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Smoking Bans as a Response to Constrained Tax Policy

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SMOKING BANS AS A RESPONSE TO CONSTRAINED TAX POLICY

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ABSTRACT

This paper presents a model which explains why governments, wishing to reduce smoking, may ban smoking in the workplace in addition to using high taxes. Individuals wish to smoke evenly and hence dislike variance in cigarette consumption. The government has two possible policies - increasing the price (imposing a tax) or limiting when the cigarettes can be consumed (imposing a ban on smoking in the workplace). The effectiveness of the tax policy is limited because the smoker can buy illegal but untaxed cigarettes on the "black market." A ban is costly as it creates variance in cigarette consumption which the smoker dislikes. We show that the optimal policy is a combination policy of "tax plus ban."

Key words: smoking, cigarette, ban, government policy

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JEL Classification: D11, H26, H75, I18

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1. INTRODUCTION

Governments - local communities, states/provinces and national governments - use many policies to discourage smoking, such as levying a high tax on cigarettes, banning smoking in the workplace and in public spaces, requiring health warnings on tobacco packages, limiting advertizing and sponsorship of sporting events by tobacco companies, and prohibiting store displays. There are many possible reasons for these policies, but certainly one reason is the ill-health smoking causes. This paper explains why a government, focusing on the ill-health caused by smoking, chooses a combination policy of levying high taxes and banning smoking in the workplace.

Cigarettes differ to most other products because they cause long-term health problems and because they are addictive. In a model of perfect foresight (e.g. Becker and Murphy (1988)), the smoker perfectly foresees the ill-health consequent on his smoking and the addictive nature of cigarettes. In this structure and in the absence of any issues associated with “second-hand” smoke, there is no market failure and the First Fundamental Welfare Theorem suggests that government policy should be limited to raising revenue and redistribution. This paper assumes a different viewpoint in which smokers incorrectly perceive the long-term health costs of smoking and government policy seeks to reduce the smoking caused by this misperception.

Concerning addiction: when a cigarette is smoked, a shot of nicotine enters into the blood and travels to the brain, providing stimulation to the smoker. However, the nicotine quickly metabolizes into cotinine, and the nicotine in the blood decreases. This decrease produces a craving; the smoker has a strong discomfort until another cigarette is lit up to restore the nicotine to its pre-existing level. Overall, the smoker seeks to maintain a steady flow of nicotine to the brain. Traditional studies of addiction stress long-term addiction in which, once a smoker has established a steady level of nicotine to the brain, it is very hard to lower this level and to quit. But there is also short-term addiction in which the smoker finds it very hard to lower his consumption during a period of the day. We focus on the individual's wish to maintain a steady level of nicotine within the day. Our modeling innovation is to model this wish by a utility function of mean-variance form, with utility depending positively on the mean of daily cigarettes and negatively on the variance of daily cigarettes. By prohibiting smoking while the smoker is at work, a ban on smoking in the workplace creates variance. If the smoker tries to replace the cigarettes he previously smoked in the workplace by smoking additional cigarettes at home, in the morning before work and in the evening after work, the variance makes such substitution costly. In consequence substitution is incomplete and the ban lowers the number of cigarettes he smokes. Put differently the ban causes cigarettes smoked at home to be an imperfect substitute for cigarettes previously smoked at work - as in the model of Jones et al. (2011).

Concerning policy. Economists have long been interested in situations in which households make inefficient choices. A well-researched example is pollution in which an individual considers only her own benefit from her action and ignores the consequences of her action on others. An important policy issue (e.g. Weitzman (1974), Hepburn (2006)) is whether the individual's action is better controlled indirectly by changing the price (implemented by imposing a tax) or directly by limiting the quantity of the activity. In our model the government can use a price policy by levying a tax on cigarettes. However, it is unable to eliminate smoking by increasing the tax on cigarettes to punitive levels because there is a black-market for cigarettes.[1] Alternatively the government can use a quantity policy which, in our model, is a ban on smoking in the workplace. Like the tax policy, the ban on smoking in the workplace cannot eliminate smoking: the smoker still smokes cigarettes at home. As discussed earlier, the ban is costly in the sense that it creates variance for the smoker which lowers his utility. Although imposing a ban is costly, we show that a ban is a component of the optimal policy or that the best policy is a combination policy of "tax plus ban."

In our model the government is paternalistic. It wants to limit smoking because smokers systematically underestimate the ill-effects of smoking on their health. We noted in the opening sentence that governments use three broad policies to discourage smoking, viz. a price policy of high taxes, a quantity policy of banning smoking in the workplace and in public places, and informational policies such as requiring health warnings on tobacco packages and limiting advertizing by tobacco companies. Although the rationale for policy is that the government has better information than the individual on the expected long-run cost of smoking, we focus on the interplay between the price and quantity policies. We do not focus on the informational policies. We do this because the psychology and behavioral economics literatures suggest that individuals have difficulty making correct and consistent decisions when events are uncertain and occur in the future.[2] These difficulties are likely to carry over into the process by which the individual uses new information to update his perception of the expected long-run risk of smoking, or an individuals' perception and hence behavior is likely to be insensitive to any new information provided by the government.[3]

As noted above, in our model the government wants to limit smoking because smokers underestimate the ill-effects of smoking on their health. Other reasons why a paternalistic government might want to limit smoking are that smokers suffer from time inconsistency

(Gruber and Koszegi (2004)), or that there is a projection bias (O'Donohue and Rabin (2001)) or that smokers are exposed to the wrong type of cue (Bernheim and Rangell (2004)). An efficiency reason to limit smoking is the externality created by "second-hand" smoke. Introducing these types of motivation for government policy would complicate our model but we do not believe it would change our results; what is important in our model is that the government wants to eliminate smoking but is unable to do so by either a tax-only or a quantity-only policy. We use the possibility of smokers switching to untaxed but illegal cigarettes as a device to limit the ability of the government to reduce smoking by taxing cigarettes. Another model might have political reasons or tax competition (as in de Bartolome (2007)) preventing the government from setting taxes which are punitively high. The devise itself is not important; what is important is that the government is unable to completely eliminate smoking by tax policy alone.

We use smoking as our motivating example but the theory may be applied when the individual's utility depends on the variance of consumption and the government wishes to limit the extent to which the product is consumed. In addition to smoking bans at the workplace (or at restaurants or other public places), our theory could be used to explain licensing laws which limit alcohol purchases to particular times; the ceiling placed on bets at some casinos (here varying the bet adds to the "thrill" so limiting the variance makes gambling less attractive); and laws which criminalize drugs, making their supply uncertain.

The paper is organized as follows. Section 2 introduces the model by describing an individual's smoking decision and the government's concern over the smoking level. Section 3 shows the effect of a smoking ban. In Sections 2 and 3, all cigarettes are taxed. Section 4 extends the discussion by introducing a "black market" or illegal market in which untaxed cigarettes may be bought. Section 5 undertakes a positive discussion when both the legal and the illegal market co-exist and when government policy is the simultaneous use of a tax and a ban on smoking in the workplace. Section 6 shows that this policy of "tax plus ban" is optimal. Section 7 concludes.

2. THE MODEL

2.1 Individual behavior

This paper is concerned with government policy towards cigarette smoking. A government policy is denoted P and is composed of two instruments, a cigarette tax (characterized by the consumer price of cigarettes q) and a potential ban on smoking in the workplace. If there is no ban on smoking in the workplace, the ban instrument is denoted as φ; if there is a ban on smoking in the workplace, the ban instrument is denoted as B. In this section the government policy is (q, φ).

The focus in this section is on considering how a smoker's utility is affected by the within-the-day variation in his smoking and how the government calculates his contribution to social welfare. Although considering these variations in a dynamic model, in which the smoker's daily utility from cigarettes and from health is determined by the path of smoking, would be more complete, it would come at the cost of much greater complexity. Instead, to make our point simply, we consider a static model.

An individual gains utility from the consumption of numeraire x, from smoking and from his health. We consider a day to have three periods; descriptively, Period 1 is the morning period before the individual goes to work, Period 2 is the period during which the individual works and Period 3 is the period after work. The individual's cigarette consumption in Period 1 is c_1, in

Period 2 is c_2 and in Period 3 is c_3 . In the absence of smoking, the individual's health is H; smoking causes ill-health or sickness S with probability Pr or his expected health is H - Pr S. Summarizing, the individual's true utility depends on x, c_1, c_2, c_3 , and on his expected health H - Pr S . We assume the individual's true utility has specific form as

$$x + v(c_1, c_2, c_3) + H - Pr\,S. \tag{1}$$

The individual's income is denoted M and the consumer price (which may include a tax) of a cigarette is denoted q. The individual potentially receives a lump-sum transfer R from the government. Budget balance for the smoker implies that his consumption of the numeraire is:

$$x = M + R - q(c_1 + c_2 + c_3). \tag{2}$$

The term $v(c_1, c_2, c_3)$ in Equation (1) represents the direct utility the individual achieves from smoking the cigarettes c_1, c_2 and c_3 . As noted in the Introduction, when a cigarette is smoked, the satisfaction is temporary; the nicotine inhaled stimulates the brain but then metabolizes into continine. The consequent decline of nicotine in the blood creates a craving for new nicotine to restore the nicotine to its pre-existing level. To assuage this craving, the smoker wants to light up another cigarette.[4] Overall, the smoker prefers a steady stream of nicotine to an unsteady stream. Therefore we model the term $v(c_1, c_2, c_3)$ as having mean-variance form, with the smoker enjoying the mean level of cigarettes and the smoker's preference for a steady stream of cigarettes being represented as a dislike of variance:

$$v(c_1, c_2, c_3) = a' \, mean(c_1, c_2, c_3) - b' \, variance(c_1, c_2, c_3).$$

The parameters a' and b' are positive. Noting that the mean can be written as

$mean(c_1, c_2, c_3) = ⅓(c_1 + c_2 + c_3)$ and the variance can be written as

$variance(c_1 , c_2 , c_3) = (2/9) (c_1^2 + c_2^2 + c_3^2 - c_1c_2 - c_1c_3 - c_2c_3)$, we write $a' = 3a$ and $b' = (9/2)b$, and individual i's direct utility achieved from smoking is:

$$v(c_1, c_2, c_3) \;=\; a(c_1 + c_2 + c_3) - b(c_1^2 + c_2^2 + c_3^2 - c_1c_2 - c_1c_3 - c_2c_3)\,. \tag{3}$$

The probability of sickness, Pr, increases as the level of smoking, $c_1 + c_2 + c_3$, increases. In addition, the severity of sickness S may depend positively on the level of smoking. We subsume the two components of expected sickness, Pr and S, into a single expression which we assume has quadratic form:

$$Pr\,S \;=\; \alpha(c_1 + c_2 + c_3) \;+\; \beta(c_1 + c_2 + c_3)^2 \tag{4}$$

where the parameters α and β are positive.[5] Substituting Equations (2), (3) and (4) into Expression (1), the individual's *true* expected utility is:

$$\begin{aligned} u^T(c_1,c_2,c_3) \;=\; & M + R - q(c_1+c_2+c_3) + a(c_1+c_2+c_3) - b(c_1^2+c_2^2+c_3^2-c_1c_2-c_1c_3-c_2c_3) \\ & + H - (\alpha(c_1+c_2+c_3) + \beta(c_1+c_2+c_3)^2)\;. \end{aligned} \tag{5}$$

Any model of smoking with policy implications must explain what seems unexplainable - why people choose to smoke when the induced health risks make it, to most outside observers, such a poor choice. We are using a static model but in the background we envision the individual as having a lifetime in which most of his consumption of cigarettes is smoked in the early and middle years, and the sickness potentially occurs in the later years after most of the cigarettes have been smoked. As such, when the level of cigarettes is being chosen, the associated sickness is a probabilistic event occurring in the future. As noted in the Introduction, there is considerable evidence from psychology and behavioral economics that individuals have difficulty making correct and consistent decisions when events are uncertain and occur in the future. Therefore, when choosing his cigarette consumption, the individual *i* perceives his expected sickness not as *Pr S* but as

$$\theta^i \, Pr\, S$$

where θ^i is his perception parameter. The individual's *perceived* expected utility is

$$x + v(c_1, c_2, c_3) + H - \theta^i \, Pr\, S; \tag{6}$$

substituting Equations (2), (3) and (4) into Expression (6), the individual's *perceived* utility is

$$u^P(c_1, c_2, c_3) = M + R - q(c_1 + c_2 + c_3) + a(c_1 + c_2 + c_3) - b(c_1^2 + c_2^2 + c_3^2 - c_1c_2 - c_1c_3 - c_2c_3)$$

$$+ H - \theta^i(\alpha(c_1 + c_2 + c_3) + \beta(c_1 + c_2 + c_3)^2) \,. \tag{7}$$

The difference between true utility and perceived utility lies in the perception parameter θ^i. We assume that all individuals know that smoking is detrimental to health, or $\theta^i > 0$. When $\theta^i = 1$, the individual correctly perceives his expected sickness. When $\theta^i < 1$ (> 1) the individual is underestimating (overestimating) the expected sickness associated with smoking.[6]

The individual chooses his cigarette consumption to maximize his *perceived* expected utility or solves

$$\max_{c_1 \geq 0,\, c_2 \geq 0,\, c_3 \geq 0} \; M + R - q(c_1 + c_2 + c_3) + a(c_1 + c_2 + c_3) - b(c_1^2 + c_2^2 + c_3^2 - c_1c_2 - c_1c_3 - c_2c_3)$$

$$+ H - \theta^i(\alpha(c_1 + c_2 + c_3) + \beta(c_1 + c_2 + c_3)^2) \; .$$

There is a potential constraint that expenditure on cigarettes cannot exceed income, or $q(c_1 + c_2 + c_3) \leq M + R$. We assume that income M is sufficiently large that the smoker buying the most cigarettes does not spend all of his income on cigarettes; the necessary restriction is shown in Appendix A. Therefore this constraint is not imposed as part of the individual's maximization problem.

The first-order condition for the choice of c_1 is:

$$\textit{either } c_1 = 0 \text{ and } \quad -q + a - b(2c_1 - c_2 - c_3) - \theta^i(\alpha + 2\beta(c_1 + c_2 + c_3))\,\big|_{c_1 = 0} \leq 0;$$

$$\textit{or } \quad c_1 > 0 \text{ and } \quad -q + a - b(2c_1 - c_2 - c_3) - \theta^i(\alpha + 2\beta(c_1 + c_2 + c_3)) \; = 0 \; . \tag{8}$$

Using the symmetry of the problem, $c_1 = c_2 = c_3$ and hence:

$$\textit{either } \; a - q - \theta^i\alpha \leq 0 \quad \text{and} \quad c_1 = c_2 = c_3 = 0 \;;$$

$$\textit{or } \quad a - q - \theta^i\alpha > 0 \quad \text{and} \quad c_1 = c_2 = c_3 = \frac{a - q - \theta^i\alpha}{6\theta^i\beta}.$$

This is rewritten as:

$$\text{If } \; \theta^i \geq \frac{a-q}{\alpha} \text{, the individual does not smoke: } c_1 = c_2 = c_3 = 0 \;; \tag{9a}$$

$$\text{If } \; \theta^i < \frac{a-q}{\alpha} \text{, the individual smokes: } c_1 = c_2 = c_3 = \frac{a - q - \theta^i\alpha}{6\theta^i\beta} > 0. \tag{9b}$$

In words, the individual smokes only if he has a sufficiently low perception of the negative effect of smoking on his health.

DEFINITION 1: *When the government policy is P, $\theta_1(P)$ is the perception parameter of the individual who is indifferent between not smoking and smoking taxed cigarettes.*

In this section, government policy is (q, φ). From Equations (9):

$$\theta_1(q, \varphi) = \frac{a - q}{\alpha}. \tag{10}$$

The perception parameter θ^i is distributed on $[\underline{\theta}, \overline{\theta}]$ with density $f(\theta^i)$. We assume:

(1) All individuals know that smoking is detrimental to health or $\underline{\theta} > 0$.

(2) Denote the resource cost of a cigarette as p (assumed fixed). When cigarettes are untaxed so that competition leads the consumer price to be p, there are some smokers and some non-smokers, or

$$\underline{\theta} < \theta_1(p, \varphi) < \overline{\theta}.$$

(3) Smoking is a "bad" in the sense that, when cigarettes are untaxed, an individual who correctly perceives the associated expected sickness (i.e. for whom $\theta^i = 1$) chooses not to smoke, or

$$\theta_1(p, \varphi) < 1.$$

We summarize these assumptions with Assumption 1:

ASSUMPTION 1: $0 < \underline{\theta} < \theta_1(p, \varphi) = \dfrac{a - p}{\alpha} < \min[1, \overline{\theta}]$.

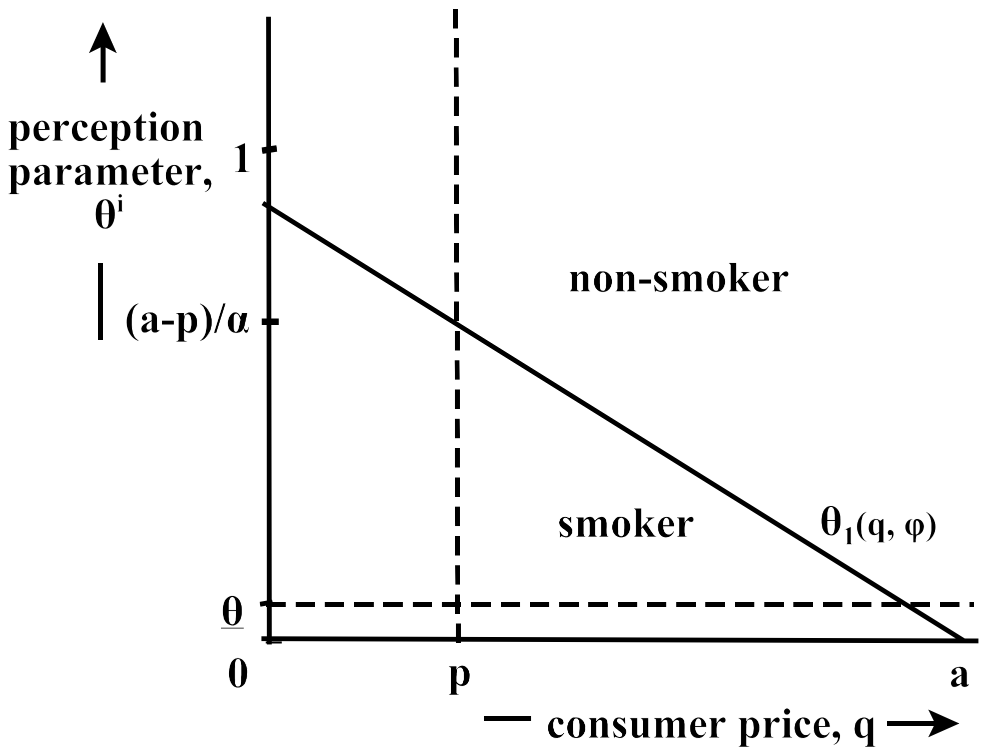

Figure 1: smoking participation as a function of θ^i and q

Figure 1 shows how the population is divided between smokers and non-smokers. The downward sloping line shows $\theta_1(q, \varphi)$ as a function of the consumer price q; the line is downward sloping because higher prices discourage smoking. At given q, individuals with θ^i above the line put sufficient weight on ill-health as to not smoke. Individuals with θ^i below the line underestimate the effect of smoking on their health to such an extent that they smoke. The individual with $\theta^i = 1$ correctly perceives the effect of cigarettes on his health and, at $q = p$, does not smoke: hence, when $q = p$, the line lies below $\theta^i = 1$.

A nice feature of the model is that the perception parameter may be symmetrically distributed and centered on $\theta^i = 1$ so that there is no systematic bias in the population's perceptions. In contrast, smokers are located in the lower range of the distribution, systematically underestimating the ill-effect of smoking on their health.

2.2 Government policy

Equation (9) shows an individual's cigarette consumption when the government policy is (q, φ). More generally, if government policy is P, denote the cigarette consumption chosen by the individual with perception parameter θ^i as $c_1(\theta^i; P)$, $c_2(\theta^i; P)$, and $c_3(\theta^i; P)$. We normalize the population to unity. Tax revenue is returned to all individuals as the lump-sum transfer R, or

$$R = \int_{\underline{\theta}}^{\overline{\theta}} (q - p)(c_1(\theta^i;P) + c_2(\theta^i;P) + c_3(\theta^i;P)) f(\theta^i)\, d\theta^i \tag{11}$$

so that R is a function of P.

The government is paternalistic. It knows that smokers are systematically underestimating the expected sickness associated with smoking, and uses policy to correct for this error. In particular, the government evaluates individual utility using the *true* expected health of the individual. Denote the true expected utility of the individual, which is a function of his perception parameter θ^i and the policy P, as $U^T(\theta^i, P)$:

$$U^T(\theta^i, P) = u^T(c_1(\theta^i;P), c_2(\theta^i;P), c_3(\theta^i;P)).$$

The government calculates social welfare as the sum of all individual true utilities, or social welfare W under policy P is

$$W(P) = \int_{\underline{\theta}}^{\overline{\theta}} U^T(\theta^i;P) f(\theta^i)\, d\theta^i. \tag{12}$$

Since social welfare is the sum of true utilities, it is maximized when individuals do not smoke.

3. SMOKING BAN IN WORKPLACE

This section considers the policy *(q, B)* or the government levies a cigarette tax and bans smoking in Period 2. With no smoking allowed in the workplace, the individual maximizes his perceived utility (Equation (7)) with the additional restriction that $c_2 = 0$, or:[7]

$$\max_{c_1 \geq 0,\, c_3 \geq 0} \; M + R(q, B) - q(c_1 + c_3) + a(c_1 + c_3) - b(c_1^2 + c_3^2 - c_1 c_3) + H - \theta^i(\alpha(c_1 + c_3) + \beta(c_1 + c_3)^2).$$

We note that, with $c_1 = c_3 > 0$, the ban has introduced variance into the smoker's utility. The first-order condition is:

$$\text{either } c_1 = 0 \text{ and } \quad -q + a - b(2c_1 - c_3) - \theta^i(\alpha + 2\beta(c_1 + c_3)) \,|_{c_1 = 0} \leq 0;$$

$$\text{or } \quad c_1 > 0 \text{ and } \quad -q + a - b(2c_1 - c_3) - \theta^i(\alpha + 2\beta(c_1 + c_3)) = 0. \tag{13}$$

By symmetry, set $c_1 = c_3$; hence

$$\textit{either} \quad \theta^i \geq \frac{a - q}{\alpha} \quad \text{and} \quad c_1 = c_3 = 0; \tag{14a}$$

$$\textit{or} \quad \theta^i < \frac{a - q}{\alpha} \quad \text{and} \quad c_1 = c_3 = \frac{a - q - \theta^i \alpha}{4\theta^i \beta + b}. \tag{14b}$$

We make several observations. Concerning participation: comparing Equations (14) and with Equations (9), we see, perhaps surprisingly, that the ban does not change the value of θ^i of the marginal smoker who is indifferent between not smoking and smoking, or does not cause any smoker to quit. Why is this? Consider the change in perceived utility when the smoker smokes his first cigarette, increasing his smoking from $c_1 = c_2 = c_3 = 0$ by a marginal cigarette (arbitrarily assumed to be smoked in Period 1). Recognizing that the left-hand side of Equation (8) is the perceived marginal utility if there is no ban,

$$\frac{\partial u^P}{\partial c_1}\Big|_{c_1 = c_2 = c_3 = 0} = a - q - \theta^i\alpha.$$

Similarly, the left-hand side of Equation (13) is the perceived marginal utility if there is a work-place ban,

$$\frac{\partial u^P}{\partial c_1}\Big|_{c_2 = 0,\, c_1 = c_3 = 0} = a - q - \theta^i\alpha\ .$$

In both cases, the marginal variance is zero and these are the same expressions.[8] So the individual, who perceives positive utility from smoking some cigarettes in Period 1 when there is no workplace ban, also perceives positive utility from smoking some cigarettes in Period 1 when there is a workplace ban. We summarize this observation below:

OBSERVATION 1: *Imposing a ban on smoking in the workplace does not cause any smoker to quit, or* $\theta_1(q, \varphi) = \theta_1(q, B)$.

Because Observation 1 is perhaps surprising, we note that the recent empirical work of Adda and Cornaglia (2010) and Jones et al. (2011) find that smoking bans do not affect participation.[9]

Concerning inter-period substitution: Adda and Cornaglia (2010) and Jones et al. (2011) stress that smoking bans induce smokers to increase their smoking in other periods of the day. In our model, the smoking ban in Period 2 may induce the smoker to increase his smoking in Periods 1 and 3. In particular, if the rule setting $c_2 = 0$ is introduced but the levels of c_1 and c_3 remain at their pre-existing levels (so that total cigarettes are reduced), the ban reduces the perceived health cost and increases the variance cost of a marginal increase in c_1 or c_3. If the reduction in the marginal perceived health cost is greater than the increase in the marginal variance cost, the cost associated with a marginal increase in c_1 or c_3 is reduced and the smoker offsets the ban by substituting into Period 1 and Period 3 cigarettes. Formally, comparing Equations (9b) and (14b), the ban induces a smoker to increase his cigarettes in Periods 1 and 3 if

$$4\theta^i\beta + b < 6\theta^i\beta \ ;$$

or if

$$b < 2\theta^i\beta$$

i.e. provided the dislike of variance term is not "too strong". We summarize this observation:

OBSERVATION 2: *Imposing a smoking ban in Period 2 increases the number of cigarettes smoked in Periods 1 and 3 unless the dislike of variance is "too strong."*

Concerning the total number of cigarettes smoked: if, when the ban is introduced, there is full substitution into Period 1 and Period 3 cigarettes (so that the total quantity of cigarettes is the pre-existing level), the marginal effect of an increase in c_1 or c_3 on perceived health is unchanged, but there is increased marginal variance. This increased marginal variance leads the smoker to lower his cigarette consumption. This is formalized below.

With no ban, the total cigarettes smoked is (using Equation (9b)):

$$c_1 + c_2 + c_3 \ = \ 3\,\frac{a - q - \theta^i\alpha}{6\theta^i\beta} \ ;$$

With a ban, the total cigarettes smoked is (using Equation (14b)):

$$c_1 + c_3 = 2\,\frac{a - q - \theta^i \alpha}{4\theta^i \beta + b}\ .$$

Hence, $b > 0$ ensures that the ban lowers the total number of cigarettes smoked by a smoker. This leads to Observation 3:

OBSERVATION 3: Imposing a ban on smoking in the workplace reduces the total number of cigarettes smoked by a smoker.

Irvine and Nguyen (2011) find that smokers subject to smoking bans in the workplace smoke 9% fewer cigarettes per day than smokers not subject to such bans. Workplace bans are effective only during working days, so Observation 3 suggests that smokers who are impacted by a workplace ban smoke less cigarettes on weekdays than on weekends, and this effect is likely to be more pronounced for heavy smokers than for light smokers (as the variance term is larger for heavy smokers). These predictions are confirmed by Nguyen (2012).

Concerning the change in social welfare if the ban is imposed: the government calculates social welfare as the sum of the true utility of each individual. From the government's perspective, the reduction in the number of cigarettes smoked is beneficial. However, the ban creates variance in the cigarettes consumed, lowering the smoker's true utility and this cost is also included in the government's calculation of social welfare. It might therefore appear that the government faces a trade-off. However, Lemma 1 shows that the benefit dominates. To make the comparison with Lemma 2 later, Lemma 1 restates that all cigarettes are bought legally.

LEMMA 1: If all cigarettes are bought legally, a ban on smoking in the workplace increases social welfare, or $W(q, B) > W(q, \varphi)$ with $q \geq p$.

PROOF: see Appendix B.

Utility has consumer surplus form and social welfare is utilitarian. In consequence social welfare is unchanged if, instead of tax revenue being returned as a uniform lump-sum transfer, the tax revenue paid by a smoker is returned to the smoker as an individual-specific lump-sum transfer. With this construction, the ban reduces the cigarettes smoked by each smoker but leaves the resources consumed by each individual unchanged. Lemma 1 is proved by showing that, with the tax being returned in this way, the ban increases the utility - as calculated by the government - of *each* smoker. This is formalized in the Corollary.

COROLLARY: If the tax revenue paid by a smoker is returned to the smoker as an individualistic lump-sum transfer, a ban on smoking in the workplace increases the true utility of each smoker.

4. TAX ONLY

Government welfare is highest when individuals do not smoke and hence, to discourage smoking, the government may impose a tax on cigarettes. In this section, we assume that the government only uses a tax and does not use a smoking ban, or the policy is (q,φ) . From Equation (10), if an individual i is choosing between not smoking and buying taxed cigarettes at consumer price q, he chooses not to smoke if

$$\theta^i \geq \theta_1(q, \varphi) = \frac{a - q}{\alpha}.$$

In addition, $\theta^i > \underline{\theta}$. Putting these inequalities together, no individual buys taxed cigarettes if

$$\underline{\theta} \geq \frac{a - q}{\alpha};$$

or if

$$q \geq a - \underline{\theta}\alpha.$$

Hence, if the individual's choice is only between not smoking and buying taxed cigarettes, the government can achieve its objective of stopping smoking by imposing a sufficiently high tax. However, a high tax favors the development of a "black market" in which individuals can buy untaxed but illegal cigarettes. This section explores the individual's choice in the presence of parallel markets for legal taxed and illegal untaxed cigarettes.

For the smoker, the advantage of buying on the illegal market is the lower price. However, participation in an illegal activity imposes a utility cost. This cost may be a psychological cost or the expected cost of being apprehended by law-enforcement and punished. We assume that the utility cost of buying illegal cigarettes is the fixed cost F.[10] The smoker chooses the illegal market if the benefit of the lower price exceeds F. The benefit of the lower price increases with the cigarettes smoked, or the smokers buying on the illegal market have low perception parameters θ^i.[11]

Formally, consider a smoker i choosing between buying legal taxed cigarettes and illegal untaxed cigarettes. If the individual buys legal cigarettes, the consumer price is q and, using Equation (9b) in Equation (7), his perceived utility is:

$$M + R - q\,3\,\frac{a - q - \theta^i\alpha}{6\theta^i\beta} + a\,3\,\frac{a - q - \theta^i\alpha}{6\theta^i\beta} + H - \theta^i\left(\alpha\left(3\,\frac{a - q - \theta^i\alpha}{6\theta^i\beta}\right) + \beta\left(3\,\frac{a - q - \theta^i\alpha}{6\theta^i\beta}\right)^2\right). \quad (15)$$

If he participates in the illegal market, the consumer price is p but he pays the fixed cost F. He receives the same lump-sum transfer R and chooses his cigarette consumption to maximize his perceived utility[12] :

$$\max_{c_1 \geq 0,\, c_2 \geq 0,\, c_3 \geq 0} \; M + R - F - p(c_1 + c_2 + c_3) + a(c_1 + c_2 + c_3)$$

$$- b(c_1^2 + c_2^2 + c_3^2 - c_1 c_2 - c_1 c_3 - c_2 c_3) + H - \theta^i(\alpha(c_1 + c_2 + c_3) + \beta(c_1 + c_2 + c_3)^2).$$

With the assumed utility function, the demand for cigarettes does not depend on the smoker's income and hence, setting $q = p$ in Equation (9b), his cigarette choice is

$$c_1 = c_2 = c_3 = \frac{a - p - \theta^i \alpha}{6\theta^i \beta};$$

substituting for c_1, c_2 and c_3 , his perceived utility is

$$M + R - F - p3\frac{a-p-\theta^i\alpha}{6\theta^i\beta} + a3\frac{a-p-\theta^i\alpha}{6\theta^i\beta} + H - \theta^i\left(\alpha\left(3\frac{a-p-\theta^i\alpha}{6\theta^i\beta}\right) + \beta\left(3\frac{a-p-\theta^i\alpha}{6\theta^i\beta}\right)^2\right). \quad (16)$$

The smoker buys in the illegal market if he achieves more utility thereby. Comparing Expressions (15) and (16), the smoker chooses the illegal market if

$$\theta^i < \frac{(a-p)^2 - (a-q)^2}{2\alpha(q-p) + 4\beta F} \quad . \qquad (17)$$

DEFINITION 2: When the government's policy is P, $\theta_2(P)$ is the perception parameter of the individual who is indifferent between buying legal taxed cigarettes and illegal untaxed cigarettes.

From Inequality (17),

$$\theta_2(q, \varphi) = \frac{(a-p)^2 - (a-q)^2}{2\alpha(q-p) + 4\beta F} . \tag{18}$$

Differentiating

$$\frac{d\theta_2(q, \varphi)}{dq} = \frac{8\beta F(a-q) - 2\alpha(q-p)^2}{(2\alpha(q-p) + 4\beta F)^2} . \tag{19}$$

We note:

(1) $\theta_2(p, \varphi) = 0$. With no tax, $q = p$ and all individuals smoke legal cigarettes. If there were a smoker with $\theta^i = 0$, he would consume an infinite quantity of cigarettes and hence even the smallest tax would induce him to buy in the illegal market. [13]

(2) $\theta_2(p, \varphi) = 0$ and, by assumption, $\theta_1(p, \varphi) > 0$. Hence $0 = \theta_2(p, \varphi) < \theta_1(p, \varphi)$. By continuity, for small increases in q above p , $\theta_2(q, \varphi) < \theta_1(q, \varphi)$. Using Equations (10), (18) and (19):

$$\theta_2(q, \varphi) < \theta_1(q, \varphi) \text{ implies } \frac{d\theta_2(q, \varphi)}{dq} > 0 .$$

Intuitively, as the tax increases, the benefit of buying a fixed quantity of cigarettes in the illegal market increases, so the indifferent smoker buys less cigarettes or has higher θ^i. In addition,

$$\theta_2(q, \varphi) = \theta_1(q, \varphi) \text{ implies } \frac{d\theta_2(q, \varphi)}{dq} = 0 .$$

(3) Denote the consumer price to be $\hat{q}$ such that $\theta_2(\hat{q}, \varphi) = \theta_1(\hat{q}, \varphi)$. At this price, the smoker with perception parameter $\theta_2(\hat{q}, \varphi)$ is indifferent between not smoking, smoking legal cigarettes and smoking illegal cigarettes. If the consumer price increases beyond $\hat{q}$, the utility this smoker gets from not smoking and from smoking illegal cigarettes is unchanged (but the utility he achieves by smoking legal cigarettes decreases). Therefore

above $\hat{q}$ the value of θ^i which makes the smoker indifferent between not smoking and smoking is $\theta_2(\hat{q};\, \varphi)$ which does not depend on q.

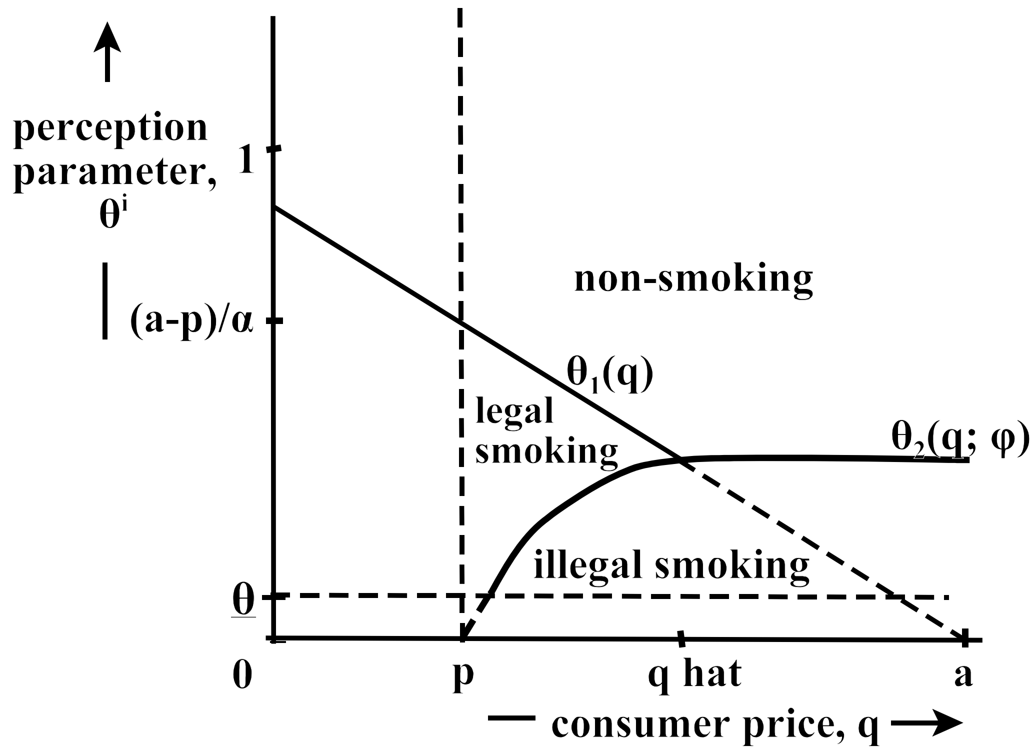

Figure 2: the division of individuals between non-smokers, smokers of legal cigarettes and smokers of illegal cigarettes

Figure 2 uses the notes above to show the $(\theta^i,\, q)$ regions at which individuals do not smoke, smoke legal cigarettes and smoke illegal cigarettes when the policy is $(q,\, \varphi)$. In the subsequent discussion, we assume that $\theta_2(\hat{q};\, \varphi) > \underline{\theta}$ (as shown). Intuitively, F must be sufficiently small to ensure that at $\hat{q}$ there are smokers buying illegal untaxed cigarettes: the necessary restriction is derived in Appendix C.

ASSUMPTION 2: $\theta_2(\hat{q};\, \varphi) > \underline{\theta}$.

Assumption 2 ensures that there are individuals for whom $\theta^i < \theta_2(\hat{q};\, \varphi)$, or the government is unable to eliminate smoking by raising the tax rate.

In Figure 2 $\theta_2(q, \varphi) = \underline{\theta}$ occurs at a value of q which is strictly greater than p. Put differently, at low tax rates, all cigarettes are bought legally; even smokers with perception parameter $\underline{\theta}$ do not buy enough cigarettes to make it worthwhile to buy illegal cigarettes.

5. COMBINATION POLICY OF TAX PLUS BAN: POSITIVE ANALYSIS

In this section we consider a positive description of imposing a ban when there are parallel legal and illegal markets. In the next section we undertake a normative analysis and show that, with parallel markets, the policy of "tax plus ban" is optimal. First, as noted in Observation 1, the perception parameter of the individual who is indifferent between not smoking and smoking legal cigarettes does not depend on whether there is a smoking ban at the workplace. Second, consider the smoker who, if there is no smoking ban, is indifferent between buying legal taxed cigarettes and buying illegal untaxed cigarettes. If a ban is enacted, this smoker reduces his consumption of cigarettes and hence the tax saved by buying in the illegal market is reduced: this smoker shifts into buying his cigarettes strictly in the legal market, or the perception parameter of the smoker who is indifferent between buying in the two markets falls.

Formally, with the ban enacted, consider the smoker with perception parameter θ^i. If he buys legal cigarettes, from Equation (14b) he buys $2(a-q-\theta^i\alpha)/(4\theta^i\beta + b)$ cigarettes to achieve perceived utility

$$M + R(q, B) - q2\frac{a-q-\theta^i\alpha}{4\theta^i\beta + b} + a2\frac{a-q-\theta^i\alpha}{4\theta^i\beta + b}$$

$$- b\left(\frac{a-q-\theta^i\alpha}{4\theta^i\beta + b}\right)^2 + H - \theta^i\left(\alpha\left(2\frac{a-q-\theta^i\alpha}{4\theta^i\beta + b}\right) + \beta\left(2\frac{a-q-\theta^i\alpha}{4\theta^i\beta + b}\right)^2\right).$$

Similarly, if he buys illegal untaxed cigarettes, he buys $2(a-p-\theta^i\alpha)/(4\theta^i\beta+b)$ cigarettes to achieve perceived utility

$$M + R(q,B) - F - p2\frac{a-p-\theta^i\alpha}{4\theta^i\beta+b} + a2\frac{a-p-\theta^i\alpha}{4\theta^i\beta+b}$$

$$- b\left(\frac{a-p-\theta^i\alpha}{4\theta^i\beta+b}\right)^2 + H - \theta^i\left(\alpha\left(2\frac{a-p-\theta^i\alpha}{4\theta^i\beta+b}\right) + \beta\left(2\frac{a-p-\theta^i\alpha}{4\theta^i\beta+b}\right)^2\right).$$

The smoker with perception parameter $\theta_2(q, B)$ achieves the same perceived utility from buying in the legal and illegal markets or, setting $\theta^i = \theta_2(q, B)$ and equating the utilities above,

$$\theta_2(q,B) = \frac{(a-p)^2 - (a-q)^2 - bF}{2\alpha(q-p)+4\beta F}. \tag{20}$$

Comparing Equations (18) and (20),

$$\theta_2(q,B) < \theta_2(q,\varphi).$$

Taking the derivative, we can show that

$$\theta_2(q,B) < \theta_1(q,B) \quad \text{implies} \quad \frac{d\theta_2(q,B)}{dq} > 0;$$

and

$$\theta_2(q,B) = \theta_1(q,B) \quad \text{implies} \quad \frac{d\theta_2(q;B)}{dq} = 0 .$$

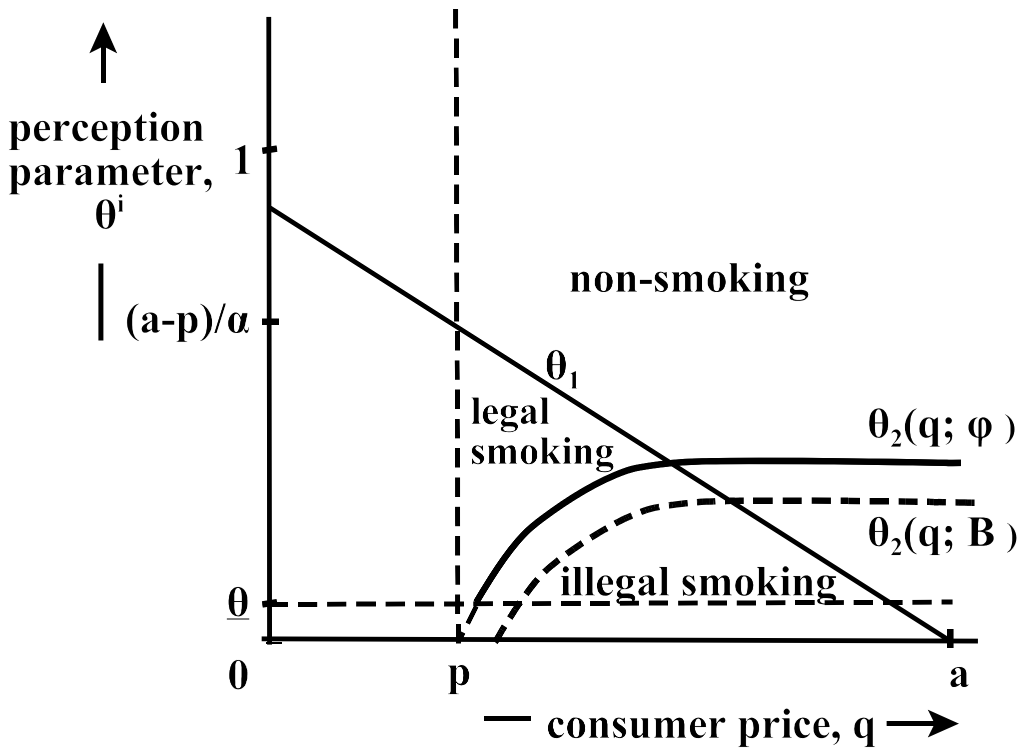

Figure 3: the division of individuals between non-smokers, smokers of legal cigarettes and smokers of illegal cigarettes with no ban and with ban in Period 2.

Figure 3 redraws Figure 2 to include the effect of the smoking ban: the $\theta_2(q, B)$ curve lies below the $\theta_2(q, \varphi)$ curve. With the ban reducing the cigarettes smoked, the ban extends the range of tax rates for which all smokers buy legal taxed cigarettes.

6. COMBINATION POLICY OF TAX AND BAN: NORMATIVE ANALYSIS

In this section we show that, in the full model with parallel legal and illegal markets, the optimal policy is "tax plus ban." Consider first imposing a ban on smoking in the workplace when the tax rate is described by the consumer price q and $q \geq p$. As noted in the discussion preceding the Corollary, the form of the individual's utility function and of the government's social welfare function implies that social welfare can be calculated “as if” any tax paid by a smoker is returned to that smoker as an individualistic lump-sum transfer. Using this construction, there are four groups of individuals:

(1) Non-smokers, $\theta_1(q, \varphi) = \theta_1(q, B) < \theta^i$. This group is unaffected when the ban is imposed.

(2) Smokers of legal cigarettes under either policy, $\theta_2(q, \varphi) \leq \theta^i \leq \theta_1(q, \varphi)$. The effect of the ban on each smoker in this group is to reduce the cigarettes he consumes but to increase his variance. Overall, the Corollary states that the benefit of the reduced cigarettes dominates, or the true utility of the individual increases.

(3) Smokers who smoked illegal cigarettes when there is no ban but shift into smoking legal cigarettes when the ban is imposed, $\theta_2(q, B) \leq \theta^i \leq \theta_2(q, \varphi)$. There are three sources of gain from each of these smokers. First, the fixed cost is saved. Second, the price he pays is increased so that he smokes less cigarettes. Third, the ban reduces the cigarettes smoked; the variance is increased but (from the Corollary) the overall effect is beneficial.

(4) Smokers of illegal cigarettes under either policy, $\theta^i \leq \theta_2(q, B)$. This group is descriptively similar to the smokers of legal cigarettes under either policy, except that the consumer price is *p*. The Corollary states that the benefit of the less smoking dominates the increased variance, or the true utility of the individual is increased.

Since true utility is increased for each smoker, the overall effect is to increase welfare. This is formalized in Lemma 2

LEMMA 2: If legal taxed and illegal untaxed cigarettes are available, a ban on smoking in the workplace increases social welfare, or $W(q, B) > W(q, \varphi)$ with $q \geq p$.

PROOF: See Appendix D.

Lemma 2 extends Lemma 1 to the full model with parallel markets. By establishing that, at any q: $q \geq p$, , the ban increases social welfare, Lemma 2 establishes that a ban on smoking in the workplace is part of the optimal policy.

Now consider whether, in the presence of a ban, it is always desirable to impose a strictly positive tax rate or $q > p$. As noted at the end of Section 4, imposing a sufficiently small tax rate pushes no smoker into buying illegal untaxed cigarettes. But such a tax reduces participation and reduces the cigarettes smoked by every smoker; both of these effects increase social welfare. This is formalized in Lemma 3 below.

LEMMA 3: $\frac{dW(q,B)}{dq}\Big|_{q=p} > 0$

PROOF: See Appendix E.

Lemma 2 and Lemma 3 together confirm that a "tax plus ban" is the optimal policy.

PROPOSITION: when smokers can buy either legal taxed or illegal untaxed cigarettes, a "tax plus ban" is the best policy, or

$$\max_{q:q>p} W(q,B) > W(p,\varphi)\,,\, W(p,B)\,,\, \max_{q:q\geq p} W(q,\varphi)\,.$$

PROOF: $\max\limits_{q:q>p} W(q,B) > W(p,B) > W(p,\varphi)$

where the first inequality comes from Lemma 3 and the second inequality comes from Lemma 2. Denote the q value which maximizes $W(q,\varphi)$ as q^A. Hence

$$\max_{q:q>p} W(q,B) \geq W(q^A,B) > W(q^A,\varphi) \equiv \max_{q:q\geq p} W(q,\varphi)$$

where the first inequality follows because q^A is allowable in the maximization on the left-hand side, the second inequality follows from Lemma 2. The final identity follows from the definition of q^A.

7. CONCLUSION

This paper considers public policy towards smoking. Some individuals smoke because they underestimate the effect of smoking on their health and they would like to smoke evenly during the day. The government knows the true health effects of smoking and would like to stop smoking. The government has two partial tools. The first tool is to impose a tax but a black market prevents the government from being able to use sufficiently high tax rates to stop all smoking. The second tool is to ban smoking during part of the day. Although the ban increases the individual's variance in consumption, it induces the individual to lower his consumption of the product at any tax rate, so that the best policy is a combination policy of "tax plus ban."

It should be emphasized that our result - two policies are better than one - does not automatically follow from the general principal that increasing the number of instruments available to the government improves the government's ability to achieve a goal. In our model, the workplace smoking ban has two effects: it deceases the quantity of cigarettes consumed and it increases the variance of the consumption stream. Both of these impacts - the former being beneficial and the latter being detrimental - are included in the government's welfare function. The paper shows that the former effect outweighs the latter effect in the government's calculus so that the ban is always a useful tool. By showing that the best policy is "tax plus ban", we hope to add to the literature on smoking policy and mor generally to add to the "price *v.* quantity" debate on how to best control some socially undesirable activities.

APPENDIX A: RESTRICTION ON M

When calculating the individual's choice of cigarettes, we do not include the constraint that the individual's income is at least as large as his expenditure on cigarettes. This is valid if the smoker's cigarette consumption, as expressed in Equation (9b), is affordable, or

$$q \; 3 \; \frac{a - q - \theta^{i}\alpha}{6\theta^{i}\beta} \;\leq\; M + R;$$

or

$$M \;\geq\; \frac{(a - q)q - \theta^{i}\alpha q}{2\theta^{i}\beta} \;-\; R.$$

R constitutes returned tax revenue, or is positive. The constraint is tightest for the smoker with the lowest θ^{i}, or with $\theta^{i} = \underline{\theta}$. Hence a sufficient condition is:

$$M \;\geq\; \frac{(a - q)q - \underline{\theta}\alpha q}{2\underline{\theta}\beta}. \tag{A.1}$$

In subsequent sections the tax, characterized by the consumer price q, is chosen by the government, and we require that the above inequality is satisfied for all q, $q \geq p$ where p is the firm price of a cigarette (which competition sets equal to the resource cost of a cigarette), or

$$M \ \geq \ \max_{q\,:\,q \geq p} \ \frac{(a - q)q - \underline{\theta}\alpha q}{2\underline{\theta}\beta}.$$

The right-hand side is maximized when

$$\frac{d}{dq}\frac{(a-q)q - \underline{\theta}\alpha q}{2\underline{\theta}\beta} \ = \ \frac{a - 2q - \underline{\theta}\alpha}{2\underline{\theta}\beta} \ = \ 0;$$

or when

$$q \ = \ \frac{a - \underline{\theta}\alpha}{2}.$$

Substituting into Inequality (A.1)

$$M \ \geq \frac{(a - \underline{\theta}\alpha)^2}{8\underline{\theta}\beta}.$$

APPENDIX B: PROOF OF LEMMA 1

(i) Introduction

The true utility achieved by the individual may be written in consumer surplus form:

$$U^T(\theta^i;P) \ = \ M + R(P) + H + G(\theta^i;P)$$

where the true consumer surplus from smoking is

$$\begin{aligned} G(\theta^i;P) \ \equiv \ & -q\big(c_1(\theta^i;P) + c_2(\theta^i;P) + c_3(\theta^i;P)\big) + a\big(c_1(\theta^i;P) + c_2(\theta^i;P) + c_3(\theta^i;P)\big) \\ & - b\big(c_1^2(\theta^i;P) + c_2^2(\theta^i;P) + c_3^2(\theta^i;P) - c_1(\theta^i;P)c_2(\theta^i;P) - c_1(\theta^i;P)c_3(\theta^i;P) - c_2(\theta^i;P)c_3(\theta^i;P)\big) \\ & - (\alpha\big(c_1(\theta^i;P) + c_2(\theta^i;P) + c_3(\theta^i;P)\big) + \beta\big(c_1(\theta^i;P) + c_2(\theta^i;P) + c_3(\theta^i;P))^2\big). \end{aligned} \qquad \text{(B.1)}$$

where $c_t(\theta^i;\, P)$ is the cigarettes smoked in Period t by the individual with perception parameter

θ^i under policy P. Using Equation (12), social welfare under policy P is

$$W(P) = \int_{\underline{\theta}}^{\bar{\theta}} \left(M + R(P) + H + G(\theta^i;P) \right) f(\theta^i)\, d\theta^i . \tag{B.2}$$

Denote the total tax paid by a smoker with perception parameter θ^i as $r(\theta^i; P)$,

$$r(\theta^i; P) = (q - p)(c_1(\theta^i; P) + c_2(\theta^i; P) + c_3(\theta^i; P)) . \tag{B.3}$$

Using Equation (11)

$$R(P) = \int_{\underline{\theta}}^{\bar{\theta}} r(\theta^i; P)\, f(\theta^i)\, d\theta^i .$$

Noting $R(P) = \int_{\underline{\theta}}^{\bar{\theta}} R(P)\, f(\theta^i)\, d\theta^i$ and inserting into Equation (B.2),

$$W(P) = \int_{\underline{\theta}}^{\bar{\theta}} \left(M + H + r(\theta^i;P) + G(\theta^i;P) \right) f(\theta^i)\, d\theta^i . \tag{B.4}$$

Social welfare is unchanged if tax revenue, instead of being shared equally across all individuals, is shared so that each smoker receives an individualistic lump-sum payment equal to the tax he pays.

(ii) Change in social welfare if ban is imposed

Using Equation (B.4), under policy (q, φ) social welfare is

$$W(q,\varphi) = \int_{\underline{\theta}}^{\theta_1(q,\varphi)} \left(M + H + r(\theta^i;q,\varphi) + G(\theta^i;q,\varphi) \right) f(\theta^i)\, d\theta^i + \int_{\theta_1(q,\varphi)}^{\bar{\theta}} (M + H)\, f(\theta^i)\, d\theta^i \;\; ;$$

under policy (q,B) social welfare is

$$W(q,B) = \int_{\underline{\theta}}^{\theta_1(q,B)} \big(M + H + r(\theta^i;q,B) + G(\theta^i;q,B)\big)\, f(\theta^i)\, d\theta^i + \int_{\theta_1(q,B)}^{\bar{\theta}} (M+H)\, f(\theta^i)\, d\theta^i \ .$$

From Observation 1, $\theta_1(q, \varphi) = \theta_1(q, B)$, and hence the increase in welfare consequent on the imposition of a ban is

$$W(q,B) - W(q,\varphi) = \int_{\underline{\theta}}^{\theta_1(q,\varphi)} \big(r(\theta^i;q,B) + G(\theta^i;q,B) - r(\theta^i;q,\varphi) - G(\theta^i;q,\varphi)\big)\, f(\theta^i)\, d\theta^i . \quad \text{(B.5)}$$

Using Equations (9b), (14b) to substitute for $c_1(\theta^i;.)$ $c_2(\theta^i;.)$, and $c_3(\theta^i;.)$ in Equations (B.1), (B.3)

$$\begin{aligned} & r(\theta^i;q,B) + G(\theta^i;q,B) - r(\theta^i;q,\varphi) - G(\theta^i;q,\varphi) \\ & \quad = (q-p)\,2\,\frac{a-q-\theta^i\alpha}{4\theta^i\beta + b} + (a-q-\alpha)\,2\,\frac{a-q-\theta^i\alpha}{4\theta^i\beta + b} - (b+4\beta)\left(\frac{a-q-\theta^i\alpha}{4\theta^i\beta + b}\right)^2 \\ & \qquad - (q-p)\,3\,\frac{a-q-\theta^i\alpha}{6\theta^i\beta} - (a-q-\alpha)\,3\,\frac{a-q-\theta^i\alpha}{6\theta^i\beta} + \beta\left(3\,\frac{a-q-\theta^i\alpha}{6\theta^i\beta}\right)^2; \end{aligned}$$

or, after simplifying,

$$\begin{aligned} & r(\theta^i;q,B) + G(\theta^i;q,B) - r(\theta^i;q,\varphi) - G(\theta^i;q,\varphi) \\ & = -\frac{2b}{4\theta^i\beta(4\theta^i\beta + b)}(a-p-\alpha)(a-q-\theta^i\alpha) + \frac{4b^2\beta + 16b\theta^i\beta^2(2-\theta^i)}{(4\theta^i\beta)^2(4\theta^i\beta + b)^2}(a-q-\theta^i\alpha)^2. \quad \text{(B.6)} \end{aligned}$$

From Assumption 1, $a - p - \alpha < 0$. For $\theta^i < \theta_1(q, \varphi)$, (a) $\theta^i < (a - q)/\alpha$ or $a - q - \theta^i\alpha > 0$ and (b) $\theta^i < 1$. Hence, for $\underline{\theta} \le \theta^i < \theta_1(q,\varphi)$, $r(\theta^i; q,B) + G(\theta^i; q,B) - r(\theta^i; q,\varphi) - G(\theta^i; q,\varphi) > 0$.

Using Equation (B.5), this implies $W(q, B) - W(q, \varphi) > 0$.

APPENDIX C: RESTRICTION ON F

We now establish the restriction on the fixed cost F such that $\theta_2(\hat{q}, \varphi) > \underline{\theta}$. $\hat{q}$ solves $\theta_2(q, \varphi) = \theta_1(q, \varphi)$, or

$$\frac{(a-p)^2 - (a-\hat{q})^2}{2\alpha(\hat{q}-p) + 4\beta F} = \frac{a-\hat{q}}{\alpha}$$

or

$$\alpha\hat{q}^2 + (4\beta F - 2\alpha p)\hat{q} + \alpha p^2 - 4a\beta F = 0. \tag{C.1}$$

The left-hand side of Equation (C.1) is a parabola. From Assumption 1, $p < a$. When $\hat{q} = p$, the left-hand side of Equation (C.1) is negative. We know $\hat{q} > p$ and hence we accept the larger root, or

$$\hat{q} = \frac{-(4\beta F - 2\alpha p) + \sqrt{(4\beta F - 2\alpha p)^2 - 4\alpha(\alpha p^2 - 4a\beta F)}}{2\alpha}.$$

We want $\theta_2(\hat{q}, \varphi) > \underline{\theta}$, or

$$\frac{(a-p)^2 - (a-\hat{q})^2}{2\alpha(\hat{q}-p) + 4\beta F} > \underline{\theta}.$$

Substituting for the value of $\hat{q}$, and rearranging:

$$F < \frac{(a - p - \alpha\underline{\theta})^2}{4\beta\underline{\theta}}.$$

APPENDIX D: PROOF OF LEMMA 2

We consider first the most complicated case which is when the tax rate is sufficiently large that $\theta_2(q, B) \geq \underline{\theta}$ or some illegal cigarettes are bought even with the smoking ban imposed . From Appendix B(i), social welfare can be calculated by assuming that all tax paid by a smoker is returned to the smoker as an individual-specific lump-sum transfer or, using Equation (B.4),

$$W(q,B) \ = \ \int_{\underline{\theta}}^{\theta_2(q,B)} \big(M + H - F + G(\theta^i;p,B)\big) f(\theta^i)\, d\theta^i$$

$$+ \ \int_{\theta_2(q,B)}^{\theta_1(q,B)} \big(M + H + r(\theta^i;q,B) + G(\theta^i;q,B)\big) f(\theta^i)\, d\theta^i \ + \ \int_{\theta_1(q,B)}^{\bar{\theta}} (M+H) f(\theta^i)\, d\theta^i;$$

and

$$W(q,\varphi) \ = \ \int_{\underline{\theta}}^{\theta_2(q,\varphi)} \big(M + H - F + G(\theta^i;p,\varphi)\big) f(\theta^i)\, d\theta^i$$

$$+ \ \int_{\theta_2(q,\varphi)}^{\theta_1(q,\varphi)} \big(M + H + r(\theta^i;q,\varphi) + G(\theta^i;q,\varphi)\big) f(\theta^i)\, d\theta^i \ + \ \int_{\theta_1(q,\varphi)}^{\bar{\theta}} (M+H) f(\theta^i)\, d\theta^i.$$

Noting that $\theta_2(q, B) < \theta_2(q, \varphi)$ and $\theta_1(q, \varphi) = \theta_1(q, B)$,

$$W(q,B) - W(q,\varphi) \ = \ \int_{\underline{\theta}}^{\theta_2(q,B)} \big(G(\theta^i;p,B) - G(\theta^i;p,\varphi)\big) f(\theta^i)\, d\theta^i$$

$$+ \ \int_{\theta_2(q,B)}^{\theta_2(q,\varphi)} \big(r(\theta^i;q,B) \ + \ G(\theta^i;q,B) + F - G(\theta^i;p,\varphi)\big) f(\theta^i)\, d\theta^i$$

$$+ \ \int_{\theta_2(q,\varphi)}^{\theta_1(q,\varphi)} \big(r(\theta^i;q,B) \ + \ G(\theta^i;q,B) - r(\theta^i;q,\varphi) - G(\theta^i;q,\varphi)\big) f(\theta^i)\, d\theta^i. \quad \text{(C.1)}$$

We consider each integral in turn.   From Equation (B.6), each term under the first and third integrals is positive. Considering the second integral, using Equations (B.1), (B.3), (9) and (14)

$$r(\theta^i; q, B) + G(\theta^i; q, B) + F - G(\theta^i; p, \varphi)$$

$$= (q-p)2\,\frac{a-q-\theta^i\alpha}{4\theta^i\beta + b} + (a-q-\alpha)\,2\,\frac{a-q-\theta^i\alpha}{4\theta^i\beta+b} - (b+4\beta)\left(\frac{a-q-\theta^i\alpha}{4\theta^i\beta+b}\right)^2$$

$$+ F - (a-p-\alpha)\,3\,\frac{a-p-\theta^i\alpha}{6\theta^i\beta} + \beta\left(3\,\frac{a-p-\theta^i\alpha}{6\theta^i\beta}\right)^2. \tag{C.1}$$

This can be rewritten as:

$$r(\theta^i; q, B) + G(\theta^i; q, B) + F - G(\theta^i; p, \varphi)$$

$$= F + (a-p-\alpha)\left(\frac{2(a-q-\theta^i\alpha)}{4\theta^i\beta+b} - \frac{a-p-\theta^i\alpha}{2\theta^i\beta}\right) + \frac{\beta}{4\theta^{i^2}\beta^2}(a-p-\theta^i\alpha)^2 - \frac{b+4\beta}{(4\theta^i\beta+b)^2}(a-q-\theta^i\alpha)^2$$

$$= F + \frac{a-p-\alpha}{(4\theta^i\beta+b)2\theta^i\beta}\Big(-4\theta^i\beta(q-p) - b(a-p-\theta^i\alpha)\Big)$$

$$+ \frac{1}{4\theta^{i^2}\beta^2(4\theta^i\beta+b)^2}\Big((4\theta^i\beta+b)^2\beta(a-p-\theta^i\alpha)^2 - 4\theta^{i2}\beta^2(b+4\beta)(a-q-\theta^i\alpha)^2\Big).$$

$F > 0$ or the first term is positive. Considering the second term: From Assumption 1, $a - p - \alpha < 0$. For a smoker, $\theta^i < \theta_1(q, \varphi)$ or $a - p - \theta^i\alpha > a - q - \theta^i\alpha > 0$ so that the second term is positive. In addition, $\theta^i < 1$ implies $(4\theta^i\beta + b)^2\beta = (16\theta^{i2}\beta^2 + 8\theta^i\beta b + b^2)\beta > (16\theta^{i2}\beta^2 + 4\theta^{i2}\beta b)\beta = 4\theta^{i2}\beta^2 (b+4\beta)$ and hence the third term is positive. The right-hand side is positive.

If q is such that $\theta_2(q, B) < \underline{\theta} < \theta_2(q, \varphi)$, the first term of Equation (C.1) is absent and the lower bound of the second term is $\underline{\theta}$; if q is such that $\theta_2(q, \varphi) < \underline{\theta}$, the first and second terms of

Equation (C.1) are absent and the lower bound of the third term is $\underline{\theta}$ (this is the case if $q=p$). As we have shown above, each term is positive and hence, in these two additional cases, $W(q,B) - W(q, \varphi) > 0$.

APPENDIX E: PROOF OF LEMMA 3

Around $q = p$ there are only non-smokers and smokers of legal cigarettes and, using Equation (B.4),

$$W(q,B) = \int_{\underline{\theta}}^{\theta_1(q,B)} \left(M + H + r(\theta^i;q,B) + G(\theta^i;q,B)\right) f(\theta^i)\, d\theta^i + \int_{\theta_1(q,B)}^{\bar{\theta}} (M+H) f(\theta^i)\, d\theta^i ;$$

Substituting for $r(\theta^i; q,B)$, $G(\theta^i; q,B)$ and $\theta_1(q, B)$,

$$W(q,B) = \int_{\underline{\theta}}^{\frac{a-q}{\alpha}} \left(M + H + (a-p-\alpha) 2 \frac{a-q-\theta^i\alpha}{4\theta^i\beta + b} - (b+4\beta)\left(\frac{a-q-\theta^i\alpha}{4\theta^i\beta+b}\right)^2 \right) f(\theta^i)\, d\theta^i + \int_{\frac{a-q}{\alpha}}^{\bar{\theta}} (M+H) f(\theta^i)\, d\theta^i$$

Differentiating with respect to q

$$\frac{dW(q,B)}{dq} = \int_{\underline{\theta}}^{\frac{a-q}{\alpha}} \left(-(a-p-\alpha)\frac{2}{4\theta^i\beta+b} + (b+4\beta)\, 2\, \frac{a-q-\theta^i\alpha}{(4\theta^i\beta+b)^2} \right) f(\theta^i)\, d\theta^i$$

$$+ \left((a-p-\alpha) 2 \frac{a-q-\theta^i\alpha}{4\theta^i\beta+b} - (b+4\beta)\left(\frac{a-q-\theta^i\alpha}{4\theta^i\beta+b}\right)^2 \right)\Big|_{\theta^i = \frac{a-q}{\alpha}} \left(-\frac{1}{\alpha}\right)$$

From Assumption 1, $a - p - \alpha < 0$ and, for $\underline{\theta} \le \theta^i \le (a-q)/\alpha$, $0 \le a - q - \theta^i\alpha$; the first term is positive. The second term is zero. Hence $dW(q,B)/dq\,|_{q=p} > 0$.

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ENDNOTES

1. In Canada in 2008, estimates suggest that 31% of the cigarettes smoked are purchased on the black market (Physicians for a Smoke Free Canada, 2011). In the US, Thursby and Thursby (2000) find that "commercial smuggling [by firms to evade state taxes] accounted for three to four percent of all cigarettes sold during the 1970s, a decade of high excise tax differentials." For the UK, HM Revenue and Customs (2006) estimates that in 2004-05 the market share for illicit cigarettes was between 10% and 19%, and for hand-rolled tobacco was between 53% and 64%.

2. Chapter 2 of Congdon, Kling and Mullainathan (2011) provides an excellent summary.

3. An empirical example of this reasoning is Fryer (2013) who finds that, when middle-school students are given daily text messages advising them of the general benefits of education, they do not change their effort as measured by test scores, attendance or behavioral incidents (although they are able to recall the information contained in the texts).

4. The model suggests that a slower decay rate of nicotine into cotinine implies lower cigarette consumption. As empirical support for this way of thinking, we note that Chinese-Americans have slower decay rates and smoke fewer cigarettes than Americans of European ancestry (Benowitz et al. (2002)).

5. There is a restriction that $Pr \leq 1$. We can impose this restriction by imposing restrictions on α and β . Or we can impose a limit on the number of cigarettes which can be smoked in a day, $c_1 + c_2 + c_3 \leq C_{max}$ and $Pr(C_{max}) \leq 1$. Or we can make ill-health sufficiently sensitive to the number of cigarettes smoked that $Pr(c_1 + c_2 + c_3) \leq 1$ is never binding. For ease of presentation, we assume that the constraint $Pr(c_1 + c_2 + c_3) \leq 1$ does not bind and ignore it.

6. We expect that more educated individuals are likely to be better informed of the expected ill-health from smoking, or that the perception parameters of more educated individuals are more tightly centered on unity, or that (if an individual with $\theta^i = 1$ does not smoke) more educated individuals smoke less. As empirical support for this way of thinking, Irvine and Nguyen (2011) find that smokers who have completed post-secondary education smoke 12% fewer cigarettes than smokers who have failed to complete secondary education.

7. Considering the potential constraint that income exceed cigarette expenditure. We will see that the ban reduces the total cigarettes smoked by a smoker (Observation 3). It follows that, if endowed income M exceeds cigarette expenditure in the absence of a ban, M exceeds cigarette expenditure in the presence of a ban and the constraint need not be imposed.

8. Put differently, with $c_2 = c_3 = 0$, as $c_1 \rightarrow 0$ the variance tends to zero faster than the mean.

9. In contrast, Evans et al. (1999) find that smoking bans reduce participation by 5%.

10. In a more general formulation, the utility cost of buying illegal cigarettes is likely to have a fixed and variable component, $F + V(c_1+c_2+c_3)$. What matters in the subsequent analysis is that $F > 0$. For simplicity and with little loss of generality, we set $V(.) = 0$.

11. The prediction that buyers of illegal cigarettes are heavier smokers has empirical support. Using the Canadian Tobacco Use Monitoring Survey 2008 we find that smokers who do not admit to having made an illegal purchase smoke on average 79 cigarettes per week but smokers who do admit to having made an illegal purchase smoke on average 92 cigarettes per week.

12. Because our assumed utility function implies that cigarette demand does not depend on income, the illegal cigarettes smoked equals the legal cigarettes smoked if the legal price is p. Hence, if F is a utility cost, restricting M as in Appendix A ($M \geq (a-\underline{\theta}\alpha)^2/8\underline{\theta}\beta$) is sufficient to ensure that cigarette expenditure does not exceed income. Alternatively, if M is a monetary cost, the restriction $M\text{-}F \geq (a-\underline{\theta}\alpha)^2/8\underline{\theta}\beta$ is sufficient to ensure that cigarette expenditure does not exceed income.

13. $\theta_2(p, \varphi) = 0$ is a limiting result: $\underset{q \rightarrow p}{Lim}\ \theta_2(q, \varphi) \rightarrow 0$.

An actual smoker has $\theta^i \geq \underline{\theta} > 0$ and $\underline{\theta}$ is such that every smoker spends less on cigarettes than his income. In consequence, the potential constraint $q(c_1+c_2+c_3) \leq M + R$ has not been imposed. The statement "If there were a smoker with $\theta^i = 0$, he would consume an infinite quantity of cigarettes..." should be interpreted as: "if there were a smoker who had $\theta^i=0$ and if he were unconstrained in his cigarette expenditure (i.e. if he could potentially consume negative numeraire), he would at consumer price q consume an infinite number of cigarettes. As $q \rightarrow p$, the utility difference between buying in the legal and in the illegal markets vanishes."