The mental representation of universal quantifiers: Evidence from verification

Tyler Knowlton (Maryland), Paul Pietroski (Rutgers), Justin Halberda (Johns Hopkins), Jeffrey Lidz (Maryland)
tzknowlt@umd.edu

The meaning of sentences like *every circle is blue* could be represented in speakers' minds in terms of individuals and their properties (e.g., for each thing that's a circle, it's blue) or in terms of relations between groups (e.g., the blue things include the circles). In other words, both the tools of first-order logic, as in (1-2), and the tools of second-order logic, as in (3-4), can be used to represent the meaning of universally quantified statements.

(1) $\forall$x:Circle(x)[Blue(x)] (3) Circles $\subseteq$ Blue-Things
(2) $\neg\exists$x:Circle(x)[$\neg$Blue(x)] (4) Circles = Circles $\cap$ Blue-Things

We offer evidence that this formal distinction is psychologically realized in a way that has detectible symptoms. Specifically, we argue that *each* is represented in first-order terms – like (1) or (2) – but *every* and *all* are represented in second-order terms, like (3) or (4).

Participants (12 per experiment) were presented with 272 quantificational statements (e.g., "every big dot is blue") and asked to judge their truth with respect to dot displays (see Fig A). Then they were asked to guess the cardinality of a subset (e.g., "how many big dots were there?"). If a quantifier Q has a first-order meaning, *Q big dot(s)* should prompt speakers to represent the big dots as individuals. In doing so, they won't automatically build a cardinality representation for that group. If Q has a second-order meaning, *Q big dot(s)* should bias speakers to represent the big dots, taken together. One consequence of representing a group is automatically building an estimate of its summary statistics, one of which is cardinality (e.g., Ariely, 2001; Alvarez, 2011). Therefore, second-order quantifiers should lead to better performance on the relevant "how many" questions than their first-order counterparts.

To judge how well participants knew a set's cardinality, responses were fit with a standard psychophysical model of number estimation, which allows for comparisons of accuracy and precision (i.e., amount of variability in responses) (Odic et al., 2016). Experiment 1 compared *most*-statements like "most of the big dots are blue" to existential-statements like "there is a big dot that's blue" and "there are big dots that are blue". Because *most* is necessarily second-order (Rescher, 1962; Barwise & Cooper, 1981), evaluating *most*-statements should lead to group representations of the restrictor set (e.g., big dots). Consequently, participants should demonstrate good estimates of its cardinality. We find that participants were more accurate (t_{11}=4.57, $p<0.001$) and more precise (t_{11}=3.02, $p<0.05$) at estimating the cardinality of this set after *most*-statements than after existential-statements. For unmentioned (distractor) sets' cardinalities (e.g., small dots) however, participants showed poor performance regardless of which statement they evaluated (Fig B).

Experiment 2 (Fig C) pitted *each* and *all* against each other with statements like "each/all of the big dots are blue". Participants were more accurate (t_{11}=3.75, $p<0.01$) and more precise (t_{11}=5.34, $p<0.001$) at guessing the cardinality of the restrictor set following *all*-statements compared to *each*-statements. This suggests that *all* has a second-order representation, whereas *each* has a first-order representation. Experiment 3 (Fig D) finds the same result when comparing *every*- to *each*-statements: *every* patterns like *all* and *most* in terms of relative accuracy (t_{11}=2.39, $p<0.05$) and precision (t_{11}=4.82, $p<0.01$) boosts. Experiment 4 (Fig E) directly compared *every*- with *all*-statements. Given the results of experiments 2 and 3, we predicted no effect of quantifier. These predictions were borne out in both accuracy (t_{11}=0.43, p=0.67) and precision (t_{11}=0.44, p=0.67), as performance was nearly identical across blocks.

Given the identity of the task and materials in all respects besides the difference in quantifier, our results are well explained by appeal to meaning. Namely, *each* has a first-order representation but *every* and *all* have second-order representations, despite the truth-conditional equivalence of all three. More broadly, our results support the idea that quantifier meanings are mentally represented at a finer grain size than truth-conditions.

Figure A.

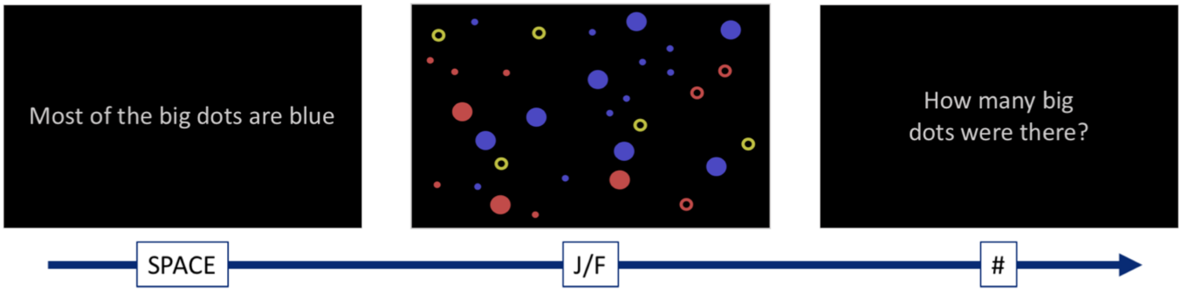

Figure B. Experiment 1

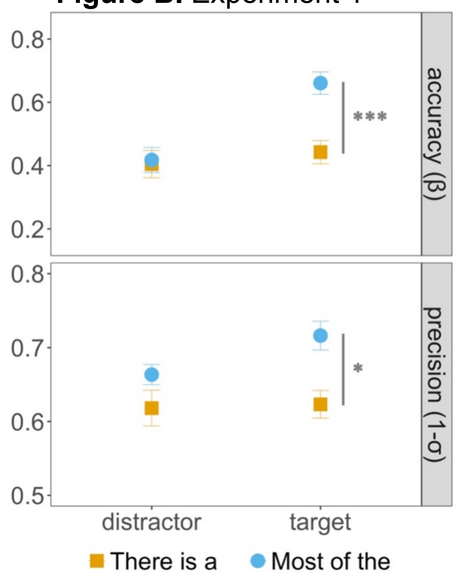

Figure C. Experiment 2

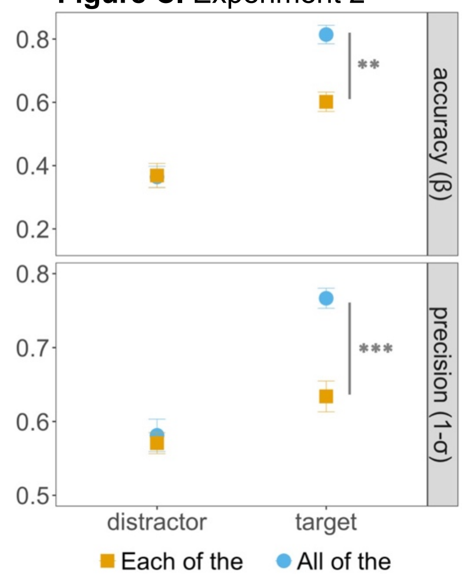

Figure D. Experiment 3

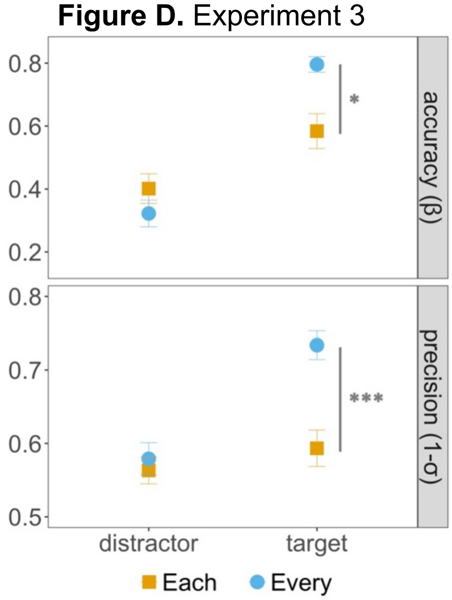

Figure E. Experiment 4