Dynamics of Gyroscopes

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Gyroscopes

- **Basic Definition:** A gyroscope measures angular orientation either directly, or through integrating a measured rotational rate or acceleration.

- **Uses:**
  - Navigation
  - Control Systems (i.e. anti-skid in cars)
  - Combine with accelerometer to form IMU

- **Desirable Qualities:**
  - High Sensitivity—Minimum detectable rate.
  - Stability—Over time, or environmental change.
  - Broad Range—Accuracy at both low and high rates.
Types of Gyroscopes

- **Classic Spinning Disk**
  - Based on gyroscopic effect of a of rotating object with angular momentum.
  
  \[
  \tau = \frac{dh}{dt}
  \]

- **Vibratory**
  - Based on Coriolis force on an object in a rotating reference frame.
  
  \[
  F_{\text{coriolis}} = 2m(\vec{v} \times \vec{U})
  \]

- **Laser (Ring, IFOG)**
  - Based on “timing” the travel of light through a course whose length varies with the rotation of the course.

  \[
  \text{Speed of Light} = c = \text{Constant}
  \]
Interferometric Fiber Optic Gyroscope (IFOG)

- Coherent (laser) light travels in opposite directions around a fiber optic coil.
- Rotation of the coil creates a path difference between the signals.
- Measuring the phase shift between the signals provides a rotation rate measurement.
Path Difference

- Distance Traveled by Clockwise Signal:
  \[ ct_- = 2\pi R - R\dot{\theta} t_- \Rightarrow ct_- = \frac{2\pi c R}{c + R\theta}. \]

- Distance Traveled by Counterclockwise Signal:
  \[ ct_+ = 2\pi R + R\dot{\theta} t_+ \Rightarrow ct_+ = \frac{2\pi c R}{c - R\theta}. \]
Path Difference

- Take difference of distance traveled by each signal.

\[ dL = c(t_+ - t_-) = 2\pi cR \left[ \frac{1}{(c - R\dot{\theta})} - \frac{1}{(c + R\dot{\theta})} \right] \]

\[ dL = 2\pi cR \left[ \frac{(c + R\dot{\theta})}{(c^2 - R^2\dot{\theta}^2)} - \frac{(c - R\dot{\theta})}{(c^2 - R^2\dot{\theta}^2)} \right] = 2\pi cR \left[ \frac{2R\dot{\theta}}{(c^2 - R^2\dot{\theta}^2)} \right] \]

\[ dL = \frac{4\pi cR^2\ddot{\theta}}{c^2 - R^2\dot{\theta}^2} \]

- Path Difference:

\[ dL = \frac{4\pi R^2\ddot{\theta}}{c} \]

\[ c >> R^2\dot{\theta}^2 \]
Light Interference

- The coherent light exiting after traveling different distances have a phase difference proportional to rotation rate.

\[ dL = \frac{4\pi R^2 \dot{\theta}}{c} \]

\[ dL = \lambda \frac{d\phi}{2\pi} \]

\[ \Rightarrow \dot{\theta} = \frac{\lambda c}{8\pi^2 R^2} d\phi \]
IFOG Summary

• Advantages
  – No mechanical parts
  – Resistant to shock and vibration
  – Long-lived
  – Accurate
  – **Commercially Available**: First Used in Boeing 777

• Disadvantages
  – Speed of light is “fast” and thus requires many loops of fiber optic fiber to create a detectable phase angle.
Coriolis Force Gyroscope

\[ F_{\text{coriolis}} = 2m(\vec{\nu} \times \vec{U}) \]

a.) Excite Primary Mode

b.) Coriolis Force Develops

c.) Excites Secondary Mode
Coriolis Force Gyroscope

• To characterize the dynamics of a vibratory gyroscope we need to:
  – Create a simple model to account for the principle features.
  – Find generalized coordinates.
  – Calculate system energy as a function of the generalized coordinates.
  – Develop the equations of motion by the Lagrangian method.
  – Evaluate the E.O.M. to determine the system response to an input rotation rate.

• Simple Model: One Axis Gyroscope
  – Vibrating mass with two orthogonal vibration modes (one for forcing and the other for sensing).
  – Rotation axis perpendicular to vibration plane.
Generalized Coordinates Review

- A set of coordinates \((q_1, q_2, \ldots)\) which:
  - Fully describes the position and orientation of the system at any time.
  - Each coordinate is independent from every other coordinate. In other words each coordinate can be set independently without violating the physical system.

\[ q_1 \neq f(q_2, q_3, \ldots) \]

- Example: Find Coordinates for a Compound Pendulum (2 D.O.F.)

- Correct?
Generalized Coordinates Review

• Fulfills condition 1. but not 2. (Note dependence of coordinates):

\[ y_1 = \sqrt{L_1^2 - x_1^2} \]

• Instead Consider Angles (generalized coordinates are often non-unique)
Generalized Coordinates For One Axis
Coriolis Gyroscope

- Generalized coordinates $\theta$, $x$, $y$ with $x,y$ relative to frame.
- Rotation about $z$-axis, with steady oscillation along $x$-axis.
- Response develops along $y$-axis.
Rotating Reference Frame

- Inertial frame XYZ and rotating frame xyz.
- Position/velocity in rotating reference frame:
  \[
  \vec{r}_{xyz} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}
  \]
  \[
  \vec{v}_{xyz} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k}
  \]
- The velocity in the fixed frame can be found by treating the unit vectors as variable:
  \[
  \vec{r}_{XYZ} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} + r_x \ddot{i} + r_y \ddot{j} + r_z \ddot{k}
  \]
Rotating Reference Frame

\[ \dot{\vec{r}}_{XYZ} = \dot{\vec{r}}_{xyz} + r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \]

- Consider \( r = \text{constant} \)

\[ \dot{\vec{r}}_{XYZ} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \]

- Velocity of Pure Angular Motion:

\[ \dot{\vec{r}}_{XYZ} = \Omega \times \vec{r}_{xyz} \]

\[ \Rightarrow \dot{\vec{r}}_{XYZ} = \dot{\vec{r}}_{xyz} + \Omega \times \vec{r}_{xyz} \]

\[ \vec{v} = \vec{v}_{\text{rotating}} + \Omega \times \vec{r}_{\text{rotating}} \]
System Energy

K.E. and P.E. as a Function of Generalized Coordinates:

\[ r_{\text{rotating}} = x \hat{i} + y \hat{j} \quad v_{\text{rotating}} = \dot{x} \hat{i} + \dot{y} \hat{j} \]

\[ v = v_{\text{rotating}} + \Omega \times r_{\text{rotating}} \]

\[ v = (\dot{x} \hat{i} + \dot{y} \hat{j}) + \dot{\theta} \hat{k} \times (x \hat{i} + y \hat{j}) \]

\[ v = (\dot{x} - \theta \dot{y}) \hat{i} + (\dot{y} + \theta \dot{x}) \hat{j} \]

\[ T = \frac{1}{2} m \left[ (\dot{x} - \theta \dot{y})^2 + (\dot{y} + \theta \dot{x})^2 \right] \]

\[ V = \frac{1}{2} k_x x^2 + \frac{1}{2} k_y y^2 \]
Lagrangian

- Three generalized coordinates $x, y, \theta$.
  - The angular displacement is imposed on the device, while the $x$ direction is forced with sinusoidal oscillation amplitude of $x_0$.
  - We are interested in the response occurring in the $y$ coordinate, so let's examine the equation of motion for $q_i=y$.

$$ \mathbf{L} = T - V $$

$$ L = \frac{1}{2} m \left[ (\dot{x} - \dot{\theta} y)^2 + (\dot{y} + \dot{\theta} x)^2 \right] - \left[ \frac{1}{2} k_x x^2 + \frac{1}{2} k_y y^2 \right] $$

$$ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i $$

$$ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{y}} \right) - \frac{\partial L}{\partial y} = Q_y $$
Formulate Equation of Motion

\[ L = \frac{1}{2} m \left[ (\dot{x} - \theta \dot{y})^2 + (\dot{y} + \theta \dot{x})^2 \right] - \left[ \frac{1}{2} k_x x^2 + \frac{1}{2} k_y y^2 \right] \]

- \( y \) coordinate:

\[ \frac{\partial L}{\partial \dot{y}} = m (\dot{y} + \theta \dot{x}) \]

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{y}} \right) = m (\ddot{y} + \ddot{\theta} x + \dot{\theta} \dot{x}) \]

\[ -\frac{\partial L}{\partial y} = -m (\dot{x} - \theta \dot{y})(-\theta) + k_y y \]

\[ m (\ddot{y} + \ddot{\theta} x + \dot{\theta} \dot{x}) + m \dot{\theta}(\dot{x} - \theta \dot{y}) + k_y y = Q_y \]

\[ m \ddot{y} + 2 m \dot{\theta} \dot{x} - m \dot{\theta}^2 y + m \ddot{\theta} x + k_y y = Q_y \]
Formulate Equation of Motion

- Add non-conservative damping forces.

\[ \delta W_{\text{non-consv.}} = F_{\text{damper}} \delta y = -c_y \dot{y} \delta y \]

\[ Q_y = -c_y \dot{y} \]

\[ m \ddot{y} + 2m \dot{\theta} \dot{x} - m \dot{\theta}^2 y + m \ddot{\theta} \cdot x + k_y y = -c_y \dot{y} \]

\[ m \ddot{y} + c_y \dot{y} + 2m \dot{\theta} \dot{x} - m \dot{\theta}^2 y + m \ddot{\theta} \cdot x + k_y y = 0 \]

- Assume for our case the Coriolis force dominates the other introduced inertial forces. (rotation rate small and steady)

**Coriolis Force**

**Additional Inertial Forces Due to Rotating Ref. Frame**
Find Response Amplitude

• Generally interested in the response amplitude not the full solution.
• Consider the Coriolis force as an external forcing function:

\[ m\ddot{y} + c_y \dot{y} + 2m\dot{\theta} \dot{x} + k_y y = 0 \]

\[ \Rightarrow m\ddot{y} + c_y \dot{y} + k_y y = -2m\dot{\theta} \dot{x} \]

• Invoke complex analysis of forced (particular) response.
• Control scheme provides a sinusoidal oscillation in x with amplitude of \( x_0 \) and frequency \( \omega_d \) (Drive Frequency).

\[ x = x_0 \sin(\omega_d t) \Rightarrow \dot{x} = x_0 \omega_d \cos(\omega_d t) \]

\[ e^{\pm i\theta} = \cos(\theta) \pm i \sin(\theta) \]

\[ \dot{x} = \text{Re}[x_o \omega_d e^{i\omega_d t}] \]
Find Response Amplitude

- Complex representation of forcing function:
  \[ F_{\text{coriolis}} = \text{Re}\left[-2m\dot{\theta} x_o \omega_d e^{i\omega_d t}\right] \]

- Particular solution has the form:
  \[ y_p = \text{Re}\left[Y e^{i\omega_d t}\right] \]

- Insert these equations back into the equation of motion:
  \[ \dot{y}_p = \text{Re}\left[iY\omega_d e^{i\omega_d t}\right] \quad \ddot{y}_p = \text{Re}\left[-Y\omega_d^2 e^{i\omega_d t}\right] \]

- Leave out Re symbol for now (just remember we want the real part)
  \[
  m\left(-Y\omega_d^2 e^{i\omega_d t}\right) + c_y \left(iY\omega_d e^{i\omega_d t}\right) + k_y \left(Y e^{i\omega_d t}\right) = -2m\dot{\theta} x_o \omega_d e^{i\omega_d t} \\
  Y\left(-m\omega_d^2 + ic_y \omega_d + k_y\right) = -2m\dot{\theta} x_o \omega_d
  \]
Find Response Amplitude

- Continued:

\[ Y \left( -m \omega_d^2 + ic_y \omega_d + k_y \right) = -2 \dot{x}_o \omega_d \]

\[ c_y = 2m \xi \omega_{n-y} \]

\[ Y \left( -\omega_d^2 + i2\xi \omega_{n-y} \omega_d + \omega_{n-y}^2 \right) = -2 \dot{x}_o \omega_d \]

\[ Y = \frac{-2 \dot{x}_o \omega_d}{\left[ \left( -\omega_d^2 + \omega_{n-y}^2 \right) + i2\xi \omega_{n-y} \omega_d \right]} \]

- Multiply top and bottom by:

\[ \left[ \left( -\omega_d^2 + \omega_{n-y}^2 \right) - i2\xi \omega_{n-y} \omega_d \right] \]

\[ Y = \frac{-2 \dot{x}_o \omega_d \left[ \left( -\omega_d^2 + \omega_{n-y}^2 \right) - i2\xi \omega_{n-y} \omega_d \right]}{\left[ \left( -\omega_d^2 + \omega_{n-y}^2 \right)^2 + \left( 2\xi \omega_{n-y} \omega_d \right)^2 \right]} \]
Find Response Amplitude

- We now have complex amplitude \( Y \):
  \[
  Y = a + ib
  \]
  \[
  y_p = \text{Re}[Ye^{i\omega_d t}]
  \]
- Complex Identity:
  \[
  a + ib = Ae^{i\phi}, \quad A = \sqrt{a^2 + b^2}
  \]
  \[
  y_p = A \text{Re}[e^{i\omega_d t + \phi}]
  \]
- Thus, the real response has an amplitude of \( A \).

\[
A^2 = \left(-2\dot{x}_o \omega_d\right)^2 \left[ \frac{\left(-\omega_d^2 + \omega_{n-y}^2\right)^2}{\left(-\omega_d^2 + \omega_{n-y}^2\right)^2 + \left(2\xi \omega_{n-y} \omega_d\right)^2} \right] + \left[ \frac{\left(2\xi \omega_{n-y} \omega_d\right)^2}{\left(-\omega_d^2 + \omega_{n-y}^2\right)^2 + \left(2\xi \omega_{n-y} \omega_d\right)^2} \right]
\]

\[
A^2 = \left(-2\dot{x}_o \omega_d\right)^2 \left[ \frac{\left(-\omega_d^2 + \omega_{n-y}^2\right)^2 + \left(2\xi \omega_{n-y} \omega_d\right)^2}{\left(-\omega_d^2 + \omega_{n-y}^2\right)^2 + \left(2\xi \omega_{n-y} \omega_d\right)^2} \right]
\]
Gyroscopic Response (Accuracy)

- Simplification yields the response amplitude for small, steady rotation input $\dot{\theta}$.

$$A = \frac{-2\dot{\theta} x_o \omega_d}{\sqrt{\left(\omega_{n-y}^2 - \omega_d^2\right)^2 + \left(2\xi \omega_{n-y} \omega_d\right)^2}}$$

- Equation extremely useful in designing a gyroscope with maximum response amplitude (increased accuracy).
- Foremost notice the amplitude decreases if the drive frequency differs from the natural frequency in the y-direction.
- Since one would select the drive frequency roughly equal to the natural frequency in the x-direction (maximize $x_o$), an optimal gyroscope has matched modes.
Gyroscopic Response (Accuracy)

• Now consider a matched mode gyroscope:
  \[ \omega_d = \omega_{n-x} = \omega_{n-y} \]
  \[ A = \frac{-\dot{\theta} x_o}{\xi \omega_n} \]

• Introduce the Quality (Q) Factor

\[ Q = \frac{1}{2\zeta} \]

\[ Q = \frac{\omega_{center}}{\text{bandwidth}} \]
Gyroscopic Response (Accuracy)

- Amplitude becomes:

\[ A = \frac{-2Qx_0 \dot{\theta}}{\omega_n} \]

- Two Key Observations
  - Want to maximize the quality factor (MEMS resonators can have Q-factors > 20,000, which makes MEMS gyroscopes interesting)
  - Want to decrease the fundamental frequency of the device. Usually set at ~20 kHz to avoid interaction with low frequency environmental noise.
  - This last fact leads to some interesting dimensions for MEMS structures.
Group Work (Two Teams)

- MUMPs Gyroscope
- Find \( L, w \) of beams and \( L_{\text{plate}} \) such that the gyroscope has a fundamental frequency of \(~20\, \text{kHz}\).
- Useful Facts
  - \( E=160\, \text{GPa}, \rho=2330\, \text{kg/m}^3 \)
  - Thickness=2\( \mu \text{m} \)
  - Assume cantilever spring
  - \( k_{eq} = \frac{3EI}{L^3} \)
  - \( I = \frac{1}{12} wt^3 \)
  - \( \omega = \sqrt{\frac{2k_{\text{cantilever}}}{m_{\text{plate}}}} \)
  - Neglect spring mass
  - Plate to substrate separation is 1.5\( \mu \text{m} \).
  - Minimum feature size also 2\( \mu \text{m} \).
Rigorous Amplitude Derivation

- Equations of Motion for x, y Coordinates:

\[
\begin{bmatrix}
 m & 0 \\
 0 & m
\end{bmatrix}
\begin{bmatrix}
 \ddot{x} \\
 \ddot{y}
\end{bmatrix}
+ \begin{bmatrix}
 c_x & -2m\dot{\theta} \\
 2m\dot{\theta} & c_y
\end{bmatrix}
\begin{bmatrix}
 \dot{x} \\
 \dot{y}
\end{bmatrix}
+ \begin{bmatrix}
 k_x & 0 \\
 0 & k_y
\end{bmatrix}
\begin{bmatrix}
 x \\
 y
\end{bmatrix}
= \begin{bmatrix}
 f_x \\
 0
\end{bmatrix}
\]

- Complex Representation:

\[
x_j = X_j e^{i\omega_d t} \quad f_j = F_j e^{i\omega_d t}
\]

\[
\begin{bmatrix}
 -m\omega_d^2 + ic \omega_d + k_x \\
 2m\dot{\theta} \omega_d
\end{bmatrix}
\begin{bmatrix}
 X \\
 Y
\end{bmatrix}
= \begin{bmatrix}
 F_x \\
 0
\end{bmatrix}
= \begin{bmatrix}
 k_x x_0 \\
 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
 Z_{11} & Z_{12} \\
 Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
 X \\
 Y
\end{bmatrix}
= \begin{bmatrix}
 F_{10} \\
 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
 X \\
 Y
\end{bmatrix}
= \begin{bmatrix}
 Z_{11} & Z_{12} \\
 Z_{21} & Z_{22}
\end{bmatrix}^{-1}
\begin{bmatrix}
 F_{10} \\
 0
\end{bmatrix}
\]
Rigorous Amplitude Derivation

\[ Y = \frac{-Z_{12}F_{10}}{Z_{11}Z_{22} - Z_{21}Z_{12}} \]

\[ Y = \frac{-2m\dot{\omega} d F_x}{m^2 \left[ \left( -\omega_d^2 + \omega_{n-x}^2 + i2\xi_x\omega_{n-x}\omega_d \right) \left( -\omega_d^2 + \omega_{n-y}^2 + i2\xi_y\omega_{n-y}\omega_d \right) + \left( 2\dot{\theta} \omega_d \right)^2 \right]} \]

\[ Y = \frac{-2\dot{\theta} \omega_d \omega_{n-x}^2 x_0}{\left[ \left( -\omega_d^2 + \omega_{n-x}^2 + i2\xi_x\omega_{n-x}\omega_d \right) \left( -\omega_d^2 + \omega_{n-y}^2 + i2\xi_y\omega_{n-y}\omega_d \right) + \left( 2\dot{\theta} \omega_d \right)^2 \right]} \]

- **Matched Modes Case:**

\[ Y = \frac{-\dot{x}_0}{2\omega \left[ \left( -\xi_x\xi_y \right) + \left( \frac{\dot{\theta}}{\omega} \right)^2 \right]} \]
How Steady?

- Derivation assumed a small and steady rotation rate.
- To provide a feel for how steady, find the time required to reduce the homogenous solution to 10% its original value.
- Assumptions:
  - \( Q = 10,000 \)
  - \( \omega = 20 \text{ kHz} = 125,000 \text{ rad/sec} \)

\[
Q = \frac{1}{2\zeta}
\]
Coriolis Gyroscope Summary

• Advantages
  – Scales down with increased accuracy
  – MEMS implementation (small and light)
  – MEMS devices can be integrated directly with associated electronics
  – No relative motion (i.e. no bearings, shafts, gears)
  – Low wear

• Disadvantages
  – Lower accuracy due to difficulty inherent in measuring motion of MEMS devices.
  – Devices often “flimsy” with length to thickness (slenderness) ratios often approaching 1000.
  – Susceptible to shock.
  – As with most MEMS devices the reliability is low due to stiction.
  – Still in research phase
Research Project

• Recall, for a Coriolis gyro., the accuracy is proportional to Q-factor.

\[ A = \frac{-2Qx_0 \dot{\theta}}{\omega_n} \]

• The total quality factor combines the losses attributed to friction, thermoelastic, air, and anchor dissipation mechanisms.

\[ \frac{1}{Q} = \sum \frac{1}{Q_i} = \frac{1}{Q_{TED}} + \frac{1}{Q_{Volumetric}} + \frac{1}{Q_{Surface}} + \frac{1}{Q_{Air}} + \frac{1}{Q_{Anchor}} \]

• Despite the often limiting nature of anchor loss, this source has received insufficient study.
Analytical Anchor Loss Relationship

Analytical model derived by assuming an equivalent single d.o.f. resonator:

\[ Q_{\text{anchor}} = 2\pi \frac{W_0}{\Delta W} \]

\[ W_0 = \frac{1}{2} k_r \bar{x}^2 \]

\[ P_a(\omega) = \int_0^T W dt = \frac{\Delta W}{T} \Rightarrow \Delta W = TP_a(\omega) \]

\[ H_s(\omega) = \frac{(k_s - m_s \omega^2) - ic_s \omega}{(k_s - m_s \omega^2)^2 + (c_s \omega)^2} \]

\[ P_a(\omega) = \frac{1}{2} \text{Re}\left[i\omega F_0^* H_s(\omega) F_0\right] \]

\[ P_a(\omega) = \frac{1}{2} \omega k_r^2 \bar{x}^2 \left( \frac{c_s \omega}{(k_s - m_s \omega^2)^2 + (c_s \omega)^2} \right) \]

\[ Q_{\text{anchor}} = \frac{1}{\omega k_r \left( \frac{c_s}{(k_s - m_s \omega^2)^2 + (c_s \omega)^2} \right)} \]

\[ F(t) = F_0 \sin(\omega t) \]

\[ Q_{\text{anchor}} = \frac{1}{k_r \text{Im}[-H_s(\omega)]]} \]
Parametric Examples-Cantilever

- A Cantilever example highlights the general trends apparent in many resonator structures.
  
  \[ k_{\text{cantilever}} = \frac{E \cdot w \cdot th^3}{4L^3} \]

  \[ Q_{\text{anchor-cantilever}} = \frac{1}{\text{Im}[H_s(\omega)]} \cdot \frac{4}{Ew} \cdot \frac{L^3}{th^3} \]

- Relationship reveals an anchor loss dependency upon resonator slenderness, width, Young’s modulus, and frequency dependent substrate properties.
- Alternative substrate materials offer significant rigidity improvement.

![Graph showing Young's Modulus and Strength](Image)
Combing anchor loss model with modal amplitude equation yields additional insight unique to gyroscopes.

\[ A = \frac{-2Qx_0 \dot{\theta}}{\omega_n} \]

\[ A = \beta (\dot{\theta} x_o) \left( \frac{1}{E} \right) \left( \frac{1}{w \left( \frac{L}{th} \right)^3} \right) \left( \frac{1}{\omega_n} \right) \left( \frac{1}{\text{Im}[H_s(\omega)]} \right) \]

Five relevent terms to work with in maximizing the sense amplitude; an input term, a ligament material term, a ligament geometry term, a frequency term, and a substrate term.
MUMP's Test Chip

- METAL
- POLY2
- HOLE2
- ANCHOR2
- POLY1_POLY2_VIA
- POLY1
- HOLE1
- ANCHOR1
- POLY0

1 mm
MUMPs Test Chip

- Series of cantilever structures to test width, frequency, and slenderness dependence of anchor loss.
- Results applicable not only to gyroscopes but to other applications involving resonators (filters).
Conclusions

• Examining the dynamics of two new classes of gyroscopes provides the design intuition to:
  – Identify the design parameters which control the sensitivity.
  – Maximize the sensitivity to input rotation through parametric design.
  – Identify deficiencies in current understanding and develop new research projects.
  – Point out the relative strengths and weakness of each, and what applications are best suited for each.