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RAPID TRAJECTORY DESIGN IN THE EARTH-MOON EPHEMERIS SYSTEM VIA AN INTERACTIVE CATALOG OF PERIODIC AND QUASI-PERIODIC ORBITS

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Abstract

Upcoming missions and prospective design concepts in the Earth-Moon system are extensively leveraging multi-body dynamics that may facilitate access to strategic locations or reduce propellant usage. To incorporate these dynamical structures into the mission design process, Purdue University and the Goddard NASA Flight Space Center have initiated the construction of a trajectory design framework to rapidly access and compare solutions from the circular restricted three-body problem. This framework, based upon a 'dynamic' catalog of periodic and quasi-periodic orbits within the Earth-Moon system, can guide an end-to-end ephemeris design. In particular, the inclusion of quasi-periodic orbits further expands the design space, potentially enabling the detection of additional orbit options. To demonstrate the concept of a 'dynamic' catalog, a prototype graphical interface is developed. Strategies to characterize and represent periodic and quasi-periodic information for interactive trajectory comparison and selection are discussed. A sample application is explored to demonstrate the efficacy of a 'dynamic' catalog for rapid trajectory design and validity in higher-fidelity models.

INTRODUCTION

With the increasing complexity of space missions, there is significant interest in trajectory design approaches that require fewer resources and deliver results sustainable over long term scenarios. Such goals may be achieved by leveraging the natural dynamical structures in the Earth-Moon system to guide the selection of a baseline trajectory. A well-informed trajectory design process may be particularly beneficial for several upcoming mission concepts including exoplanet observatories, in-situ exploration of asteroids as well as redirect concepts, and lunar cubesat missions.[1, 2, 3, 4, 5] The design of a baseline trajectory is nontrivial in a dynamically sensitive environment. In fact, in a higher-fidelity multi-body regime, the comparison of a large set of candidate solutions demands significant, and often prohibitive, time and computational resources. However, the well-studied Circular Restricted Three-Body Problem (CR3BP) can provide a reasonable approximation to the actual dynamical environment. The dynamical structures available in this model have been successfully leveraged by several missions in the Sun-Earth system as well as in early demonstrations in the Earth-Moon system.[6, 7, 8]

For rapid trajectory design in a multi-body regime, knowledge of the dynamical structures in a simplified model may facilitate a better understanding of the design space than a set of point solutions in the complete ephemeris model. Many software packages, for example, Systems Tool Kit (STK) and NASA's General Mission Analysis Tool (GMAT), offer a graphical environment for trajectory design incorporating gravitational fields at various levels of fidelity.[9, 10] However, the focus is generally directed towards the delivery of trajectory point designs and other operational mission support capabilities. Thus, they may be not specifically structured to offer guidance and insight into the available dynamical structures throughout the region. To supply a framework for incorporating knowledge of the dynamical accessibility in the Earth-Moon system, Purdue University and NASA Goddard Space Flight Center have been developing an interactive adaptive design process exploiting a reference catalog of solutions from the CR3BP to enhance efficient trajectory design in such complex environments. In this simplified model, periodic and quasi-periodic orbits govern the underlying dynamics and are approximately retained in higher-fidelity models. The current effort is devoted to creating direct links between the problem understanding and its practical application by exploring the Earth-Moon design space. There exists a wide array of known orbits with significant potential for parking, staging and transfers within the Earth-Moon system. Previously developed software tools, e.g., AUTO, can also supply a selection of these solutions as well as some insight into the local dynamics and the evolution of a set of orbits along any family.[11] In particular, AUTO enables the computation of periodic orbits and their numerical continuation into orbit families, as well as the detection and analysis of bifurcations. Nevertheless, such tools do not provide a basic "blueprint" to support rapid, efficient and well-informed decisions regarding the use of fundamental solutions in multi-body dynamical environment for any mission prior to an end-to-end trajectory design.

To overcome the challenges associated with identifying candidate trajectories in a chaotic multi-body regime, the available dynamical structures may be explored interactively. Previous studies on the application of interactive visual analytics to trajectory design has been conducted by Schlei for various applications in multi-body regimes [12]. In addition, a prototype software to assemble trajectories via point-and-click arc selection in multi-body scenarios is introduced by Haapala et al.[13] This design suite, Adaptive Trajectory Design (ATD), offers an interactive interface to facilitate exploration of mission design options. First, the dynamics in the Earth-Moon and Sun-Earth systems are approximated using the CR3BP. Dynamical structures in the form of periodic and quasi-periodic orbits and manifolds may be computed on-demand to construct an initial guess for an end-to-end trajectory, along with maneuvers. Ultimately, the constructed initial guess can be corrected both in the CR3BP and in an ephemeris model, and even exported to NASA's GMAT. [10] As a supplement to ATD, a 'dynamic' catalog has been constructed to identify and characterize periodic orbit that may aid in trajectory design and selection within the Earth-Moon system.[14] This information and a preliminary classification of orbits are compiled into a graphical environment, allowing the user to directly interact with data that can not be adequately represented by a static set of tabular data. As a result, a 'dynamic' and interactive catalog may overcome some of the challenges associated with constructing a predefined trade space to analyze a large set of solutions for a general mission concept. [15]

In this investigation, the Earth-Moon catalag of periodic solutions is expanded to include nearly bounded motion. Quasi-periodic motion, which inherits the behavior of a nearby periodic orbit, further expands the set of design options, thereby allowing identification of trajectories that may satisfy the mission requirements when transitioned to an ephemeris model. Families of quasi-periodic solutions are precomputed numerically and sampled to construct a representative set, which the user can immediately access in the catalog. [16, 17] While a quasi-periodic orbit may partially retain the characteristics of a nearby periodic solution, it may also possess unique and independent features that may be exploited in the mission design process. Accordingly, quasi-periodic motions are characterized in terms of quantities that may be used for a preliminary evaluation of the mission constraints. The utilization of this information within a user graphical interface is discussed and a prototype is demonstrated via application to a sample mission concept.

DYNAMICAL BACKGROUND

The rapid and intuitive exploration of the dynamical structures in the Earth-Moon system is first based on the CR3BP. This dynamical model, which serves as a reasonable approximation to the actual gravitational field, reflects the motion of a massless spacecraft under the influence of the point-mass gravitational attractions of the Earth and Moon. These two primary bodies are assumed to move in circular orbits about their mutual barycenter. The motion of the vehicle is described relative to a coordinate frame, $\hat{x}\hat{y}\hat{z}$, that rotates with the motion of the Earth and Moon. In this frame, the spacecraft is located by the nondimensional coordinates (x, y, z). By convention, quantities in the CR3BP are nondimensionalized such that the Earth-Moon distance, as well as the mean motion of the primaries, are both equal to a constant value of unity. In addition, the Earth and Moon have nondimensional masses equal to $1-\mu$ and μ, respectively, where μ equals the ratio of the mass of the Moon to the total mass of the system. In the rotating frame, the equations of motion for the spacecraft are written as:

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x},\ \ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y},\ \ddot{z} = \frac{\partial U}{\partial z} \tag{1}$$

where the pseudo-potential function,

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{d} + \frac{\mu}{r} \quad ,$$

while

$$d = \sqrt{(x+\mu)^2 + y^2 + z^2}$$

and

$$r = \sqrt{(x-1+\mu)^2 + y^2 + z^2} \quad .$$

This gravitational field admits five equilibrium points: the collinear points L_1, L_2, and L_3 are located along the Earth-Moon line; and two equilateral points, L_4 and L_5, form equilateral triangles with the two primaries. Since the CR3BP is autonomous, a constant energy integral exists in the rotating frame and is equal to the Jacobi constant, JC:

$$JC = 2U - \dot{x}^2 - \dot{y}^2 - \dot{z}^2 \tag{2}$$

At any specific value of the Jacobi constant, there are infinite possible trajectories exhibiting a wide array of behaviors. However, any trajectory may be generally classified as one of four types of solutions: equilibrium point, periodic orbit, quasi-periodic orbit, and chaotic motion. Each of these solutions can be identified using numerical techniques and subsequently characterized using concepts and quantities from dynamical systems theory.

CATALOG OF PERIODIC ORBITS

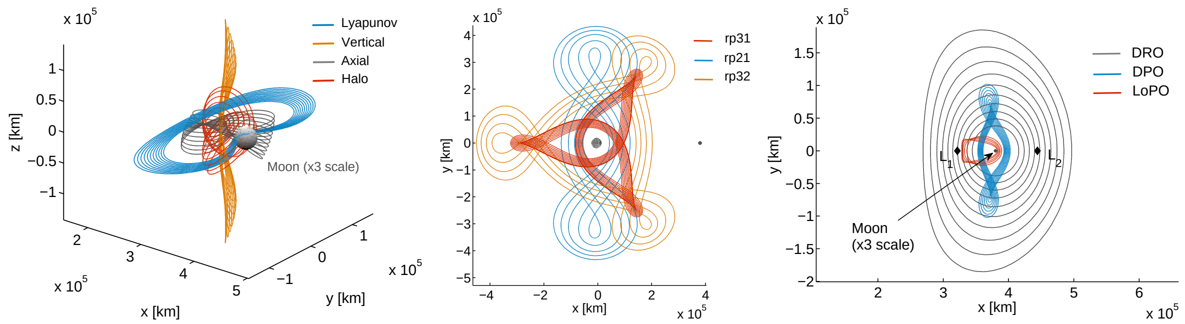
To capture the dynamical structures available in the CR3BP, families of periodic orbits are exploited. The characteristics of a periodic orbit generally reflect qualities of the nearby dynamics, potentially indicating the presence of additional structures, such as nearby manifolds or bounded motions. The characterization and classification of families of periodic solutions is, therefore, valuable in creating an efficient framework for mission design and preliminary mission trade-offs.[15]

The catalog adopts the classification system for CR3BP periodic orbits, the one most currently accepted in the astrodynamics community. Families of periodic orbits in the CR3BP are gathered into four classes: Libration Point Orbits (LPO), Resonant Orbits (RES), Moon-Centered Orbits (P2), and Earth-Centered Orbits (P1). Classes are designated according to the dynamical origin of the families. Orbits identified as LPO emanate from the vicinity of the equilibrium points, such as the sample of axial, halo, Lyapunov, and vertical families in Figure 1(a). Resonant orbits, i.e., RES families, are each derived from an integer ratio between the orbital period and the period of the Moon's motion around the Earth; the resonance is denoted $p : q$, where p is the number of Moon revolutions about the Earth by the time that the vehicle accomplishes q orbits along the reference. Some representative orbits from the 3:1, 3:2, 2:1 families are depicted in Figure 1(b). Orbits classified as P2 originate from the Moon's dynamical neighborhood; this class includes distant prograde orbits, low prograde orbits and distant retrograde orbits such those displayed in Figure 1(c). Similarly, P1 families originate from the Earth's dynamical neighborhood. The families currently included in the catalog are listed in Table 1 along with the abbreviations adopted to identify each set of orbits. Presently, conic arcs and P1 families are not included in the catalog, but may be considered for future expansion of the data set.

Table 1: List of families of periodic orbits currently available in the catalog.

Class	Family	Tag
LPO	L_i Lyapunov	Lyi
	L_i Halo	Hi
	L_i Axial	Ai
	L_i Vertical	Vi
	L_i Short Period	SPi
	L_i Long Period	LPi
	L_i Horseshoe	HS
RES	Planar Resonant n:m	rpnm
	Spatial Resonant n:m	rsnm
Moon-Centered	Planar Distant Retrograde	DRO
	Spatial Distant Retrograde	DRO3D
	Distant Prograde	DPO
	Low Prograde	LoPO

For each family of periodic orbits, characteristic parameters can be analyzed by the user to identify orbits that exhibit a desired behavior. Such quantities enable the creation of user-defined design spaces for the rapid performance of simple trades. The application of Keplerian parameters to a preliminary design framework may be ineffectual due to the time variation along a generic arc and the dependence on the selected central body. A characterization of periodic orbits in the CR3BP for mission design is discussed in [14]. The focus is a set of quantities that includes the geometrical amplitudes of the orbit, its orbital period and Jacobi constant. As a first approximation, the geometrical amplitudes may be useful in evaluating preliminary size and constraints, while the period of the orbit provides an approximate time scale for use in maneuver and communications planning. In addition, the Jacobi constant, as defined in Eq. (2), indicates the energy level of the trajectory and can be linked to the minimum transfer cost between two orbits. These quantities are complemented by estimates for the operational costs. Accordingly, simple periodic orbit insertion and station keeping costs for a large variety of families are included, en-



(a) Libration Point Orbits. (b) Resonant Orbits. (c) Moon-Centered Orbits.

Figure 1: Sample members of well-known orbit families in the Earth-Moon system, plotted in the rotating frame.

abling rapid estimation of the deterministic DV. Such insight is valuable in preliminary identification of regions in the Earth-Moon system that are accessible for a given mission scenario. In the catalog, a selected set of periodic orbit insertion costs are based on simple straightforward transfers from LEO, including direct transfers and powered flyby transfers. The station keeping maneuvers implemented to predict the orbit maintenance costs are based on the long-term strategy discussed in [15].

The trade spaces in the 'dynamic' catalog use simple statistical indexes, such as mean, range or standard deviation to provide a compact global representation of a large set of orbits. For example, consider Figure 2. Each box in the figure represents a family of orbits (identified by the associated label); the boxes are portrayed in a two dimensional design space, where the horizontal axis corresponds to an estimate of the annual station-keeping cost and the vertical axis represents the direct insertion transfer DV. Within this space, each box is centered at the mean value of the associated quantities along the family. The size of each box is proportional to the standard deviation of the quantities computed along the family. Using this simplified representation, the user can simultaneously estimate and compare the station-keeping and transfer costs for multiple families of orbits. Since this representation is implemented in an interactive graphical interface, the user can modify the visible information, such as the quantities on each axis or the families included in the plot. For a complete representation of the characteristic quantities along the family, the actual characteristic curve can be displayed on-demand. Allowing the user to display information only when desired, reduces the complexity in visualizing multiple characteristic curves that are overlaid on a single two-dimensional plot. A more detailed description of this representation of periodic orbits and its implementation appear in [15].

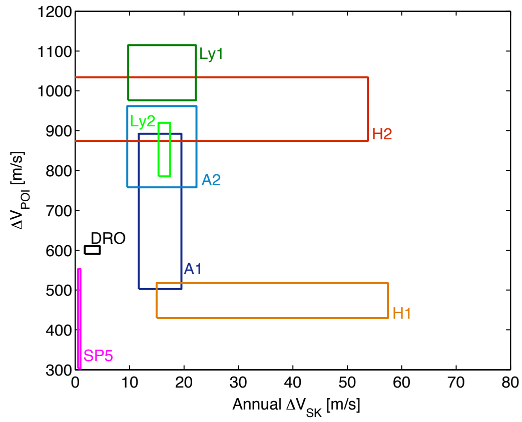


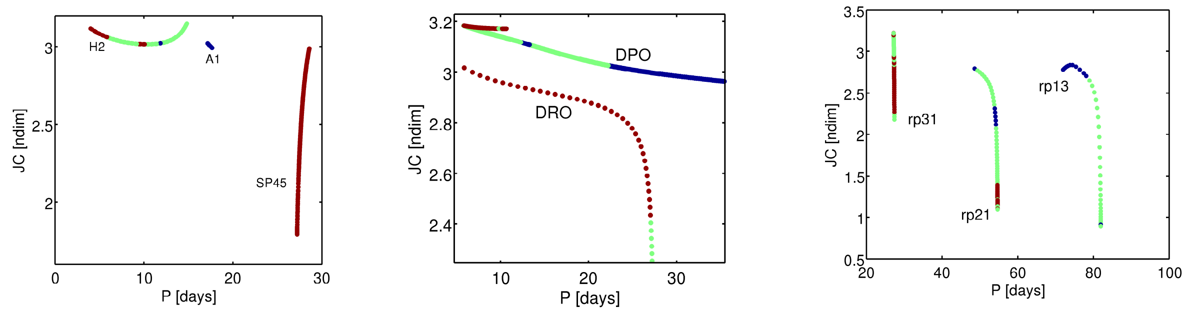
Figure 2: Simplified representation of transfer and station-keeping cost of each family using boxes.

EXISTENCE OF QUASI-PERIODIC ORBITS

In the CR3BP, the existence of important dynamical structures, such as manifolds and quasi-periodic orbits, is associated with the linear stability of a periodic orbit. Upon linearization of the equations of motion for the CR3BP, in Eq. (1), the general solution to the linear variational equations is written as

$$\boldsymbol{\delta x}(t) = e^{A(t-t_0)}\boldsymbol{\delta x}(t_0) \quad , \tag{3}$$

where $\boldsymbol{\delta x}(t)$ is the variation relative to some reference solution, $A = A(t)$ is the Jacobian matrix of first partial derivatives of Eq. (1) evaluated along the reference (A is generally not constant). The state transition matrix (STM) is defined as $\Phi(t, t_0) = e^{A(t-t_0)}$, essentially indicating (in linear approximation) the



(a) Libration Point Orbits. (b) Moon-Centered Orbits. (c) Resonant Orbits.

Figure 3: Number of nontrivial modes associated to a center subspace across sample families in the CR3BP denoted by color (Blue = 0, Green = 1, Red = 2).

sensitivity of a given state along the reference path at time t to any variations in the initial state at t_0. For a periodic orbit, the STM integrated for exactly one orbital period, known as monodromy matrix, is leveraged to predict the orbital stability.

The eigenvalues λ_i of the monodromy matrix, which occur in reciprocal pairs, reveal the stability of the reference periodic orbit and, therefore, the behavior of the nearby flow. For each eigenvalue, there is a corresponding eigenvector, $\boldsymbol{v}_i$; together, all 6 eigenvectors span $\mathbb{R}^6$. In the CR3BP, the monodromy matrix for a periodic orbit possesses two trivial eigenvalues equal to unity, due to periodicity of the solution and the existence of a family. The eigenvectors associated with $|\lambda_i| > 1$ define an unstable invariant subspace, where the nearby trajectories depart the vicinity of the orbit. The eigenvectors corresponding to $|\lambda_i| < 1$, however, identify a stable invariant subspace, with trajectories that asymptotically approach the orbit. Each of these subspaces include manifolds that may connect the orbit to other regions of the Earth-Moon system. Families of quasi-periodic orbits are incorporated in an invariant center subspace, EC, which is predicted by the existence of eigenvalues that lie along the unit circle, i.e., $|\lambda_i| = 1$. Each quasi-periodic motion in the center subspace traces out a closed surface or torus. Although these trajectories do not exactly repeat over time, they are bounded solutions. If the monodromy matrix possesses a pair of nontrivial eigenvalues that lie on the unit circle, one family of quasi-periodic orbits exists. Two pairs of unitary nontrivial eigenvalues, however, indicate the existence of two quasi-periodic families. In this case, nearby motions at the same energy level neither depart nor approach the reference, but, rather remain in its vicinity indefinitely. Similar to the evolution of periodic orbits along a family, quasi-periodic orbits also evolve along a family. In designing trajectories that satisfy a given set of mission constraints, quasi-periodic orbits supply structures that may offer a better alternative than the corresponding periodic orbit. Furthermore, the boundedness of a torus may be retained in an ephemeris model when combined with small maintenance maneuvers.

As the stability of a periodic orbit evolves along a family, so too does the number of associated families of quasi-periodic orbits. In Figure 3, the number n_m of pairs of unitary nontrivial eigenvalues is summarized, and, equivalently, the number of quasi-periodic families. In this figure, each plot corresponds to sample members of a class of periodic orbits. In Figure 3, each periodic orbit is represented by a single marker in a two-dimensional space: the vertical location corresponds to the Jacobi constant of the periodic orbit, while the horizontal axis indicates the associated period. Marker colors indicate the value of n_m and, therefore, the number of quasi-periodic families corresponding to a periodic orbit: none (blue), 1 (green), and 2 (red). Examination of Figure 3 provides a simple overview of the qualitative stability of sample of periodic orbits in the CR3BP for the Earth-Moon system. First, consider the resonant 1:3 planar family (rp13). The absence of red markers indicates that each member of the resonant 1:3 planar family possesses stable/unstable manifolds, potentially enabling relative low ΔV transfers to the orbits. Furthermore, green markers identify members of the resonant 1:3 planar family with nearby quasi-periodic motion, that may be exploited during mission design. Conversely, the L_4/L_5 short period family (SP45) is comprised solely of members with two quasi-periodic family in their vicinity. The L_1 axial orbits, however, possess no quasi-periodic motion along the entire family. Accordingly, L_1 axial orbits (A1) do not provide feasible options for missions that leverage the boundedness of motion along a torus, e.g., a formation of spacecraft. Considering these examples, the description in Figure 3 enables preliminary identification of reference orbits that produce quasi-periodic motions, that may be incorporated into the mission design process.

COMPUTATION OF QUASI-PERIODIC ORBITS

Given the complexity involved in numerically defining a torus in a chaotic dynamical system, the computation of quasi-periodic orbits in the CR3BP also presents numerous challenges. One of the earliest proposed methods for locating quasi-periodic orbits near libration points leverages analytical approximations (from first- to fourth-order).[18, 19, 20] However, these approximations are only accurate in a small neighborhood of the libration point, and are not applicable in other regions of configuration space. Several numerical algorithms have been proposed by previous researchers to compute quasi-periodic orbits in any region of the CR3BP.[21, 20, 22] However, the range of validity of existing numerical methods for computing families of quasi-periodic orbits is limited and depends on both the family and the complexity of the nearby flow. Nevertheless, two recently developed strategies are employed in combination to compute quasi-periodic orbits for this investigation. The first method, introduced by Pavlak, involves only enforcing continuity constraints to compute bounded motion that likely lies near a quasi-periodic orbit.[16] Although the computed solutions are not precisely quasi-periodic, this methodology yields relatively accurate predictions of the existence and shape of tori within a family of quasi-periodic orbits. A second approach employed by Olikara and Scheeres, recovers the exact tori that emanate from a periodic orbit with little computational difficulty. However, the numerical detection of precisely quasi-periodic solutions is limited to tori that can be described by two nonresonant frequencies. Thus, such limitations may yield few quasi-periodic motions in a region with nearby higher-order periodic orbits. These two approaches can be used in combination to produce a vast selection of quasi-periodic orbits for use in a dynamical catalog of ordered motions in the Earth-Moon system.

The differential corrections algorithm described by Pavlak supplies a relatively robust and computationally inexpensive method for computing approximately quasi-periodic motions in various regions of configuration space.[16] In particular, this strategy is based on a similar scheme used to compute periodic orbits, and can be straightforwardly extended to higher-fidelity models. This methodology begins with a periodic orbit, possessing a nontrivial center manifold, that is decomposed into several arcs. These arcs are repeated several times and result in multiple revolutions of the periodic orbit as the initial guess. Next, a perturbation is applied to the initial state along the first arc in either the y- or z-direction of the rotating frame. An approximately quasi-periodic solution can be generated from this slightly perturbed solution by using a multiple shooting algorithm that enforces continuity between neighboring arcs and the value of the Jacobi constant. For subsequent solutions along the family, a natural parameter continuation scheme is employed, and the initial state along the first arc is increasingly displaced from the original periodic orbit. Since the converged solutions are not precisely quasi-periodic, the last several revolutions along the solution are discarded. Nevertheless, the resulting trajectory generally reflects the structure and behavior of quasi-periodic orbits along a given family.

An alternative method for locating a quasi-periodic orbit in the CR3BP relies on numerical computation of the corresponding two-dimensional invariant torus using a stroboscopic mapping, as presented by Olikara and Scheeres [23]. In particular, each torus is described by two frequencies. One frequency, ω_0, corresponds to motion along the associated periodic orbit and possesses a value that is near the inverse of the orbital period. The second frequency, ω_1, indicates rotation in the transverse direction and is related to the complex eigenvalue of the limiting periodic orbit. This numerical method employs the concept of an invariant circle, formed by the intersection of the torus and a higher dimensional plane. States that lie along this invariant circle, when propagated forward for a time equal to $T_0 = 2\pi/\omega_0$, return to the circle and also experience a rotation related to the transverse frequency, ω_1. To numerically compute the two-dimensional torus using this stroboscopic mapping constraint, an odd number of states along the invariant circle are differentially corrected, along with the corresponding frequencies. Although this procedure produces an exact torus, it experiences numerical difficulties when the frequencies pass through low-order resonances. In such cases, alternate numerical strategies may be employed. Nevertheless, to construct an initial guess, a periodic orbit that possesses a nontrivial center subspace, along with the eigenvalues and eigenvectors of its monodromy matrix, is employed. In particular, at a selected location along the reference periodic orbit, an eigenvector that corresponds to the center subspace is scaled by a small number and rotated by 360 degrees, forming an initial guess for the invariant circle. Upon computing a small torus near the reference periodic orbit, the differential corrections process is continued to produce additional tori along the family of quasi-periodic orbits at a given energy level. Both procedures by Pavlak and Olikara are implemented to compute quasi-periodic orbits at various energy levels for different families in the catalog, including halo, Lyapunov, vertical, distant retrograde, 2:1 and 3:1 resonant trajectories.

CHARACTERIZATION OF QUASI-PERIODIC ORBITS

In addition to families of periodic orbits within the Earth-Moon system, the existence of nearby quasi-periodic motions significantly expands the design space. In particular, these tori possess additional degrees of freedom, while still retaining the boundedness that may be valuable in the design of long-term space applications. Quasi-periodic orbits along a family may exhibit a larger range of in-plane and out-of-plane amplitudes than their periodic counterparts, enabling significant flexibility in the trajectory design process. In fact, while a periodic orbit may fail to meet a desired set of mission requirements, arcs along a torus may provide viable alternatives. Furthermore, the boundedness of quasi-periodic motion can be approximately retained when the selected arcs are transitioned to a higher-fidelity model, such as ephemeris. For instance, NASA's ARTEMIS mission successfully exploited segments of quasi-periodic trajectories near L_1 and L_2 in the Earth-Moon system for the science orbits of its two spacecraft.

Similar to the inclusion of periodic orbits in an efficient trajectory design framework, the characterization of quasi-periodic motions is warranted to facilitate their inclusions in simple trade spaces. First consider a geometric description of a quasi-periodic orbit, essentially describing the shape and size of the associated torus. Such geometric quantities may include orbit box sizes, cone angles, periapsis and apoapsis distances. This information may be useful in the design of trajectories for missions that are constrained by requirements on altitude or even directionality for communications, line-of-sight or shadow avoidance. Each of these quantities that may characterize quasi-periodic orbits can be equally applied to the exactly periodic motion.

To characterize the size of a quasi-periodic orbit in configuration space, a simple geometric box is be employed. In particular, consider the smallest box that encloses the entire quasi-periodic orbit with each side aligned and parallel to the $\hat{x}$, $\hat{y}$, $\hat{z}$ unit vectors in the CR3BP, as depicted in Figure 4. The length of each box dimension is defined as B_x, B_y and B_z, representing the maximum end-to-end excursion of the trajectory in each spatial direction as defined in the rotating frame. Note that, by this definition, each box dimension does not directly correspond to the orbit amplitudes A_y and A_z, which are defined as the maximum magnitudes of the y and z variables along the trajectory. For example, consider an L_1 northern halo orbit, which is not symmetric across the xy-plane. In this case, the value of A_z is equal to the maximum excursion of the trajectory in the positive $\hat{z}$ direction. This quantity is greater than the maximum extension of the trajectory in the negative $\hat{z}$ direction. In contrast, the value of B_z is equal to the difference between the most positive and most negative values of z along the trajectory. Accordingly the value of B_z is not equivalent to twice the amplitude A_z, i.e., $B_z \neq 2A_z$. Despite the simplicity of this geometric representation, B_x, B_y, and B_z, supply a straightforward characterization of the 'size' of the quasi-periodic solution. Consider, for example, quasi-periodic orbits within the center subspace of an L_1 vertical orbit. As the family evolves away from the periodic orbit, the trajectories expand in the $\hat{y}$ direction. This variation in the size of the quasi-periodic orbit may be straightforwardly visualized by an increasing value of B_y. Similarly, a planar periodic orbit may possess an associated family of quasi-periodic orbits that extend out of the Earth-Moon plane. The parameter B_z offers an estimate of the maximum variation of z along each trajectory. Such information may be valuable in preliminary evaluation of geometrical mission constraints.

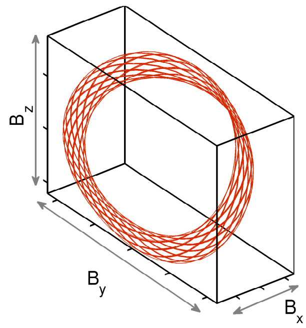


Figure 4: Box sizes schematics.

In addition to size, the maximum angular deviation from a given direction is included in the geometrical description of a quasi-periodic orbit. Such information can be represented using cone angles, with an axis of symmetry aligned along the $\hat{x}$ axis of the

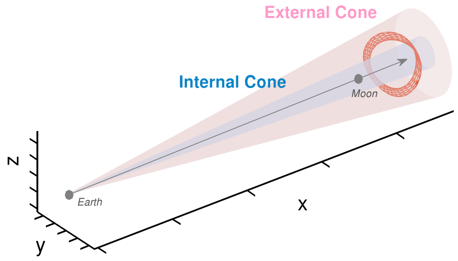


Figure 5: Cone angles schematics.

rotating frame. The base point for each cone is located at one of the primary bodies: either the Earth or the Moon. Using this reference point, two cones are defined: one external cone and one internal cone. The external cone, as depicted in Figure 5 is constructed as the smallest conical surface that encompasses the entire trajectory. The corresponding cone angle is equivalent to the largest angular deviation of the orbit with respect to the x-axis of the rotating frame. Similarly, the internal cone is constructed as the largest cone about the x-axis that lies inside the entire trajectory path. As portrayed in Figure 5, the associated cone angle denotes the minimum angular deviation of the orbit with respect to the x-axis. For orbits that pass directly through the x-axis, an internal cone cannot be constructed. Additionally, if the base point of the cone lies within the orbital path, neither the internal nor external cones can be computed. Despite the simplicity of this geometric representation, cone angles supply valuable information for preliminary assessment of directional constraints such as visibility requirements, possibly due to science, communication or power requirements. For example, consider an L_2-based orbit. A constraint on continuous communication can be translated to a requirement on lunar occultation avoidance. To maintain this direct link to the Earth the spacecraft must, therefore, remain outside a small 0.25 degree internal cone. By characterizing quasi-periodic motions via cone angles, such constraints can be rapidly evaluated for a large set of trajectories, thereby allowing the user to explore the design space.

Characterization of quasi-periodic motions via simple geometrical quantities facilitates the comparison of a large set of periodic and quasi-periodic orbits. To visualize that comparison, plots such as Figure 6 are employed. Displayed on this plot, dots correspond to periodic orbits in a given family, colored by the corresponding Jacobi constant. The value of B_z for each periodic orbit, i.e., a dot in the figure, is plotted as a function of an integer index that simply represents the order the trajectories appear in the dataset, without any sorting. For the case of the DROs, each member is planar, resulting in a series of periodic orbits that aligns with the horizontal axis, $B_z = 0$. Using Figure 6 as a reference, each family of quasi-periodic solutions is represented as a single vertical line originating at the periodic orbit and encompasses the range of values of B_z along the computed portion of the quasi-periodic family. The color of each vertical bar reflects the number of nontrivial complex modes n_m within the center subspace. By employing simple representations such as the plot in Figure 6, the potential for quasi-periodic motions to satisfy a given set of mission constraints may be rapidly evaluated. For instance, three-dimensional periodic DROs only exist for a limited range of y-amplitudes, Jacobi constants and periods. However, three-dimensional quasi-periodic DROs can significantly expand the set of solutions that both exhibit the motion typical of the DRO family and extend out of the xy-plane.

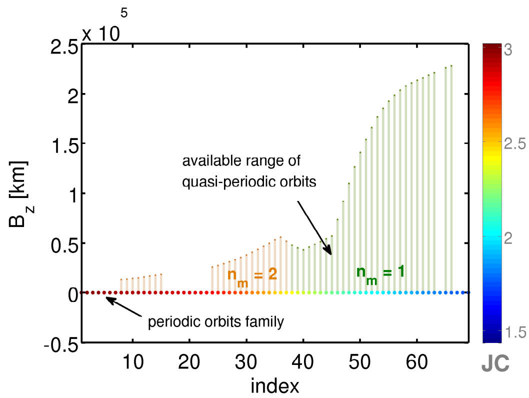


Figure 6: Comparison of the out-of-plane excursion in terms of B_z for periodic and quasi-periodic DRO orbits.

Arcs along the complete quasi-periodic trajectory are often more valuable than the entire torus. As demonstrated in previous applications, history profiles for selected variables are an effective way to identify portions of a quasi-periodic orbit that may satisfy certain mission requirements.[24] These time history plots depict the temporal evolution of a parameter of interest; a high number of revolutions is warranted to sufficiently capture the global behavior of the quasi-periodic motion. Mission requirements can often be translated into thresholds on variables of interest. Such variables may vary significantly along a quasi-periodic orbit, especially in comparison to the reference periodic solution. For example, Figure 7 depicts the time history for the y-component of the state along an L_2 vertical quasi-periodic orbit. The y-component oscillates within a [-40000,40000] km range for the quasi-periodic solution, which is remarkably larger than the [-13000,13000] km range for the periodic motion. For some time intervals along the quasi-periodic orbit, the variable may remain within some of the given thresholds. Such intervals are quickly identified on a time history plot. An approximate envelope curve may also be numerically computed and overlaid on the time history, as in Figure 7. To calculate the envelope the extreme values over 1-3 revolutions are detected. Then, the envelope is estimated using a spline interpolating those extreme values. The spline approximation becomes inaccurate at the boundaries of the history profile, due to extrapolation from the last available interpolating point. In general, the selected time intervals correspond to arcs of the quasi-periodic orbit that meet some of the mission specifications and can aid in the construction of a feasible trajectory.

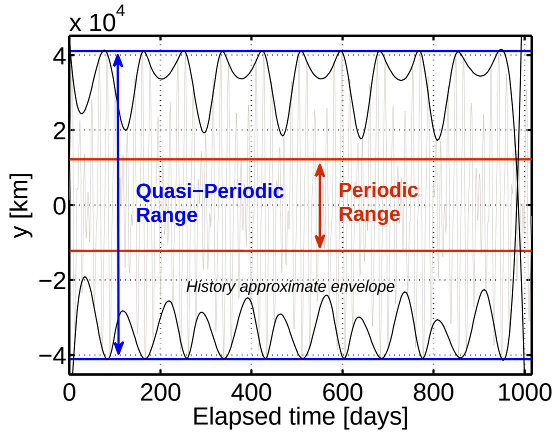


Figure 7: Representative history plot for a L_2 vertical quasi-periodic orbit.

IMPLEMENTATION IN INTERACTIVE GRAPHICAL INTERFACE

Using a simple set of characteristic quantities, quasi-periodic solutions are incorporated into an efficient framework for orbit comparison and design. This additional framework extends a pre-existing interactive catalog of periodic orbits for the quick selection of trajectories. The interactive catalog serves as a resource to obtain preliminary arcs prior to the use of an end-to-end design tool for complete path construction and transition to ephemeris. For demonstration, the catalog is implemented as a prototype Graphical User Interface (GUI).[15] The prototype is assembled in the MATLAB® environment. A supplemental and interactive module is built to visualize and compare quasi-periodic solutions from the CR3BP, as displayed in Figure 8. The analysis originates in the upper-left panel, where selected characteristics for a family of periodic orbits and their corresponding quasi-periodic motions are displayed in the form of a representation similar to Figure 6. Relevant characteristics are displayed on the vertical axis and can be selected by the user. The subplot within this panel displays the number of quasi-periodic families associated with each periodic orbit. A selector, such as a sliding bar on the graph allows the user to select the quasi-periodic family of interest. A quasi-periodic trajectory that is representative of the selected family of tori appears in the upper-right panel and is plotted in configuration space. The lower panel reports the time history for a selected quantity of interest. Using the functions associated with this panel, the user can set thresholds on a given variable and enable automatic detection of the time intervals along a quasi-periodic orbit satisfying these pre-defined limits. Time intervals of interest can also be manually selected. Ultimately, the time history enables the selection of segments or arcs along a quasi-periodic solution that satisfy the mission requirements. Following arc selection, the user can export any candidate solutions for use in an advanced trajectory design suite for further examination. Given the iterative nature of the trajectory design process, such a catalog may be used interactively to explore additional candidate orbits.

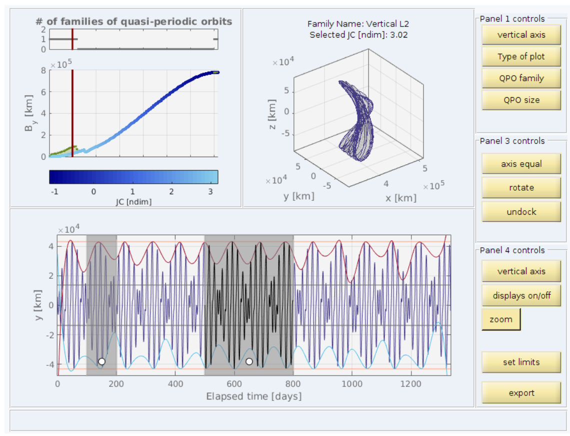


Figure 8: Screenshot of the GUI prototype for the quick comparison and selection of quasi-periodic arcs.

SAMPLE APPLICATIONS

Since the dynamical environment in the Earth-Moon system is reasonably approximated by the CR3BP, an interactive catalog of periodic and quasi-periodic solutions can be used for preliminary trajectory design in various mission scenarios. Given a mission concept and a set of constraints on the path of the spacecraft, the interactive catalog can be employed to reveal a variety of orbital options, thereby alleviating the complexity of searching the design space in a higher-fidelity ephemeris model. Based on the intended region in the Earth-Moon space for the spacecraft, the user can first reduce the set of candidate orbits by identifying feasible regions of configuration space, e.g., in the vicinity of a given libration point or near the Moon. The user can then simultaneously explore two-dimensional trade-spaces involving each family to examine whether any of the members satisfy the given mission constraints. For periodic orbit families that possess a nontrivial center subspace, the user may also explore one family at a time and analyze the characteristics of any nearby quasi-periodic motion. Through this selection strategy, the user can identify and export orbit options for analysis in a higher-fidelity model. This process is demonstrated for two mission design applications.

L_2 Gateway Operations

Consider a long-term Multi-Purpose Lunar Fractionated System (MPLFS) consisting of a formation of spacecraft stationed near the Earth-Moon L_2 gateway. The spacecraft comprising the MPLFS could provide various services for operations in cislunar and/or interplanetary space. Locating the MPLFS near L_2 could enable the exploitation of natural multi-body dynamics either for a spacecraft that may rendezvous with the formation or upon deployment of any of the individual modules to their final destination. For instance, an MPLFS that may facilitate lunar operations could consist of several spacecraft including a fueling depot, multiple on-orbit storage vehicles, as well as a communications relay. Since the MPLFS must be located near the L_2, the set of candidate orbits for this mission can be significantly reduced. Depending upon the mission scenario, it may be crucial to maintain continuous contact with the Earth. This requirement can be translated into a geometrical constraint on the path, such that the MPLFS should maintain a constant line-of-sight to the Earth. In addition, the L_1 gateway may be used as a low-cost transfer mechanism between the Earth and lunar vicinities. Accordingly, a vehicle that originates near the Earth may pass through the L_1 gateway in order to rendezvous with the MPLFS. To ensure that any such transfers for a crewed vehicle, for example, are possible and not prohibitive in terms of fuel consumption, a loose constraint can be placed on the energy level of the candidate orbit for the MPLFS. Specifically, the Jacobi constant of the MPLFS orbit should be comparable to the Jacobi constant value of L_1. Operationally, it is also convenient for all the vehicles to remain in a close formation. Orbit options that satisfy each of these constraints can be rapidly identified using the interactive catalog.

The interactive catalog enables rapid identification of a preliminary baseline solution for the MPLFS concept. First, consider the ranges of Jacobi constant values for the L_1 and L_2 families of periodic orbits available in the catalog. The simple bar plots in the catalog reveal that the JC ranges along these families generally overlap.[15] This correspondence in Jacobi constant is indicative of potentially low-cost transfers between the L_1 and L_2 regions. Thus, each of the families of L_2 orbits possesses members that satisfy the loose constraint on the Jacobi constant. Next, the type of baseline motion for the MPLFS - periodic or quasi-periodic - is selected. For multiple spacecraft that lie along the same periodic orbit, there are limited orbital geometries that enable a configuration where the first and last spacecraft remain close over time. Alternatively, placing several spacecraft along the surface of a quasi-periodic orbit may ensure that each member of the spacecraft formation remains in sufficiently close proximity to the other modules. The feasibility of leveraging quasi-periodic motion in formation flight has been demonstrated by previous researchers.[25] Accordingly, the quasi-periodic selection tool within the catalog is employed. Recall from Figure 3, that quasi-periodic solutions are only available along the L_2 Lyapunov, vertical, and halo orbits. Since the L_2 axial family does not possess any members with a center subspace, these orbits are discarded from the set of candidate solutions. Instead, each of the L_2 Lyapunov, vertical, and halo orbit families are analyzed individually to identify any solutions that satisfy the continual line-of-sight constraint.

The quasi-periodic orbits that lie in the center subspace of the L_2 Lyapunov family are examined using the catalog to identify whether any solutions can provide continuous communications to the Earth. A continuous line-of-sight to the Earth from a spacecraft in the MPLFS is possible when the formation lies outside of the lunar Earth shadow. This requirement is straightforwardly translated to a simple geometric constraint that the candidate orbit must not pass within the angular radius of the Moon as observed from Earth, i.e., the orbit must remain outside a 0.25 degree cone constructed with an axis of symmetry aligned with the x-axis and its base point located at the Earth. Consider first an L_2 Lyapunov periodic orbit. Since this orbit lies within the xy-plane, it periodically passes through the lunar shadow every half revolution. The out-of-plane oscillations associated with nearby quasi-periodic orbits may enable repeated passages that are outside of this shadow cone. In fact, Figure 9 depicts the out-of-plane extension via the box size, B_z, for the largest available quasi-periodic solutions associated to each L_2 Lyapunov orbit with a center subspace. For this family of orbits, the evolution of the nearby quasi-periodic orbits exhibits one of two behaviors. For Jacobi constant values within the range $JC = [3.15, 3.17]$, the corresponding families of quasi-periodic tori connect a member of the L_2 Lyapunov orbit to a member of the L_2 vertical family at the same energy level. Accordingly, quasi-periodic orbits along a family initially resemble a Lyapunov orbit. As the family evolves, however, the tori gradually transform to resemble a vertical orbit, as depicted in Figure 9(a). For an interval of Jacobi constant values $JC = [2.95, 3.00]$, the L_2 Lyapunov and vertical orbits are no longer linked via a family of tori. Rather, the quasi-periodic orbits evolve towards the geometry depicted in Figure 9(b). For brevity, only quasi-periodic orbits with a Jacobi constant in the interval $JC = [3.15, 3.17]$ are investigated to identify orbits that satisfy the continuous line-of-sight requirement for the MPLFS mission.

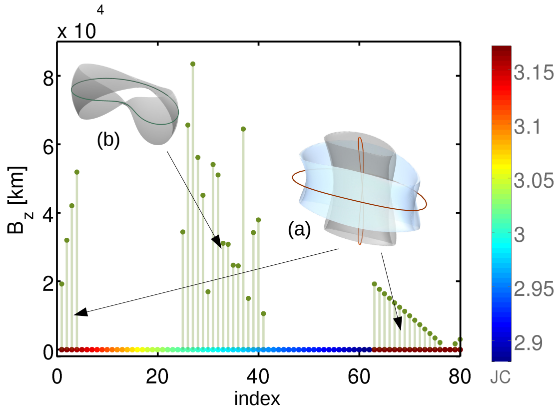
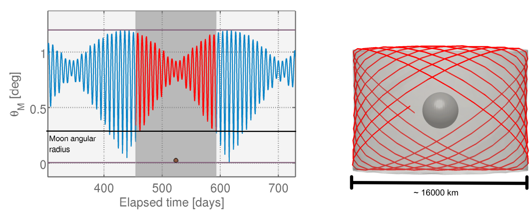


Figure 9: Out-of-plane extension and geometry of available quasi-periodic solutions associated to different L_2 Lyapunov orbits (indentified by the catalog index).

To assess whether L_2 Lyapunov quasi-periodic motion may satisfy the mission constraints for the MPLFS, the angular time histories for the tori that exist at $JC = 3.17$ are rapidly examined using the catalog. In addition to enabling visualization of the configuration space, the catalog interface also displays the time history of a user-selected variable of interest. In this sample mission scenario, the instantaneous angular deviation of the spacecraft from the x-axis, as measured from the Earth, is leveraged to determine whether continuous communication to Earth is possible for a given quasi-periodic orbit. For the selected torus, the catalog includes a capability for automatic detection of segments of the orbit that satisfy a given constraint on one of the included characteristic parameters. As an example, this automatic detection is applied to an L_2 quasi-periodic orbit that exists at a Jacobi constant of $JC = 3.17$, as displayed by the gray toroidal surface in Figure 10(b). The time history for the angular deviation of motion along this torus is displayed in Figure 10(a). A segment of the quasi-periodic orbit that remains outside of the lunar shadow cone is highlighted in red. Recall that for the MPLFS mission scenario, the baseline orbital motion must allow a continuous line-of-sight to the Earth. However, the highlighted segment in Figure 10(a) only spans 150 days. For a different mission scenario, 150 days may be a sufficiently long visibility window and, therefore, render this quasi-periodic orbit a candidate solution. Nevertheless, for the MPLFS example, the L_2 Lyapunov quasi-periodic orbits do not provide a solution that remains permanently outside of the lunar shadow cone.

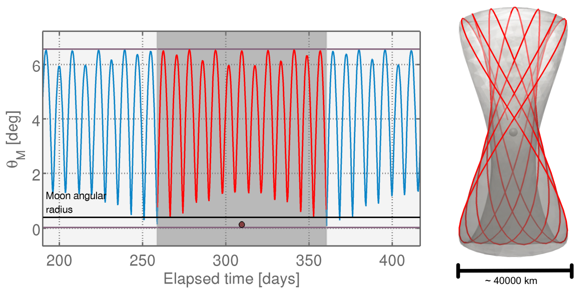


(a) Angle separation history.

(b) $y - z$ view of the torus.

Figure 10: L_2 Lyapunov quasi-periodic arc that satisfies the continuous coverage requirement for the MPLFS mission example.

Quasi-periodic motions emanating from the three-dimensional L_2 vertical families may also provide candidate orbits for the MPLFS spacecraft formation. Using the catalog interface, several L_2 vertical quasi-periodic motions are examined. Similar to the L_2 Lyapunov family, quasi-periodic orbits near some members of the L_2 vertical family only slightly deviate from the originating periodic solution. Other L_2 vertical orbits, however, possess nearby tori that exhibit significantly different geometries across the family. Despite this variability in the behavior of the L_2 quasi-periodic orbits, each torus still passes behind the Moon. Accordingly, the MPLFS formation would pass within the lunar shadow cone. For instance, consider a sample L_2 vertical quasi-periodic orbit as displayed in Figure 11. The corresponding time history for this quasi-periodic motion is depicted in Figure 11(a). Since the angular deviation along the trajectory passes within the lunar shadow cone angle twice within the plotted time period, only a 100 day segment along the L_2 vertical quasi-periodic orbit satisfies the line-of-sight constraint. Accordingly, the L_2 vertical family does not provide a viable candidate solution for the MPLFS mission concept.



(a) Angle separation history.

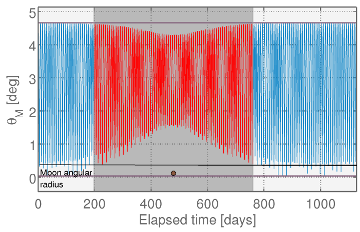
(b) $y-z$ view of the torus.

Figure 11: L_2 vertical quasi-periodic arc that satisfies the continuous coverage requirement for the MPLFS mission example.

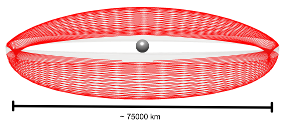
Additional candidate quasi-periodic orbits emanate from the L_2 halo family. Since the majority of members in this family that possess a nontrivial center subspace also satisfy the continuous line-of-sight constraint for the MLPFS mission concept, quasi-periodic solutions near the reference periodic orbit also remain continuously visible from Earth. As a family of L_2 halo quasi-periodic orbit evolves away from the reference periodic solution, however, some tori may pierce the lunar shadow cone. Accordingly, an interactive catalog of periodic and quasi-periodic solutions provides a straightforward and rapid method for identifying the space of feasible solutions. Consider the large L_2 halo quasi-periodic orbit in Figure 12(b), which possesses a Jacobi constant of $JC = 3.15$. For this orbit, the catalog automatically highlights trajectory segments that provide a line-of-sight to the Earth. As portrayed in Figure 12(a), the MPLFS can maintain a 550 day-long Earth-visibility window. While this time interval may be feasible for many mission scenarios, the selected quasi-periodic orbit does not provide continuous line-of-sight for a multi-year mission. To identify a baseline trajectory with indefinite visibility at this same energy level, the torus size may be reduced. For instance, consider the smaller torus displayed in Figure 13(b). As evident from the angular deviation time history in Figure 13(a), motion along the quasi-periodic orbit never passes within the lunar shadow cone. Accordingly, this quasi-periodic orbit supplies a candidate baseline orbit for the MPLFS mission scenario. To identify an alternate candidate orbit that satisfies the continuous visibility requirements, tori near other members of the L_2 halo family are explored. Near rectilinear halo orbits, for example, produce viable candidate tori. For instance, motion along the torus depicted in Figure 14(a) remains entirely outside the lunar shadow cone. The quasi-periodic motion along this torus, identified and exported from the catalog, can be transition to a higher-fidelity design environment such as ATD. The resulting solution, differentially corrected for continuity in an point mass ephemeris model of the Earth, Moon and Sun, is plotted in Figure 14(b) and requires no maintenance maneuvers. As demonstrated through this example, a catalog of periodic and quasi-periodic solutions in the CR3BP facilitates exploration of orbit options that satisfy the constraints associated with a general mission concept in the Earth-Moon system. Furthermore, the resulting analysis reflects the dynamical structures that are retained in a high fidelity ephemeris model.

In-Orbit Lunar Facility

Rapid access and visualization of families of quasi-periodic solutions also supports a well-informed examination of natural trajectories that enables mission scenarios for a single spacecraft. Consider, for example, the preliminary selection of a baseline orbit for a long-term human habitat in the lunar vicinity. In addition to establishing a long-term lunar presence, exploration of the polar regions of the Moon has recently garnered interest for its scientific return. Accordingly, a space-based infrastructure that orbits the Moon should also support manned excursions to the lunar poles. This requirement can be translated into a constraint that the spacecraft must be able to view the north (or south) polar regions from above (or below). Due to their favorable stability properties, members of the DRO family are frequently proposed as reference orbits for lunar infrastructure. Smaller



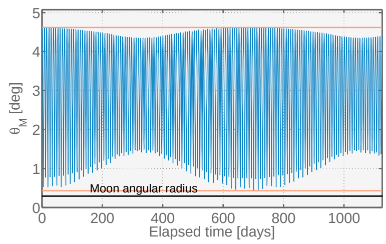
(a) Angle separation history.



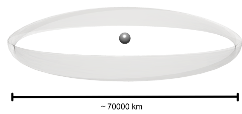
(b) $y-z$ view of the torus.

Figure 12: L_2 halo quasi-periodic arc that satisfies the continuous coverage requirement for the MPLFS mission example.

periodic members of the DRO family lie entirely within the Earth-Moon orbital plane, limiting the visibility of the polar regions. In a preliminary analysis of orbit options, the interactive catalog is used to identify any nearby quasi-periodic DRO orbits that regularly possess an out-of-plane extension above the lunar radius. Thus, the geometric size of a candidate quasi-periodic solution in the z-direction, B_z, should be greater than the diameter of the Moon. In fact, some quasi-periodic members of the DRO family do extend out-of-plane, as depicted by the nonzero values of B_z corresponding to the shaded regions in Figure 15. While periodic members of the DRO family may not support the described lunar infrastructure mission, a catalog of orbits and their characteristics rapidly reveals that quasi-periodic orbits can certainly meet the mission requirements. Consider, for example, a small DRO with an orbital period of 13.2 days, similar to the proposed reference orbit for the asteroid redirect mission.[2] Since this orbit possesses a two-dimensional center subspace, there are three-dimensional quasi-periodic orbits that exist in its vicinity. Using the catalog, a large torus from this family of quasi-periodic orbits is selected. The automatic detection feature in the quasi-periodic module of the catalog is employed to identify arcs along the quasi-periodic trajectory that pass above the lunar radius in the z-direction. These segments are highlighted in the z-component time history of the three-dimensional quasi-periodic motion, portrayed in Figure 16(a). As evident in this figure, a spacecraft in

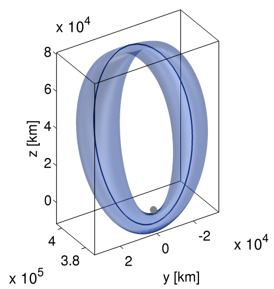


(a) Angle separation history.

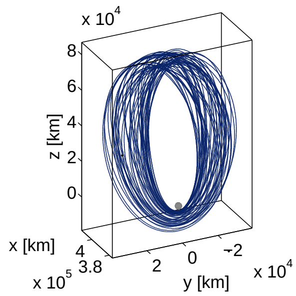


(b) $y-z$ view of the torus.

Figure 13: L_2 halo quasi-periodic torus that satisfies the continuous coverage requirement for the MPLFS mission example.



(a) Quasi-periodic torus in CR3BP.



(b) Solution converged in ephemeris.

Figure 14: Comparison of near rectilinear halo trajectory for different Earth-Moon system models. Ephemeris solution converged in ATD for Jan 1, 2023, including Earth, Moon, and Sun gravity.

this reference orbits frequently passes above the north pole, potentially supporting crewed or robotic excursions to these regions. In fact, these passages provide 8 days of continuous coverage every 10 days. A sample segment along the gray torus is highlighted in red in Figure 16(b). Longer or more frequent polar viewing windows may be obtained by selecting an alternate torus, or varying the reference periodic DRO. This complex design space is rapidly explored using a catalog of periodic and quasi-periodic solutions. The resulting analysis can be verified by correcting the selected solutions in a higher-fidelity ephemeris model.

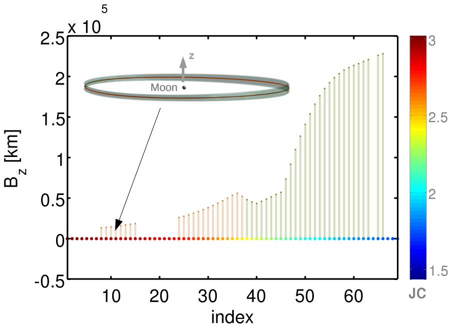
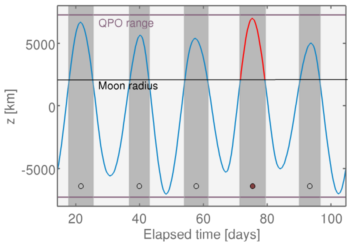
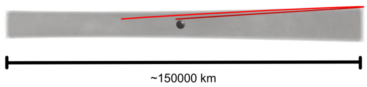


Figure 15: Out-of-plane extension and geometry of available quasi-periodic solutions associated to different DROs (indentified by the catalog index).



(a) Out-of-plane component history.



(b) $x - z$ view of the torus.

Figure 16: DRO quasi-periodic arc that enables coverage of the lunar poles for an in-orbit space habit near the Moon.

CONCLUDING REMARKS

To support the design of trajectories in the Earth-Moon system, a graphical environment for the comparison and selection of candidate orbits is explored. This framework is based on an interactive catalog of solutions for the CR3BP, a simplified gravitational model that enables preliminary trajectory design the Earth-Moon system. The set of readily available solutions incorporated in the catalog include both periodic and quasi-periodic orbits. These bounded motions are straightforwardly characterized via simple geometrical quantities that reflect both the size and behavior. Along with interactive visualizations of the trade space, these characteristics parameters are useful in identifying ordered motions that satisfy a given mission constraint. In fact, the capabilities of an interactive framework for exploring and conducting trade-offs during the preliminary mission design phase are apparent in the design of mission concepts that support operations within either cislunar or interplanetary space. Furthermore, this early analysis of candidate orbits in the CR3BP is validated when transitioned to an ephemeris model, thereby demonstrating the utility of a catalog framework.

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