Spatial Visualization

Workshop 4: Two-Axis Rotations

Overview

Grade level: undergraduate engineering students

Estimated time required: 2 hours

Expendable materials: isometric paper printouts, scrap paper and pencils

Reusable materials: workbooks, software, computers, snap cubes (used for all workshops)

See Materials List (below) for details.

Summary

Students learn about two-axis rotations with a specific emphasis on how to rotate objects both physically and mentally about two axes. A two-axis rotation is a rotation of an object about a combination of x, y or z-axes, as opposed to a single-axis rotation, which is about a single x, y or z-axis. This workshop follows the same format as that of the one-axis rotation workshop. Students practice drawing two-axis rotations through an exercise using cube blocks to create shapes, and then drawing on isometric paper the shapes from various x-, y- and z-axis rotation perspectives. Then they individually apply the same rotation concepts in a workbook exercise. They also pair up with a partner to test their exclusively mental spatial visualization abilities at a station with no rotating built objects. Lastly, students further solidify two-axis rotation concepts via computer software exercises.

Engineering Connection

Rotating objects is a spatial visualization technique that enables engineers to visualize complicated assemblies in mechanisms and other systems in physics, chemistry, mathematics and engineering fields. Spatial visualization is an essential and learnable skill that engineers use to clearly communicate their ideas to other people so the ideas can evolve and become real-world products, structures and systems. Many engineering applications require that objects be depicted from multiple viewpoints, such as mechanical drawings used to manufacture components, architectural drawings, and chemical compounds. To produce these views, objects must be rotated across multiple axes, called two-axis rotations. This skill is similar to lifting up an object and looking at it from all sides, which is not always possible since some objects/components are too large, heavy or inaccessible for physical manipulation. Instead, engineers must be able to visualize what various viewpoints look like without physically manipulating the objects. Typically, engineers use computer-aided design (CAD) software to help visualize complicated rotations of objects. This lesson helps students develop their two-axis rotation skills.

Pre-Requisite Knowledge

Before taking part in this spatial visualization lesson, students should have learned about spatial visualization in the previous three workshops. They should have a firm knowledge about isometric drawing, how to use isometric paper and coded plans (as taught in the previous lesson, *Isometric Drawings and Coded Plans*), as well as orthographic views (as taught in the lesson, *Orthographic Views*). They should also understand how to visualize, perform and draw one-axis rotations with the help of the right-hand rule. Additionally, students should be familiar with the Cartesian coordinate system (x-, y-, z-axes) and degrees of rotation.

Learning Objectives

After this workshop, students should be able to:

- ✓ Rotate objects about two axes.

- ✓ Translate the rotation of one object to a second object.

- ✓ Use the right-hand rule to explore rotations of objects.

Materials List

| Item	Description & Amount	Source/Supplier/Cost	Station #s
Isometric Paper	Isometric, three-dot paper for sketching, 2+ per student	Isometric Paper (pdf); print double-sided as needed	1, 2, 3
Snap cubes	Interlocking cubes, 8 cubes per student	Such as: Higher Ed Services ($3 for 15 cubes) or Amazon ($85 for 200)	1, 3
Pencil with eraser	Any erasable pencil; pens are not recommended	School/students provide	1, 2, 3
Colored pencils	To associate drawings to the snap cubes; to share among all students	Such as: Crayola colored pencils at Amazon ($8 for 50)	1, 3
Developing Spatial Thinking *Workbook & Instructional Software*	1 per student pair; includes web-based software used with workbook	$40 for workbook/software from Higher Ed Services	2 (workbook) 4 (software)
Computer or laptop	1 per student pair	School/students provide	4
Workbook answers	For the instructor and for students to check their work	Workbook Answer Key (pdf)	2
PowerPoint® presentation	For the instructor, slides 28-31	Spatial Visualization Presentation (pptx)	NA

Introduction & Motivation

Have the slide presentation up and displayed to the class, starting with slide 28.

Today, we are going to explore rotations about two axes. You experience two-axis rotations throughout your everyday lives. For example, how many times must you rotate a U-lock when taking it off your bike? Objects can be rotated about the three axes—x, y and z—and any *combination* of the three. When we perform rotations about more than just one axis, we are performing multi-axis rotations.

Show students slide 29, which is the same as Figure 1.

Do you see that the first block on the top left is the same block as on the top right? We are simply looking at it from another angle. How would you describe this rotation? Take a look at this figure and determine your answer. Now explain to a partner what you think is the correct answer and why. What are some methods you used to determine how the block was rotated? How would you define the rotation in terms of a positive or negative rotation about the x-, y- and/or z-axes? *(Answer: Negative rotation about the y-axis; negative rotation about the z-axis. Teacher note: Other combinations of rotations may exist that solve this problem, as well as many other two-axis rotations.)* Notice that this takes two rotations about different axes rather than just one.
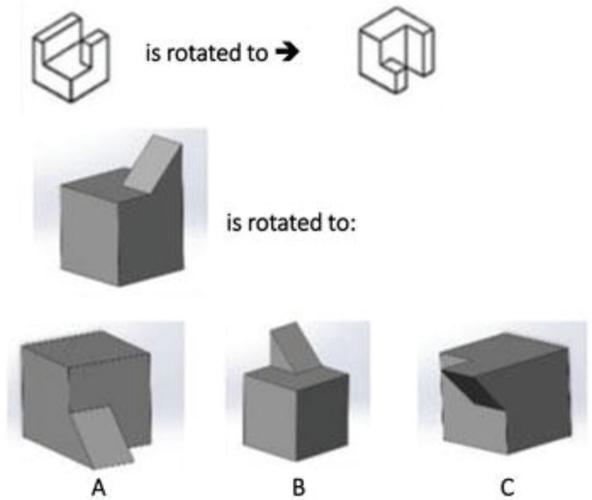
Figure 1. *Which rotation of the gray object is analogous to the rotation of the white object?*

Now show students slide 30.

An important part of two-axis rotations is that the order the rotations are performed drastically affects the final object position. Notice that when we perform a negative z-axis rotation and then a negative x-axis rotation, the original shape ends up as something completely different than when we perform a negative x-axis rotation and then a negative z-axis rotation. In summary, two-axis rotations are not commutative; order matters! Throughout this workshop, we will only be performing two-axis rotations. Remember that clockwise implies a negative rotation and counter-clockwise implies a positive rotation. Use the right-hand rule to assist you in performing these rotations.
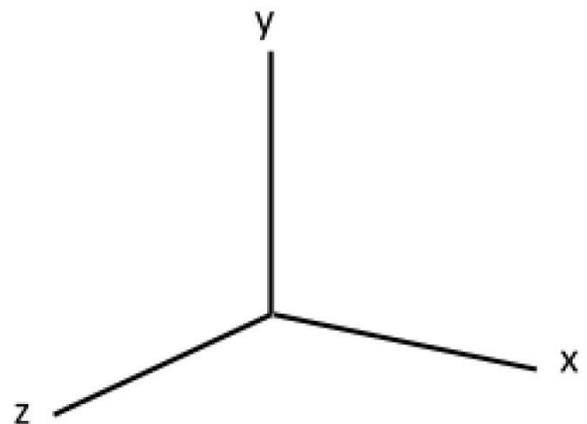
For additional explanation, if needed, a review of the right-hand rule is provided next.

Let's do a quick recap of the right-hand rule.

Refer to slide 25 as a review from the previous lesson.

It works as follows: Point your thumb parallel to the axis you are rotating about and curve your fingers naturally towards the palm of your hand. The direction your fingers move is the same way that the object will move. The axes shown in slide 24 (same as Figure 2) are the positive axes, and if you flip each axis 180°, you get the negative axes.

Slide 31 recaps the four workshop stations. Leave slide 31 up so students know what to accomplish at each station.

Figure 2. *Three positive axes, x, y and z.*

Vocabulary

Term	Definition
coded plan	A two-dimensional depiction of an object that defines the volume based on the top view of the object.
isometric	Of or having equal dimensions. The isometric view of an object is the angle at which an equal angle (120°) exists between all axes (such as looking down a corner of the object).
isometric paper	A grid of dots arranged equidistant from one another. Used in making isometric sketches. Also called three-dot paper.
one-axis rotation	The result of rotating an object solely about a single axis. In this workshop, the rotations only occur in 90º increments. Also called single-axis rotation.
orthographic view	A way to draw an object that shows three views of an object from the three planes in an orthogonal (right-angle) coordinate system. The views represent the exact shape of an object as seen from one side at a time as you are looking perpendicularly to it. Depth is not shown. An orthographic drawing is also called a multi-view drawing.
right-hand rule	A useful memory tool in the rotation of objects that uses a person's right hand and fingers to help in understanding orientation conventions for vectors in three dimensions. Often used in physics and math.
two-axis rotation	The result of rotating an object about two axes. Can take place around the x, y, or z axes at 90º, 180º or 270º.

Lesson Procedure

Before the Workshop

✓ Gather materials.

✓ Make copies of the Isometric Paper. Print double-sided to reduce paper use and increase drawing area per page.

✓ Prepare a way to digitally get students the web-based software link they will need for this lesson such as by email or by preparing a workshop website with the link.

✓ Prepare to project the Spatial Visualization Presentation, a PowerPoint® file, and use its content to aid in your instruction, as makes sense for your class. Slides 27-30 support this lesson. The slides are animated so a mouse or keyboard click brings up the next graphic or text.

✓ Prepare the four stations, as described in slide 31 and Figure 3.

Station 1: Block Relay

- Build an object with 6 blocks
- Define the **axes** and a **two-axis rotation**
- Draw the **isometric views** before and after rotation
- Then pass the object to a neighbor and repeat
- Check drawing for accuracy with neighbor

Station 2: Workbook Drill

- Individually, answer questions on **ODD** pages rot2-5 to rot2-17 on scrap paper
- Check your answers with the answer key

Station 3: Frozen

- In pairs, build an object of any size
- Define the **axes** and a **two-axis rotation**
- Draw the **isometric views** before and after rotation
- Then pass the object to a neighbor and repeat
- Check drawings for accuracy with neighbor

Station 4: Computer-Aided Visualization

- Read *Module 8 - Rotations about Two Axes*
- Complete the **rotations about two axes** exercises

Figure 3. *The four stations for Workshop Four.*

With the students: Introduction

✓ Present to the class the Introduction & Motivation content, supported by slides 28-31.

✓ Divide the class into four groups that will rotate through the four stations.

✓ Give each student 2 pieces of isometric paper and 8 cubes.

✓ Explain that students are to primarily work independently on their drawings, but feel free to share and troubleshoot with their partners.

✓ Direct students to work on one piece of isometric paper until it is full.

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Lesson Procedure Continued

Station 1: Block Relay

Students begin by building a shape with six blocks. Next, they define the axes and a two-axis rotation. Then they draw the isometric views before and after the defined rotation. Then they pass the object to a neighbor and repeat. After students have completed drawing both their own objects and their neighbors' objects, have them collaborate and check their drawings for accuracy.

Station 2: Workbook Drill

Have students individually answer workbook questions on odd pages rot2-5 to rot2-17, recording their answers on scrap paper. After they have completed the problems, have them check their answers with the answer key. Then have them communicate with neighboring students about answers they may have gotten wrong and collaborate on ways to understand and solve the problems.

Station 3: Frozen

Working in pairs, have students each build an object of any size (best to start off with objects composed of 5-8 cubes and then add more). Have them define the axes about the shape and a two-axis rotation. Then switch objects with a partner and draw the isometric view before and after the rotation *without moving the object*—consider it frozen! Then have them compare drawings with their partners and repeat.

Station 4: Computer-Aided Visualization

Have students individually read *Module 8 – Rotations about Two Axes* from the *Developing Spatial Thinking Software* and complete all rotations about two axes exercises.

Instructor Tips

• In Station 2, the workbook pages are more difficult than those in the previous workshops. For additional help, have students draw the problems on scrap paper to aid in solving them. It may also help to build the shapes for the first few problems as they get used to rotations.

• For complex shapes, have students focus on just one part of the shape throughout a rotation and visualize where that part ends up.

• In Station 3, because the object is frozen, it is crucial that the axis it is being rotated about is well defined. To avoid confusion, have students define the axes to their partner visually and out loud.