

Empowering Undergraduate Students Through Mathematical Thinking and Learning

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This presentation concentrates on the aspects and experiences of undergraduate students' personal empowerment. It considers the powerful role of mathematics in enhancing undergraduate students' general thinking and learning skills, creativity, independence, self-confidence and personal agency. These features of empowerment will be outlined theoretically and also by using the data from interviews with undergraduate students gathered both in The United States and in Finland. Additionally, some elements of mathematics instruction needed in this kind of an empowerment will be discussed.

Personal Empowerment and Learning

Aspects Involved in Personal Empowerment

There are many more notions of empowerment in the literature than clear definitions for the term. More often than among education researchers, this term is used by social scientists and in sociological theories and it is frequently applied also by educational sociologists. The roots of the word relate to the term *power and power relations* used especially within sociological literature. The goals of this kind of *social empowerment* through individuals are not only to increase people's academic and social skills in order to enable them to take better use of their social and economical environment, or to learn to act along with the structures of a society, but also to actively impact on the development of these structures and their everyday social and cultural environment (Berry, 2005; Safford, 2000). This latter viewpoint is closely connected to the notions of inclusion against exclusion in education, and also to central ideas presented in critical learning theories (e.g., Giroux, 1983), feministic studies (Walkerdine, 1998), and transformational learning (Mezirow, 1991).

In an education context, these kinds of studies focus on capacity building through education among minorities or in undeveloped countries and, more recently, on the social interactions, variation of positioning and voices, negotiation of identities and meanings, or power constructions through discursive practice in classroom contexts (e.g., Evans, Morgan, & Tsatsaroni, 2006; Walkerdine, 1998; Yackel & Cobb, 1996). We will here concentrate on the more individual kind of a perspective and *personal empowerment* in the form of enhancing students' personality or identity, skills, knowledge and experiences through mathematics thinking and learning. These two perspectives (personal vs. social empowerment) are clearly related but the focus in this paper is more to capture those aspects of learning mathematics that constitute students' personally meaningful events or experiences that increase their personal satisfaction or

well-being, identity, and the development of their personal thinking and skills — even though they may have already good mathematical skills and positive mathematical identity.

One definition of empowerment that is also applied in mathematics education research relates to Cummins's (2000) model of power relations and his distinction between coercive and collaborative relations of power. In contrast to coercive power relations that impose one particular perspective over others in relation to language, culture, identity or what counts as knowledge or mathematics, *collaborative power relations* arise from a view of power as being 'generated' for all participants through interaction (Barwell, 2005). Cummins (2000) calls this process *empowerment*, in which the voices of all are listened to and respected. Collaborative relations of power, for example, enable minority students to engage meaningfully and to actively take part in the interactive processes of teaching and learning (Barwell, 2005). More traditionally, this definition is connected to notions and theories of collaborative and cooperative learning. These viewpoints stress collaborative knowledge construction in which both responsibility and knowledge are actively shared (e.g., Goodsell, 1992; Qin, Johnson, & Johnson, 1995). Behind them is the idea of the enhancing power of synergy achieved by true sharing, communication, and construction of ideas, knowledge, and skills with the emphasis on the active interaction between the students (e.g., Prince, 2004). This kind of social context and collaborative relations should make a significant contribution also for personal empowerment.

Most commonly the notions of empowerment in mathematics education literature refer to the instructional efforts, curricular reforms, or classroom practices that are suggested or applied in order to increase the level of students' mathematical knowledge and skills. These perspectives can be seen to support the idea of students' *mathematical empowerment*. Often the main goal in these efforts or related classroom experiments has been to increase the level of students' mathematics achievements and outcomes within a particular group or school context, within a state, or within a whole nation. Strong socio-cultural or economic needs and current educational policies have guided these suggestions and efforts, but also much of the vast research on mathematical cognitions and problem solving relates to this goal. Many mathematics education researchers focus on the improvement of students' problem solving skills and various descriptions of these skills are presented (e.g., Schoenfeld, 1992). Mathematics education standards and education policy makers use such terms as exploration, conjecturing, reasoning logically and the ability to use a variety of mathematical methods effectively to solve non-routine problems for getting mathematical power (e.g., NCTM, 1989). Mathematicians may again use such terms as mathematical maturation, deep mathematical thinking or mathematical sophistication.

Another important feature of personal empowerment relates to the term *self-empowerment*. One example of the use of this term is the Self-Empowerment Theory of Achievement offered by Tucker (1999) and applied, for example, by Berry (2005) in their respective studies of African-American males' experiences of mathematics learning. In order for students to be successful, various forms of self-empowerment were considered necessary. These included high levels of self-motivation, perceived self-control, adaptive skills, engagement in successful behaviors, and self-praise. Within a wider perspective, we may point to the so-called self-theorists' views of achievement

and motivation that consider self-concept, self-esteem, self-confidence or self-determination as important determinants of learning outcomes and experiences (e.g., Deci & Ryan, 2002; Dweck, 2000). Numerous studies of mathematics education also indicate the importance of these self-referenced cognitions for studying mathematics, mathematical performances, and mathematical problem solving (see Malmivuori, 2001; McLeod, 1992; Seegers & Boekaerts, 1996). Empowerment of students in learning mathematics and solving mathematical problems thence significantly involves not only increasing their mathematical knowledge and skills but also enhancement of their positive self-judgments and strengthening their confidence and control experiences.

Perspectives of this Study

In this section we will introduce the applied theoretical constructs for studying undergraduate students' personal empowerment. The recent process-oriented views of learning and education research on self-systems, metacognition and self-regulated learning have brought to the centre the idea or emphasis on active, intentional, and independent learners (e.g., Boekaerts, 1995; Zimmerman & Schunk, 2001). Such perspectives stress students' proactive rather than reactive roles in their learning processes. The personal processes that confirm an efficient state of functioning and learning, together with positive learning experiences and performances are then emphasized (e.g., Bandura, 1986; Manigault, 1997). Our theoretical perspective links these personal features and processes especially to the idea of personal agency and power with respect to doing and learning mathematics (Malmivuori, 2001, 2007). According to McCombs and Marzano (1990), when students are aware of the self as agent, a sense of efficacy, internalized goals for learning, self-regulated learning, and experience of competence are produced.

The role of self-concept in mathematics achievement has long been studied both by mathematics education researchers and by educational psychologists. Self-confidence and self-efficacy represent more specific self-constructs which are also widely applied in mathematics education studies. Both of these constructs have to do with students' experienced self-worth or self-esteem and self-control that appear to be the core experiences for any important personal life or learning event (Malmivuori, 2001). They can be considered to reflect core features of both personally meaningful learning and powerful affective experiences in difficult learning or achievement situations (Malmivuori, 2006). *Positive self-perceptions* intertwined with assurance, confidence, reflection, positive expectancies, and *positive emotion* help create a context for efficient cognitive processes and engagement in mathematics learning and problem solving situations. They are significant features of self-empowerment. Results from these are then measured in terms of students' more efficient cognitive strategy use or deeper processing strategies, and in efficient use of self-regulatory strategies in performances (Bandura, 1986; 1997; Boekaerts, 1995; Taylor & Corrigan, 2005). These aspects again relate to empowerment of students' *thinking and problem solving skills*, and further to their mathematical empowerment.

Thus, self-confidence and self-efficacy are involved in the most significant and powerful personal learning experiences and often determine the level of students' motivation, their possibility for success, and the quality of their affective experiences with mathematics. Development of mathematical identity is tightly connected to these experiences and perceptions of the self in mathematics performance situations. The

positive aspects of self-motivation have then to do with the activation or maintenance of positive expectations, higher commitment, and higher personal goals for learning, performances, and self-regulation in studying mathematics (Malmivuori, 2001). In turn, willingness to study mathematics leads to more frequent mathematical successes and higher persistence, better outcomes, and more frequent choices of advanced mathematics courses and careers. Mathematics learning is then accepted by students as a central domain for *enhancing their personal identity* and development.

Our understanding of personal empowerment comes close to the characterizations of *self-regulated learners* within socio-cognitive, constructivist or phenomenological theories of learning and motivation. For example, Zimmerman (1989) describes self-regulated students as those who are metacognitively, motivationally, and behaviorally active participants in their own learning. Such characteristics as independence, responsibility for one's own learning and achievement outcomes, control over choices of strategies and volition, intrinsic interest in and goals for learning, extraordinary effort and persistence during learning and performance can all be attached to self-regulated learners. Furthermore, personal agency, active self-regulation, and empowerment involve not only students' active and persistent learning but also their ability to select, structure and create environments that optimize their learning (De Corte, Verschaffel & Eynde, 2000; McCombs & Marzano, 1990). Self-regulated learners approach tasks with confidence, diligence, and resourcefulness. These kind of aspects enhance personal learning experiences and achievements through personal enablement and empowerment (Paris & Byrnes, 1989). Self-regulated learners report high self-efficacy, self-related attributions for outcomes, intrinsic task interest, and positive emotion (Borkowski, Carr, Rellinger, & Pressley, 1990; Schunk, 1990).

Study of Undergraduate Students' Personal Empowerment

Applied Interview Data and the Subjects

The empirical results of this study are based on data from both interviews with Finnish and American undergraduate students. The Finnish data consists of individual interviews with 12 beginning undergraduate mathematics students at the University of Helsinki. The data were gathered at the end of the students' first university semester within a larger research framework dealing with these students' first experiences with university mathematics, their personal histories and school experiences with mathematics, and their views and beliefs about mathematics. The interview data with American undergraduate mathematics students was gathered as part to the IBL (Inquiry-Based Learning) project, which focuses on undergraduate mathematics teaching and learning through discovery and active classroom interactions. The preliminary data was gathered through personal and group interviews from five participating university campuses. The students' experiences with university mathematics varied from less than one year to two and three years. All the interviewed students at these campuses had taken one or more mathematics courses that applied IBL teaching and learning strategies. There were a total of 23 USA students involved in the interviews.

Data Analysis and the Constructed Themes

The idea of content analysis was applied in constructing the themes from student interview data. The coding scheme was mainly based on the theoretical constructs

introduced above but the subcategories were driven also by the interview data. All the student interviews were first transcribed with a focus on the content of the discussions, without specifications for intonations, breaks or other silent signs. All the statements in episodes and sentences referring to students' personal enhancing experiences with school or university mathematics were examined. In these episodes, students were describing the features, moments, and experiences of mathematics or of studying mathematics that made them feel positive about themselves or that were otherwise positive or enhancing aspects of mathematics. Afterwards, the selected expressions and sentences were categorized under several themes. In reporting the results and examples from students' expressions, we use codes FF1, ..., FF6 to denote Finnish female students and codes FM1, ..., FM4 to indicate the Finnish male students. In turn, AF1, ..., AF11 denote the American female students and the codes AM1, ..., AM12 refer to the American male students.

In accordance with the introduced theoretical perspectives, we distinguished between students' self-empowerment and their mathematical empowerment. The latter aspect was included especially in the first category of the second broader theme of empowering thinking and learning skills. Statements tightly connected to the powerful moments or experiences of positive self-esteem, enhanced self-confidence, and high satisfaction with their own performance or achievements were grouped under the theme of *empowerment by experienced high personal ability and self-worth*. This category seemed to represent powerful authentic experiences with a strong memory of positive feelings like joy, self-respect and experienced strong inner satisfaction or gratitude. Other kinds of positive feelings or *powerful emotions* (e.g., aha experiences, enjoyment, liking) were experienced with mathematics as a whole or after hard effort and struggle or at the moments of discovering a hint or an idea in mathematical problems. These were not connected (by students) directly to experiences of enhanced self-esteem or self-worth. Another category of students' experiences was named *building mathematics identity*. This theme represented student statements that dealt with their views about knowing and doing of mathematics as an important feature of identity development. The theme of *personal agency and self-regulation* gathered statements about the forms of activity and experiences that indicated the quality and development of students' independence, self-reflection, and active control of their own thinking and learning while studying mathematics. These four themes are reported as forms of self-empowerment.

The second main theme of empowering thinking and learning skills consisted of three categories. The category of *enhancement of thinking and problem solving skills* is used here to denote all those student statements referring to the cognitive, socio-cognitive or metacognitive skills and knowledge that they consider highly important and as gains after learning mathematics and solving mathematical problems in a social environment. Other categories of this theme have to do with such aspects as *increased skills in learning and understanding*, and *empowerment through involved creativity and flexibility*. The first category gathers those expressions and ideas related directly to enhancement of their general thinking skills or ability to solve problems in various contexts. Creativity and flexibility refer to the creative aspects of learning to think mathematically, to the creative and intuitive moments of doing mathematics and to acquired flexible thinking skills. Increased skills in learning and understanding include students' improved skills in learning and thinking about other subjects, their ability to

apply mathematical knowledge and skills in other contexts, and their deeper understanding.

Results

Self-Empowerment

Experiences of high personal ability, self-esteem and self-worth.

In this section, we will deal with undergraduate students' experiences of studying and knowing mathematics intertwined with their positive self-perceptions, enhanced self-esteem or self-confidence with pride, satisfaction or other accompanied positive feelings of the self. These aspects have to do with students' *empowerment by experiencing high self-worth and personal ability*. As the interviews with Finnish students had a broader focus than those with American students, these expressions came more frequently from Finnish undergraduates. They told us about their personal mathematical history, and often their most powerful memories and experiences involved in positive experiences of the self when performing well in mathematics. These experiences were characterized by positive feelings of joy and self-respect, strong inner satisfaction or gratitude. This was more apparent in Finnish male students' experiences. As an example, below is a Finnish male student's responses in assessing the most positive features of studying mathematics and his personal relation to studying mathematics. His expressions reflect the high importance of mathematics for his self-perception.

FM1: I have always been quite good in mathematics, and that's what makes you feel good. I have never thought about why is it mathematics. I have always liked that, and in math classes I have liked that I know the things. ... It has not been important at all whether I'm bad in other classes. In mathematics I have always wanted to be among the best.

Another two Finnish male students' related expressions told us again about how closely assessments of one's own self-worth and personal ability with respect to others can be linked to students' experiences in everyday mathematics classrooms throughout their school years.

FM2: I have never needed to look up to anyone in math classes.

FM3: Always I had the chance to show others that, yes, now I could do this.

This kind of self-empowerment with school mathematics was not only experienced by male students. Even though this was not so clearly linked to females' assessment of their personal worth as compared to the males', self-respect played an important role also among female students' school experiences with mathematics.

FF1: It is nice when you can write down something in mathematical symbols so that you can think that no outsider can understand any of it, and still you know that thing.

FF2: The instructional situation, it has been the most positive, and that I have succeeded in solving the problems and I have recognized that I can succeed at those and that I can do mathematics.

Mathematics examinations represent common situations where students experience powerful assessment of one's own ability and worth. This is reflected in a Finnish female's report of her significant experiences involved in mathematics examinations.

FF2: I was always insecure. I had tension every time when I went to a math exam. I was very nervous whether I can solve the problems. ... Then, it was so nice when I noticed that now I can do these all. I always liked to, I thought that fabulous, these are right, these have to be right.

Also USA students' responses referred to high self-respect or self-esteem and their experiences of satisfaction while engaged in succeeding in (university) mathematics. They were engaged in cooperative IBL mathematics instruction with increased group work. Their expressions thence reflected a kind of collaborative self-empowerment while experiencing positive esteem and pride in solving difficult problems.

AF1: And it's really tricky, but it's kind of satisfying when you do. And it makes us feel smart.

AF2: Yeah. In another class, kind of fun. Especially if most groups didn't get the answer and your group did, you feel proud.

Powerful positive emotions with mathematics.

The most powerful emotions have to do with students' judgments of their personal ability, skills and worth with respect to mathematics achievement and performance situations (Malmivuori, 2007, 2004). As noted above, deep experiences of self-worth are often expressed as various kinds of powerful emotions like pride or joy. But, there are also other kind of powerful affective experiences with mathematics that are not so apparently connected to assessment of one's own mathematical ability or skills. These may include powerful feelings of enjoyment, short term responses such as so-called "aha"-experiences when succeeding after a difficult problem-solving episode, or just overall liking of mathematics. In this data, powerful emotions were experienced after understanding or completion of mathematical problems, after hard effort and struggle, or at the moments of discovering a hint or an idea in mathematical problems.

Again, due to the interview situation and research context, the Finnish students in our study reported more of this kind of affective responses, and males more frequently than females. Most of the Finnish students interviewed said that they had liked mathematics through all their school years, and this influenced their decision to study mathematics at the university.

FF2: Yes, I have always liked mathematics. For example, in high school, I might just do math homework even for 6 hours. ... It has always been so pleasant.

FM1: I have liked mathematics from the beginning of my school years. It's fun. We used to solve problems with my friends, and I think that maybe the enthusiasm of some of them increased through me.

Some students offered more specific contexts for powerful positive experiences including problem solving situations, responses after a success or understanding the topics, or particular aspects or forms of doing mathematics. For example:

AM1: I enjoy it because I enjoy mathematics, enjoy writing up the proofs, and learning new things.

FF3: Yes, it gets you in a very good mood, so that yes, I succeeded.

FM3: I enjoy of that when I notice that, aha, two hours went by. ... No, it is not a waste of time. It is so incredibly good feeling if you can solve the problem.

FF2: What I enjoyed was that when I went to math class, then the problems, they always felt just sort of that we'll do something really exciting and interesting when we'll puzzle out mathematical problems. That's what I liked.

Students reported also moments of discovery and a kind of "flow" experience while engaged in doing mathematics. These moments or situations of deep involvement and discovery were experienced as powerful positive states that made it possible to solve even hard mathematical problems.

FF4: You get tremendously lot out of it, and in a way you probably get the most positive experiences and, in turn, the worst experiences out of it. In that way it is a really important thing.

FM3: Yes, you in a way fall into a kind of a trance. You don't notice that time goes ... And then, suddenly, you notice that it was solved, and then you are sweaty all over.... Yes, you got just that, you can in a way concentrate. You don't even notice what's happening around you.

Building mathematical identity.

This section deals with students' understanding of mathematical thinking and learning as significantly involved in the development of their personal identity. Statements in this group relate to the opportunities, situations, and processes through which students may either check, verify or confirm their identity or worth, or gain confidence and self-esteem. The examples describe students' ideas and experiences of mathematics as an important framework for their self-reflection and development of their (academic) self-perceptions. Again, more references for these aspects appeared in Finnish students' interview data. More often than USA students, Finnish students discussed the significance of school mathematics learning in testing or increasing their self-esteem, worth and confidence level.

FF1: Yes, success in mathematics is quite important. I consider it a kind of measure of intelligence, and I want to succeed in it.

FF4: I have always had this thing that I have to succeed in things and especially in math. So that if things didn't work, I had to do them again in a different way.... I have always had a little of this kind of a perfectionism.

FM2: It has been important that I have always wanted to prove myself that I am good at least in this.

Generally, both males and females discussed this kind of empowerment in their identity by learning and studying mathematics. But, Finnish male students more often referred to competition or social comparison in mathematics.

FM3: I have usually this thing that I'll succeed much better if I'll have somebody against whom I can compete ... For example, in our school, there were a few classmates against whom I had a big competition. Some of them got the full points and another got one point less. There was a terrible war going. The same thing happened in high school.

But competition and comparison can have a significant role also in the development and testing of identity among female students.

FF3: I was with one girl in the same class from the beginning of the first elementary grade until the last of high school years, for all those years. It began already in elementary school that we competed for whom of us is better (in mathematics). We have never been good friends. We can't tolerate each other.

USA students talked about the role of the social environment in an IBL class for gaining or verifying their self-esteem or self-confidence. These focused especially on the personal challenge of presenting and explaining solutions to other students in front of the class.

AM2: Also confidence. Speaking in front of people – I was uncomfortable with this before, it was hard at first.

AF4: Sometimes I'll get up to explain my proof and be like, - Oh gosh, how do I justify this stuff? - and then you remember. Cause you kind have to defend yourself up there.

Another male student said a bit more about the process of gaining more self-confidence by presenting mathematics to others.

AM4: I think that one thing for me is going up in front of people and talking, 'cause there are some people that like to do that, but I don't think a lot of people do. I don't. The first time he called me, I was really fidgety and looking around and stuff. But he makes you feel comfortable, and then the more you do it, you kind of see how it is for a teacher. And then you just become used to it after a while. It's not a big deal.

Going to the blackboard and presenting a solution also constitutes a big challenge and important context for the development of Finnish students' self-perceptions and confidence.

FM2: I hope that I do not need to go to the blackboard if I am not completely sure that my solution is correct. ... I necessarily do not want to go there if I don't know that it is certainly true.

FM2: With trivial problems I didn't care if someone else could do that on the blackboard. I don't need to prove it to the others if I myself know that I can do that problem. That is enough for me.

Personal agency and self-regulation.

The students' experiences reflected both motivational self-empowerment and empowerment in personal agency and self-regulatory skills accompanied by self-confidence. The number of such statements was higher among USA male students than

among USA females, but the opposite was true for Finnish students. The USA students' experiences in their IBL classes seemed in particular to give rise to their active reflection about own learning and problem solving activities. They described independence, self-knowledge, self-instruction, responsibility, and active regulation of their own learning. Aspects of the class had enhanced their need and ability to be reflective and active in studying mathematics, and they mostly found these to be highly positive features of studying mathematics.

AM2: It made me want to be a math major. I know what I know, very well.

AM3: And it really helps to reestablish what you learned.

AM1: Writing out homework, you had to be sure you understood exactly what they said. If you saw it in class, you had to know how to do it yourself. You're responsible for everything.

Some of the students offer direct hints about the processes of empowerment by confidence, independence and efficacy.

AF3: I think it helps with your self-confidence, in math and in general, because when you do figure out something by yourself, it makes you feel really proud of yourself, and it shows you that you really are capable of all of this different math. It definitely helps you feel comfortable, if you're going into another, harder class.

AM6: ...because you realize that when you try to teach something to yourself, you're learning it in a different way, and it stays with you longer than when you just study it for a test and then you don't look at it again.

Finnish students talked more about the beneficial or motivational aspects involved in own effort, self-knowledge, and regulation of own learning.

FF4: It has been the most important in math that I have understood the things and in a way known that I know the things in it.

FF2: I probably see it that it is quite independent work. It is a sort of development of yourself. It is a kind of brain work.

Students described how the active regulation of their thought works for them and what are the results of their own effort and persistence.

FM3: Yes, quite actively it can take few hours. I may try different forms of solving the problem. So that, aha, this didn't work. Just all over again, from the beginning, for as long as it is solved. So it comes then.

Several students asserted that doing mathematics on your own and believing in your own efforts and persistence is what is needed in mathematics.

FF2: You need to be quite independent. And, studying it, you need to believe in what you are doing. Yes, it takes hard work before you can know the things.

FF4: It is a sort of subject that you can not feed in, as it is particularly a sort of understanding and just a kind of insight.... You need to figure it out by yourself.

Empowering thinking and learning skills

In this section, we concentrate on the personal empowerment that is involved in enhanced thinking and problem solving skills, creativity and usefulness of mathematical knowledge and skills for other subjects, skills or areas of everyday life. The examples illustrate what kind of experiences and ideas students have about the quality and development of their knowledge and thinking in doing mathematics.

Enhancement of thinking and problem solving skills.

Data from the interviews indicate that thinking and reasoning are strong features of doing and studying mathematics. But mathematical problem solving or construction of proofs not only requires careful reasoning skills but also serve to enhance students' general thinking skills such as remembering or logical reasoning and broadening their perspectives. This intellectual endeavor seems to appeal to and to fascinate undergraduate students in their study of mathematics. First, we present an example of students' ideas about the types of thinking needed to succeed in mathematics.

FM2: You just need to understand the way to read it.... It may demand that you have to change your way of thinking completely. ... It needs then hard work and motivation.

FM2: In mathematics it is important to learn a kind of a model for problem solving, a model that is based on logic.... You should not let anything to bluff you.

The enhanced general thinking skills through studying and doing mathematics were most often described by terms related to logical thinking, both by Finnish and USA students.

FF4: I think that mathematics has developed a sort of logical thinking.

AM10: I just like the way that it's making me think and it's making me look at things, which is improving my logical thinking abilities.

AF4: You're like, - I know I had a reason for this! - But usually it comes to you and makes you stronger with your logic.

Mathematical problem solving and making proofs were considered as powerful exercise for brains and for the development of thinking.

FF4: Even simple problems develop something in you, and even in those you may make mistakes. Then you learn in a way to pay attention to the right things so that you'll have fewer mistakes in the future.

AF6: It trains you to look at things from all possible angles. Not necessarily from a political sense, but more from a problem solving sense, just seeing every approach. A lot of it is in math, but it extends a bit outside math as well.

AM9: And when you start to prove things, you get a glimpse at the underlying structure of all mathematics, and see how things relate, and start thinking about things in a different way.

The USA students talked how presenting and explaining mathematics in the IBL classroom enhanced their thinking.

AF3: I think it really facilitates problem solving skills. ... And I've become so much better of a thinker.

AM6: Before, it used to be (id) think of something linearly. And this way, you break something down and find the cause to everything and find many solutions. I think that really made a big difference. You don't see it now. You see it later. But it does do that type of thing to you.

AF8: You learn to understand other people's thought processes, how to explain to them. You are learning how classmates think. Alternate ways to consider an idea – you see different proof styles.

Increased skills in learning and understanding.

Both USA and Finnish students found various other contexts for applying their knowledge and skills acquired from studying mathematics. In addition to the general usefulness of mathematics, students reported an increased ability to do better in other subjects, to understand new or difficult ideas, and increased social learning skills. The perceived general high value of studying mathematics in order to act in everyday life was described especially by Finnish students. In turn, USA students spoke about their enhanced social learning skills, due to their IBL experience.

FM1: Well, mathematics is actually everything. It's everywhere and with everyone.

FF4: In mathematics you really need to think a lot. So, it develops you. I really believe that in a way it develops thinking, and not only mathematical thinking but in relation to everything.

Both USA and Finnish students reported their increased ability to do and succeed in other subjects or other areas of everyday life.

FF4: When you are able to apply what you have learned also to other subjects. In a way, when you notice that you can use those also in more difficult things.

FF4: Math has been useful (in a humanistic subject) in thinking so that I can combine things.

AF6: That really helps writing Philosophy papers.

AM6: I think in general what it does is, I saw that when I go home and I was taking care of the family business, whenever you have a problem - and I didn't realize it until I was talking to my mother and she told me - now when there's a problem, I sit there and I look at, you try to find every possible way to find a solution to a problem. And this is a more practical, real life way it affected me.

A smaller number of comments relate to a kind of deeper understanding through abstraction, gained by studying university mathematics.

AM9: Then you can start abstracting it. And you'll understand the reasons why you completely understood it, and why it's necessary to have abstraction.

AM9: Because it is hard to start abstracting things, and you need to know-. Like, looking at a number one, I know immediately what that is. And you have that concept and some others down, then you can put them together and completely understand something.

AM4: ...and you really internalize it once you do find your own proof.

Development of understanding and social skills in learning and dealing mathematics in an active IBL classroom context was clearly noted by USA students. They reported increases in their ability to take and to offer critique to other students, their ability to understand and support others' efforts and thinking, and the positive aspects involved in learning to help each other.

AM11: Seeing people in class present and having to think. What they're thinking is really interesting, because it helps you get math from other peoples' perspectives.

AM1: Finding [the flaws] in everybody else's argument - I became a lot more critical of myself that semester.

AF11: Everyone goes up, everyone has to do it. You're all in the same shoes, so you learn how to support it each other.

AF6: So that's just a good feeling about, someone helps you, and maybe you help everyone, and maybe it could be you wrote something on the board that someone else didn't have. Maybe you wrote something that no one else had. You don't know. It's just a good feeling to know you're helping the class.

Empowerment through involved creativity and flexibility.

Mathematical thinking offers opportunities for creativity and flexible or multifold use of one's own mental resources. Creativity may be accompanied by independent thinking and the ability to find one's own ways of understanding and solving problems. All these features are further closely intertwined with increased personal agency, efficacy and inner satisfaction. Both USA and Finnish students spoke of these issues. Finnish students discussed flexibility and creative aspects of studying university mathematics.

FF2: Math differs from other subjects so that in it you can develop in a way, it is so multilayered. So that, you can explain one thing in many ways and in some way, it is not at all restricted to anything but in it you always get to different things.

FF2: You need to absorb those terms that may be a bit difficult at some times and then just in a way create by yourself...

FF4: School mathematics was not so concerned with applications but particular solution and things. It was not as free activity (as university mathematics)... There are no strict rules how to apply (mathematical knowledge). In principle you may do anything as long as you can prove that to be valid.

A Finnish male student viewed mathematics as a powerful and essentially creative activity in nature. He described people who need mathematics as:

FM3: Those who develop this world ahead. Or, not necessarily forward but to some direction at least. Just those who probably will make some new inventions or the like. Somehow, these utopians.

Some students described the highly positive experiences of being able to see and use their own ideas in doing mathematics. Creative and intuitive features of doing mathematics were important for students' enjoyment of mathematics.

FF1: It is fun when you immediately get an insight, a small intuitive thought that it should go like this and this, when you further notice that it goes like this.

AM4: (It's) my favorite math class I've had, because there's a lot of emphasis on trying to solve things on your own, and it sounds cheesy, but there is kind of a joy in discovering something on your own, vs. reading some answer.

FM2: I do not want to study mathematics by learning to use particular rules and formulas. I do not get anything out of that. I would rather start from the creative aspect of doing mathematics. For example, some problems in which you'll face a new situation and for which you need to develop the logic in that situation or the main point in the problem.

AM3: ... You take the time to do the proof without previous examples, without any help, so it makes you think through the problem as you go through it. It's kind of like a process of learning how to walk again.

Some USA students discussed the positive effects following a creative style of studying in an active mathematics class with discovery and inquiry.

AM11: ... Since you have to create your own way of doing things, and see other people create their own way of doing things, it can help with creativity in an area where there really isn't as much.

AF10: Math books are more accessible now. You can have more intuition, about what's in them, because you have had to do it yourself. You have to find the intuition yourself.

Discussion

The idea in this paper was to focus on students' enhancing experiences and development while studying and doing mathematics. This *personal empowerment* relates to those features of mathematics learning and problem solving that have a powerful impact on the development of students' thinking and learning skills, their self-perceptions or identity and, further, their willingness to study mathematics. Undergraduate students' experiences of self-empowerment as well as enhanced thinking and learning skills were described from interview data with both USA and Finnish undergraduate students. The results reveal the important role of mathematics learning and thinking in the development of students' personal skills and identities. Studying mathematics in a social classroom context constitutes a significant frame of reference for the quality and development of students' perceptions of their own ability and skills. Uniquely powerful experiences of success enhanced students' self-confidence, satisfaction and positive emotions. This was even more obvious after exerting hard effort or facing difficulty in doing mathematics. This kind of self-empowerment was experienced also as a result of group effort and success. In turn, increases in mathematical knowledge and thinking skills seemed to strongly impact students' general thinking skills, creativity, deeper understanding, and also their learning skills. Increase in creative thinking and flexibility were reported especially by the USA

students who had taken courses applying active teaching and learning strategies with discovery and group discussions. In addition to enhanced thinking skills, these students told about their increased skills in communicating and learning other subjects. But both the Finnish and the USA students described studying and doing mathematics as significantly related to independence and ability to think on one's own. These characteristics enhanced their perceptions of capability but also awareness of their own knowledge and learning. Studying mathematics constituted a real empowering element for these undergraduate students' development of identity.

Conclusions

The USA students had all taken college mathematics courses that applied active and discovery learning methods called IBL (inquiry-based learning). Their experiences indicated the very powerful effects of this kind of learning context with its emphasis on inquiry, independence, and active cooperation with other students. These students learned to face the challenge of doing mathematics and their own thinking and understanding. Explaining and writing mathematics to others enhanced students' thinking, self-confidence and deeper understanding. Responsibility and meaningful learning activities represent important aspects of active learning (e.g., Prince, 2004). On the other hand, the interaction among the students was related to collaborative power relations, in which the voices of all are listened to and respected. These seemed to enhance students' social learning skills and the development of critical thinking. Such aspects as creativity, responsibility and highly powerful positive emotions and states while doing mathematics are not directly related to increases in mathematical content knowledge but rather extend to broader situations of learning and everyday life. For example, such aspects as dispositions toward wondering, problem finding and investigating, and ability to be metacognitive and intellectually careful are seen by Perkins and Tishman (2001) to provide the best leverage on the kinds of thinking and learning challenges that young people in our society face.

What features of mathematics instruction and learning may foster this kind of personal development? The experiences of the students that we interviewed pointed to features such as social support, positive encouragement, challenging but meaningful learning and problem solving, and to freedom to think on their own but also to opportunities to reflect their own ideas and discuss them with other students. Case studies of effective undergraduate mathematics programs again point to the climate established by the faculty and attitudes such as respecting students, caring about the students' academic and general welfare, and enjoying one's career as a collegiate educator (Tucker, 1995). Enhancing elements of active or cooperative learning environment include features such as encouraging students to dialogue together, successful cooperative groups with positive interdependence, challenging students' thinking or promoting higher-order thinking, and encouraging students to accept responsibility for their own learning (Gillies, 2007). Pedagogical practices and teachers applying these elements seem to achieve high educational gains as well as enhance students' social skills (Gillies, 2007; Jordan & Le Métais, 1997; King, 2002). In turn, problem-based instruction is seen to address many desirable outcomes of an undergraduate education including critical thinking and ability to analyze and solve complex problems, efficient use of learning resources, cooperative work, and demonstration of versatile and effective communication skills (Duch, Groh, & Allen, 2001). In other studies, inquiry appears to

be more effective than traditional instruction at improving academic achievement and the development of thinking and problem solving (Prince & Felder, 2007). The results of this study also indicate that these features of classroom practices are highly efficient for undergraduate students who study mathematics. Research on the active forms of teaching and instruction often deals with the role and activities of the teacher. More research is needed to show which of the mathematics classroom practices and forms of interaction or inquiry are the most effective for students and in which classroom contexts and for which student groups.

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