How do gravity and ice temperature affect the performance of thermal melting probes?

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Julia Kowalski, Kai Schüller and the EnEx Team
A roadmap to exploring the Ocean World Enceladus

• before 2012 robotic student project
• 2012 – 2015 EnEx-MIDGE collaboration - advancing technologies and subglacial sample return in Antarctica
• 2015 further development of key technologies
Modeling the dynamics of melting probes – the 0D approach

Engineering model:

Aamot, CRREL-TR-194. 1967

Energy balance yields:

\[
V = \frac{P_m}{A \rho (h_m + c_p(T_m - T_{\text{ice}}))}
\]

\(V\): melting velocity  \(P_m\): input power  
\(A\): crosssection of the probe  \(\rho\): density of the ice

\(c_p\): specific heat capacity of the ice  
\(T_m\): melting temperature  
\(T_{\text{ice}}\): ice temperature  
\(h_m\): melting enthalpy of ice

How do melting probes perform in low gravity conditions?

The IceMole’s 2015 design:
Modeling the dynamics of melting probes – the 4D approach

The current state of the probe is given by its center-of-mass and its attitude:

\[ \xi(t) := \begin{bmatrix} X(t) \\ Q(t) \end{bmatrix} \]

First derivative yields translational and angular velocity:

\[ \frac{d}{dt} \xi(t) = \begin{bmatrix} V(t) \\ \omega(t) \end{bmatrix} \]

The **Euler-Newton equations** allow to determine the trajectory based on applied forces:

\[
\begin{align*}
    m \frac{d}{dt} V(t) &= F(t, u(t)) \\
    I \frac{d}{dt} \omega(t) &= T(t, u(t)) - \omega \times I \omega
\end{align*}
\]

The forces depend on the position of the liquid-solid interface, hence the ambient state \( u \).

The current ambient state of the ambient is given by temperature, velocity and pressure:

\[ \begin{bmatrix} T(t, x) \\ v(t, x) \\ p(t, x) \end{bmatrix} \]

It is subject to a PDE operator

\[ \frac{\partial}{\partial t} u(t, x) = \mathcal{L}(u(t, x), \xi(t), \frac{d}{dt} \xi(t)) \]

that depends on position and attitude of the probe, as well as its melting velocity.
Modeling the dynamics of melting probes – the smart way

Microscale melt film determines the probe’s macroscale dynamics

\( \nabla \cdot \mathbf{u} = 0 \)
\( \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} \)
\( \partial_t T_l + (\mathbf{u} \cdot \nabla) T_l = \alpha_l \Delta T_l \)

\( \Omega_s: \) Heat equation in the solid ice
\( \partial_t T - (\nabla \cdot \nabla) T_s = \alpha_s \Delta T_s \)

Water-ice interface conditions:
- no-slip
- inflow according to melting rate
- melting temperature
- Stefan condition

Heat source surface:
- no-slip
- no inflow
- temperature or heat flux
- Newton’s third law:

\[ \int_{\text{surface}} p \, d\sigma = F \]
Hybridized computational model: SimCoMet – Simulating Contact Melting

Energy balance:
\[\epsilon Pe (u \partial_x T + w \partial_z T) = \partial^2_z T\]

Transformation:
\[
\begin{align*}
\xi &= x \\
\eta &= z / \delta(x)
\end{align*}
\]

\[
\begin{align*}
u \partial_x T + w \partial_z T &= u \partial_\xi T + \frac{1}{2} (w - w_0 \partial_\xi \delta) \\
\partial_z T &= \partial_\xi T \left( \frac{d \xi}{dz} \right)^2 + 2 \partial_\xi T \left( \frac{d \eta}{dz} \right) + \left( \frac{d \eta}{dz} \right)^2 \\
\partial_\eta T &= \left( \frac{d \eta}{dz} \right)^2 + \frac{1}{\delta^2} \partial_\eta T
\end{align*}
\]

Solved using FDM

Stefan condition (for each x):
\[\partial_z T|_{z=\delta(x)} = \frac{\rho v(x, \delta(x))}{\kappa} (h_m + c_p (T_m - T_x))\]

Initial guess:
- melting velocity
- temperature

Solve Reynolds equation (1D/2D)

Force equilibrium?

Solve energy balance (2D/3D)

Stefan condition?
Contact melting – some fundamental results

Rotational melting modes

Spatially varying heat flux distribution
Leverage contact melting theory to study performance in extreme cryoenvironments.

Consider melt film (red) as a closed system:

\[ \dot{Q}_H - \dot{Q}_E - \dot{Q}_C = 0 \]

Convective loss \( \dot{Q}_E \)
Input power \( \dot{Q}_H \)
Utilized power \( \dot{Q}_C \)

Melt film

Allows us to:
- study the melting velocity / efficiency
- determine the critical refreezing length

for a reference probe (1m long / 0.06m radius / 25kg)
in a representative cryoenvironments:
- Mars: 210 K / 3.7 m/s²
- Enceladus: 150 K / 0.1 m/s²
- Europa: 100 K / 1.3 m/s²
Melting velocity over contact force

Melting velocity of 1kW probe:

Melting velocity of a 5kW probe:
Implications for the melting probe's design

![Diagram of melting probe with input powers of 3.5 kW and 1.5 kW]

**Enceladus (150K)**

- **Critical Refreezing length [m]**
- **Input power at the melting head [W]**
- **Melting velocity [m/s]**

<table>
<thead>
<tr>
<th>Input Power</th>
<th>Critical Refreezing Length</th>
<th>Melting Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 kW</td>
<td>0.5 m</td>
<td>3.5 m/s</td>
</tr>
<tr>
<td>1.5 kW</td>
<td>0.25 m</td>
<td>2 m/s</td>
</tr>
</tbody>
</table>

Equations:

- $F^* = 1.37 N$
- $F^* \rightarrow \infty$

Schüller, Kowalski, Icarus (accepted)
Conclusions and Outlook

Conclusions

• We developed a flexible micro-scale contact melting simulation model

• We gained further insight into the behavior of melting probes in extreme cryoenvironments

• We contributed to the fundamental understanding of contact melting processes

• First validation experiments have been promising

Next steps

• Trajectory model for the IceMole (trajectory control will be tested in 2018 field campaign)

• High altitude rocket and vacuum chamber experiments due soon (VIPER / EnEx- nExT)

• In proposal phase: Smart process model and data integration for ice exploration
Thanks ...