Inclusive and Equitable Mathematics Education: Active Learning is Necessary, but not Sufficient

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Numerous studies have demonstrated that active learning can increase student learning and reduce achievement gaps; research has also shown that active learning in undergraduate mathematics is not consistently equitable. These findings highlight a gap in what we know about active learning and indicate the need for a deeper understanding of the relationship between equity, inclusion, and active learning. Drawing on research about inclusive and equitable mathematics learning environments across secondary and postsecondary contexts, in concert with what is known about active learning in undergraduate mathematics classrooms, I present a theoretical argument that active learning is a necessary but insufficient condition for mathematics learning communities to be inclusive and equitable. I close by suggesting potential strategies for ensuring active learning is implemented in ways that are inclusive and equitable.

Keywords: active learning, inquiry, equitable, inclusive, critical

The use of active learning and inquiry-based mathematics education (IBME, a form of active learning) is becoming increasingly common in undergraduate mathematics courses (Stains, 2018). Active learning has been shown to result in improved student learning outcomes (Deslauriers et al., 2019; Freeman et al., 2014, Laursen et al. 2014) as well as significantly reduced achievement gaps between women and men (Laursen et al, 2014) and between students who are members of underrepresented and overrepresented identity groups (Theobald et al., 2019). Freeman et al. (2014) suggest that research supports "active learning as the preferred, empirically validated teaching practice in regular classrooms" (p. 8410), and Theobald et al. (2019) calls for evidence-based active-learning course designs to replace traditional lecturing across the STEM disciplines" (p. 6476). Active learning was found to be one of the common characteristics in a study of successful calculus programs at five doctoral degree-granting mathematics departments deemed to be exemplary based on persistence rates and students' reported enjoyment and confidence in mathematics (Rasmussen, Ellis, Zazkis & Bressoud, 2014; Rasmussen, Ellis & Zazkis, 2014). Abundant evidence of student-centered instruction supporting increased student success in mathematics at the primary and secondary levels (e.g., Boaler, 2006; Matthews et al., 2021) further reinforces the claim that active learning is a better way to teach mathematics than traditional lecture.

And yet, performance (Reinholz et al., 2022; Johnson et al., 2020) and participation (Reinholz et al., 2022) gaps between majority identity groups and those who are underrepresented in mathematics persist in some active learning classrooms. Johnson et al. (2020) showed that achievement gaps between women and men increased in a set of linear algebra classrooms using IBME. Reinholz et al. (2022) found performance differences and differences in participation rates between women and men in undergraduate mathematics classes taught using inquiry-oriented instruction and suggested that "simply implementing active learning is insufficient… for improving gender equity in mathematics" (p. 204). Research has shown that mathematics classrooms using active learning can be inclusive and equitable, while also demonstrating that not all active learning mathematics classrooms achieve this standard.

The relationship between equity, inclusion, active learning and IBME is under-researched, and scholars have not yet explained the distinctions between inclusive and equitable and exclusive and/or inequitable active learning. In this theoretical report I draw on research about inclusive and equitable mathematics learning environments in secondary and tertiary contexts, along with what is known about active learning and IBME in undergraduate mathematics classrooms, to answer the question: *What will it take to cultivate undergraduate mathematics learning environments that are reliably equitable and inclusive?* I present the argument that instructors' use of active learning in general, and IBME specifically, is a necessary but insufficient condition for mathematics learning to be inclusive and equitable. Then I propose additional criteria that may, when used in addition to IBME, contribute to creating reliably inclusive and equitable undergraduate mathematics classrooms.

Theoretical Perspectives: What is Inclusive and Equitable Mathematics?

Research showing the positive effects for students of learning mathematics through active learning or IBME is abundant and convincing. The strength of the case in favor of active learning can make it seem confusing or implausible when evidence is presented showing some settings exhibiting persistent inequity and/or exclusion. To explain how the extensive body of research supporting active learning can be correct while also failing to explain why some active learning settings are exclusive and/or inequitable, I leverage theoretical perspectives. A clear theoretical perspective names the researchers' commitments and perspectives; it provides a context that helps explain why certain decisions are made in conducting research, including the questions that are asked, the data that is collected and how that data is analyzed and interpreted. This section describes the dominant perspective that has shaped most of the existing research on active learning in undergraduate mathematics and the critical perspective that may help to usher in a new chapter focused on inclusion and equity in mathematics education. This is followed by a commitment to intersectionality and attending to the interplay of lived experiences across identity group memberships, and a description of the inclusive and equitable teaching practices that shape my thinking about the nature of inclusive and equitable undergraduate mathematics.

Critical Perspective

Gutiérrez (2007) and Martin (2003) called for mathematics education researchers to adopt a critical perspective, and yet much of the research on active learning in undergraduate mathematics continues to reflect a strongly dominant perspective. Gutiérrez's (2009) framework for understanding equity in mathematics provides a useful way to understand and interpret the body of research on active learning. This framework includes two axes. The dominant axis consists of the dimensions of access and achievement, and the critical axis consists of the dimensions of identity and power. The majority of the existing research on the effect of active learning in undergraduate STEM settings addresses questions about achievement – either learning outcomes or achievement gaps – or about access to learning opportunities that arise from engaging actively with course content.

Numerous mathematics education researchers across secondary and tertiary settings have called for paying greater attention to the role and impact of identity and power (Adiredja, 2015; Gutiérrez, 2009, 2013) – the critical axis of equity in mathematics – in mathematics learning spaces. Attending to the role of identity and power in mathematics classrooms means adopting a different vision if what it means for mathematics education, in this case specifically active learning, to be successful. Mathematics instruction that supports the "mathematical identities, excellence and literacies of marginalized students" (Gutiérrez, 2008, p. 357) may differ from that which leads to increased test scores and reduced participation gaps. Gutiérrez's framework helps to illuminate that the existing research on active learning has demonstrated active learning's success along the dominant axis. Active learning has been shown to increase *achievement* and *access* to learning opportunities. Gutiérrez's framework also highlights what remains to be investigated: by attending closely to *identity* and *power* in mathematics classrooms we may learn what helps students who are members of marginalized identity groups to succeed and thrive.

Framework for Understanding the Social Space of Mathematics

Leyva et al.'s (2022) proposed framework for understanding mathematics as a white cisheteropatriarchal space explains how sociomathematical (Yackel & Cobb, 1996; Leyva, 2017) and sociohistorical (Leyva, 2021) norms are at play in mathematics classrooms in ways that relate to identity including but not limited to gender, race and class. Leyva (2017) calls for researchers to "carefully attend to mathematics learning contexts and the interplay of students' multiple identities (including race or ethnicity, culture, class, gender, and sexuality)" (p. 406). Crenshaw (1991) suggests that intersectionality could provide the means for understanding and addressing experiences of marginalization that are shared across identity groups (p. 1299).

Inclusive and Equitable Instruction

Examples of instructional practices that have been shown to contribute to inclusive learning environments and equitable outcomes for students who are members of underrepresented or marginalized groups in mathematics include equitable teaching practices (Boaler, 2006) and belonging centered instruction (Matthews et al., 2021). Additionally, culturally relevant (LadsonBillings, 1995) and sustaining (Alim, Paris & Wong, 2020; Ladson-Billings, 2014; Paris, 2012; Paris & Alim, 2014) pedagogy supports inclusive and equitable learning environments that are not specific to mathematics. Describing each of these approaches to inclusive and equitable instruction in detail is beyond the scope of this paper. I will, instead, describe cross-cutting themes, since teaching strategies that appear in multiple of these instructional approaches are especially likely to be impactful across settings.

Teaching strategies that emerge as common across these instructional approaches include holding high expectations (Boaler, 2006; Ladson-Billings, 1995; Matthews et al., 2021), assigning competence (Boaler, 2006; Ladson-Billings, 1997) and decentering teacher authority (Matthews et al., 2021). Just as importantly, additional themes are present across these approaches that relate to interpersonal interactions, social dynamics, and community. Boaler (2006) described relational equity as a teaching approach that "valued different insights, methods, and perspectives in the collective solving of particular problems" (p. 45), an approach that supported students learning to "appreciate the contributions of different students, from many different cultural groups and with many different characteristics, and perspectives" (p. 45). Belonging centered instruction includes an interpersonal domain that attends to community, empathy, and social and emotional aspects of students' classroom experience (Matthews et al., 2021). Ladson-Billings (1997) emphasizes the "need to develop caring and compassionate relationships with students" (p. 707).

Review of the Literature: Three Plus Decades of Inquiry and Active Learning

The dominant perspective is evident in active learning and IBME in how researchers have sought to understand their impacts, and in the origins of these instructional practices. Active learning, inquiry-oriented instruction and inquiry-based learning all developed as strategies for increasing student engagement and student learning in undergraduate science, technology, engineering, and mathematics (STEM). While they each differ somewhat in their commitments and orientations, curriculum development and classroom instruction that adheres to principles of inquiry and/or active learning is consistently oriented toward a common goal of supporting students to understand STEM course content more deeply and to succeed in their undergraduate STEM courses. Over time there has been a trend toward coalescing around common definitions and principles, as described below.

Active Learning

Definitions and conceptions of active learning have consistently centered around the idea that students benefit from being actively involved and engaged in learning instead of passively listening to lecture. Prince (2004) defined "the core elements of active learning to be introducing activities into the traditional lecture and promoting student engagement" (p. 225). Bonwell and Eison (1991) suggested that students should be engaged "in such higher-order thinking tasks as analysis, synthesis, and evaluation" (p. *iii*), and they proposed defining active learning as "instructional activities involving students in doing things and thinking about what they are doing" (p. *iii*). In 2016, the Active Learning in Mathematics Research Action Cluster of the Mathematics Teacher Education Partnership synthesized the research on active and inquiry based learning available at that time and developed a set of five design principles for active learning that included 1) mathematical coherence, 2) instructional activities that promote "active construction of meaning" and "sense-making," 3) norms for classroom discourse that encourage students to share "reasoning in process," 4) an instructional environment that includes multiple modes of instruction, and 5) instructional decision-making in which "the choices made in lesson design and adaptation should favor the perspective of the learners." (Webb, 2016, p. 2).

Inquiry Based Mathematics Education

Laursen and Rasmussen (2019) named four pillars of Inquiry Based Mathematics Education (IBME), the term they proposed to unify inquiry-oriented instruction and inquiry-based learning, and to situate it as a specific form of active learning. The four pillars are as follows:

1. Students engage deeply with coherent and meaningful mathematical tasks.
2. Students collaboratively process mathematical ideas.
3. Instructors inquire into student thinking.
4. Instructors foster equity in their design and facilitation choices. (p. 138)

There is significant overlap between the four pillars of IBME as explicated by Laursen and Rasmussen (2019) and the design principles for active learning. One result has been that the four pillars have increasingly been taken up as a definition of active learning. A team of researchers from the Student Engagement in Mathematics through an Institutional Network for Active Learning (SEMINAL) project analyzed data from interviews with 115 stakeholders (administrators, members of client disciplines, coordinators, leaders, instructors and learning assistants) who were asked about their conceptualizations of active learning. The research team compared stakeholders' responses to the four pillars of IBME and found that instructors' conceptualizations of active learning aligned closely with the first three of the four pillars (Williams et al., 2022). I conjecture that the inquiry and active learning instructional contexts for which research has shown increased student learning outcomes and reduced achievement gaps (Deslauriers et al., 2019; Freeman et al., 2014; Laursen et al., 2014; Theobald et al., 2020), particularly "high-intensity" (Theobold et al., 2020) forms of active learning, are closely aligned with the first three pillars of IBME.

Inclusive and Equitable Undergraduate Mathematics Education: What does it take?

The call for mathematics education researchers to investigate the nature and effect of inclusive and equitable instruction in undergraduate mathematics is becoming increasingly powerful. Hagman (2019) called for an eighth characteristic of successful calculus programs – diversity, equity, and inclusion practices. Laursen and Rasmussen (2019) included an aspirational fourth pillar specifying the need to foster equity in the design and facilitation of IBME and noted that it is not yet entirely clear how to accomplish this in inquiry-based classrooms. Theobald et al., (2020) posited that the large variation in efficacy observed across studies in their data set might result, in part, from variations in the culture of inclusion (p. 6479). Reinholz et al., (2022) suggested that implementing active learning alone is insufficient to improve gender equity in mathematics. And Leyva et al. (2022) analyzed thirty-four undergraduate Black and Latin* students' perceptions of "supportive-for-all practices" which include such active learning aligned practices as creating "space for questions and mistakes" (p. 339), discourse, discussion, and student-thinking centered instruction. They found that "supportive-for-all practices" were perceived by students of color to be "necessary yet insufficient to cultivate equitable opportunities for classroom participation and access to content" (p. 339).

Next I will describe the existing relationship between active learning and equity and argue that IBME is a necessary part of implementing inclusive and equitable instruction in undergraduate mathematics. I will propose instructional practices that are compatible with IBME, and which have potential for resulting in reliably inclusive and equitable IBME learning environments. Finally, I will close by calling for research that adopts a critical perspective to study the nature and effectiveness of inclusive and equitable active learning in mathematics.

Active Learning and Equity: An Under-Investigated Relationship

When researchers shared the seven characteristics of successful calculus programs (Rasmussen, Ellis, Zazkis & Bressoud, 2014; Rasmussen, Ellis & Zazkis, 2014), their definition of "successful" attended to students' experiences by considering reported levels of "enjoyment" and "confidence," but they did not consider how students who were members of underrepresented identity groups fared in the mathematics departments they described. Hagman (2019) identified this failure and called for the addition of an eighth characteristic – diversity, equity, and inclusion practices – not because it was found to have been present, but rather because that study had not been conducted with attention to the ways students' identities impacted their experiences of their calculus programs. The dominant perspective guiding that research project left the dimensions of identity and power uninvestigated.

The fourth pillar of IBME, "instructors foster equity in their design and facilitation choices" (Laursen & Rasmussen, 2019, p. 138), demonstrates commitment to currently under-investigated practices for supporting equity and inclusion. But it is presented with a caveat: "the research base in undergraduate mathematics education does not reveal just how to accomplish this in inquirybased college classrooms" (p. 138). The authors observe that, while research has shown the potential for inquiry classrooms to be equitable, "this is not automatic" (p. 138). They also point out that research on secondary contexts (e.g., Boaler 2006) provides potentially useful direction. In essence, Laursen and Rasmussen are not suggesting that IBME is equitable as it is typically enacted, but rather that IBME can and should be equitable when ideally implemented. Williams et al.'s (2022) findings show that instructors' conceptualizations of active learning align with the first three pillars of IBME but engage minimally, if at all, with the fourth pillar of fostering equity. These findings align with the aspirational nature of the fourth pillar of IBME. Active learning, as described by undergraduate mathematics faculty and instructors, appears to be accurately described by the first three pillars of IBME. For naming the critical goal of achieving equity and inclusion I appreciate Laursen and Rasmussen's inclusion of the fourth pillar that should motivate and inspire further research to describe exactly what is necessary to achieve inclusive and equitable undergraduate mathematics education. The remainder of this paper proposes what inclusive and equitable undergraduate mathematics education might require.

What Are Equitable Teaching Practices in Active Learning Classrooms?

First and foremost, I address the necessity of recognizing and including IBME as a central component, or perhaps a foundational building block, of inclusive and equitable mathematics education. I will then propose instructional practices that research across secondary and tertiary mathematics contexts suggests might be missing components of inclusive and equitable mathematics education. Finally, I call for research grounded in a critical perspective that increases our collective knowledge in the field related to creating the conditions necessary for students to succeed, thrive and to experience full membership in the mathematics community.

IBME: Is it necessary? Let's imagine mathematics instruction that *do<u>es not</u>* meet any one of the first three pillars of IBME. What would this imply regarding equity and/or inclusion in such a classroom? I will take up each of the first three pillars individually.

1. A mathematics classroom in which students *do not* engage deeply with coherent and meaningful mathematical tasks denies students the experience of high expectations (Boaler, 2006, p. 44; Ladson-Billings, 1995; Matthews et al., 2021) and access to the rich learning experiences necessary to achieve high levels of learning. Mathematics instruction that fails to incorporate the first pillar of IBME fails to support the dominant axis dimensions of access and achievement and is not equitable.
2. A mathematics classroom in which students *do not* collaboratively process mathematical ideas misses crucial opportunities for students to experience equitable teaching practices of "assigning competence" (Boaler, 2006, p. 43) and decentering teacher authority (Matthews et al., 2021), and fails to support equitable access to learning opportunities.
3. A mathematics classroom in which instructors *do not* inquire into student thinking fails to exhibit equitable teaching practices of "assigning competence" (Boaler, 2006, p. 43), "student responsibility" (Boaler, 2006, p. 43-44) and decentering teacher authority (Matthews et al., 2021). Mathematics instruction that fails to incorporate the third pillar of IBME fails to provide access to learning opportunities. Failure to inquire into student thinking firmly upholds traditional power dynamics in which the instructor's ideas and thinking override other ways of thinking about or understanding mathematical content.

Such instruction fails to address the critical axis dimension of power.

This suggests that active learning that omits any of the first three pillars of IBME is inconsistent with inclusive and/or equitable mathematics instruction, demonstrating the necessity of including these practices in order to achieve inclusive and equitable mathematics classrooms. In fact, Mathematics instruction that results in students engaging deeply with coherent and meaningful mathematical tasks, collaboratively processing mathematical ideas, and in which instructors consistently inquire in student thinking is not only necessary, but it is an excellent place to start. Next, I will suggest what inclusive and equitable instructional practices could potentially be missing in an IBME classroom.

What else will it take? To consider what might need to be added to the first three pillars of IBME to cultivate reliably inclusive and equitable mathematics classrooms I return to the teaching strategies that I shared earlier in this paper. These included holding high expectations, assigning competence, and decentering teacher authority, all of which are somewhat aligned with the first three pillars of IBME. I also noted that inclusive and equitable teaching includes attention to interpersonal interactions, social dynamics, and community. In the socially interactive environments of active learning classrooms these social and community aspects of inclusive and equitable teaching are especially salient. Furthermore, the first three pillars of IBME do not explicitly call for attention to the community nature of active learning classrooms. The second pillar – students collaboratively process mathematical ideas – may imply the need for instructors to attend to social interactions; however, without careful and explicit attention to establishing norms of interaction, the group tasks that are often intended to foster collaboration are equally likely to reinforce preexisting and deeply established sociohistorical and socio-mathematical norms that are often marginalizing for students who are members of under-represented identity groups in mathematics. With a focus on the critical axis dimensions of identity and power as they are relevant in active learning mathematics classrooms, I suggest the following as potential strategies to support inclusive and equitable mathematics instruction:

1) co-development and ongoing revision of class norms of participation
2) opportunities for students to provide feedback about their experiences at regular intervals throughout the course
3) normalizing the experience of not knowing, particularly as it applies to “prerequisite” knowledge
4) flexible course structures that provide students multiple opportunities to demonstrate their learning.

Ultimately, applying a critical perspective and attaining the goal of supporting students to consistently develop positive mathematics identities that include a sense of belonging in mathematics and a sense of being a capable and skilled doer of mathematics should be central to achieving the aspirational fourth pillar of IBME: Instructors foster equity in their design and facilitation choices (Laursen & Rasmussen, 2019, p. 138).

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