THE JOURNAL OF THE LEARNING SCIENCES, *14*(1), 35–67

Shifting Engagements in Figured Worlds: Middle School Mathematics Students' Participation in an Architectural Design Project

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Project-based curricula have the potential to engage students' interests. But how do students become interested in the goals of a project? This article documents how a group of 8th-grade students participated in an architectural design project called the Antarctica Project. The project is based on the imaginary premise that students need to design a research station in Antarctica. This premise is meant to provide a meaningful context for learning mathematics. Using ethnography and discourse analysis, the article investigates students' engagement with the imaginary premise and curricular tasks during the 7-week project. A case study consisting of scenes from main phases of the project shows how the students took on concerns and responsibilities associated with the *figured world* proposed by the Antarctica Project and how this shaped their approaches to mathematical tasks (Holland, Lachicotte, Skinner, & Cain, 1998). Participating in the figured world of Antarctica and evaluating situations within this world was important for how students used mathematics meaningfully to solve problems. Curricular tasks and classroom activities that facilitated students in assuming and shifting between roles relevant to multiple figured worlds (i.e., of the classroom, Antarctica, and mathematics) helped them engage in the diverse intentions of curricular activities.

During a discussion about temperature conversions, a student asks, "Why would we care?" The teacher responds that they are doing a math unit in which the theme is Antarctica and architecture. The math is "embedded in

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> some type of story thing to make it more interesting"; otherwise she would have to do the entire year out of the text. She ends by saying that these are her "golden opportunities to do the fun stuff" like designing. (Content log, November 12, 1996)
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> The classroom researchers have just finished showing the class a video of architects at work. Kids clap and a student asks what the "point" is. The researcher explains that the students will be learning about new things in this project, including mathematics, what Antarctica is like, and what architects do. (Content log, November 13, 1996)

These excerpts represent classroom participants' initial attempts to make sense of using an architectural design project. Students saw the project as a possibly interesting but uncertain endeavor; the teacher considered it to be a more entertaining alternative to using a textbook for learning and teaching mathematics; and classroom researchers viewed it as an opportunity to engage students in authentic practices of mathematics.

Research has suggested that project-based curricula have the potential to engage students' interests and provide them with opportunities to work on contextualized problems, which can support them in making connections between what they learn in school and their experiences outside of school (Boaler, 1998; Blumenfeld et al., 1991; Brown, Collins, & Duguid, 1989). But how do students become interested in the goals and intended academic content of a project? Why should they "care"? This analysis illuminates these questions by focusing on the process through which one group of students made sense of how to participate in and learn through an extended project.

The article documents how a group of four eighth-grade students engaged in an architectural design project called the Antarctica Project. The project is based on the imaginary premise that students are hired as architects to design a research station on the Antarctic coast. The imaginary premise is meant to provide a meaningful context for learning and using mathematics. Using a combination of ethnography and discourse analysis, the article investigates the students' engagement with both the imaginary premise and intended curricular tasks over the course of the 7-week unit. A case study consisting of scenes from the main phases of the project shows how the students took on concerns and responsibilities associated with the figured world proposed by the Antarctica Project. Figured worlds are defined as a "socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al., 1998, p. 52). Figured worlds such as those of traditional schooling, academia, and romance propose characters, activities, and motivations that serve to mediate and influence one's behaviors and perceptions (Holland & Eisenhart, 1990). As shown in this analysis, participating in the figured world of Antarctic building design (Antarctica for short) and coming to understand how to act and evaluate situations within this world was important for how the focal students used mathematics as a resource for solving problems.

USING PROJECTS TO SUPPORT LEARNING

The question of how to build on and extend students' experiences to further learning in schools is perennial in educational research (Dewey, 1938). One response is the use of project-based curricula that involve students' use of disciplinary concepts and methods to investigate problems they find meaningful (Greeno & Middle School Mathematics through Applications Project Group [MMAP], 1998). Interest in using project-based curricula, particularly those that include design activities, to support mathematics learning has been renewed in recent years as an effort to engage students in real-world problem solving (National Council of Teachers of Mathematics [NCTM], 2000).

Projects involving design activities differ in their particular emphases but share some features, including the notion that design is an iterative process that involves the creation, evaluation, and redesign of solutions; designed artifacts are resources for thinking and working with other people and technologies; and peer collaboration can help students develop sophisticated solutions to problems (Kolodner et al., 2003). Many project-based curricula also include the use of software and hands-on classroom activities to support students' mathematical and scientific inquiries (Greeno & MMAP, 1998; Hmelo, Holton, & Kolodner, 2000; Kafai & Ching, 2001).

The project-based curriculum studied in this analysis, created by MMAP, is organized around simulated real-world problems that provide opportunities for students to engage in standards-based math topics (NCTM, 2000). In MMAP units, students are confronted with complex problems that they need to explore to determine how they might use mathematics as a resource for solving them. The simulated real-life problems are designed to provide a framework students can use to understand and organize their mathematical activities (Cognition and Technology Group at Vanderbilt, 1992; Goldman, Knudsen, & Latvala, 1998; Newman, 1985/1997).

The premise of the Antarctica Project is that an architectural firm has hired students as architects to design a cost-efficient research station in Antarctica that scientists can live and work in for 20 years. Memos sent from the architectural firm guide students through the project and introduce challenges that involve using mathematics to solve problems. By providing a meaningful context in which students use mathematical concepts and methods to create floor plans and determine the best levels of insulation for their buildings, the Antarctica Project can become an effective environment for learning about proportion, functional relations between variables, and the work involved in designing buildings.

Although research has shown that project-based curricula can engage students productively in problem-solving activities, engaging students deeply in the intended tasks of a project poses a difficult task for teachers and curriculum designers (Barron et al., 1998; Blumenfeld et al., 1991; Krajcik et al., 1998; Marx, Blumenfeld, Krajcik, & Soloway, 1997; Schauble, Glaser, Duschl, Schulze, & John, 1995). A number of classroom practices have been identified that facilitate students' learning, including the establishment of participant structures that encourage student participation, the use of presentations to peers and visitors to reflect on the goals of the overall project, and the construction of inscriptions that allow students to investigate and understand situations (Barron et al., 1998; Hall, 2000; Kafai & Ching, 2001; Kolodner et al., 2003; Penner, Lehrer, & Schauble, 1998). Learning how to participate successfully in project activities is an ongoing and necessary process for both students and teachers. Although this analysis focuses on how students made sense of their participation in a project, it also considers how the teacher and classroom research team mediated students' relations with the project.

SETTING, PARTICIPANTS, AND METHODS

The data presented in this article was collected as part of the Math-at-Work Project,[1] an ethnographic research project that compared the development and organization of mathematical practices across school and work settings in which design was a leading activity (Hall, 1999). This study focuses on a group of four students working on an architectural design project in an eighth-grade mathematics classroom in northern California. The school serves a socioeconomically and ethnically diverse student population. Thirty-eight percent of the students at the school receive free or reduced-priced lunches. In the classroom studied, 60% of the students were African American, 18% were Caucasian, 18% were Asian American, and 4% were Hispanic.

The classroom teacher, Ms. Alessi,[2] had used short inquiry activities in the classroom, but the architectural design project she used as part of this research project was the most extensive she had taught in her 5 years of teaching. Ms. Alessi received ongoing support from the research team in terms of using the units and thinking about students' work on the projects. Prior to and throughout the school year, she participated in after-class debriefing sessions and weekly research project meetings in which members of the research team (including myself) discussed curricular activities and assessments, redesigned activities, and reviewed and discussed videotaped classroom interactions. Ms. Alessi also attended institutes hosted by the curriculum designers in which she discussed teaching issues and shared student work with other teachers who were using MMAP units in their classrooms.

[1]The Math-at-Work Project conducted ethnographic research in two middle school mathematics classrooms, which used the same curriculum units at the same time, and in two workplaces (an architectural firm and a biology research station).

[2]All proper names are pseudonyms.

The focal student group was chosen based on the teacher's recommendations of who would be most likely to engage in the activities of the project. The members of the group included four boys: Gento, Patrick, Andre, and James. Based on classroom observations and a content-level analysis of all groups' participation in a design review in which students presented their in-progress research station designs to visiting architects, the focal group was found to be representative of the class.[3]

Primary data collected in this study include videotapes of 7 weeks of daily classroom activities focusing on the focal student group, fieldnotes, and examples of student work. Analysis also drew on video and audiotapes of the after-class meetings with the teacher and classroom researchers. These conversations served as an index to significant classroom and group events and provided evidence of ongoing assessments of student work, which shaped the subsequent design and implementation of the unit's activities.

TRACKING ENGAGEMENTS IN FIGURED WORLDS

Figured worlds is a useful concept for understanding how students become engaged in simulated, real-world projects because it provides a way to understand how students assume orientations necessary to participate in collectively imagined situations. Figured worlds are simplified interpretive frames that describe characters who are inspired by a particular set of concerns to participate in a narrow range of meaningful activities. Artifacts play an important role in figured worlds because they can serve as *pivots* (Vygotsky, 1978), which shift the frame of an activity and evoke or "'open up' figured worlds" (Holland et al., 1998, p. 61). The resources provided by figured worlds, including its characters, their concerns, and relevant artifacts and activities enable one to develop a sense of self and meaning in relation to the figured world (Boaler & Greeno, 2000). Through extended participation in a figured world, one comes to inhabit this imagined space, embody its perspectives, and act according to its local order. An analysis of how students engage in figured worlds over the course of an extended project can help us appreciate how they build understandings of their activities in and through interpretations of their experiences.

To locate and analyze students' engagement in figured worlds, shifts in their engagement, and the consequences of this for developing a relation to the activities of the design project, this analysis employs insights and procedures from the ethnographic microanalysis of interaction (Erickson, 1992; Erickson & Shultz, 1981). Analysis began with the creation of content logs of classroom videotapes documenting the activities of the focal student group during the project. Content

[3]The focal group's research station design was comparable to other groups' designs in terms of the level of detail in their floor plans, the degree of attention to design constraints, and their level of sophistication in presenting their design.

logs included summary descriptions of what happened in the classroom and short analytic notes discussing events at a more theoretical level. Memos were written to develop analytic categories and link together theoretically related events and to construct a longitudinal analysis of learning and development. Using the constant comparative method developed by Glaser and Strauss (1967), categories and themes were compared, refined, and their validity confirmed or disconfirmed through reference to content logs, video, and audiotapes.

Selected events were transcribed to analyze in detail how classroom participants oriented themselves to particular figured worlds in interaction. The sociolinguistic notion of a participation framework was employed to examine how students and the teacher used talk, embodied activity, and inscriptions to position themselves and others in relation to activities and ideas (Erickson, 1982; Goffman, 1981; Hall & Rubin, 1998). Documenting how participants used these resources to assume "footings" or stances vis-à-vis one another provided a way to study the characters and activities to which participants were orienting and how they evaluated situations (Goffman, 1981). Evidence of shifts in footing or changes in orientation to the ongoing activity included the content of an utterance, the relation of an utterance to the context of talk, and tone of voice. Furthermore, because multiple figured worlds or activity frames are always present in a situation, the notion of *lamination* was used to describe how activity frames that proposed different characters and concerns were layered onto one another (Goffman, 1981). Different layers of lamination were identified through the analysis of emergent participation frameworks. This approach to studying classroom interaction supported an analysis of students' shifting and, at times, simultaneous engagements with various figured worlds over the course of the project.

CLASSROOM WORLD, ANTARCTICA WORLD, AND USING MATHEMATICS

Analysis of the focal students' participation during the design project revealed that the students engaged primarily in two figured worlds: the figured world of the traditional classroom and the deliberately crafted figured world of Antarctic building design. The figured world of the classroom, which has a long history in traditional schooling, was a familiar world for the students, as evidenced in Patrick's recommendation to Gento about doing homework:[4]

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[4]Parentheses indicate transcriber comments, actions are indicated with a number inside a parentheses in a turn at talk, and action descriptions are provided in the right-hand column. Contiguous utterances are indicated with equal signs, and elongated syllables are indicated with double colons.

December 4, 1996

Patrick: Did you do yours (the homework)?
Gento: Huh? I didn't—I didn't even know that you had to have squared (graph) paper.
Patrick: You didn't—you didn't have to.
Gento: I need some squared paper.
Patrick: You don't need it.
Gento: Well, it's on the sheet. It's on the (reads from the worksheet) "use squared paper" I don't have any.
Patrick: Just get it from her (the teacher), dude.
Gento: I KNOW, but I didn't even know that I had to get it.
Patrick: You should probably just do it (homework) no matter what though.

Patrick's advice to "do it (homework) no matter what" reveals his orientation to the figured world of the classroom and its associated concerns with completing assignments. Students ("smart" and "dumb") and teachers populate the figured world of the traditional classroom in which students are concerned with completing assignments and getting "good" (or good enough) grades. Assignments, worksheets, homework, and report cards are among the most relevant artifacts in this world.

The students also oriented to the figured world of Antarctica as illustrated in the following content log excerpt:

> Andre reviews the floor plan and cost of the group's research station design and exclaims, "A toilet does not cost two hundred dollars!" He adds that this is an "expensive building" and that they should be sued for this "overpriced stuff." (Content log, December 4, 1996)

Andre's assessment of what he thinks are exorbitant costs for a toilet and the group's research station reveals that he is oriented to the concerns proposed by the figured world of Antarctica. This world is populated with architects, scientists, clients, and the employers of an architectural firm who are interested in the design of a functional and cost-efficient research station. The floor plan of the focal students' research station design and the tools used to determine costs (e.g., ArchiTech™ software) were key artifacts that supported entrance into the figured world of Antarctica.

Each of these figured worlds, as discussed in this analysis, differently mediated how students approached, participated in, and evaluated the activities of the design project. Studies of how mathematical activities are organized in out-of-school contexts suggest that the roles and relations one has to activities lead to different ways of identifying and approaching solutions to mathematical problems (de la Rocha, 1986; Lave, Murtaugh, & de la Rocha, 1984; Saxe, 1991). The MMAP curriculum designers recognize this and created the Antarctica Project to engage learners in roles other than “students” who “learn to carry out pre-existing algorithms to solve problems that are carefully matched to the algorithm, not necessarily understanding the algorithm or the problem” (MMAP, 1996, p. 5). The Antarctica Project suggests new roles for learners to play within the figured world of Antarctica (e.g., architects who need to design a research station), which are intended to position students in a more meaningful relation to problems that may involve mathematics.

Two interrelated questions frame this analysis:

1. How do the focal students come to be engaged in the figured world of Antarctica, and what does the development of this engagement look like over the course of the project?
2. How does engagement in the figured worlds of the classroom and Antarctica mediate the students’ use of mathematics?

The three scenes presented in this case study illustrate how students engaged with, shifted between, and simultaneously inhabited the figured worlds of the classroom and Antarctica as they made sense of curricular activities. The analysis focuses on how classroom interactions and the use of pivotal artifacts mediated how students came to take on the perspectives of characters within the figured world of Antarctica. Particular attention is paid to the tensions that emerged as students engaged in these figured worlds to understand how they came to participate in and learn through the design project (Engeström, 1999).

THE ANTARCTICA PROJECT

There were three main phases of the Antarctica Project as enacted in the classroom: (a) predesign research and investigations, (b) research station design, and (c) cost analysis. These phases overlapped to some extent to support the revisiting of ideas and methods as students progressed through the project. Students used a computer-assisted design tool called ArchiTech™ to create floor plans, set the parameters of their building designs (e.g., level of roof insulation), and analyze the costs of their designs.

Some of the mathematical concepts students were meant to encounter in the project included distinguishing independent from dependent variables and reasoning about related functions. The project also aimed to engage students in activities that resemble the work of architects, including the iterative design of a floor plan for a research station. In these ways, the Antarctica Project provided opportunities for students to engage in practices that are valued by current mathematics education reformers, such as solving real-world problems and using inscriptions to communicate about mathematics (NCTM, 2000).

Scene 1: "Doing Mathematics" and "Doing School" in a Design Project

This scene, from the first phase of the project, illustrates tensions created by the new expectations proposed by the curriculum regarding how to engage in mathematical activities.

Intended activity. "Area and Perimeter" is part of the predesign phase of the Antarctica Project. The purpose of the activity, as described by the curriculum unit, is to "give students a chance to do an investigation, some pattern recognition, and then create 'theorems' about the relationship between the area and perimeter of closed polygons" (MMAP, 1996, p. 157). A second intention of the activity is to encourage students to think about how an investigation of area and perimeter can help them design a research station.

The worksheet asks students to choose a floor area (e.g., 12 square units) and determine all the different perimeters for rectangles with whole number length walls. After determining all the possible perimeter measures for the chosen area, groups are supposed to make "conjectures" about the general patterns relating area and perimeter. The term *conjecture* is defined on the worksheet as "your best guess, based on evidence. You don't know for sure that it is true, but you have reason to think it might be."

The last question of the assignment asks students to write about how their conjecture might help them create a building design. A sample conjecture offered by the curriculum unit is that "you get the smallest perimeter by making a square" (MMAP, 1996, p. 157). Recognizing this could help students design a building with a small perimeter, which would help reduce the costs of constructing the building.

Enacted activity. At the start of their work on the assignment, Gento connected the investigation of area and perimeter with the design of the research station. He commented to himself (and for the benefit of the researchers) as he worked on the assignment:

November 13, 1996

Gento: Hope they (the researchers filming his group) get a close up on this. So this is how we can save money, y'know. Make it boxy, but then it's really ugly.

Although Gento did not elaborate on how a "boxy" (or square-like) design might save money, he is correct that a building with fewer exposed surfaces would be cheaper to build and heat than one with many. In making this observation about a general relation between area, perimeter, and costs, Gento made the kind of intended connection between the mathematical investigation and the building design activity. His analysis, in fact, went beyond the intended mathematical considerations of cost efficiency and considered the aesthetics of a building design. Gento's awareness of the value of his comment is evidenced in that he hoped his work and observation were captured in the official video record of the activity ("Hope they get a close up on this"). He did not, however, share his analysis with the members of his group.

For portions of the next two class periods, the group was immersed in creating figures that had the same area but different perimeters. To make figures with an area of 12 square units, they would draw a figure made up of 12 squares representing one square unit each, and to find its perimeter they would count its exterior surfaces. Not all the figures they created adhered to the main constraint given by the assignment, which was that all figures should be rectangles. In transforming the task in this way, the students moved away from its intended mathematical purpose and created a more complex solution space.

The search for new configurations ended when the teacher directed students to put all their figures on a poster and make a conjecture about the relation between area and perimeter. In the following excerpt, Patrick shares his conjecture with the teacher:

Patrick: Do you want to hear my congestion (conjecture)?
Ms. Alessi: Yes.
Patrick: There's no perimeter less than the area.
Ms. Alessi: Okay. Good. Try to get more.

Following the teacher's positive evaluation of the conjecture, Gento wrote the conjecture on the group's poster and the students stopped working on the assignment, despite the teacher's encouragement to continue.

Discussion. This scene illustrates one of the focal group's first encounters with the Antarctica Project. The students' activities reveal an orientation toward the figured world of the traditional classroom with its associated concerns with completing tasks and obtaining the teacher's approval, which are part of receiving institutional approval in the form of grades (Becker, Geer, & Hughes, 1968; John, Torralba, & Hall, 1999). Although Gento's initial comment about the activity suggests that he connected the mathematical activity of investigating area and perimeter relations with building design and the concerns of the figured world of Antarctica, this insight was lost in the group's subsequent work on the assignment.

One clue to the group's interpretation of the task was how the students and the teacher held each other accountable in the accomplishment of the activity of "conjecturing" (McDermott, Gospodinoff, & Aron, 1978). Consider Patrick's exchange with Ms. Alessi. The exchange resembles, but does not exactly follow, the format of an initiation-response-evaluation (I-R-E) sequence (Mehan, 1979). In an I-R-E sequence, the teacher initiates the exchange by asking a question, a student responds, and the teacher evaluates the response. A positive evaluation indicates a correct response and signals an end to the exchange, and a negative evaluation indicates an undesirable response, which leads to a re-initiation of the sequence.

In this exchange, Patrick initiated the sequence by asking the teacher if she wanted to hear his "congestion." She agreed and was thereby positioned as wanting to hear his conjecture. Patrick then offered his conjecture in the "response" slot. The exchange ends, as it would in a standard I-R-E sequence, when Ms. Alessi evaluated Patrick's response positively.

Initiation
Patrick: Do you want to hear my congestion?
Ms. Alessi: Yes.

Response
Patrick: There's no perimeter less than the area.

Evaluation
Ms. Alessi: Okay. Good.

Patrick strategically used a standard form from traditional classroom discourse (i.e., the I-R-E sequence) to position himself as a "good" student who has completed the required task. His request is an attempt to display his knowledge for the teacher, and, as he may expect, the teacher's evaluation will indicate whether he and his group mates have an adequate conjecture. The teacher's positive response suggests to the group, as evidenced in their subsequent activities, that they have a correct answer and are finished with the activity.

The activity posed by the "Area and Perimeter" worksheet was meant to engage students in what might be described as a figured world of mathematics. Such a world is populated by mathematical agents who use mathematical concepts and symbols (e.g., area, perimeter, and numbers) to observe, identify, and represent patterns. These agents are motivated by a desire to understand and explain the order of the social and physical world. In this scene, the worksheet and the creation of a final poster evoked the figured world of the classroom for the students. As a result, the students turned what was meant to be the mathematical activity of conjecturing with which they were not familiar into the more familiar classroom activity of "getting an answer." The teacher's concluding suggestion to "try to get some more" indicates that she too focused on conjectures as a product rather than conjecturing as a process of mathematical inquiry.

Analysis of the videotaped interactions and written assignments of the focal group produced during the predesign phase of the project revealed that the students typically did not discuss or interpret the significance of the patterns identified in their "investigations." The interactions described in this scene were found to be representative of the focal students' work early in the project.

Scene 2: Designing a Research Station as Students and Architects

This second scene, from the research station design phase of the project, describes how the students began to engage with the figured world of Antarctica. It highlights a tension that emerged in the group regarding the relevance of Antarctic building design activities in the figured world of the classroom.

Intended activity. Students started designing their research stations by writing requirements lists, which were meant to "answer the question: What should the Antarctica station be like?" (MMAP, 1996, p. 81). The curriculum designers intended the list to serve as a way for students to imagine the needs of clients and as a set of constraints for the students' research station designs. Using their requirements lists, students were to produce hand-drawn floor plans, which they would transfer to the ArchiTech™ software and revise as necessary. To help students understand their role as building designers, the research team introduced the students to the tools and work practices of architects. In addition to the software tools and literature provided by the MMAP unit, students explored architectural artifact kits, which included common tools architects use in their work (e.g., trace paper and floor plans), watched videotapes of architects working on an earthquake retrofit project, and presented their research station designs to architects and graduate students studying architecture in a design review session.

Enacted activity. Gento and Andre were the only members of the focal group who created hand-drawn floor plans. In the following excerpt, Gento compares his design to Andre's, explaining that from the perspective of a user, his design is more "organized." At the start of the conversation, Patrick is away from the group, but he returns to make the final decision regarding whose design the group will use:

November 12, 1996

Gento: I like your design and everything, um, and all the details, but when you walk in, you have to walk in straight through the kitchen to get to the first lab. So you have to walk through all of these things, the kitchen =
Andre: Hmm, but right there
Gento: = into lab two, but what if there's a special thing in lab one?
Andre: Uh, okay. I like your design too.
Gento: I think mine is more organized though cause
Patrick: Yeah, we should do his (Gento's design), Andre.

In this comparison of designs, Gento and Andre began to imagine what it would be like for someone to inhabit the building they were designing. Gento animated a person walking through the building and scientists using the lab space. Patrick's comment, which was not informed by this discussion or a prior review of the designs, cut short Gento and Andre's spontaneous design review, which was the kind of conversation about the benefits and drawbacks of students' designs in which the curriculum designers and the teacher wanted students to engage. Following Patrick, the group stopped comparing designs and proceeded to transfer Gento's hand-drawn research station to the computer.

In the process of transferring the design to ArchiTech™, the students made on-the-spot revisions to Gento's original design based on information they gathered from their research on the environment of Antarctica. For example, the students decided not to put windows in bedrooms because it is "light all day" during the Antarctic summers (Content log, November 22, 1996).

During the design phase of the project, the focal students attended to aspects of the intended curricular task such as producing a building that met the needs of its clients and could withstand the harsh conditions of Antarctica. The students also attended to aspects of their design that were beyond the basic requirements of the task, which they thought would add to the aesthetic appeal and functionality of their research station.

Tensions regarding the relevance of the building design activities in the classroom also shaped the group's design decisions. Consider the following excerpt from the students' conversation about finalizing their requirements list so they could include it in the next day's design review. The point of the design review or the "Architect Judging," according to a handout the students had just received, was that architects were going to "be looking for the best design for the lowest cost." In this exchange, the students are debating whether they should include a tractor in their final requirements list, because including the tractor would increase the total cost of their design. Andre insists vigorously that they need a "SNOW tractor" because a "truck won't work" in Antarctica. Gento, who thinks they don't need one, shouts at Andre, "You are too loud and you can't afford too many ground things!" The following discussion ensues:

December 4, 1996

Andre: We nee::d a tractor.
Patrick: What does it matter?
Gento: First of all
Patrick: Who cares?
Gento: It's a hundred and eighty four thousand one hundred and ninety one (the total cost of their design)
Patrick: First of all, this isn't real. We're only doing it in class so it doesn't matter.
Andre: There are cars that cost more than that and people buy them.
Gento: (Puts hands up as if in surrender) Never mind, never mind, never mind.

As this excerpt shows, members of the group had different orientations to the activity of creating a requirements list. Andre and Gento debated how to manage the tension between creating a research station design that includes everything an inhabitant might need and keeping it reasonably priced; these are concerns proposed by the figured world of Antarctica. Their intense argument about the tractor, which was accompanied by raised and annoyed voices, indicates that they were engaging in the figured world of Antarctica. Meanwhile, Patrick dismissed Andre's and Gento's concerns (and those of the figured world of Antarctica) by emphasizing that they are "only doing it in class so it doesn't matter." This comment indicates an orientation to the figured world of the classroom in which responses to assignments do not need to adhere to reality. Patrick's comment also suggests he thinks Andre and Gento believe that arguing about tractors and the price of their building design "matter" and are acting as though these concerns are "real." This gives further evidence that he and they are not orienting to the same worlds. The discussion concluded with the inclusion of a tractor on their requirements list.

The following day, the group presented their research station designs to visiting architects and graduate students in architecture.[5] During the review session, student groups took turns presenting their research station designs to their classmates and visiting architects. Architects would then ask questions about, comment on, and critique groups' building designs. The event was organized to help students understand and employ architects' perspectives on building design and give students a chance to discuss and compare their research station designs with each other.

Through the organization of the design review and their conversations with the architects, students were introduced to a common discourse strategy used by architects, which was constructing reasoned and persuasive arguments about how a building design meets and possibly exceeds a client's expectations. As one of the architects put it, students need to have a "sales pitch" to convince a potential client of the uniqueness and practicality of their design. In the following exchange, members of the focal group are asked to explain a design decision they had not discussed explicitly in any recorded conversation:

December 5, 1996

Architect: Why do you have two labs?
Patrick: Cause, um
Architect: What are you guys studying?
Gento: Just for extra research space.
Patrick: Yeah. I think that we might need—if you had two ideas it'd
James: One might be too crowded.
Gento: Just lots of space, lots of space to a make research on.
Patrick: One's kind of a computer room and one's kind of a small research room.

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[5]Architects from the architectural firm studied by the Math-at-Work Project participated in the design review.

This exchange illustrates how the conversations with the architects encouraged students to describe the intentions behind their design decisions, which is what an architect would need to do when proposing a design.

The presentations also provided opportunities for students to understand how professional architects made sense of their design solutions, which allowed them to appreciate new problems and ways of approaching their solutions. Some of the issues the architects raised involved mathematics; the mathematics-relevant moments were not, however, straightforward. They were shaped by multiple concerns, including the need to balance clients' needs and cost efficiency, understand patterns of building use, and consider how environmental conditions might affect building design.

The architects' main comment about the students' designs was that they needed to consider how clients would actually use their research stations. For example, a group created a building design that had no hallways. This led one of the architects to use trace paper and colored pens to redraw the group's design so she could highlight and explain the significance of the ratio of "circulation space" (e.g., hallways) to "lived-in space" (e.g., rooms). In another conversation, an architect noted that the group's building had bedrooms of different sizes, including a penthouse suite. He asked the students if they considered how this might affect the "social dynamics" of the scientists living in the building. How would they decide who lived in the biggest room? The architect suggested creating a more "democratic" design in which all the rooms were the same size.

In reviewing the focal group's design, the architects noted that the building was well organized because it included a main corridor that allowed people to get to and from rooms without having to walk through every room in the building, but that they did not include any spaces in which the scientists could relax, other than the bedrooms. The focal group took the advice of the architects and decided to include a living room in their final research station design (see Figure 1).

With the advice of the architects, the student groups finalized their building designs before entering the cost-analysis phase of the Antarctica Project.

Discussion. The focal students' activities during the research station design phase contributed greatly to the emerging relevance of the figured world of Antarctica to their work on the design project. The students came to appreciate this figured world and their roles within it through the design of their research station and conversations with architects.

The focal students oriented to the figured world of Antarctica during the design phase of the project because it was then that they had to create their research station. The research station design is a central artifact in this world, and it served as a pivot that shifted the frame of the students' activities from the figured world of the classroom and their accompanying understandings of their roles and responsibilities as students to an interest and engagement with the figured world of Antarctica.

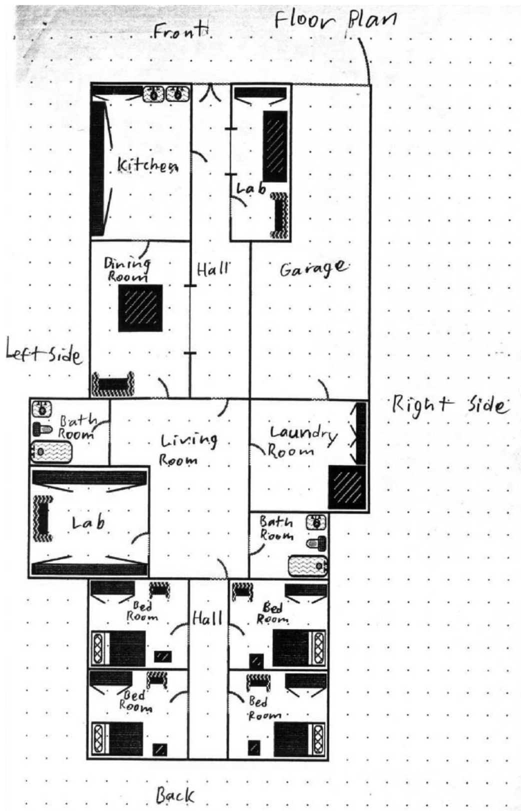

FIGURE 1 The focal group's final research station design.

In designing their research station, for example, the students considered the building's potential inhabitants and drew on their personal experiences and knowledge of the Antarctic environment.

The students' conversation with the architects, which was anchored in their research station design, was also important because it allowed the students to appreciate the "professional vision" and discourse practices of architects (Goodwin, 1994). The conversation helped the students to act as architects in terms of providing justifications for design decisions, animating the use of designed space, and comparing relevant quantities (e.g., circulation and living space). As the architects emphasized in their conversations with the students, animating a building design by imagining hypothetical scenarios is a necessary aspect of designing and "pitching" a design. In these ways, the research station design and the conversations with the architects mediated the students' relation with the increasingly more elaborate figured world of Antarctica.

As Patrick's comment about what "matters" suggests, the students were not always fully engaged in the figured world of Antarctica; they shifted in and out of this world as necessary to manage, for example, social relations in the group. This did not preclude that the students were able to engage meaningfully with some of the tasks proposed from within the figured world of Antarctica. Rather it suggests that managing what matters and when in a classroom is a complex activity.

In the next scene, the focal students combine aspects of the figured worlds of Antarctica and the classroom to use mathematics as a resource for solving a design problem.

Scene 3: Interpreting Mathematical Patterns to Make Design Decisions

The final scene, from the cost-analysis phase of the project, examines how Patrick and Gento used their understandings of mathematics along with an appreciation for their roles within the figured worlds of the classroom and Antarctica to interpret graphs during a public presentation.

Intended activity. "Making the Best of It" is an optimization problem designed to help students determine the level of insulation that will provide the lowest building and heating costs for their research station over 20 years (see the appendix for the "Making the Best of It" worksheet). The activity is intended to provide an opportunity for students to conduct analyses of how changes in one aspect of their research station designs (e.g., roof insulation levels) will affect other aspects of their design (e.g., building costs), to investigate these functional relations using tables and graphs, and to explain the meaning of these relations.

As part of the assignment, students used ArchiTech™ to collect data on how changing roof insulation levels would affect the costs to build and heat their research stations over 20 years. Determining which level of roof insulation is the most economical for their building is a complex mathematical problem, because the relation between insulation level and the cost to construct a building is linear and the relation between insulation level and the cost to heat the structure is inversely proportional. To help students investigate this problem, Ms. Alessi required each group to create a table indicating the level of roof insulation used in their particular research station (from R5 to R60 in increments of 5), and its associated building cost, heating cost over 20 years, and total cost (building cost + heating cost over 20 years; see Table 1).

She also asked each group to construct a set of three graphs: roof insulation level versus building cost, roof insulation level versus heating cost over 20 years, and roof insulation level versus total cost.

After collecting data on various costs for their research stations, students were given a worksheet titled "Making the Best of It." The worksheet was designed to

TABLE 1
Data Collected by the Focal Group on the Building Costs, Heating Costs Over 20 Years, and the Total Costs of Their Research Station Design Using Different Levels of Roof Insulation

Roof Insulation	*Building Cost*	*Heating Cost (20 Years)*	*Total Cost*
60	177,273	44,160	221,433
55	175,163	45,120	220,283
50	173,053	46,080	219,133
45	170,943	47,520	218,463
40	168,833	49,200	218,033
35	166,723	51,360	218,083
30	164,613	54,240	218,853
25	162,503	58,320	220,823
20	160,393	64,560	224,953
15	158,283	74,640	232,923
10	156,173	95,280	251,453
5	154,063	156,480	310,543

help students “figure out strategies for” finding the most effective level of insulation for their station (MMAP, 1996, p. 167). It asked students to discuss the meaning of building and heating cost graphs for a research station design (not their own) and directed them to consider heating, building, and total costs for their own building to choose the best insulation value.

The public presentation was the first of its kind to be held in Ms. Alessi’s class; it was uncommon for students to discuss what they were doing or compare their work in a public setting and, in these ways, this occasion was unique. Based on conversations with members of the classroom research team, the teacher made decisions about the organization of the presentation, including which groups would present, what they would discuss, and the material environment in which students would present.

She chose two student groups (including the focal group) to discuss their graphs in the public presentation. The groups were selected because they had completed their graphs and because their research station designs differed so much from each other that the teacher and researchers thought the variation would be worth discussing in terms of costs. Each presenting group was given a set of questions that the teacher produced based on conversations with the research team (see Figure 2).

In arranging the material environment of the public presentation, Ms. Alessi constructed large (2 × 3 foot) versions of each of the presenting groups’ three graphs (see Figure 3).

The teacher decided to use the same scale for the two presenting groups’ corresponding graphs (e.g., the two groups’ building cost graphs used the same scale) to facilitate students in comparing across graphs so they could see that the same gen-

1. How do your graphs compare with the other group that is presenting?
2. How does the Building Cost graph compare with the Heating Cost graph?
3. Why is the Building Cost graph linear (a straight line)?
4. Why is the Heating Cost graph curved?
5. How did we get the third graph (the Total Cost graph) from the first two graphs?
6. How did the Total Cost graph get that shape from the other two graphs?

FIGURE 2 The teacher's questions about presenting groups' building, heating, and total cost graphs.

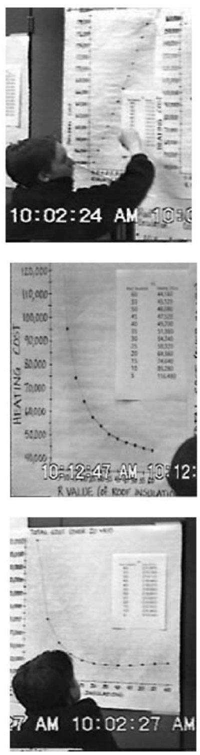

FIGURE 3 The three graphs made by the teacher based on data collected by the focal group. The top graph shows the roof insulation versus building cost, the middle graph shows the roof insulation versus heating cost over 20 years, and the bottom graph shows the roof insulation versus total cost.

eral patterns held across the building, heating, and total cost graphs for different research station designs despite their specific values.

Enacted activity. Prior to the presentation, neither Patrick nor Gento[6] knew how to respond to the teacher's questions about the graphs, labeling them "stupid" and "crazy":

January 9, 1997

Gento: Why do the heating cost and the uh
Patrick: How do the what?
Gento: Why do the heating cost and the uh and the uh never mind, that's a stupid question. Okay, how do the total cost and the HEY Patrick … How does the total cost graph get the shape from the other two graphs?
Patrick: I don't know.
Gento: You don't know. I don't know either.
Patrick: How did the total cost graph get its shape from the—IT DIDN'T.
Gento: It didn't, I know.
Patrick: That's a crazy question.

Gento and Patrick worried about what they would say about the graphs and, as Patrick put it, whether they were "ready" and knew "how" to present. To prepare for the presentation, Gento looked over the "Making the Best of It" worksheet, which he had not completed. The worksheet questions, which focused on how to find the "best" level of insulation for a building design, led Gento to examine the group's total cost graph. Standing in front of their group's enlarged set of graphs, he and Patrick located the lowest point on the total cost graph and determined that this was the best level of roof insulation for their building (40R). Identifying the best deal using the graphs was apparently a more comprehensible goal than discussing the meaning of their shapes.

Ms. Alessi introduced the presentation to the class by asking students to recall their responsibilities as architects, to consider the shapes of the graphs, and to all take part in the presentation:

Ms. Alessi: When you're going to sell a design to a client, you do have to have money, the cost of it as a consideration because your client will, not always, but almost always, money will be a consideration. … I want us to look at the three graphs

[6]Gento and Patrick were the only members of the focal group to present the group's graphs to the class. Andre was absent on the day of the presentation, and James, although present, was absent for a number of days prior to the presentation, and it was decided (without discussion) that he would not participate in the presentation.

and how they're shaped and what produces what, as many ideas as we can get about this and I want us all to participate.

On their way to the front of the room to make their presentation, Gento told Patrick under his breath that he should do all the talking. Thus Patrick, somewhat reluctantly, took the lead in discussing the meaning of the graphs. He used Ms. Alessi's framing of the presentation in terms of the figured world of Antarctica to organize his interpretation of the graphs. He positioned himself as an architect presenting his group's research station design to an audience of potential clients or people concerned with finding a cost-effective building design (see Figure 4).

Patrick: You probably wouldn't want to have (1) an R value of just five cause it'd be, be pretty cold, plus uh heh (2) your heating costs would be high. So, and also (3) but, if you have 60 uh your heating cost is lower (4) but, also (5) your building cost is higher so you want to look kind of at a neutral price and the kind of price that we found is kind of 40 (6).

(1) Points back and forth from the *y*-axis and *x*-axis for an R5 on the building cost graph,
(2) sweeps hand over the heating cost graph starting from the uppermost portion of the *y*-axis,
(3) points to R60 on *x*-axis,
(4) points to R60 on *y*-axis,
(5) points toward upper region of the *y*-axis on the building cost graph,

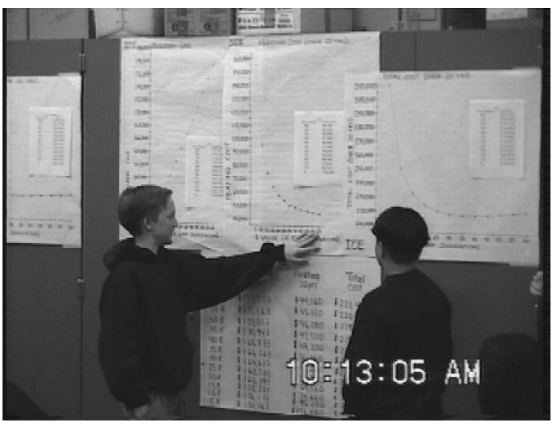

FIGURE 4 Patrick, standing on the left, discussing the tradeoffs between the building and heating costs for a research station using extremely low and high levels of roof insulation. The graphs from the left are (a) the building cost graph, (b) the heating cost graph, and (c) the total cost graph.

(6) points to R40 on the graphed line on the heating cost graph.

Patrick compared the use of the most extreme levels of insulation investigated in the model (i.e., an R value of 5 versus an R value of 60) on building costs, heating costs over 20 years, and the living conditions in Antarctica. He crafted his interpretation as a sales pitch in which he demonstrated an appreciation for his audience's particular problems and emphasized the value of what his group had determined. Patrick ascribed desires and concerns to his listeners-as-potential-clients that corresponded to those elaborated in the figured world of Antarctica. On the one hand, he argued that using the lowest level of roof insulation (i.e., an R value of 5) would make for a cold research station and expensive heating bills. On the other hand, using the highest level of roof insulation (i.e., an R value of 60) would lead to savings on heating bills but expensive building costs.

Patrick directed the audience to *look at* and *through* the graphs to understand how their values could be used to make a decision about insulation levels (Forman & Ansell, 2002; Hall, 2000; John, Luporini, & Lyon, 1997). Looking through the graphs involves making sense of the numerical values and graphical patterns in terms of their meaning in the world of Antarctic building design. Learning how to use inscriptions (e.g., graphs) to move flexibly between the representing and represented worlds is an explicit aim of the MMAP units (Greeno & Hall, 1997). This type of discourse practice is especially important when using models to imagine and explore situations that do not yet exist or to which one cannot gain access.

In response to the teacher's request to talk about the shapes of the graphs, Gento took the floor to explain how the graphs represented the relations between insulation levels and costs in a physical form. In the following excerpt, he discusses how the total cost graph obtained its shape from the building and heating costs graphs:

Gento: On this one, both of the costs combined, this one, it's like this motion I mean it's like this form 'cause the more, the less R value, the more the building costs over over prices the heating costs

Although the details are incorrect, Gento's explanation indicates that he was thinking about how the functional relation between costs and insulation levels was represented on the total costs graph. That this was the first time he had attempted to articulate this understanding may explain his confusion. In the explanation, Gento used his own term, "over prices," to describe the general pattern in which building costs exceed heating costs when the R value of insulation is low (i.e., less than R40), thus producing the peak of the graph. Then he stated that building costs would "over price" (or exceed) heating costs when the R value is low; however, in that situation it is the heating costs that would exceed the building costs. Gento's earlier comment about not knowing how the total cost graph "got its shape" along with the confusion and numerous false starts evidenced in this transcript suggest that he is developing a new understanding of the relations represented on the total cost graph.

Half an hour after he and Patrick finished their presentation, Gento made a request to add something else to his account of the shapes of the graphs. Because it was unusual for students in this class to offer to say something regarding mathematical content without being asked, Gento's addition suggests that he was interested in working on the problem and that he identified the public presentation as a forum in which he could share his ideas. He returned to the graphs and explained (see Figure 5):

Gento: I think I know why the heating cost is in a (1) curvy motion. Um outside it's negative 40 degrees, that's how we did the sliders. And 40 (2) or around 40. The 40 area resistance insulation um is enough (3) and a little more (insulation) wouldn't really matter. And so (4) that's why it starts to slow down going (5) downwards.

(1) Runs finger along line of the heating cost graph,
(2) circles values on the *x*-axis around R40,
(3) opens and raises both hands in a final motion as if to offer,
(4) moves finger from R35 to R40,
(5) points down.

Gento's explanation focused on the physical features of the graph (i.e., its shape, specific points) to show and explain why the graph has the shape it does and to explain the meaning of the graph in terms of the design of a research station. He explained why the graph has a "curvy motion," that is, why the line of the graph goes down sharply as roof insulation levels increase and then goes down less dra-

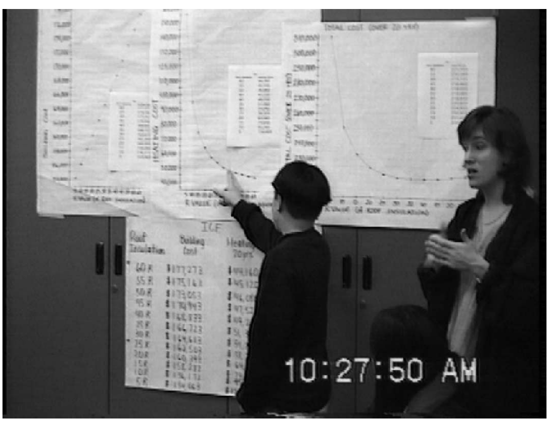

FIGURE 5 Gento, standing on the left (action 4), explaining why the heating cost graph is in a "curvy motion." The adult at right is a sign language aide who assisted two hearing-impaired students in the class.

matically after a certain point. Gento began by stating that R40 is enough roof insulation for his group's research station and used talk and gesture to explain how the shape of the graph shows this. He moved his hand over the curve of the graph and pointed to the location on the graph where the curve begins to decrease less sharply, which is around the R40 level of insulation. The effects of increasing roof insulation levels beyond R40, Gento stated, would not reduce heating costs that much more and that is why the downward curve of the graph is less steep than it is before R40. In his words, R40 is "enough."

Gento addressed the question about why the heating cost graph is curved, and, in accordance with the teacher's questions, he focused on how the relation between insulation level and heating costs are represented physically on the graph. In addition to using the questions, his analysis that R40 is "enough" suggests that he considered a prospective client's desire to find the most efficient level of insulation.

As these excerpts demonstrate, Patrick and Gento engaged creatively with the figured world of Antarctica to communicate their understandings of mathematical patterns to a real and imagined audience. Based on their analysis of the graphs, the focal group revised their research station design by setting R40 as their level of roof insulation to create a more cost-efficient building design.

Discussion. The organization and enactment of the public presentation encouraged the focal students to participate in multiple embedded and interpenetrating figured worlds. There were at least three layers of lamination co-constructed by the students and their audience in the context of the presentation: the figured worlds of Antarctica, the classroom, and mathematics. These figured worlds were realized and used to draw others into particular forms of participation through the use of talk, embodied activity, and physical representations in moment-to-moment interaction.

Both Ms. Alessi and Patrick invoked the figured world of Antarctica during the presentation. When Ms. Alessi mentioned "selling a design to a client" in her introduction, she echoed the architect's suggestion to the students during the design review (i.e., to have a sales pitch) and positioned the students as architects. Engaging with the figured world of Antarctica encouraged Patrick to address his analysis of the graphs to a potential client's concerns about the cost efficiency of his group's research station design. In the immediate context of his talk, Patrick addressed an audience of his classmates, but his interpretation was constructed as a dialogue with an imagined client. Specifically, audience members were positioned as potential clients who had particular desires and concerns regarding the design of a research station by the way Patrick designed his utterances for recipients (Goodwin & Heritage, 1990).

In addition to orienting to the figured world of Antarctica, the focal students attended to the figured world of the classroom and their responsibilities as students. Gento and Patrick were apprehensive as they prepared for the presentation, which indicates that they were concerned with how they were going to perform. That

Gento asked Patrick to be the spokesperson, and then asked to speak in the public forum after they had "finished" is further evidence that he cared about what was said in the presentation. Concerns with classroom accountability and maintaining face during the public presentation compelled the students to try to give adequate answers to the teacher's questions during the presentation.

The teacher's questions about the "representing world" (Hall, 2000) of the graphs pointed the students to the figured world of mathematics, which includes symbols, conventions, and rules that define appropriate actions by mathematical agents. Although Gento and Patrick initially thought the teacher's questions were incomprehensible, the list of questions along with the group's graphs served as pivots that enabled the students to enter into and explore this figured world of mathematics in which they could examine systematically the effects of changing insulation levels on costs (John et al., 1997; Nemirovsky, 1994; Ochs, Jacoby, & Gonzales, 1994). For example, in response to the teacher's request to discuss the shape of the total cost graph, Gento focused on and directed the audience's attention to how quantitative relations are represented in graphical displays. How did the teacher's questions about the graphs change from being "crazy" to being legitimate and answerable? A possible answer is that using the graphs to determine and explain how to identify the best level of insulation for a building design helped the students understand the meaning of the shapes of the graphs. In other words, the figured world of the project and its concerns with cost efficiency served to mediate the students' discussion of the graphs as mathematical forms.

The multilayered participation framework that emerged during the presentation supported the students in moving between the figured worlds of the classroom, Antarctica, and mathematics. For example, Gento's desire to add to the group's interpretation of the graphs combined concerns with explaining the shape of the graph (from the figured world of mathematics) and constructing a good presentation that responded to the teacher's questions (from the figured world of the classroom). The interpenetration of the worlds and their concerns served to expand the students' positions in any single one of these worlds and created the possibility for new forms of participation and learning (Engeström, 1999). The students' levels of commitment to each of these figured worlds and their understandings of the mathematics involved in interpreting the graphs contributed to their successful engagement with the diverse intentions of the activity.

DISCUSSION

Simulated real-world projects can support students' learning by helping them identify emergent goals within an authentic context. From a situated approach to learning, the relations between context, activity, and motivation are highly interrelated, and thus meaningful problems are shaped by one's relation to an activity (Lave, 1988; Lave & Wenger, 1991). MMAP units such as the Antarctica Project aim to expand students' often limited experiences with mathematical activities in the classroom by proposing new roles for them to take on in relation to curricular activities. As discussed in this analysis, students did not assume these roles fully or unproblematically; it was observed that the students gradually took on and shifted between the roles and concerns proposed by the imaginary world of the Antarctica Project and those of the classroom. Although the students' engagement in the figured world of Antarctica was uneven, the roles, artifacts, activities, and concerns proposed by this world helped them use mathematics in a meaningful context.

The Antarctica Project proposes an imaginary premise that includes characters, scenarios, and tools so that students can experience, through extended role-play, a version of what it is like to work as professional adult designers (something that would otherwise be impossible to experience as middle-school students). The type of virtual experiencing that role-playing can offer is most effective, as shown in this analysis, if students take on to a certain degree the concerns and responsibilities of the characters in this imagined world.

Prior research on the use of project-based curricula suggests that the use of participant structures that encourage student participation, the construction of inscriptions, and peer and expert reviews support students' learning in projects. These were effective means of supporting the focal students' participation in the Antarctica Project because, from the perspective of the students' engagement in figured worlds, participant structures that centrally involved the creation, use, and explanation of inscriptions provided a way for students to enter into, explore, and take on roles and concerns relevant to the figured world of Antarctica. More specifically, curricular tasks and classroom activities that encouraged students to draw on their experiences, know-how, and imaginations supported their meaningful participation in the design project. Imaginative resources such as acting as architects and visualizing walking through a research station built on the students' emerging knowledge of architecture and their experiences of inhabiting houses and other buildings. This helped the focal students begin to care about issues posed from within this figured world, which mediated their use of mathematics.

The students' presentations to architects and to their classmates and teacher also allowed the students to elaborate their understandings of the figured world of Antarctica and to act within this world. The presentations to the architects provided students with the opportunity to see, hear, and interact with practicing architects. This helped the students appreciate the concerns and professional vision of actual architects. Presentations to classmates and the teacher created a forum in which the students could use these understandings to act as architects and use graphs to make and explain a design decision. Both of these types of presentations were also significant in that they compelled the students to organize their materials and understandings for public and high-stakes performances within the figured world of the classroom. Classroom activities such as these supported students in shifting between roles or positions (i.e., as students and architects) in differently figured worlds. This was important because, as the data suggest, diverse ways of understanding and engaging with curricular activities can create possibilities for students to participate in potentially expansive learning environments.

It was challenging to engage students directly in mathematical activities that were not rooted in the figured world of the project. The students had difficulty engaging in the intended purposes of the conjecturing activity, which was part of the "Area and Perimeter" assignment, and the analysis of the shapes of the graphs, which was part of the "Making the Best of It" presentation. As mentioned in the analysis, students did not have extended opportunities to discuss the meaning of the mathematical patterns and findings in class until the public presentation described in the third scene. Prior to the presentation, discussion of mathematical patterns and analyses were limited to short answers to teacher-initiated questions (John [2001] described the routine organization of talk and interaction in the classroom). That Gento engaged in the analysis of the shapes of the graphs during the public presentation suggests that the public and extended discussion of the meaning of the graphs in relation to the figured world of the project supported an interpretation of their meanings as mathematical forms. Unfortunately, this type of engagement occurred at the close of the Antarctica Project.

The research project team's reflection on the use of the Antarctica Project contributed to the more successful implementation of a second MMAP unit in Ms. Alessi's classroom (Guppies). The Antarctica Project can be thought of as a "launcher unit," which served to familiarize students and teachers with the discourse practices and social organization of MMAP units to facilitate their participation in a second, similarly structured design project (Holbrook & Kolodner, 2000). Classroom interactions were organized as they were in the earlier project (e.g., local group work, design reviews with professional biologists, and public presentations); however, we arranged more presentations and discussions of mathematical patterns than we did in the Antarctica Project. This project successfully engaged the focal students in discussing the meaning and interpretation of mathematical forms as evidenced by the quality of their mathematical activities and conversations throughout the project (for a detailed discussion of the students' work in this project, see Jurow, 2002). Providing opportunities for students to enter into the figured world of mathematics as well as the imaginary world proposed by the MMAP unit appeared to support the students in using and learning mathematics in diverse and meaningful contexts.

CONCLUSION

This article considered the ways in which a simulated real-world project was engaging for a group of middle-school mathematics students. It examined how their engagement developed over the course of the 7-week project and how this supported them in taking on productive relations to mathematical activities.

The analysis focused on the Antarctica Project, an architectural design project that proposes an imaginary premise in which students are meant to take on the roles of architects who need to design a research station on the Antarctic coast. Through the process of designing, revising, and analyzing their research station building design, students are meant to learn and use mathematics to solve realistic problems. To understand how students came to care about the goals of the Antarctica Project and found them meaningful, this analysis considered how the activities of one group of students were shaped through interaction with the multiple figured worlds of the classroom, Antarctic building design, and mathematics. The notion of figured worlds provided a lens through which to study how the students were "drawn to, recruited for, and formed in these worlds, and [became] active in and passionate about them" (Holland et al., 1998, p. 49). Examining how figured worlds are mediated through language, tools, and interactions in the classroom can help us appreciate how students navigate through and develop understandings of themselves as knowledgeable agents in these worlds (Boaler & Greeno, 2000).

This research suggests that the analysis of figured worlds to document students' developing engagements in an extended project and to understand the meaning of this for learning mathematics is a productive approach. As emphasized in the analysis, the relations between the figured worlds of the classroom, Antarctica, and mathematics were not seamless; there were tensions between these worlds that were revealed over the course of the 7-week project. The members of the focal group struggled between the figured world of the traditional classroom and a view of school tasks as inconsequential and an emerging orientation to the figured world of Antarctica, a world in which making informed and economical design decisions matter. We began to see the possibility in the students' participation across the scenes of new and more educationally effective ways of participating in curricular activities.

The approach described in this article points to the need to recognize the multiple figured worlds in which students participate and are asked to participate in when we ask them to engage in projects. Identifying and distinguishing sometimes overlapping figured worlds, roles, and relations in classroom interaction is a challenging but promising analytic task. It requires developing a deep appreciation for the various figured worlds that are invoked in a setting. What kinds of characters populate these figured worlds? In what kinds of activities do the characters engage, and what motivates their behaviors? Appreciating these culturally and historically shaped frames of interpretation provides a way to understand how people organize and interpret their participation in a setting. Studying how figured worlds are evoked and transformed in interaction calls for a combination of methods such as ethnography and discourse analysis that enable one to study participants' purposeful ways of interpreting, acting, and meaning using various semiotic resources (Skinner, Valsiner, & Holland, 2001).

The longitudinal perspective taken in this analysis provided a way to examine how students developed understandings of their activities in terms of their experiences with the figured worlds of the classroom, an imaginary premise, and mathematics. Studying how students negotiate tensions between different ways of engaging the manifest curriculum in terms of participating in alternatively figured worlds requires further study. A future line of analysis, not addressed in this article, might explore how students' personal histories mediate engagement in figured worlds and how repeated engagement in such worlds supports the development of particular kinds of learners. A focus on students' engagement with differently figured worlds can help us better understand how they experience and reconfigure the learning environment of the classroom.

ACKNOWLEDGMENTS

This research was funded, in part, by grants from the National Science Foundation (RED–9553648) and the University of California Berkeley's Graduate School of Education.

Thanks to my collaborators on the Math-at-Work Project. I am especially grateful to Rogers Hall for his guidance in conducting my dissertation research, which is the basis for this article, and in developing this analysis. Thanks also to Noel Enyedy, Frederick Erickson, Jordan Jurow, Yasmin Kafai, Janet Kolodner, Bill Sandoval, Laura Weishaupt, and three anonymous reviewers for their thoughtful advice and encouragement in preparing this article. Special thanks to Ms. Alessi and her students for their participation in this study.

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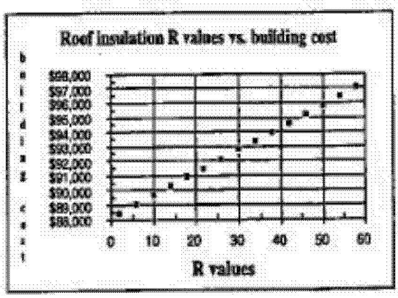
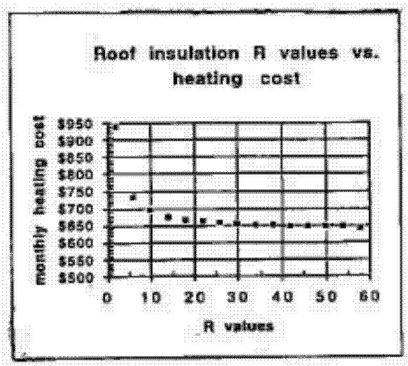
APPENDIX
"Making the Best of It" Worksheet

MAKING THE BEST OF IT

Memo 3 asks you to find the best insulation for your station. These activities will help you figure out strategies for doing that.

1 These two graphs are based on data collected from ArchiTech **for a 310 square meter design:** For each graph write a paragraph that tells what they say. Then write a sentence or two that compares the two graphs.





For 2–4, use your own design. Don't change the shape or size; only use the sliders.

2 Test out the heating and building costs for R 2 and R 60 for your design. Write down what you get.

3 Predict what the heating and building costs will be if you set the insulation value at 31 R. Now test your prediction, and explain what you find out.

3 Predict what the heating and building costs will be if you set the insulation value at 31 R. Now test your prediction, and explain what you find out..

4 Memo 3 asks you to figure out "the R value that will give *the lowest total building and heating costs for 20 years.*"

Explain, in words, symbols or pictures, how to calculate the total cost for heating and building for twenty years.

5 **Make a table** that shows the effect of increasing the R value of one kind of insulation on the total cost for heating and building. Make sure you don't change anything else during this activity.

Roof Insulation	Heating Cost	Building Cost	20 year total
58	642.88	$97,015	$251,306
54	643.65	$96,397	$250,873
50	644.56	$95,779	$250,473

6 **Describe** at least two patterns that you see in your table. Use this table to help you **choose the most effective insulation value** for *that kind of insulation* for your design. Explain how you made your choice.

HINT

To learn more about insulation how it works and how it is measured look at the student handout, ArchiTech Information.