3BP, and are valuable for gaining physical intuition.¹⁶ In the construction of a trajectory design procedure for WFIRST, a surface of section is defined using an *x*-coordinate in the Sun-Earth rotating frame and lying between the Earth and SE L_2 . Flow that approaches and departs the surface of section with a transverse

crossing is then analyzed to identify arcs of interest. For the purpose of designing low-cost trajectories with impulsive maneuvers, Poincaré maps support the identification of solutions that intersect this hyperplane at nearby positions with a small difference in velocity.

TRAJECTORY DESIGN FOR WFIRST

With application to the WFIRST mission, a trajectory design procedure is developed and implemented as a graphical user interface within MATLAB – incorporated, as an additional module, into the ATD design environment.^{6,7} By leveraging dynamical systems techniques, a low-cost transfer is constructed from a specified set of initial conditions along a LEO to a candidate mission orbit. Given the general itinerary for a transfer to the SE L_2 vicinity, the trajectory is split into four components, used to guide the design process:

- 1. An arc along a LEO, propagated from a given initial state vector and time.
- 2. Following the implementation of a maneuver, a post-LEO arc that terminates at the intersection of the trajectory with a hyperplane located between the Earth and SE L_2 .
- 3. An arc that approaches a selected candidate mission orbit with the introduction of a maneuver.
- 4. Multiple revolutions of the candidate mission orbit.

Arcs in each of these four phases are selected in the CR3BP using dynamical systems techniques and are evaluated for their potential to satisfy the mission constraints. These arcs are then assembled to construct an initial guess trajectory which is corrected to recover a continuous solution. Next, this path is transitioned to the ephemeris model and exported to an operational level modeling environment for subsequent analysis.

Mission Constraints

Prior to designing a trajectory that delivers the WFIRST spacecraft to the operational orbit, the mission constraints must be expressed quantitatively or geometrically. In particular, the constraints that most significantly impact the trajectory design procedure for WFIRST include the following:

- 1. The initial condition along the trajectory is set equal to a state labeled $\bar{x}_{E,IC}$ and is specified by the launch vehicle provider in Earth-fixed coordinates. Constraining the initial condition, specified in Earth-fixed coordinates is straightforward to implement using the SPICE toolkit. In fact, this toolkit, accessible in MATLAB, includes a function that converts an Earth-fixed state to Earth-centered J2000 coordinates at a specified epoch. The output information is then used to fix the initial condition along the trajectory via a straightforward equality constraint, such that $\bar{x}_{E,1} - \bar{x}_{E,IC} = \bar{0}$, where $\bar{x}_{E,1}$ is the state at the first node along the trajectory. When correcting a trajectory in the CR3BP, the initial state is transformed into a Sun-Earth rotating frame. Then, the equality constraint is rewritten entirely in this coordinate system.
- 2. The maneuver to depart LEO, $\Delta \bar{V}_{LEO}$, must occur within 10 to 60 minutes beyond the initial condition. The time at which the $\Delta \bar{V}_{LEO}$ maneuver is applied is explicitly incorporated into the procedure for generating LEO departure arcs.
- 3. The maneuver to depart LEO, $\Delta \bar{V}_{LEO}$, must be directed predominantly within the orbital plane. The requirement on the maneuver direction is introduced as an equality constraint during the ephemeris corrections process. This equality constraint is written as:

$$\left|\sin^{-1}\left(\frac{\Delta \bar{V} \cdot \hat{h}}{|\Delta \bar{V}|}\right)\right| - \alpha_{LEO} + \beta_{LEO}^2 = 0$$

where \hat{h} is the orbit normal evaluated prior to the application of the maneuver, β_{LEO} is a slack variable used to convert an inequality constraint to an equality constraint, and α_{LEO} is the user-specified maximum angular deviation of the $\Delta \bar{V}_{LEO}$ vector out of the spacecraft plane of motion.

4. The mission orbit must possess a SEL2V angle that is less than 36 degrees, while the transfer must deliver a SEL2V angle that is less than 33 degrees. The constraints on the maximum SEL2V angles along both the transfer trajectory and mission orbit are geometrically represented by a cone, impacting the maximum y- and z- excursion of a path in the Sun-Earth rotating frame. In fact, the maximum

angular constraint on the mission orbit is equivalent to a 36 degree angle cone originating at the Earth and centered on the x-axis in the Sun-Earth rotating frame, as depicted in Figure 1. As illustrated, at a location along a trajectory where the x-component of the state is equal to that of L_2 , the norm of the position vector projected into the yz-plane must be less than 1.09×10^6 km. A similar cone is also constructed to represent the maximum SEL2V angle constraint along the transfer trajectory.



Figure 1. Translation of maximum SEL2V angle requirement to a cone constraint, impacting the maximum y- and z- extensions of a path as viewed in the rotating frame.

5. The mission orbit must not cross the Earth or lunar shadow over a period of ten years. To avoid the Earth shadow, the spacecraft must remain outside of both the umbral and penumbral shadow cones. These shadow regions are depicted in Figure 2 with the Sun, Earth and L_2 located along the x-axis in the Sun-Earth rotating frame, the umbral shadow shaded grey and the penumbral shadow colored blue. From trigonometry, the tip of the umbral shadow cone intersects the x-axis on the Earth-side of L_2 . Accordingly, the penumbral shadow cone dominates characterization of the boundaries of the Earth shadow region near L_2 . Trigonometry is also employed to determine the radius of the penumbral shadow cone at L_2 to equal approximately 13,000 km. This Earth shadow avoidance constraint is approximately incorporated into the orbit selection process by ensuring that any candidate orbits in the CR3BP do not pierce a 0.51° cone centered along the Earth to L_2 line, thereby corresponding to a requirement on the minimum SEL2V angle of a candidate mission orbit. Once a transfer that likely avoids the penumbral and umbral shadows of the Earth is identified, the lunar shadow constraint is evaluated in the General Mission Analysis Tool (GMAT) or Systems Tool Kit (STK) due to its precise timing dependency.



Figure 2. Translation of Earth shadow avoidance requirement to a cone angle constraint, with umbral shadow region shaded gray and penumbral shadow colored blue.

6. The magnitude of the orbit insertion maneuver, $\Delta \bar{V}_{L2OI}$, must be less than 15 m/s. This constraint is first introduced implicitly into the trajectory design procedure by leveraging stable manifold arcs as an initial guess for constructing a low-cost transfer that approaches an unstable candidate mission orbit. Then, the maneuver magnitude is incorporated as an equality constraint during the corrections process.

Each of these constraints, represented mathematically, are incorporated into the trajectory design process during the identification of the natural dynamical structures that are used to assemble an initial guess and, then, during corrections. By locating individual arcs that satisfy the mission constraints, the trajectory designer can bias the recovered continuous solution to retain a given geometry while also reducing the required computational effort and time for identification of a feasible path when transitioning to a higher-fidelity model.

Overview of Design Procedure

The application of dynamical systems theory enables the development of a systematic and well-informed trajectory design procedure for the WFIRST mission. This procedure consists of the following steps:

- 1. Families of periodic orbits that exist in the CR3BP near the SE L_2 point are computed. Orbits, discretely sampled along each family, that satisfy the Earth shadow and 36 degree SEL2V angle constraints are retained and saved as candidate mission orbits in a .mat file. Note that quasi-periodic orbits are not included in the candidate orbit set to reduce the complexity of visualization and computation. However, periodic orbits offer a sufficient starting point for recovering quasi-periodic mission orbits during the corrections process.
- 2. Stable manifolds that reach the Earth vicinity in negative time are generated for a user-specified range of candidate mission orbits along a selected L_2 libration point orbit family to support identification of transfers that potentially require a small ΔV_{LEO} maneuver. Trajectories that lie on the manifolds are integrated backwards in time in the Sun-Earth CR3BP until they pierce a hyperplane defined by an *x*-coordinate between the Earth and L_2 . Manifolds arcs that intersect this hyperplane, along with information identifying their generating periodic orbit, are stored.
- 3. Paths that depart the initial LEO are generated. A set of states, sampled to lie along a LEO and that occur between 10 and 60 minutes beyond a pre-specified initial condition, i.e., state vector and epoch, is constructed. Then, a range of tangential $\Delta \bar{V}_{LEO}$ maneuvers, defined by a range of magnitudes, are applied to each state within this set. After application of a $\Delta \bar{V}_{LEO}$, each state is then integrated forward in time in the Sun-Earth CR3BP until piercing a hyperplane defined by an *x*-coordinate between the Earth and L_2 . This hyperplane is defined to be consistent with the previous step. Paths that depart the specified LEO and pierce the hyperplane, along with the maneuver direction and epoch, are stored.
- 4. An initial guess is constructed using Poincaré mapping. An interactive graphical environment enables the selection of a LEO departure path along with multiple revolutions of the mission orbit and an associated stable manifold arc. Arcs along each segment of the trajectory itinerary are assembled and discretized into a series of nodes, thereby forming an initial guess.
- 5. The constructed initial guess is corrected in the Sun-Earth CR3BP to ensure full state continuity, while also incorporating the two maneuvers, $\Delta \bar{V}_{LEO}$ and $\Delta \bar{V}_{L_2OI}$. The initial condition is constrained to equal the pre-specified state as provided in Earth-fixed coordinates, and the epoch is allowed to vary. During this step, the magnitude of the $\Delta \bar{V}_{L_2OI}$ maneuver, as well as the maximum SEL2V angle along the transfer, are gradually reduced until satisfying the mission constraints. Note that the midcourse correction maneuver is not incorporated into the design of the reference trajectory but, rather, reserved for mitigating the impact of any errors in the large $\Delta \bar{V}_{LEO}$ maneuver.
- 6. The entire trajectory is corrected within a point-mass ephemeris model. In this step, the initial state is constrained in Earth-fixed coordinates, while the magnitude of the $\Delta \bar{V}_{L_2OI}$ maneuver and the maximum SEL2V angle along the transfer is gradually reduced. Furthermore, SRP can be incorporated into the dynamical model during the corrections process.
- 7. The corrected trajectory is used to generate similar solutions at various epochs across a user-defined launch window. The resulting trajectories are stored and GMAT scripts are automatically generated.
- 8. A trajectory, designed using dynamical systems techniques and corrected in a point mass ephemeris model, is exported to GMAT and then STK. The GMAT script writes to a text file including the epoch and the state corresponding to waypoints along the reference trajectory. Then, this text file is read into MATLAB and input into STK via MATLAB's Component Object Model (COM) connection. The designed trajectory is recovered via targeting in a higher-fidelity model within STK with the application of regular station-keeping maneuvers. Note that this step is explored in the final section of this paper.

The procedure, as summarized above, is implemented via a graphical user interface within MATLAB as an additional module to Purdue University's ATD trajectory design environment. Each step is accessed in separate windows, with the computed data saved as a .mat file for use in subsequent steps. Using this module, the general procedure is leveraged to design trajectories to either Sun-Earth L_2 orbits or, with some modifications, Sun-Earth L_1 orbits. For the purposes of this paper, however, each of these eight steps is demonstrated in the context of the WFIRST mission.

Step 1: Identify Candidate Periodic Orbits

To identify candidate mission orbits for WFIRST, families of simply-periodic orbits that lie within the vicinity of the SE L_2 point are examined. In particular, orbit families considered within this analysis include SE L_2 Lyapunov orbits, SE L_2 halo orbits, SE L_2 vertical orbits, and SE L_2 axial orbits. Orbits that lie along these families are generated on-demand and viewed within the window displayed in Figure 3. In this window, the left panel offers a guide to each step in the candidate orbit identification process; the central control panel enables the user to select numerical parameters, run the orbit generation procedure and customize the graphical display. Then, the right workspace panel displays candidate and infeasible mission orbits in the top plot within configuration space, and their associated descriptive parameters in the bottom plot.



Figure 3. Graphical user interface implementing, in MATLAB, Step 1 in the WFIRST trajectory design procedure: the identification of candidate mission orbits.

The candidate orbit identification process begins with the definition of the maximum feasible SEL2V angle along a mission orbit, as well as a specification of the parameters describing the computation of orbits along each available family. Each of these parameters is defined in the central control panel. Specifically, the user selects the family along which a discrete set of orbits is computed. An orbit within this family is constructed using a multiple shooting procedure, requiring the user to specify the number of arcs for discretization along the orbit.⁸ Then, a pseudo-arclength continuation method generates a given number of orbits along the family, originating from a precomputed initial orbit stored within the module. To guide this process, the user also specifies the continuation step size, essentially a measure of the distance between two orbits along the specified family. A discrete set of periodic orbits along the family is then sorted into one of two sets: candidate orbits and infeasible orbits. For an orbit to be designated a candidate orbit, it must not pierce a 36° cone representing the maximum SEL2V angle constraint. Furthermore, a candidate orbit must not pierce a 36° cone representing the maximum SEL2V cone constraint. Orbits that satisfy both of these constraints are considered for further investigation.

Once the computation and candidate orbit identification process is completed, each set of orbits is then displayed in both configuration space and in a two-dimensional parameter space. In the top plot of the right workspace panel depicted in Figure 3, the interactivity of the graphical user interface enables the display of either all the computed orbits or a subsampled set. In this plot, candidate orbits are colored blue and infeasible orbits are plotted in red. To ensure clear visualization, the density of orbits from each set that is plotted can be iteratively updated. The plots viewed in configuration space at the top of the workspace panel can also be configured to display additional graphical objects such as: the Earth; the libration points; a cone illustrating the maximum SEL2V angle constraint for mission orbits; and the penumbral and umbral shadow cones. Overlaying these graphical objects on a plot of the candidate and/or infeasible mission orbit sets enables vi-

sual verification that members of the family violate the mission constraints and the specific cause. To further support the analysis of candidate orbits, parameters including the amplitude, orbital period, Jacobi constant, periapsis and apoapsis radii, as well as minimum and maximum SEL2V angles along the family, appear in the bottom right plot. These quantities, useful during exploration of the trade space, are displayed on the horizontal and vertical axes in the bottom of the workspace panel and selected via a dropdown menu. The constructed graphical user interface supports iterative updates to the parameters defining the orbit computation process, as well as the family of interest, thereby enabling an exploration of the mission orbit design space. For each family of interest, candidate orbits are stored in a .mat file for use in subsequent phases of the trajectory design process.

Using this interactive candidate orbit identification process, each of the symmetric and simply periodic orbit families in the vicinity of SE L_2 is examined with application to WFIRST. First, the L_2 Lyapunov orbit family supplies only infeasible orbits. Since each orbit within this family lies solely within the plane of motion of the primaries, as displayed in Figure 4(a), the penumbral shadow region is pierced twice along each revolution. Similarly, the three-dimensional L_2 vertical family, plotted in Figure 4(b), crosses the x-axis in the Sun-Earth rotating frame twice per revolution, intersecting the penumbral shadow region. Accordingly, each member of this family violates the mission orbit constraints. Next, the L_2 axial orbit family that bifurcates from the L_2 Lyapunov family, possesses only members with a maximum SEL2V angle that is greater than 36 degrees, as displayed in Figure 4(c), rendering them infeasible mission orbits. Finally, the L_2 halo family is examined. Plotted in Figure 4(d), this family contributes both candidate mission orbits (blue) and infeasible orbits (red). In particular, a small range of members in this halo family, those that are located close to the planar bifurcation from the L_2 Lyapunov orbits, pierce the penumbral shadow region, thereby violating the mission constraints. Furthermore, orbits with a sufficiently large z-amplitude pierce the maximum SEL2Vangle constraint cone. Between these sets of infeasible orbits, however, is a range of northern and southern L_2 halo orbits, colored blue, that may serve as candidate mission orbits for further analysis with application to the WFIRST mission. Note that, in the vicinity of these periodic orbits, quasi-periodic orbits may also exist. These dynamical structures could offer feasible alternatives to a mission orbit that avoids the Earth shadow region, while maintaining an SEL2V angle below the maximum allowable value of 36 degrees. These dynamical structures exist within a higher dimensional family, presenting significant challenges for straightforward computation and visualization. Such nonperiodic and ordered motion is, however, recovered during corrections from an initial guess comprised of multiple revolutions along a periodic orbit.

Step 2: Generate Candidate Orbit Stable Manifolds

Given an unstable candidate orbit that satisfies the Earth shadow avoidance and maximum SEL2V angle constraints, the associated stable manifold structure is leveraged to identify potentially low-cost transfers from LEO. This step within the trajectory design process is completed via the window displayed in Figure 5. Within the control panel at the center of the window, a set of candidate orbits, computed in the previous step, are accessed. Once loaded, these orbits are plotted in configuration space and viewed in the top plot



Figure 4. Selected feasible (blue) and infeasible (red) periodic orbits in the (a) L_2 Lyapunov family, (b) L_2 vertical family, (c) L_2 axial family and (d) L_2 halo family, overlaid with shadow regions (gray) and maximum SEL2V constraint (orange).

of the workspace panel. Then, this candidate set is iteratively subsampled to define a set of orbits for which the stable manifolds are computed (if the orbits possess stable and unstable modes). The selected orbits are plotted in the workspace panel and iteratively refined until a sufficient quantity and density of candidate orbits is selected. Selecting too few orbits for stable manifold generation reduces the options in a search that exposes arcs to be leveraged for recovery of low-cost transfers to a feasible mission orbit; however, selecting too many orbits increases the computational load for manifold generation, visualization and analysis. Once the candidate orbits are subsampled, the number of nodes for discretizing the periodic orbit during manifold generation must be defined.



Figure 5. Graphical user interface implementing, in MATLAB, Step 2 of the WFIRST trajectory design procedure: the generation of stable manifolds associated with selected candidate mission orbits.

For a discrete set of unstable periodic orbits, trajectories that lie along the stable manifold and reach the Earth vicinity are identified. In fact, to support construction of an initial guess for a low-cost transfer from LEO to a feasible mission orbit, the manifolds are generated until they pierce a hyperplane defined by a nondimensional x-coordinate, x_H , that lies between the Earth and L_2 such that $x_H = 0.7(1 - \mu) + 0.3L_{2,x}$, where $L_{2,x}$ is the nondimensional x-coordinate of L_2 in the Sun-Earth rotating frame. This hyperplane is selected as a consequence of its location between the initial and final orbit. Furthermore, the hyperplane lies sufficiently far from the Earth to mitigate the challenges associated with correcting discontinuities close to one of the singularities in the CR3BP. Following manifold generation, the integration time and state information corresponding to the intersection of each trajectory along the stable manifold of a periodic orbit and the hyperplane are then stored. This process is repeated for each of the selected periodic orbits. Then, the computed manifolds are plotted in configuration space and viewed in the top plot of the workspace panel and on a (y, z) map in the bottom of the workspace panel; the manifold representations are colored by an integer index corresponding to the originating periodic orbit. On the plot depicting the map, the (x, y)-components of velocity at the intersection of the stable manifold with the hyperplane are also displayed via arrows. As an example, the stable manifolds for fifteen selected L_2 northern halo orbits appear in the right workspace panel in Figure 5 and are viewed in configuration space in the top plot and represented on a (y, z) map in the bottom plot. This manifold information reflects the geometry of various arcs that approach an L_2 northern halo orbit and satisfy the mission constraints. The selected orbit set and the density of trajectories along the stable manifold can be iteratively refined until a sufficiently dense set of arcs is available. These stable manifold arcs are then stored in a .mat file for further analysis.

Step 3: Generate LEO Departure Arcs

Initial guess construction is supported by the identification of arcs that depart LEO following a tangential and impulsive maneuver implemented within a constrained time interval following the initial epoch. To

identify these arcs, a pre-defined initial condition is supplied in the form of both the state vector and a range of initial epochs. In this demonstration, an epoch of September 11 2025 is used and is derived from a previous iteration in the design of the WFIRST mission concept. For a single short coast or a long coast state at this epoch, both the position and velocity components must be supplied in an Earth-fixed frame; this information appears at the top of the central control panel of the window displayed in Figure 6. This exact initial condition is leveraged due to the high sensitivity of states that lie near the Earth during the corrections process. For a specified epoch, the initial state is then converted to instantaneous orbital elements for verification. Next, the LEO associated with this initial state is visualized in configuration space within the top plot of the workspace panel. This LEO is discretized to identify states that occur 10 to 60 minutes beyond the specified initial condition. Each of these states is considered a potential location for the LEO departure maneuver, $\Delta \bar{V}_{LEO}$. However, due to the time-dependency of the transformation from an Earth-fixed frame to either an Earthinertial or Sun-Earth rotating frame, integrating this state from two different initial epochs may produce distinctly different trajectories. Accordingly, arcs that depart the specified LEO are generated for a variety of initial epochs within a user-specified range. Furthermore, $\Delta \bar{V}_{LEO}$ is assumed to be tangential to the instantaneous spacecraft velocity which reduces the size and complexity of the set of trajectories that depart the specified LEO. Recall that the only constraint on the maneuver is a direction predominantly within the spacecraft plane of motion. Accordingly, the precise direction of the maneuver is adjusted during corrections. Nevertheless, the user specifies a range of values for the magnitude of ΔV_{LEO} . Then, the LEO departure paths are generated in the CR3BP for each initial epoch and each maneuver magnitude consistent with the following itinerary:

- 1. The initial condition is specified via an Earth-fixed state and an epoch within the specified range.
- 2. The initial condition is integrated for a time interval of between 10 and 60 minutes in the CR3BP.
- 3. A tangential LEO departure maneuver, with a magnitude within the user-specified range, is applied.
- 4. The post-maneuver state, transformed to the rotating frame, is integrated forward in time in the CR3BP until it pierces the same hyperplane as used in Step 2, defined by the x-coordinate, x_H , that lies between the Earth and SE L_2 .

By generating these arcs in the CR3BP, both the computational time and the complexity of the trajectory design space are reduced, while still providing sufficient information to predict the geometry of solutions in a higher-fidelity ephemeris model. Once a set of trajectories that depart the LEO and follow the specified itinerary is generated, this information is visualized on a two-dimensional map in (y, z) coordinates. Each of the crossings on this map is colored by the z-component of velocity, with additional arrows indicating the (x, y)-components of velocity at the intersection with the hyperplane. A sample data set of map crossings corresponding to trajectories that originate from a given Earth-fixed state and depart towards SE L_2 is displayed in the bottom right plot of Figure 6. This data set, representing the intersections of the LEO departure path with the hyperplane, along with the initial epoch, maneuver magnitude and time are stored in a .mat file for use in the construction of an initial guess.



Figure 6. Graphical user interface implementing, in MATLAB, Step 3 of the WFIRST trajectory design procedure: the generation of LEO departure paths.

Step 4: Construct Initial Guess

The candidate mission orbits, associated stable manifold arcs, and the LEO departure paths, all generated previously during the trajectory design procedure, are leveraged to construct an initial guess for the WFIRST trajectory. To support the identification of arcs to assemble an initial guess for a potentially low-cost transfer between LEO and a candidate mission orbit in the SE L_2 vicinity, Poincaré mapping is employed. As an example, consider the map displayed in Figure 7(a) which enables simultaneous visualization of the flow departing the constrained LEO after a tangential maneuver (blue) as well as the stable manifold structures approaching candidate L_2 northern halo orbits (red). The crossing of each trajectory with the hyperplane from both Step 2 and Step 3 is displayed on the map via a filled circle located by the y- and z- configuration space coordinates and colored by the z-component of velocity. Then, the x- and y- components of velocity are represented in the horizontal and vertical dimensions, respectively, corresponding to the colored arrows attached to each map crossing. Map crossings representing the stable manifolds corresponding to candidate periodic orbits are indicated with red arrows, while the crossings of the LEO departure paths are displayed with blue arrows. Poincaré mapping enables visualization of the flow structures that depart the LEO and those that approach candidate periodic orbits, thereby supporting identification of trajectories in each data set that possess close crossings of the hyperplane. These nearby arcs are leveraged to construct the initial guess for the WFIRST transfer path. However, analysis apparent in Figure 7(a) reveals that simply overlaying the hyperplane crossings of the data generated in Step 2 and Step 3 produces a map that is difficult to analyze. To overcome these challenges, interactive filtering is employed to reduce the data set and to focus only on potentially close crossings on the map. The first step in the data filtering process involves automated elimination of crossings corresponding to LEO departure paths (indicated with blue arrows) that do not lie within a box encompassing the crossings of trajectories on the stable manifold and defined in both position and velocity, to within a small tolerance. The reduced data set is plotted in Figure 7(b), illustrating a clearer view of the candidate flow structures that can potentially be incorporated into the construction of an initial guess. Additional filtering steps are later employed to further reduce the set of candidate arcs for efficient assembly of an initial guess.



Figure 7. Sample map representation of LEO departure paths (blue arrows) and stable manifolds of candidate SE L_2 northern halo orbits (red arrows) in (y, z) position components in the Sun-Earth rotating frame (a) prior to filtering and (b) after the first step of filtering. Arrows indicate x- and y- components of velocity and points are colored by the z-component of velocity.

Interactive filtering and examination of the relevant flow structures for constructing an initial guess is achieved via a graphical user interface. In particular, both loading and filtering of the map crossing data is supported in the top panel of the window displayed in Figure 8. Then, the (y, z) map from Step 4 of the trajectory design process is displayed in the bottom left panel of the window in Figure 8. An additional filtering option, accessible in the middle of the top panel, further simplifies the visualization. Specifically, filtering is applied to crossings in each of the two data sets, i.e., the crossings corresponding to stable manifolds of candidate periodic orbits and the crossings associated with LEO departure paths. A crossing from one of the data sets is discarded from the map if it does not possess a position or velocity within a user-specified threshold of any crossings in the other data set. Such reduction in the data sets produces a map resembling the bottom left

plot in Figure 8 for a LEO to L_2 northern halo orbit transfer. Next, interactive selection commences on a pair of blue and red arrows that lie close in the (y, z) map and possess arrows of similar length and direction; the result is a selection of arcs to construct an initial guess for a potentially low-cost transfer. With the selection of one blue and one red arrow from the map, an arc that departs the LEO and an arc that approaches a candidate mission orbit, respectively, are identified and plotted in the three-dimensional configuration space representation in the bottom right of the window in Figure 8 using a consistent color scheme. This transfer is analyzed further in three dimensions using MATLAB's plot rotation and zoom functions. Additional graphical objects such as the shadow or SEL2V cones are added to this plot to support visualization of the mission constraints and comparison to the constructed trajectory. With this graphical interface, interactive selection of the map crossings enables exploration of the constrained design space and the assembly of arcs with the potential to lie close to feasible low-cost transfers. As an example, two map crossings are selected in the bottom left plot of Figure 8 and indicated by black asterisks. These map crossings are located nearby in configuration space, while also possessing a similar velocity due to the close alignment and similar length of their arrows. The corresponding transfer to an L_2 northern halo orbit is displayed in the bottom right plot, exhibiting only a small discontinuity at the crossing of the hyperplane, indicating the potential for a small maneuver, ΔV_{L_2OI} , to enable recovery of a nearby, continuous solution. Multiple revolutions of the candidate mission orbit, associated with the selected stable manifold arc are then concatenated to the end of the designed transfer and plotted in the bottom right plot in purple. For the WFIRST application, one mission constraint is that the spacecraft must not cross an Earth or lunar shadow for at least 10 years; thus, 20 revolutions of the candidate halo orbit, with a period of approximately 180 days, are added to the initial guess. Following construction of a desired mission trajectory, each arc is discretized, using the the right side of the top panel in Figure 8, into a series of nodes for use in a multiple shooting corrections algorithm. Once the constructed initial guess is discretized, the state information corresponding to each node, as well as the initial epoch, are stored as a .mat file for use in subsequent steps of the design process.



Figure 8. Graphical user interface implementing, in MATLAB, Step 4 of the WFIRST trajectory design procedure: constructing an initial guess via Poincaré mapping.

Step 5: Correct Trajectory in CR3BP

The constructed initial guess is updated via a multiple shooting corrections algorithm to recover a twomaneuver continuous solution that retains the geometry of the original design, while also satisfying the relevant mission constraints. The window displayed in Figure 9 illustrates the implementation of this corrections procedure. While the left panel offers a guide to the corrections procedure, the central panel enables user control and a summary of information describing the recovered solutions. Then, the right panel displays a configuration space representation of the recovered solution, with the option to add graphical objects representing influential mission constraints. An illustration of this methodology is achieved using the initial guess constructed in the previous step of the trajectory design process, linking a state in LEO to a candidate L_2 northern halo orbit with application to the WFIRST mission.



Figure 9. Graphical user interface implementing, in MATLAB, Step 5 of the WFIRST trajectory design procedure: corrections within the CR3BP.

Once the initial guess is loaded, a plot appears in the right panel in configuration space. Then, two maneuvers, located at the departure from LEO and the first crossing of the mission orbit with the (x, z) plane, are activated via checkboxes. To add a constraint on the maximum SEL2V angle along the transfer arc, limited to a maximum value of 33 degrees, the node closest to the maximum excursion of the transfer in the y and z position variables is identified. Once this node is selected, initial corrections recover a solution that is continuous in the CR3BP with the two impulsive maneuvers, ΔV_{LEO} and ΔV_{L_2OI} . During the corrections step, the direction of the first maneuver, $\Delta \bar{V}_{LEO}$, is constrained via an inequality constraint to be aligned within the orbital plane of the LEO to within a 0.1 degree tolerance, satisfying the requirement that the maneuver be directed predominantly within the plane of motion of the spacecraft. After the corrections process has converged on a continuous solution, to within an acceptable tolerance, the magnitude of the maneuvers, as well as the maximum SEL2V angle along the transfer are displayed in the middle of the central panel in Figure 9. If the maneuver magnitudes and/or the maximum SEL2V angle exceed the limits specified by the mission constraints, these parameters are gradually reduced via successive iterations of the corrections algorithm, augmented by additional equality constraints. Once a feasible continuous solution is recovered, it is plotted in configuration space in the right panel of the window in Figure 9, as displayed for the example of an L_2 northern halo mission orbit. For this single trajectory, successive reduction of the orbit insertion maneuver magnitude, ΔV_{L_2OI} , to a value of 9.38 m/s results in the revolutions of the periodic orbit appearing to separate, resembling a quasi-periodic orbit near the original candidate halo orbit over a ten-year time interval. In fact, the boundedness of quasi-periodic trajectories that satisfy the maximum SEL2V angle constraint offer an alternative mission orbit. However, due to the additional complexity of computation and visualization of two-dimensional families of tori, these orbits are not directly incorporated into the candidate mission orbit identification step. Rather, such solutions are straightforwardly recovered from multiple revolutions of a nearby periodic orbit, simplifying the trajectory design process and reducing the computational load. Incorporating this solution as a reference, the shadow and transfer cone constraints, as well as the maneuver locations, are easily visualized and examined using MATLAB's rotation and zoom functions. With an acceptable trajectory, the continuous solution is stored in a .mat file for subsequent updates.

Step 6: Correct Trajectory in Ephemeris with Solar Radiation Pressure

The next step in the trajectory design process involves correction of the current solution in a point mass ephemeris model that also captures the perturbation due to solar radiation pressure, while enforcing the mission constraints. This procedure is implemented via the graphical user interface displayed in Figure 10. With a similar layout to the windows employed for previous steps in the trajectory design process, the central panel is used to control the corrections algorithm. Once the solution produced in Step 5, one that is continuous in the CR3BP, is loaded into this new window, each node is transformed from a state representation in the Sun-Earth rotating frame to an Earth-centered J2000 inertial frame using the initial epoch and subsequent times



Figure 10. Graphical user interface implementing, in MATLAB, Step 6 of the WFIRST trajectory design procedure: corrections of an end-to-end trajectory in a point mass ephemeris model including the Sun, Earth and Moon, plus the SRP per-turbation.

between nodes. This conversion is implemented using the SPICE toolkit to produce a state description for each node in nondimensional position and velocity coordinates. Once each node is described using inertial state coordinates, integration forward in time occurs in a point mass ephemeris model including the Sun, Earth and Moon. Then, the discontinuous initial guess is plotted in blue in the Sun-Earth rotating frame within the right panel of the window in Figure 10. The locations of the maneuvers, ΔV_{LEO} and ΔV_{LOOI} , are specified along with the node located at the maximum SEL2V angle along the transfer. Once these nodes are selected, additional properties of the corrections process are specified, including the presence of solar radiation pressure. Then, the constraints on the maximum SEL2V angle and the component of the ΔV_{LEO} maneuver normal to the orbital plane of the specified LEO are activated. With these additional properties selected, the corrections procedure is initiated to produce a solution that is continuous in a point mass ephemeris model of the Sun, Earth and Moon with, potentially, the influence of solar radiation pressure, a significant perturbing force acting on the WFIRST spacecraft. This corrected solution is plotted in black in the Sun-Earth rotating frame in the right panel of the window displayed in Figure 10. A summary of the maneuver magnitudes and maximum SEL2V angle along the transfer are displayed in the bottom half of the central panel in Figure 10. Following inspection of these parameters, the maneuver magnitudes and maximum SEL2V angle are gradually reduced, via the control panel, to a user-specified value within the upper bounds enforced by the mission constraints. At each step during this interactive corrections process, the current solution is analyzed and compared to a graphical representation of the shadow and SEL2V angle constraints using the plot customization features at the bottom of the central panel as well as MATLAB's plot rotation and zoom functions. A sample continuous trajectory for WFIRST is plotted in black in the Sun-Earth rotating frame in Figure 10. This particular transfer, one that approximately retains the designed geometry, requires a $\Delta \bar{V}_{LEO}$ magnitude equal to 3.24 km/s and a $\Delta \bar{V}_{L_2OI}$ maneuver magnitude of 9.35 m/s, while maintaining a maximum SEL2V angle along the transfer equal to 30.9 degrees. Once an acceptable trajectory is recovered, the solution is stored for a subsequent launch window analysis.

Step 7: Compute Similar Solutions Across Launch Window

Given a feasible trajectory, continuation is employed to identify trajectories with a similar geometry at various epochs across a user-defined launch window. The graphical user interface for this step of the trajectory design process is displayed in Figure 11. Consistent with the windows constructed for the previous steps, the central panel is used to control the launch window analysis while the right panel displays the trajectories in the Sun-Earth rotating frame. To begin the recovery of similar solutions across the launch window, the corrected trajectory computed in Step 6 is loaded and plotted in the right workspace panel, with the initial epoch corresponding to the solution also displayed for reference. Next, maneuver locations as well as limitations



Figure 11. Graphical user interface implementing, in MATLAB, Step 7 of the WFIRST trajectory design procedure: identifying trajectories with a similar geometry across a launch window.

on the direction of the first maneuver, $\Delta \bar{V}_{LEO}$, are defined. Prior to any corrections, the incorporation of SRP into the dynamical model is also selected along with any constraints on the orbit insertion maneuver and maximum SEL2V angle. Once the launch window is defined, solutions resembling the previously computed trajectory are also identified at user-specified increments across the selected launch window. At each epoch within this window, the corresponding trajectory is plotted in the Sun-Earth rotating frame and the maneuver summary is updated. The computed solutions are then saved to .mat files as well as GMAT scripts for export. To demonstrate the capability for export to higher-fidelity modeling software, consider the reference trajectory that connects a LEO to an L_2 northern quasi-halo orbit, appearing in black in Figure 11. This trajectory is input and subsequently analyzed in GMAT via the script generated upon completion of the launch window analysis. The resulting transfer is plotted in a Sun-Earth rotating frame in Figure 12. Such capability for direct export to operational-level modeling software is also useful for higher-fidelity analysis in STK through MATLAB's COM interface.



Figure 12. End-to-end trajectory exported to GMAT and propagated in an ephemeris model including the Sun, Earth and Moon and also reflecting the influence of solar radiation pressure. This transfer is displayed in a pulsating Sun-Earth rotating frame.

PRELIMINARY STATION-KEEPING ANALYSIS

Although the reference trajectory for WFIRST is designed in a point mass ephemeris model that also captures an approximate measure of the solar radiation pressure, additional perturbations due to higher-order gravitational contributions from the Earth, the true nonspherical shape and time-dependent attitude of the spacecraft, as well as regular momentum unloads may impact the actual motion of the spacecraft. In fact, the WFIRST spacecraft is currently designed to require regular momentum unloads that impart an equivalent $\Delta \bar{V}_{MU}$ on the order of millimeters per second, occurring several hours to days apart. The exact direction of each momentum unload is not known a priori, regularly introducing additional uncertainty into the motion of the WFIRST spacecraft beyond those due to insertion errors, imperfect thruster performance, and navigation errors. In the chaotic dynamical regime in the SE L_2 vicinity, such uncertainties can cause a significant deviation in the path of a spacecraft over time. Accordingly, recovery of a path for WFIRST that approximately follows the desired trajectory over the entire mission lifetime in a higher-fidelity model requires the regular application of station-keeping maneuvers; for this mission, the time interval between station-keeping maneuvers is currently designed to equal 21 days. However, due to the chaotic nature of the motion in the vicinity of SE L_2 , computation of the maneuvers is a non-trivial task. Accordingly, a station-keeping strategy that leverages knowledge of the underlying dynamics in multi-body regimes is employed for preliminary analysis of the required maneuvers.

To compute station-keeping maneuvers in a higher-fidelity ephemeris modeling environment, a strategy presented by Pavlak and Howell that leverages dynamical systems techniques via a mode analysis is employed.⁵ This methodology originates with the computation of the STM associated with each station-keeping maneuver location for a time that is approximately equal to one revolution along the mission orbit, i.e., the monodromy matrix. In a higher-fidelity ephemeris model, each element in the STM is determined using firstorder finite differencing.¹⁷ Once the complete STM is computed for approximately one revolution, stable and unstable modes are identified. Note that real eigenvalues from the STM with a magnitude less than unity correspond to stable modes. Previous analysis for the Acceleration Reconnection Turbulence and Electrodynamics of Moon's Interaction with the Sun (ARTEMIS) mission by Pavlak has demonstrated that, given an STM propagated for one revolution, the associated stable eigenvector is closely aligned with an optimal solution for a station-keeping maneuver.¹⁷ Note that this mode analysis technique, originally derived from periodic orbit stability analysis, is exploited in this example for the approximate computation of the stable mode along a quasi-periodic reference trajectory. Such insight, gained via dynamical systems techniques, enables a computationally-efficient determination of a reasonably good initial guess for the direction of a stationkeeping maneuver for the WFIRST spacecraft during preliminary analysis, while supporting a reduction in the parameter space during a constrained optimization procedure.

To implement a stable mode strategy for computing station-keeping maneuvers in STK's operational-level modeling environment, MATLAB's COM interface is exploited. This computational structure enables access to MATLAB's complex mathematical operations and control of iterative procedures, while leveraging STK's object-oriented programming interface. To frame the station-keeping analysis procedure, a single trajectory, designed in the ATD module developed in this investigation, is discretized into a sequence of waypoints. These points of reference, each stored as a state vector and an epoch, include the initial condition, the space-craft state following application of the LEO departure maneuver, and each subsequent crossing of the xz-plane as defined in the Sun-Earth rotating frame over a ten-year time interval. This information, along with the direction and magnitude of the ΔV_{LEO} and ΔV_{L_2OI} , is stored in a data file that is accessed by a script written in MATLAB. The script, which first opens up a COM channel with STK, creates an STK scenario via definition and parameterization of the three-dimensional visualization, coordinate frames, dynamical models, satellite objects, mission control sequence in Astrogator, maneuvers, and differential corrections procedures.

Segmentation of the STK simulation into multiple spacecraft supports computationally efficient customization and computation within the STK scenario. In fact, to implement the mode analysis employed for stationkeeping maneuvers, three satellites are introduced:

- Spacecraft 1 to represent the WFIRST spacecraft with maneuvers and propagation segments iteratively added to the corresponding mission control sequence,
- Spacecraft 2 for computation of the STM in the higher-fidelity dynamical model, and
- Spacecraft 3 for calculation of the station-keeping maneuver via STK's design explorer optimizer.

First, $\Delta \bar{V}_{LEO}$ and $\Delta \bar{V}_{L_2OI}$ are recalculated and applied to the main WFIRST spacecraft in a higher-fidelity dynamical environment that models the Earth gravity with degree and order equal to eight, along with the point mass contributions of the Sun and Moon, and SRP via a spherical model of the spacecraft. Once the WFIRST spacecraft has reached the first crossing of the xz-plane in the Sun-Earth rotating frame near the mission orbit and the $\Delta \bar{V}_{L_2OI}$ maneuver is applied, the momentum unload cycle is implemented. This cycle is assumed to consist of four pairs of 8.7 mm/s momentum unloads applied in a random direction followed by a coast segment for 130 hours; each cycle is appended to the mission control sequence for the WFIRST spacecraft. After each momentum unload cycle is complete, a station-keeping maneuver may be applied. To compute this maneuver, the initial conditions for Spacecraft 2 and Spacecraft 3 are updated to possess the same state and epoch as the WFIRST spacecraft, i.e., Spacecraft 1. Then, Spacecraft 2 is propagated for approximately one revolution of the mission orbit. For the SE L_2 quasi-halo recovered during trajectory design within the ATD module, the time for one revolution around the orbit is calculated as the average between each positive intersection of the reference trajectory with the xz-plane in the Sun-Earth rotating frame, equal to 180.25 days. Then, small perturbations are independently applied to the initial state in each of the six position and velocity components and propagated. The corresponding perturbed trajectories are then used to construct the STM via a first-order forward finite differencing procedure. While propagation of the motion of a spacecraft and the frame conversion is completed in STK, MATLAB is used to construct an STM and compute the associated eigenvalues and eigenvectors, followed by identification of the stable mode. Once this information is recovered, and the unit vector corresponding to the stable mode is normalized, the (x, y, z) components of the eigenvector are imported as an initial guess for the direction of the station-keeping maneuver. The x-coordinate of the spacecraft's natural path at its first intersection with the xz-plane, relative to that of the reference trajectory, is employed in a straightforward test to determine whether the maneuver should be parallel or anti-parallel to the eigenvector computed in MATLAB. Then, the magnitude of this initial guess for the maneuver is estimated to be the same order of magnitude as the momentum unloads. This station-keeping maneuver is introduced as an initial guess for Spacecraft 3. Optimization, available within STK via the design explorer optimizer, is then applied to recover a station-keeping maneuver that alters the motion of Spacecraft 3 to intersect the xz-plane at its next crossing to within a 100 km tolerance of the corresponding crossing of the reference trajectory. This optimization scheme minimizes the magnitude of the station-keeping maneuver, while limiting each of the azimuth and elevation angles defining the direction of the maneuver to within 5 degrees of the corresponding angles for the computed stable eigenvector, enabling an informed process for constraining the parameter space and reducing the computational time. Once a solution is available, the station-keeping maneuver is appended to the mission control sequence for the WFIRST spacecraft, i.e., Spacecraft 1. Then, the momentum unload cycle and station-keeping analysis is repeated for the remainder of the ten-year time interval.

Using the process developed for this analysis, a preliminary station-keeping analysis is completed for the reference trajectory previously constructed for WFIRST, recovering a sequence of low-cost maneuvers over a ten-year time interval. The magnitudes of the computed station-keeping maneuvers are displayed in Figure 13(a) as a function of an index identifying the maneuver number. Then, the corresponding direction of each maneuver is displayed in Figure 13(b) as a scaled black arrow with a basepoint at the location of the applied maneuver, as viewed in the Sun-Earth rotating frame. Overlaid on this figure are blue and green arrows that indicate the stable eigenvector direction in the $+\hat{x}$ and $-\hat{x}$ directions, respectively. In general, the station-keeping strategy employed in this investigation recovers locally optimal maneuvers with a direction



Figure 13. (a) Magnitude of each station-keeping maneuver following insertion and (b) comparison of the direction of each station-keeping maneuver (black) to the stable eigenvectors directed in the $+\hat{x}$ -direction (blue) and $-\hat{x}$ -direction (green).

that is closely aligned with the stable eigenvectors of the STM propagated over one revolution, reducing the computational effort required for preliminary analysis. These maneuvers produce a trajectory that remains within 100 km of the reference path at each of the xz-plane crossings. In general, the majority of the computed maneuvers are aligned with the stable eigenvector direction to within one degree; however, approximately 11% of the maneuvers possess directions that are close to the upper bound on the angular deviation from the stable eigenvector direction imposed during the corrections process. Such outliers are potentially due to the inaccuracy of a numerically-calculated STM over approximately one revolution along a quasi-periodic trajectory in precisely recovering the stable mode, or the presence of multiple local minima in the constrained optimization problem. Nevertheless, the station-keeping strategy does recover a trajectory for the WFIRST spacecraft that remains close to the designed reference solution and bounded over a ten-year time interval, with individual maneuvers that are on the order of 1 cm/s. Accordingly, the total station-keeping maneuver cost over the entire mission lifetime is approximately 2.27 m/s, which is feasible to accommodate in the total ΔV budget for the mission. The trajectory associated with the computed station-keeping maneuvers is plotted in Figure 14 in a pulsating Earth-centered Sun-Earth rotating frame (a) from above the xy-plane and (b) looking down the x-axis towards the Sun. In each plot, the green trajectory represents the path between application of the two maneuvers, $\Delta \bar{V}_{LEO}$ and $\Delta \bar{V}_{L_2OI}$. Then, each of the alternating magenta and yellow arcs corresponds to a single momentum unload cycle connected by station-keeping maneuvers, while the white arrow indicates the direction of motion. In addition to the station-keeping maneuvers, this trajectory, corrected in a high-fidelity ephemeris model within STK, requires an orbit insertion maneuver, $|\Delta V_{L_2OI}|$, of 9.38 m/s. Furthermore, the eclipse report function in STK verifies that both Earth and lunar shadows are avoided over the entire mission lifetime. Accordingly, the sample trajectory displayed in Figure 14 retains the designed characteristics, while also satisfying the WFIRST mission constraints. Thus, the use of dynamical systems techniques, combined with early and systematic incorporation of the mission constraints, via the graphical user interface, enables the rapid and well-informed design of a trajectory for the WFIRST mission. Using this trajectory as a reference, additional insight from dynamical systems theory supports a preliminary station-keeping analysis within a more representative operational-level modeling environment.



Figure 14. End-to-end trajectory recovered in STK following preliminary stationkeeping analysis viewed (a) down on the xy-plane of the Sun-Earth rotating frame and (b) down the Earth- L_2 line towards the Earth.

CONCLUDING REMARKS

For the upcoming WFIRST mission, several mission constraints pose key challenges for trajectory designers. When these constraints are incorporated towards the end of the design process and within a higher-fidelity ephemeris model, identification of a feasible transfer is cumbersome. However, by employing dynamical systems techniques along with early incorporation of the mission constraints, a well-informed trajectory design procedure is constructed and implemented as an additional module to Purdue University's ATD tool. In this approach, the natural underlying dynamical structures that exist within an approximate and autonomous CR3BP model of the Sun-Earth are analyzed. Candidate arcs that may satisfy the mission constraints, while also recreating the general itinerary of a transfer between LEO and a mission orbit near SE L_2 , are then identified and assembled to construct an initial guess. This initial guess is corrected first in the CR3BP and then in

a point mass ephemeris model that captures the perturbative effect of SRP. During this corrections procedure, mission constraints on the geometry of the transfer and associated maneuvers are enforced. The resulting feasible transfer is then exported to GMAT and/or STK for subsequent analysis. Using MATLAB and STK, in combination with further use of dynamical systems techniques, a preliminary station-keeping analysis within a high-fidelity ephemeris model that also incorporates regular momentum unloads is performed. This procedure, demonstrated for the WFIRST mission, enables rapid and well-informed recovery of feasible transfers within the chaotic dynamical regime of the Earth, Sun and Moon.

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