



Spatial Visualization

Workshop 5: Inclined Planes & Curved Surfaces

Overview

Grade level: undergraduate engineering students

Estimated time required: 2 hours

Expendable materials: isometric and orthographic paper printouts, playdough, scrap paper and pencils

Reusable materials: workbooks, software, computers, snap cubes (used for all workshops)

See *Materials List (below)* for details.

Summary

In this workshop, students focus on expanding and applying the knowledge learned in workshops 1-4 to shapes with inclined planes and curved surfaces. A full explanation of how orthographic and isometric views apply to such shapes is provided. Students then practice by building shapes with an inclined plane or curved surface and drawing the isometric views. Then they individually answer workbook problems. They also practice creating orthographic views for inclined planes and curved surfaces in an interactive *Orthographic Telephone* exercise. For additional explanation of these concepts, students also read and complete a software exercise.

Engineering Connection

Visualizing and drawing inclined planes and curved surfaces is a valuable technique that enables engineers to create complicated designs, plans and assemblies across all engineering disciplines. Most engineering applications require the depiction of complicated shapes with many inclined planes and curved surfaces. The real world cannot be described by orthogonal shapes! For example, when an engineer is calculating the volume of a 3D shape it is rarely a simple cube. They must use advanced calculus techniques to perform multiple integrals in order to find the volume of the object. Imagine a mechanical engineer aiming to minimize the volume of a component to reduce the material cost. Picture the way a chemical engineer describes how two proteins interact in a cell. Inclined planes and curved surfaces exist within all fields of engineering and engineers need to be able to visualize these from all angles to optimize designs.

Pre-Requisite Knowledge

Before taking part in this spatial visualization lesson, students should have learned about spatial visualization in the previous four workshops on orthographic and isometric drawings, and one- and two-axis rotation. They should have a firm knowledge about isometric drawing, how to use triangle-dot paper and coded plans (as taught in the lesson, *Isometric Drawings and Coded Plans*), as well as orthographic views (as taught in the lesson, *Orthographic Views*). An understanding of how to visualize, perform and draw one- and two-axis rotations is recommended but not required. Additionally, students should be familiar with the Cartesian coordinate system (x-, y-, z-axes) and degrees of rotation.



Learning Objectives

After this workshop, students should be able to:

- ✓ Recognize and define inclined planes and curved surfaces.
- ✓ Build and draw isometric views of objects with both inclined planes and curved surfaces.
- ✓ Draw the three different orthographic views of objects with inclined planes and curved surfaces.
- ✓ Decipher the difference between flat surfaces, inclined planes and curved surfaces delineated in orthographic views.

Materials List

Item	Description & Amount	Source/Supplier/Cost	Station #s
Isometric paper	Isometric, three-dot paper for sketching, 2+ per student	Isometric Paper (pdf); print double-sided as needed	1, 2
Orthographic paper	Orthographic paper for sketching, 2+ per student	Orthographic Paper (pdf); print double-sided as needed	2, 3
Snap cubes	Interlocking cubes, 8 cubes per student	Such as: Higher Ed Services (\$3 for 15 cubes) or Amazon (\$85 for 200)	1, 3
Pencil with eraser	Any erasable pencil; pens are not recommended	School/students provide	1, 2, 3
Colored pencils	To associate drawings to the snap cubes; to share among all students	Such as: Crayola colored pencils at Amazon (\$8 for 50)	1, 3
Clay or modeling compound	Traditional playdough, for modeling shapes	Such as: 10-pack of 2-oz cans of Play-Doh for \$8 from Amazon	1
Developing Spatial Thinking Workbook & Instructional Software	1 per student pair; includes web-based software used with workbook	\$40 for workbook/software from Higher Ed Services	2 (workbook) 4 (software)
Computer or laptop	1 per student pair	School/students provide	4
Workbook answers	For the instructor and for students to check their work	Workbook Answer Key (pdf)	2
PowerPoint® presentation	For the instructor, slides 32-34	Spatial Visualization Presentation (pptx)	NA



Introduction & Motivation

Have the slide presentation up and displayed to the class, starting with slide 32.

Welcome back! Today we will be learning about inclined planes and curved surfaces. Let's begin by defining what they are and how they appear around us. You may have noticed that spatial visualization is a skill that can be applied to all shapes, not just cubes. Everyday objects have inclined planes, such as architectural structures like the Pyramids of Giza or even the small ramp-shaped door stop that props open a door. We see curved surfaces everywhere—from our coffee cups to our writing utensils. These objects and shapes can be depicted with the same methods that we have been using for orthogonal objects.

Move to slide 33, which is the same as Figure 1.

Inclined planes are sloped surfaces, meaning they are not parallel to any other planes within a cubic shape. In orthographic views, an inclined plane appears in one of the three views as an angled edge and as a plane in the other two. Typically, it appears smaller to the viewer because the two plane views do not show its dimensions fully. A curved surface can appear on any part of an object and takes on an organic form. In orthographic views, a curved surface appears as a circular or partially circular edge in one of the views and often reveals the true size and shape of the arc. In the remaining views, it appears as a rectangle. In this case, it is often shown with centerlines to indicate that it is curved rather than flat.

This image (on slide 33, same as Figure 1) displays the three orthographic views and an isometric view for a common inclined plane. Notice how the L-shape appears angled in the front view in the bottom left corner and as a plane in the other two views.

Slide 34 recaps the four workshop stations. Leave slide 34 up so students know what to accomplish at each station.

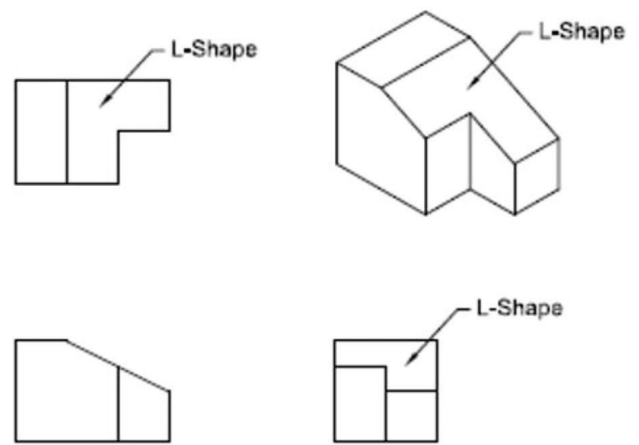


Figure 1. Orthographic views of a 3D object with an L-shaped inclined plane.



Vocabulary

Term	Definition
coded plan	A two-dimensional depiction of an object that defines the volume based on the top view of the object.
curved surface	A curved surface is a non-orthogonal and non-planar surface.
edge	A change in depth on the surface plane that is represented by a solid line.
front view	The two-dimensional depiction of an object from the z-axis perspective.
hidden edge	A change in depth within the surface plane that may be hidden from the specific view being drawn. Represented by a dotted line.
inclined plane	Inclined planes are sloped surfaces, meaning they are non-orthogonal to other planes within a cubic shape.
isometric	Of or having equal dimensions. The isometric view of an object is the angle at which an equal angle (120°) exists between all axes (such as looking down a corner of the object).
isometric paper	A grid of dots arranged equidistant from one another. Used in making isometric sketches. Also called three-dot paper.
orthographic view	A way to draw an object that shows three views of the object from the three planes in an orthogonal (right-angle) coordinate system. The views represent the exact shape of an object as seen from one side at a time as you are looking perpendicularly to it. Depth is not shown. An orthographic drawing is also called a multi-view drawing.
projection	The process of mapping 3D points onto a 2D plane. For example, imagine the silhouette or shadow of a tall building.
right-hand rule	A useful memory tool in the rotation of objects that uses a person's right hand and fingers to help in understanding orientation conventions for vectors in three dimensions. Often used in physics and math.
side view	The two-dimensional depiction of an object from the x-axis perspective.
top view	The two-dimensional depiction of an object from the y-axis perspective.



Lesson Procedure

Before the Workshop

- ✓ Gather materials.
- ✓ Make copies of the Isometric Paper and Orthographic Paper. Print double-sided to reduce paper use and increase drawing area per page.
- ✓ Prepare a way to digitally get students the web-based software link they will need for this lesson such as by email or by preparing a workshop website with the link.
- ✓ Prepare to project the Spatial Visualization Presentation, a PowerPoint® file, and use its content to aid in your instruction, as makes sense for your class. Slides 32-34 support this lesson. The slides are animated so a mouse or keyboard click brings up the next graphic or text.
- ✓ Prepare the four stations, as described in slide 34 and Figure 2.

Station 1: Block Relay	Station 3: Orthographic Telephone
<ul style="list-style-type: none"> • Use cubes to build a shape with an inclined plane or curved shape • Use Play-Doh to add inclined and curved shapes to the blocks • Draw the isometric view • Pass object to a neighbor and repeat (draw the isometric view of your neighbor's object) • Check both drawing for accuracy with neighbor 	<ul style="list-style-type: none"> • Build an object with an inclined plane or curved surface • Pass object to neighbor; the neighbor draws orthographic views of object and disassembles it • Second student passes orthographic drawings to different neighbor who rebuilds the object • Continue in circle until object arrives at originating student • <i>Did final version match original object?</i>
Station 2: Workbook Drill	Station 4: Computer-Aided Visualization
<ul style="list-style-type: none"> • Individually, answer questions on ODD pages inc1 to inc15 • Check your answers with the answer key 	<ul style="list-style-type: none"> • Read <i>Module 5 - Inclined Planes and Curved Surfaces</i> • Complete the inclined planes and curved surfaces exercises

Figure 2. The four stations for Workshop Five.

With the students: Introduction

- ✓ Present to the class the Introduction & Motivation content, supported by slides 32-34.
- ✓ Divide the class into four groups that will rotate through the four stations.
- ✓ Give each student 2 pieces of isometric paper, 2 pieces of orthographic paper, and 8 cubes.
- ✓ Explain that students are to primarily work independently on their drawings, but feel free to share and troubleshoot with their partners.
- ✓ Direct students to work on one piece of paper until it is full.



Lesson Procedure Continued

Station 1: Block Relay

Students begin by using the cubes to each build a shape with an inclined plane or curved surface; use playdough to add inclined and curved surfaces to the blocks. Next, draw the isometric view. Then they pass the object to a neighbor and repeat. After students have completed drawing both their own objects and their neighbors' objects, have them collaborate and check their drawings for accuracy.

Station 2: Workbook Drill

Students individually answer the workbook questions on odd pages inc1 to inc15, recording their answers on scrap paper. After they have completed the problems, have them check their answers with the answer key. Then have them communicate with neighboring students about answers they may have gotten wrong and collaborate on ways to understand and solve the problems.

Station 3: Orthographic Telephone

Students each build an object with an inclined plane or curved surface and pass the object to a neighbor. Then the neighbor draws the orthographic views of the object and disassembles it. Next, the second student passes the orthographic drawings to a different neighbor who rebuilds the object. This process continues in a circle until the object arrives at the originating student. As a class, have students share their experiences and discuss the results. Did the final version match the original object? What were any differences? If so, how might those errors be prevented (by improved drawings)?

Station 4: Computer-Aided Visualization

Have students individually read *Module 5 – Inclined Planes and Curved Surfaces* from the *Developing Spatial Thinking Software* and complete all inclines planes and curved surfaces exercises.

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Supporting Programs

Engineering Plus Degree Program, University of Colorado Boulder
Engineering Excellence Fund, University of Colorado Boulder

Acknowledgments

This workshop was developed by the Engineering Plus degree program in the College of Engineering and Applied Science at the University of Colorado Boulder, initially as a component of a first-year engineering design course. This workshop is part of a seven-workshop series developed and taught by Dr. Jacob L. Segil for undergraduate engineering students. The authors express their appreciation to Dr. Sheryl Sorby for her extensive work on this topic, which provided a foundation on which we could build.

Instructor Tips

- Inclined planes and curved surfaces can be intimidating to draw at first. This lesson involves thinking outside of the box, literally. Students might feel hesitant about creating organic lines and shapes in their drawings. Assure them this is okay by working through an example with them.
- Station 3 is a fun activity that works best with limited communication between students. Encourage students to draw silently on their own and then communicate after.