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Abstract

We propose a bootstrap ARDL test. By applying the appropriate bootstrap method, some weaknesses underlying the Pesaran et al. (2001) test are addressed including size and power properties and the elimination of inconclusive inferences. In addition, inferences based solely on the significance of the F-test and single t-test of Pesaran et al. (2001) are not sufficient to avoid degenerate cases. The bootstrap ARDL provides an additional test on the significance of coefficients on lagged levels of the regressors, which provides a better insight on the cointegration status of the model.

Keyword: Bootstrap method, ARDL bounds test, degenerate cases
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JEL classification numbers: C15, C32, C52

1. Introduction

In the early 2000s, Pesaran et al. (2001) (PSS hereafter) developed a cointegration test, namely, the Autoregressive-Distributed Lag Bounds Test (ARDL hereafter), for dealing with models that involve time series with mixed orders of integration. This approach has gained popularity due to several advantages over other cointegration testing methods. However, many researchers apply this test in environments that violate the underlying assumptions of the bounds testing framework. For example, the bounds test assumes that there is no feedback at the levels from the dependent variable to independent variables. Some researchers implicitly violate this assumption by treating each of the variables as the dependent variable in a sequence of regressions on the others. This implicitly allows two or more variables to be (weakly) endogenous in violation of the assumptions underlying the distributions of the test statistics presented in PSS.

Therefore, one of the objectives of this paper is to evaluate the performance of the test under violation of the assumption of no feedback from the dependent variable to the independent variables. We employ Monte Carlo simulations to investigate the size and power properties of the PSS ARDL bounds test under a range of environments, including those violating the assumption of weak exogeneity of the regressors. In addition we employ these same Monte Carlo experiments to investigate the performance of ARDL cointegration tests using the bootstrap procedure, providing a comparison of the performance of the bootstrap and asymptotic tests in the ARDL model. This adds to the existing research on bootstrapping in time series models (for example, Li and Maddala, 1997; Chang and Park, 2003; Palm et al., 2010; Ko, 2011). It is increasingly common to use the bootstrap to perform hypothesis tests in econometric analysis, as bootstrap test statistics' critical values are often more accurate than asymptotic critical values (Singh, 1981; Beran, 1988). Palm et al. (2010) (PSU hereafter) prove the consistency of the bootstrap test of cointegration in a conditional ECM, and then demonstrate with simulation the improved properties of the bootstrap test statistics relative to the asymptotic ones. Their consistency proof supports the methodology used in this investigation since both studies work within the ECM framework. One difference is that their study does not consider the endogeneity problem underlying the bounds cointegration test.

Defined by PSS, cointegration under the bounds test can be found if and only if two tests individually reject their respective null hypotheses, together with the condition that the dependent variable is known to be $I(1)$. Some researchers draw conclusions based solely on the joint significance of the first test in PSS, the F-test on the lagged levels of all variables. They neglect to perform the second test, which is the t-test on the coefficient of the lagged level of the dependent variable (for example, Alhassan and Fiador, 2014; Garg and Dua, 2014; Jiang and Nieh, 2012; Muscatelli and Spinelli,2000 and Getnet et al., 2005). In this situation, incorrect conclusions may be drawn from a degenerate case, which PSS define as arising when the overall F-statistic is significant, but the coefficient on the lagged level of the dependent variable is not significantly different from zero. In this degenerate case, there is no cointegration among the series in the model.

PSS rules out another degenerate case by assuming the dependent variable is integrated of order one.[1] However, researchers applying the ARDL bounds test do not always check for unit roots, recognizing that this test is explicitly designed for cases of mixed and unknown orders of integration *among the several explanatory variables.* But, this flexibility does not extend to the dependent variable.[2] Furthermore, the low power problem of unit root tests may lead one to incorrectly conclude that a dependent variable is $I(1)$ and proceed with the ARDL tests. For example, Goh and McNown (2015) examined whether Malaysia's interest rate was cointegrated with that of the US during the recent managed floating exchange rate regime. They found that despite the significant F and t-statistics for the two tests in PSS, the t-statistic for the lagged *independent* variable (i.e. US interest rate) was insignificant. Hence, this suggests that the lagged level of the dependent variable is the sole source for the overall statistical significance of the lagged levels. In this case the ARDL equation reduces to a generalized Dickey-Fuller equation, the dependent variable is actually stationary, and cointegration between the two series does not exist.

Instead of assuming the dependent variable to be $I(1)$ in order to rule out the degenerate case, we propose an explicit test on the lagged level of the independent variable(s) to have a full picture of the cointegration status between the dependent and independent variables. By application of all three ARDL tests - the joint F-test on all lagged level terms, the test of significance on the coefficient of the lagged level of the dependent variable, and the new test on the lagged level(s) of the independent variable(s) - we can have a better insight into whether the relationship between the dependent variable and independent variables is one of cointegration, non-cointegration, or a degenerate case. The critical values for this additional test may be generated through the bootstrap procedure in any empirical application.

A further advantage of the bootstrap ARDL test is the elimination of inconclusive inferences with the bounds test. To determine the existence of a long-run relationship between the dependent variable and its regressors, PSS presented a pair of tests: an F-test of the joint significance of the coefficients of lagged level variables, and a t-test on the single coefficient on the lagged level of the dependent variable. Critical values for these tests are generated based on two alternative data-generating processes, with all regressors $I(0)$ in one case and all $I(1)$ in the other. The values generated with $I(0)$ regressors provide lower bounds to the critical values whereas values produced by the $I(1)$ regressors establish upper bounds to the critical values. If the computed test statistics fall outside the bounds, conclusive inference is made without knowing the integration orders of the underlying regressors. Otherwise the test is inconclusive. However, through the bootstrap procedure, the critical values are generated based on the specific integration properties of each data set, and the possibility of an indeterminate test is eliminated.

In summary, this study offers several contributions to the ARDL testing methodology. From the Monte Carlo simulations, there is evidence that the endogeneity problem has only minor effects on the size and power properties of the ARDL bounds testing framework using the asymptotic critical values. In addition, if the resampling procedure is applied appropriately, the bootstrap test performs better than the asymptotic test in the ARDL bounds test based on size and power properties. Furthermore, the bootstrap procedure has the additional advantage of eliminating the possibility of inconclusive inferences. Finally, we present an extension of the ARDL testing framework for the alternative degenerate case, with critical values generated by the bootstrap procedure. In an empirical application we demonstrate the occurrence of this alternative degenerate case when both the joint F test and t test on the lagged level of the dependent variable are significant, but the coefficient on the lagged level of the independent variable is insignificant. We find occurrence of this alternative degenerate case even when the dependent variable appears to be $I(1)$ based on standard unit root tests. Therefore, the proposed bootstrap ARDL test provides a better insight on the cointegration status of the series in the model.

This paper is constructed as follows. Section 2 explains the degenerate cases in greater detail. Section 3 discusses the data generating process (DGP), model, and simulation setup of the Monte Carlo experiments. Next, we present our simulation results in Section 4. Section 5 describes an empirical application and illustrates the degenerate cases, and Section 6 concludes the study.

2. Assumption 3 of Bounds Testing Approach and the Existence of Degenerate Case

2.1 Assumption 3 of Bounds Testing Approach

PSS used 5 assumptions as the foundation for the bounds testing approach. Some researchers may miss one of the crucial assumptions i.e. Assumption 3 that is spelled out by PSS, page 293. Consider a (k+1)-VAR model of order p:

$$\mathbf{\Phi}(L)(\mathbf{z}_t - \boldsymbol{\mu} - \boldsymbol{\gamma} t) = \boldsymbol{\varepsilon}_t, t = 1, 2, \ldots \tag{1}$$

where L is the lag operator, $\{\mathbf{z}_t\}_{t=1}^{\infty}$ is a (k+1) random process that can be partitioned into $(y_t, \mathbf{x}_t)'$, μ and γ are unknown (k+1)-vectors of intercept and trend coefficients. The $(k+1, k+1)$ matrix of lag polynomials $\mathbf{\Phi}(L)$ is equal to $\mathbf{I}_{k+1} - \sum_{i=1}^{p} \mathbf{\Phi}_i (L)^i$, where $\mathbf{I}_{k+1}$ is an identity matrix of order k+1, with $\{\Phi_i\}_{i=1}^{p}$ $(k+1, k+1)$ matrices of unknown coefficients. Applying PSS Assumptions 1 and 2, the VAR(p) in Equation (1) is expressed as a conditional ECM system:

$$\Delta y_t = c_0 + c_1 t + \pi_{yy} y_{t-1} + \boldsymbol{\pi}_{yx} \mathbf{x}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\psi}_i' \Delta \mathbf{z}_{t-1} + \boldsymbol{\omega}' \Delta \mathbf{x}_t + u_t, \ t = 1, 2, \ldots \tag{2}$$

$$\Delta \mathbf{x}_t = \mathbf{c}_0 + \mathbf{c}_1 t + \boldsymbol{\pi}_{xy} \mathbf{y}_{t-1} + \boldsymbol{\pi}_{xx} \mathbf{x}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_{xi}' \Delta \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_{xt}, \ t = 1, 2, \ldots \tag{3}$$

where π denotes a long-run coefficient matrix or vector, $\boldsymbol{\psi}_i'$ and $\boldsymbol{\Gamma}_{xi}'$ are matrices of short-run multipliers, Δ is the difference operator, $\boldsymbol{\omega}'$ contains the coefficients on $\Delta \mathbf{x}_t$, and u_t and $\boldsymbol{\varepsilon}_{xt}$ are i.i.d. errors.[3] PSS states Assumption 3 as: the k-vector $\boldsymbol{\pi}_{xy} = \mathbf{0}$, i.e. there is no feedback from the level of y_t in the conditional unrestricted ECM for $\mathbf{x}_t$, but it does not impose similar restrictions on the short-run multipliers in the equations for $\mathbf{x}_t$. Under Assumption 3, Equation (3) becomes

$$\Delta \mathbf{x}_t = \mathbf{c}_0 + \mathbf{c}_1 t + \boldsymbol{\pi}_{xx} \mathbf{x}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_{xi}' \Delta \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_{xt},$$

This assumption restricts the vector $\mathbf{x}_t$ to be long run *forcing variables* for $\{y_t\}_{t=1}^{\infty}$. Under Assumption 3, the conditional ECM (2) now becomes

$$\Delta y_t = c_0 + c_1 t + \pi_{yy} y_{t-1} + \boldsymbol{\pi}_{yx.x} \mathbf{x}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\psi}_i' \Delta \mathbf{z}_{t-1} + \boldsymbol{\omega}' \Delta \mathbf{x}_t + u_t.^{4} \tag{4}$$

This is the crucial assumption that supports the PSS methodology. By incorporating Assumption 3 together with other assumptions, one can detect cointegration irrespective of the level of integration of the regressors.

The PSS framework assumes weak exogeneity of the regressors. These regressors are not impacted by the dependent variable in the long-run, but this does not preclude the existence of cointegrating relationships among the regressors; nor does it assume the absence of (short run) Granger causality from the dependent variable to the regressors. Since the asymptotic distributions presented by PSS build on Assumption 3, its violation may invalidate test results. Unfortunately, some researchers have ignored this assumption in their empirical applications of the ARDL bounds test, for instance, as in Shahbaz et al. (2013), Satti et al. (2014), Blotch et al. (2015), Baharumshah et al. (2009), Guru-Gharana (2012) and Ahmed et al. (2007). These studies tested for cointegration by treating each variable sequentially as the dependent variable, changing the classification of the dependent and forcing variables for each different specification. This implicitly assumes that all variables are endogenous and PSS Assumption 3 is violated. The findings of cointegration (or long-run relationship) in each of these studies is therefore suspect. It is common to assume all the variables as endogenous in contemporary macroeconometrics, and regressors in ARDL equations may not generally be weakly exogenous. Since this problem could be pervasive, an important question addressed in this study is how the violation of Assumption 3 affects the size and power of the ARDL bounds testing procedure.

2.2 The Existence of Degenerate Cases

The hypotheses behind the F and t-tests in the ARDL bounds procedure are the following. In the unrestricted error-correction model of Equation (4), PSS introduced the test for the absence of any level relationship between y_t and $\mathbf{x}_t$ by defining the null hypothesis as $H_0 = H_0^{\pi_{yy}} \cap H_0^{\pi_{yx.x}}$, where $H_0^{\pi_{yy}} : \pi_{yy} = 0, \quad H_0^{\pi_{yx.x}} : \boldsymbol{\pi}_{yx.x} = \mathbf{0}'$, while the alternative hypothesis is defined as $H_1 = H_1^{\pi_{yy}} \cup H_1^{\pi_{yx.x}}$ and it covers not only $H_1 : \pi_{yy} \neq 0, \ \boldsymbol{\pi}_{yx.x} \neq \mathbf{0}'$ but also permits $H_1 : \pi_{yy} \neq 0$, $\boldsymbol{\pi}_{yx.x} = \mathbf{0}'$ or $H_1 : \pi_{yy} = 0, \quad \boldsymbol{\pi}_{yx.x} \neq \mathbf{0}'$. Cases when $H_1 : \pi_{yy} \neq 0, \quad \boldsymbol{\pi}_{yx.x} = \mathbf{0}'$ and $H_1 : \pi_{yy} = 0$, $\boldsymbol{\pi}_{yx.x} \neq \mathbf{0}'$ PSS refers to as degenerate level relationships between y_t and $\mathbf{x}_t$. We call the former degenerate case #1 and degenerate case #2 for the latter. Degenerate cases imply no cointegration.

To establish the existence of a levels relationship between y_t and $\mathbf{x}_t$, the F statistic on π_{yy} and $\boldsymbol{\pi}_{yx.x}$ must be significant. However, to conclude the existence of a level relationship, using the significance of the F test solely is insufficient because it only rejects the null of $H_0 = H_0^{\pi_{yy}} \cap H_0^{\pi_{yx.x}}$ in favor of the alternative hypothesis $H_1 = H_1^{\pi_{yy}} \cup H_1^{\pi_{yx.x}}$. However, this alternative hypothesis includes the possibility of a degenerate level relationship as well. For this reason, the t-test of $H_0 : \pi_{yy} = 0$ against its alternative $H_1 : \pi_{yy} \neq 0$ must be applied to ensure the outcome is free of degenerate case #2. Although PSS presents the necessary critical value bounds for testing the lagged level of the dependent variable, some studies fail to apply this test.[5]

To rule out degenerate case #1, one also needs to make sure that the dependent variable is integrated of order one or $I(1)$. If the F test is significant and the dependent variable is $I(1)$, then the coefficient $\boldsymbol{\pi}_{yx.x}$ must be significant, ruling out degenerate case #1. If the dependent variable is $I(0)$ and the independent variables are $I(1)$, then there cannot be cointegration. The significance of the overall F-statistic in this case results from the stationarity of the dependent variable. The ARDL test in this case is equivalent to a generalized unit root test with the difference of the dependent variable regressed on its lagged level plus additional irrelevant lagged levels of the independent variables. For instance, Morley (2006) in his study using conventional ADF tests found that the immigration variables from three countries were borderline $I(0)/I(1)$ processes. If immigration is actually $I(0)$, then the ARDL test with immigration as the dependent variable is non-informative since an $I(0)$ series cannot be cointegrated with $I(1)$ regressors.

Despite that, as mentioned previously, ruling out this degenerate case by using the assumption of dependent variable as $I(1)$ can lead to incorrect conclusions. We would suggest that when conducting the bounds test, the F-test must be complemented with tests of both degenerate cases as well. These tests will be able to identify whether there is cointegration or a degenerate case, as will be shown through Monte Carlo simulations and illustrated in the empirical applications.

3. The Data-Generating Process (DGP), Model and Bootstrap Procedures

3.1 The Data-Generating Process (DGP)

PSS has listed five different cases of interest according to the choice of deterministic components. For this investigation, we focus on a bivariate version of Case III, i.e. the model with unrestricted intercepts but no trend in the long run relation.[6] The DGP setting is similar to PSU but with some modifications that fit the needs of this study. To accommodate a variety of cases of cointegration, non-cointegration, and degenerate cases, the most general model is an unrestricted bivariate ECM(1,1):

$$\Delta y_t = \tau^y - \alpha^y\left(\beta_1^y y_{t-1} - \beta_2^y x_{t-1}\right) + \phi_1^y \Delta y_{t-1} + \phi_2^y \Delta x_{t-1} + \varepsilon_t^y \text{, and} \tag{5}$$

$$\Delta x_t = \tau^x - \alpha^x\left(\beta_1^x x_{t-1} - \beta_2^x y_{t-1}\right) + \phi_1^x \Delta y_{t-1} + \phi_2^x \Delta x_{t-1} + \varepsilon_t^x \text{,} \tag{6}$$

followed by generation of y_t and x_t recursively as

$$y_t = y_{t-1} + \Delta y_t \text{ and} \tag{7}$$

$$x_t = x_{t-1} + \Delta x_t \,, \tag{8}$$

for t = 3, …, *T* where *T* is the sample size. For Equations (5) and (6), parameters denoted τ are the intercept terms; α terms are the coefficients of the error correction components that capture the gravitation towards the equilibrium relationship by y_t and x_t; β terms are the coefficients on lagged level variables (where β_1 refers to the coefficient on the lagged dependent variable and β_2 represents the coefficient on the lagged independent variable), ϕ parameters are coefficients on lagged differences. The $\boldsymbol{\varepsilon}_t$ is the disturbance and is related to the structural innovations u_t and v_t by:

$$\varepsilon_t^y = \rho v_t + u_t \text{ and} \tag{9}$$

$$\varepsilon_t^x = v_t \,, \tag{10}$$

where u_t and v_t are independently and identically distributed as $N\left(0, \sigma^2\right)$

$$\begin{bmatrix} u_t \\ v_t \end{bmatrix} = IN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{u_t}^2 & 0 \\ 0 & \sigma_{v_t}^2 \end{bmatrix}\right). \tag{11}$$

Here ρ allows dependence between the equation errors and is equivalent to the contemporaneous correlation between y_t and x_t, after controlling for conditioning information. The superscript *y* denotes variables in equation for *Y*; *x* denotes variables in equation *X*.

This DGP framework allows various types of interdependencies between y_t and x_t through choice of alternative parameter values in the ECM(1,1), including allowance for unilateral or bilateral direction feedbacks from the levels of the variables, cointegrating relations, and degenerate cases. Without loss of generality, we may choose combinations of the vector β for $(y_t \quad x_t)'$ by setting it to $(0 \quad 1)$, $(1 \quad 0)$ or $(1 \quad 1)$. For the y_t equation if the vector is set to $(0 \quad 1)$, then only x_{t-1} or lagged level of independent variable appears in the error-correction term. In this case there is no cointegration, but rather the degenerate case #2 mentioned by PSS. Similarly with the vector $(1 \quad 0)$, only y_{t-1} (the lagged level of the dependent variable) appears in the error correction term, which defines degenerate case #1. y_t is not related to x_t in the long-run, and the significant F is due to the stationarity of y_t. Lastly, if the vector is $(1 \quad 1)$, y_t and x_t are cointegrated, provided the speed of the adjustment parameter is less than zero $\alpha < 0$.

In designing the direction of feedback in the levels between y_t and x_t, for unilateral feedback from x_t to y_t we omit the lagged level of y_t from the equation for Δx_t . For example, in Equations (5) and (6), with adjustment parameter $\alpha \neq 0$, specify $\beta_1^y = \beta_2^y = \beta_1^x \neq 0$ while $\beta_2^x = 0$, so that in Equation (5) Δy_t is related to both lagged levels y_{t-1} and x_{t-1}. However, in Equation (6) Δx_t only contains its own lagged level, x_{t-1}. This is consistent with PSS's Assumption 3, treating x_t as the long-run forcing variable for y_t, with no feedback from the level of y_t in the equation for Δx_t . However, in designing an environment that violates Assumption 3, allowing feedback from the dependent variable to the independent variable, we specify $\beta_1^y = \beta_2^y = \beta_2^x \neq 0$ and $\beta_1^x = 0$. In this case, because y_{t-1} is present in the Δx_t equation, x_t is no longer an exogenous long-run forcing variable to y_t. As long as y_{t-1} appears in the RHS of the Δx_t equation, Assumption 3 is violated in the use of the PSS ARDL bounds tests in the Δy_t equation.

3.2 Bootstrap Procedures

In bootstrapping time series data PSU establish the consistency of cointegration tests using the residual-based bootstrap procedure. Papers that use the residual-based bootstrap method in cointegrating regressions include Li and Maddala (1997), Harris and Judge (1998), and Chang et al. (2006). To perform the residual-based bootstrap, one can obtain the residuals from OLS estimation of either the restricted or the unrestricted model (see Remark 2, page 654, in PSU). To obtain the restricted residuals, estimate the restricted ARDL(1,1) model, $\tilde{\varepsilon}_t = \Delta y_t - \hat{c} - \delta_0 y_{t-1} - \delta_0 x_{t-1} - \hat{\theta}_1 \Delta y_{t-1} - \hat{\theta}_2 \Delta x_{t-1}$, by imposing the null hypothesis of $\delta_0 = 0$; to obtain the unrestricted residuals, estimate the residuals from the ARDL(1,1) model, $\tilde{\varepsilon}_t = \Delta y_t - \hat{c} - \hat{\delta}_1 y_{t-1} - \hat{\delta}_2 x_{t-1} - \hat{\theta}_1 \Delta y_{t-1} - \hat{\theta}_2 \Delta x_{t-1}$, under the alternative hypothesis of cointegration with $\delta_1 \neq 0, \delta_2 \neq 0$. The consistency of bootstrap tests of cointegration within the error correction model framework is established by PSU. The cointegration tests analyzed in this current study are also grounded in the ECM framework, with bootstrapped pseudo-data generated with and without the restrictions of the null hypothesis. Since the proof of consistency of these bootstrap tests is provided in PSU, it is unnecessary to repeat this proof here.

The bootstrap procedure that follows is identical to that presented in Section 3.2 of PSU. In particular, Equation *Y* from (12) below with the restriction $\delta_0 = 0$ duplicates PSU Equation (8). The equation for *X* relaxes the restriction of non-cointegration, $\delta_1^x \neq 0, \delta_2^x \neq 0$, which duplicates PSU's alternative bootstrap procedure discussed in their Remark 2, although they allow a more general case for deterministic components. As is common in most Monte Carlo studies, PSU generate series in some environments that depart from the assumptions that support the asymptotic theory. For example, they consider specifications with multivariate GARCH error processes and also models with non-constant parameters in the short run dynamics. They also analyze variations on the method of resampling the residuals in the bootstrap. They find the bootstrap procedure to be robust to these alternative specifications. Similarly, the current study analyzes the performance of the bootstrap tests in environments that depart from the narrow assumptions that support the asymptotic distribution theory. Of course, this type of robustness test is one of the primary advantages of Monte Carlo studies over sole reliance on asymptotic distributions. Reasonable size and power performance of the bootstrap tests under these alternative conditions would be evidence that the asymptotic distribution theory provides general support for the empirical bootstrap distributions.

The algorithm to obtain the bootstrap critical values is presented in 8 steps for the Equation *Y* [7]:

Step 1: Fit the restricted ARDL(1,1) model for Δy_t and unrestricted ARDL(1,1) model for Δx_t. The Δy_t equation is imposed with the null of the F test, $\delta^y = \delta_0 = 0$. Estimate both equations by OLS and save the restricted residuals defined as

$$\begin{aligned}\tilde{\varepsilon}_t^y &= \Delta y_t - \hat{c}^y - \delta_0 y_{t-1} - \delta_0 x_{t-1} - \hat{\theta}_1^y \Delta y_{t-1} - \hat{\theta}_2^y \Delta x_{t-1} \\ \tilde{\varepsilon}_t^x &= \Delta x_t - \hat{c}^x - \hat{\delta}_1^x y_{t-1} - \hat{\delta}_2^x x_{t-1} - \hat{\theta}_1^x \Delta y_{t-1} - \hat{\theta}_2^x \Delta x_{t-1}\end{aligned} \quad (12)$$

[8]

Note that this same set of restrictions applies to the t-tests on the lagged level of the dependent variable and the lagged level of the independent variable, regardless of the null hypothesis.

Step 2: Rescale and recenter the restricted residuals by the following formula:

$$\ddot{\varepsilon}_t = \hat{\varepsilon}_t - (n-q-1)^{-1} \sum_t \hat{\varepsilon}_t , \quad (13)$$

where n is the number of observations and q is the lag length of the model. In this case, the q is equal to 1. This step follows the recommendation of Davidson and MacKinnon (2005). Save the rescaled and recentered residuals.

Step 3: Resample the $\ddot{\varepsilon}_t$ with replacement to obtain the bootstrap residuals ε_t^*.

Step 4: For generating bootstrap observation t, use the model (12) in Step 1 to generate Δy_t and Δx_t, inserting the bootstrap residuals in place of the original residuals. Then generate y_t^* and x_t^* as:

$$\begin{aligned} y_t^* &= y_{t-1}^* + \Delta y_t^* \\ x_t^* &= x_{t-1}^* + \Delta x_t^* \end{aligned}.$$

Step 5: Repeat Step 4 T times to obtain T observations on the bootstrap series y_t^* and x_t^*.

Step 6: Use OLS to estimate the unrestricted ARDL(1,1) equation with the bootstrap data

$$\begin{aligned} \Delta y_t^* &= \hat{c}^y + \hat{\delta}_1^y y_{t-1}^* + \hat{\delta}_2^y x_{t-1}^* + \hat{\theta}_1^y \Delta y_{t-1}^* + \hat{\theta}_2^y \Delta x_{t-1}^* + \varepsilon_t^{y*} \\ \Delta x_t^* &= \hat{c}^x + \hat{\delta}_1^x y_{t-1}^* + \hat{\delta}_2^x x_{t-1}^* + \hat{\theta}_1^x \Delta y_{t-1}^* + \hat{\theta}_2^x \Delta x_{t-1}^* + \varepsilon_t^{x*} \end{aligned}. \tag{14}$$

Step 7: Calculate the bootstrap F and t test statistics.

For the F test of joint significance of both lagged level terms (the null hypothesis of $\delta^y = \delta_0$ **=0**), we follow PSS in the use of the F statistic version of the Wald test. The bootstrap version of this test statistic is F_y^*; similarly $t_y^{\mathrm{DV}*}$ is the bootstrap t statistic on the lagged level of the dependent variable, and $t_y^{\mathrm{IDV}*}$ is the bootstrap t statistic on the lagged level of the independent variable.

Step 8: Repeat from Step 1 to Step 7 B times to obtain bootstrap test statistics $F_{y,b}^*$, or $t_{y,b}^{\mathrm{DV}*}$ and $t_{y,b}^{\mathrm{IDV}*}$, where b = 1, 2, …, B. Construct an empirical bootstrap distribution from the ordered bootstrap test statistics, and determine the critical values from this empirical distribution. Select bootstrap critical values c_α^* or $c_{1-\alpha}^*$ as

$$c_{1-\alpha}^* = \min\left\{c : \sum_{b=1}^{B} I\left(T_b^* > c\right) \le \alpha\right\}, \text{ or} \tag{15}$$

$$c_\alpha^* = \max\left\{c : \sum_{b=1}^{B} I\left(T_b^* < c\right) \le \alpha\right\} \tag{16}$$

where T_b^* is the order bootstrap test statistic, equivalent to $(1-\alpha)$ and α-quantiles of the ordered bootstrap statistics respectively. Reject the null hypothesis if the test statistic (F_y or t_y^{IDV}) calculated from Equation (4) is greater than $c_{1-\alpha}^*$ or test statistic (t_y^{DV}) less than c_α^*, where α is the nominal level of the test. For the estimation of Equation *X*, follow the same algorithm above, except that the restriction of the null in Step 1 is imposed on the Δx_t equation. The same changes apply in Step 4.

4. Simulation

4.1 Size and Power Analyses for Asymptotic and Bootstrap Tests

In this section, we report on a set of simulations with replications of *N* = 2,000 and bootstrap replications *B* = 1,000 to investigate both asymptotic and bootstrap ARDL bounds test performances. The upper and lower bound critical values used for the experiment are adopted from PSS and Narayan (2005) and they are used for the F-test and t-test on lagged dependent variable respectively. For those cases involving *I*(1) independent variable, the upper bound critical value is used, whereas for those cases with *I*(0) independent variable the lower bound critical value is used. Experiments with sample sizes of 50 and 100, with and without contemporaneous correlation under 16 different DGP cases are examined. The 16 simulation DGPs are described in Table 1 following the notation of Equations (5) and (6). These 16 cases include designs with no-cointegration, weak exogeneity, endogeneity of the regressor, and cointegration with various types of feedback effects between y_t and x_t. The cointegration status for Equations *X* and *Y*, PSS Assumption 3 violation status and each variable's integration order are summarized in Table 2. When the cointegration status of the two equations differs, one of the two series is weakly exogenous so that the ARDL equation for that series should not show significant response to the error correction term.

First, the asymptotic test is analyzed based on the critical values defined by the bound for *I*(1) or *I*(0) regressors consistent with the DGPs in these simulations. The size and power performances of the asymptotic test are summarized in Table 3. The simulation results suggest that the asymptotic F test generally works well in all cases, even with an endogenous regressor. The rejection rates are close to the 5% nominal level, ranging between 0.028 and 0.077. Cases in which

Table 1: Parameter combinations used in the DGP simulation

Case	τ^x	α^x	β_1^x	β_2^x	ϕ_1^x	ϕ_2^x	τ^y	α^y	β_1^y	β_2^y	ϕ_1^y	ϕ_2^y
1	0.02	0	0	0	0	0	0.02	0	0	0	0	0
2	0.02	0	0	0	0	0.5	0.02	0	0	0	0.5	0
3	0.02	0	0	0	0.2	0.5	0.02	0	0	0	0.5	0.2
4	0.02	0	0	0	0	0.5	0.02	0.5	1	0	0.5	0
5	0.02	0	0	0	0.2	0.5	0.02	0.5	1	0	0.5	0.2
6	0.02	0	0	0	0	0.5	0.02	0.5	0	1	0.5	0
7	0.02	0	0	0	0	-0.5	0.02	0.5	0	1	-0.5	0
8	0.02	0	0	0	-0.2	-0.5	0.02	0.5	0	1	-0.5	-0.2
9	0.02	0.5	0	1	0	0.5	0.02	0.5	1	0	0.5	0
10	0.02	0.5	0	1	0.2	0.5	0.02	0.5	1	0	0.5	0.2
11	0.02	0.5	1	1	0	0.5	0.02	0.5	1	0	0.5	0
12	0.02	0.5	1	1	0.2	0.5	0.02	0.5	1	0	0.5	0.2
13	0.02	0.5	1	1	0	0.5	0.02	0.5	1	1	0.5	0
14	0.02	0.5	1	1	0.2	0.5	0.02	0.5	1	1	0.5	0.2
15	0.02	0	0	0	0	0.5	0.02	0.5	1	1	0.5	0
16	0.02	0	0	0	0.2	0.5	0.02	0.5	1	1	0.5	0.2

Note: Notations with superscript *x* refer to variables in Equation *X* and *y* for Equation *Y*.

Table 2: Information of the experiments

Case	**Equation *X***			**Equation *Y***		
	Status	Violation assumption	Integration order of x_t	Status	Violation assumption	Integration order of y_t
1	No-cointegration	No	$I(1)$	No-cointegration	No	$I(1)$
2	No-cointegration	No	$I(1)$	No-cointegration	No	$I(1)$
3	No-cointegration	No	$I(1)$	No-cointegration	No	$I(1)$
4	No-cointegration	No	$I(1)$	Degenerate #1	No	$I(0)$
5	No-cointegration	No	$I(1)$	Degenerate #1	No	$I(0)$
6	No-cointegration	Yes	$I(1)$	Degenerate #2	No	$I(1)$
7	No-cointegration	Yes	$I(1)$	Degenerate #2	No	$I(1)$
8	No-cointegration	Yes	$I(1)$	Degenerate #2	No	$I(1)$
9	Degenerate #2	No	$I(1)$	Degenerate #1	Yes	$I(0)$
10	Degenerate #2	No	$I(1)$	Degenerate #1	Yes	$I(0)$
11	Cointegration	No	$I(1)$	Degenerate #1	Yes	$I(0)$
12	Cointegration	No	$I(1)$	Degenerate #1	Yes	$I(0)$
13	Cointegration	Yes	$I(1)$	Cointegration	Yes	$I(1)$
14	Cointegration	Yes	$I(1)$	Cointegration	Yes	$I(1)$
15	No-cointegration	Yes	$I(1)$	Cointegration	No	$I(1)$
16	No-cointegration	Yes	$I(1)$	Cointegration	No	$I(1)$

Assumption 3 is violated do not necessarily produce important size distortions; for example, Cases 6-8 in Equation *X* show empirical sizes of 0.049 – 0.058.

On the other hand, the t-test faces a more serious undersize problem, but again this occurs regardless of the consideration of violation of Assumption 3. For Cases 1-3 in both Equations *X* and *Y*, the sizes are close to the nominal 0.05 value. However, in all other cases in which the null hypothesis is true (the coefficient on the lagged level of the dependent variable equals zero), the empirical size is below 0.05. This is particularly extreme in Cases 15 and 16 for the Equation *X* (0.006 and 0.007) and Cases 6-8 (0.006 – 0.010) in the Equation *Y*. These cases involve a mixture of Assumption 3 violations and non-violations, so there is no indication that endogeneity is the source of these size problems. In any case, the evidence of size distortions for the ARDL bounds test motivates analysis and comparison of the ARDL tests based on a bootstrap procedure. In addition, power comparisons are presented below, together with an investigation of the performance of the new test for degenerate case #1.

Table 3: Size and power of the asymptotic test (n = 50, ρ = 0.5)

Case	F_x	F_y	t_x^{DV}	t_y^{DV}
1	**0.057**	**0.050**	**0.053**	**0.047**
2	**0.067**	**0.060**	**0.065**	**0.053**
3	**0.054**	**0.077**	**0.039**	**0.056**
4	**0.044**	0.960	**0.058**	0.977
5	**0.056**	0.967	**0.068**	0.978
6	**0.049**	0.925	**0.012**	**0.006**
7	**0.058**	0.821	**0.045**	**0.008**
8	**0.055**	0.737	**0.031**	**0.010**
9	0.680	0.758	**0.050**	0.820
10	0.517	0.663	**0.039**	0.708
11	0.997	0.953	0.996	0.708
12	0.997	0.997	0.986	0.550
13	0.638	0.770	0.673	0.803
14	0.497	0.619	0.509	0.627
15	**0.028**	0.964	**0.006**	0.971
16	**0.032**	0.885	**0.007**	0.898

Note: Testing level α = 0.05. Number in bold refers to size property.

Table 4 summarizes the results for both asymptotic and bootstrap tests (the latter indicated with an * below). Generally, the F^* test performs well in all the cases. The test has similar size properties as the asymptotic test with empirical sizes close to 5%. For the t-test on the lagged dependent variable, the bootstrap test helps to correct the size distortions that existed with the asymptotic test in some cases. The undersized problem underlying the asymptotic t^{DV} test is reduced by the bootstrap test, with nominal sizes close to 5% for almost all cases (note especially Cases 15 and 16 in the Equation *X* and 6-8 in the Equation *Y*, which were especially problematic for the asymptotic test). For the t^{IDV*} test the bootstrap procedure has reasonable sizes in most cases. However, in Case 8 for the Equation *X* and Case 10 for the Equation *Y* this test is somewhat undersized with empirical rejection rates of 0.02.

Overall, the bootstrap test performs well in terms of size, overcoming the most severe size distortions shown by the asymptotic test. In addition, the Monte Carlo evidence shows that it has

Table 4: Size and power of the asymptotic and bootstrap tests (n = 50, ρ = 0.5)

Case	F_x	F_x^*	F_y	F_y^*	t_x^{DV}	t_x^{DV*}	t_y^{DV}	t_y^{DV*}	t_x^{IDV*}	t_y^{IDV*}
1	**0.057**	**0.065**	**0.050**	**0.061**	**0.053**	**0.068**	**0.047**	**0.060**	**0.051**	**0.041**
2	**0.067**	**0.071**	**0.060**	**0.062**	**0.065**	**0.076**	**0.053**	**0.066**	**0.058**	**0.056**
3	**0.054**	**0.057**	**0.077**	**0.070**	**0.039**	**0.044**	**0.056**	**0.054**	**0.038**	**0.044**
4	**0.044**	**0.053**	0.960	0.954	**0.058**	**0.057**	0.977	0.977	**0.040**	**0.037**
5	**0.056**	**0.052**	0.967	0.946	**0.068**	**0.059**	0.978	0.982	**0.050**	**0.032**
6	**0.049**	**0.061**	0.925	0.930	**0.012**	**0.061**	**0.006**	**0.036**	**0.049**	0.981
7	**0.058**	**0.058**	0.821	0.907	**0.045**	**0.047**	**0.008**	**0.039**	**0.029**	0.923
8	**0.055**	**0.068**	0.737	0.855	**0.031**	**0.048**	**0.010**	**0.040**	**0.021**	0.936
9	0.680	0.698	0.758	0.762	**0.050**	**0.057**	0.820	0.910	0.877	**0.026**
10	0.517	0.530	0.663	0.633	**0.039**	**0.067**	0.708	0.827	0.785	**0.021**
11	0.997	0.995	0.953	0.952	0.996	0.995	0.708	0.833	0.817	**0.060**
12	0.997	0.996	0.997	0.997	0.986	0.988	0.550	0.765	0.687	**0.055**
13	0.638	0.747	0.770	0.843	0.673	0.912	0.803	0.956	0.920	0.964
14	0.497	0.607	0.619	0.703	0.509	0.831	0.627	0.886	0.843	0.896
15	**0.028**	**0.057**	0.964	0.972	**0.006**	**0.048**	0.971	0.985	**0.048**	0.976
16	**0.032**	**0.065**	0.885	0.890	**0.007**	**0.061**	0.898	0.942	**0.063**	0.936

Note: Testing level α = 0.05. Number in bold refers to size property.

higher power than the asymptotic test. This can be seen from all the cases from in Table 4. For example, the power of the F-test for Equation *Y* in Case 8, is 11.8 percentage points higher than the asymptotic test, and the smallest improvement is 0.5 percentage points in Case 12 for Equation *X*. Across all cases, the average increase in power with the bootstrap F test is 3.0 percentage points, relative to the asymptotic test.

The increase of power from the bootstrap procedure is especially strong for the t-test on the lagged dependent variable. The power is higher for the bootstrap test in almost all the cases. For example, power increases as much as 32.7 percentage points (Case 12, Equation *Y*) and the average increase in power is 11.3 percentage points across all cases. For the power of the bootstrap t^{IDV*} test, Table 4 shows that its power is high with an average rejection rate of 88.8%.

Besides that, two additional bootstrap procedures were also implemented in our study besides the one presented in this paper. The first bootstrap procedure is based on the system null imposition on VAR model as suggested in PSU, and the second bootstrap follows the same procedure as presented in the paper with the imposed restriction specific to each particular test. However, these alternative bootstrap procedures do not improve the size and power properties relative to the asymptotic test, and are inferior in some cases to those based on the bootstrap design used for the results in Table 4. Although the asymptotic theory in PSU is based on the first alternative design, the similarity of results indicates that this theoretical support is robust to the specific choice of restrictions imposed in the bootstrap generating procedure.[9]

In addition, we extended our analysis for the case of pure *I*(0) series although this may not be the focus of the ARDL bounds test as presented by PSS. Their simulated bounds for the critical values area all based on data generation processes with the dependent variable as *I*(1). In any case the bootstrap tests are robust to this stationary environment, as shown in the Appendix. Therefore, reasonable size and power properties of the bootstrap tests are maintained across the full range of *I*(0) and *I*(1) combinations for dependent and independent variables.

4.2 Analyses with Different Combinations of Sample Sizes and Cross Equation Correlations

Tables 5 - 7 present size and power results for the bootstrap test in environments with zero and non-zero correlations between the equation errors and with different sample sizes. Table 5 shows the results from an experiment using a sample size of 50 and with the absence of contemporaneous correlation between series *Y* and *X*.

Table 5: Size and power of the asymptotic and bootstrap tests (n = 50, ρ = 0.0)

Case	F_x	F_x^*	F_y	F_y^*	t_x^{DV}	t_x^{DV*}	t_y^{DV}	t_y^{DV*}	t_x^{IDV*}	t_y^{IDV*}

1	**0.054**	**0.068**	**0.053**	**0.071**	**0.059**	**0.082**	**0.054**	**0.083**	**0.064**	**0.065**
2	**0.063**	**0.069**	**0.070**	**0.071**	**0.064**	**0.085**	**0.068**	**0.091**	**0.057**	**0.068**
3	**0.068**	**0.068**	**0.072**	**0.080**	**0.071**	**0.089**	**0.069**	**0.091**	**0.084**	**0.074**
4	**0.062**	**0.054**	0.993	0.977	**0.069**	**0.059**	0.997	0.990	**0.051**	**0.030**
5	**0.056**	**0.055**	0.993	0.981	**0.066**	**0.060**	0.997	0.982	**0.052**	**0.022**
6	**0.046**	**0.064**	0.982	0.965	**0.016**	**0.056**	**0.003**	**0.018**	**0.051**	0.981
7	**0.057**	**0.063**	0.857	0.945	**0.056**	**0.066**	**0.003**	**0.029**	**0.036**	0.961
8	**0.053**	**0.064**	0.832	0.915	**0.040**	**0.060**	**0.002**	**0.023**	**0.021**	0.961
9	0.872	0.891	0.896	0.911	**0.028**	**0.033**	0.945	0.972	0.971	**0.048**
10	0.789	0.794	0.791	0.757	**0.043**	**0.038**	0.845	0.898	0.945	**0.030**
11	1.000	1.000	0.943	0.959	1.000	1.000	0.861	0.933	0.968	**0.071**
12	1.000	1.000	0.993	0.993	1.000	1.000	0.783	0.886	0.913	**0.050**
13	0.945	0.966	0.942	0.973	0.958	0.991	0.958	0.995	0.992	0.996
14	0.860	0.907	0.851	0.900	0.885	0.974	0.879	0.975	0.979	0.979
15	**0.032**	**0.056**	0.999	0.999	**0.009**	**0.063**	1.000	1.000	**0.051**	0.998
16	**0.037**	**0.061**	0.996	0.995	**0.006**	**0.064**	0.997	0.996	**0.065**	0.996

Note: Testing level α = 0.05. Number in bold refers to size property.

From Table 5, the asymptotic F and t-tests performance is fairly similar in both size and power properties to the experiments with dependency between series (Tables 3 and 4; cross equation error correlation of 0.5). The same is true for the bootstrap test, which performs well in most of the cases except for the first three cases of no-cointegration. In these cases, the bootstrap F-test and t-test on the lagged dependent variable are slightly oversized compared to the asymptotic test with sizes as high as 0.09. Generally, these size differences are not large; 0.03 is the greatest difference between the test sizes in these first three environments. For the t test on lagged independent variable, except Case 3 (sizes of 0.084 and 0.074), the bootstrap test performs well with reasonable empirical sizes. In addition, the bootstrap F and t tests have higher power than the asymptotic test although comparison of tests with unequal sizes can be problematic. The t-test on the lagged independent variable has high power properties as well and with an average rejection rate of 97.2%.

Next, Table 6 presents the test results using a larger sample size (100 observations), first allowing dependence between the series. Again, the asymptotic test performs the same as in the previous experiments, showing that the flaws underlying the t-test on lagged dependent variable are not resolved by increasing the sample size, but the bootstrap tests again overcome the problem of small sizes. In Cases 15 and 16 for Equation *X* the sizes are 0.002 and 0.004 for the asymptotic t-test, versus 0.050 and 0.066 for the bootstrap test. There is similar improvement

Table 6: Size and power of the asymptotic and bootstrap tests (n = 100, ρ = 0.5)

Case	F_x	F_x^*	F_y	F_y^*	t_x^{DV}	t_x^{DV*}	t_y^{DV}	t_y^{DV*}	t_x^{IDV*}	t_y^{IDV*}
1	**0.047**	**0.063**	**0.044**	**0.060**	**0.047**	**0.066**	**0.046**	**0.063**	**0.046**	**0.043**
2	**0.050**	**0.069**	**0.053**	**0.066**	**0.056**	**0.077**	**0.041**	**0.056**	**0.050**	**0.042**
3	**0.064**	**0.064**	**0.061**	**0.062**	**0.041**	**0.046**	**0.044**	**0.050**	**0.030**	**0.039**
4	**0.053**	**0.062**	1.000	1.000	**0.057**	**0.063**	1.000	1.000	**0.046**	**0.044**
5	**0.057**	**0.063**	1.000	1.000	**0.062**	**0.072**	1.000	1.000	**0.042**	**0.038**
6	**0.053**	**0.070**	1.000	1.000	**0.005**	**0.076**	**0.004**	**0.033**	**0.041**	1.000
7	**0.064**	**0.064**	0.996	0.999	**0.035**	**0.061**	**0.005**	**0.044**	**0.032**	1.000
8	**0.044**	**0.059**	0.987	0.997	**0.019**	**0.048**	**0.009**	**0.039**	**0.020**	0.999
9	0.972	0.965	0.984	0.986	**0.046**	**0.044**	0.990	0.998	0.997	**0.037**
10	0.896	0.905	0.949	0.947	**0.041**	**0.047**	0.953	0.992	0.987	**0.037**
11	1.000	1.000	1.000	1.000	1.000	1.000	0.977	0.995	0.991	**0.057**
12	1.000	1.000	1.000	1.000	1.000	1.000	0.895	0.983	0.965	**0.060**
13	0.964	0.985	0.983	0.994	0.969	1.000	0.986	1.000	1.000	1.000
14	0.844	0.921	0.939	0.968	0.853	0.989	0.943	0.997	0.990	0.997
15	**0.023**	**0.053**	1.000	1.000	**0.002**	**0.050**	1.000	1.000	**0.052**	1.000
16	**0.038**	**0.069**	0.999	0.999	**0.004**	**0.066**	0.999	1.000	**0.063**	1.000

Note: Testing level α = 0.05. Number in bold refers to size property.

with the bootstrap test for Cases 6, 7 and 8 in the Equation *Y*. However, there is little difference in the power of the asymptotic test compared with the bootstrap, as the power for both tests is very close to a 100% rejection rate.

Finally, the results of the experiments with a larger sample size and zero correlation between the equation errors are shown in Table 7. There are no important differences between the results in Table 7 and those of the Tables 4 - 6. The bootstrap test still performs well in this environment, but large sample size does not help in resolving the size problem from Cases 1 to 3. In any case this problem is not severe; in no case is the size of any bootstrap test greater than 0.09, and the bootstrap F tests all have sizes less than 0.07. As in the previous experiment, a large sample size experiment pushes the power of the test to the extreme level and not much comparison can be made here.

Several conclusions emerge from the Monte Carlo experiments. First, the ARDL bounds test of PSS is not adversely affected by the presence of endogenous regressors. There are cases that produce undersized performance, but these are not necessarily associated with the violation of PSS Assumption 3. Second, the tests using the bootstrap show improvements in size relative to

Table 7: Size and power of the asymptotic and bootstrap tests (n = 100, ρ = 0.0)

Case	F_x	F_x^*	F_y	F_y^*	t_x^{DV}	t_x^{DV*}	t_y^{DV}	t_y^{DV*}	t_x^{IDV*}	t_y^{IDV*}
1	**0.048**	**0.062**	**0.056**	**0.066**	**0.049**	**0.082**	**0.055**	**0.081**	**0.070**	**0.062**
2	**0.057**	**0.064**	**0.059**	**0.065**	**0.053**	**0.083**	**0.062**	**0.091**	**0.078**	**0.074**
3	**0.058**	**0.060**	**0.059**	**0.063**	**0.055**	**0.076**	**0.051**	**0.082**	**0.067**	**0.073**
4	**0.058**	**0.059**	1.000	1.000	**0.053**	**0.057**	1.000	1.000	**0.044**	**0.029**
5	**0.056**	**0.050**	1.000	1.000	**0.052**	**0.052**	1.000	1.000	**0.051**	**0.023**
6	**0.036**	**0.070**	1.000	1.000	**0.005**	**0.063**	**0.005**	**0.013**	**0.049**	1.000
7	**0.069**	**0.065**	0.999	0.999	**0.048**	**0.066**	**0.001**	**0.028**	**0.026**	1.000
8	**0.042**	**0.062**	0.998	1.000	**0.020**	**0.064**	**0.001**	**0.014**	**0.023**	1.000
9	1.000	0.999	1.000	0.998	**0.033**	**0.031**	1.000	1.000	1.000	**0.060**
10	0.994	0.990	0.990	0.988	**0.032**	**0.030**	0.993	0.999	1.000	**0.052**
11	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	**0.059**
12	1.000	1.000	1.000	1.000	1.000	1.000	0.990	0.999	1.000	**0.061**
13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
14	0.998	0.999	0.997	0.998	0.998	1.000	0.997	1.000	1.000	1.000
15	**0.026**	**0.060**	1.000	1.000	**0.007**	**0.057**	1.000	1.000	**0.052**	1.000
16	**0.028**	**0.060**	1.000	1.000	**0.003**	**0.050**	1.000	1.000	**0.047**	1.000

Note: Testing level α = 0.05. Number in bold refers to size property.

the asymptotic test, especially when the asymptotic test is severely undersized. Third, the bootstrap tests produce higher power compared to the asymptotic test, without additional size distortions. Fourth, through the results of the extension analysis for the case of pure I(0) series, this reinforces the point that the bootstrap procedure produces exact critical values, not merely lower and upper bounds as in the asymptotic tests.

The proposed new ARDL bounds test based on bootstrap procedure is a more robust cointegration test than the PSS test. Finally, the new bootstrap test on the lagged level of independent variable(s) shows reasonable size and power properties, providing a more complete picture of the cointegration status among the variables. Erroneous conclusions can be reached if one does not explicitly perform a test for degenerate case #1 ($\pi_{yy} \neq 0$, $\boldsymbol{\pi}_{yx.x} = \mathbf{0}'$), and the Monte Carlo evidence above indicates that the bootstrap test is informative about this hypothesis.

5. Empirical Application

In this section, we re-examine the long run saving-investment (S-I) relation to test for international capital mobility using the bootstrap ARDL test. Identification of degenerate cases is illustrated here as well. The relation between savings and investment can serve as an indicator of international capital mobility, an idea introduced by Feldstein and Horioka (1980) (F-H henceforth). In their study of 16 OECD countries over the period 1960-1974, they found that savings and investment ratios are strongly related to each other with a slope coefficient not significantly different from one, implying very low international capital mobility. Reinterpreted in a contemporary time series context, the absence of cointegration between savings and investment ratios for a country implies considerable capital mobility. On the other hand, cointegration between these two series means that they move together in the long run, but a coefficient less than one in this relation also allows for some degree of capital mobility. Cointegration with a coefficient of unity implies strong capital immobility in the long run.

The use of the cointegration method in examining the S-I correlation was first conducted by Miller (1988), who applied the Engle-Granger two-step procedure to U.S. data over the period of 1946-1987. Our application covers 15 selected OECD countries over the period 1960-2013 using data from the World Bank database. Estimation periods vary for each country depending on data availability. The basic F-H relation is:

$$IR_t = c + bSR_t + \varepsilon_t, \tag{17}$$

Where *IR* is the investment-GDP ratio, *SR* is the saving-GDP ratio, *c* is a constant term, *b* is the coefficient measuring the degree of capital mobility, ε_t is the error term and subscript *t* is the time index. We test for cointegration applying the bootstrap ARDL test based on the equation:

$$\Delta IR_t = c + \delta_1 IR_{t-1} + \delta_2 SR_{t-1} + \sum\nolimits_{i=1}^{p-1} \theta_{1,i} \Delta IR_{t-i} + \sum\nolimits_{i=1}^{p-1} \theta_{2,i} \Delta SR_{t-i} + \sum\nolimits_{j=1}^{k} \sigma_j D_{t,j} + \varepsilon_t. \tag{18}$$

Equation (18) is an unrestricted ECM for *IR* and *SR* as the dynamic counterpart to the cointegrating Equation (17). Dummy variables, $D_{t,j}$, are included in the estimation to deal with possible structural breaks in (18). Before modelling, we have to ensure the integration orders of variables *IR* and *SR* do not exceed one to fit with the assumption made by the bounds test. For the sake of parsimony and to avoid over-parameterization, the maximum optimal lag length in this study is up to p = 4, which should be adequate for annual data. The choice of optimal lag length p is based on Akaike's Information Criteria. Estimations using *IR* as the dependent variable (denoted as Equation *IR*) following Equation (18) for the 15 countries are shown in

Table 8. All the equations passed all standard diagnostics; the critical values were obtained from the bootstrap procedure.

Based on the bootstrap overall F-test and the t-test on the lagged dependent variable, there is evidence that *IR* and *SR* are cointegrated for Australia, Iceland, Israel, South Korea, Mexico, New Zealand, Norway, Portugal, Turkey, and UK (10 countries). Conversely, there is insufficient evidence to show that SR and IR are cointegrated for Austria, Finland, Japan, Netherlands and Spain (5 countries). However, the test on the lagged independent variable fails to reject the null hypothesis that its coefficient is zero, even at a 10% significant level, for Israel, Norway, and Turkey. This suggests that these countries actually fall into degenerate case #1, and the *SR* and *IR* series are not cointegrated. The significance of the F test comes solely from the significance of the lagged dependent variable in these three cases. Furthermore, this also indicates that the dependent variable is actually $I(0)$. Notice that the possible existence of degenerate case #1 is not testable with the existing tests from the PSS methodology. To conduct the test on lagged independent variable, the bootstrap procedure has been used.

There is an additional interesting case in these estimations. The coefficients on IR_{t-1} and SR_{t-1} for Israel and the Netherlands both have negative signs. These perverse signs do not match the theory or the ECM system. For the *IR* – *SR* relation the signs of these two coefficients must be opposite, in order to adjust the system back to equilibrium. In any case it can be seen that, these countries are non-cointegration and degenerate cases. Table 9 summarizes the cointegration status for all countries.

Table 8: ARDL bounds test estimation using *IR* as dependent variable

	Australia	Austria	Finland
Dummy for Equation ΔIR_t	75, 83, 91, 01	75, 79, 82, 09	91, 09
Dummy for Equation ΔSR_t	83, 91, 10	75, 79, 09	91, 09
Period/Sample size (*n*)	1960-2012/53	1970-2012/43	1975-2012/38
$\bar{R}^2$	0.6006	0.6591	0.5242
Q-stat(12)	13.490	9.5327	10.370
LM(2)	2.1222	2.0188	0.3817
JB	1.2871	0.5632	1.7026
F-statistic	4.8561**	1.1651	1.4418
t-statistic (lagged DV)	-2.9129**	-1.5075	-1.6648
t-statistic (lagged IDV)	2.1985**	0.9890	0.7734
	Iceland	Israel	Japan
Dummy for Equation ΔIR_t	06	74, 85	09
Dummy for Equation ΔSR_t	08	71, 85, 90	09
Period/Sample size (*n*)	1979-2012/33	1965-2013/49	1977-2012/36
$\bar{R}^2$	0.4878	0.6901	0.5950
Q-stat(12)	8.4674	10.690	5.9501
LM(2)	0.2157	3.9329	0.1199
JB	1.4653	0.8862	3.5717
F-statistic	12.7310***	19.0203***	2.1713
t-statistic (lagged DV)	-4.4394***	-6.1389***	0.2152
t-statistic (lagged IDV)	3.7065***	-3.4092	-0.5847
	South Korea	Mexico	Netherlands
Dummy for Equation ΔIR_t	98, 09	86, 93	73, 81, 09
Dummy for Equation ΔSR_t	-	83, 86	73
Period/Sample size (*n*)	1976-2012/37	1979-2012/34	1970-2012/43
$\bar{R}^2$	0.8333	0.7073	0.4312
Q-stat(12)	6.3989	9.6003	14.868
LM(2)	1.4257	0.7927	3.2328
JB	1.9898	0.7021	1.2121
F-statistic	14.7517***	24.7609***	7.6571***
t-statistic (lagged DV)	-3.6370**	-6.9738***	-2.2291
t-statistic (lagged IDV)	5.3525***	3.6219***	-0.6717

Note: 75 in dummy columns denote dummy year 1975. *, **, *** denote significant at 10%, 5% and 1% level respectively based on the critical value generated from bootstrap.

Table 8: ARDL bounds test estimation using *IR* as dependent variable (Continued)

	New Zealand	Norway	Portugal
Dummy for Equation ΔIR_t	-	86	80, 89
Dummy for Equation ΔSR_t	91	86, 09	85, 96, 08
Period	1972-2012	1975-2012	1975-2012
Sample size (*n*)	41	38	38
$\bar{R}^2$	0.3846	0.4121	1
Q-stat(12)	15.687	5.5753	7.7885
LM(2)	1.2258	2.8832	0.9625
JB	0.6956	0.7897	2.5675
F-statistic	6.9562**	7.0545**	10.7534***
t-statistic (lagged DV)	-3.5296**	-3.4727**	-4.6347***
t-statistic (lagged IDV)	3.2084***	0.6384	2.8599**
	Spain	Turkey	UK
Dummy for Equation ΔIR_t	09	87, 09	76, 80, 88
Dummy for Equation ΔSR_t	81, 92, 95	87, 09	76
Period	1975-2012	1974-2012	1970-2012
Sample size (*n*)	38	39	43
$\bar{R}^2$	1	0.3938	0.5046
Q-stat(12)	5.9327	12.229	5.3859
LM(2)	1.5858	1.7747	0.8942
JB	0.7878	0.9863	3.7070
F-statistic	2.9910	5.3983**	9.4140***
t-statistic (lagged DV)	-2.3803	-2.9009**	-4.3391***
t-statistic (lagged IDV)	2.0793	1.2033	3.7250***

Note: 75 in dummy columns denote dummy year 1975. *, **, *** denote significant at 10%, 5% and 1% level respectively based on the critical value generated from bootstrap.

Table 9: Summary of cointegration status

Equation *IR*	Conclusion
Australia	Cointegration
Austria	No-cointegration
Finland	No-cointegration
Iceland	Cointegration
Israel	Degenerate #1
Japan	No-cointegration
Mexico	Cointegration
Netherland	No-cointegration
New Zealand	Cointegration
Norway	Degenerate #1
Portugal	Cointegration
South Korea	Cointegration
Spain	No-cointegration
Turkey	Degenerate #1
UK	Cointegration

6. Conclusion

The ARDL bounds testing approach has been misused by some researchers. As developed by PSS (2001), this approach to cointegration testing assumes that there is no feedback from the dependent variable to the regressors. However, in many cases it is unreasonable to assume any series in a given model is weakly exogenous. Therefore, in some applications of the ARDL bounds tests, each variable is treated as the dependent variable, sequentially, and regressed on the other variables. This is implicitly allows each variable to be endogenous, thereby violating the weak exogeneity condition of the bounds testing framework.

Another problem found in applications of the ARDL bounds test is a failure to consider and test for degenerate cases. PSS present the critical values suitable for testing the significance of the coefficient on the lagged level of the dependent variable, but some applications omit this test. In addition, another degenerate case is possible, in which the coefficients on the entire set of lagged level series are jointly significant, but this is solely due to the significance of the coefficient on the lagged level of the dependent variable. Since PSS do not provide critical values for a test of this degenerate case, this test is not found in applications of the ARDL bound testing procedure.

One of the objectives of this study is to evaluate the performances of the ARDL bounds test when the weakly exogenous regressors assumption is violated. Based on Monte Carlo simulation evidence presented here, it is found that the tests underlying the PSS ARDL bounds testing approach are not affected by the violation of this assumption.

However, the Monte Carlo simulations uncover some evidence of size distortions for these tests, especially for the t-test on the lagged dependent variable. As an alternative approach this study evaluates the ARDL test procedures based on bootstrap simulations. The Monte Carlo evidence indicates that bootstrap procedures resolve the size problems found with the PSS critical values, while also performing well in terms of power. In addition, when critical values are generated by bootstrap procedures, the possibility of inconclusive inferences from the ARDL bounds tests is reduced.

Moreover, the demonstration of the occurrence of degenerate cases is a further aspect of this study. Empirical application of the ARDL approach to savings-investment cointegration testing, shows that inferences based on the F and t-tests provided by PSS are not sufficient to uncover the degenerate case #1. The two tests of PSS are augmented by an additional bootstrap test on the significance of the lagged level(s) of the explanatory variable(s), to uncover the possibility of this degenerate case. Furthermore, this test has an additional advantage i.e. allowing one to infer the order of integration of the dependent variable. Monte Carlo simulations show that this new test has reasonable size and power properties, and the empirical applications of this test to the savings-investment relations of several countries demonstrate that this degenerate case #1 can arise in practical applications.

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Appendix

Analysis on Pure $I(0)$ Series

Our analysis extended beyond the original scope of the PSS ARDL bounds test to consider cases when the dependent variable is stationary. Although this case was not considered in the simulation of bounds for the ARDL test critical values by PSS, some researchers may want to apply ARDL cointegration testing in this environment. In addition to checking the robustness of the bootstrap test in this case, this experiment also reinforces the point that the bootstrap procedure provides exact critical values. Regardless of the orders of integration of the series involved, a single bootstrap critical value is determined (estimated) that is specific to the integration properties of the series. This establishes a further advantage of the bootstrap test over the asymptotic bounds test by eliminating the possibility of inclusive inferences.

The DGP for the series y_t and x_t as:

$$\varepsilon_t^y = \rho\varepsilon_t^x \tag{A1}$$

and followed by the autoregressive equations

$$y_t = \lambda_1^y y_{t-1} + \lambda_2^y y_{t-2} + \varepsilon_t^y$$

$$x_t = \lambda_1^x x_{t-1} + \lambda_2^x x_{t-2} + \varepsilon_t^x .$$

There are total 4 cases are considered in the analysis and the first 3 cases follow the simple DGP above. For the last case it considers feedback effect hence, its DGP replaces equation (A1) by the following equations

$$\varepsilon_t^y = -\alpha^y \left(\beta_1^y y_{t-1} - \beta_2^y x_{t-1}\right) + u_t^y \text{, and}$$

$$\varepsilon_t^x = u_t^x ,$$

where u_t^y and u_t^x are the independent innovations. Table A below shows the parameter combinations used in the DGP simulation and Table B is the results.

Table A: Parameter combinations used in the DGP simulation

Case	α^y	β_1^y	β_2^y	λ_1^x	λ_2^x	λ_1^y	λ_2^y	ρ	Integration order x_t	Integration order y_t
1	-	-	-	0.8	0	0.8	0	0	$I(0)$	$I(0)$
2	-	-	-	0.6	0.3	0.6	0.3	0	$I(0)$	$I(0)$
3	-	-	-	0.8	0	0.8	0	0.5	$I(0)$	$I(0)$
4	0.5	0	1	0.8	0	0.8	0	0	$I(0)$	$I(1)$

Table B: Size and power of the bootstrap tests

Case	F_x^*	F_y^*	t_x^{DV*}	t_y^{DV*}	t_x^{IDV*}	t_y^{IDV*}
1	**0.054**	**0.061**	**0.055**	**0.067**	**0.056**	**0.060**
2	**0.061**	**0.055**	**0.063**	**0.054**	**0.068**	**0.062**
3	**0.050**	**0.046**	**0.044**	**0.039**	**0.046**	**0.039**
4	**0.060**	0.799	**0.058**	**0.034**	**0.052**	0.903

Note: Simulations run at N = 2000 and B = 1000 replications with sample size of n = 50. Testing level α = 0.05. Number in bold refers to size property.

These cases were generated as pure $I(0)$ series for both y_t and x_t except Case 4 where series y_t is generated as $I(1)$ and x_t as $I(0)$. Table B presents evidence of reasonable size properties for the bootstrap procedure in these stationary environments. The smallest size is 0.04 and the largest is 0.07. It is also worth noting that the tests are applied with the exact critical values generated by the bootstrap; the tests are not bounds tests with possible indeterminate outcomes. Whether the regressors are $I(0)$, as in Table B, or $I(1)$, as in Tables 4 – 7, the bootstrap tests employ exact estimated critical values that will depend on the integration properties of the series in the model.

Endnotes

[1] PSS (2001) discusses this alternative degenerate case in line one, p.295, in which the differenced dependent variable depends on its own lagged level in a conditional ECM and not on the lagged level of independent variable(s). In this case the dependent variable is actually stationary. Hence, the degenerate case can be ruled out if the dependent variable is not stationary.

[2] PSS mentions that the bounds test approach is applicable regardless the integration order either $I(0)$ or $I(1)$ for regressors only, which does not include the dependent variable itself.

[3] The derivation details can be found in PSS, page 291.

[4] The notations for coefficient on $\mathbf{x}_{t-1}$ from Equation (2) and (4) are defined as $\boldsymbol{\pi}_{yx} \equiv \boldsymbol{\pi}_{y} - \boldsymbol{\omega}'\boldsymbol{\Pi}_{x}$ and $\boldsymbol{\pi}_{yx.x} \equiv \boldsymbol{\pi}_{yx} - \boldsymbol{\omega}'\boldsymbol{\Pi}_{xx}$ in PSS. Details see PSS.

[5] For example, Alhassan and Fiador, 2014; Garg and Dua, 2014; Jiang and Nieh (2012), Muscatelli and Spinelli (2000) and Getnet et al. (2005).

[6] It is common to use bivariate data to investigate a test statistic’s performances. See PSU, Kremer (1992) and Banerjee (1998).

[7] y_t is defined as the dependent variable for Equation Y, whereas x_t is defined as the dependent variable for Equation X.

[8] The bootstrap set up is general and suitable for the cases when the number of independent variables, k exceeds one. Those k independent variables follow the same setup as for Equation X.

[9] The alternative bootstraps results are not present here. These results are available from the authors upon request.