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Why Branded Firms May Benefit from Counterfeit Competition

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Abstract

A durable-good monopolist sells its branded product over two periods. In period 2, when there is entry of a counterfeiter, the branded firm may charge a high price to signal its quality. Counterfeit competition thus enables the branded firm to commit to a high price in period 2, alleviating the classic time-inconsistency problem under a durable-good monopoly. This can increase the branded firm's profit by encouraging consumer purchase without delay, despite the revenue loss to the counterfeiter. Total welfare can also increase, because early purchase eliminates delay cost and consumers enjoy the good for both periods.

JEL: D82, L11, L13

Keywords: Counterfeit, Durable Good, Quality Signaling

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1 Introduction

Counterfeits have become a fast growing multi-billion dollars business. In the 2007 OECD counterfeit report, the volume of counterfeits was around 200 billion dollars in international trade, 2% of world trade.¹ This figure does not include domestic consumption of counterfeits or digital products distributed via internet. The U.S. government estimated that counterfeit trade increased more than 17 fold in the past decade (U.S. Customs and Border Protection 2008).

Counterfeits are generally viewed as harmful to both the authentic producers and consumers, especially when they are deceptive, such as counterfeits of pharmaceutical products, eyeglasses, luxury goods or even normal textile products of famous brands.² However, some recent empirical evidence suggests that (deceptive) counterfeits could actually benefit the branded firm. In particular, Qian (2008) finds that the average profit for branded shoes in China is higher after counterfeit entry. Qian (2011) provides further evidence that the impact of counterfeits on profit depends on the quality gap between the authentic good and the counterfeit good; the branded firm benefits from counterfeits when the quality gap is sufficiently large. In this paper, I provide a theoretical explanation of why a branded firm can indeed benefit from competition of a deceptive counterfeiter when the quality difference of their

¹The Economic Impact of Counterfeiting and Piracy <http://www.oecd.org/industry/ind/38707619.pdf>

²This does not mean consumers cannot distinguish products at all. It is just hard for buyers to tell whether the good is authentic without any other information. For example, a consumer may not be able to separate a genuine Chanel bag from a fake one only by appearance. However, if one is priced at \$3,000 and the other is sold for \$50, she will know that the expensive one is more likely to be authentic ex post. On the other hand, non-deceptive counterfeits are those that consumers can easily recognize when purchasing, such as digital products.

products is large enough.

I consider a model with an authentic durable-good firm which sells in two periods. Without counterfeits, the branded durable-good monopolist faces the classic time-inconsistency problem (Coase, 1972): after selling to high-value consumers in the first period at a high price, it cannot resist cutting its price in the second period. But then rational consumers will delay their purchase, forcing the monopolist to reduce its price in the first period and lower the monopolist's overall profit. Now suppose that a counterfeiter will enter the market in the second period. In order to separate its product from counterfeits, the branded firm needs to set a high price to signal its quality. Thus the presence of counterfeits enables the branded firm to commit to a high price in period 2, providing a solution to the time-inconsistency problem. This then motivates more consumers to purchase in period 1 instead of waiting to buy in period 2, even if the first-period price is high. When the quality gap is sufficiently large, this “front-loading” effect will dominate the profit loss from competition in the second period. In terms of total welfare, counterfeits are likely to decrease surplus in the second period; however, first-period welfare increases due to front loaded purchases. Early purchases contribute twice the surplus compared to late purchases because consumers can use the good for two periods. Therefore, if the quality gap is not too large, it is possible for counterfeits to increase welfare.

The results in this paper shed light on the policy towards counterfeits. Both branded firms and consumers respond to counterfeits strategically. In the model, the authentic firm separates itself from the counterfeiter through high price when the quality gap is large enough. Therefore, consumers will not be fooled by counterfeits with extremely low quality. Moreover, knowing the later counterfeit entry, consumers are more inclined to purchase early, which benefits both the authentic firm and total welfare in a dynamic context.

The existing literature has investigated various strategies by the durable-good monopolist to resolve the commitment problem (see, e.g., Waldman, 2003 for an excellent survey). They include leasing rather than selling the durable good (Coase, 1972; Bulow, 1982), special contracts between the monopoly and consumers (Butz, 1990), offering an inferior version (Karp and Perloff, 1996; Hahn, 2006), and product-line management (Huhn and Padilla, 1996). All of these involve tactics that the monopoly adopts to alleviate the problem. The present paper suggests a novel commitment mechanism through the competition from another firm.

Several other papers have discussed the counter-intuitive result of price- or profit-increasing competition (e.g., Chen and Riordan, 2008; Gaibaix et al., 2005; Perloff et al., 2005; Thomadsen, 2007, 2012). In those papers, competition changes the demand curve of the incumbent firm. When the competitor attracts some price-elastic consumers, the incumbent can concentrate on price-inelastic consumers by charging a higher price. However, in my paper, quality signaling leads to the higher price. In addition, in these static models, competition generally will not increase a firm's profit even if prices go up, because a monopoly will always earn higher profit than a duopoly if the price is the same. However, in a dynamic model, price-increasing competition helps the monopoly to overcome the time-inconsistency problem and boosts profit.

There are other papers that discuss deceptive counterfeits. Grossman and Shapiro (1988a), for example, discuss the problem in international trade; they show that counterfeits will decrease the total welfare and the authentic firm's profit. Qian (2014) focuses on brand-protection strategies against counterfeits, including increasing price or upgrading quality, etc. She uses a vertical differentiation model similar to my modeling of second-period competition. The main difference is that I investigate the counterfeit problem in a dynamic context. This new feature yields opposite

results from hers: in her paper, the authentic firm's profit decreases with the threat of counterfeits. Also, total welfare also drops when the ratio of uninformed consumer is high. However, in the present paper, the branded firm's profit and total surplus might increase even if all consumers are uninformed.

Finally, the modeling of second-period counterfeit competition is related to the literature of duopoly signaling games. Hertzendorf and Overgaard (2001), Fluet and Garella (2002) and Yehezkel (2008) study similar games with advertising. These papers focus on the role of dissipative advertising in expanding the separating equilibrium regime while I try to answer how counterfeits influence profit and welfare. Like Qian (2014), these papers only investigate the static game while my paper incorporates the signaling game into a durable-good model.

The paper is organized as follows. Section 2 presents the model and reviews the monopoly benchmark. Section 3 investigates the effect of counterfeit competition on profit and welfare in a specific equilibrium. Section 4 shows that the main results continue to hold for other equilibria of the model under proper refinement. Section 5 concludes. All proofs are relegated to the Appendix.

2 The Model and the Monopoly Benchmark

I adapt the two-period durable-good model in Tirole (1988). A branded firm sells a durable good that can be used in two periods. The quality of its product Q_A is normalized to 1. In the second period, a counterfeiter producing a low-quality clone $Q_C = C < 1$ will enter and compete with the branded firm.³ Firms have no marginal cost to produce the good. Consumers know the quality of both products from the

³This implicitly assumes that the authentic product has a lead time advantage. Many firms have special designs on the new product so that imitators have to spend some time to learn and copy.

beginning of the game. However, they are not able to tell which good is produced by the branded firm from its appearance before their purchase.⁴ This contrasts with the standard assumption that consumers can trace the producer of the good.

There is a unit mass of heterogeneous consumer indexed by the taste parameter $\theta_l \sim U[0, 1]$. Consumer's utility has the linear function form:

$$U_l = \theta_l Q_i - p_i, i \in \{A, C\}, \text{ where } p_i \text{ is the price of firm } i$$

The discount factors of both firms and consumers are assumed to be 1.

Let $\mu_i(p_A, p_C)$ be the probability that consumers believe the good from firm i is the authentic good, given p_A and p_C . Unlike the traditional monopoly signaling model, there are two signal senders here. Consumer belief is based on price and the number of firms charging that price. Consumers are aware that two firms sell the good and one of them is the counterfeiter. Thus, $\mu_A(p_A, p_C) + \mu_C(p_A, p_C) = 1$ in equilibrium. In a pooling equilibrium, where $p_A = p_C$, consumers cannot separate two products and $\mu_A = \mu_C = \frac{1}{2}$. In a separating equilibrium, where $p_A \neq p_C$, consumers believe that the expensive good is authentic and the cheap one is counterfeit.

Given consumer's belief, the firm's profit is represented by

$$\Pi_{it}^k(p_A, p_C, \mu_i), t \in \{1, 2\}, k \in \{P, S\}$$

The subscript i, t stands for firm type and time respectively. We use the superscript k to denote equilibrium values in the second period (P for Pooling Equilibrium and S

⁴They are aware of the counterfeit quality in the first period. The assumption can be relaxed such that consumers only know the distribution of the counterfeit quality, which will not change our result qualitatively. The underling assumption is that counterfeits are deceptive and all consumers are uninformed. An alternative assumption is that part of consumers are informed. As long as the proportion of uninformed consumers are large enough, our qualitative conclusion will hold.

for Separating Equilibrium). Also, assume that the separating equilibrium is selected when profits are the same for a separating and a pooling equilibrium.

The time-line of the game is as follows: the authentic firm sets the first-period price p_1 in $t = 1$. Consumers decide whether to buy or wait. The counterfeiter enters in $t = 2$ and both firms set prices simultaneously. Then consumers observe both prices and make a purchasing decision based on their beliefs.

Before analyzing the game with counterfeit competition, let's first review the benchmark monopoly model without entry.⁵

(i) When the monopoly lacks commitment power, it has an incentive to decrease the price to reap the residual demand in $t = 2$. There is a marginal consumer θ_1 who is indifferent between buying in $t = 1$ and in $t = 2$. Therefore, the intertemporal incentive compatibility constraint for her is:

$$2\theta_1 - p_1 = \theta_1 - p_2$$

The right (left) hand side is her surplus from buying in $t = 1$ ($t = 2$), given that her expected second-period price is fulfilled in equilibrium ($E(p_2) = p_2$). In $t = 2$, the optimal price $p_2^M = \frac{1}{2}\theta_1$, combining with the intertemporal incentive compatibility constraint, the monopoly's aggregate profit can be written as:

$$\Pi = (2\theta_1 - \theta_1 + p_2)(1 - \theta_1) + p_2(\theta_1 - p_2)$$

The first (second) term is the profit from the first (second) period. Therefore, the marginal buyer and the monopoly's profit are $\theta_1^M = \frac{3}{5}$ and $\Pi^M = \frac{9}{20}$ respectively.

(ii) When the monopoly can commit to the same price, there will be no sale in $t = 2$ and $p_1 = 2\theta$. Henceforth, the monopoly's profit is as follows:

$$\Pi = 2\theta(1 - \theta)$$

⁵Since there is only one firm here, the subscript represents time and the superscript stands for the equilibrium value in monopoly case.

This gives a optimal profit $\Pi = \frac{1}{2}$ and $\theta_1 = \frac{1}{2}$. The profit in no commitment case is lower because of the standard time-inconsistency problem: high valuation consumers will anticipate the price reduction in the future and some of them postpone purchase to the second period.

3 Equilibrium Analysis With Counterfeit Competition

In this section, I will first characterize the set of Perfect Bayesian Equilibrium (PBE) under counterfeit competition. I then show that there exists an equilibrium at which the counterfeit can increase the authentic firm's profit and social welfare.⁶

Standard backward induction is applied to analyze the counterfeit game. As in the benchmark, there is a marginal consumer θ_1 , such that all consumers with taste parameter above θ_1 will purchase in the first period. The remaining consumers may purchase in the second period. θ_1 can be interpreted as the market size of the second period.

3.1 Signaling Game in Second Period

In $t=2$, there is a signaling game played between a pair of vertically differentiated firms and consumers. Consumers use market prices to update their beliefs. If both firms have the same price, counterfeits are indistinguishable ex post and a pooling equilibrium is sustained. If the counterfeiter sets a lower price than the branded firm and reveals itself, there will be a separating equilibrium where consumers know for

⁶In next section, I show all equilibria survive from the refinement have the desired result

sure which goods are counterfeits.⁷

In a pooling equilibrium, consumers are equally likely to pick a genuine product, leading the expected quality of the product to be $\frac{1+C}{2}$. The profit function is given by the following equation.

$$\Pi_{A2}(p_2, p_2, \frac{1}{2}) = \Pi_{C2}(p_2, p_2, \frac{1}{2}) = \frac{1}{2}(\theta_1 - \frac{2p_2}{1+C})p_2$$

In a separating equilibrium, profit functions of both firms are the same as under vertical price competition with complete information.

$$\begin{aligned}\Pi_{A2}(p_{A2}, p_{C2}, 1) &= (\theta_1 - \frac{p_{A2} - p_{C2}}{1-C})p_{A2} \\ \Pi_{C2}(p_{A2}, p_{C2}, 0) &= (\frac{p_{A2} - p_{C2}}{1-C} - \frac{p_{C2}}{C})p_{C2}\end{aligned}$$

The counterfeiter's best response function is always $p_{C2} = \frac{C}{2}p_{A2}$ in a separating equilibrium.

The key question is when a separating equilibrium can be sustained. In the standard monopoly signaling game, the separation is attained if the single-crossing condition is satisfied: the firm with high marginal cost is willing to distort price further than the low-cost firm because the profit depends only on its own price and consumer belief. However, in a duopoly case, a firm's profit is also affected by the other firm's price. When one sets a high price, the other one faces a trade off between favorable consumer belief and demand: if the counterfeiter decides to pool with the authentic firm, which tries to signal by pricing high, its product has 50% chance to be treated as authentic. However, the demand is low because of the uniform high

⁷Because of the asymmetric information, consumers only infer the quality of the firm from its price. Henceforth, there is another symmetric separating equilibrium where the counterfeiter charges a higher price than the branded firm. However, I will ignore that one since in this equilibrium all consumers are paying a higher price for the fake product, which is unrealistic in real life.

price in the market. Alternatively, the counterfeiter can reveal itself with a lower price, which may be better because the upward distorted price of the branded firm mitigates competition and leaves a large market for the counterfeiter. Two incentive compatibility constraints must be satisfied to support a separating equilibrium. The first equation assures that the counterfeiter does not deviate to the authentic price and the second one implies the branded firm wants to maintain the high price.

$$\Pi_{C2}(p_{A2}, p_{C2}, 0) \geq \Pi_{C2}(p_{A2}, p_{A2}, \frac{1}{2}) \quad (1)$$

$$\Pi_{A2}(p_{A2}, p_{C2}, 1) \geq \Pi_{A2}(p_{C2}, p_{C2}, \frac{1}{2}) \quad (2)$$

Lemma 1. (i) When the quality of the counterfeit is low ($C \leq C_1 \approx 0.604$), a set of separating equilibria can be sustained: $p_{A2}^S \in [\underline{p}_2(\theta_1, C), \overline{p}_2(\theta_1, C)]$; $p_{C2}^S = \frac{C}{2}p_{A2}^S$, where $\underline{p}_2(\theta_1, C) = \frac{2(1-C^2)}{C^2-3C+4}\theta_1$ and $\overline{p}_2(\theta_1, C) = \frac{(4-C)(1-C^2)}{2(2-C)(1+C)-C^2(1-C)}\theta_1$. (ii) For any quality C , there exists a set of pooling equilibria where both firms price at $p_2^P \in [0, \underline{p}_2(\theta_1, C))$.

All equilibria listed in Lemma 1 can be supported by a system of beliefs off the equilibrium path, such as the most pessimistic belief. For any separating equilibrium with $\tilde{p}_{A2} \in [\underline{p}_2, \overline{p}_2]$ and $\tilde{p}_{C2} = \frac{C}{2}p_{A2}$, if the out of equilibrium belief is that any deviating price $p' \neq \{\tilde{p}_{A2}, \tilde{p}_{C2}\}$ is conceived as a sign of counterfeits, then no firm would deviate and that particular separating equilibrium is stable. Similarly, the belief that $\mu(p', \tilde{p}_2) = 0, \forall p' \neq \tilde{p}_2$ can support all pooling equilibria.

The result is very intuitive: when the quality gap is large, the profit in a pooling equilibrium is low because of the low expected quality. The authentic firm just needs to slightly distort the price upward, which will reduce price competition and leave the counterfeiter enough profit under separating regime. For the branded firm, since price distortion is moderate, the cost of signaling is not too high. However, if two

products are close substitutes, the cost of signaling for the branded firm is so high that it would rather pool with the counterfeiter.

As in other signaling games, this model also has multiple equilibria. In some pooling equilibria with low price, counterfeit competition is detrimental to the branded firm's profit. In this section, I will show that there exists an equilibrium in which both the authentic firm and the society benefit from counterfeit entry under certain conditions. In the next section, it is proved that all equilibria surviving from the Competitive Intuitive Criterion refinement have similar properties.

The equilibrium I will focus on here is the one with the highest second-period profit for authentic firm, which is defined as the *profit-maximizing equilibrium*. It seems reasonable that consumers will believe that the authentic firm will choose the price that maximizes its second-period profit. Therefore, consumers believe the firm charging that price is the authentic firm. If both firms set that price, the good has 50% probability to be genuine. Any other price indicates a fake product. This is the pessimistic belief that supports the profit-maximizing price in $t=2$. Formally, consumer belief is defined as follow.

$$\begin{aligned}\mu_i(p_{A2}^*, p_{A2}^*) &= \frac{1}{2}; \quad \mu_A(p_{A2}^*, p_2) = 1, \forall p_2 \neq p_{A2}^*; \\ \mu_A(p_2, \cdot) &= \mu_C(\cdot, p_2) = 0, \forall p_2 \neq p_{A2}^*\end{aligned}$$

In this section, an extra asterisk is used in superscript to denote variables in the profit-maximizing equilibrium. Let p_{A2}^{S*} and p_2^{P*} be the authentic price in the optimal separating and pooling equilibrium respectively. $p_{A2}^* = \arg \max[\Pi_{A2}^{S*}, \Pi_2^{P*}] \in \{p_{A2}^{S*}, p_2^{P*}\}$ is the price that maximizes the branded firm's second-period profit, which is illustrated in the following lemma.

Lemma 2. In the profit-maximizing equilibrium: (i) if the counterfeit's quality is low enough ($C \leq C_3 \approx 0.512$), the separating equilibrium is supported as the PBE

of signaling game in $t=2$. $p_{A2}^* = p_{A2}^{S*} = \underline{p}_2(\theta_1, C)$, $\Pi_{A2}^* = \Pi_{A2}^{S*} = \frac{4(1-C)^2(1-C^2)}{C^2-3C+4}\theta_1^2$. (ii)

If the counterfeit's quality is high ($C > C_3$), the pooling equilibrium will be

selected. (a) For $C_3 < C \leq C_2 \approx 0.702$, $p_2^* = p_2^{P*} = \frac{1+C}{4}\theta_1$, $\Pi_2^* = \Pi_{A2}^{P*} = \frac{1+C}{16}\theta_1^2$; (b)

For $C > C_2$, $p_2^* = p_2^{P*} = \underline{p}_2(\theta_1, C)$, $\Pi_2^* = \Pi_{A2}^{P*} = \frac{C(1+C)(1-C^2)}{2(C^2-3C+4)^2}\theta_1^2$.

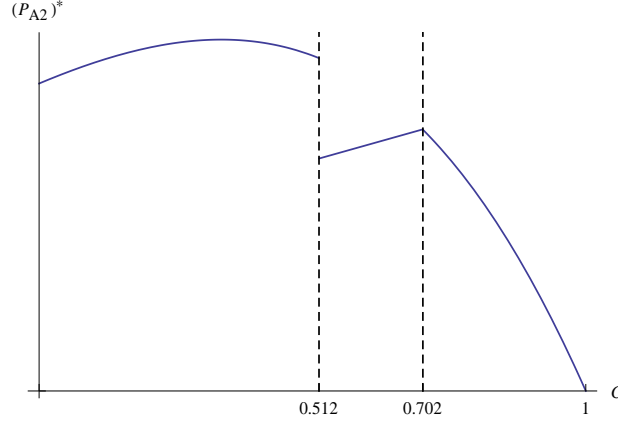
Figure 1 illustrates the second-period price scheme in the profit-maximizing equilibrium. For $C \in [0, C_3]$, the price $\underline{p}_2(\theta_1, C)$, which is the minimum price that prevents the counterfeiter from mimicking the branded firm, has an inverted-U shape with respect to C and is higher than the monopoly price in benchmark. The counterfeiter's profit in the pooling equilibrium increases faster with C than its profit in the separating equilibrium when C is close to 0.⁸ Therefore, the authentic firm is forced to increase the price in order to reduce competition and increase the competitor's profit in the separating equilibrium. As C gets larger, the condition will be reversed and the authentic firm has no need to incur a large distortion to support the separating equilibrium. Combining these two segments give us an inverted-U shape price in the separating equilibrium. When $C \in (C_3, C_2]$, the price increases with C because of higher expected quality. When C is close to 1, the game converges to Bertrand Competition of homogeneous good, and the price goes down to 0.

3.2 The Dynamic Game

In this subsection, I will analyze the dynamic game and illustrate why the entry of counterfeiter may generate higher profit for the incumbent. Given the second-period consumer surplus and the first-period price, the marginal buyer in the first period will be determined. The authentic firm's decision is to choose this marginal consumer to maximize total profit.

⁸When C is close to 0, $\frac{d\Pi_{C2}^P}{dC} = \frac{1}{(1+C)^2}p_{A2}^2 \geq \frac{d\Pi_{C2}^S}{dC} = \frac{1}{4(1-C)^2}p_{A2}^2$.

Figure 1: Equilibrium Price in t=2



Pooling Equilibrium

In the first segment of the pooling equilibrium ($C_3 < C \leq C_2$), consumer surplus in period 2 decreases because the market is flooded with counterfeits. This pushes more consumers to buy in the first period since the authentic good can be guaranteed. However, the market price is lower than the benchmark, which makes late purchase more attractive ($p_2^{P*} = \frac{1+C}{4}\theta_1 \leq \frac{1}{2}\theta_1$). Overall, consumer surplus falls below the benchmark case and the time-inconsistency problem is mitigated. We call this effect of making consumers buy early as the *Front-Loading Effect*. On the other hand, counterfeit competition will decrease the branded firm's revenue in the second period, which is the *Competition Effect*. The change of the authentic firm's profit is determined by the magnitude of these two effects.

The marginal consumer who purchases at t=1 in the pooling equilibrium is determined by the binding incentive compatibility constraint:

$$2\theta_1 - p_1 = \frac{1+C}{2}\theta_1 - p_2^{P*}$$

The authentic firm's maximization problem is:

$$\max_{\theta_1} \Pi_A^{P*}(\theta_1) = (1 - \theta_1)(2\theta_1 - \frac{1+C}{2}\theta_1 + p_2^{P*}) + \frac{1}{2}(\theta_1 - \frac{2p_2^{P*}}{1+C})p_2^{P*}$$

The marginal buyer θ_1^{P*} and equilibrium profit Π_A^{P*} are:

$$\theta_1^{P*} = \begin{cases} \frac{1 + \frac{3-C}{4}}{2(1 + \frac{11-5C}{16})} & C \in (C_3, C_2] \\ \frac{[\frac{3-C}{2} + \frac{2(1-C^2)}{C^2-3C+4}]}{2[\frac{3-C}{2} + \frac{(1-C^2)}{C^2-3C+4} + \frac{4(1-C^2)(1-C)}{(C^2-3C+4)^2}]} & C \in (C_2, 1) \end{cases}$$

$$\Pi_A^{P*} = \begin{cases} \frac{(1 + \frac{3-C}{4})^2}{4(1 + \frac{11-5C}{16})} & C \in (C_3, C_2] \\ \frac{[\frac{3-C}{2} + \frac{2(1-C^2)}{C^2-3C+4}]^2}{4[\frac{3-C}{2} + \frac{(1-C^2)}{C^2-3C+4} + \frac{4(1-C^2)(1-C)}{(C^2-3C+4)^2}]} & C \in (C_2, 1) \end{cases}$$

As Figure 2 shows, when $C \in (C_3, C_2]$, θ_1^{P*} increases with C for two reasons. Individual surplus in the second period increases as the counterfeit quality rises and more customers tend to wait, which decreases the wedge between p_1 and θ_1 . On the other hand, the branded firm balances the profit in each period to maximize total profit by properly choosing θ_1 . It is optimal to leave more customers in the second period (increase second-period market size) because the higher second-period profit increases with C .

When $C \in (C_2, 1)$, θ_1^{P*} first increases and then decreases in this range. When C gets close to 1, the front-loading effect disappears because p_2 is close to 0. The branded firm decreases the market size in period 2 due to fierce competition. It can be inferred that the incumbent does not benefit from counterfeit competition in this range.

Separating Equilibrium

In the separating equilibrium, the competition effect is not as strong as in the pooling equilibrium since the counterfeit quality is low. Also, as the high-quality

producer, the branded firm takes a larger share of the total profit compared to the head-to-head competition in the pooling equilibrium. The mechanism of the front-loading effect is slightly different. Consumers will not be fooled ex post but face a super monopoly price in the second period as Lemma 2 indicated. Now, the marginal buyer θ_1^P faces two options in the second period—buy the authentic good or the counterfeit.

$$2\theta_1 - p_1 = \max\{\theta_1 - \underline{p}_2(C, \theta_1), C\theta_1 - \frac{C}{2}\underline{p}_2(C, \theta_1)\}$$

However, the buyer who is indifferent between a genuine product and a counterfeit in the second period must be below θ_1 . Therefore, the outside option is purchasing the authentic good in $t=2$. The incumbent's profit maximization is as follow.

$$\max_{\theta_1} \Pi_A(\theta_1) = (1 - \theta_1)(\theta_1 + \underline{p}_2(C, \theta_1)) + \Pi_{A2}^{S*}(\theta_1)$$

In equilibrium,

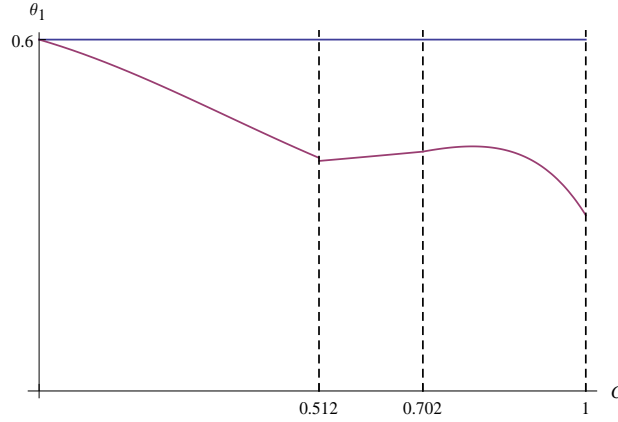
$$\begin{aligned} \theta_1^{S*} &= \frac{1 + \frac{2(1-C^2)}{C^2-3C+4}}{2[1 + \frac{2(1-C^2)(-C^2+C+2)}{(C^2-3C+4)^2}]} \\ \Pi_A^{S*} &= \frac{[1 + \frac{2(1-C^2)}{C^2-3C+4}]^2}{4[1 + \frac{2(1-C^2)(-C^2+C+2)}{(C^2-3C+4)^2}]} \end{aligned}$$

The left segment of lower curve in Figure 2 informs that θ_1^{S*} monotonically decreases with C . As the quality gap closes, the branded firm's profit in the second period decreases. It would be better to assign less weight in the second period by decreasing θ_1^{S*} .

Profit Comparison

Proposition 1. In the profit-maximizing equilibrium, the authentic firm's profit will be higher than the monopoly benchmark if the counterfeit quality is sufficiently

Figure 2: Marginal Buyer in t=1



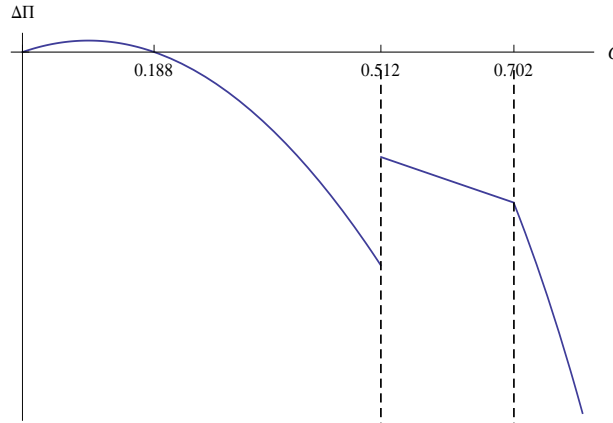
low ($C < C_4 \approx 0.188$). When the counterfeit quality is above that threshold, at any equilibrium in the second period, competition always decreases the incumbent's profit.

Figure 3 illustrates Proposition 1: when the pooling equilibrium emerges in the second period, the competition effect is too strong and always dominates the front-loading effect. The authentic firm suffers from the counterfeit entry. In the first segment of the pooling equilibrium, the front-loading effect gets weaker when the quality increases (θ_1^{P*} increases with C) and the time-inconsistency problem is reinforced. However, the high-quality counterfeit also weakens the competition effect and raises the second-period profit. In the second segment, the competition effects gets too strong and the front-loading effect disappears.

However, if the separating equilibrium is sustained, the branded firm's profit has an inverted-U shape and can be higher than the monopoly benchmark. When the counterfeit quality is 0, the result with counterfeit competition is the same as the monopoly benchmark. In the first period, since the high second-period price makes consumers less likely to wait, the front-loading effect will be stronger when $\underline{p}_2(\theta_1, C)$

is high. Recall that $\underline{p}_2(\theta_1, C)$ has an inverted-U shape, which implies the branded firm's profit will have the same curvature. On the other hand, the magnitude of negative competition effect monotonically increases with C . Therefore, when the quality of counterfeits is low, the combination of a strong front-loading effect and a weak competition effect raises the branded firm's profit above the benchmark. As C increases, this condition will be reversed and the incumbent's profit falls below the monopoly case.

Figure 3: Profit Difference



3.3 Welfare and Policy Implication

In terms of welfare, the conventional wisdom is that the deceptive counterfeit is harmful, because it fools consumers into buying the low-quality product at a relatively high price; this rationale led to trademark policy aiming to prevent consumer confusion. Grossman and Shapiro (1988a) show deceptive counterfeits decrease welfare with free entry in trade. However, this paper shows that the impact on welfare can be quite different in a dynamic context.

In the monopoly benchmark, total surplus is given by the following equation.

$$TS^M = \int_{\theta_1^M}^1 2\theta d\theta + \int_{\theta_2^M}^{\theta_1^M} \theta d\theta$$

The first (second) term represents the surplus created by first (second) period transaction⁹. The total surplus decreases with θ_1 , which is implied by the fact that early buyer enjoy double surplus. Given the marginal buyer in each period, $TS^M = 0.775$.

The welfare in the presence of deceptive counterfeit competition is a piecewise function.

$$TS(C) = \begin{cases} TS^{S*}(C) = \int_{\theta_1^{S*}}^1 2\theta d\theta + \int_{\bar{\theta}_2^{S*}}^{\theta_1^{S*}} \theta d\theta + \int_{\underline{\theta}_2^{S*}}^{\bar{\theta}_2^{S*}} C\theta d\theta & \text{if } C \leq C_3 \\ TS^{P*}(C) = \int_{\theta_1^{P*}}^1 2\theta d\theta + \int_{\theta_2^{P*}}^{\theta_1^{P*}} \frac{1+C}{2} \theta d\theta & \text{if } C > C_3 \end{cases}$$

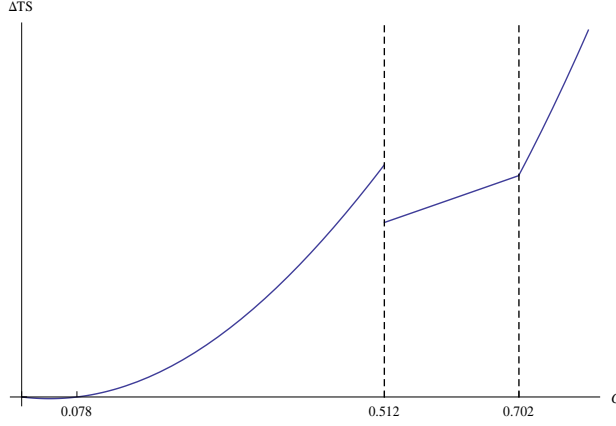
In the separating equilibrium, there are two marginal consumers in the second period. $\bar{\theta}_2^{S*}$ denotes the marginal consumer who is indifferent between the genuine good and the counterfeit. $\underline{\theta}_2^{S*}$ stands for the one who is indifferent between buying the counterfeit and buying nothing. Surplus is discounted by C if the counterfeit is purchased. In the pooling equilibrium, expected surplus is discounted by $\frac{1+C}{2}$ for all consumers because of confusion. Comparing welfare under two cases yields the next result.

Proposition 2. The entry of deceptive counterfeits increases total welfare if and only if the counterfeit quality is not too low ($C \geq C_5 \approx 0.078$).

Deceptive counterfeits have two effects on welfare. Firstly, the second-period surplus decreases because of competition with incomplete information, which is the

⁹Surplus is attributed to the trading period. First-period buyer enjoys surplus in both periods but the purchase is made in the first period, therefore all surplus belongs to the first period.

Figure 4: Welfare Difference



typical criticism against counterfeits. However, if the first-period welfare is taken into account, the result will be quite different. As Figure 2 shows, there are always more sales in $t = 1$ once $C > 0$. The front-loading effect pushes consumers to buy in $t = 1$ either because the higher price or lower expected quality in $t = 2$. The competition effect also forces the incumbent to reduce the market size in $t = 2$ by decreasing first-period price and expanding the market in $t = 1$. Consumers who purchase in the first period provide “double” contribution to surplus since they are guaranteed with high quality for two periods, which is the reason that total welfare could be higher under bad competition.

In Figure 4, the middle segment demonstrates the welfare difference under the pooling equilibrium with $C \in (C_3, C_2]$. The downward pressure on welfare decreases with C because consumer confusion problem is alleviated. Since θ_1^{P*} increases with C in this range, the positive effect also decreases with C . Overall, the social welfare is higher for all quality levels that sustain the pooling equilibrium in the second period. In the right segment of Figure 4, the second-period price decreases with C , which implies more trade and higher welfare.

The left segment is the welfare in the separating equilibrium. In Figure 2, as the counterfeit quality improves, the positive effect increases with C roughly at the same speed ($\frac{d^2\theta_1^{S*}}{dC^2}$ is close to 0). The second-period welfare decreases because of upward distorted prices. Since the second-period price has an inverted-U shape, the welfare in that period will be an U shape curve. Combining these two effects, it is clear why total welfare also has a U shape. When the counterfeit quality is 0, the model coincides with the benchmark. When C is small, unlike the pooling equilibrium, θ_1 is close to the benchmark value and decreases slower compared to the second-period welfare. Therefore, when the counterfeit quality is sufficiently low, the overall welfare effect is negative.

This proposition implies that deceptive counterfeits may have a positive effect on welfare in a dynamic context, which is contrary to the traditional argument. What is more surprising is that welfare is significantly higher when counterfeits are indistinguishable ex post. The result reminds us to think deeply about the counterfeit problem. Firstly, branded firms actively adopt strategies against clones. Although counterfeits are deceptive ex ante, whether they can be recognized ex post is endogenized. If the quality of clones is low, in which case consumer confusion induced by counterfeits has a strong negative effect on welfare, the authentic firm will signal by price and rational consumer will not be fooled. If consumers cannot distinguish counterfeits from authentic goods ex post, it must be that the quality gap is close enough. Even if consumers are diverted to counterfeits in that case, the welfare loss is relatively small. Secondly, consumers respond rationally to the problem. In the present paper, they are aware that surplus associated with future purchase is lowered by the counterfeit competition. Thus, more people buy earlier, which is beneficial for both the branded firm and welfare. However, as I point out, when the authentic firm decides to separate itself by distorted price, the counterfeiter

can also charge a higher price in the second period. This “price collusion” created by quality signaling might decrease welfare.

4 Equilibrium Refinement and Robustness

The profit-maximizing equilibrium discussed above is only one of equilibria in our model. In this section, the Intuitive Criterion (Cho and Kreps, 1987) is applied to refine equilibria. Since there are two signal senders here, I will use a competitive version as Bontems et al. (2005) and Yehezkel (2008). We will show that all pooling equilibria are eliminated with a tiny adjustment. The refinement is not applicable to separating equilibria because both firms’ prices are informative.¹⁰ However, it is proved that our general conclusion that counterfeit competition may increase the branded firm’s profit and social welfare holds in all separating equilibria.

In previous discussion, both firms are assumed to have zero marginal cost. Now, let the authentic firm has a slightly higher marginal cost $\epsilon > 0$ which is arbitrarily close to 0. This is just a tie-breaker that helps us to eliminate all pooling equilibria. By continuity of all functions in the paper, this modification will not alter any of our results except for the existence of pooling equilibria. For convenience, I only explicitly state this adjustment in the refinement.

¹⁰The Intuitive Criterion requires unilateral deviation. However, since the other firm charges the equilibrium price, consumers can use that information to construct the out of equilibrium belief. Therefore, I cannot simply assume a belief towards the deviating firm while the other one prices at the equilibrium path. Bester and Demuth (2011), Bontems et.al (2006) and Hertzendorf and Overgaard (2001) have discussed this issue.

4.1 Equilibrium Refinement

Pooling Equilibrium

The basic logic of the Intuitive Criterion is equilibrium dominance: an equilibrium should be eliminated if there exists an out-of-equilibrium price such that given consumer's most favorite belief, one type of firm would be better off by deviating from the equilibrium price to that out-of-equilibrium price, while the other type of firm cannot benefit from such deviation.

In terms of pooling equilibria, the Competitive Intuitive Criterion requires that there is no p' , such that

$$\Pi_{A2}(p', p_2^P, 1) \geq \Pi_{A2}(p_2^P, p_2^P, \frac{1}{2}) \quad (3)$$

$$\Pi_{C2}(p_2^P, p', 1) < \Pi_{C2}(p_2^P, p_2^P, \frac{1}{2}) \quad (4)$$

However, for every pooling equilibrium, there must exist a p' such that both equations hold, which means all pooling equilibria are eliminated. The reason is similar to the refinement in the monopoly signaling game: The authentic firm with a higher marginal cost ϵ , no matter how small it is, has a lower cost to signal its quality. Since the profit function satisfies single-crossing property, I can always find an upward distorted price such that the authentic firm is willing to deviate to that price if consumers believe its high quality, while the counterfeiter is not willing to deviate even if people believe it produces genuine products at that price. The detail can be found in Proof of Proposition 3.

Separating Equilibrium

Since the Intuitive Criterion cannot be applied to separating equilibria, Herten-dorf and Overgaard (2001) and Yehezkel (2008) use a stronger refinement named

Resistance to Equilibrium Defections (REDE) to select the unique and most intuitive separating equilibrium in the duopoly signaling game, which is similar to the unprejudiced equilibrium in Bagwell and Ramey (1991).¹¹ Only the least distorted equilibrium survives that refinement, which is the profit-maximizing equilibrium investigated in the previous section. However, we don't need to impose that extra refinement since our main results hold in all separating equilibria, which is proved in next subsection.

4.2 Robustness of Results

We have shown that separating equilibria survive the refinement and pooling equilibria are eliminated. The question is whether our main conclusions regarding incumbent's profit and social welfare still hold in other separating equilibria. Let's first investigate the incumbent's profit in all separating equilibria. In the previous section, the profit-maximizing equilibrium is discussed in detail, which is the one with lowest second-period price among all separating equilibria. For any C , it can be proved that the branded firm's profit increases with the second-period price among equilibria because the front-loading effect grows faster than the competition effect. Since the branded firm can benefit from counterfeits under the equilibrium with lowest second-period price, the result will hold under all other equilibria. Therefore, if

¹¹Basically, REDE assumes that consumers can still make reasonable inductions from the equilibrium behavior of one sender even if they see out of equilibrium behavior from the other sender. Mathematically, if consumers observe that one good is sold at a price $\tilde{p} \in [\underline{p}_2(C, \theta_1), \overline{p}_2(C, \theta_1)]$ and the other one is priced at $p \in [0, \underline{p}_2(C, \theta_1))$ but $p \neq \frac{C}{2}\tilde{p}$, then they will believe the one with \tilde{p} is genuine and the other one is counterfeit. This gives the authentic firm an incentive to unilaterally deviate to the price that will maximize its profit within the separating equilibrium range. The counterfeiter never deviates because any deviation cannot fool consumers.

the counterfeit quality is below C_4 , the authentic firm's profit is always higher with the presence of counterfeits, no matter which separating equilibrium emerges in the second period.

In terms of the impact on welfare, there is not such a nice monotonicity property among equilibria because the welfare in $t = 2$ may be too low when the price is high in that period. However, it is verified that if C is higher than a threshold, the social welfare is higher with counterfeit competition in all equilibria. The economic intuition is the same as the last section. All equilibria have higher second price (more distortion) than the one that maximizes second-period profit. Thus, the incumbent's second-period profit in other equilibria is lower than that one. When the branded firm maximizes the profit, it tends to reduce the weight on the second period (lower θ_1). Therefore, more consumers purchase in the first period and the welfare increases.

Proposition 3. All pooling equilibria are eliminated by the Competitive Intuitive Criterion. In every separating equilibrium, when $C \leq C_4$, the authentic firm's profit is higher with counterfeit competition. When $C \geq C_6 \approx 0.248$, the social welfare is higher in the presence of counterfeits.

4.3 Provision of a Damaged Good by the Branded Firm

This paper points out that the entry of a low-quality competitor can actually benefit the incumbent. The downside to the brand is that it takes away part of the revenue. An interesting question is whether the branded firm can overcome the competition effect by offering an inferior version itself and earn higher profit? We do observe many examples of damaged goods. Armani has a premium ready-to-wear line marketed as Giorgio Armani, relatively cheaper lines as Armani Collezioni and Emporio Armani, as well as lines distributed in shopping malls like Armani Jeans and Armani

Exchange.

Firstly, the incumbent has no incentive to provide an inferior good in the second period. Deneckere and MacAfee (1996) points out the linear utility function fails the condition that damaged goods help to raise profit. In my model, no matter what inferior quality the branded firm chooses, the optimal decision is to sell zero damaged version in $t=2$. The second-period price and profit are the same as monopoly benchmark. Since the price is not higher than the monopoly price, the front-loading effect does not exist. Therefore, the total profit can never be higher than the benchmark. If there is any fixed cost associate with product line introduction, the profit is always lower than the monopoly case.

Secondly, damaged goods introduced in the first period is not profitable as well. Hahn (2006) discusses the benefit of introducing damaged goods in durable-good model. In his paper, part of high (low) type consumers buy a high (low) quality good in each period, which changes the ratio of consumer type. Since some low types have purchased damaged goods in earlier period, the firm has less incentive to decrease price sharply later, which relaxes the competition between two versions and alleviates the time-inconsistency problem. However, with continuous consumer type, it can be proved that if anyone buys a damaged good in the first period, then all consumers with higher θ must purchase (a damaged or premium version) in that period as well. Therefore, introducing damaged version in $t = 1$ only makes some higher type consumers who would purchase the premium version select damaged version, which decreases profit for sure.¹² The mechanism that helps to solve Coase Conjecture in Hahn (2006) disappears in my model and the firm would rather just offer the original version.

¹²The detailed proof is available upon request

5 Conclusion

While the conventional wisdom is that deceptive counterfeits are always harmful for the authentic firm and social welfare, this paper argues that the opposite can hold in durable-good markets. Despite the business-stealing effect, deceptive counterfeits mitigate the time-inconsistency problem for the incumbent. It is demonstrated that the effect of counterfeits crucially depends on their quality. When the quality gap is sufficiently small, pooling equilibria are sustained in the second period. The front-loading effect cannot cover the loss from the competition and the authentic firm's profit decreases with counterfeit entry. However, if the quality gap is sufficiently large, the low-quality counterfeiter only incurs a mild competition which is dominated by increased sales in the first period, and the branded firm benefits from counterfeit competition in this case. Moreover, the incumbent cannot earn higher profit by offering a damaged good because then the front-loading effect then disappears. In terms of welfare, contrary to traditional arguments, it is shown that in a large quality range, the deceptive counterfeiter is actually beneficial to the society due to more earlier purchases. Surprisingly, if counterfeits remain indistinguishable ex-post, total surplus unambiguously increases.

There are several directions for future research. For instance, I only investigate the short-term effects of counterfeits. An interesting question is how the counterfeit entry affects the incumbent's incentive to innovate. Given that counterfeits may increase the branded firm's profit, there is a possibility that they promote innovation as well. Another extension is to endogenize the counterfeit entry by explicitly modeling public policies that affect its entry cost. Moreover, famous brands face many counterfeiters with different qualities in reality. It would be interesting to study how counterfeiters compete with each other and their impact on the branded firm.

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A Appendix

Proof of Lemma 1 To sustain a separating equilibrium, the incentive compatibility constraint for the counterfeiter requires that:

$$\begin{aligned} \left(\frac{p_{A2} - \frac{C}{2}p_{A2}}{1 - C} - \frac{\frac{C}{2}p_{A2}}{C} \right) \frac{C}{2}p_{A2} &\geq \frac{1}{2} \left(\theta_1 - \frac{2p_{A2}}{1 + C} \right) p_{A2} \\ p_{A2} &\geq \frac{2(1 - C^2)}{C^2 - 3C + 4} \theta_1 = \underline{p}_2(\theta_1, C) \end{aligned}$$

This equation is derived from Eq(1) by plugging the best response function of the counterfeiter. Similarly, the incentive compatibility constraint for the authentic firm requires that:

$$\begin{aligned} (\theta_1 - \frac{p_{A2} - \frac{C}{2}p_{A2}}{1-C})p_{A2} &\geq \frac{1}{2}(\theta_1 - \frac{C}{1+C}p_{A2})\frac{C}{2}p_{A2} \\ p_{A2} &\leq \frac{(4-C)(1-C^2)}{2(2-C)(1+C) - C^2(1-C)}\theta_1 = \bar{p}_2(\theta_1, C) \end{aligned}$$

Therefore, when $\bar{p}_2(\theta_1, C) \geq \underline{p}_2(\theta_1, C)$, a separating equilibrium exists. Otherwise, only pooling equilibria can be supported.

$$\frac{(4-C)(1-C^2)}{2(2-C)(1+C) - C^2(1-C)}\theta_1 \geq \frac{2(1-C^2)}{C^2 - 3C + 4}\theta_1$$

This implies that when $C \leq C_1 \approx 0.604$, a separating equilibrium can be supported.

For pooling equilibria, as long as Eq(1) is violated, the counterfeiter is willing to pool with the authentic firm. Therefore, $\forall C$, if $p_{A2}^P = p_{C2}^P = p_2^P \in [0, \underline{p}_2(C, \theta_1))$, a pooling equilibrium can be sustained by certain out of equilibrium beliefs. *Q.E.D.*

Proof of Lemma 2 In separating equilibria, it can be easily shown that all authentic prices are higher than the unconstrained optimal price. Since the profit function is a concave parabola, $p_{A2}^{S*} = \underline{p}_2(\theta_1, C)$. In any separating equilibrium, the branded firm's profit decreases with the counterfeit quality in the second period because of intensified competition.

In pooling equilibria, when $C < C_2 \approx 0.702$, the unconstrained optimal price is always less than $\underline{p}_2(\theta_1, C)$. Therefore, the optimal price is the unconstrained optimal, $p_2^{P*} = \frac{1+C}{4}\theta_1$. Fixing the market size, the authentic firm's profit increases with C within this range. That is because consumer confusion is alleviated, which enables the firm to raise the price. However, when $C > C_2$, the quality gap is small and the

competition is intense. The unconstrained optimal is higher than $\underline{p}_2(\theta_1, C)$. Since the profit function is a concave parabola as well, $p_2^{P*} = \underline{p}_2(\theta_1, C)$.

As Lemma 1 indicates, when $C \leq C_1$, both types of equilibria exist and $\Pi_{A2}^* = \text{Max} \{ \Pi_{A2}^{S*}, \Pi_{A2}^{P*} \}$. Given $C_1 < C_2$, the price of the optimal pooling equilibrium is $p_2^{P*} = \frac{1+C}{4}\theta_1$. Since $\frac{d\Pi_{A2}^{S*}}{dC} < 0$ and $\frac{d\Pi_{A2}^{P*}}{dC} > 0$, there is a cut-off quality $C_3 \approx 0.512$ such that the optimal separating equilibrium is chosen if $C \leq C_3$ and the pooling equilibrium would be selected for $C_3 < C \leq C_1$. When the counterfeit quality is low, the profit in a separating equilibrium is high because of moderate distortion while the profit in a pooling equilibrium is low due to low expected quality. As the quality of the fake good increases, the profit associates with the pooling equilibrium increases. For $C > C_1$, separating equilibria cannot exist and the only candidate is pooling equilibria. When $C_1 < C \leq C_2$, the equilibrium price is the unconstrained optimal price. If $C > C_2$, the price is the binding price $\underline{p}_2(C, \theta_1)$. *Q.E.D*

Proof of Proposition 1

(1) When $C > C_3$, the pooling equilibrium is sustained. There are two second-period prices given different C , both of which can be written as $p_2^{P*} = F(C)\theta_1^{P*}$. Firstly, I will prove $\forall C \in (C_3, 1)$, $\frac{\partial \Pi_A^{P*}}{\partial F(C)} > 0$.

$$\frac{\partial \Pi_A^{P*}}{\partial F(C)} = \frac{(\frac{3-C}{2} + F(C))[(\frac{1}{2} - \frac{F(C)}{1+C}) + \frac{F(C)}{2}]}{4(\frac{3-C}{2} + \frac{F(C)}{2} + \frac{F(C)^2}{1+C})^2}$$

Since $0 < F(C) \leq \frac{1+C}{4}$, $\frac{\partial \Pi_A^{P*}}{\partial F(C)} > 0$.

Given this property, it can be shown that even with the larger second-period price $p_2^{P*} = \frac{1+C}{4}\theta_1, \forall C \in (C_3, 1)$, counterfeit competition in the pooling equilibrium still cannot increase the branded firm's profit.

If $p_2^{P*} = \frac{1+C}{4}$,

$$\frac{d\Pi_A^{P*}}{dC} = \frac{(1 + \frac{3-C}{4})(-2 + \frac{35C-53}{64})}{(1 + \frac{11-5C}{16})^2} < 0, \forall C \in (C_3, 1)$$

Therefore, $\forall C \geq C_3, \Pi_A^{P*}(C) \leq \Pi_A^{P*}(C_3)$. Since $\Pi_A^{P*}(C_3) < \Pi^M$, we have $\Pi_A^{P*}(C) < \Pi^M, \forall C \geq C_3$.

(2) When $C \leq C_3$, the separating equilibrium is supported in the second period. If $C = 0$, the model is degenerated to the monopoly benchmark. $\Pi^M = \Pi_A^{S*}$.

Let $\Delta\Pi(C) = \Pi_A^{S*} - \Pi^M$, then $\frac{d\Delta\Pi(C)}{dC}|_{C=0} = 0.045 > 0$. So there must exist some C that is low enough such that the authentic firm's profit would increase under the competition.

On the other hand, the only root $C_4 \in (0, 1]$ of $\Delta\Pi(C) = 0$ is $C_4 \approx 0.188$. Henceforth, $\Delta\Pi(C) \geq 0$ if $C \leq C_4$ and vice versa. *Q.E.D.*

Proof of Proposition 2.

(1) In the pooling equilibrium, similar to proof of proposition 1, I can write $p_2^{P*} = F(C)\theta_1^{P*}$. Firstly, I will show that $\frac{\partial TS^{P*}(C)}{\partial F(C)} < 0$ for any $C > C_2$.

$$TS^{P*}(C) = (1 - (\theta_1^P)^2) - \frac{1+C}{4}[1 - \frac{4}{(1+C)^2}F(C)^2]$$

$$\frac{\partial TS^{P*}(C)}{\partial F(C)} = 2(\theta_1^{P*})^2[\frac{\partial \theta_1^{P*}(C)}{\partial F(C)}(\frac{1+C}{4} - \frac{F(C)^2}{1+C} - 1) - \frac{F(C)}{1+C}\theta_1^{P*}]$$

Plugging in θ_1^{P*} and $\frac{\partial \theta_1^{P*}(C)}{\partial F(C)}$, it is easy to verify that $\frac{\partial TS^{P*}(C)}{\partial F(C)} < 0$.

Given this property, it is proved that with the larger second-period price $p_2^{P*} = \frac{1+C}{4}\theta_1, \forall C \in (C_3, 1)$, counterfeit competition in pooling equilibrium still increases the

total welfare. When $p_2^{P*} = \frac{1+C}{4}\theta_1$, $\theta_2^{P*} = \frac{1}{2}\theta_1^{P*}$.

$$\begin{aligned}
TS^{P*}(C) &= \int_{\theta_1^{P*}}^1 2\theta d\theta + \int_{\theta_2^{P*}}^{\theta_1^{P*}} \frac{1+C}{2}\theta d\theta \\
&= 1 - (\theta_1^{P*})^2 \left(1 - \frac{3(1+C)}{16}\right) \\
\Delta TS^{P*}(C) &= TS^{P*}(C) - TS^M(C) = \frac{5}{8}(\theta_1^M)^2 - (\theta_1^{P*})^2 \left(1 - \frac{3(1+C)}{16}\right) \\
\frac{d\Delta TS^{P*}(C)}{dC} &= \frac{8(1+C)}{(27-5C)^3} > 0
\end{aligned}$$

Therefore, $\Delta TS^{P*}(C) \geq \Delta TS^{P*}(C_3)$, $\forall C > C_3$. Since, $\Delta TS^{P*}(C_3) > 0$, deceptive counterfeits always yield a higher welfare under the pooling equilibrium.

(2) In the separating equilibrium,

$$\bar{\theta}_2^{S*} = \frac{2-C}{2(1-C)}\underline{p}_2(C, \theta_1), \theta_2^{S*} = \frac{1}{2}\underline{p}_2(C, \theta_1)$$

$$\begin{aligned}
TS^{S*}(C) &= \int_{\theta_1^{S*}}^1 2\theta d\theta + \int_{\frac{(2-C)(1+C)}{C^2-3C+4}\theta_1^{S*}}^{\theta_1^{S*}} \theta d\theta + \int_{\frac{1-C^2}{C^2-3C+4}\theta_1^{S*}}^{\frac{(2-C)(1+C)}{C^2-3C+4}\theta_1^{S*}} C\theta d\theta \\
&= 1 - \frac{1}{2}(\theta_1^{S*})^2 \left[1 + \frac{(1+C)^2(4-3C)(1-C)}{(C^2-3C+4)^2}\right]
\end{aligned}$$

Since $\frac{d\Delta TS^{S*}(C)}{dC}|_{C=0} < 0$, if C is sufficiently low, $TS^{S*}(C) < TS^M(C)$. Moreover, there is only one root $C_5 \in (0, 1]$ such that $\Delta TS^{S*}(C) = 0$. Therefore, $\forall C \leq C_5$, $TS^{S*}(C) \leq TS^M(C)$ and vice versa. Henceforth, the counterfeit entry increases total welfare if its quality $C \geq C_5 \approx 0.078$. *Q.E.D.*

Proof of Proposition 3.

(i) The elimination of all pooling equilibria.

Firstly, $\forall p \in [0, \underline{p}_2(C, \theta_1))$, there exists a $p < p' < p + (1-C)\theta_1$, such that (3) is binding. Choosing a δ that is arbitrarily close to 0. Then $\Pi_{A2}(p + \delta, p, 1) >$

$\Pi_{A2}(p, p, \frac{1}{2})$ and $\Pi_{A2}(p + (1 - C)\theta_1, p, 1) = 0 < \Pi_{A2}(p, p, \frac{1}{2})$. Therefore, by the continuity of profit function, there must exist a $p < p' < p + (1 - C)\theta_1$ that makes $\Pi_{A2}(p', p, 1) = \Pi_{A2}(p, p, \frac{1}{2})$.

Plug p' and Eq(3) into Eq(4),

$$\begin{aligned} & \Pi_{C2}(p, p', 1) - \Pi_{C2}(p, p, \frac{1}{2}) \\ &= (\theta_1 - \frac{p' - p}{1 - C})p' - \frac{1}{2}(\theta_1 - \frac{2p}{1 + C})p \\ &= \epsilon(\theta_1 - \frac{p' - p}{1 - C})(\frac{p - p'}{p - \epsilon}) < 0 \end{aligned}$$

Hence, for every pooling equilibrium, there is a price p' that the authentic firm wants to deviate and the counterfeiter does not given consumer's best belief.

Now let's make some preliminary definition for separating equilibria

$$\underline{p}_2(C, \theta_1) = \frac{2(1 - C^2)}{C^2 - 3C + 4}\theta_1, \bar{p}_2(C, \theta_1) = \frac{(4 - C)(1 - C^2)}{2(2 - C)(1 + C) - C^2(1 - C)}\theta_1$$

For convenience, let

$$\underline{K}(C) = \frac{2(1 - C^2)}{C^2 - 3C + 4}, \bar{K}(C) = \frac{(4 - C)(1 - C^2)}{2(2 - C)(1 + C) - C^2(1 - C)}$$

In any separating equilibrium, the authentic firm's price is between $\underline{K}(C)\theta_1$ and $\bar{K}(C)\theta_1$.

(ii) For incumbent's profit:

$$\begin{aligned} \Pi_A^S &= \frac{1}{4} \frac{[1 + K(C)]^2}{[1 + \frac{2-C}{2(1-C)}K(C)^2]} \\ \frac{\partial \Pi_A^S}{\partial K(C)} &= \frac{[1 + K(C)][1 - \frac{2-C}{2(1-C)}K(C)]}{2(1 + \frac{2-C}{2(1-C)}K(C)^2)^2} \end{aligned}$$

Since $1 - \frac{2-C}{2(1-C)}K(C) \geq 1 - \frac{2-C}{2(1-C)}\bar{K}(C) > 0$, $\frac{\partial \Pi_A^S}{\partial K(C)} > 0$. The profit-maximizing equilibrium is the one that yields lowest total profit for the incumbent. In that equilibrium, when $C \leq C_4$, the profit with counterfeit entry is higher. Therefore no matter which separating equilibrium is sustained in the second period, $\Delta \Pi_A^S(C) \geq 0$ if $C \leq C_4$.

(iii) For total welfare:

$$\Delta TS^S(C, K(C)) = 0.225 - \frac{1}{8} \frac{[1 + K(C)]^2 [1 + \frac{4-3C}{4-4C} K(C)^2]}{[1 + \frac{2-C}{2-2C} K(C)^2]^2}$$

When $C = 0$,

$$\Delta TS^S(0, K(0)) = 0.225 - \frac{1}{8} \frac{[1 + K(0)]^2}{1 + K(0)^2}$$

Since $\frac{[1+K(0)]^2}{1+K(0)^2}$ increases with $K(0)$,

$$\Delta TS^S(0, K(0)) \leq 0.225 - \frac{1}{8} \frac{[1 + \underline{K}(0)]^2}{1 + \underline{K}(0)^2} = 0$$

When $C = C_1, \forall K(C_1)$,

$$\begin{aligned} \Delta TS^S(C_1, K(C_1)) &= 0.225 - \frac{1}{8} \frac{[1 + K(C_1)]^2 [1 + \frac{4-3C_1}{4-4C_1} K(C_1)^2]}{[1 + \frac{2-C_1}{2-2C_1} K(C_1)^2]^2} \\ &> 0.225 - \frac{1}{8} \frac{[1 + K(C_1)]^2}{[1 + \frac{2-C_1}{2-2C_1} K(C_1)^2]} \\ &\geq 0.225 - \frac{1}{8} \frac{[1 + \bar{K}(C_1)]^2}{[1 + \frac{2-C_1}{2-2C_1} \bar{K}(C_1)^2]} > 0 \end{aligned}$$

Therefore, when $C = 0$, the welfare differences under all separating equilibria are negative. However, when $C = C_1$, the welfare differences under all separating equilibria are strictly positive. By continuity of the welfare difference function, there must exist a threshold C_6 such that as long as $C \geq C_6$, the welfare is higher in presence of deceptive counterfeits for any separating equilibrium. Numerically, I find that $C_6 \approx 0.248$. *Q.E.D.*