Spatial Visualization

Workshop 6: Reflections & Symmetry

Overview

Grade level: undergraduate engineering students

Estimated time required: 2 hours

Expendable materials: isometric paper printouts, scrap paper and pencils

Reusable materials: workbooks, software, computers, snap cubes (used for all workshops)

See Materials List (below) for details.

Summary

In this workshop, students explore the spatial visualization techniques of reflection and symmetry. This workshop serves as an add-on to the (previous) *Inclined Planes and Curved Surfaces Workshop* with the goal to advance students' learned spatial visualization skills. It begins with an explanation of both reflection and symmetry. Then students explore reflections by building objects from cube blocks, defining a reflection plane, and drawing the object before and after a reflection. Then they individually practice reflection and symmetry workbook problems. They also have the opportunity to ask the instructor questions in an open-ended help station called "Meet the Expert."

Engineering Connection

Reflection and symmetry are an everyday part of nature and a prevalent theme in architecture, chemistry and engineering. For example, the external human body and even parts of the internal body are symmetrical. This is extremely useful to biomechanical engineers who design prosthetic parts and must understand body parts, how they interact and replicate natural forms. When designing replacement parts using computer-aided design (CAD), they might use these symmetries to expand their designs. This is only possible, however, using the correct spatial visualization techniques. Similarly, civil engineers might design symmetrical bridges. Understanding the symmetries within a bridge design may simplify the design process by only needing to do structural load calculations for half the bridge components. This lesson helps students develop an eye for and skill for these important reflection and symmetry techniques.

Pre-Requisite Knowledge

Before taking part in this spatial visualization lesson, students should have learned about spatial visualization in the previous five workshops. They should have a firm knowledge about isometric drawing, how to use isometric paper and coded plans (as taught in the lesson, *Isometric Drawings and Coded Plans*), as well as orthographic views (as taught in the lesson, *Orthographic Views*). They should also understand how to visualize, perform and draw two-axis rotations with the help of the right-hand rule. Additionally, students should be familiar with the Cartesian coordinate system (x-, y-, z-axes) and degrees of rotation.

Learning Objectives

After this workshop, students should be able to:

- Define a reflection plane and draw a reflection about the plane.
- Identify symmetric objects from non-symmetric ones.
- Identify different planes of symmetry.

Materials List

Item	Description & Amount	Source/Supplier/Cost	Station #s
Isometric paper	Isometric, three-dot paper for sketching, 2+ per student	Isometric Paper (pdf); print double-sided as needed	1, 2, 3
Snap cubes	Interlocking cubes, 8 cubes per student	Such as: Higher Ed Services ($3 for 15 cubes) or Amazon ($85 for 200)	1, 3
Pencil with eraser	Any erasable pencil; pens are not recommended	School/students provide	1, 2, 3
Colored pencils	To associate drawings to the snap cubes; to share among all students	Such as: Crayola colored pencils at Amazon ($8 for 50)	1, 3
Developing Spatial Thinking *Workbook & Instructional Software*	1 per student pair; includes web-based software used with workbook	$40 for workbook/software from Higher Ed Services	2 (workbook) 4 (software)
Computer or laptop	1 per student pair	School/students provide	4
Workbook answers	For the instructor and for students to check their work	Workbook Answer Key (pdf)	2
PowerPoint® presentation	For the instructor, slides 35-39	Spatial Visualization Presentation (pptx)	NA

Introduction & Motivation

Have the slide presentation up and displayed to the class, starting with slide 35.

Today we are going to explore the concepts of reflection and symmetry. Reflection and symmetry are an everyday part of nature and a common theme in architecture, chemistry and engineering.

Move to slide 36.

Let's begin by defining object reflections. The drawings on this slide show how a plane acts as a "mirror" to create a new orientation—or reflection—of an object. A reflection is simply a geometric transformation across a mirrored plane.

Slide 37

The best way to draw reflections is to begin by defining a plane. Then project the object onto the reflection plane and use the corresponding points to determine where edges or surfaces are reflected. Start with the edges closest to the plane and then move further away.

Slide 38

Symmetry occurs when two sides of an object are mirror images of one another. If an object has symmetry, a plane can divide the object in half such that the two sides are mirror images of one another. Symmetrical objects have one or more planes that create identical shapes. It is important to note that a 180° rotation about the plane of symmetry creates that same shape. which is a good way to check to determine whether an object has symmetry.

Slide 39 recaps the four workshop stations. Leave slide 39 up so students know what to accomplish at each station.

Vocabulary

Term	Definition
coded plan	A two-dimensional depiction of an object that defines the volume based on the top view of the object.
isometric	Of or having equal dimensions. The isometric view of an object is the angle at which an equal angle (120°) exists between all axes (such as looking down a corner of the object).
plane	A flat surface within space (think of the xy plane, xz plane, and zy plane). When thinking about reflections, planes act as an invisible sheet between an object and its reflection.
reflection	A geometric transformation in which an object is transformed across a mirrored plane. Any point of the original object has a matching point on its reflection, just like a mirror reflection.
symmetry	Symmetry occurs when two sides of an object are mirror images of one another. If an object has symmetry, a plane can divide the object in half such that the two sides are mirror images of one another. Symmetrical objects have one or more planes that create identical shapes. A 180° rotation about the plane of symmetry creates that same shape.

Lesson Procedure

Before the Workshop

- Gather materials.
- Make copies of the Isometric Paper. Print double-sided to reduce paper use and increase drawing area per page.
- Prepare a way to digitally get students the web-based software link they will need for this lesson such as by email or by preparing a workshop website with the link.
- Prepare to project the Spatial Visualization Presentation, a PowerPoint® file, and use its content to aid in your instruction, as makes sense for your class. Slides 35-39 support this lesson. The slides are animated so a mouse or keyboard click brings up the next graphic or text.
- Prepare the four stations, as described in slide 39 and Figure 1.

Station 1: Block Relay

- Build an object with 6 blocks and define a **reflection plane**
- Draw **isometric views** of the object before and after reflection
- **Pass object and reflection plan** to neighbor and repeat
- Check drawing for accuracy with neighbor

Station 2: Workbook Drill

- Individually, answer questions on **EVEN** pages reflx/sym04 to reflx/sym14
- Check your answers with the answer key

Station 3: Meet with the Expert

- Bring your questions to the expert(ish)

Station 4: Computer-Aided Visualization

- Read *Module 9 - Object Reflections on Symmetry*
- Complete the **reflections and symmetry** exercises

Figure 1. *The four stations for Workshop Six.*

With the students: Introduction

- Present to the class the Introduction & Motivation content, supported by slides 35-39.
- Divide the class into four groups that will rotate through the four stations.
- Give each student 2 pieces of isometric paper and 8 cubes.
- Explain that students are to primarily work independently on their drawings, but feel free to share and troubleshoot with their partners.
- Direct students to work on one piece of paper until it is full.

Lesson Procedure Continued

Station 1: Block Relay

Students begin by each building a shape with six blocks. Next, they define a reflection plane. Then they draw the isometric views before and after the reflection. Then they pass the object to a neighbor and repeat. After students have completed drawing their own objects and their neighbors' objects, have them collaborate and check their drawings for accuracy.

Station 2: Workbook Drill

Have students individually answer workbook questions on even pages reflx/sym04 to reflx/sym14, recording their answers on scrap paper. Have them check their answers as they go, using the answer key. Direct them to communicate with neighboring students about answers they may have gotten wrong and collaborate on ways to understand and solve the problems.

Station 3: Meet with the Expert

This is an opportunity for students to bring their questions to the instructor. Encourage students to ask any spatial visualization questions they have on this workshop and the previous five workshops. Work through problems with them and provide examples.

Station 4: Computer-Aided Visualization

Have students individually read *Module 9 – Object Reflections on Symmetry* from the *Developing Spatial Thinking Software* and complete all object reflections and symmetry exercises.

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Instructor Tips

- In Station 2, the symmetry workbook exercises typically challenge students. Provide examples of how an object might have multiple planes of symmetry. For example, a rectangular prism has five, while a cylinder has an infinite number.
- For the first few problems in Station 2, have students draw the different planes of symmetry and explain why each is a plane of symmetry.
- During Station 3, if students hesitate to ask questions, use your best judgement to identify which concepts they may need extra help with and work through more examples. With encouraged interaction, it is likely that questions will arise.