Spatial Visualization

Workshop 7: Write a Rule

Overview

Grade level: undergraduate engineering students

Estimated time required: 2 hours

Expendable materials: scrap paper and pencils; (optional) colored pencils

Reusable materials: practice tests and workbooks; (optional) snap cubes

See Materials List (below) for details.

Summary

Students learn a new spatial visualization test-taking technique called “write a rule” and apply it, along with their pre-requisite knowledge gained during the previous workshops/lessons in this series, to practice answering test problems. The write a rule technique is a process in which the rotation of one object is simply defined in order to determine the position of a second object after the same rotation. It is a method used to simplify spatial visualization problem solving and has been proven to be effective for many students. Students practice the technique on the first eight rotating cube problems of the provided practice test. Then they apply it to workbook problems. After conducting this lesson/workshop, the instructor may want to administer a spatial visualization test such as the *Purdue Spatial Visualization Test: Rotations* (PSVT:R) to measure student knowledge and skills.

Engineering Connection

As seen in the previous workshops in this series, rotating objects is a spatial visualization technique that enables engineers to visualize complicated assemblies in mechanisms and other systems in fields such as physics, chemistry, mathematics and engineering. Spatial visualization is an essential and learnable skill that engineers use to clearly communicate their ideas to other people so the ideas can ultimately be turned into real-world products, structures and systems.

The write a rule technique has real-world applications. It is a simple and useful way to quickly make spatial visualization observations and comparisons on the job. For example, imagine working on a bridge construction site where consecutive horizontal I-beams need to be oriented the same way as a vertical beam. As you watch the crew rotate the first beam from its flat horizontal position to its vertical orientation, you observe “the bottom become the side” —the “rule.” Now you can apply this rule to each of the beams to make sure they are all placed in the correct position. While this is a simple example, the same technique can be applied to rotations at all scales and complexities. This brings efficiency to many engineering projects and reduces the confusion that often comes with spatial visualization.

Pre-Requisite Knowledge

Before taking part in this spatial visualization lesson, students should have learned about spatial visualization in the previous six workshops. They should have a firm knowledge about isometric drawing, how to use isometric paper and coded plans (as taught in the lesson, *Isometric Drawings and Coded Plans*), as well as orthographic views (as taught in the lesson, *Orthographic Views*). They should also understand how to visualize, perform and draw one- and two-axis rotations with the help of the right-hand rule. Additionally, students should be familiar with the Cartesian coordinate system (x-, y-, z-axes) and degrees of rotation.

Learning Objectives

After this workshop, students should be able to:

- Compare rotated objects more efficiently.
- Apply the rotation pattern of one object to a second object.

Materials List

Item	Description & Amount	Source/Supplier/Cost	Station #s
Practice Test	For students to develop test-taking techniques	SV Practice Test (pdf)	1
Practice Test answers	For students to check their work	SV Practice Test Answer Key (pdf)	1
Snap cubes (optional)	Interlocking cubes, ~8 cubes per student	Such as: Higher Ed Services ($3 for 15 cubes) or Amazon ($85 for 200)	1, 2
Pencil with eraser	Any erasable pencil; pens are not recommended	School/students provide	1, 2
Colored pencils (optional)	To associate drawings to the snap cubes; to share among all students	Such as: Crayola colored pencils at Amazon ($8 for 50)	1, 2
Developing Spatial Thinking *Workbook*	1 per student pair; includes web-based software used with workbook	$40 for workbook/software from Higher Ed Services	2 (workbook)
Computer or laptop	1 per student pair	School/students provide	4
Workbook answers	For the instructor and for students to check their work	Workbook Answer Key (pdf)	2
PowerPoint® presentation	For the instructor, slides 40-43	Spatial Visualization Presentation (pptx)	NA

Introduction & Motivation

Have the slide presentation up and displayed to the class, starting with slide 40. The slides are animated, so a mouse or keyboard click brings up the next graphic or text.

Click to slide 41, an example question from the Purdue Spatial Visualization Test: Rotations [PSVT:R]. Ask students to tell you the answer to this example problem. Note that the format is similar to SAT questions, but with shapes instead of words.

Today we are going to explore a test-taking technique called "write a rule" that is also quick and easy for real-world problems, too. Even though you have acquired a great set of spatial visualization skills, you have one more important technique to learn.

Show slide 42, which is the same as Figure 1.

You have mastered two-axis rotations, but here is another way to tackle these types of problems. For example, in the cube diagram shown, we could define a two-axis rotation as a negative rotation about the y-axis and a negative rotation about the z-axis. *(Note that many different solutions to this problem exist.)* Or, we could write a rule! What we mean by "rule," is to identify how the first object is rotated—that's the "rule"—and then apply that "rule" to the second object to determine what it looks like when rotated the same way as the first object.

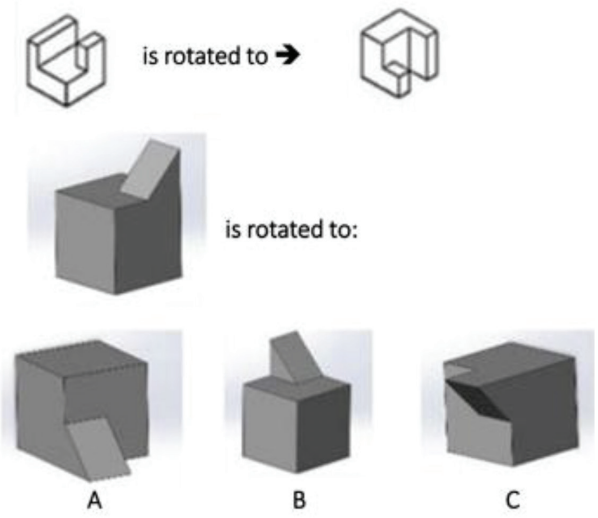

Figure 1. *Which rotation of the gray object is analogous to the rotation of the white object?*

This is how it works: In the first object, we pick a distinctive surface that we are able to identify after any rotation. The front side of the object (left side) is L-shaped and no other surfaces are similar to it on the object. After rotation, the L-shape is on the top. In this case, the rule is "front to top." Applying this same rule to the second object (dark gray shape, below), we see that the front side is a square and the only rotated view that has a square as the top is answer A. So, the answer is A.

This example rule is only one option; you could write many different "rules" for each individual problem.

It is also important to note that while this technique is often successful, it is not flaw-proof. Make sure you write your rules with caution. For instance, if you rotate a pyramid using the rule "the top becomes a triangle," you are more likely to be mistaken since almost every side is a triangle. So, it often helps to pick a distinctive side shape when making a "rule" to follow. And if not sure, verify by making and testing another rule using a different side.

Slide 43 recaps the two workshop stations. Leave slide 43 up so students know what to accomplish at each station.

Vocabulary

Term	Definition
coded plan	A two-dimensional depiction of an object that defines the volume based on the top view of the object.
isometric	Of or having equal dimensions. The isometric view of an object is the angle at which an equal angle (120°) exists between all axes (such as looking down a corner of the object).
isometric paper	A grid of dots arranged equidistant from one another. Used in making isometric sketches. Also called three-dot paper.
one-axis rotation	The result of rotating an object solely about a single axis. In this workshop, the rotations only occur in 90º increments. Also called single-axis rotation.
orthographic view	A way to draw an object that shows three views of an object from the three planes in an orthogonal (right angle) coordinate system. The views represent the exact shape of an object as seen from one side at a time as you are looking perpendicularly to it. Depth is not shown. An orthographic drawing is also called a multi-view drawing
right-hand rule	A useful memory tool in the rotation of objects that uses a person's right hand and fingers to help in understanding orientation conventions for vectors in three dimensions. Often used in physics and math.
two-axis rotation	The result of rotating an object about two axes. Can take place around the x, y, or z axes at 90º, 180º or 270º.
write a "rule"	A spatial visualization test-taking technique in which a "rule" is identified to describe the rotation of one object and then applied to a second object to determine its parallel rotation.

Lesson Procedure

Before the Workshop

- Gather materials.
- Make copies of the SV Practice Test, one per student, and the SV Practice Test Answer Key, about one for every five students, which they can share for checking their answers.
- Prepare to project the Spatial Visualization Presentation, a PowerPoint® file, and use its content to aid in your instruction, as makes sense for your class. Slides 40-43 support this lesson. The slides are animated so a mouse or keyboard click brings up the next graphic or text.
- Prepare the four stations, as described in slide 43 and Figure 2.

Station 1: Write a Rule Practice	Station 2: Workbook Drill
• Individually, complete the **first 8 problems** on the practice test • For each problem, **write a rule** and then select the answer, writing it on scrap paper • Check your answers with the answer key	• Individually, answer questions on **EVEN** pages rot2-1 to rot2-17 on scrap paper • Check your answers with the answer key

Figure 2. *The four stations for Workshop Seven.*

With the students: Introduction

- Present to the class the Introduction & Motivation content, supported by slides 40-43.
- Divide the class into two groups that will rotate through the two stations.
- Hand out the practice tests and workbooks to student pairs; direct them to work on scrap paper so the practice tests and workbooks can be reused.
- Make scrap paper available to all students.

Lesson Procedure Continued

Station 1: Write a Rule Practice

Have students individually complete the first eight problems of the practice test. For each problem, they write a rule and then select an answer and write it on a piece of scrap paper. When finished, have them check their answers with the answer key. Once this is done, they are welcome to share their "rules" and problem-solving techniques with a partner.

Station 2: Workbook Drill

Have students individually answer the workbook questions on even pages rot2-1 to rot2-17, recording their answers on scrap paper. After they have completed the problems, have them check their answers with the answer key. Then have them communicate with neighboring students about answers they may have gotten wrong and collaborate on ways to understand and solve the problems.

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Instructor Tips

- Initially, "writing a rule" can be intimidating for students since it is a creative process with no specific solution.
- Encourage students to choose the methods most obvious to them and practice creating their own rules that apply to each problem. While another student's rule might be obvious once it's revealed, it is more helpful for each student to practice this exercise on his/her own.