Author Accepted Manuscript

Accepted for publication in
International Journal of Research in Undergraduate Mathematics Education

What's in a name?
Framing struggles of a mathematics education reform community

Concerns over the quality and diversity of the future workforce in science, technology, engineering and mathematics (STEM) in the United States have led to repeated calls to reform undergraduate education. This paper focuses on the efforts of one mathematics education reform movement in higher education to frame itself and its inquiry-based learning (IBL) teaching methods. Since its inception, members of this IBL movement struggled to negotiate their historical connection to the legacy of the late mathematician R.L. Moore and their need to modernize to recruit new mathematics instructors. A strong initial connection to Moore and his now-troubling racial and sexist views led to intense disputes over the framing, or branding, of the movement that at first inhibited the growth of the movement. Over time, however, as newer members joined the movement these framing disputes increasingly reflected a new generation understanding of IBL that embraces a broader range of methods and people that has slowly transformed the movement.

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Keywords: inquiry-based learning; undergraduate mathematics education reform; framing; social movements; social identity; history of mathematics education

Acknowledgments: We thank the study participants for their time and candor. We also thank the Educational Advancement Foundation (EAF) for sharing documents from their archives, and the US National Science Foundation for support of this study under award DUE-1347669. We thank a number of advisors who provided thoughtful comments on this manuscript and the public presentation of our findings to practitioners. All findings and opinions are those of the authors and not the EAF or the funder.

What's in a name?

Framing struggles of a mathematics education reform community

Concerns over the quality of the future workforce in science, technology, engineering and mathematics (STEM) in the United States have led to repeated calls to reform undergraduate STEM education to improve the numbers, preparation, and diversity of STEM graduates and workforce (Boyer 1990; Boyer Commission 1998; Brint 2011; NGA 1986; NSF 1993, 1996; U.S. Department of Education 2016). Common targets of such calls are mathematics courses, which serve as gateways into other STEM majors, but also as barriers, because poor educational experiences in introductory mathematics courses lead many students to switch out of STEM degree programs (Bressoud 2015; Ellis, Kelton, & Rasmussen 2014; Hurtado, Eagan, Pryor, Whang, & Tran 2012; Kuh 2008; Larson, Glover, & Melhuish 2015; Seymour & Hewitt 1997). Research shows that active, student-centered instruction in STEM courses can improve student achievement, retention, and sense of professional identity (Freeman, Eddy, McDonough, Smith, Okoroafor, Jordt, & Wenderoth 2014; Hu, Kuh, & Li 2008; Kwon, Rasmussen, & Allen 2005; Lee 2011; Rasmussen et al. 2006), and may be especially important for women and for students of color who are underrepresented in these fields (Ellis, Rasmussen, & Duncan 2013; Fullilove & Treisman 1990; Kogan & Laursen 2014; Laursen et al. 2014). In mathematics, examples of such student-centered instruction include inquiry-based learning (Retsek 2013; Yoshinobu & Jones 2013), inquiry-oriented instruction (Rasmussen & Kwon 2007; Rasmussen, Kwon, Allen, Marrongelle, & Burtch 2006), and process-oriented guided inquiry learning (Straumanis, Beneteau, Guadarrama, Guerra, Lenz, & the POGIL Project 2014).

In US higher education, many instructional reform efforts develop as grassroots projects, designed and led by educators themselves in informally organized communities (Kezar & Gehrke, 2015). In contrast, in European models and U.S. school education, reform often emerges from governments, institutions, or as the result of research (Schmidt, McKnight & Raizen 2007; Artigue & Blomhøj 2013; Krainer & Zehetmeier 2013; Wake & Burkhardt 2013). Absent government mandates or institutional directives to change, the spread of a reform may depend on leaders' ability to persuade others to adopt their approach and join their work. Thus it is important to understand when and how these community-based efforts at reform develop, gather adherents, and take hold in classrooms (Kezar & Gehrke 2015). Education reform in the context of university mathematics is especially interesting to study, given persistent cultural myths that mathematicians produce their best research work when they are young (Henrion 1997) and that giving serious thought to teaching is for faculty members whose careers are in decline (Bass 2005), and arguments for the efficiency and pedagogical value of "sage on the stage" lecturing (e.g., Wu 1999).

Describing a Movement for IBL in Mathematics

This paper is the result of a broader historical sociological research project (Haberler forthcoming) that focuses on one specific community of reformers who have sought to spread

their student-centered pedagogy to other instructors in the United States since the late 1990s. Historically the particular reform movement we analyze is connected to the "Moore method," named after the distinct teaching style of renowned University of Texas topologist R. L. Moore (W. S. Mahavier 1999; W. T. Mahavier 1997; Parker 2005). Although student-centered pedagogies, both in concept and practice, appeared in the United States and Europe well before the 1990s (Artigue & Blomhøj 2013), this Moore-derived movement developed somewhat independently of those concepts and practices. In its earliest days, this group of mathematics educators received no formal pedagogical training and did not engage with the breadth of educational research that would have introduced them to the ideas of thinkers such as John Dewey or Hans Freudenthal. Instead, their earliest understandings and framings of student-centered teaching in mathematics were rooted in reflections by former Moore students on what he did in his classroom and how this influenced them as students. Today most members of this movement identify their teaching method under a much broader term, inquiry-based learning (IBL), rather than as "Moore method" teaching. As we trace in this paper, this shift in terms itself reflects ongoing struggle within the movement over these teaching methods and the values used to frame and identify them.

Depending on who is asked, the proximity to Moore is a blessing or a curse. Moore was a topologist who completed his doctoral work at the University of Chicago in 1905. He taught at several universities before moving to the University of Texas at Austin, where he remained from 1920 until his retirement in 1969 (Parker 2005). Moore's distinctive way of teaching challenged students to solve a series of problems completely on their own without any external sources of assistance or information. Many of Moore's successful students found their experiences in his courses to be empowering or transformative (Albers, Alexanderson, & Reid 1990; Whyburn 1970; Wilder 1976).

What became this Moore-derived IBL movement began in the late 1990s through the efforts of several of Moore's former students. Working with the Educational Advancement Foundation (EAF), itself founded by a former Moore student, this group of empowered former Moore students began to work together to preserve Moore's legacy. They established a yearly conference in his memory and laid the groundwork for plans to spread his teaching style to more instructors at universities and colleges across the country.

Moore himself did not refer to his teaching as inquiry-based learning. However, leaders of the fledgling movement saw the resemblance between Moore's teaching and their generalized perception of the inquiry-based teaching that was gaining momentum across higher education at the turn of the 21st century (Boyer Commission 1998; NSF 1996; Brint 2011). With the support of EAF, community members created a series of workshops, faculty mentoring and small grants programs to support new IBL instructors, and developed four IBL "centers" at select research universities to disseminate these pedagogies to future mathematicians (Haberler forthcoming).

Yet, despite awareness of pedagogical needs in mathematics and a growing body of scholarship that supports IBL as an effective way to teach mathematics, this movement has

experienced mixed results in spreading IBL to new institutions and instructors. The recent history of the movement is instructive in showing the important connection between instructor identity and beliefs (Brownell & Tanner 2012) and the framing of an educational reform movement in higher education. Movement members have increasingly wrestled with the negative perceptions of Moore's history of well-documented racism, anti-Semitism, and sexism (Albers, Alexanderson, & Reid 1990; Corry 2007; Hersh 2011; Parker 2005; Ross 2007; Starbird 2015), and these perceptions are real because they have had negative consequences for the growth of the movement (Thomas & Thomas 1928). For most of this IBL movement's existence, its members have struggled intensely over the nature and limits of inquiry-based learning methods, and with questions of which students should have the opportunity to participate in inquiry-based learning classes. These struggles have both constrained and enabled their ability to grow the IBL reform movement.

In this paper, we present results of a qualitative study based primarily on interview data that describes this IBL movement in mathematics. This paper is not a philosophical argument about the proper definition and application of IBL in mathematics education, though that topic is a subject of keen interest and ongoing investigation, debate and scholarship (Ernst, Hodge, & Yoshinobu 2017; Katz & Thoren 2017a,b; Kuster, Johnson, Keene, & Andrews-Larson 2017; Rasmussen, Marrongelle, Kwon, & Hodge 2017). In fact, within the community, recent scholarship has shown that even the concept of inquiry is understood and applied in nuanced ways by different scholars (Katz & Thoren 2017a,b). While our interview sample is large, we do not provide an exhaustive narrative of the experiences of all participants in the community or a complete history of the IBL movement. This paper takes a historical and synthetic approach to describe the perspectives of movement members and to identify the framing issues they have collectively encountered in defining their movement's goals, audiences, and messages, particularly as they relate to diversity and inclusion. In doing so, it illustrates how such framing can help or hinder the progress of other educational movements.

Conceptual Framework

In this study, we conceptualize the IBL mathematics community as a type of social movement because the group has organized with the goal of continually challenging the status quo—in this case the dominance of lecture methods of instruction in higher education (Snow, Soule, & Kriesi 2004). This group is not alone to do so; indeed, inquiry is a broad label, so it is important to clearly define this movement as one that was initially populated by instructors with direct connection to R. L. Moore and grew as others converged with them over time. Other mathematics educators may teach in similar ways but are not conceptualized as part of this specific group, although some were sampled in the interview set. Hereafter, mentions of "the IBL movement" or "community" will refer to the specific group that began to organize around the Moore conference on an annual basis.

The interpretive framework for this study draws heavily on the social movement mobilization and framing literature. A central issue in this body of scholarship is the extent to

which individuals can positively identify with the social causes, changes, or innovations proposed by a particular social movement. Such identification influences the actions of potential and existing movement members alike.

In considering how potential members are mobilized to join a movement, van Stekelenberg and Roggeband (2013) suggest that a successful movement will recognize the potential for change in relevant target communities or organizations, fulfill the individual needs of potential members, and strategically use discursive framing so that more people can positively identify with its work (see also Klandermans 2013). All of these mobilization strategies reflect a desire to connect with the identities of individuals. Branding, labeling, and attempts to explain IBL are all examples of mobilization strategies employed by this movement that have the potential to create or limit the engagement of current and potential movement members based on their ability to identify with these framings of IBL.

Discursive framing, or what Strang and Soule (1998, p.276) call "interpretive work," refers to the ways that different discourses used by the movement interact to frame the movement's purposes, values, and actions. These discourses include formal public statements or advertising in the media as well as informal, in-person discussions between movement members (or non-members) about the movement's work. This framework assumes that current and prospective movement members have multiple identities based on their various social group and role affiliations in different parts of their lives (Stryker 1968; Stryker, Owens, & White 2000). As a result, individuals may experience identity conflicts between their non-movement and movement identities, conflicts that result from the discursive framing process (Stryker 2000). For example, in this study, we will show that many interview participants who joined the movement after 2005 reported prior interests in equity issues in their teaching. Remarks by these instructors reflect pre-existing identities that came into conflict with elements of the dominant discursive frame about the IBL movement that was in place when they arrived and centered on Moore's influence on the group.

In some cases, individuals negotiate a positive movement identity through frame alignment and amplification processes: As they focus more on key aspects of the movement's dominant frame that resonate with their pre-existing identities, the potential for identity conflict diminishes (Benford & Snow 2000; Snow, Rocheford, Worden, & Benford 1986). In this study, as we will show, some newer members stressed the importance of student-centered instruction when talking about what drew them to the movement. Student-centered instruction, and the ability to engage with other instructors who shared interest in that practice, offered them something positive to cling to when aligning their own identity with the framing of the movement.

In other cases, identity conflicts regarding the dominant movement frame may lead to the development of sub-groups within the movement and to framing disputes over its purposes, values, and actions (Benford 1993; Stryker et al. 2000). Current and prospective movement members negotiate their own multiple understandings and identities related to the purpose of the

movement throughout their time as members (Givan, Roberts, & Soule 2010). In this study, for example, many community members used generational labels to refer to subgroups in the community, reflecting the connection between identity and framing. As more young instructors joined the movement, it became easier for new members to remain in the movement and advocate for their own framing of the movement. As a result, the potential for conflict over framing increased.

This symbolic work, establishing and maintaining the boundaries of key concepts and what it means to be a member of a movement, is a dynamic, ongoing process. This perspective focused our attention on how the active movement participants understood and identified with IBL, and how their perceptions resonated or conflicted with the dominant framing of the IBL movement. Because the focus is on participant perceptions—their understandings of IBL, their identifications with it and the IBL movement—and the consequences of these perceptions for the movement's success, this study does not rely upon a formal, philosophical definition of IBL nor on explicit identification of IBL teaching practices. When these topics do appear, they are raised by respondents because of their importance to instructor identity or the IBL movement's framing processes. That is, we focus on the discursive processes of this educational reform movement, not on its intellectual content.

Context for the Study

The study was conducted as a component of a larger study on the people, structures and ideas that influenced the formation and growth of an IBL mathematics reform movement, defined as above. This community interested us as scholars because of its distinctive origins and growth during a period when active learning approaches were beginning to take hold in undergraduate education beyond an initial group of innovators in mathematics and other STEM disciplines (Handelsman, Ebert-May, Beichner, Bruns, Chang, DeHaan, Gentile, Lauffer, Stewart, Tilghman, & Wood 2004). While we cannot document exact numbers of people active in this loosely bound network, the available records show a growth in participation over two decades. For example, the first conference drew 88 participants from nine states, over 80% of whom were from Texas. By 2015, the conference had grown to over 300 participants, and the mailing list reached over 3000 participants from all 50 states and several countries.

Scheme 1 illustrates selected markers of growth in the numbers and participation in movement-related IBL activities. While many of these activities were sponsored by the Educational Advancement Foundation (EAF 2015), the movement has also been supported by federal funding and reliant upon a strong informal network of grassroots leaders (Haberler forthcoming). Other manifestations include the publication of textbooks, teacher guides, and special journal issues focused on IBL practice, well-attended symposia on IBL practice at professional society meetings, incorporation of an IBL Special Interest Group in the Mathematics Association of America (MAA), and linkages to the MAA's Project NExT (New Experiences in Teaching) professional development program for early-career instructors. An extensive professional development program for instructors seeking to implement IBL in their

own contexts has trained 40-75 instructors annually since 2010, with documented success (Hayward, Kogan & Laursen 2016; Hayward & Laursen 2016). By all these measures, we argue that the particular community of educators connected to this strand of inquiry learning in mathematics must be considered a robust and lively educational reform movement whose successes and challenges are worth understanding.

Scheme 1: Markers of Growth of the IBL Mathematics Movement

1998	2008	2015
First Legacy of R. L. Moore conference Tens of participants	Legacy conferences continue Moore biography circulates Presence at the Joint Mathematics Meetings Periodic IBL workshops Four IBL Centers established (2004) Journal of IBL Math begins (2007) Mentoring program available (there were different versions, the earliest of which began in the early 2000s) Formal research study of IBL begin (2008) Hundreds of participants	IBL/Legacy conference continues Presence at the Joint Mathematics Meetings and MathFest Externally funded IBL workshops Four IBL Centers engage grad students, postdocs & faculty & have served thousands of undergraduates Journal of IBL Math has 46 refereed sets of notes Academy of IBL established (2009) Small grants (2009) are awarded Mentoring program is ongoing Research studies disseminated (2011) and published (2014) Formal links to MAA and Project NExT Thousands of participants

While teaching methods and student outcomes are not the focus of this study, some characterization of IBL teaching is useful context for this study. Many IBL educators have described their practice (see, e.g., Coppin, Mahavier, May & Parker 2009; Ernst, Hodge & Yoshinobu 2017; Hotchkiss, Ecke, Fleron & Renesse 2014, and references therein; Katz &

Thoren 2017a,b, and references therein; Yoshinobu & Jones 2013). Researchers too have characterized the range of teaching practices used by self-described IBL practitioners working in this particular tradition (Laursen 2013; Laursen, Hassi & Hough 2016; Laursen, Hassi, Kogan, Hunter & Weston 2011; Laursen, Hassi, Kogan & Weston 2014). Typically, IBL courses follow a carefully scaffolded sequence of problems rather than a textbook, emphasizing student construction, communication, analysis, and critique of mathematical arguments. Through this process, student ideas and explanations define and drive progress through the curriculum and build toward student discovery of big ideas, new to them, of their mathematical topic. In some IBL classes, an observer would see students going to the board to present previously prepared solutions or proofs, while classmates make comments and refinements to the presented work. In other classrooms, students might work in structured small groups, then compare their approaches in a whole-group discussion, while instructors manage and monitor this process. These IBL approaches have been adapted to nearly all courses in the mathematics curriculum, from first-year to advanced courses, and in courses for mathematics-focused students, for general education of humanities and arts students, and for preservice elementary teachers.

Researchers have also reported the shorter- and longer-term outcomes of IBL instruction for various student groups, including STEM majors and pre-service K-12 teachers (Hassi & Laursen 2015; Kogan & Laursen 2014; Laursen 2013; Laursen, Hassi & Hough 2016; Laursen, Hassi, Kogan, Hunter & Weston 2011; Laursen, Hassi, Kogan & Weston 2014). Collectively these studies point to positive outcomes of IBL instruction for students, including deepened understanding, confidence in their mastery of mathematics, heightened communication and collaboration skills, and appreciation for mathematics as a way of knowing. Students feel empowered by an IBL experience, especially women, and students' grades in later courses suggest lasting benefits to students' abilities to study mathematics successfully, particularly for initially weaker students. Compared to lecture-based courses, IBL helps to level an uneven playing field by improving the outcomes for all and closing gaps for groups often underserved by mathematics education. While these results are similar to those seen for other forms of active learning in STEM disciplines (e.g., Freeman et al. 2014), they are important here because the data were gathered from undergraduate mathematics classrooms where instructors used IBL course materials and instructional practices very representative of this particular IBL tradition. Indeed these instructors often identified themselves, to varying degrees, as participants in the IBL movement we discuss.

Research Methods and Data Analysis

Data were gathered primarily through 34 conversational, semi-structured interviews (Burgess 1984; Seidman 2006) that took place by telephone (32) or in person (2) with core members of the community. To identify a preliminary list of participants, we used knowledge of the community gained from past evaluation projects as well as participation in movement activities such as organizing IBL workshops or the annual conference. We began with key members of this movement, including members who were part of the community from its

beginning and others identified as current leaders, and generated additional names through a snowball technique as data gathering unfolded (Merriam 1998; Mason 2002). Factors such as time within the community, extent of participation and leadership, and prominence within the community were considered in seeking a balanced sample of interviews. As this purposive sample was developed, preliminary impressions of the data suggested a need to interview some instructors who taught in similar ways as the main IBL community but did not fully identify with that group. A short list of names was created from interviewee recommendations and pre-existing knowledge from past evaluation work, and three of these individuals agreed to participate (these three interviews are considered part of the core interview set).

Potential interviewees were sent a solicitation email with background information about the study and how their interview would contribute. Interviews were 60-90 minutes in length. They covered interviewees' knowledge of the history of the IBL movement, what IBL meant to them and how they used it in their own courses, their perspectives on the values and shared behaviors of the IBL community, and their hopes for the future of the movement. In all, 34 people participated in these full-length interviews and together constitute the "core" data set.

To complement these interviews of established community members, we also interviewed 15 new and potential members of the IBL community. We solicited these 15-20-minute interviews at the annual IBL conference through public announcements explaining who we wanted to talk to and why, and at an exhibit table in the conference hotel. This set of supplementary interviews focused on how newer members learned about IBL, what they thought about IBL so far, and their experiences at their first IBL conference.

Institutional review board approval was obtained for this study prior to data gathering. Due to the size and nature of the IBL movement, identifying information (including gender) is minimized in reporting the findings to protect participant identities. All interviews were digitally recorded, transcribed verbatim, and coded manually using the data analysis software *NVivo 10*. The emergent coding process was informed by Miles and Huberman (1994), particularly the strategies of pattern coding, clustering, identifying intervening variables, and making conceptual coherence. The first stage of coding utilized concepts from the social movement literature and included prominent codes based on instructor comments about IBL, their relationship to the framing of the movement, and the presence of frame disputes, as Table 1 shows.

Table 1: Prominent themes in first-stage coding (49 interviews)

Codes	**Number of References**
IBL	997
Moore Method	490
General framing references	370
Framing disputes	108

Secondary coding schemes were developed through analysis and exploration of patterns in the relationships between different clusters of initial codes. Grouping by self-identified generations within the IBL movement emerged as the most helpful way to cluster data for this part of the analysis, and the number of coding references resulting from this clustering process is presented in Table 2.

Table 2: Prominence of generational themes in secondary coding (49 interviews)

Codes	Number of References
General remarks on generation	103
"New", "Young," "Younger" generation	317
"Old" generation	91

In addition to interview data, hundreds of documents provided contextual background information for the analysis. These documents served as references for data triangulation (Denzin 1989, 2006; Flick 2009) and historical context (Flick 2009; Marius & Page 2002; Tosh 2002), particularly in the chronology and development of movement activities—for example, in assessing the purpose, character, and size of the early movement. The archival documents collected with permission from the Educational Advancement Foundation included written and electronic correspondence, mission statements, board minutes, conference video recordings, and conference agendas. Unless explicitly noted, all claims and quotations come from interview data.

Findings

The ways in which this IBL movement's founders framed and labeled their movement, and consequently the pedagogical innovation they were hoping to disseminate, constrained its growth for several years. One reason for this difficulty was rooted in the identities, individual and group-based, of old and new generation movement members and in the connections between those identities and the frames they developed for their movement. In the beginning, the movement and the pedagogy it promoted were tightly connected to R. L. Moore because of his role in inspiring the original members, but the perspectives of newer members often did not resonate with these original frames. In this analysis, we explore the characteristics and perspectives of the original frames of the movement, the perspectives of newer movement members, the conflicts between old and new generation ideas about IBL, and the recent shift toward new generation ideas as dominant in framing the IBL movement. Because of the nature of the data, the discussions referenced in the findings often did not include sophisticated philosophical definitions of IBL, but instead highlighted key principles and activities.

The Generations of IBL

Throughout the history of this IBL movement, the label inquiry-based learning (IBL) has masked extensive disagreements and variety in actual teaching practice. At the core of IBL, however, has been a focus on student-centered instruction. Despite extensive discussion over the

years about which students should receive IBL and what it actually looks like in practice, this general focus provided the foundation for a framing concept that kept the group together.

Permeating respondents' discussions of what IBL means and looks like in practice was the recognition of generational differences within the movement. The discursive use of 'generation' as a grouping category by movement members during their interviews reflects the prominence of academic heritage for some members and is analytically useful in highlighting important differences in perspectives about IBL. Members used "old" generation to refer to the direct academic descendants of Moore, mostly White men who received their doctoral training from Moore at the University of Texas or from one of his students. "New" generation was a term participants used to refer to mostly "younger" scholars who joined the movement in the last decade. These people never met Moore and most did not know about him until after joining the community.

While the interviewees mostly referred to the generations of IBL in a general, binary way in conversation—"old" versus "new," "young," or "younger"—this hardly reflects the actual make-up of the movement. In reality, there are at least three generations in the movement. Most members of the "old" generation have died or were not able to participate in the study, however, the strength of their preferred framing remained strong among middle generation members. The exploration of the framing discourse that follows is not a representation of the movement as it actually is, but as participants explained it in interviews, which underlines the dynamics of identity formation and maintenance at work within this movement. However, these explanations reflected underlying perceptions about the nature of the movement that translated into actions with real consequences throughout the history of the movement (Thomas & Thomas 1928). Most frequently, interviewees described generational conflict in discussing those areas where they undertook identity negotiations to fit the dominant framing of the movement (as they perceived it) or where they had to amplify certain aspects of the framing to attain a sense of positive identity. In Table 3 we categorize interview participants broadly by their alignment with generational framing, using common terminology and omitting the 15 interviews with newly joined members who did not yet have broad experience with the community.

Table 3: Classification of interviewees by generation
(Based on interviews with 34 core members)

Generation of interviewee	Number of interviews
"Old"	7
"New," "Young," "Younger"	17
Hard to tell	10

Old Generation IBL: Remembering Moore

For old generation members, the IBL movement began as a way to remember the educational legacy of R. L. Moore. The older generation framing for the movement has elevated Moore's legacy since the beginning and it was supported by the social and financial backing of the Educational Advancement Foundation (EAF), which organized and funded all movement events. In their eyes, the Moore method was still the "purest" form of IBL, and all variations of what today would be called IBL in mathematics grew out of the Moore method through a series of adaptations. They advocated for a definition of IBL that was as close to what Moore did in his courses as possible. In our interviews, some "old generation" instructors still used the label "Moore method" extensively and emphasized its centrality when speaking about the movement and its teaching methods. As an example of how deeply some revered Moore's teaching, several interviewees commented that "the only person who ever truly taught Moore method courses was Moore himself." Indeed, Moore's descendants began talking about the Modified Moore method to denote their own adaptation in their courses. In this way, the old generation framing of the movement could accommodate adaptation to the method and change within the movement in general, so long as the nominal reference to Moore remained intact.

Despite acknowledging a need to adapt Moore's approach to their own personality and context, the old generation still held "pure" Moore method as the gold standard of this style of teaching. Depending on the level of the course, Moore would begin by "carefully selecting the members of the class" in order to create a class "as homogenously ignorant as possible" (Jones 1977, p. 274. See also Albers, Alexanderson, & Reid 1990). Moore did not assign textbooks in his classes and forbade students from consulting outside sources or working with each other. Instead, students were asked to solve a carefully constructed series of mathematical proofs on their own, using only logic and knowledge gained earlier in the course. One old generation interviewee explained that Moore "used the minimum of symbols and… posed problems in English sentences too. You were to go home, with no help from anyone, and see if you could figure it out." In class, Moore would ask one student at a time—beginning with lower ability students—if he (seldom she) could solve the proof for that day (Jones 1977). If the student answered affirmatively, he went to the board to show the class his proof. Some interviewees remembered that Moore also asked students who were not ready to solve the proof to wait outside. In this way, each student was solely responsible for her or his own progress through the course. Because of the combination of these practices, the old generation understood "pure" Moore method as best suited for students with high ability and promise in mathematics.

Old generation members who had first-hand experience of this approach valued the sense of accomplishment and "transformation" they gained from solving the proofs on their own. However, many also reported experiencing a period of intense frustration early in the course. One shared that, while he initially hated those courses, once the mathematics clicked in his mind, he took all the courses he could from professors who taught using the Moore method. Another old generation speaker remembered struggling in a year-long Moore method course. Halfway

through the course, his professor decided that "[h]e was going to meet with the three 'hot shots' at a different time, and continue to meet with the other five of us at the regular hour. I was really furious about this." In fact, however, with the three high-achieving students removed to their own mini-class, the classroom dynamic for the other five improved dramatically, and they caught up in the course. This account characterizes the powerful experience that students could have, and in turn accounts for the older generation's reverence for Moore and his teaching. Old generation descriptions of the Moore method align with published accounts of some of Moore's former students (Albers, Alexanderson, & Reid 1990; Jones 1977; Whyburn 1970; Wilder 1976).

The annual Legacy of R. L. Moore conference, which became a focal meeting point for movement members, began as an attempt to share and spread the "gospel" of Moore. The conference's name made the initial framing of this movement unmistakable: it was about legacy. The legacy or lineage focus of the early movement members traced back to the 1970s when many of Moore's former students began publishing scholarship remembering his life's work. In 1972, D. Reginald Traylor and William Bane collaborated on a self-published work entitled *Creative Teaching: The Heritage of R. L. Moore*, which chronicled Moore's life and sought to identify all of the mathematicians who were part of Moore's academic lineage. EAF documents revealed that such efforts to identify the vast reach of Moore's 'lineage' continued into the 2000s. The introduction to the EAF-promoted biography of Moore identified 50 students who received their Ph.D. under Moore's guidance, leading to "1,678 doctoral descendants" (Parker 2005, p. vii).

Nearly all interviewees agreed that the earliest years of the Legacy Conference, spanning the late 1990s and early 2000s, felt more like a "family reunion" than an academic conference. Video recordings of the earliest conferences verify these accounts. For example, at the very first conference, the camera pans across the room at several points. The viewer notices few students in the room, and few women. Rather, those visible are mostly White men, all connected to Moore in some way, taking turns remembering different aspects of Moore's personality and classroom demeanor and how they responded to it (EAF 1998). Early conference agendas also support this general framing of the movement at that time. Consider, for example, conference talk titles from the first (1998) and third (2000) conferences (Scheme 2):

Scheme 2: Selected talk titles from early Legacy of R. L. Moore Conferences

1998	2000
RLM: The Big Picture	Moore Method in the Math-For-People-Who-Don't-Want-To-Take-Math Course
RLM's Influence on Other Theories of Mathematics Instruction	On Three Crucial Elements of Texas-Style Teaching as Shown to be Successful in the Secondary Mathematics Classroom
RLM's Calculus Class	Mathematics immersion, inquiry-based learning and Hungarian semesters
Relations to UT Colleagues and Adaptation of the RLM Method to Other Classes	Why are Moore Method Courses Effective?
	Adapting Moore's Method to the Distance Education Classroom

These lists are not exhaustive but they are instructive. In 1998, all topics centered on Moore. In 2000, IBL appears for the first time in any conference program, but it only appears once, while ten talks directly or indirectly referenced Moore. "Texas-style," for example, was another way of referring to the Moore Method at that time because the method also became associated with the University of Texas ("Past Event Reports" n.d.). The "remembering Moore" type of talk was very common at these conferences into the late 2000s.

Explanations of how one was a Moore "descendant" were common at the early conferences, and many old and mid-generation interviewees still understood, or at least explained, their place in the community through their academic "heritage." For example, one old generation member introduced himself by delineating his pedigree: "I'm now one of the growing few who had the good fortune to learn mathematics through the Moore method many years ago. I… studied with... students of R. L Moore and his colleagues in Texas, and so I pretty much got it straight from the horse's mouth." One consequence of the group's emphasis on academic heritage and the role of Moore was that the movement appeared to be a closed group. Some used words like "religious" or "dogmatic" to describe the sentiment in those years: The focus of the movement's frame for the old generation was on keeping the topics of discussion in line with traditional Moore method teaching, and, as subsequent analysis explains, aggressively policing deviations from it. All aspects of this framing—a focus on academic heritage and an approach to student-centered teaching tied so closely to Moore's method—made it challenging for a new generation to join the movement without some internal identity conflict, because few had both the academic pedigree to feel wholly welcome and views on teaching that wholly matched with Moore's.

New Generation IBL: Equity and Collaboration

In our interviews, the new generation often defined itself in direct contrast to the old generation. Nearly all recognized Moore's role in getting this specific movement started. Many even saw value in his method in specific contexts, such as graduate courses where the goal is to encourage independent mathematical research and create professional mathematicians. However, these speakers also viewed Moore and his teaching as out of place in today's society; a poor match to the needs of their classrooms and to how they experienced professional mathematics research. As a result, the then-dominant framing of the movement did not resonate well with new generation members. Instead, newer members came to use the broader term 'IBL' almost exclusively when identifying their teaching; they also emphasized "social justice" and "collaboration" when discussing their understanding of both the IBL movement and its pedagogy. This framing of IBL as socially inclusive and collaborative teaching stands in stark contrast to the old generation's framing of the method as Moore-centric and as providing solitary learning for future research mathematicians.

New generation members saw themselves as teaching more diverse students than Moore taught. Though the language of "social justice" was explicitly identified by only one participant in the study in referring to instructors' concerns for equity in learning opportunities, most new generation instructors identified as instructors who sought to teach all students and who embraced their role teaching students with diverging skill levels and trajectories in mathematics. One new generation interviewee explicitly connected social justice to the waves of new members who entered the movement in recent years, comparing them to the old generation: "I feel like another gap is that maybe the younger people feel that social justice is part of their calling. I don't think the Moore descendants do." Other new generation speakers articulated their social justice perspective, and the context requiring it, in discussing the courses and students they taught. For example, one speaker captured the range of experiences for many of his peers who were not at elite universities and did not teach graduate courses in speaking about not being able to use the "pure" Moore method because it poorly matched his students:

> It certainly wouldn't work for the students that I work with, because I work with the weaker students. I think they would find it intimidating. I think they would find it pointless.... I know I, myself, say, 'Okay, I've got somebody who's gonna get a degree in heating, ventilation, and air conditioning maintenance.... They have to take my math course.... What are they gonna walk away [with], and why should they be taking it?'

For many in the new generation, teaching undergraduate courses, especially courses for non-math majors, broadened the scope of their identities as instructors and their goals for teaching. Rather than seeing their courses as a way to weed out weaker students, many in the new generation sought to connect with all students, including those not actively seeking a professional academic future in mathematics. This contrasted sharply with an older generation framing that advocated education for a narrower population of students with futures as research mathematicians.

A second key component of new generation thinking about IBL connects directly with instructors' positioning of themselves as educators who desire to reach all students. Most new generation IBL instructors in our sample saw student peer collaboration as central to their classrooms. As one instructor succinctly stated, "Class is not where things are taught. Class is where the individuals collaborate." An important aspect of collaboration for new generation IBL instructors is the role it plays in opening up mathematics education to all students. For example, one instructor described using collaborative learning activities—which Moore opposed—because there are "students... who lack confidence enough [that they] get comfort from the other people telling them that they're correct before they go to the board." Other new generation IBL practitioners likewise used collaborative work to ensure that shy students or traditionally lower-achieving students had ample opportunities to succeed in their math courses. One speaker emphasized that students "should be responsible for validating the ideas presented.... They're not just looking to me as the sole authority for when things are true. I want them to actually be responsible and engaged in that process of validating ideas." A second participant stressed the power such collaborative engagement around mathematics brought to student learning and the development of skills that could help them beyond mathematics.

> From my point of view, that's the thing that's important. You're asking them to both articulate carefully their own thoughts in ways that are intelligible to their peers, and you're also asking them to listen carefully to their peers who are struggling to do the same thing. In the best situation, there's a mutual help going on where you're asking questions, say, "Wait a minute. I don't quite understand," or, "Did you mean this?" They're trying to help each other craft a way to express complex ideas to each other. That's a powerful learning tool, but it's also simply an absolutely powerful and essential tool for success no matter what field you have. Because whatever you're doing, ultimately—if you can't communicate what you're thinking and what you're working about to another individual, then it's not really gonna do anybody much good.

A third speaker placed similar importance on collaboration, adding that "education research is showing this to be a reasonable thing." Identifying educational research evidence as a reason for doing IBL was not common across all interviews, but it was non-existent among old generation interviewees who needed only the memory of Moore to justify their pedagogical choices.

A fourth participant articulated a rationale for the use of collaboration that stood in stark contrast to the traditional Moore Method argument that emphasized independent learning as preparation for professional mathematical research. This instructor instead perceived collaboration as an essential activity of mathematical work.

> I think the collaborative process in mathematics is something that's hidden from early students. When you're a professional working mathematician, yes, you do close your door and sit in your office at your desk and work with your proving theorems. [But] then you go to seminars and you talk about your work. [A]nd students don't see that. They are worried that if they collaborate, it's going to somehow diminish what they learned.

> Trying to get them to see that and articulating that to them is an important value that I want to add to their experience in the class.

The speaker's tone here reflects an awareness that the archetype of the isolated mathematician permeates the discipline, extending further than the IBL movement's concerns about the role of collaboration in framing their teaching. Implied here is the notion that new generation instructors are transforming the movement but also the perceptions of mathematics held by their students. For this group of IBL practitioners, collaboration is the most powerful tool for that purpose.

The new generation of IBL users shared the old generation's focus on student-centered instruction, despite differences, and they valued the opportunities offered to them for engaging with others who shared that value. These similarities were important enough that small numbers of new generation instructors stayed active in the early movement. That is, at least some new generation members were able to amplify these aspects of movement framing in order to attain a positive movement identity and thereby sustain their engagement with the group. But not all new generation members agreed. In fact, one key aspect of the new generation frame for the IBL movement is diversity itself, in both pedagogical and social senses. This generation's strong focus on 'social justice' approaches to college teaching, and their preference for collaborative instructional activities and styles, did not disappear. Rather, they informed the conflicts that the movement would eventually endure as more new generation instructors joined the movement over time.

IBL Framing Disputes

Attracted by prospects of community with like-minded colleagues, particularly others who shared interests in the possibilities of IBL as a means to make mathematics more inclusive, younger instructors joined the movement slowly but consistently. Over the course of several years, as old and new generation ideas about IBL collided, the IBL movement encountered two main framing disputes. Both disputes connected to identity issues for movement members. First, the movement's attachment to the memory of R. L. Moore linked it to a personality and social perspective that many potential members found troubling. Moore's racist, anti-Semitic, and sexist perspectives are commonly known (Albers, Alexanderson, & Reid 1990; Hersh 2011; Parker 2005; Starbird 2015). The new generation members in the movement struggled to deal with these aspects of Moore's legacy. For example, one new generation IBL instructor spoke about how Moore's legacy prevented the spread of IBL to other like-minded instructors.

> [P]art of Moore's legacy is racist, and... there are people who would be on board with [our] ideas and wouldn't have any trouble being part of the community at the conference, but because his name is still associated with it, sort of on principle, [they] wouldn't consider it.

A second new generation member expressed frustration that Moore's legacy prevented growth of the community.

> I didn't know [Moore]. I mean, the stories are that he did not treat all groups equally, that he was selective and exclusionary in some of the people he allowed into his classes. I know for a fact that there are colleagues of mine... who will not attend the Legacy Conference because Moore's name is attached to it, even though they do IBL.

A third compared Moore to Thomas Jefferson—both a promoter of American liberty and a slaveholder— as a central yet divisive historical figure: "R. L. Moore is similar in that sense. Some people can't separate the guy's ideas from his personality and his personal views, and that's understandable. That's an unfortunate part of the history." A fourth interviewee simply recounted actual conversations with colleagues: "I asked why they don't come and they said, 'I will not go because Moore was a bigot.'" When asked how widespread these concerns were outside of the movement, this interviewee continued, emphasizing how easy it might be for White, male faculty to downplay these issues that are crucial to other potential new members.

> That's really hard to say. I think [these concerns] are reasonably common among female faculty and minority faculty. I don't know—I hate to stereotype people—but I don't know if white male faculty worry about these things quite as much. I don't think it's unknown.

Information connecting Moore's social views and the IBL community spread through informal and formal means of communication to many potential new members. One outsider—an instructor who shared many interests with the IBL movement but had struggled to feel like a part of the movement despite attending movement events—described hearing about Moore's troubling personal views from colleagues as well as reading about it in a mathematics journal.

> I heard that from people, and then somebody wrote an article about it that came out and was published somewhere—an MAA journal or something, I don't remember. Someone wrote an article about it that I've seen it in print since.

A second outsider interviewed also learned about Moore's personal views via word of mouth.

> What I remember being told was something like, 'R. L. Moore was actually racist and had these kind of issues…' I don't know. Yeah. I don't know. I don't know anything. I don't know what was true, what's not true, and what was said, and what was not said, but I was given that sort of impression in the last year or so.

The lack of certainty explained here underlines the importance of the issue to potential new members of the IBL movement: Even the impression of a connection to Moore's racism was a challenge to positively identifying with the values and teaching strategies of the movement.

In attempting to spread IBL to a new generation of instructors, then, movement founders had failed to recognize the potential challenges of Moore's racism and sexism to the population they were targeting for growth. One newer generation acknowledged that diversity and inclusion represented the movement's biggest challenge, but also its most promising area for growth.

> I think getting minority [faculty] involvement is a challenge. I think diversity and inclusion is probably our biggest challenge and, I think, our biggest potential, to be honest. I mean, I definitely think it's there. We're great with gender. Our gender equality is wonderful, but what we don't have is, we don't have a lot of ethnic minorities. We don't have a lot of people that come from poor families, working class families. We don't have a lot of people who are first-generation college students. That's where I think we could do better. That's our biggest challenge right now.

Second, the movement's original framing of the pedagogy as the "Moore method" and eventually "Modified Moore method" was problematic because, in addition to connecting to Moore's racist legacy, it prevented potential adopters from understanding or identifying with the teaching method itself. One new generation member recalled resistance from colleagues who felt "that it's somewhat cliquish or somewhat exclusive... that if you haven't had sufficient contact with the right people, or you're not part of the 'cool kids' crowd," then the movement was not for you.

In extreme cases, though these were exceptional, some new generation interviewees shared that they had never heard of the Moore method until they attended their first IBL conference or workshop. One speaker acknowledged that "the kind of mathematics [topology] that Moore did is not so popular anymore," and so few would know the reference.

In cases where the new generation had actually heard of the Moore method, they learned of it in graduate school and generally had negative impressions. One new generation interviewee recalled thinking that the Moore method "sounded boring from a student's perspective." Another had attended an institution where one undergraduate course was taught using the Moore method and remembered, "The word on the street was that the students hated it. It was awful." Such reputations made the process of recruiting and persuading new members to join this IBL movement difficult and slow. Analysis of younger generation interviews (n=17) suggest that it took substantial consideration for many of them to join the movement, as it required them to weigh the perceived benefits of joining a group of their colleagues who taught in appealing ways against the costs associated with joining a movement that, in many of their own words, was "dogmatic," "off-putting," "clique-ish," "a fairly closed community," or "a cult of personality" as well as connected to the exclusive and elitist social legacy of R.L. Moore. The early growth of the movement, then, was due largely to new members' own determination to negotiate their identification with the movement by emphasizing a more inclusive vision of IBL and, for some, a desire to change the movement from the margins.

Compounding the framing issues connected to Moore's name has been the independent development of IBL practitioners from outside of the Moore-centric movement. One speaker explained that more people have joined the movement over the years without any connection to Moore and "they develop their own method.... That is, they teach themselves how to teach inquiry-based learning." Additionally, a growing number of instructors came into contact with other threads of inquiry-based mathematics education (Artigue & Blomhøj 2013) through

training or independent research and were seeking a community to sustain them. These newer, less Moore-centric practitioners entered the movement over time and provided a competing frame of historical reference, one that had little use for the troubled legacy of R. L. Moore.

The struggle between the founding members and the slowly expanding younger membership over the labels, values, and purposes that contributed to the framing of the movement initially placed a strong constraint on its early growth. The old framing fostered ongoing identity conflicts for current and potential new generation members, yet the new framing was not strong enough to fully replace it. Many new generation interviewees expressed feeling out of place or not fully welcome the first time they attended the annual conference. One instructor suggested, "There's something about the structure and the history of the group that, for one reason or another, has not resonated with everyone, I think. Certainly, it was the case at my first Legacy meeting. It was sort of a cult [of] personality meeting." Another agreed, offering a stark criticism of the movement's culture: "It was extraordinarily off-putting the way that the older members of the community would talk about their bloodline, their genealogy, their purity, if you will; with whom they studied and which descendant of whom they took courses from." While the older generation championed the value of their experiences with Moore or his descendants, the language they used when they spoke about their experiences also ostracized potential new members: new members struggled to fully and positively identify with the community if they did not share that same academic heritage.

In addition to fostering interactions that new members found unwelcoming, the annual conference was also the setting for many discursive conflicts over what teaching methods appropriately fit under IBL. Older generation members fought for fidelity to the "pure" Moore Method framing and sought to police deviations from it. One interviewee recollected what he called a particularly "cruel attack" that occurred at a Legacy conference. Moore's practice was to introduce mathematical definitions and theorems in ordinary language. In this case, the "cruel attack" came when one presenter talked about using symbols in presenting mathematical definitions and theorems, and a member of the old generation quickly, fiercely, and repeatedly reminded all present that this was not what Moore did in his courses. Another older member recounted a similar conflict at a conference over students' role in class:

> I remember some ferocious arguments... over people talking about whether or not the [students] could agree to a proof being correct. Then this all got confounded with somebody else mentioning social constructivism. Of course, they [the older generation] didn't understand any of this conversation at all, but the notion that the [students] would agree to the correctness of a proof would just absolutely send [them] into an outrage.

Because old generation members came to their understanding of student-centered teaching from Moore and his descendants, they were largely unaware of similar pedagogical ideas, based in social constructivist learning theories that had developed outside of mathematics. The ideas entering the movement through newer members, such as transferring authority in the classroom

to students, stood in stark contrast with the traditional Moore method idea that the instructor retained authority over the "rightness" of knowledge discussed in class.

Older generation instructors also stood by Moore's insistence that it was ultimately more empowering if students arrived at answers on their own and not through collaboration. One interviewee attempted to explain the importance of the "pure Moore method" and why some people who practiced IBL didn't advocate for it.

> At the purest end—the Moore method would be considered the absolute purest form. ...[P]eople who have not experienced it, or have tried it and seen their students get frustrated, and therefore believe that this method is too hard for students…or not understanding enough of our current selfie generation… are highly prejudiced against it.... They've never experienced that transformational experience, so they do not understand it.

This speaker's insistence on the importance of experiencing the Moore method first-hand points to a hope that students would have equally empowering experiences, but nonetheless connects back to the old generation's fondness for academic heritage and the need to know Moore to truly understand or replicate his pedagogical approach to teaching.

In contrast, newer members of the movement argued that a focus on methodological purity was offensive; it was an impediment to positive identification with the IBL movement. It reminded them of Moore's racist legacy: one speaker characterized the word "pure" as problematic because "it triggers some... 'that sounds almost racist' responses." Instead, they advocated for more diversity within IBL, philosophically and socially. Additionally, this group argued that allowing student collaboration was more effective and fostered a better classroom culture in today's increasingly diverse classrooms.

When speaking about collaboration, and the inherent increase in communication that is required in collaborative classrooms, younger generation movement members often explicitly opposed their position to the traditional Moore Method, as these quotations from three different interviewees illustrate.

> I definitely don't practice Moore Method in my teaching, mostly because I feel strongly that students collaborating is a good thing. I think it's a skill I want my students to develop, so I encourage them to work together. I notice the ones in isolation tend to be the most frustrated and getting the least out of the class.

> One of the differences between me and what was happening early on in traditional R. L. Moore was, immediately, [in my classes] there was a large amount of collaborative learning.

> First of all, I heavily encourage collaboration. I think a lot of people who use a pure Moore method, or who have experienced it, would tell that you one of the advantages is that students must rely on themselves if they're not collaborating… but I feel like I'm

> providing more students with a pretty darn good experience than I would if I did anything else. I think probably other people feel the same way.

In addition to positioning themselves against a Moore Method framing of IBL, new generation members rejected the connection of that framing to the preparation of professional mathematicians. One interviewee's discussion exemplifies rejection of such notions of how research mathematicians should be taught.

> I think it's brutal…. You're absolutely isolated, forbidden to talk to people. It's one of the things I find the most confusing about that pure version of [Moore Method]. Don't look at any books. Don't talk to anybody. Work absolutely isolated on your own. I think that's wrong for how a mathematician works.

For such speakers, the idea of the isolated mathematical researcher did not ring true; their embrace of engaging with others' ideas at a deep level was in turn reflected in their approach to IBL. Exposed to identity conflict regarding how their teaching differed from Moore Method, these newer generation members embraced collaboration as something they and others valued, and as resonating with their professional identities as social justice educators for all students. There was strength in numbers: The more new generation members joined the community, the easier it was to make this argument and still feel a sense of belonging.

In recent years, the ongoing conflict over the framing of the IBL movement has transitioned to a point of change in the community. The persistence, and gradually increasing numbers, of movement members who hold a broader definition of IBL has seemingly won the day. These quotes from four different movement members collectively describe how the conference has changed in recent years:

> The audience that comes to the meeting now is the younger crowd, no experience with [Moore method] for the most part, so they really want to just talk about teaching. They are much more likely to use the phrase "IBL" than they are "Moore method."

> I think interpretation of IBL is one of the [issues]…. The first few Legacy conferences I went to, it felt like there was a divide. There was this group of people that was really devoutly following Moore and what Moore did, and that [thought] what he did was very important and it should be preserved. Then there was another group of people who were trying to introduce new ideas, and I feel like this group of people that were open to new ideas—seemed like the minority the first time I crashed the conference. Then, as the years went on, they started becoming the vocal majority, they were the ones organizing [now].

> It was sort of a gradual process, and a back-and-forth between people, the older more senior people who would have their ideas—so it would be a sort of balancing of two different ideas at times that would go on at the conference. Now, it's almost exclusively in the hands of the younger people and nobody's complained.

> I think there've been some changes. I think there was a sense when the Legacy meetings were started out—they were going for a few years before I joined them—it was almost like an alumni meeting… or a veterans meeting of people that'd fought a war, something like this. I think that aspect is gone now.

As a result of this trend toward the framing emerging from the younger membership, some leaders of the movement were considering decreasing their nominal association with Moore. Several younger generation interviewees spoke about their hopes to remove Moore's name from the title of the annual conference as a symbolic gesture to underline the changes that were already taking place. They wanted to foster a positive identity for the movement that focused more on student-centered teaching, social justice, and collaboration, and thereby to connect with greater numbers of contemporary instructors. However, this transition took years to develop and resulted from conflict as original and new members confronted their initial differences over the framing of the movement. Furthermore, despite the emergence of a new dominant frame for the community, the potential for ongoing conflict over the definitions of IBL and the values championed by this movement still exists.

Conclusions

The original framing of the movement experienced a high degree of frame resonance (Snow & Benford 1988) initially. As a mechanism for remembering the legacy of R.L. Moore, it connected with the experiences and perspectives of the instructors who founded the movement. According to four participants with intimate knowledge of the earliest years, the desire to grow was always part of the movement. However, our interview data, as well as documents from the first few annual conferences, indicate that growth was secondary to legacy in the first few years. By the early 2000s, however, spreading IBL to more institutions and recruiting younger mathematics instructors into the movement became more important, as evidenced in several programs supported by EAF in the early 2000s: IBL workshops for new instructors, university IBL Centers to teach more undergraduates using these methods and share course models, small grants and a journal to support writing and sharing of IBL curricula, and a fledgling mentoring program. With more programs and activities connected to the movement, and thus new opportunities to explain IBL to a widening circle, the content of these outreach efforts escalated in importance. As a result, the original, Moore-centric frame was a problem: It did not resonate with the movement's "targets of mobilization" (Benford & Snow 2000, p. 621) because younger instructors did not derive their sense of self from their academic heritage. Further, they found any framing aligned with Moore offensive because of the racism, anti-Semitism, and sexism tied to his name. The ensuing framing disputes over Moore's place in the movement, the appropriate definition of IBL, and the appropriate students for IBL for a while stunted the growth of the movement but eventually resulted in a period of movement transformation that is still underway. In this way, both initial and later framings have had an impact on the movement's progress: Independent of their correctness, these perceptions of the IBL movement have been real in their consequences (Thomas & Thomas 1928).

The overall growth of the IBL movement was not solely due to the efficacy of IBL teaching methods in the classroom. True, the anecdotal claims from the lives of Moore's students and descendants as well as the recent influx of research about the value of student-centered teaching (Kwon et al. 2005; Rasmussen et al. 2006) and IBL-specific teaching (Kogan & Laursen 2014; Laursen et al. 2014) in mathematics have played roles in the framing of the movement. But in the early years, the limitations caused by the exclusionary language and values in the framing of the movement overpowered stories about the efficacy of the teaching methods. Only recently has a more inclusive framing developed (Hayward, Kogan & Laursen 2016) that has the potential to include a more diverse range of IBL teaching methods and instructors.

A big tent philosophy of IBL has taken a strong hold over a critical mass of younger, energetic, and more inclusive instructors who are interested in teaching all students at all levels of mathematics (Special Interest Group of the MAA on IBL in Mathematics, 2017). This younger generation is busy creating newer IBL programs that reflect the emerging dominant framing, reaching out to instructors at community colleges and minority-serving institutions, taking steps to acknowledge and discuss the movement's history, and building bridges with other communities of mathematics educators. If successful, these most recent developments in the movement could lead to significant changes in undergraduate mathematics across the country for the next generation of STEM students.

As researchers studying this movement, our findings also have a role to play in its process of transformation. We recognize that these research findings raise issues that are troubling for community members to confront, and potentially divisive, but we also accept our ethical duties to share these findings with the community that we studied and to assist them in making sense and good use of the findings. As we wrote and edited this manuscript, we found that most who read this paper in draft form experienced sometimes-strong internal conflicts or emotional reactions related to our findings. Ironically, these underline the very power that framing issues have on individuals within and proximal to this IBL movement. We thus consulted many community members about how to best to share this work and developed a set of materials to share the findings directly with these practitioners, including a short non-academic essay and video message describing key findings, and a suite of responses from community leaders suggesting different possible responses to the findings (Laursen, Haberler & Hayward 2017). We will track with interest the ways these may be helpful—or not—to those taking the movement forward.

Indeed, both in its subject matter, and in our efforts to bring these findings to practitioners, this study serves as an example of research that addresses equity issues in mathematics education, as called for recently in this journal (Adiredja & Andrews-Larson 2017) and elsewhere (Aguirre, et al. 2017). This study does not focus on student identities and discourses, however, but rather attends to the "organizational structures that shape access and opportunity in mathematics" (Battey 2013, p. 332) by studying how these gendered and

racialized structures are developed, perpetuated, and challenged (Battey 2013; NCSM & TODOS 2016). In the case of this IBL movement, instructor buy-in and access to the movement—two factors that, in turn, influenced how many students would be exposed to IBL nationwide over time—were influenced by the gendered and racialized organizational structure and discursive framing that were already in place through the connection to Moore's prejudicial legacy. Many new generation instructors saw enough value in the IBL movement and enough promise in using IBL in their classrooms to challenge that legacy and push for a more equitable framing of this approach to teaching.

Implications

Results of this study raise important implications for understanding reform efforts in higher education. Common arguments for why reforms of teaching are slow to implement or die off altogether focus on well-known contextual issues related to faculty conservatism, faculty rewards systems, departmental resistance, and student resistance. But this study reminds researchers and practitioners that framing matters, and it refocuses research and policy on the centrality of faculty to curricular and instructional change processes (Bastedo 2005; Cuban 1999; Finnegan 1997; Stark & Lattuca 1997, p. 85). The ways that movements, groups, or individuals label and talk about proposed reforms enables or constrains how reforms spread and continue. It is simply not enough to implement and institutionalize teaching methods that engage students or promote learning, although the promise to achieve these desired outcomes is important in any effort to reform undergraduate teaching. Reform movements and their respective innovations connect with pre-existing instructor identities (Benford & Snow 2000; Snow, Rocheford, Worden, & Benford 1986), values, and perceived needs of their students. Reform efforts that conflict with or ignore these preexisting identities and values may yield anemic results. In the example of this movement, younger instructors who self-identified as interested in social justice and collaborative teaching methods found those identities at odds with the framing of the movement when they joined. In contrast, old generation members of the movement were so transformed by their first-hand experiences as students in a Moore method class that they hoped to preserve and spread the use of the method however they could. Both illustrate how personal pedagogical choices can be for instructors. Framings of educational innovations that ignore that individuality in pedagogical choice will likely experience limited adoption, dissemination, and growth. One repercussion of this point is the potential for pedagogical change to occur as different generations enter and progress through their careers as instructors.

Finally, frames that constrain and enable instructors to positively identify with reform movements such as IBL are examples of the "virtual" or symbolic barriers that have profound implications for diversity in higher education and in the STEM disciplines specifically (Whittaker & Montgomery 2012). In higher education, the stakes of improving the retention and educational experiences of all students, and especially students who have been historically underrepresented, are high. There is ample scholarship to support the use of teaching methods that can improve the retention and learning of these students, yet traditional lecture methods

persist (Freeman, et al. 2014; Hake 1998; Kuh 2008; Ruiz-Primo, Briggs, Iverson, Talbot, & Shepard 2011). The struggle to frame IBL in mathematics in a more modern, socially inclusive way serves as a reminder that proven educational innovations cannot affect students if they do not first resonate with the instructors who teach them on a personal and professional level.

Conflict of Interest Statement

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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