DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 09-13

Search, Heterogeneity, and Optimal Income Taxation

Nikolay Dobrinov

University of Colorado

November 2009

Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

Search, Heterogeneity, and Optimal Income Taxation*

WORKING PAPER

Nikolay Dobrinov
University of Colorado – Boulder

November 9, 2009

Abstract

I derive the optimal income tax schedule on imperfect labor markets with search. In the search framework workers and vacancies decide how intensively to search for partners, and whether to match with a potential partner when they meet one. The private choice on intensity of search affects not only the private expected income of the decision maker, but also the rate at which partners meet and match, as well as the distributions of productivity types among the actively searching workers and vacancies. A searching agent does not take into account these latter, external effects, and the level of her search intensity is not socially optimal. As a result, the level of total production in the economy, is suboptimal. Income taxation can restore the socially optimal search intensities. I show that an optimal income tax system, designed to both control for externalities and raise positive government revenue: (1) rewards/punishes an agent for the externalities she imposes on the rest of the actively searching agents on the market; and (2) takes into account the Ramsey's (1927) elasticity rule to allocate the burden of taxation among agents in the economy.

1 Introduction

The traditional incidence and welfare analysis of income taxation assumes perfect labor markets. In recent years, however, widespread unemployment in Europe led researchers to reconsider the implications of taxes on income within the framework of imperfect labor markets[1]. On imperfect labor markets with search[2], the search intensity choice of a worker affects the matching opportunities of the rest of the workers and vacancies on the market: a worker who

*I am grateful to my advisers Charles De Bartolome, Jeffrey Zax, Yongmin Chen, Donald Waldman, and Thomas Thibodeau for their invaluable advice and suggestions. I also want to thank Anna Rubinchik and Martin Boileau for their help with technical issues.

[1]Søorensen (1997), Van Der Ploeg (1998)). Lockwood and Manning (1993), Bovenberg and van der Ploeg (1994), Holmlund and Kolm (1995), Koskela and Vilmulen (1996), and Kolm (1997) study wage taxation in union bargaining models. Hoel (1990), Pisauro (1991), Fuest and Huber (1997), Stiglitz (1999), and Kleven and Sørensen (1999) in efficiency wage models, and Pissarides (1983, 1985, 1990), Millard and Mortensen (1996), Shi and Wen (1999) and Boone and Bovenberg (2002) in search models. Pissarides (1998) and Sørensen (1999) investigate wage taxation in all of these three types of models.

[2]Imperfect labor markets are modeled in three contexts: union bargaining models, efficiency wage models, and search models. The results of my analysis apply only to search models. As Pissarides (1998) shows, the effects of tax policy and tax reform are different when studied in different contexts. For more details on search models see Diamond (1981, 1982a,b), Mortensen (1982a,b), Pissarides (1984a,b), Hosios (1990), Burdett and Coles (1997), Burdett and Coles (1999), Mortensen and Pissarides (1999), Acemoglu and Shimer (1999), Shimer and Smith (2000), Shimer and Smith (2001), Boone and Bovenberg (2002).

searches more intensively makes it easier for vacancies to meet workers, and more difficult for other workers to meet vacancies. In addition to this, a more productive worker is a preferred partner for a searching vacancy. Because output is shared after the search costs are sunk, the worker is not appropriately awarded for her search efforts. This leads to an equilibrium where low productivity workers search too hard, while high productivity workers do not search hard enough. As a result, the level of total production is sub-optimal. This paper explores the role of the tax system to alleviate labor-market imperfections and to optimally raise revenue. I find that the optimal revenue-generating income tax schedule takes into account the externalities imposed by searching agents on the rest of the participants on the market. In particular, an agent who imposes a net positive externality is awarded by sharing less of the burden of raising the required by the government revenue. In doing so, the tax system restores the search intensity efforts at their socially optimal levels, and still raises, the required for the production of the public good, revenue.

The literature on labor taxation has focused largely on tax reform, whereas I study the optimal design of the tax system. While recognizing that a more progressive tax system may cut unemployment (Koskela and Vilmulen (1996), Pissarides (1998)), but may also raise costs, the literature on taxation in imperfect labor markets has rarely discussed the optimal trade-off between the costs and the benefits of a more progressive tax system[3]. A rare exception are Boone and Bovenberg (2002) who explore optimal income taxation in a search model with homogeneous in productivity workers and vacancies. They show that the externality controlling task of the tax system is independent from the revenue generating task. Furthermore, they find that the government can successfully distribute the tax burden between firms and vacancies by taxing each worker(firm) at a rate proportional to the inverse of its elasticity of supply/demand.

This optimal trade-off between equity and efficiency, when designing a tax system, and the characteristics of the optimal tax schedule are studied very extensively in the context of perfect labor markets. In these models workers are assumed to differ in productive skill, which is not observable to the government in the process of designing the optimal income tax schedule[4]; the government designs the tax system using endogenous variables like income and consumption. Because productivity is not observed, a worker can pretend to be of different productivity type to lower her exposure to the tax. The self-selection constraints that the

[3]For exceptions see Sørensen (1999), and Boone and Bovenberg (2002)

[4]Mirrlees (1971), Sheshinski (1972), Cooter (1978), Phelps (1973), Feldstein (1973), Stiglitz (1981), Stiglitz (1987), and Diamond (1998), among others.)

government has to consider when designing the tax system, lead to a distortion associated with redistributing any significant amount of resources from the more able to the less able. Mirrlees (1971) finds that there is a clear trade-off between efficiency and equity, and less support for the progressivity of the optimal income tax than predicted by Edgeworth (1897)[5]. The main feature of the results is that the optimal tax schedule depends on the distribution of skills within the population, and on the labor-consumption preferences of the population, in such a complicated way that it is not possible to say in general whether marginal tax rates should be higher for high-income, low-income or intermediate-income groups[6].

Some of the strongest results that emerge from the literature on optimal income taxation (See Cooter (1978)) are that: a worker with higher productive skill enjoys at least as high utility as a person with low productive skill; the marginal tax rate on income is less than one; the marginal tax rate on income is nil at the top and bottom of the skill distribution; with respect to income levels there is a zone with increasing marginal tax rates and a zone with decreasing marginal tax rates; the optimal tax on any good is inversely proportional to its elasticity of demand[7].

I build on the search literature and the literature on optimal income taxation on perfect labor markets. In my model, workers and vacancies are heterogeneous in productive skill and the government does not observe the productivity type of each agent when designing the optimal income schedule. I can identify three main contributions to the literature on imperfect labor markets.

First, I simplify the workhorse search model of Mortensen and Pissarides[8] by formulating a static, one-shot, game to facilitate interpretation of the results. I further simplify the model by sidestepping the matching dimension, and focusing on the search externalities that arise when workers and vacancies decide how intensively to search for partners. These simplifications make the derivations of the optimal tax system tractable, while the main failure of labor markets, as described in the dynamic models, is still preserved.

Second, I expand on the model of Boone and Bovenberg (2002) by allowing workers and

[5]Ignoring the incentive effects associated with taxation, Edgeworth tried to show that Utilitarianism implied progressivity: if all individuals had the same utility of income functions, which exhibited diminishing marginal utility, then the decrease in social welfare from taking a dollar away from a poor person was more than the decrease in social welfare from taking a dollar away from a rich person.

[6]Mirrlees (1971) assumes a utility function $U = log(x) + log(1 - y)$, where x is consumption and y is labor supply, and log-normal distribution of skills, and finds that the tax schedule looks close to linear. However, Sheshinski (1972) and Diamond (1998) conclude that simulation results are sensitive to both the utility function and the family of distributions of skills assumed, which opens up the possibilities of different conclusions.

[7]Ramsey (1927). In the case of optimal income taxes see Diamond (1998), and Boone and Bovenberg (2002).

[8]Again, see Mortensen (1982a,b), Pissarides (1984a,b), Mortensen and Pissarides (1999)

agenscies to be heterogeneous in productivity type. This extension allows me to more deeply study the externalities that arise on imperfect labor markets, some of which are missing on markets with homogeneous in productivity workers. When a worker increases her intensity of search she makes it more difficult for other workers to meet vacancies (the congestion externality), and makes it easier for vacancies to meet workers (the thick-market externality). When workers and vacancies are of different productivity types, however, the externalities imposed by a searching worker are more involved, because by marginally increasing her intensity of search the worker also makes it more difficult for the vacancy to meet a worker of the other type - a congestion externality if the worker is of low productivity type, and a thick-market externality if the worker is of high productivity type.

Discussing optimal income taxes is also more meaningful when workers differ in productivity. Note that in a model with homogeneous in productivity type workers and firms, Mortensen (1982), Hosios (1990) and Acemoglu and Shimer (1999) identify that equating the agent's bargaining power to the elasticity of the matching function (her contribution to the match), ensures efficient levels of search intensities on both sides of the market. However, as demonstrated by Shimer and Smith (2001) and by the analysis in this paper, when workers and firms are of ex-ante different productivity types, a generalized output sharing rule is not always sufficient to decentralize the social optimum. In the absence of externality-correcting taxes the decentralized equilibrium is often inefficient.

The assumption of heterogeneous in productivity type agents also allows me to study the progressivity of the optimal income tax in the context of imperfect labor markets. I assume that both supply and demand are elastic. When workers and vacancies differ in productive skill, they search with different intensities, and the elasticities of their search effort, with respect to the rewards of search, depend on the productive skill of the worker or vacancy. The set of elasticities can tell us something about the progressivity of the tax system. In my model the elasticity of supply/demand is lower for the workers/vacancies who search more intensively in equilibrium. It turns out that in all equilibria higher productivity types search more intensively, which suggests a progressive element in the optimal tax system if the relative elasticities are inversely related to relative tax rates.

Third, I provide new intuition on the usefulness of optimal income taxation in alleviating labor market imperfections in search models. By efficiently allocating bargaining power, the tax system acts as a substitute for complete contracts in protecting the optimal incentives for search activities, while still raising revenue. More importantly, my analysis reveals how the externality alleviating role of the tax system interacts with the revenue generating task of the system. Boone and Bovenberg (2002), in their model with homogeneous in productivity agents, find that the externality controlling part of the tax can be separated from the revenue raising part of the tax. I study the optimal total tax rate, and show that the externality controlling part of the tax rate is incorporated within the total tax rate, and is a natural part of what determines the tax burden faced by a worker or vacancy of a given skill type.

Using Pigou income taxes, I find that there are two main externalities that arise in my model. The first externality is related to the ability of an agent to create a match. A more able agent is not rewarded fully for her contribution in creating the match, because the bargaining process depends only on the predetermined bargaining power of each potential partner. This sends a wrong signal to the worker on the return to search. Pigou taxes reward agents who are more productive in creating a match and punishes agents who have too much bargaining power (inconsistent with their ability to create a match).

The second externality that arises in the search process, is related to the effect of the intensity of search on the distributions of productivity types on each side of the market. When a worker of high type increases her intensity of search, she changes the distribution of actively searching workers in a favorable way from the point of view of the vacancy, because it increases the probability that the vacancy will meet a highly productive worker. The opposite holds for workers of low type. Because search efforts are held up, high types under-search and low types over-search in the private equilibrium, leading to suboptimal levels of production. Pigou taxation restores the socially optimal levels of search intensity while retaining a balanced budget.

Using linear income taxes to decentralize the social optimum and raise a predetermined level of government revenue I show that the optimal tax system is composed of an element that restores the socially optimal level of search intensities, and an element that raises the required revenue. Since high productivity type imposes a net positive externality, and a low productivity type imposes a net negative externality, the element that restores efficiency on the search market suggests a more regressive tax system.

The second major result that arises from optimal income taxation with positive government revenue is that the relative tax rates are inversely related to the relative elasticities of search activity. High productivity agents search more intensively in the social optimum, and because the elasticity of search activity decreases in the equilibrium search intensity, the revenue raising element of the optimal tax suggests a progressive tax system.

Whether the optimal tax system on imperfect labor markets with search is actually progressive or regressive depends on the shape of the search intensity cost function (preferences), and on the shape of the production function. The slower the search costs rise, and the larger the difference between the marginal contribution to a partnership by a high productivity type and the marginal contribution to a partnership by a low productivity type, the more dominant the regressive component will be.

2 Model

The economy is populated by workers and vacancies, which within their own group differ in productive skill. For simplicity the productive skill types on each side of the market are assumed to be two - high (H) and low (L) type. The exogenous number of workers in the economy is l_k, for $k = H, L$, and the exogenous number of vacancies is q_m, for $m = H, L$. Workers and vacancies have two options each period - either to participate on a labor market and form bilateral partnerships to produce an exogenously determined flow output of $y_{km} > 0$, or to not participate on the market and receive an income of zero. There is no restriction to the production function of the partnership except the meaning we imply by the notion of difference in the productivity of the partnership, $y_{Hm} > y_{Lm}$.

In the beginning of each period workers search for vacancies at a self-selected search intensity $\delta_k \in [0, 1]$, which can be interpreted as the probability of search during the period. By searching workers incur a search cost $c_w(\delta_k)$, which increases in their intensity of search. To ensure that a worker selects a unique and positive search intensity in equilibrium, and that this intensity is lower than unity, the cost function is assumed to be continuous and strictly convex, with $c_w(0) = 0$, $c'_w(0) = 0$, and $\lim_{\delta_k \to 1} c'_w(\delta_k) = +\infty$. Similarly, in the beginning of each period vacancies search with an intensity v_m, and incur a search cost $c_\pi(v_m)$ sharing the same characteristics as the search cost function of workers.

A worker or vacancy who searches with positive intensity meets at most one potential partner from the opposite side of the market within the period. The probability that a worker meets a vacancy during the period is λ, and is positively related to the number of vacancies on the market, and negatively related to the number of workers on the market. Similarly, the probability that a vacancy meets a worker during the period is ϕ, and is negatively related to the number of vacancies on the market, and positively related to the number of workers on the market.

Once a worker meets a vacancy the parties perfectly observe the potential output of the partnership and the shares each of them receives, and match for sure (since the outside market alternative is absent). At this stage the search process ends and the matched pairs produce until the end of the period, while the unmatched agents stay idle. At the end of the period all matches dissolve. The game repeats the next period and workers choose their strategies independently from the strategies played, and outcomes reached, in previous periods.

2.1 The matching technology

The probabilities of encounter, λ and ϕ, are determined by the matching technology, which describes the relation between inputs, search and recruiting activity, and the output of the matching process, the number of encounters and matches per period.

The assumption that each prospective worker meets prospective employers with probability λ implies that the expected aggregate number of unemployed workers who meet vacant jobs within the period is equal to $\lambda \sum_k \delta_k l_k$. Similarly, the assumption that each vacancy is visited by workers with probability ϕ, implies that the expected aggregate number of vacancies who are visited by unemployed workers within the period is equal to $\phi \sum_m v_m q_m$. In equilibrium the number of workers who meet vacancies must be equal to the number of vacancies who meet workers within the period. The identity $\lambda \sum_k \delta_k l_k = \phi \sum_m v_m q_m$ requires that the probabilities of encounter, λ and ϕ, are functions of the measures of market participation, $\sum_k \delta_k l_k$ and $\sum_m v_m q_m$.

The problem is solved by introducing an encounter function $N(\sum_k \delta_k l_k, \sum_m v_m q_m, \alpha, \beta)$, which measures the number of encounters/matches in the economy per period, and is such that

$$\lambda \sum_k \delta_k l_k = N(\sum_k \delta_k l_k, \sum_m v_m q_m, \alpha, \beta) = \phi \sum_m v_m q_m.$$

The encounter function N depends on the number of actively searching workers in the economy, $\sum_k \delta_k l_k$, the number of actively searching vacancies in the economy, $\sum_k \delta_k l_k$, the effectiveness of workers to create matches, α, and the effectiveness of vacancies to create matches, β. The encounter function is increasing in each of its arguments and can take various functional forms.

The functional form can be derived from genuine specifications of the meeting process. The most common functional form is the constant returns to scale Cobb-Douglass matching function, $N = A\left(\sum_k \delta_k l_k\right)^{\alpha} \left(\sum_m v_m q_m\right)^{1-\alpha}$, where $0 < \alpha < 1$ measures the effectiveness of workers in creating matches, and $1-\alpha$ measures the effectiveness of vacancies in creating matches. As Mortensen and Pissarides (1999) note, the meeting process that might generate such an encounter function is not known. However, Pissarides (1996) and Blanchard and Diamond (1989) provide empirical justification for a widely used Cobb-Douglass CRS encounter function with $\alpha \sim 0.5$. Since a Cobb-Douglass matching function fits the data well, I assume that the matching function is of the Cobb-Douglass form.

Key Assumption 1: *The encounter function takes the form*

$$N = A\left(\sum_k \delta_k l_k\right)^{\alpha}\left(\sum_m v_m q_m\right)^{1-\alpha} \quad \textit{with } 0<\alpha<1, \ \textit{ and } A \leq 1$$

In my discrete setting this function models a matching probability that is less than unity; an entering the market pair of potential partners increases the number of matches, M, by less than one.

Denoting $\theta = \sum_m v_m q_m / \sum_k \delta_k l_k$ to measure the market tightness, the ratio of actively searching vacancies to actively searching workers on the market, we can express the probabilities of encounter as:

$$\lambda = \frac{N}{\sum_k \delta_k l_k} = A\left(\frac{\sum_m v_m q_m}{\sum_k \delta_k l_k}\right)^{1-\alpha} = M(\theta) \tag{1}$$

$$\phi = \frac{N}{\sum_m v_m q_m} = A\left(\frac{\sum_m v_m q_m}{\sum_k \delta_k l_k}\right)^{1-\alpha} / \frac{\sum_m v_m q_m}{\sum_k \delta_k l_k} = M(\theta)/\theta, \tag{2}$$

where M is a function of θ.

One can then show that the elasticity of the matching function, with respect to the number of actively searching vacancies on the market, is

$$1-\alpha = \frac{M'(\theta)}{M(\theta)} / \frac{1}{\theta}, \tag{3}$$

and represents the effectiveness of vacancies in creating matches.

2.2 Output sharing

When a worker and a firm meet they immediately observe the level of joint output, and match if each of them receives more from production than from their outside option. I assume that any partnership produces positive output and that the outside option is zero, so that all partners match when they meet.

The payoff generated by the partnership is split by a Nash bargain. The parties bargain over the total output y_{km}, with the worker receiving a wage w_{km} and the firm receiving a profit $\pi_{km} = y_{km} - w_{km}$, when the output is not taxed. Importantly, the sequence of decisions is such that wages are negotiated after the search cost (and efforts) of the worker have been sunk. The intensity of search of each partner has no role in the bargaining process. The wage and profit that maximize the Nash bargaining function $w_{km}^{\psi_{km}}(y_{km} - w_{km})^{1-\psi_{km}}$ are

$$\begin{aligned} w_{km} &= \psi_{km} y_{km} \\ \pi_{km} &= (1 - \psi_{km}) y_{km}, \end{aligned} \tag{4}$$

where $\psi_{km} \in [0, 1]$ denotes $k's$ share of the total output y_{km}, and $1 - \psi_{km} \in [0, 1]$ denotes the share of the vacancy. In the search-and-matching literature, the share parameter ψ_{km} is interpreted as the bargaining power of a worker of a skill type k in a partnership with a firm of skill type m. Under individual rationality and a Nash Bargain a necessary and sufficient condition for a match to form is that $y_{km} \geq 0$.

2.3 Sequence of events

The model is static and the sequence of events within a period is the following: In a stage zero, the government sets a tax policy, with the zero stage non-existent in a laissez-fair equilibrium. In the first stage, workers and vacancies choose intensity of search. In the second stage, potential partners meet and match. In the third stage, the matched agents produce jointly, the unmatched agents do not produce and exit the market. In the fourth stage, all matches formed in stage three dissolve.

2.4 Private expected utility functions

The probability that the firm encountered by a worker is of, say, high type is $v_H q_H / \sum_m q_m$. The expected utility of a worker of type k within a period is

$$U_k = -c_w(\delta_k) + \delta_k \left\{ M(\theta) \left(\frac{v_H q_H}{\sum_m v_m q_m} \psi_{kH} y_{kH} + \frac{v_L q_L}{\sum_m v_m q_m} \psi_{kL} y_{kL} \right) + (1 - M(\theta))0 \right\} + (1 - \delta_k)0$$

$$U_k = -c_w(\delta_k) + \delta_k M(\theta) E_{(m)} \psi_{km} y_{km}, \tag{5}$$

where $E_{(m)}$ denotes the expectation under the distribution of skill types among vacancies in the economy. A worker chooses her intensity of search δ, which determines her cost of searching, $c(\delta)$ and the probability of being on the market, δ. Once on the market, the worker meets a potential partner with a probability $M(\theta)$, and the type of the vacancy she meets depends on the distribution of vacancy productivity types in the economy. If, with probability $1 - M(\theta)$, the worker searches intensively, but does not meet a vacancy once on the market, she does not produce and exits the market. If the worker searches with itensity of zero, she receives no income. Since the outside-market option of the worker is zero, whenever the worker encounters a potential partner she matches for sure. Note that the utility function of the worker can be viewed as a classical separable utility function in consumption and labor supplied. The first part of the function describes the dis-utility of the worker from giving up leisure, $[-c(\delta_k)]$, and the second part of the function is her expected consumption given how much labor she supplies to the market, $[\delta_k M(\theta) E_{(m)} \psi_{km} y_{km}]$.

Similar logic applies in determining the expected utility of a vacancy. The proportion of actively searching workers of high type, among all actively searching workers on the market, is $\delta_H l_H / \sum_k \delta_k l_k$. The expected utility of a vacancy of type m within a period is

$$\begin{aligned} V_m = & -c_\pi(v_m) \\ & + v_m \left\{ \frac{M(\theta)}{\theta} \left[\frac{\delta_H l_H}{\sum_k \delta_k l_k} (1 - \psi_{Hm}) y_{Hm} + \frac{\delta_L l_L}{\sum_k \delta_k l_k} (1 - \psi_{Lm}) y_{Lm} \right] + \left(1 - \frac{M(\theta)}{\theta} \right) 0 \right\} \\ & + (1 - v_m)0 \end{aligned}$$

$$V_m = -c_\pi(v_m) + v_m \frac{M(\theta)}{\theta} E_{(k)} (1 - \psi_{km}) y_{km} \tag{6}$$

A vacancy meets a potential partner with a probability $\phi = M(\theta)/\theta$, and the type of worker she meets depends on the distribution of actively searching types of workers. Since the outside-market option of the vacancy is zero, whenever the vacancy encounters a potential partner she matches for sure.

3 Optimal search intensity and market inefficiencies

In this section I derive the optimal search intensities in the social optimum and decentralized equilibrium, which do not coincide in general due to uninternalized externalities in the decentralized equilibrium.

3.1 Social Optimum

A social planner maximizes a laissez-fair Utilitarian welfare function - a sum of the expected utilities of all participants in the economy per period, with respect to search intensities:

$$W = \max_{\delta,v}\left\{\sum_k l_k U^k + \sum_m q_m V^m\right\}$$
$$\text{s.th.} \quad \delta_k \geq 0, \quad v_m \geq 0.$$

Using (1), (2), (5), and (6), and re-arranging gives[9]

$$W = \max_{\delta,v}\left\{\sum_k l_k[-c_w(\delta_k)] + \sum_m q_m[-c_\pi(v_m)] + NE_{(k)}E_{(m)}y_{km}\right\} \quad \text{s.th.} \quad \delta_k \geq 0, \quad v_m \geq 0 \tag{7}$$

where $NE_{(k)}E_{(m)}y_{km}$ represents the total social benefit (TSB) from search (total output if search intensities are at the socially optimal levels), and $\sum_k l_k[-c(\delta_k)] + \sum_m q_m[-c_\pi(v_m)]$ represents the total social cost of search (TSC).

[9]Using the definitions of λ and ϕ from equations (1) and (2), one can rewrite the objective function as

$$W = \max_{\delta,v}\left\{\sum_k l_k[-c_w(\delta_k)] + \sum_m q_m[-c_\pi(v_m)] + \frac{N}{\sum_k \delta l}\sum_k \delta_k l_k E_{(m)}\psi_{km}y_{km} + \frac{N}{\sum_m vq}\sum_m v_m q_m E_{(k)}(1-\psi_{km})y_{km}\right\}$$
$$= \max_{\delta,v}\left\{\sum_k l_k[-c(\delta_k)] + \sum_m q_m[-c_\pi(v_m)] + NE_{(k)}E_{(m)}\psi_{km}y_{km} + NE_{(k)}E_{(m)}(1-\psi_{km})y_{km}\right\}.$$

The Kuhn-Tucker conditions with respect to δ_k are[10]

$$-c_w'(\bar{\delta}_k) + M(\theta)[E_{(m)}y_{km} - (1-\alpha)E_{(k)}E_{(m)}y_{km}] \leq 0 \qquad \text{for } k=H,L \tag{8}$$

$$\bar{\delta}_k \geq 0 \qquad \text{for } k=H,L \tag{9}$$

$$\left(-c'(\bar{\delta}_k) + M(\theta)[E_{(m)}y_{km} - (1-\alpha)E_{(k)}E_{(m)}y_{km}]\right)\bar{\delta}_k = 0 \qquad \text{for } k=H,L, \tag{10}$$

where $\bar{\delta}$ and $\bar{\theta}$ denote the socially optimal search intensity of a worker and the socially optimal market tightness. Except for some special production functions $\bar{\delta}_k > 0$, for $k = H, L$, and equations (8) hold with a strict equality.

It is still possible, however, that low type[11] is not hired in the social optimum, $\bar{\delta}_L = 0$. This is the case for some production functions. Consider the following example from Shimer and Smith (2006): $y_{HH} = 1$, $y_{LL} = \epsilon$, $y_{LH} = \epsilon(1+\epsilon)$, with a convex search cost function $c(0) = c'(0) = 0$. If ϵ is sufficiently small it is not optimal for low type workers to search at all.

Proposition 1a. *In the social optimum the search intensities of workers are determined by*

$$\begin{aligned} c_w'(\bar{\delta}_H) &= M(\bar{\theta})[E_{(m)}y_{Hm} - (1-\alpha)E_{(k)}E_{(m)}y_{km}] \\ c_w'(\bar{\delta}_L) &= M(\bar{\theta})[E_{(m)}y_{Lm} - (1-\alpha)E_{(k)}E_{(m)}y_{km}] \end{aligned} \quad \textit{for} \quad \left| \begin{array}{l} \bar{\theta} \lesseqgtr 1, \\ \bar{\delta}_H > 0,\ \bar{\delta}_L > 0 \end{array} \right. , \tag{11}$$

$$\begin{aligned} c_w'(\bar{\delta}_H) &= M(\bar{\theta})[E_{(m)}y_{Hm} - (1-\alpha)E_{(k)}E_{(m)}y_{km}] \\ c_w'(0) &\geq M(\bar{\theta})[E_{(m)}y_{Lm} - (1-\alpha)E_{(k)}E_{(m)}y_{km}] \end{aligned} \quad \textit{for} \quad \left| \begin{array}{l} \bar{\theta} \lesseqgtr 1, \\ \bar{\delta}_H > 0,\ \bar{\delta}_L = 0 \end{array} \right. . \tag{12}$$

To easily interpret equation (11) observe that the effective (socially optimal) wage of the

---

[10]
$$\begin{aligned} \frac{\partial W}{\partial \delta_H} &= l_H(-c_w'(\delta_H)) + N\frac{l_H}{\sum_k \delta l}\alpha E_{(k)}E_{(m)}y_{km} \\ &+ N\left[\frac{l_H}{\sum_k \delta l}E_{(m)}y_{Hm} - \frac{l_H}{\sum_k \delta l}E_{(k)}E_{(m)}y_{km}\right], \end{aligned}$$

noting that $\partial M/\partial \delta_H = M(l_H/\sum_k \delta l)\alpha$.

[11] If a worker of a given type is not hired in the economy it must be the worker of low productive skill, because she generates a lower level of expected output in the economy; for the same cost function, the marginal social benefit of search intensity of low type is always below the marginal social benefit from increasing the intensity of high type.

worker can be written as

$$E_{(m)}y_{km} - (1-\alpha)E_{(k)}E_{(m)}y_{km} = \alpha E_{(k)}E_{(m)}y_{km} + E_{(m)}y_{km} - E_{(k)}E_{(m)}y_{km}.$$

A worker of high type (low type) receives a share of the expected per match output proportional to the average ability α of workers to create matches ($\alpha E_{(k)}E_{(m)}y_{km}$), plus the difference (extra income (or loss) for the economy) between the generated output from a partnership with this type of worker, $E_{(m)}y_{km}$, and the output generated by the average partnership in the economy, $E_{(k)}E_{(m)}y_{km}$. Thus, a social planner considers both the ability of the worker to create matches and the ability of the worker to favorably (or negatively) affect the distribution of skills among actively searching workers.

Similarly one can derive the socially optimal search intensities of vacancies:

Proposition 1b. *In the social optimum the search intensities of vacancies are determined by*

$$\begin{aligned} c'_{\pi}(\bar{v}_H) &= \frac{M(\bar{\theta})}{\bar{\theta}}[E_{(k)}y_{kH} - \alpha E_{(k)}E_{(m)}y_{km}] \\ c'_{\pi}(\bar{v}_L) &= \frac{M(\bar{\theta})}{\bar{\theta}}[E_{(k)}y_{kL} - \alpha E_{(k)}E_{(m)}y_{km}] \end{aligned} \qquad \mathit{for} \qquad \left| \begin{array}{l} \bar{\theta} \lesseqgtr 1, \\ \bar{v}_H > 0,\ \bar{v}_L > 0 \end{array} \right., \tag{13}$$

$$\begin{aligned} c'_{\pi}(\bar{v}_H) &= \frac{M(\theta)}{\theta}[E_{(k)}y_{kH} - \alpha E_{(k)}E_{(m)}y_{km}] \\ c'_{\pi}(0) &\geq \frac{M(\theta)}{\theta}[E_{(k)}y_{kL} - \alpha E_{(k)}E_{(m)}y_{km}] \end{aligned} \qquad \mathit{for} \qquad \left| \begin{array}{l} \bar{\theta} \lesseqgtr 1, \\ \bar{v}_H > 0,\ \bar{v}_L = 0 \end{array} \right.. \tag{14}$$

3.2 Decentralized equilibrium

A worker of type k maximizes her expected utility by choosing her privately optimal intensity of search

$$\begin{aligned} \max_{\delta_k} \ U_k = &-c_w(\delta_k) + \delta_k M(\theta)E_{(m)}\psi_{km}y_{km} \\ &\text{s.th.} \quad \delta_k \geq 0, \end{aligned} \tag{15}$$

where $PC_k = c_w(\delta_k)$ is the personal cost of search, and $PB_k = \delta_k M(\theta)E_{(m)}\psi_{km}y_{km}$ is the personal benefit from search.

The Kuhn-Tucker conditions with respect to δ_k are[12]

$$\begin{aligned} -c'_w(\tilde{\delta}_k) + M(\tilde{\theta})E_{(m)}\psi_{km}y_{km} &\leq 0 \\ \tilde{\delta}_k &\geq 0 \\ (-c'_w(\tilde{\delta}_k) + M(\tilde{\theta})E_{(m)}\psi_{km}y_{km})\tilde{\delta}_k &= 0, \end{aligned} \tag{16}$$

where $\tilde{\delta}_k$ and $\tilde{\theta}$ denote the privately optimal search intensities and market tightness, in the decentralized equilibrium. The worker takes as given the observed on the market probability of meeting a vacancy, $M(\theta)$, and does not internalize the externality she imposes on all actively searching agents on the market by changing the equilibrium market tightness. Furthermore, since a worker does not take into consideration how her behavior affects the utility of a vacancy, in her decision to increase her intensity of search she does not consider how she affects the distribution of productive skills of the actively searching workers in the economy.

In the decentralized equilibrium all types of workers search with strictly positive intensities. To see this note that $\psi_{LL} \neq 0$ (even if $\psi_{LH} = 0$), and because we assumed that $y_{km} > 0$, the personal marginal benefit from increasing the intensity of search is always positive. Given that the marginal cost of search is zero only at search intensity of zero, $c'(0) = 0$, then $\tilde{\delta}_k > 0$. Using Kuhn-Tucker condition (16) we can state the most important result for this section

Proposition 2a. *In the decentralized equilibrium the search intensities of workers are determined by*

$$\left. \begin{aligned} c'_w(\tilde{\delta}_H) &= M(\tilde{\theta})E_{(m)}\psi_{Hm}y_{Hm} \\ c'_w(\tilde{\delta}_L) &= M(\tilde{\theta})E_{(m)}\psi_{Lm}y_{Lm} \end{aligned} \quad \text{for} \quad \right| \begin{aligned} &\tilde{\theta} \lesseqgtr 1, \\ &\tilde{\delta}_H > 0,\ \tilde{\delta}_L > 0 \end{aligned} \quad . \tag{17}$$

Similarly, the search intensities privately selected by vacancies are:

[12]The personal marginal benefit of search with respect to search intensity is

$$\begin{aligned} PMB_k &= \lambda E_{(m)}\psi_{km}y_{km} + \delta_k \left(M'(\theta)\frac{\partial\theta}{\partial\delta_k} \right) E_{(m)}\psi_{km}y_{km} \\ &= \lambda E_{(m)}\psi_{km}y_{km}. \end{aligned}$$

The last equality follows from the assumed in the externalities literature notion, that in the competitive equilibrium a worker perceives herself too small, compared to the whole economy, to be able to affect the probability of encounter for workers, $M(\theta)$, by her decision to change her private intensity of search, $\partial\theta/\partial\delta_k = 0$.

Proposition 2b. *In the decentralized equilibrium the search intensities of vacancies are determined by*

$$\left.\begin{aligned} c'_\pi(\tilde{v}_H) &= \frac{M(\tilde{\theta})}{\tilde{\theta}} E_{(k)}(1-\psi_{kH})y_{kH} \\ c'_\pi(\tilde{v}_L) &= \frac{M(\tilde{\theta})}{\tilde{\theta}} E_{(k)}(1-\psi_{kL})y_{kL} \end{aligned} \quad \text{for} \quad \right| \begin{aligned} &\tilde{\theta} \lesseqgtr 1, \\ &\tilde{v}_H > 0,\ \tilde{v}_L > 0 \end{aligned} \quad . \tag{18}$$

Corollary 3. *The decentralized equilibrium is unique.*

See the proof to Corollary 3 in the Appendix. In her decision how intensively to search, by not being able to affect the market encounter rate, a worker employs her strictly dominant strategy (given the optimal strategies employed by the rest of the workers and vacancies on the market), considering only her personal payoff from her strategy. This is unlike in the social optimum, where the social marginal benefit of the worker accounts for (at least part of) the surplus enjoyed by the vacancy, and the effect of the worker's choice on the distribution of skill types among workers. Still it is not clear whether a worker of a given type under or over-searches in the decentralized equilibrium as this depends on a set of parameters.

Proposition 4. *For a given, constant across types, output share, ψ, workers of high type are favored in the economy and search more intensively than the less favored, low type workers, in both the decentralized equilibrium and the social optimum.*

4 Employing optimal income taxes to decentralize the social optimum

Because output shares are determined after search efforts are sunk, uninternalized externalities lead to discrepancies between the resulting social optimum and market equilibrium. In the language of the search literature search efforts are held up. To directly discuss the externalities that arise we need to be able to directly compare the first order conditions (11) to (17), and (12) to (18). This requires a knowledge on the resulting socially optimal and decentralized equilibrium market tightness, $M(\bar{\theta}) \lesseqgtr M(\tilde{\theta})$, which is hardly possible. It is convenient to describe externalities via Pigou taxation, which sets $\bar{\theta} = \tilde{\theta}$, $\bar{v} = \tilde{v}$, and $\bar{\delta} = \tilde{\delta}$. In what follows I first discuss the feasibility of income taxes on imperfect labor markets with heterogeneous in productivity workers and vacancies. Next, I use Pigou taxation to describe the externalities that arise, assuming that the government can perfectly observe productivity types and can use lump sum transfers as an instrument to return the generated revenue(or to raise the needed net subsidy) from the Pigou tax. Last, I derive optimal income taxes that serve two purposes: to decentralize the social optimum and raise a positive government revenue. In this last part, I assume that the government does not observe productivity types and can not use lump sum transfers.

In the income taxation literature a worker of type k (a vacancy of type m) varies her labor supply (labor demand) in response to the imposed income tax. Similarly, in this model the first order conditions that determine search intensities can be interpreted as the labor supply (labor demand) functions of the worker (vacancy) in the market equilibrium or social optimum. By choosing her intensity of search, the worker actually chooses what proportion of her one unit of labor to supply to the market. The government (the social planner) does not observe the labor supply or the contracted wage rate. Instead, the social planner only observes the total income received from a worker and can only use total income as a tax base.

Since a worker meets a low or a high type vacancy with certain probabilities, the payoff from a match is match-contingent and the government observes the income of a worker from the match with the particular vacancy. Ideally we would expect the government to levy match-based income taxes, and the worker to face some form of an expected tax, based on her expected income. However, only expected after-tax income plays a role in private strategic decisions, and in this version of the model, for simplicity I assume that the social planner observes the expected income per period and uses expected income as a tax base. To see that this is a plausible assumption observe first that when high type worker does not pretend to be a low type worker her expected income is $\delta_H M(\theta) E_{(m)} w_{Hm}$. When (and if) a high type worker pretends to be a low type worker her expected income is $\left(\delta_L \frac{E_{(m)} w_{Lm}}{E_{(m)} w_{Hm}}\right) M(\theta) E_{(m)} w_{Hm} = \delta_L M(\theta) E_{(m)} w_{Lm}$, searching with an intensity $\left(\delta_L \frac{E_{(m)} w_{Lm}}{E_{(m)} w_{Hm}}\right)$ to receive the expected income of low type $\delta_L M(\theta) E_{(m)} w_{Lm}$[13]. The government then observes exactly two levels of expected income for workers in the economy, $\delta_H M(\theta) E_{(m)} w_{Hm}$ and $\delta_L M(\theta) E_{(m)} w_{Lm}$, and can design the tax schedule to offer only two tax levels: τ_H^w, when the observed expected income is $\delta_H M(\theta) E_{(m)} w_{Hm}$, and τ_L^w, when the observed expected income is $\delta_L M(\theta) E_{(m)} w_{Lm}$.

The feasibility of income taxes on expected income also depends on the information set

[13] A high type worker, for example, has to search less intensively than a low type worker to achieve the same expected income as a low type worker.

shared by potential partners during the bargaining process: each side must perfectly observe the tax rates used by the government on the match-based income of their partner. This requires an employer to perfectly observe the search intensity of the worker she bargains with. Delipalla and Keen (1992), as well as Boone and Bovenberg (2002), show that in contrast to competitive labor markets, on imperfect labor markets ad valorem and specific taxes lead to different allocative effects; tax incidence is shared only with an ad valorem tax or with specific taxes on both workers and employers[14]. Using specific taxes, levied on each side of the market, the worker and vacancy effectively bargain over the pre-tax output and pay their taxes based on their pre-tax output shares, which are not altered by taxation. Thus a bargaining employer need not observe the intensity of search of the worker to determine the sharing rules. To see this, assume that in a partnership km the worker pays taxes at a specific tax rate[15] τ_k^w and the employer pays taxes at a specific tax rate τ_m^π. Then the after-tax wage, $\ddot{w}_{km}$, and after-tax profit, $\ddot{\pi}_{km}$, are

$$\begin{aligned}\ddot{w}_{km} &= w_{km} - \frac{\tau_k^w \delta_k M(\theta) w_{km}}{\delta_k M(\theta)} = (1-\tau_k^w) w_{km} \\ \ddot{\pi}_{km} &= (y_{km} - w_{km}) - \frac{\tau_m^\pi v_m (M(\theta)/\theta)\,(y_{km} - w_{km})}{v_m (M(\theta)/\theta)} = \left(y_{km} - \frac{\ddot{w}_{km}}{1-\tau_k^w}\right)(1-\tau_m^\pi)\end{aligned}$$

The parties bargain over the total after-tax output choosing an optimal after-tax wage rate:

$$\max_{\ddot{w}_{km}} \, (\ddot{w}_{km})^{\psi_{km}} \left[\left(y_{km} - \frac{\ddot{w}_{km}}{1-\tau_k^w}\right)(1-\tau_m^\pi)\right]^{1-\psi_{km}}. \tag{19}$$

This problem however is equivalent to the one where the potential partners choose the pretax wage rate to maximize the post-tax output

$$\max_{w_{km}} \, (w_{km})^{\psi_{km}} (y_{km} - w_{km})^{1-\psi_{km}} \left[(1-\tau_k^w)^{\psi_{km}} (1-\tau_m^\pi)^{1-\psi_{km}}\right],$$

and as a result the pre-tax wages and profits do not depend on taxes[16]:

[14] To see this, consider a simple example where demand for labor is given by $D = a_1 - b_1 w$ and supply of labor is given by $S = a_2 + b_2 w$. When wages are taxed the supply function is $S = a_2 + b_2 w(1-\tau)$. On competitive markets pre-tax wage rate is determined where $D = S$, and increases in the tax rate, while on imperfect labor markets employers do not take into consideration the search intensity of the worker when contracting the wage rate. As a result, on imperfect labor markets, the worker bears the whole incidence from the specific tax on wages, and the employer bears the whole incidence from the specific tax on profits.

[15] I assume linear tax functions for tractability in the derivation of the optimal income tax schedule, when the social planner attempts to simultaneously control for externalities and raise positive government revenue.

[16] Though pretax wages and profits are not affected by taxation this is not true about the effective bargaining power of each side. The effective bargaining power of the vacancy, for instance, is measured by the share of the post tax output, $(y_{km} - \tau_k^w w_{km} - (y_{km} - w_{km})\tau_m^\pi)$, received by the vacancy. Note that the revenue of the

$$w_{km} = \psi_{km} y_{km}$$
$$\pi_{km} = (1-\psi_{km}) y_{km}.$$

Since π_{km} is independent from the private behavior of the worker, the government observes only two levels of the expected revenue to vacancies and chooses only two levels of the tax rates for vacancies, τ_H^π and τ_L^π.

In what follows I adopt the following notation: $w_k = E_{(m)} w_{km} = E_{(m)} \psi_{km} y_{km}$ is the expected pre-tax wage rate of a worker of type $k = H, L$; $z_k^w = \delta_k M(\theta) w_k$ is the expected pre-tax income of a worker of type $k = H, L$; $\pi_m = E_{(k)} \pi_{km} = E_{(k)} (1-\psi_{km}) y_{km}$ is the expected pre-tax profit rate of a vacancy of type $m = H, L$; and $z_m^\pi = v_m \frac{M(\theta)}{\theta} \pi_m$ is the expected pre-tax revenue of a vacancy of type $m = H, L$. One can, then, write the after tax expected utility of a worker of type k as

$$\begin{aligned} U_k = & -c_w(\delta_k) + LS \\ & + \delta_k \left\{ M(\theta) \left(\frac{v_H q_H}{\sum_m v_m q_m} \psi_{kH} y_{kH} + \frac{v_L q_L}{\sum_m v_m q_m} \psi_{kL} y_{kL} \right) (1-\tau_k^w) + (1-M(\theta))0 \right\} + (1-\delta_k)0 \end{aligned}$$

$$U_k = -c_w \left(\frac{z_k^w}{M(\theta)\, w_k} \right) + LS + (1-\tau_k^w) z_k^w, \tag{20}$$

where LS is a lump sum transfer, when lump sum transfers are an available to the government tax instrument.

The first order condition of private optimization, with respect to worker's intensity of search given the tax function, is

$$c_w'(\ddot{\tilde{\delta}}_k) = M(\ddot{\tilde{\theta}})(1-\tau_k^w) w_k, \tag{21}$$

where $\ddot{\tilde{\delta}}$ and $\ddot{\tilde{\theta}}$ denote the privately optimal search intensity and the decentralized equilibrium

vacancy is $(1-\psi_{km})(1-\tau_m^\pi) y_{km}$, which after re-arrangement, $(\pm(1-\psi_{km})(\tau_m^\pi - \tau_k^w) w_{km})$, leads to

$$[y_{km} - \tau_k^w w_{km} - (y_{km} - w_{km})\tau_m^\pi] \left[\frac{(1-\psi_{km})(1-\tau_m^\pi) y_{km}}{y_{km} - \tau_k^w w_{km} - (y_{km} - w_{km})\tau_m^\pi} \right].$$

The effective bargaining power of the vacancy is $\left[\frac{(1-\psi_{km})(1-\tau_m^\pi) y_{km}}{y_{km} - \tau_k^w w_{km} - (y_{km} - w_{km})\tau_m^\pi} \right]$ and since $w_{km} = \psi_{km} y_{km}$ one can show that the effective bargaining power of the vacancy decreases in τ_m^π and increases in τ_k^w. Higher tax on wages decreases the after-tax wage of the worker and lowers her incentive to bargain. Correspondingly the incentive of a vacancy to bargain is lowered when the tax on revenue is high.

market tightness in the presence of income taxes. One can similarly derive the first order conditions that determine the privately selected search intensities of vacancies in the presence of taxes on expected revenue.

Lemma 4. *In the presence of income taxes the decentralized equilibrium search intensities of workers and vacancies are determined by*

$$\begin{aligned} c'_w(\ddot{\tilde{\delta}}_k) &= M(\ddot{\tilde{\theta}})(1-\tau^w_k)w_k \\ c'_\pi(\ddot{\tilde{v}}_m) &= \frac{M(\ddot{\tilde{\theta}})}{\ddot{\tilde{\theta}}}(1-\tau^\pi_m)\pi_m \end{aligned} \qquad \text{for} \qquad \left| \begin{array}{l} \ddot{\tilde{\theta}} \lesseqgtr 1, \\ \ddot{\tilde{\delta}}_k > 0,\ \ddot{\tilde{v}}_m > 0 \end{array} \right. , \tag{22}$$

$$\begin{aligned} c'_w(0) &\geq M(\ddot{\tilde{\theta}})(1-\tau^w_L)w_L \\ c'_\pi(0) &\geq \frac{M(\ddot{\tilde{\theta}})}{\ddot{\tilde{\theta}}}(1-\tau^\pi_L)\pi_L \end{aligned} \qquad \text{for} \qquad \left| \begin{array}{l} \ddot{\tilde{\theta}} \lesseqgtr 1, \\ \ddot{\tilde{\delta}}_L = 0,\ \ddot{\tilde{v}}_L = 0 \end{array} \right. . \tag{23}$$

4.1 Characterizing externalities through Pigou taxes

Suppose the government, perfectly observes search intensities and uses a Pigou tax on expected income of workers $(\check{\tau}^w_H, \check{\tau}^w_L)$, and a Pigou tax on expected revenue of vacancies $(\check{\tau}^\pi_H, \check{\tau}^\pi_L)$. Lump sum transfers are an available to the government instrument and the collected revenue (or raised subsidy) $\check{R}$, from the Pigou tax, is returned to all parties via a lump sum, LS:

$$\begin{aligned} \check{R} &= (\textstyle\sum_k \delta l)M(\theta)\left[\frac{\delta_H l_H}{\sum_k \delta l}\check{\tau}^w_H w_H + \frac{\delta_L l_L}{\sum_k \delta l}\check{\tau}^w_L w_L + \frac{v_H q_H}{\sum_m vq}\check{\tau}^\pi_H \pi_H + \frac{v_L q_L}{\sum_m vq}\check{\tau}^\pi_L \pi_L\right] \\ 0 &= \check{R} - \left(\sum_k l_k + \sum_m l_m\right) LS, \end{aligned}$$

where $(\sum_k \delta l)M(\theta) = N$ is the number of matches in the economy in equilibrium.

The after tax expected utility of a worker of type k is

$$U_k = -c_w\left(\frac{z^w_k}{M(\theta)\, w_k}\right) + LS + (1-\check{\tau}^w_k)z^w_k \tag{24}$$

The lump sum, LS, enters the utility function of the worker additively and does not affect private behavior. The first order condition of private optimization, with respect to intensity of search, is

$$c'_w(\check{\check{\delta}}_k) = M(\check{\check{\theta}})(1 - \check{\tau}^w_k)w_k, \tag{25}$$

where $\check{\check{\delta}}$ and $\check{\check{\theta}}$ denote the privately optimal search intensity and the decentralized equilibrium market tightness in the presence of Pigou income taxes.

The Pigou income tax rate of a worker of type k sets $\check{\check{\delta}}=\bar{\delta}$ and $\check{\check{\theta}}=\bar{\theta}$, and using (11), (12), and (25), is determined by

$$(1 - \check{\tau}^w_k)E_{(m)}\psi_{km}y_{km} = E_{(m)}y_{km} - (1-\alpha)E_{(k)}E_{(m)}y_{km}. \tag{26}$$

Analogously we can derive the conditions that determine optimal Pigou taxes on employers' revenues

Proposition 5. *The type specific Pigou income tax that decentralizes the socially optimal search intensity of a worker of type k, and a vacancy of type m are*

$$\begin{aligned} 1-\check{\tau}^w_k =& \frac{E_{(m)}y_{km} - (1-\alpha)E_{(k)}E_{(m)}y_{km}}{E_{(m)}\psi_{km}y_{km}} \\ 1-\check{\tau}^\pi_m =& \frac{E_{(k)}y_{km} - \alpha E_{(k)}E_{(m)}y_{km}}{E_{(k)}(1-\psi_{km})y_{km}} \end{aligned} \quad \text{for} \quad \left| \begin{aligned} & \check{\check{\theta}} = \bar{\theta} \lesseqgtr 1, \\ & \check{\check{\delta}}_k = \bar{\delta}_k > 0, \ \check{\check{v}}_m = \bar{v}_m > 0 \end{aligned} \right. , \tag{27}$$

$$\begin{aligned} \check{\tau}^w_L =& 1 \\ \check{\tau}^\pi_L =& 1 \end{aligned} \quad \text{for} \quad \left| \begin{aligned} & \check{\check{\theta}} = \bar{\theta} \lesseqgtr 1, \\ & \check{\check{\delta}}_L = \bar{\delta}_L = 0, \ \check{\check{v}}_L = \bar{v}_L = 0 \end{aligned} \right. . \tag{28}$$

Furthermore, for constant returns to scale Cobb-Douglass encounter function, the budget is balanced, $\check{R}=0$.

See the proof to Proposition 5 in the Appendix. To interpret equations (27) and (28), consider the Pigou tax on workers from equation (27). The tax rate: (1) decreases in the contribution of the worker to the total output in the economy, $E_{(m)}y_{km}$; increases in the bargaining power of the worker, $E_{(m)}\psi_{km}y_{km}$; and decreases in the ability of the worker to create matches, α. The tax rate controls for the ability of the worker to create matches, as well as for the ability of the worker to change the distribution of worker productive skills among the actively searching workers. Since a high type worker makes the partnership more productive, high type worker is more desirable as a partner, and the latter aspect of the Pigou tax rate favors workers of high type.

The intuition for the balanced government budget (see the proof to Proposition 5) is as follows. With a CRS matching function, the output of the matches is exhausted exactly in providing the correct marginal incentives to workers and vacancies; the tax policy only redistributes income from agents with excessive bargaining power in the laissez fair market equilibrium, to agents with not enough bargaining power. With decreasing returns to scale agents are on average over-rewarded in the laissez fair market equilibrium, and Pigou taxation generates positive revenue, which can be transfered back to workers and vacancies without distorting their search incentives. With increasing returns to scale agents are on average under-rewarded in the laissez fair market equilibrium, because the partnership output is not enough to reward the searching parties for their efforts in creating a match. In this case the government runs a deficit, which can be financed via the lump sum tax, LS, without distorting incentives.

Note also that since low type agents are less desirable in the economy, the tax policy forces them to subsidize the efforts of high type agents ($g = 0$). With production functions that generate minimal output when one of the partners is of low type, low type agents may not be desirable in the economy at all. In this case all the income of low type agents is extracted by the Pigou tax, $\check{\tau}_L^w = \check{\tau}_L^\pi = 1$.

Hosios (1990) shows that when workers and vacancies are homogeneous in productive skill, the hold up problem can be solved by equating the bargaining power of each side to the elasticity of the matching function with respect to their intensity of search. Shimer and Smith (2001) show that Hosios's condition can not be applied in a dynamic search-and-matching model with heterogeneous in productivity workers. I confirm the Shimer and Smith's (2001) result for the static model in this paper: externalities on imperfect labor markets with heterogeneous in skill type workers and vacancies, can only be corrected by taxation. To easily see this, first note that we are particularly interested whether a type contingent output share specified as

$$\begin{aligned} \psi_{ji} &= 1 - \psi_{ij} \quad \text{so that} \quad \psi_{HL} = (1 - \psi_{LH}) \\ \psi_{ii} &= 1 - \psi_{ii} \quad \text{so that} \quad \psi_{HH} = \psi_{LL} = \frac{1}{2} \end{aligned} \tag{29}$$

can decentralize the social optimum. For a decentralized equilibrium to coincide with the social equilibrium, ψ_{km} must be chosen in such a way that the first order conditions of the laissez-fair private and social maximization problems coincide. However, when $\psi_{LL} = \frac{1}{2}$ and

$y_{LL} > 0$, albeit very small, low type worker always searches with positive intensity in the decentralized equilibrium, but is forced out of the market in the social optimum.

Proposition 6. *When workers differ in productive skill, the social optimum can not be always decentralized by carefully assigned bargaining power.*

4.2 Optimal income taxes with positive government revenue

In this section I study the optimal income tax schedule, which simultaneously raises revenue and decentralizes the socially optimal search intensities. For this purpose I introduce a positive government revenue requirement R, which finances the production of a public good[17].

The social planner chooses tax rates to maximize a Utilitarian welfare function

$$W = \left\{ \sum_k l_k U^k + \sum_m q_m V^m \right\},$$

a sum of the expected utilities of all participants in the economy per period.

Using (1), (2), (5), and (6), the definitions $w_k = E_{(m)}\psi_{km}y_{km}$, $z_k^w = \delta_k M(\theta) w_k$, $\pi_m = E_{(k)}(1-\psi_{km})y_{km}$, and $z_m^\pi = v_m \frac{M(\theta)}{\theta}\pi_m$ from Section 4, and re-arranging, we can write the welfare function as

$$W = \sum_k l_k \left(-c_w\!\left(\frac{z_k^w}{M(\theta)\, w_k} \right) \right) + \sum_m q_m \left(-c_\pi\!\left(\frac{z_m^\pi}{\frac{M(\theta)}{\theta}\, \pi_m} \right) \right) + (\textstyle\sum_k \delta l) M(\theta)\, E_{(k)} E_{(m)} y_{km},$$

where $(\sum_k \delta l) M(\theta) = N$ is the number of matches in the economy in equilibrium.

If the revenue requirement is R, output accrues to workers, employers and government

$$R \leq (\textstyle\sum_k \delta l) M(\theta) \left[\frac{\delta_H l_H}{\sum_k \delta l} \tau_H^w w_H + \frac{\delta_L l_L}{\sum_k \delta l} \tau_L^w w_L + \frac{v_H q_H}{\sum_m vq} \tau_H^\pi \pi_H + \frac{v_L q_L}{\sum_m vq} \tau_L^\pi \pi_L \right], \tag{30}$$

where $(\sum_k \delta l) M(\theta) = M$ is the total number of matches. Using equation (30), in its strict

[17] The public good, even if valued by consumers, does not affect their choice on search intensity.

equality form, the welfare function can be further expanded as

$$W = \sum_k l_k \left(-c_w\!\left(\frac{z_k^w}{M(\theta)\, w_k} \right) \right) + \sum_m q_m \left(-c_\pi\!\left(\frac{z_m^\pi}{\frac{M(\theta)}{\theta} \pi_m} \right) \right)$$
$$+ (\textstyle\sum_k \delta l) M(\theta) \left[\frac{\delta_H l_H}{\sum_k \delta l}(1-\tau_H^w) w_H + \frac{\delta_L l_L}{\sum_k \delta l}(1-\tau_L^w) w_L \right.$$
$$\left. + \frac{v_H q_H}{\sum_m vq}(1-\tau_H^\pi)\pi_H + \frac{v_L q_L}{\sum_m vq}(1-\tau_L^\pi)\pi_L \right] + R.$$

Since the social planner does not observe search intensities, but only expected income/revenue, the social planner chooses tax rates to maximize the utilitarian welfare function subject to a revenue generation constraint, and subject to incentive compatibility constraints for workers and vacancies. The incentive compatibility constraint is that the selected by a worker (vacancy), of a given productivity type, labor supply (labor demand) maximizes utility given the tax function. The simplest way to proceed is to replace the self-selection constraints with the first order conditions for individual choice, (22) (see Cooter (1978), and Diamond (1998) among others). The tax function must be such that it gives a higher utility to a high type worker, when a high type worker self-selects to not mimic the behavior of a low type worker, and a low type worker self-selects to not mimic the behavior of a high type worker. The intuition for this result is that a high type worker can always achieve the expected income of a low type worker by providing less labor than a low type worker.

Lemma 7 *High type worker/vacancy receives a larger utility than a low type worker/vacancy in the presence of income taxes.*

See the proof to Lemma 7 in the Appendix. Expanding θ, and substituting the first order conditions (22) into the welfare function, the maximization problem can be written as

$$\max_{\tau_k^w, \tau_m^\pi} W = \sum_k l_k \left(-c_w\!\left(\frac{z_k^w}{M(\theta)\, w_k} \right) \right) + \sum_m q_m \left(-c_\pi\!\left(\frac{z_m^\pi}{\frac{M(\theta)}{\theta} \pi_m} \right) \right)$$
$$+ \delta_H l_H c_w'\!\left(\frac{z_H^w}{M(\theta)\, w_H} \right) + \delta_L l_L c_w'\!\left(\frac{z_L^w}{M(\theta)\, w_L} \right) + v_H q_H c_\pi'\!\left(\frac{z_H^\pi}{\frac{M(\theta)}{\theta} \pi_H} \right) + v_L q_L c_\pi'\!\left(\frac{z_L^\pi}{\frac{M(\theta)}{\theta} \pi_L} \right)$$

$$\text{s.th.} \qquad R \le (\textstyle\sum_k \delta l) M\!\left(\frac{\sum_m vq}{\sum_k \delta l} \right) \left[\frac{\delta_H l_H}{\sum_k \delta l} \tau_H^w w_H + \frac{\delta_L l_L}{\sum_k \delta l} \tau_L^w w_L + \frac{v_H q_H}{\sum_m vq} \tau_H^\pi \pi_H + \frac{v_L q_L}{\sum_m vq} \tau_L^\pi \pi_L \right], \tag{31}$$

and the Lagrangian is

$$\max_{\tau_k^w,\tau_m^\pi} L = \sum_k l_k \left(-c_w\left(\frac{z_k^w}{M(\theta)\, w_k}\right)\right) + \sum_m q_m \left(-c_\pi\left(\frac{z_m^\pi}{\frac{M(\theta)}{\theta}\pi_m}\right)\right)$$
$$+\delta_H l_H c_w'\left(\frac{z_H^w}{M(\theta)\, w_H}\right) + \delta_L l_L c_w'\left(\frac{z_L^w}{M(\theta)\, w_L}\right) + v_H q_H c_\pi'\left(\frac{z_H^\pi}{\frac{M(\theta)}{\theta}\pi_H}\right) + v_L q_L c_\pi'\left(\frac{z_L^\pi}{\frac{M(\theta)}{\theta}\pi_L}\right)$$
$$+\mu\left(\left(\textstyle\sum_k \delta l\right)M(\theta)\left[\frac{\delta_H l_H}{\sum_k \delta l}\tau_H^w w_H + \frac{\delta_L l_L}{\sum_k \delta l}\tau_L^w w_L + \frac{v_H q_H}{\sum_m vq}\tau_H^\pi \pi_H + \frac{v_L q_L}{\sum_m vq}\tau_L^\pi \pi_L\right] - R\right), \tag{32}$$

where μ is the marginal cost of public funds. Note that we do not impose non-negativity constraints on optimal taxes because the *full* tax rates, τ, incorporate a component that raises revenue, and a component which controls for the uninternalized by the agent search externalities. The first order conditions to the maximization of the above Lagrangian are derived in the proof to Proposition 8 (below) in Appendix A, and their final forms are given by equations (60)-(63).

Let $\varepsilon_k^w = \frac{1/(z_k^w/M(\theta)\, w_k)}{c_w''/c_w'}$ denote the elasticity of search intensity (supply of labor) of a worker of high type with respect to the rewards to search, and recall that the elasticity $(1-\alpha) = \frac{M'(\theta)}{M(\theta)}/\frac{1}{\theta}$ measures the effectiveness of vacancies in generating matches.

4.2.1 The marginal cost of public funds

The next result characterizes the cost to raising public funds when the social planner simultaneously controls for the externalities.

Proposition 8. *When the social planner chooses optimal income taxes to correct for the search externalities and raise a fixed level of government revenue, R, the marginal cost of funds is*

$$\mu = \left(1 - \frac{E_{(s)} w_{Hs}\tau_H^w + E_{(s)} w_{Ls}\tau_L^w + E_{(s)}\pi_{sH}\tau_H^\pi + E_{(s)}\pi_{sL}\tau_L^\pi}{\frac{1-\tau_H^w}{\varepsilon_H^w}w_H + \frac{1-\tau_L^w}{\varepsilon_L^w}w_L + \frac{1-\tau_H^\pi}{\varepsilon_H^\pi}\pi_H + \frac{1-\tau_L^\pi}{\varepsilon_L^\pi}\pi_L}\right)^{-1} = [1 - f(\tau,\varepsilon)]^{-1}, \tag{33}$$

where the expectations with respect to s in the numerator have weights $\left(\frac{\delta_H l_H}{\sum_k \delta l}\frac{v_L q_L}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l}\frac{v_H q_H}{\sum_m vq}\right)$ *when* $s \neq k$ *or* $s \neq m$, *and* $\left(\frac{\delta_H l_H}{\sum_k \delta l}\frac{v_H q_H}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l}\frac{v_L q_L}{\sum_m vq}\right)$ *when* $s = k$ *or* $s = m$, τ *(with* $f_\tau' > 0$*) is the average tax rate with weights being a function of pretax incomes/revenues, and* ε *(with* $f_\varepsilon' > 0$*) is the average elasticity of labor supply/demand with weights being a function of post-tax incomes/revenues.*

See the proof to Proposition 8 in the Appendix. The marginal cost of public funds depends on both the government revenue requirement, through the average tax burden τ, and the sensitivity of private behavior with respect to the rewards to search, through the average level of elasticity of demand and supply of labor. In particular, the marginal cost of public funds is increasing in the average tax burden and also increasing in the average of the demand and supply elasticities. Intuitively, labor market behavior is more sensitive to taxation when elasticities are large.

The marginal cost of public funds is unity iff either demand or supply of labor is inelastic, $\varepsilon_k^w = 0$ or $\varepsilon_m^\pi = 0$. In this case revenue generating taxation does not distort incentives. It is not possible to say whether the marginal cost of funds approaches zero if the government revenue requirement, R, approaches zero, by just considering equation (33). The equilibrium tax rates are the rates that generate the government revenue. However, these rates are different from the equilibrium tax rates in an economy where the search for partners does not generate externalities. If the government revenue requirement is zero, the tax rates only control for the externalities, and balance the government budget if the matching function, M, is of constant returns to scale (see the proof to Proposition 5 in Appendix A). However, it is not clear whether pure externality-controlling taxes set the marginal cost of funds to unity, when the matching function is not characterized by constant returns to scale.

If demand elasticity is infinite, $\varepsilon_m^\pi = \infty$, as it would be with free entry of vacancies, then demand elasticity does not appear explicitly in the marginal cost of funds function, μ. Intuitively it must be that income taxes are not distorting employers' behavior. This is only possible if the revenue generating taxes for employers are zero when $\varepsilon_m^\pi = \infty$, and revenue is raised only by taxing workers. The externality correcting taxes for employers, however, must still be in effect so that employers face the correct search incentives.

4.2.2 The optimal income tax structure

In this subsection I discuss the characteristics of the optimal income tax structure when the government is raising revenue and is simultaneously correcting for search externalities. The exact expressions for each tax rate are too complex to reveal any intuitions, however, the first order conditions to problem (32) reveal enough information to discuss the main characteristics of the optimal tax system. The first result is in line with Cooter (1984), and states that an agent never chooses a search intensity such that more than her marginal income is taxed away.

Proposition 9. *The optimal marginal income tax rate is weakly lower than unity, $\tau \leq 1$.*

This result follows immediately from the first order conditions determining search intensity in the presence of income taxes, (22) and (23). In an income interval where $\tau > 1$, an increase in search intensity leads to a decrease in the after-tax income. No one will choose their search intensity in this interval. By conditions (22) and (23), and the definition of the search cost function, the worst one can do is not search at all.

The second result shows that the optimal income tax system supports more actively searching agents of high productivity type and a less actively searching agents of low productivity type.

Conjecture 10. *The optimal income tax system is such that, in the optimum, high type worker/vacancy searches with higher intensity than low type worker/vacancy.*

See the proof to Conjecture 10 in the Appendix. We already proved in Lemma 7 that in equilibrium high productivity agents enjoy larger utility than low productivity agents. The result in Conjecture 10 is in line with the literature on optimal income taxation, where the labor market is not explicitly modeled, however, it is also in line with our results on the externalities imposed in the economy by a high type agent, as discussed in Section 4.1. The self selection constraint, and the fact that the marginal rate of substitution between consumption and labor is lower for the more productive individual, generate an optimal tax schedule where a more productive worker supplies labor more intensively and enjoys larger consumption (see Stiglitz (1987)). Our model further suggests that on imperfect labor markets the search intensity of the more productive worker is subsidized to reach the socially efficient level, which, as we know from Section 3.1, is larger for the more productive worker.

The next result is in support of the Ramsey (1927) rule, that a more elastic behavior should be taxed at a lower rate. In our model two comparisons can be made on how tax rates associate with elasticities. The first one relates the relative tax rates, faced by a high type worker and a low type worker, to their relative elasticities. The second one relates the relative tax rates on supply and demand to their relative elasticities. In our model the relationship between relative tax rates and relative elasticities is not as exact as in Ramsey (1927), because the tax rates also incorporate a term that controls for externalities. However, the optimal marginal tax rate at some income level depends on the elasticity of supply/demand at this income level (even if the skill level is not observed by the social planner), since this is important for marginal distortions (see also Diamond (1998)). The first order conditions to the social planner's problem reveal that (Proposition 11i), when the relative elasticity of supply of a high type worker increases, the relative marginal tax rate of high type worker decreases.

Proposition 11 (below) also reveals that the optimal marginal tax schedule is such that the tax burden is born by the side of the market whose labor market participation is less elastic; when the elasticity of supply increases relative to the elasticity of demand, a larger portion of the tax burden is allocated to the demand side. Furthermore, when demand of labor is perfectly elastic, the whole burden is born by the supply side (see Proposition 8)[18]. This is in contrast to the celebrated production efficiency result of Diamond and Mirrlees (1971) that the entire tax burden should be put on the supply side of the market. However, it is in support of the findings of Boone and Bovenberg (2002), that on imperfect labor markets with homogeneous in productivity agents, the relative elasticities of demand and supply of labor are inversely related to the relative tax rates on supply and demand.

Proposition 11. *The optimal income tax system is such that in the optimum*

$$i)\ \partial\left(\frac{\tau_H^w}{\tau_L^w}\right) / \partial\left(\frac{\varepsilon_H^w}{\varepsilon_L^w}\right) < 0 \tag{34}$$

$$ii)\ \partial\left(\frac{\tau^w}{\tau^\pi}\right) / \partial\left(\frac{\varepsilon^w}{\varepsilon^\pi}\right) < 0, \tag{35}$$

where τ^w, τ^π, ε^w, and ε^π are the tax rates on labor income and profit and the elasticities of labor market participation, when the distributions of productivity types on each side of the market are collapsed to a constant[19].

See the proof to Proposition 11 in the Appendix. In the proof to Conjecture 10 I show that, for convex search intensity cost functions, the elasticity of supply/demand decreases in the search intensity. Because the socially optimal search intensity increases in type, the elasticity rule described in the first part of Proposition 11 suggests a progressive tax system.

[18] Such would be the case with free entry of vacancies.

[19] To determine the rule under which the tax burden is allocated to each side of the market we need not consider productivity types, but only differentiate between workers and employers.

The last two results discuss the externality-correcting components of the tax system. As described in Proposition 5, there are two main uninternalized channels through which the worker's choice on intensity of search affects vacancies: the first is the effectiveness of the worker in favorably altering the distribution of productivity types of workers, faced by a vacancy; and, the second is the effectiveness of workers in creating matches as measured by the elasticity of the matching function, α.

Proposition 12. *The optimal income tax system is characterized by*

$$i)\ \partial\left(\frac{\tau_H^w}{\tau_L^w}\right) / \partial\left(\frac{E_{(m)}\pi_{Hm}}{E_{(m)}\pi_{Lm}}\right) < 0 \tag{36}$$

$$ii)\ \partial\left(\frac{\tau^w}{\tau^\pi}\right) / \partial(\alpha) < 0 \tag{37}$$

See the proof to Proposition 11 in the Appendix. Because more productive workers change the distribution of productive skill among workers in a favorable for vacancies direction, the externality correcting part of the optimal tax rates suggests a more regressive tax system (as also suggested by Proposition 5). Whether the tax system is actually progressive or regressive depends on the shape of the search intensity cost function (preferences), and on the shape of the production function. The slower the search costs rise, and the larger the difference between the marginal contribution to a partnership by a high type and the marginal contribution to a partnership by a low type, the more dominant the regressive component will be.

Proposition 12 also suggests that the more effective is a given side of the market in creating matches, the more encouraged this side should be to participate. This result is in line with Boone and Bovenberg (2002) and suggests that part of the tax should work to eliminate the disparity between the bargaining power of an agent and her ability to create a match.

5 Conclusion

This paper develops a static model of search, where workers and vacancies of different productivity types search and match to produce. In the process of search workers and vacancies do not consider all the effects from their search activity. This leads to inefficient levels of the search intensities, and in particular, markets on which in equilibrium low productivity agents are over-represented and high productivity agents are under-represented. I show that optimal income taxes can be employed to correct for the arising inefficiencies in search and at he same time raise a positive government revenue. The optimal income tax schedule is composed of an externality controlling element and a revenue raising element. These elements usually work in opposite directions, making it difficult to determine the optimal progressivity of the optimal income tax system. To complete the analysis, a study on the effects of the optimal income tax schedule on equilibrium market tightness is necessary. This will shed more light on equilibrium unemployment levels, and is considered as a next step in the analysis.

References:

Acemoglu, D. and R. Shimer (1999) "Holdups and Efficiency with Search Frictions," *International Economic Review*, 40, 827-849.

Boone, J., and L. Bovenberg (2002) "Optimal Labor Taxation and Search," *Journal of Public Economics*, 85, 53-97.

Bovenberg, A., and F. van der Ploeg (1994) "Effects of the Tax and Benefit System on Wage Formation and Unemployment," Working Paper, Tilburg University.

Burdett, K., and M.Colles, (1997) "Marriage and Class," *The Quarterly Journal of Economics*, 112, 141-168.

Burdett, K., and M.Colles, (1999) "Long-term Partnership Formation: Marriage and Employment," *The Economic Journal*, 109, 307-334.

Cooter, R. (1978) "Optimal Tax Schedules and Rates: Mirrlees and Ramsey," *American Economic Review*, 68, 756-768.

Diamond, P. (1981) "Mobility Costs, Frictional Unemployment and Efficiency," *Journal of Political Economy*, v.89, 798-812.

Diamond, P. (1982a) "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, v.90, 881-894.

Diamond, P. (1982b) "Wage Discrimination and Efficiency in Search Equilibrium," *Review of Economic Studies*, v.49, 217-227.

Diamond, P. (1998) "Optimal Income Taxation: An Example With U-Shaped Pattern of Optimal Marginal Tax Rates," *The American Economic Review*, v.88, 83-95.

Edgeworth, F. (1897) "The Pure Theory of Taxation," *Economic Journal*, 7, 46-70.

Feldstein, M. (1973) "On the Optimal Progressivity of the Income Tax," *Journal of Political Economics*, 2, 357-376.

Fuest, C., and B. Huber (1997) "How Does Tax Progression Affect Employment? The Case of a Small Open Economy,"University of Munich Working Paper.

Hoel, M. (1990) "Efficiency Wages and Income Taxes," *Journal of Economics*, 51, 8999.

Holmlund, B., and A. Kolm (1995) "Progressive Taxation, Wage Setting, and Unemployment," Working Paper, Department of Economics, Uppsala University.

Hosios, A.J. (1990) "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57, 279-298.

Kolm, A. (1997) "Taxation, Wage Formation, Unemployment and Welfare," *Economics Studies*, 32, Department of Economics, Uppsala University.

Koskela, E., and J. Vilmunen (1996) "Tax Progression is Good for Employment in Popular Models of Trade Union Behavior," *Labour Economics*, 3, 6580.

Kleven, H., and Srensen, P. (1999) "Labor Tax Reform, The Good Jobs and the Bad Jobs," *Economic Policy Research Unit, Copenhagen*, Working Paper.

Lockwood, B., and A. Manning (1993) "Wage Setting and the Tax System Theory and Evidence for the United Kingdom," *Journal of Public Economics*, 52, 129.

Millard, S., and Mortensen, D. (1996) "The Unemployment and Welfare Effects of Labor Market Policy: A comparison of the U.S. and the U.K.," *In: Snower, D., and D. de la Dehesa, (Eds.), Unemployment Policy: How Should Governments Respond to Unemployment.*, Oxford University Press, Oxford.

Mirrlees, J. (1971) "An Exploration in the Theory of Optimum Income Taxation," *The Review of Economic Studies*, 38/2, 175-208.

Mortensen, D. T. (1982a) "The Matching Process as a Non-cooperative/Bargaining Game," *in: J.J. McCall, ed., The Economics of Information and Uncertainty*, University of Chicago Press, Chicago, IL, 233-254.

Mortensen, D. T. (1982b) "Property Rights and Efficiency in Mating, Racing, and Related Games," *American Economic Review*, v.72, No.5., 968-979.

Mortensen, D., and Ch. Pissarides (1999) "New Developments in Models of Search in the Labor Market," *Handbook of Labor Economics*, v.3, ch.39.

Phelps, E. (1973) " Taxation of Wage Income for Economic Justice," *Quarterly Journal of Economics*, 87/3, 331-354.

Pisauro, G. (1991) " The Effects of Taxes on Labour in Efficiency Wage Models," *Journal of Public Economics*, 46, 329345.

Pissarides, C. (1983) "Efficiency Aspects of the Financing of Unemployment Insurance and Other Government Expenditure," *Review of Economic Studies*, 50, 5769.

Pissarides, C.A. (1984a) "Short-run Equilibrium Dynamics of Unemployment, Vacancies and Real Wages," *American Economic Review*, v.75, 676-690

Pissarides, C.A. (1984b) "Efficient Job Rejection," *Economic Journal Conference Papers*, v.94, 97-108

Pissarides, C. (1985) "Taxes, Subsidies and Equilibrium Unemployment," *Review of Economic Studies*, 52, 121133.

Pissarides, C. (1998) "The Impact of Employment Tax Cuts on Unemployment and Wages: the Role of Unemployment Benefits and the Tax Structure" *European Economic Review*, v.42, 155-183

Pissarides, C. (1990) "Equilibrium Unemployment Theory," Blackwell, Oxford.

Sheshinski (1972) "The Optimal Linear Income tax," *The Review of Economic Studies*, 39/3, 297-302.

Shi, S., and Q. Wen (1999) "Labor Market Search and the Dynamic Effects of Taxes and Subsidies," *Journal of Monetary Economics*, 33, 457495.

Shimer, R., and L. Smith (2000) "Assortative Matching and Search," *Econometrica*, 68, 343-369.

Shimer, R., and L. Smith (2001) "Matching, Search, and Heterogeneity" *Advances in Macroeconomics*, 1(2001), Iss. 1, Art. 5.

Srensen, P. (1997) "Public Finance Solutions to the European Unemployment Problem?," *Economic Policy*, 25, 223264.

Srensen, P. (1999) "Optimal Tax Progressivity in Imperfect Labor Markets," *Labor Economics*, 6, 435452.

Stiglitz, J. (1987) "Pareto Efficient and Optimal Taxation and the New New Welfare Economics," *In: Handbook of Public Economics*, v.2.

Stiglitz, J. (1999) "Self-selection and Pareto Efficient Taxation," *Journal of Public Economics*, 6, 239262.

Stiglitz, J. (1999) "Taxation, Public Policy and the Dynamics of Unemployment.," *International Tax and Public Finance*, 6, 239262.

F. van der toPloeg (1998) "Unemployment and Public Finance in Europe," *In: Srensen, P.B. (Ed.), Public Finance in a Changing World.*, MaxMillan, London.

Appendices:

A Proofs of the main results

Proof of Corollary 3.

First note that the vector of search intensities is unique for a given market tightness, $\tilde{\theta}$. Suppose there exists another equilibrium market tightness, $\tilde{\theta}_1 \neq \tilde{\theta}$. In particular suppose $\tilde{\theta}_1 > \tilde{\theta}$, then $\tilde{\delta}_H(\tilde{\theta}_1) > \tilde{\delta}_H(\tilde{\theta})$, $\tilde{\delta}_L(\tilde{\theta}_1) > \tilde{\delta}_L(\tilde{\theta})$, $\tilde{v}_H(\tilde{\theta}_1) < \tilde{v}_H(\tilde{\theta})$, and $\tilde{v}_L(\tilde{\theta}_1) < \tilde{v}_L(\tilde{\theta})$. However this means that $\tilde{\theta}_1 < \tilde{\theta}$, which is a contradiction. One can similarly show that there does not exist a second equilibrium with market tightness, $\tilde{\theta}_1 < \tilde{\theta}$. □

Proof of Proposition 5.

To see the first part, set equation (11) equal to equation (25), and equation (12) equal to equation (25). The result follows immediately because Pigou taxes set $\check{\tilde{\delta}} = \bar{\delta}$ and $\check{\tilde{\theta}} = \bar{\theta}$. Follow the same steps to determine the tax rates for vacancies.

To see the second result, write the revenue function dropping the assumption of CRS matching function. For that purpose write $1-\alpha=\beta$

$$\check{R} = N \begin{bmatrix} \frac{\delta_H l_H}{\sum_k \delta l}\left[(1-\alpha)E_{(k)}E_{(m)}y_{km} - E_{(m)}(1-\psi_{Hm})y_{Hm}\right] \\ +\frac{\delta_L l_L}{\sum_k \delta l}\left[(1-\alpha)E_{(k)}E_{(m)}y_{km} - E_{(m)}(1-\psi_{Lm})y_{Lm}\right] \\ +\frac{v_H q_H}{\sum_m vq}\left[(1-(1-\alpha))E_{(k)}E_{(m)}y_{km} - E_{(k)}\psi_{kH}y_{kH}\right] \\ +\frac{v_L q_L}{\sum_m vq}\left[(1-(1-\alpha))E_{(k)}E_{(m)}y_{km} - E_{(k)}\psi_{kL}y_{kL}\right] \end{bmatrix}$$

$$= N \begin{bmatrix} \frac{\delta_H l_H}{\sum_k \delta l}\left[(1-\alpha)E_{(k)}E_{(m)}y_{km} - E_{(m)}(1-\psi_{Hm})y_{Hm}\right] \\ +\frac{\delta_L l_L}{\sum_k \delta l}\left[(1-\alpha)E_{(k)}E_{(m)}y_{km} - E_{(m)}(1-\psi_{Lm})y_{Lm}\right] \\ +\frac{v_H q_H}{\sum_m vq}\left[(1-\beta)E_{(k)}E_{(m)}y_{km} - E_{(k)}\psi_{kH}y_{kH}\right] \\ +\frac{v_L q_L}{\sum_m vq}\left[(1-\beta)E_{(k)}E_{(m)}y_{km} - E_{(k)}\psi_{kL}y_{kL}\right] \end{bmatrix}$$

$$\check{R} = N\left[1-(\alpha+\beta)\right]$$

The government budget exactly balances when the matching function is characterized by constant returns to scale; a decreasing returns to scale matching function generates some revenue that has to be redistributed, and increasing returns to scale matching function requires the government to raise revenue to cover a net subsidy. If the encounter function takes other functional forms the budget may not be balanced (for example the budget is not balanced for the Leontief encounter function).□

Proof of Lemma 7.

In the presence of income taxes, the utility of a worker of type k is

$$\begin{aligned} U_k &= -c_w(\delta_k) + \delta_k M(\theta)(1-\tau_k^w)w_k \\ &= -c_w\left(\frac{z_k^w}{M(\theta)\,w_k}\right) + (1-\tau_k^w)z_k^w \end{aligned}$$

The partial derivative of the utility function with respect to productivity w_k is

$$\frac{\partial U_k}{\partial w_k} = -c_w' \frac{1}{M(\theta)\,w_k}\frac{\partial z_k^w}{\partial w_k} - c_w' \frac{z_k^w}{M(\theta)}\left(-\frac{1}{w_k^2}\right) + \frac{\partial z_k^w}{\partial w_k}(1-\tau_k^w) = c_w' \frac{z_k^w}{M(\theta)\,w_k} > 0,$$

where the last equality follows from the Envelope Theorem. □

Proof of Proposition 8.

In what follows I adopt the following notation: $w_k = E_{(m)} w_{km} = E_{(m)} \psi_{km} y_{km}$ is the expected pre-tax wage rate of a worker of type $k = H, L$; $z_k^w = \delta_k M(\theta) w_k$ is the expected pre-tax income of a worker of type $k = H, L$; $\pi_m = E_{(k)} \pi_{km} = E_{(k)}(1 - \psi_{km}) y_{km}$ is the expected pre-tax profit rate of a vacancy of type $m = H, L$; and $z_m^\pi = v_m \frac{M(\theta)}{\theta} \pi_m$ is the expected pre-tax revenue of a vacancy of type $m = H, L$. I log-linearize the first order conditions of private behavior in the presence of taxes, (22)

$$c_w'\left(\frac{z_k^w}{M(\theta)\, w_k}\right) = M(\theta)\,(1 - \tau_k^w) w_k,$$
$$c_\pi'\left(\frac{z_m^\pi}{\frac{M(\theta)}{\theta}\pi_m}\right) = \frac{M(\theta)}{\theta}(1 - \tau_m^\pi)\pi_m,$$

with respect to z_H^w, z_L^w, z_H^π, z_L^π, τ_H^w, τ_L^w, τ_H^π, τ_L^π. The goal is to first derive the rate of change of each income/revenue level with respect to changes in each tax rate: $\frac{dz_k^w}{d\tau_k^w}$, $\frac{dz_k^w}{d\tau_m^\pi}$, $\frac{dz_m^\pi}{d\tau_k^w}$, and $\frac{dz_m^\pi}{d\tau_m^\pi}$, for $k = H, L$ and $m = H, L$. In the second step I substitute these rates of change in the first order conditions for the maximization of the welfare function with respect to tax rates. For brevity I very often write δ_k instead of $z_k^w / M(\theta)\, w_k$, and v_m instead of $z_m^\pi / \frac{M(\theta)}{\theta}\, \pi_m$. This approach in deriving the first order conditions for the optimal income tax schedule is suggested by Sheshinski (1972), and Boone and Bovenberg (2002).

Log-linearize the private first order conditions

From the first order condition of a worker of high type we have

$$\begin{aligned}
c_w'' \frac{dz_H^w}{M(\theta)\, w_H} &= M'(\theta) \left(\frac{\frac{q_L dz_L^\pi}{\frac{M(\theta)}{\theta}\pi_L} + \frac{q_H dz_H^\pi}{\frac{M(\theta)}{\theta}\pi_H}}{\sum_k \delta l} - \frac{\frac{l_L dz_L^w}{M(\theta)\, w_L} + \frac{l_H dz_H^w}{M(\theta)\, w_H}}{\sum_k \delta l}\theta \right)(1 - \tau_H^w) w_H \\
&\quad - M(\theta)\, w_H d\tau_H^w \\
c_w'' \frac{dz_H^w}{M(\theta)\, w_H} \frac{1}{M(\theta)\, w_H} &= \frac{M'(\theta)}{M(\theta) w_H}(1 - \tau_H^w) w_H \theta \left[E_{(m)}\left(\frac{dz_m^\pi}{z_m^\pi}\right) - E_{(k)}\left(\frac{dz_k^w}{z_k^w}\right) \right] - d\tau_H^w \\
\frac{M(\theta)\,(1 - \tau_H^w) w_H}{c_w'} \frac{z_H^w}{z_H^w} c_w'' \frac{dz_H^w}{M(\theta)\, w_H} \frac{1}{M(\theta)\, w_H} &= \frac{M'(\theta)}{M(\theta)\, w_H}(1 - \tau_H^w) w_H \theta \left[E_{(m)}\left(\frac{dz_m^\pi}{z_m^\pi}\right) - E_{(k)}\left(\frac{dz_k^w}{z_k^w}\right) \right] - d\tau_H^w \\
\frac{1}{\varepsilon_H^w} \frac{dz_H^w}{z_H^w} &= (1 - \alpha) \left[E_{(m)}\left(\frac{dz_m^\pi}{z_m^\pi}\right) - E_{(k)}\left(\frac{dz_k^w}{z_k^w}\right) \right] - \frac{d\tau_H^w}{1 - \tau_H^w},
\end{aligned}$$

where $\varepsilon_H^w = \frac{1/(z_H^w / M(\theta)\, w_H)}{c_w''/c_w'}$ is the elasticity of search intensity of a high type worker with respect to the rewards to search, $(1 - \alpha) = \frac{M'(\theta)}{M(\theta)} / \frac{1}{\theta}$ is an elasticity that measures the effectiveness of vacancies in generating matches, and $E_{(k)}$ and $E_{(m)}$ denote expectations with respect to the distribution of worker productive skills, and the distribution of vacancy productive skills respectively. From the first order conditions, for high and low type workers, for optimal intensity of search in the market equilibrium, we have

$$\frac{dz_H^w}{z_H^w}\left(\frac{1}{\varepsilon_H^w}+(1-\alpha)\frac{\delta_H l_H}{\sum_k \delta l}\right)=(1-\alpha)\left[E_{(m)}\!\left(\frac{dz_m^\pi}{z_m^\pi}\right)-\frac{\delta_L l_L}{\sum_k \delta l}\left(\frac{dz_L^w}{z_L^w}\right)\right]-\frac{d\tau_H^w}{1-\tau_H^w} \tag{38}$$

$$\frac{dz_L^w}{z_L^w}\left(\frac{1}{\varepsilon_L^w}+(1-\alpha)\frac{\delta_L l_L}{\sum_k \delta l}\right)=(1-\alpha)\left[E_{(m)}\!\left(\frac{dz_m^\pi}{z_m^\pi}\right)-\frac{\delta_H l_H}{\sum_k \delta l}\left(\frac{dz_H^w}{z_H^w}\right)\right]-\frac{d\tau_L^w}{1-\tau_L^w}. \tag{39}$$

Similarly one can show that from the first order conditions, for high and low type vacancies, for optimal intensity of search in the market equilibrium we have

$$\frac{dz_H^\pi}{z_H^\pi}\left(\frac{1}{\varepsilon_H^\pi}+\alpha\frac{v_H q_H}{\sum_m vq}\right)=\alpha\left[E_{(k)}\!\left(\frac{dz_k^w}{z_k^w}\right)-\frac{v_L q_L}{\sum_m vq}\left(\frac{dz_L^\pi}{z_L^\pi}\right)\right]-\frac{d\tau_H^\pi}{1-\tau_H^\pi} \tag{40}$$

$$\frac{dz_L^\pi}{z_L^\pi}\left(\frac{1}{\varepsilon_L^\pi}+\alpha\frac{v_L q_L}{\sum_m vq}\right)=\alpha\left[E_{(k)}\!\left(\frac{dz_k^w}{z_k^w}\right)-\frac{v_H q_H}{\sum_m vq}\left(\frac{dz_H^\pi}{z_H^\pi}\right)\right]-\frac{d\tau_L^\pi}{1-\tau_L^\pi}. \tag{41}$$

I next solve the system of equations (38)-(41) to derive the equations that relate changes in each income/revenue level to the changes in each tax rate, $\frac{dz_k^w}{d\tau_k^w}$, $\frac{dz_k^w}{d\tau_m^\pi}$, $\frac{dz_m^\pi}{d\tau_k^w}$, and $\frac{dz_m^\pi}{d\tau_m^\pi}$, for $k=H,L$ and $m=H,L$. Subtract equation (39) from equation (38) to get

$$\frac{dz_H^w}{z_H^w}=\left(\frac{dz_L^w}{z_L^w}\frac{1}{\varepsilon_L^w}+\frac{d\tau_L^w}{1-\tau_L^w}-\frac{d\tau_H^w}{1-\tau_H^w}\right)\varepsilon_H^w. \tag{42}$$

Substituting equation (42) in equation (39) gives

$$\frac{dz_L^w}{z_L^w}\left(\frac{1}{\varepsilon_L^w}+(1-\alpha)\frac{\delta_L l_L}{\sum_k \delta l}\right)=(1-\alpha)E_{(m)}\!\left(\frac{dz_m^\pi}{z_m^\pi}\right)-$$
$$(1-\alpha)\frac{\delta_H l_H}{\sum_k \delta l}\left(\frac{dz_L^w}{z_L^w}\frac{1}{\varepsilon_L^w}+\frac{d\tau_L^w}{1-\tau_L^w}-\frac{d\tau_H^w}{1-\tau_H^w}\right)\varepsilon_H^w-\frac{d\tau_L^w}{1-\tau_L^w}.$$

Re-arranging this leads to

$$\frac{dz_L^w}{z_L^w}\frac{1}{\varepsilon_L^w}=\frac{(1-\alpha)E_{(m)}\!\left(\frac{dz_m^\pi}{z_m^\pi}\right)-\frac{d\tau_L^w}{1-\tau_L^w}\left(1+(1-\alpha)\frac{\delta_H l_H}{\sum_k \delta l}\varepsilon_H^w\right)+\frac{d\tau_H^w}{1-\tau_H^w}(1-\alpha)\frac{\delta_H l_H}{\sum_k \delta l}\varepsilon_H^w}{\Delta_1}, \tag{43}$$

where $\Delta_1 = 1+(1+\alpha)E_{(k)}\varepsilon_k^w$. Substituting equation (43) in equation (42) gives the counterpart to equation (43) that relates to a worker of high type

$$\frac{dz_H^w}{z_H^w}\frac{1}{\varepsilon_H^w}=\frac{(1-\alpha)E_{(m)}\!\left(\frac{dz_m^\pi}{z_m^\pi}\right)+\frac{d\tau_L^w}{1-\tau_L^w}(1-\alpha)\frac{\delta_L l_L}{\sum_k \delta l}\varepsilon_L^w-\frac{d\tau_H^w}{1-\tau_H^w}\left(1+(1-\alpha)\frac{\delta_L l_L}{\sum_k \delta l}\varepsilon_L^w\right)}{\Delta_1}. \tag{44}$$

The relevant conditions for low and high type vacancies are then

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{\varepsilon_H^\pi} = \frac{\alpha E_{(k)}\Big(\frac{dz_k^w}{z_k^w}\Big) + \frac{d\tau_L^\pi}{1-\tau_L^\pi}\alpha\frac{v_L q_L}{\sum_m vq}\varepsilon_L^\pi - \frac{d\tau_H^\pi}{1-\tau_H^\pi}\left(1+\alpha\frac{v_L q_L}{\sum_m vq}\varepsilon_L^\pi\right)}{\Delta_2} \tag{45}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{\varepsilon_L^\pi} = \frac{\alpha E_{(k)}\Big(\frac{dz_k^w}{z_k^w}\Big) - \frac{d\tau_L^\pi}{1-\tau_L^\pi}\left(1+\alpha\frac{v_H q_H}{\sum_m vq}\varepsilon_H^\pi\right) + \frac{d\tau_H^\pi}{1-\tau_H^\pi}\alpha\frac{v_H q_H}{\sum_m vq}\varepsilon_H^\pi}{\Delta_2}, \tag{46}$$

where $\Delta_2 = 1+\alpha E_{(m)}\varepsilon_m^\pi$. Adding equation (45) to equation (46) we get

$$E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi}\Big) = \frac{\alpha E_{(k)}\Big(\frac{dz_k^w}{z_k^w}\Big)E_{(m)}\varepsilon_m^\pi - E_{(m)}\Big(\varepsilon_m^\pi\frac{d\tau_m^\pi}{1-\tau_m^\pi}\Big)}{\Delta_2}, \tag{47}$$

and similarly adding equation (43) to equation (44) we have

$$E_{(k)}\left(\frac{dz_k^w}{z_k^w}\right) = \frac{(1-\alpha)E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi}\Big)E_{(k)}\varepsilon_k^w - E_{(k)}\Big(\varepsilon_k^w\frac{d\tau_k^w}{1-\tau_k^w}\Big)}{\Delta_1}. \tag{48}$$

From equations (47) and (48) one can express $E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi}\Big)$ and $E_{(k)}\Big(\frac{dz_k^w}{z_k^w}\Big)$ as a function of only elasticities, tax rates, and tax rate changes

$$E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi}\Big) = -\frac{(\Delta_2-1)E_{(k)}\Big(\varepsilon_k^w\frac{d\tau_k^w}{1-\tau_k^w}\Big) + \Delta_1 E_{(m)}\Big(\varepsilon_m^\pi\frac{d\tau_m^\pi}{1-\tau_m^\pi}\Big)}{\Delta_1+\Delta_2-1} \tag{49}$$

$$E_{(k)}\Big(\frac{dz_k^w}{z_k^w}\Big) = -\frac{\Delta_2 E_{(k)}\Big(\varepsilon_k^w\frac{d\tau_k^w}{1-\tau_k^w}\Big) + (\Delta_1-1)E_{(m)}\Big(\varepsilon_m^\pi\frac{d\tau_m^\pi}{1-\tau_m^\pi}\Big)}{\Delta_1+\Delta_2-1} \tag{50}$$

Substituting equation (49) in equations (43) and (44), and equation (50) in equations (45) and (46) we derive the final four equations that relate the change of each income/revenue level to the changes in all tax rates:

$$\begin{aligned}\frac{dz_H^w}{z_H^w}\frac{1}{\varepsilon_H^w} =& -\frac{(1-\alpha)\left[(\Delta_2-1)E_{(k)}\Big(\varepsilon_k^w\frac{d\tau_k^w}{1-\tau_k^w}\Big)+\Delta_1 E_{(m)}\Big(\varepsilon_m^\pi\frac{d\tau_m^\pi}{1-\tau_m^\pi}\Big)\right]}{\Delta_1(\Delta_1+\Delta_2-1)}\\ &+\frac{(\Delta_1+\Delta_2-1)\left[\frac{d\tau_L^w}{1-\tau_L^w}(1-\alpha)\frac{\delta_L l_L}{\sum_k \delta l}\varepsilon_L^w - \frac{d\tau_H^w}{1-\tau_H^w}\left(1+(1-\alpha)\frac{\delta_L l_L}{\sum_k \delta l}\varepsilon_L^w\right)\right]}{\Delta_1(\Delta_1+\Delta_2-1)},\end{aligned} \tag{51}$$

$$\begin{aligned}\frac{dz_L^w}{z_L^w}\frac{1}{\varepsilon_L^w} =& -\frac{(1-\alpha)\left[(\Delta_2-1)E_{(k)}\Big(\varepsilon_k^w\frac{d\tau_k^w}{1-\tau_k^w}\Big)+\Delta_1 E_{(m)}\Big(\varepsilon_m^\pi\frac{d\tau_m^\pi}{1-\tau_m^\pi}\Big)\right]}{\Delta_1(\Delta_1+\Delta_2-1)}\\ &+\frac{(\Delta_1+\Delta_2-1)\left[-\frac{d\tau_L^w}{1-\tau_L^w}\left(1+(1-\alpha)\frac{\delta_H l_H}{\sum_k \delta l}\varepsilon_H^w\right)+\frac{d\tau_H^w}{1-\tau_H^w}(1-\alpha)\frac{\delta_H l_H}{\sum_k \delta l}\varepsilon_H^w\right]}{\Delta_1(\Delta_1+\Delta_2-1)},\end{aligned} \tag{52}$$

$$
\begin{aligned}
\frac{dz_H^\pi}{z_H^\pi}\frac{1}{\varepsilon_H^\pi} =& -\frac{\alpha\left[\Delta_2 E_{(k)}\Big(\varepsilon_k^w \frac{d\tau_k^w}{1-\tau_k^w}\Big) + (\Delta_1 - 1)E_{(m)}\Big(\varepsilon_m^\pi \frac{d\tau_m^\pi}{1-\tau_m^\pi}\Big)\right]}{\Delta_2(\Delta_1+\Delta_2-1)} \\
&+ \frac{(\Delta_1+\Delta_2-1)\left[\frac{d\tau_L^\pi}{1-\tau_L^\pi}\alpha\frac{v_L q_L}{\sum_k vq}\varepsilon_L^\pi - \frac{d\tau_H^\pi}{1-\tau_H^\pi}\left(1+\alpha\frac{v_L q_L}{\sum_k vq}\varepsilon_L^\pi\right)\right]}{\Delta_2(\Delta_1+\Delta_2-1)},
\end{aligned}
\tag{53}
$$

$$
\begin{aligned}
\frac{dz_L^\pi}{z_L^\pi}\frac{1}{\varepsilon_L^\pi} =& -\frac{\alpha\left[\Delta_2 E_{(k)}\Big(\varepsilon_k^w \frac{d\tau_k^w}{1-\tau_k^w}\Big) + (\Delta_1 - 1)E_{(m)}\Big(\varepsilon_m^\pi \frac{d\tau_m^\pi}{1-\tau_m^\pi}\Big)\right]}{\Delta_2(\Delta_1+\Delta_2-1)} \\
&+ \frac{(\Delta_1+\Delta_2-1)\left[-\frac{d\tau_L^\pi}{1-\tau_L^\pi}\left(1+\alpha\frac{v_H q_H}{\sum_k vq}\varepsilon_H^\pi\right) + \frac{d\tau_H^\pi}{1-\tau_H^\pi}\alpha\frac{v_H q_H}{\sum_k vq}\varepsilon_H^\pi\right]}{\Delta_2(\Delta_1+\Delta_2-1)},
\end{aligned}
\tag{54}
$$

From conditions (51)-(54) we derive the final forms of the partial derivatives of each income/revenue level with respect to each tax rate.

$$
\begin{aligned}
\frac{dz_H^w}{d\tau_H^w}\frac{1}{z_H^w} &= \frac{\varepsilon_H^w}{1-\tau_H^w}\left[\frac{(1-\alpha)\frac{\delta_H l_H}{\sum_k \delta l}\varepsilon_H^w - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\right] \\
\frac{dz_L^w}{d\tau_H^w}\frac{1}{z_L^w} &= \frac{\frac{\varepsilon_H^w}{1-\tau_H^w}\frac{\delta_H l_H}{\sum_k \delta l}\varepsilon_L^w(1-\alpha)}{\Delta_1+\Delta_2-1} \\
\frac{dz_H^\pi}{d\tau_H^w}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_H^w}{1-\tau_H^w}\frac{\delta_H l_H}{\sum_k \delta l}\varepsilon_H^\pi\alpha}{\Delta_1+\Delta_2-1} \\
\frac{dz_L^\pi}{d\tau_H^w}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_H^w}{1-\tau_H^w}\frac{\delta_H l_H}{\sum_k \delta l}\varepsilon_L^\pi\alpha}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
\frac{dz_H^w}{d\tau_L^w}\frac{1}{z_H^w} &= \frac{\frac{\varepsilon_L^w}{1-\tau_L^w}\frac{\delta_L l_L}{\sum_k \delta l}\varepsilon_H^w(1-\alpha)}{\Delta_1+\Delta_2-1} \\
\frac{dz_L^w}{d\tau_L^w}\frac{1}{z_L^w} &= \frac{\varepsilon_L^w}{1-\tau_L^w}\left[\frac{(1-\alpha)\frac{\delta_L l_L}{\sum_k \delta l}\varepsilon_L^w - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\right] \\
\frac{dz_H^\pi}{d\tau_L^w}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_L^w}{1-\tau_L^w}\frac{\delta_L l_L}{\sum_k \delta l}\varepsilon_H^\pi\alpha}{\Delta_1+\Delta_2-1} \\
\frac{dz_L^\pi}{d\tau_L^w}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_L^w}{1-\tau_L^w}\frac{\delta_L l_L}{\sum_k \delta l}\varepsilon_L^\pi\alpha}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{56}
$$

$$
\begin{aligned}
\frac{dz_H^w}{d\tau_H^\pi}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_H^\pi}{1-\tau_H^\pi}\frac{v_H q_H}{\sum_m vq}\varepsilon^w_H(1-\alpha)}{\Delta_1+\Delta_2-1} \\
\frac{dz_L^w}{d\tau_H^\pi}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_H^\pi}{1-\tau_H^\pi}\frac{v_H q_H}{\sum_m vq}\varepsilon^w_L(1-\alpha)}{\Delta_1+\Delta_2-1} \\
\frac{dz_H^\pi}{d\tau_H^\pi}\frac{1}{z_H^\pi} &= \frac{\varepsilon_H^\pi}{1-\tau_H^\pi}\left[\frac{\alpha\varepsilon^\pi_H\frac{v_H q_H}{\sum_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\right] \\
\frac{dz_L^\pi}{d\tau_H^\pi}\frac{1}{z_L^\pi} &= \frac{\frac{\varepsilon_H^\pi}{1-\tau_H^\pi}\frac{v_H q_H}{\sum_m vq}\varepsilon^\pi_L\alpha}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{57}
$$

$$
\begin{aligned}
\frac{dz_H^w}{d\tau_L^\pi}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_L^\pi}{1-\tau_L^\pi}\frac{v_L q_L}{\sum_m vq}\varepsilon^w_H(1-\alpha)}{\Delta_1+\Delta_2-1} \\
\frac{dz_L^w}{d\tau_L^\pi}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_L^\pi}{1-\tau_L^\pi}\frac{v_L q_L}{\sum_m vq}\varepsilon^w_L(1-\alpha)}{\Delta_1+\Delta_2-1} \\
\frac{dz_H^\pi}{d\tau_L^\pi}\frac{1}{z_H^\pi} &= \frac{\frac{\varepsilon_L^\pi}{1-\tau_L^\pi}\frac{v_L q_L}{\sum_m vq}\varepsilon^\pi_H\alpha}{\Delta_1+\Delta_2-1} \\
\frac{dz_L^\pi}{d\tau_L^\pi}\frac{1}{z_L^\pi} &= \frac{\varepsilon_L^\pi}{1-\tau_L^\pi}\left[\frac{\alpha\varepsilon^\pi_L\frac{v_L q_L}{\sum_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\right]
\end{aligned} \tag{58}
$$

Maximization of the welfare function with respect to taxes

I next maximize the welfare function with respect to taxes, subject to the positive revenue requirement and the self-selection constraints as discussed in text. The Lagrangian, as shown in the text, can be written as

$$
\begin{aligned}
\max_{\tau_k^w,\tau_m^\pi} W = {} & \sum_k l_k\left(-c_w\!\left(\frac{z_k^w}{M(\theta)\,w_k}\right)\right)+\sum_m q_m\left(-c_\pi\!\left(\frac{z_m^\pi}{\frac{M(\theta)}{\theta}\pi_m}\right)\right) \\
& +\delta_H l_H c_w'\!\left(\frac{z_H^w}{M(\theta)\,w_H}\right)+\delta_L l_L c_w'\!\left(\frac{z_L^w}{M(\theta)\,w_L}\right)+v_H q_H c_\pi'\!\left(\frac{z_H^\pi}{\frac{M(\theta)}{\theta}\pi_H}\right)+v_L q_L c_\pi'\!\left(\frac{z_L^\pi}{\frac{M(\theta)}{\theta}\pi_L}\right)+R \\
& +\mu(\textstyle\sum_k \delta l)M(\theta)\left[\frac{\delta_H l_H}{\sum_k \delta l}\tau_H^w w_H+\frac{\delta_L l_L}{\sum_k \delta l}\tau_L^w w_L+\frac{v_H q_H}{\sum_m vq}\tau_H^\pi\pi_H+\frac{v_L q_L}{\sum_m vq}\tau_L^\pi\pi_L\right],
\end{aligned}
$$

where μ is the marginal cost of funds. Denote

$$
a=\frac{\delta_H l_H}{\sum_k \delta l}\tau_H^w w_H+\frac{\delta_L l_L}{\sum_k \delta l}\tau_L^w w_L \qquad \text{and} \qquad b=\frac{v_H q_H}{\sum_m vq}\tau_H^\pi\pi_H+\frac{v_L q_L}{\sum_m vq}\tau_L^\pi\pi_L.
$$

Since, as it will be shown below, the marginal cost of funds is greater than one, the social planner chooses to spend exactly R. To avoid clutter I assume this is so, and from the Kuhn-Tucker conditions I present only the relevant case for $\mu > 0$. The first order condition with respect to τ_H^w is

$$
\begin{aligned}
&\frac{\partial L}{\partial \tau_H^w} = \\
&= \sum_k l_k \left(-\frac{c_w'}{M(\theta)\, w_k} \frac{dz_k^w}{d\tau_H^w} \right) + \sum_m q_m \left(-\frac{c_\pi'}{\frac{M(\theta)}{\theta} \pi_m} \frac{dz_m^\pi}{d\tau_H^w} \right) \\
&+ \frac{dz_H^w}{d\tau_H^w} \frac{1}{M(\theta)\, w_H} l_H c_w' \left(\frac{z_H^w}{M(\theta)\, w_H} \right) + \delta_H l_H c_w'' \left(\frac{z_H^w}{M(\theta)\, w_H} \right) \frac{1}{M(\theta)\, w_H} \frac{dz_H^w}{d\tau_H^w} \\
&+ \frac{dz_L^w}{d\tau_H^w} \frac{1}{M(\theta)\, w_L} l_L c_w' \left(\frac{z_L^w}{M(\theta)\, w_L} \right) + \delta_L l_L c_w'' \left(\frac{z_L^w}{M(\theta)\, w_L} \right) \frac{1}{M(\theta)\, w_L} \frac{dz_L^w}{d\tau_H^w} \\
&+ \frac{dz_H^\pi}{d\tau_H^\pi} \frac{1}{\frac{M(\theta)}{\theta} \pi_H} q_H c_\pi' \left(\frac{z_H^\pi}{\frac{M(\theta)}{\theta} \pi_H} \right) + v_H q_H c_\pi'' \left(\frac{z_H^\pi}{\frac{M(\theta)}{\theta} \pi_H} \right) \frac{1}{\frac{M(\theta)}{\theta} \pi_H} \frac{dz_H^\pi}{d\tau_H^\pi} \\
&+ \frac{dz_L^\pi}{d\tau_L^\pi} \frac{1}{\frac{M(\theta)}{\theta} \pi_L} q_L c_\pi' \left(\frac{z_L^\pi}{\frac{M(\theta)}{\theta} \pi_L} \right) + v_L q_L c_\pi'' \left(\frac{z_L^\pi}{\frac{M(\theta)}{\theta} \pi_L} \right) \frac{1}{\frac{M(\theta)}{\theta} \pi_L} \frac{dz_L^\pi}{d\tau_L^\pi} \\
&+ \mu \left[\sum_k \left(\frac{l_k}{M(\theta)\, w_k} \frac{dz_k^w}{d\tau_H^w} \right) M(\theta) + (\textstyle\sum_k \delta l) M(\theta) \left(\frac{\sum_m \left(\frac{q_m}{\frac{M(\theta)}{\theta} \pi_m} \frac{dz_m^\pi}{d\tau_H^w} \right)}{\sum_k \delta l} - \frac{(\sum_m vq) \sum_k \left(\frac{l_k}{M(\theta)\, w_k} \frac{dz_k^w}{d\tau_H^w} \right)}{(\sum_k \delta l)^2} \right) \right] (a+b) \\
&+ \mu (\textstyle\sum_k \delta l) M(\theta) \left[\begin{array}{l}
\frac{l_H}{M(\theta)\, w_H} \frac{dz_H^w}{d\tau_H^w} \frac{\tau_H^w w_H}{\sum_k \delta l} + \frac{l_L}{M(\theta)\, w_L} \frac{dz_L^w}{d\tau_H^w} \frac{\tau_L^w w_L}{\sum_k \delta l} - \left(\frac{\sum_k \left(\frac{l_k}{M(\theta)\, w_k} \frac{dz_k^w}{d\tau_H^w} \right)}{\sum_k \delta l} \right) a + \frac{\delta_H l_H}{\sum_k \delta l} w_H \\
+ \frac{\delta_H l_H}{\sum_k \delta l} \tau_H^w \left(\frac{v_H q_H}{\sum_m vq} \frac{1}{z_H^\pi} \frac{dz_H^\pi}{d\tau_H^w} w_{HH} + \frac{v_L q_L}{\sum_m vq} \frac{1}{z_L^\pi} \frac{dz_L^\pi}{d\tau_H^w} w_{HL} - \frac{\sum_m \left(\frac{q_m}{\pi_m} \frac{dz_m^\pi}{d\tau_H^w} \right)}{\sum_m vq} w_H \right) \\
+ \frac{\delta_L l_L}{\sum_k \delta l} \tau_L^w \left(\frac{v_H q_H}{\sum_m vq} \frac{1}{z_H^\pi} \frac{dz_H^\pi}{d\tau_H^w} w_{LH} + \frac{v_L q_L}{\sum_m vq} \frac{1}{z_L^\pi} \frac{dz_L^\pi}{d\tau_H^w} w_{LL} - \frac{\sum_m \left(\frac{q_m}{\pi_m} \frac{dz_m^\pi}{d\tau_H^w} \right)}{\sum_m vq} w_L \right) \\
+ \frac{q_H}{\frac{M(\theta)}{\theta} \pi_H} \frac{dz_H^\pi}{d\tau_H^w} \frac{\tau_H^\pi \pi_H}{\sum_m vq} + \frac{q_L}{\frac{M(\theta)}{\theta} \pi_L} \frac{dz_L^\pi}{d\tau_H^w} \frac{\tau_L^\pi \pi_L}{\sum_m vq} - \frac{\sum_m \left(\frac{q_m}{\frac{M(\theta)}{\theta} \pi_m} \frac{dz_m^\pi}{d\tau_H^w} \right)}{\sum_m vq} b \\
+ \frac{v_H q_H}{\sum_m vq} \tau_H^\pi \left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{1}{z_H^w} \frac{dz_H^w}{d\tau_H^w} \pi_{HH} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{1}{z_L^w} \frac{dz_L^w}{d\tau_H^w} \pi_{LH} - \frac{\sum_k \left(\frac{l_k}{w_k} \frac{dz_k^w}{d\tau_H^w} \right)}{\sum_k \delta l} \pi_H \right) \\
+ \frac{v_L q_L}{\sum_m vq} \tau_L^\pi \left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{1}{z_H^w} \frac{dz_H^w}{d\tau_H^w} \pi_{HL} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{1}{z_L^w} \frac{dz_L^w}{d\tau_H^w} \pi_{LL} - \frac{\sum_k \left(\frac{l_k}{w_k} \frac{dz_k^w}{d\tau_H^w} \right)}{\sum_k \delta l} \pi_L \right)
\end{array} \right] = 0
\end{aligned}
\tag{59}
$$

Take the first six rows from the above expression and re-arrange

$$\delta_H l_H\, c_w''\!\left(\frac{z_H^w}{M(\theta)\,w_H}\right)\frac{1}{M(\theta)\,w_H}\frac{dz_H^w}{d\tau_H^w}+\delta_L l_L\, c_w''\!\left(\frac{z_L^w}{M(\theta)\,w_L}\right)\frac{1}{M(\theta)\,w_L}\frac{dz_L^w}{d\tau_H^w}$$

$$+v_H q_H\, c_\pi''\!\left(\frac{z_H^\pi}{\frac{M(\theta)}{\theta}\pi_H}\right)\frac{1}{\frac{M(\theta)}{\theta}\pi_H}\frac{dz_H^\pi}{d\tau_H^w}+v_L q_L\, c_\pi''\!\left(\frac{z_L^\pi}{\frac{M(\theta)}{\theta}\pi_L}\right)\frac{1}{\frac{M(\theta)}{\theta}\pi_L}\frac{dz_L^\pi}{d\tau_H^w}$$

$$+\mu(\textstyle\sum_k \delta l)M(\theta)\left[\frac{\sum_k\left(\frac{l_k}{M(\theta)\,w_k}\frac{dz_k^w}{d\tau_H^w}\right)}{\sum_k \delta l}+\frac{M'(\theta)}{M(\theta)}\theta\left(\frac{\sum_m\left(\frac{q_m}{\frac{M(\theta)}{\theta}\pi_m}\frac{dz_m^\pi}{d\tau_H^w}\right)}{\sum_m vq}-\frac{\sum_k\left(\frac{l_k}{M(\theta)\,w_k}\frac{dz_k^w}{d\tau_H^w}\right)}{\sum_k \delta l}\right)\right](a+b)=$$

$$=\delta_H l_H\, c_w''\!\left(\frac{z_H^w}{M(\theta)\,w_H}\right)\frac{1}{M(\theta)\,w_H}\frac{dz_H^w}{d\tau_H^w}\frac{z_H^w}{z_H^w}\frac{M(\theta)\,(1-\tau_H^w)w_H}{c_w'(z_H^w/M(\theta)\,w_H)}$$

$$+\delta_L l_L\, c_w''\!\left(\frac{z_L^w}{M(\theta)\,w_L}\right)\frac{1}{M(\theta)\,w_L}\frac{dz_L^w}{d\tau_H^w}\frac{z_L^w}{z_L^w}\frac{M(\theta)\,(1-\tau_L^w)w_L}{c_w'(z_L^w/M(\theta)\,w_L)}$$

$$+v_H q_H\, c_\pi''\!\left(\frac{z_H^\pi}{\frac{M(\theta)}{\theta}\pi_H}\right)\frac{1}{\frac{M(\theta)}{\theta}\pi_H}\frac{dz_H^\pi}{d\tau_H^w}\frac{z_H^\pi}{z_H^\pi}\frac{M(\theta)}{\theta}\frac{(1-\tau_H^\pi)\pi_H}{c_\pi'(z_H^\pi/\pi_H)}$$

$$+v_L q_L\, c_\pi''\!\left(\frac{z_L^\pi}{\frac{M(\theta)}{\theta}\pi_L}\right)\frac{1}{\frac{M(\theta)}{\theta}\pi_L}\frac{dz_L^\pi}{d\tau_H^w}\frac{z_L^\pi}{z_L^\pi}\frac{M(\theta)}{\theta}\frac{(1-\tau_L^\pi)\pi_L}{c_\pi'\!\left(z_L^\pi/\frac{M(\theta)}{\theta}\pi_L\right)}$$

$$+\mu(\textstyle\sum_k \delta l)M(\theta)\left[\alpha\frac{\sum_k\left(\frac{l_k}{M(\theta)\,w_k}\frac{dz_k^w}{d\tau_H^w}\right)}{\sum_k \delta l}+(1-\alpha)\frac{\sum_m\left(\frac{q_m}{\frac{M(\theta)}{\theta}\pi_m}\frac{dz_m^\pi}{d\tau_H^w}\right)}{\sum_m vq}\right](a+b)=$$

$$= \delta_H l_H \frac{1}{\varepsilon_H^w} \frac{dz_H^w}{d\tau_H^w} \frac{1}{z_H^w} M(\theta)\,(1-\tau_H^w) w_H + \delta_L l_L \frac{1}{\varepsilon_L^w} \frac{dz_L^w}{d\tau_H^w} \frac{1}{z_L^w} M(\theta)\,(1-\tau_L^w) w_L$$

$$+ v_H q_H \frac{1}{\varepsilon_H^\pi} \frac{dz_H^\pi}{d\tau_H^\pi} \frac{z_H^\pi}{z_H^\pi} \frac{M(\theta)}{\theta} (1-\tau_H^\pi)\pi_H + v_L q_L \frac{1}{\varepsilon_L^\pi} \frac{dz_L^\pi}{d\tau_L^\pi} \frac{z_L^\pi}{z_L^\pi} \frac{M(\theta)}{\theta} (1-\tau_L^\pi)\pi_L$$

$$+ \mu(\textstyle\sum_k \delta l) M(\theta) \left[\alpha \frac{\sum_k \left(\frac{l_k}{M(\theta)\, w_k} \frac{dz_k^w}{d\tau_H^w} \right)}{\sum_k \delta l} + (1-\alpha) \frac{\sum_m \left(\frac{q_m}{\frac{M(\theta)}{\theta} \pi_m} \frac{dz_m^\pi}{d\tau_H^w} \right)}{\sum_m vq} \right] (a+b).$$

Substitute this back into the first order condition (59). Divide the whole equation (59) by $(\sum_k \delta l) M(\theta)$ and re-arrange, noting that $a + b = R/(\sum_k \delta l) M(\theta) = \bar{R}$,

$$\begin{aligned}
\frac{\partial L}{\partial \tau_H^w} &= \frac{dz_H^w}{d\tau_H^w} \frac{1}{z_H^w} \frac{\delta_H l_H}{\sum_k \delta l} \left[\frac{1-\tau_H^w}{\varepsilon_H^w} w_H + \mu(w_H \tau_H^w + E_{(m)} \pi_{Hm} \tau_m^\pi - (1-\alpha)\bar{R}) \right] \\
&+ \frac{dz_L^w}{d\tau_H^w} \frac{1}{z_L^w} \frac{\delta_L l_L}{\sum_k \delta l} \left[\frac{1-\tau_L^w}{\varepsilon_L^w} w_L + \mu(w_L \tau_L^w + E_{(m)} \pi_{Lm} \tau_m^\pi - (1-\alpha)\bar{R}) \right] \\
&+ \frac{dz_H^\pi}{d\tau_H^w} \frac{1}{z_H^\pi} \frac{v_H q_H}{\sum_m vq} \left[\frac{1-\tau_H^\pi}{\varepsilon_H^\pi} \pi_H + \mu(\pi_H \tau_H^\pi + E_{(k)} w_{kH} \tau_k^w - \alpha \bar{R}) \right] \\
&+ \frac{dz_L^\pi}{d\tau_H^w} \frac{1}{z_L^\pi} \frac{v_L q_L}{\sum_m vq} \left[\frac{1-\tau_L^\pi}{\varepsilon_L^\pi} \pi_L + \mu(\pi_L \tau_H^\pi + E_{(k)} w_{kL} \tau_k^w - \alpha \bar{R}) \right] \\
&+ \frac{\delta_H l_H}{\sum_k \delta l} w_H \mu \\
&= 0.
\end{aligned}$$

Using (55), this is

$$\begin{aligned}
&(\Delta_1 + \Delta_2 - 1) \left[(1-\mu) \frac{1-\tau_H^w}{\varepsilon_H^w} w_H + \mu(w_H \tau_H^w + E_{(m)} \pi_{Hm} \tau_m^\pi - (1-\alpha)\bar{R}) \right] = \\
&= (1-\alpha) E_{(k)} \left(\varepsilon_k^w \left[\frac{1-\tau_k^w}{\varepsilon_k^w} w_k + \mu(w_k \tau_k^w + E_{(m)} \pi_{km} \tau_m^\pi - (1-\alpha)\bar{R}) \right] \right) \\
&- \alpha E_{(m)} \left(\varepsilon_m^\pi \left[\frac{1-\tau_m^\pi}{\varepsilon_m^\pi} \pi_m + \mu(\pi_m \tau_m^\pi + E_{(k)} w_{km} \tau_k^w - \alpha \bar{R}) \right] \right).
\end{aligned} \tag{60}$$

Analogously we can write the final forms of the first order conditions with respect to τ_L^w,

τ_H^π, and τ_L^π

$$
\begin{aligned}
&(\Delta_1+\Delta_2-1)\left[(1-\mu)\frac{1-\tau_L^w}{\varepsilon_L^w}w_L+\mu(w_L\tau_L^w+E_{(m)}\pi_{Lm}\tau_m^\pi-(1-\alpha)\bar{R})\right]=\\
&=(1-\alpha)E_{(k)}\left(\varepsilon_k^w\left[\frac{1-\tau_k^w}{\varepsilon_k^w}w_k+\mu(w_k\tau_k^w+E_{(m)}\pi_{km}\tau_m^\pi-(1-\alpha)\bar{R})\right]\right)\\
&-\alpha E_{(m)}\left(\varepsilon_m^\pi\left[\frac{1-\tau_m^\pi}{\varepsilon_m^\pi}\pi_m+\mu(\pi_m\tau_m^\pi+E_{(k)}w_{km}\tau_k^w-\alpha\bar{R})\right]\right), \qquad (61)
\end{aligned}
$$

$$
\begin{aligned}
&(\Delta_1+\Delta_2-1)\left[(1-\mu)\frac{1-\tau_H^\pi}{\varepsilon_H^\pi}\pi_H+\mu(\pi_H\tau_H^\pi+E_{(k)}w_{kH}\tau_k^w-\alpha\bar{R})\right]=\\
&=-(1-\alpha)E_{(k)}\left(\varepsilon_k^w\left[\frac{1-\tau_k^w}{\varepsilon_k^w}w_k+\mu(w_k\tau_k^w+E_{(m)}\pi_{km}\tau_m^\pi-(1-\alpha)\bar{R})\right]\right)\\
&+\alpha E_{(m)}\left(\varepsilon_m^\pi\left[\frac{1-\tau_m^\pi}{\varepsilon_m^\pi}\pi_m+\mu(\pi_m\tau_m^\pi+E_{(k)}w_{km}\tau_k^w-\alpha\bar{R})\right]\right), \qquad (62)
\end{aligned}
$$

$$
\begin{aligned}
&(\Delta_1+\Delta_2-1)\left[(1-\mu)\frac{1-\tau_L^\pi}{\varepsilon_L^\pi}\pi_L+\mu(\pi_L\tau_L^\pi+E_{(k)}w_{kL}\tau_k^w-\alpha\bar{R})\right]=\\
&=-(1-\alpha)E_{(k)}\left(\varepsilon_k^w\left[\frac{1-\tau_k^w}{\varepsilon_k^w}w_k+\mu(w_k\tau_k^w+E_{(m)}\pi_{km}\tau_m^\pi-(1-\alpha)\bar{R})\right]\right)\\
&+\alpha E_{(m)}\left(\varepsilon_m^\pi\left[\frac{1-\tau_m^\pi}{\varepsilon_m^\pi}\pi_m+\mu(\pi_m\tau_m^\pi+E_{(k)}w_{km}\tau_k^w-\alpha\bar{R})\right]\right). \qquad (63)
\end{aligned}
$$

To derive the marginal cost of funds, μ, add equations (60) through (63)

$$
\mu=\left(1-\frac{\left[\begin{array}{l} w_H\tau_H^w+w_L\tau_L^w+\pi_H\tau_H^\pi+\pi_L\tau_L^\pi \\ +E_{(m)}\pi_{Hm}\tau_m^\pi+E_{(m)}\pi_{Lm}\tau_m^\pi+E_{(k)}w_{kH}\tau_k^w+E_{(k)}w_{kL}\tau_k^w \end{array}\right]-2\bar{R}}{\frac{1-\tau_H^w}{\varepsilon_H^w}w_H+\frac{1-\tau_L^w}{\varepsilon_L^w}w_L+\frac{1-\tau_H^\pi}{\varepsilon_H^\pi}\pi_H+\frac{1-\tau_L^\pi}{\varepsilon_L^\pi}\pi_L}\right)^{-1}.
$$

Expanding terms and re-arranging, this is

$$
\mu=\left(1-\frac{\left[\begin{array}{l} \frac{\delta_L l_L}{\sum_k \delta l}(w_H\tau_H^w+E_{(m)}\pi_{Hm}\tau_m^\pi)+\frac{\delta_H l_H}{\sum_k \delta l}(w_L\tau_L^w+E_{(m)}\pi_{Lm}\tau_m^\pi) \\ \frac{v_L q_L}{\sum_m vq}(\pi_H\tau_H^\pi+E_{(k)}w_{kH}\tau_k^w)+\frac{v_H q_H}{\sum_m vq}(\pi_L\tau_L^\pi+E_{(k)}w_{kL}\tau_k^w) \end{array}\right]}{\frac{1-\tau_H^w}{\varepsilon_H^w}w_H+\frac{1-\tau_L^w}{\varepsilon_L^w}w_L+\frac{1-\tau_H^\pi}{\varepsilon_H^\pi}\pi_H+\frac{1-\tau_L^\pi}{\varepsilon_L^\pi}\pi_L}\right)^{-1}.
$$

Further this can be rearranged as

$$\mu = \left(1 - \frac{\begin{bmatrix} \left[\left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{v_L q_L}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{v_H q_H}{\sum_m vq} \right) w_{HH} + \left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{v_H q_H}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{v_L q_L}{\sum_m vq} \right) w_{HL} \right] \tau_H^w \\ + \left[\left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{v_H q_H}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{v_L q_L}{\sum_m vq} \right) w_{LH} + \left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{v_L q_L}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{v_H q_H}{\sum_m vq} \right) w_{LL} \right] \tau_L^w \\ + \left[\left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{v_L q_L}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{v_H q_H}{\sum_m vq} \right) \pi_{HH} + \left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{v_H q_H}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{v_L q_L}{\sum_m vq} \right) \pi_{HL} \right] \tau_H^\pi \\ + \left[\left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{v_H q_H}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{v_L q_L}{\sum_m vq} \right) \pi_{LH} + \left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{v_L q_L}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{v_H q_H}{\sum_m vq} \right) \pi_{LL} \right] \tau_L^\pi \end{bmatrix}}{\frac{1-\tau_H^w}{\varepsilon_H^w} w_H + \frac{1-\tau_L^w}{\varepsilon_L^w} w_L + \frac{1-\tau_H^\pi}{\varepsilon_H^\pi} \pi_H + \frac{1-\tau_L^\pi}{\varepsilon_L^\pi} \pi_L} \right)^{-1},$$

$$\mu = \left(1 - \frac{E_{(s)} w_{Hs} \tau_H^w + E_{(s)} w_{Ls} \tau_L^w + E_{(s)} \pi_{sH} \tau_H^\pi + E_{(s)} \pi_{sL} \tau_L^\pi}{\frac{1-\tau_H^w}{\varepsilon_H^w} w_H + \frac{1-\tau_L^w}{\varepsilon_L^w} w_L + \frac{1-\tau_H^\pi}{\varepsilon_H^\pi} \pi_H + \frac{1-\tau_L^\pi}{\varepsilon_L^\pi} \pi_L} \right)^{-1}, \tag{64}$$

where the expectations with respect to s in the numerator have weights $\left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{v_L q_L}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{v_H q_H}{\sum_m vq} \right)$ when $s \neq k$ or $s \neq m$, and $\left(\frac{\delta_H l_H}{\sum_k \delta l} \frac{v_H q_H}{\sum_m vq} + \frac{\delta_L l_L}{\sum_k \delta l} \frac{v_L q_L}{\sum_m vq} \right)$ when $s = k$ or $s = m$. □

Proof of Conjecture 10.

From the first order conditions (60) and (61) we can derive an equivalent expression for the marginal cost of public funds

$$\mu = \left(1 - \frac{w_H \tau_H^w - w_L \tau_L^w + E_{(m)} \pi_{Hm} \tau_m^\pi - E_{(m)} \pi_{Lm} \tau_m^\pi}{\frac{1-\tau_H^w}{\varepsilon_H^w} w_H - \frac{1-\tau_L^w}{\varepsilon_L^w} w_L} \right)^{-1}. \tag{65}$$

From Proposition 8 we know that

$$\frac{w_H \tau_H^w - w_L \tau_L^w + E_{(m)} \pi_{Hm} \tau_m^\pi - E_{(m)} \pi_{Lm} \tau_m^\pi}{\frac{1-\tau_H^w}{\varepsilon_H^w} w_H - \frac{1-\tau_L^w}{\varepsilon_L^w} w_L} > 0. \tag{66}$$

Suppose that $\frac{1-\tau_H^w}{\varepsilon_H^w} w_H < \frac{1-\tau_L^w}{\varepsilon_L^w} w_L$. Since $w_H > w_L$ by assumption, the last inequality can hold if $(1 - \tau_H^w) w_H < (1 - \tau_L^w) w_L$ and $\varepsilon_H^w \geq \varepsilon_L^w$. From the first order condition for search intensity, in the presence of income taxes (22), it follows that $\delta_H < \delta_L$. To see that this condition is also compatible with the second inequality, $\varepsilon_H^w \geq \varepsilon_L^w$, note the following examples which show that for a very general form of the search cost function the elasticity ε_k^w weakly decreases in search intensity.

Example 1: $c = A\delta^\beta$ with $\beta \geq 3$, $c' = A\beta(\delta)^{\beta-1}$, $c'' = A\beta(\beta-1)(\delta)^{\beta-2}$ and elasticity of

δ with respect to c'

$$\varepsilon = \frac{1}{\delta}/\frac{c''}{c'} = \frac{1}{\beta - 1}.$$

The elasticity is independent of δ, does not change for A, but decreases in β: the elasticity decreases for steeper cost functions.

Example 2: $c = A(\delta^\gamma + \delta^\beta)$ with $\beta \geq 3$, $\gamma < \beta$, $c' = A(\gamma\delta^{\gamma-1} + \beta\delta^{\beta-1})$,

$$\begin{aligned} c'' =& A(\gamma(\gamma-1)\delta^{\gamma-2} + \beta(\beta-1)\delta^{\beta-2}) > 0, \\ \varepsilon =& \frac{\gamma\delta^{\gamma-2} + \beta\delta^{\beta-2}}{\gamma(\gamma-1)\delta^{\gamma-2} + \beta(\beta-1)\delta^{\beta-2}}. \end{aligned}$$

The elasticity does not depend on A but depends on δ, γ, and β. The elasticity decreases in the intensity of search

$$\frac{\partial\varepsilon}{\partial\delta} = -\delta^{\beta+\gamma-5}(\beta-\gamma)^2 < 0.$$

Now suppose $\beta \geq 3$ and $\gamma = \beta - 1$. Then the elasticity decreases in β. Further, one can show that $\epsilon < 1$ for $\beta = 3$ and $\gamma = 2$, and $\epsilon > 1$ for $\beta = 2$ and $\gamma = 1$.

Under the assumption $\frac{1-\tau^w_H}{\varepsilon^w_H} w_H < \frac{1-\tau^w_L}{\varepsilon^w_L} w_L$, the numerator of equation (66) must be negative. However for $(1-\tau^w_H)w_H < (1-\tau^w_L)w_L$ it must be that $\tau^w_H > \tau^w_L$ and in the numerator of equation (66) $\tau^w_H w_H > \tau^w_L w_L$. Next consider the term $(E_{(m)}\pi_{Hm}\tau^\pi_m - E_{(m)}\pi_{Lm}\tau^\pi_m)$. Since by assumption $\pi_{HH} > \pi_{LH}$ and $\pi_{HL} > \pi_{LL}$ then it is easy to show that $(E_{(m)}\pi_{Hm}\tau^\pi_m - E_{(m)}\pi_{Lm}\tau^\pi_m > 0)$. However this is a contradiction to the fact that the ratio (66) is positive. Then it must be that in the social optimum with income taxes $(1-\tau^w_H)w_H > (1-\tau^w_L)w_L$, $\delta_H > \delta_L$, and $\varepsilon^w_H \leq \varepsilon^w_L$. □

Proof of Proposition 11.

The first part of the proposition follows immediately from equation (65). To see the second result first note that we want to derive the distribution of the tax burden between workers and employers. Then, the productivity skill is not relevant for this comparison, and we can collapse the first order conditions (60)-(63) to describe a market where the distributions of productivity skill on each side of the market collapse to a constant. The relevant first order conditions in such a market are only two, one that determines the optimal tax rate to a worker, and one that determines the optimal tax rate to a vacancy. Dropping all subscripts and expectation operators from equations (60) and (61) these conditions are

$$
\begin{aligned}
&(\Delta_1+\Delta_2-1)\left[(1-\mu)\frac{1-\tau^w}{\varepsilon^w}w+\mu(w\tau^w+\pi\tau^\pi-(1-\alpha)\bar{R})\right]= \\
&=(1-\alpha)\left[(1-\tau^w)w+\mu(w\tau^w\varepsilon^w+\pi\tau^\pi\varepsilon^w-(1-\alpha)\bar{R}\varepsilon^w)\right] \\
&-\alpha\left[(1-\tau^\pi)\pi+\mu(\pi\tau^\pi\varepsilon^\pi+w\tau^w\varepsilon^\pi-\alpha\bar{R}\varepsilon^\pi)\right]
\end{aligned}
\tag{67}
$$

$$
\begin{aligned}
&(\Delta_1+\Delta_2-1)\left[(1-\mu)\frac{1-\tau^\pi}{\varepsilon^\pi}\pi+\mu(\pi\tau^\pi+w\tau^w-\alpha\bar{R})\right]= \\
&=-(1-\alpha)\left[(1-\tau^w)w+\mu(w\tau^w\varepsilon^w+\pi\tau^\pi\varepsilon^w-(1-\alpha)\bar{R}\varepsilon^w)\right] \\
&+\alpha\left[(1-\tau^\pi)\pi+\mu(\pi\tau^\pi\varepsilon^\pi+w\tau^w\varepsilon^\pi-\alpha\bar{R}\varepsilon^\pi)\right],
\end{aligned}
\tag{68}
$$

where $\Delta_1+\Delta_2-1=1+(1-\alpha)\varepsilon^w+\alpha\varepsilon^\pi$. Add the first order conditions (67) and (68), and re-arrange, noting that $w\tau^w+\pi\tau^\pi=\bar{R}$

$$
\frac{-(\mu-1)\frac{1-\tau^w}{\varepsilon^w}w+\mu(1-(1-\alpha))\bar{R}}{-(\mu-1)\frac{1-\tau^\pi}{\varepsilon^\pi}\pi+\mu(1-\alpha)\bar{R}}=const \tag{69}
$$

The second part of Proposition 11 follows immediately by noting that the higher is the relative elasticity of demand for labor, $\varepsilon^\pi/\varepsilon^w\uparrow$, the lower is the relative tax burden on vacancies, $\tau^\pi/\tau^w\downarrow$.$\square$

Proof of Proposition 12.

To prove the first part of Proposition 12, consider again the first order conditions (60) and (61), and in particular the derived form of the marginal cost of public funds in Conjecture 10, equation (65). Expand the expression $E_{(m)}\pi_{Hm}\tau_m^\pi-E_{(m)}\pi_{Lm}\tau_m^\pi$

$$
E_{(m)}\pi_{Hm}\tau_m^\pi-E_{(m)}\pi_{Lm}\tau_m^\pi=\begin{array}{l}+\frac{v_Hq_H}{\sum_m vq}\pi_{HH}\tau_H^\pi+\frac{v_Lq_L}{\sum_m vq}\pi_{HL}\tau_L^\pi\\ -\frac{v_Hq_H}{\sum_m vq}\pi_{LH}\tau_H^\pi-\frac{v_Lq_L}{\sum_m vq}\pi_{LL}\tau_L^\pi\end{array} \tag{70}
$$

When $\pi_{HH}-\pi_{LH}$ and/or $\pi_{HL}-\pi_{LL}$ increase, then $E_{(m)}\pi_{Hm}\tau_m^\pi-E_{(m)}\pi_{Lm}\tau_m^\pi$ also increases. Then, for any vector of tax rates $\tau^\pi(\tau_H^\pi,\tau_L^\pi)$ it must be true that $E_{(m)}\pi_{Hm}-E_{(m)}\pi_{Lm}$ also increases. For any vector of after tax wages $w(w_H,w_L)$, such an increase in $E_{(m)}\pi_{Hm}-E_{(m)}\pi_{Lm}$ must be occompanied by a decrease in $\tau_H^w/\tau_L^w\downarrow$.

To see the second part of Proposition 12, consider again the first order conditions (67) and (68), and in particular condition (69), derived in Proposition 11. The second part of Proposition 12 follows immediately by noting that the higher is the contribution of a vacancy in creating a match, as measured by the elasticity $(1-\alpha)$, $(1-\alpha)/\alpha \uparrow$, the higher is the tax burden on workers as measured by the tax rate, $\tau^{\pi}/\tau^{w} \downarrow$.$\square$