# AN INTERACTIVE TRAJECTORY DESIGN ENVIRONMENT LEVERAGING DYNAMICAL STRUCTURES IN MULTI-BODY REGIMES

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# ABSTRACT

Trajectories that exist within a multi-body dynamical environment offer low-cost options to develop a long-term human presence within the solar system. Representing the dynamical accessibility of regions in a multi-body regime, as well as enabling access to specific solutions, is nontrivial for dynamically sensitive environments such as the Earth-Moon and Sun-Earth systems. The concept of an interactive design enviroment is explored and analyzed as a framework to enable rapid and well-informed construction of complex trajectories that leverage natural arcs. The design environment is practically implemented in the form of a graphical user interface that consists of several modules that offer guidance into the active selection of known dynamical structures. The capabilities for the prototype trajectory design suite are demonstrated via an application to a preliminary design concept, i.e., preliminary trajectory selection and construction within a multibody regime for a mission to a variety of Earth-Moon and Sun-Earth libration point orbits.

*Index Terms*— Multi-body systems, Three-body problem, Trajectory design, Libration points, Periodic solutions

#### 1. INTRODUCTION

For rapid trajectory design in a multi-body regime, knowledge of the dynamical structures in a simplified model may facilitate a better understanding of the design space than a set of point solutions in the complete ephemeris model. Many software packages, such as Systems Tool Kit (STK) and NASA's General Mission Analysis Tool (GMAT), offer a graphical environment for trajectory design incorporating various gravitational fields at various levels of fidelity [1, 2]. However, the focus is generally directed towards the delivery of trajectory point designs and other operational mission support capabilities. Thus, they may not be specifically structured to offer guidance and insight into the available dynamical structures throughout the region. To supply a framework for incorporating knowledge of the sensitive dynamics in the Earth-Moon and Sun-Earth systems, Purdue University and NASA Goddard Space Flight Center are collaborating to develop the interactive trajectory design environment, Adaptive Trajectory Design (ATD), to exploit natural dynamical structures from the Circular Restricted Three-Body Problem (CR3BP) with the capability to add fidelity during the design process. In the simplified model, periodic and quasi-periodic orbits, as well as any associated manifolds, govern the underlying dynamics and are approximately retained in higher-fidelity models. The current effort is devoted to creating direct links between the problem understanding and its practical application by exploiting these structures within the Earth-Moon and Sun-Earth systems. In particular, there exist a wide array of known orbits with significant potential for parking, staging and transfers within both systems. Previously developed software tools, such as AUTO, can also supply a selection of these solutions as well as some insight into the local dynamics and the evolution of a set of orbits along any family [3]. In fact, AUTO enables the computation of periodic orbits and their numerical continuation into orbit families, as well as the detection and analysis of bifurcations. However, such tools do not offer a basic "blueprint" to support rapid, efficient and well-informed decisions regarding the use of fundamental solutions to construct end-to-end trajectories within multi-body dynamical environments for various mission scenarios.

To overcome the challenges associated with identifying candidate trajectories in a chaotic multi-body regime, available dynamical structures can be actively incorporated into the trajectory construction process. This process is demonstrated via the ATD design suite, which offers an interactive environment to assemble trajectories via point-and-click arc selection for exploration of mission design options within multi-body systems [4]. First, the fundamental dynamics in the Earth-Moon and Sun-Earth systems are approximated using the CR3BP. Dynamical structures in the form of periodic and quasi-periodic orbits, as well as their associated manifolds, may be computed on-demand to construct an initial guess for an end-to-end trajectory, including impulsive maneuvers. This initial guess can then be corrected both in the CR3BP and in an ephemeris model, and even exported to GMAT [2]. As a supplement to ATD, a 'dynamic' catalog has also been created to identify and characterize periodic and quasi-periodic orbits that may aid in trajectory design and selection within the Earth-Moon system [5]. This information is compiled into a graphical environment, allowing the user to directly interact with data that cannot be adequately represented by a static database. As a result, a dynamic and interactive catalog may overcome some of the challenges associated with constructing a predefined trade space to analyze a large set of solutions for a general mission concept [6]. Together, these capabilities enable guided exploration of the solution space for trajectory design within chaotic multi-body systems and, potentially, the identification of innovative orbital options. The various modules available within ATD are demonstrated via a sample mission application.

### 2. EXAMPLE MISSION

To demonstrate the capabilities of ATD, consider a potential formation of spacecraft located near the Earth-Moon  $L_2$  libration point, labeled a Multi-Purposed Lunar Fractionated System (MPLFS). The vehicles that constitute the MPLFS could conduct operations in cislunar and/or interplanetary space: Fueling depots, multiple on-orbit storage vehicles, as well as a communications relay are possible services provided by a MPLFS architecture. Furthermore, stationing the MPLFS near the  $L_2$  gateway could be strategic for either rendezvous with the formation or commissioning any of the individual modules to their final destination. Depending upon the mission scenario, it may be necessary to establish continuous communications with Earth or, equivalently, to include a geometrical constraint on the path such that the MPLFS should maintain a constant line-of-sight to the Earth. Additionally, a vehicle departing the Earth vicinity may leverage a lowenergy transfer that passes through the  $L_1$  gateway to rendezvous with the MPLFS. To enable such links, the energy level for the MPLFS orbit may be loosely constrained. Specifically, the Jacobi constant values (which is analogous to the trajectory energy level within the CR3BP) for the MPLFS orbit and orbits near the  $L_1$  libration point should be similar. Since the MPLFS is comprised of multiple vehicles, bounded motion is desirable to retain a formation. Orbit configurations that meet the mission requirements and enable formation flying may be constructed by leveraging periodic and quasiperiodic dynamical structures that are rapidly selected within the ATD design suite.



**Fig. 1**. Comparison for the range of Jacobi constant values for families of periodic orbits near  $L_1$  and  $L_2$ 

# 3. SELECTING A MISSION ORBIT

The challenges associated with directly exploring a generic mission concept in a high-fidelity ephemeris model may be offset by an interactive design environment that grants access to known dynamical structures, thereby revealing a variety of orbital options. To aid in the selection of natural structures, a dynamic catalog of known solutions is included as a module within the design environment. This catalog facilitates rapid and guided identification of periodic and quasi-periodic orbits that may be leveraged by a baseline trajectory for a given mission concept. Within the Catalog module, user-defined trade spaces are available to guide decisions during the selection process. For example, statistical representations of quantities of interest that reflect desired orbit characteristics may be compared between various families of orbits. For an application to the MLPFS mission concept, for example, the range of Jacobi constant values for families of periodic orbits near the  $L_1$  and  $L_2$  libration points within the Earth-Moon system, including halo, Lyapunov, and vertical orbits, may be explored and compared. A simple bar plot from the catalog module, depicted in Figure 1, reveals that the values of Jacobi constant ranges along the families generally overlap, indicating the potential existence of low-cost transfers between the  $L_1$  and  $L_2$  regions. Thus, several families, as identified within the catalog, may satisfy the loose constraint on the Jacobi constant value for the MPLFS. The user may also select alternate statistical representations and quantities, such as the orbital period or representative station-keeping and transfer costs, to obtain a preliminary comparison between a large set of orbits.

In addition to exploring a large set of trajectories via a statistical description, the evolution of selected quantities along a family of orbits can be examined. Within the catalog, a family of orbits may be visualized in a two-dimensional trade space. Information on a third parameter may be displayed by color. Using this representation, Figure 2 reflects the dimension of the center subspace of candidate families of periodic orbits



**Fig. 2**. Number of nontrivial modes associated to a center subspace across sample families in the CR3BP denoted by color (Blue = 0, Green = 1, Red = 2)

that may be employed for the MPLFS mission concept. The dimension of the center subspace supplies a preliminary indication of the existence of nearby manifold or quasi-periodic structures. Within the MPLFS scenario, stable and unstable manifolds may be employed to construct low-cost transfers to other destinations. Additionally, a quasi-periodic torus may serve as dynamical support framework for the deployment of a formation of servicing vehicles. As displayed within Figure 2, a user-defined trade space within the catalog reveals the existence of tori near the  $L_2$  Lyapunov, vertical, and halo families. A quasi-periodic selection tool is also available within the catalog. Accordingly, each candidate family may be analyzed individually to identify any solutions that satisfy the line-of-sight constraint, and thus have the potential for constant communications with Earth. To satisfy this constraint, the cone angle from Earth relative to the x-axis in the rotating frame must be larger than 0.25° to avoid occultation behind the Moon. An interactive interface, as depicted in Figure 3, enables the user to rapidly verify this simple mission requirement on several quasi-periodic structures that exist within the CR3BP, and may be approximately retained within a higherfidelity model. As detailed in [7], quasi-periodic halo orbits may enable a formation of vehicles to maintain a constant line-of-sight with the Earth. Accordingly, a quasi-periodic  $L_2$ halo orbit with a Jacobi constant equal to  $C \approx 3.14$  is selected to facilitate the demonstration of the ATD software tool.

# 4. EXPLOITING INVARIANT MANIFOLD ARCS

Within ATD, the user can design and construct transfers that actively leverage natural manifold structures from the CR3BP. To demonstrate this process, consider a scenario that requires a vehicle from the MPLFS to transfer from the selected Earth-Moon  $L_2$  quasi-halo orbit to the Earth-Moon



Fig. 3. Analysis and selection of quasi- periodic arcs



**Fig. 4.** A selection of unstable manifold arcs (magenta) departing an Earth-Moon  $L_2$  halo and stable manifold arcs (green) approaching an  $L_1$  halo orbit, viewed in the Earth-Moon rotating frame

 $L_1$  vicinity; such an example could include servicing a malfunctioning spacecraft or delivering resources to a depot. To construct a low-cost transfer from the selected  $L_2$  holding orbit to the  $L_1$  vicinity, invariant manifold structures are employed. Although manifolds may be computed directly from the quasi-halo, they can be challenging and computationally intensive to generate. As a simpler approach, manifold arcs associated with a nearby  $L_2$  halo orbit at the same Jacobi constant value as the quasi-halo are employed. These arcs can then be corrected to link directly to the quasi-periodic structure. A selection of unstable manifold arcs departing the  $L_2$  halo are depicted in magenta in Figure 4. An  $L_1$  halo orbit with the same Jacobi value as the reference quasi-halo is selected as the destination orbit; stable manifold arcs that approach this orbit appear in green in Figure 4.

A transfer between the  $L_2$  and  $L_1$  regions is facilitated by locating an intersection between an unstable and stable manifold arc. Once these arcs are selected, each can be trimmed, or "clipped", within the design environment to minimize the dis-



**Fig. 5**. An end-to-end, initial transfer design between an Earth-Moon  $L_2$  quasi-halo structure (gold) and an  $L_1$  halo orbit (magenta), represented in the Earth-Moon rotating frame

tance between the end points. The transition from the quasihalo structure to the  $L_2$  halo manifold is enabled by clipping the quasi halo such that its end-point approximately coincides with the beginning of the unstable manifold arc. Similarly, an arc is constructed to link the stable manifold arc to the initial state along the  $L_1$  halo orbit. An initial guess for an end-toend transfer, in Figure 5, is assembled by re-ordering the arc segments in the design environment to be consistent with the following itinerary:

- 1. Loiter for 270 days in an  $L_2$  quasi-halo structure
- 2. Depart  $L_2$  quasi-halo along an unstable manifold arc
- 3. Transfer to stable manifold arc to approach  $L_1$  halo
- 4. Arrive at  $L_1$  halo orbit to conduct operations

This process can be followed within the ATD design environment to produce various initial guesses for transfers that satisfy the design constraints.

## 5. DIFFERENTIAL CORRECTIONS PROCESSES

Once an end-to-end trajectory is constructed in the CR3BP Design module, it can be discretized and loaded into the CR3BP Corrections Module. The discretized transfer is represented by a series of nodes and arc segments, as displayed in Figures 6 and 7. Depending upon the trajectory requirements, a variety of constraints can be applied to the discretized transfer. The user may fix the state, altitude, or Jacobi constant value corresponding to any of the nodes. Additionally, nodes can be constrained to be apses relative to either primary and can incorporate a maneuver, i.e., a  $\Delta V$ . Finally, the total  $\Delta V$  and time-of-flight of the end-to-end transfer can be constrained to be equal to user-specified values.

For the MPLFS-based example, consider the following constraints: first, the  $L_1$  halo orbit is constrained to be periodic by enforcing perpendicular crossings of the xz-plane



**Fig. 6.** CR3BP Corrections module: The discretized transfer design is decomposed into individual segments (colored lines), each beginning at a node (colored dots). The interface supplies tools to constrain nodes and adjust the corrections algorithm's settings



Fig. 7. Constraints are enforced during corrections: two state constraints (gold circles) to preserve  $L_1$  halo periodicity, one Jacobi Constant constraint to preserve the  $L_2$  quasi-halo (magenta circle), and three maneuver locations (red asterisks)

at two distinct points. Second, the  $L_2$  quasi-halo structure is required to maintain a constant value of the Jacobi constant, which allows spatial flexibility in the corrections process without compromising the energy of the quasi-halo. Finally, maneuvers are located at three nodes: a node at the beginning of the clipped unstable manifold arc, the node at the interface between the stable and unstable manifold arcs, and a node at the end of the clipped stable manifold segment. These constraints and maneuver locations are illustrated in the design environment in Figure 7. The state constraints at the perpendicular crossings are represented by gold circles, the Jacobi constant constraint by a magenta circle, and the maneuver locations by red asterisks.

The CR3BP Corrections module employs a Newton-Raphson process in the form of a multiple-shooting algorithm to adjust the baseline solution such that all constraints are satisfied, along with continuity. This module includes an option to use Matlab's built-in fsolve algorithm or ATD's



**Fig. 8.** A corrected transfer in the Earth-Moon CR3BP with a total  $\Delta V$  of 39 m/s including an *xy*-projection (top) and *xz*-projection (bottom)

custom multiple-shooting algorithm. An initial application of the corrections scheme yields a solution with a total  $\Delta V$ of over 120 m/s. A constraint on the total  $\Delta V$  is imposed to reduce the transfer cost to 100 m/s. Subsequent corrections with a decreasing allowable  $\Delta V$  reduce the transfer cost to 39 m/s. This corrected design is plotted in Figure 8.

Next, the corrected CR3BP solution is transitioned to the Ephemeris Corrections module. This module numerically integrates an imported trajectory under the influence of a variety of gravitational fields. Since the transfer of interest has been designed in the Earth-Moon CR3BP, both the Earth and Moon are automatically included in the ephemeris model. Additional bodies, such as the Sun and other planets, can be added to the model to increase its fidelity. The ATD package employs NAIF SPICE data sets to compute the precise locations of these bodies during the propagation [8]. Accordingly, the user selects an epoch associated with the initial node in the CR3BP design. For this example, let the initial epoch be January 1, 2020, 00:00:00.00 UTC.

Similar to the CR3BP Corrections Module, the Ephemeris Corrections module offers a number of node constraint options. The state, apsis, altitude, total  $\Delta V$ , and total time-offlight constraints retain the same functionality as described previously. In addition, the inclination of a node can be fixed relative to the equatorial plane for any body included in the model, and the epoch time associated with a node can also be fixed. Note that it is no longer possible to constrain the Jacobi constant value associated with a node as the ephemeris model admits no such integration constant. Furthermore, the ephemeris corrections scheme also employs



Fig. 9. Ephemeris Corrections Module

a multi-dimensional Newton-Raphson method to satisfy all constraints. The partial derivatives that form the Jacobian matrix for this process can be computed analytically, numerically, or by either method on a node-by-node basis. To demonstrate the ephemeris corrections scheme, three maneuvers are allowed in the same locations as in the CR3BP corrections process and the transfer is corrected for continuity; analytical derivatives are employed to maximize computational speed. Perturbations from solar gravity are included in the model, with perturbations from all other celestial bodies neglected. The corrected transfer, plotted in the Ephemeris Corrections module in Figure 9, retains a geometry that resembles the original solution constructed within the CR3BP. Although the total  $\Delta V$  has increased to approximately 125 m/s, this transfer cost is reduced by iteratively decreasing the maximum allowable  $\Delta V$  and by exploring alternate epochs.

## 6. TRANSITION TO OPERATIONAL EPHEMERIS

As a final step in the corrections process, the Ephemeris Corrections module creates a script that can be opened by GMAT, which provides operational-level ephemeris modeling and is fully verified and validated. By default, the GMAT script produced by ATD includes gravity harmonics of degree and order one for the central body in the force model and treats all other bodies as point masses. Thus, in this example, simple lunar gravity harmonics are included and the Earth and Sun are incorporated as point masses. These settings are straightforwardly modified by editing the default script and further perturbations such as solar radiation pressure can be incorporated. To recreate the transfer, GMAT performs minor corrections on each arc segment to enforce continuity, with small maneuvers permitted in the same locations as specified in the ATD design. A converged solution is depicted in the xy- and xz-planes in Figure 10. Note that the number of revolutions around the quasi-halo structure have been reduced to lessen the numerical difficulties associated with transitioning the solution from ATD to GMAT; additional revolutions along the quasi-periodic structure may be recovered in GMAT by propagating in reverse time. As in the ATD ephemeris-corrected



**Fig. 10**. Trajectory in GMAT: *xy*-projection (top) and *xz*-projection (bottom), Earth-Moon rotating coordinates

solution, the geometry of the GMAT solution resembles the desired baseline trajectory. With this export capability, the advanced suite of tools available in GMAT can be leveraged to further analyze and validate the transfer design within a higher-fidelity dynamical environment.

## 7. TRANSFERS BETWEEN CR3BP SYSTEMS

A scenario that requires a vehicle from the MPLFS to transfer from the Earth-Moon  $L_2$  vicinity to the  $L_1$  region has been designed in ATD and transitioned to an operational ephemeris tool. This transfer exists solely within the Earth-Moon vicinity and does not leverage Sun-Earth dynamical structures. To demonstrate additional design capabilities incorporated within ATD, consider a similar mission that requires a spacecraft to transfer from the Earth-Moon  $L_2$  quasi-halo to the Sun-Earth  $L_2$  vicinity to service an observation platform. The System Blending module in ATD facilitates such system-tosystem transfer designs and is employed here to construct the transfer. As an example, assume the destination orbit is a Sun-Earth  $L_2$  southern halo. Recall that manifold arcs from an Earth-Moon  $L_2$  halo supply reasonable approximations for the quasi-halo manifolds. These manifold arcs are once again leveraged to depart the quasi-halo and are represented by magenta contours in Figure 11. A set of stable manifold arcs, in green, asymptotically approach the destination Sun-Earth halo orbit. A transfer is constructed by linking an Earth-Moon  $L_2$  halo unstable manifold arc to a Sun-Earth  $L_2$  halo



Fig. 11. System Blending Module: Unstable manifolds (magenta) emanate from an Earth-Moon  $L_2$  halo orbit and flow into stable manifolds (green) that asymptotically approach a Sun-Earth  $L_2$  halo orbit

stable manifold. In contrast to the halo-halo transfer within the Earth-Moon system, as described in the previous sections, these manifold arcs originate in two different CR3BP systems. In fact, the unstable manifold arcs exist naturally in the Earth-Moon system, while the stable arcs occur in the Sun-Earth system. The module reflected in Figure 11 includes two views of the design space: an Earth-Moon rotating view on the left, and a Sun-Earth rotating view on the right. The Sun-Earth view provides a clearer and more intuitive representation of the transfer. The spacecraft originates on the magenta structure and proceeds counter-clockwise to the "mouth" of the tube-like structure, where the spacecraft may leverage a maneuver to transition onto the green manifold tube to approach the Sun-Earth halo orbit.

Identifying a connection between the two manifold structures is nontrivial due to the non-planar nature of the manifold arcs and the time-dependent geometry of the problem. In particular, during one synodic month, the Earth-Moon system completes one revolution relative to the Sun-Earth line (the x-axis in the Sun-Earth rotating frame). Thus, a variety of relative geometries can be achieved by selecting an appropriate epoch. Accordingly, a strategy capable of exploring multiple geometries is required to identify suitable links between manifold segments. This example demonstrates the use of a higher dimensional Poincaré map to locate connections. A "hyperplane" is employed and defined as a physical plane normal to the xy-plane; in the Sun-Earth system the xy-plane coincides with the ecliptic and in the Earth-Moon system, it represents the Earth-Moon orbital plane. The hyperplane is constrained to include a common point between systems and is oriented by some angle  $\theta$  relative to the positive x-axis in the working frame. In this case, the Earth represents a common point between the Sun-Earth and Earth-Moon systems. Let  $\Sigma_1$  represent the Earth-Moon hyperplane with orientation  $\theta_1$  relative to the Earth-Moon line, as illustrated in Figure 12, and let  $\Sigma_2$  represent the Sun-Earth hyperplane with orientation  $\theta_2$  relative to the Sun-Earth line. The angles  $\theta_1$  and  $\theta_2$  are



**Fig. 12**. Two hyperplanes,  $\Sigma_1$  and  $\Sigma_2$  are chosen to construct an inter-system Poincaré map

selected independently and an appropriate epoch is identified to achieve a system geometry such that the two hyperplanes coincide with one another.

A map is constructed by recording the points where the Earth-Moon unstable manifold arcs cross  $\Sigma_1$  and then noting the points where the Sun-Earth stable manifold arcs cross  $\Sigma_2$ . The two sets of points are then overlaid and compared to identify links between manifold segments. However, due to the six-dimensional nature of the problem, a single 2D map is insufficient to represent the full state of the map crossings. Additional information is represented on the Poincaré map via glyphs, e.g., vectors or arrows as seen in Figure 13 [9]. The choice of a hyperplane reduces the full state from six dimensions to five. Vectors represent an additional four states: the vector base point represents the y and z map crossing coordinates, and the horizontal and vertical vector components represent  $\dot{y}$  and  $\dot{z}$ , respectively. Note that these states are all represented in the Sun-Earth rotating coordinate frame. The final dimension is incorporated by coloring each crossing by its Jacobi constant value in the Sun-Earth system. The Sun-Earth manifold arcs all possess the same Jacobi value in this system, but the Earth-Moon manifold arcs, which do not occur naturally in the Sun-Earth CR3BP, span a range of Jacobi constant values.

A Poincaré map is now constructed for the Earth-Moon to Sun-Earth transfer with  $\theta_1 = 270^\circ$  and  $\theta_2 = 300^\circ$ . The Sun-Earth manifold crossings, plotted in Figure 13 as red-brown vectors with square base points, exist at a Jacobi constant value of approximately 3.0009. The Earth-Moon manifold crossings, on the other hand, span a range of energies from below 3.0005 to 3.0009 and are represented by colored dots and vectors on the map. Candidate low-cost connections between manifold arcs are identified by locating map crossings that meet the following criteria:

- 1. Map crossings, represented by squares and circles, are near each other on the map
- 2. The direction and magnitude of the vectors associated with nearby points are similar



**Fig. 13**. Poincaré mapping tool to facilitate the identification of links between Earth-Moon and Sun-Earth manifold arcs

### 3. The color of the candidate map crossings are similar

If all three criteria are met, the manifold arcs represented by the glyphs are guaranteed to pass near each other in space with similar velocities. A zoomed view of the Poincaré map appears in Figure 14 with a potential low-cost link between manifold arcs. The long, red-brown vector located in the center of the map represents a Sun-Earth manifold arc. Several Earth-Moon manifold arcs cross the map nearby with velocities oriented similarly in the  $(\dot{y}, \dot{z})$  space. Additionally, the similarity in color between the points indicates that the states along the two manifolds are at approximately the same energy level on the map; since the map crossings are nearly collocated spatially, a common Jacobi constant value indicates that the velocity magnitudes are very similar. Thus, an interface between the two manifold arc segments requires only a change in velocity direction.

The design environment allows the user to click on map crossings and subsequently investigate candidate transfers. The aforementioned transfer candidate is selected and plotted in configuration space in Figure 15. In the Earth-Moon rotating view (left), a gold unstable manifold arc departs the blue Earth-Moon  $L_2$  quasi-halo and links to a light blue Sun-Earth manifold arc directly below the Earth in the xy-plane, i.e., at  $\theta_1 = 270^\circ$ . The same motion is also depicted in the Sun-Earth rotating view (right). The gold Earth-Moon unstable manifold arc interfaces with the blue Sun-Earth stable manifold arc on a plane oriented at  $\theta_2 = 300^\circ$  relative to the Sun-Earth x-axis. The blue, stable manifold arc proceeds to asymptotically approach the Sun-Earth  $L_2$  halo orbit, completing the transfer.

The system transfer design can be transitioned to an ephemeris model by following the steps similar to those in the previous sample mission scenario. To complete the blending, the Ephemeris Corrections Module is leveraged with a Sun-Earth-Moon model to constrain and correct the path. Following ATD ephemeris corrections, the blended transfer design is again transitioned to GMAT or another operational



**Fig. 14**. A zoomed-in view of the higher-dimensional Poincaré map: circles represent Earth-Moon manifold crossings and squares represent Sun-Earth manifold crossings

trajectory design software for final adjustments.

## 8. CONCLUDING REMARKS

Representing the dynamical accessibility of various regions in a multi-body regime, as well as enabling access to specific solutions, is nontrivial for dynamically sensitive environments such as the Earth-Moon and Sun-Earth systems. Trajectory design in such dynamical environments is facilitated by the Adaptive Trajectory Design (ATD) tool, developed by Purdue University and NASA Goddard. ATD includes an interactive catalog of solutions for the CR3BP, a simplified gravitational model that facilitates preliminary trajectory design in the Earth-Moon and Sun-Earth systems. To demonstrate the capabilities of this tool, sample trajectories for a Multi-Purposed Lunar Fractionated System (MPLFS) are designed. First, the catalog of Earth-Moon periodic and quasi-periodic structures is leveraged to identify 'holding' orbits that meet specific mission constraints. Next, an end-toend trajectory is constructed from a selected quasi-halo structure near the Earth-Moon  $L_2$  libration point to an  $L_1$  halo orbit within the Earth-Moon system. Invariant manifold arcs are rapidly generated and leveraged to demonstrate low-cost transfer options between the originating and destination structures. Constraints on the transfers are applied and enforced in a multiple-shooting corrections scheme within the CR3BP. The end-to-end trajectory is then transitioned to an ephemeris module within ATD for higher-fidelity corrections. The trajectory design is exported to and corrected in NASA's General Mission Analysis Tool (GMAT), which supplies operationslevel ephemeris tools. As a final demonstration of the tools available in ATD, a transfer from the Earth-Moon quasi-halo



**Fig. 15**. An end-to-end transfer from an Earth-Moon  $L_2$  quasi-halo structure to a Sun-Earth  $L_2$  halo orbit.

to a Sun-Earth  $L_1$  halo is designed. Poincaré maps are constructed and leveraged to identify connections between Earth-Moon and Sun-Earth manifold arcs. Such system-to-system transfers can be corrected in the ephemeris model and transitioned to operational tools such as GMAT. Ultimately, the design framework available as part of ATD facilitates rapid and well-informed construction of complex trajectories within a multi-body regime.

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