DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 09-11

Advanced-Purchase Premiums versus Discounts in the Presence of Capacity Constraints

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November 2009

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Wednesday 4^{th} November, 2009

Abstract

When examining the pricing behavior of airlines, concerts, and sports tickets one observes that in some cases advanced sales are made at a discount while other times a premium is charged. While these markets are all characterized by advanced sales and capacity constraints, the firms advanced pricing strategies differ. This paper first develops a simple example where charging an advanced-purchase premium is optimal. Then examines when an advance-purchase discount is optimal versus when a premium can be charged for advanced-purchases. Discounts will tend to prevail when most customers are uncertain as to their future preferences. Premiums will be charged when most consumers know their preferred good in advance.

*I would like to thank my adviser Yongmin Chen for providing much needed direction and many helpful comments and Jennifer Lamping for her detailed suggestions.

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1 Introduction

When an individual purchases an airline ticket, it is advisable to make the purchase well in advance of the flight to take advantage of the airline's advanced-purchase discounts. Waiting until closer to the time of the flight often results in paying a higher price for the same flight. Since there are typically many flights to a given destination throughout the course of a day, a consumer purchasing far in advance is not likely to always know for certain which flight will be preferred once the departure date arrives. When one chooses to purchase a ticket to a popular concert or show in advance, a higher price is often paid than if one were to walk up to the ticket window right before the show. The higher price often takes the form of a service charge by some market intermediary like Ticket Master or Fandango. In this case, a consumer is often choosing between concerts on different days or by different bands. As such, he is more likely to know for certain which performance he will prefer. Both these industries are characterized by capacity constraints and firms selling tickets in advance as well as close to the time of consumption. In the airline industry advanced-sales discounts are used while for concerts and shows advanced-sales premiums are observed.

This paper will show that the decision by a capacity constrained firm to offer an advance-purchase discount or premium depends on the certainty of the consumers preferences between two substitute goods and the cost of ending up with the less preferred good. First a simple example under which capacity constraints lead to advanced-purchase premiums is developed. Then, in a model expanding on the Gale and Holmes (1992) framework of advance-purchase discounts, the critical elements from the advanced-purchase premium example will be added.

Gale and Holmes (1992) and Gale and Holmes (1993) start with a monopolist airline offering a pair of flights to a group of consumers who don't know their preferences in advance. The consumers have differing costs of being mismatched with the ex-post less preferred flight. This creates an opportunity to price discriminate by offering an advanced-purchase discount.

Those consumers with a low cost of consuming the wrong good will take the discount and buy in advance, leaving only the high-cost consumers remaining in the market. These consumers can then be charged a higher price for their preferred flight once their preferences become known.

I extend their model to include three consumer types: those who are uncertain as to their preferences, those who prefer good one, and those who prefer good two. Each consumer will have a cost of consuming their less preferred good. This cost is continuous from having no preference between the goods to having no value for the less preferred good. With a capacity constrained monopolist, preference-uncertain consumers can be forced to separate into relatively high-cost and relatively low-cost consumers being offered an advanced-purchase discount. The low-cost, preference-uncertain consumers will be willing to take the discount since the risk of purchasing the wrong good is not a large concern. The high-cost, preference-uncertain consumers are forced to wait as the discount does not cover the risk of getting the wrong good. In the case of a discount, all preference-certain consumers will purchase their preferred good in advance. The effectiveness of the discount will depend on both the number of preference-uncertain consumers and the distribution of the costs of mismatching.

On the other hand, if a premium is charged, preference-uncertain consumers will be unwilling to pay it[1] but the high-cost preference-certain consumers will (if the premium is not too large). These customers will purchase in advance to ensure themselves from the good being sold out if they wait. The low-cost preference-certain consumers will not pay the premium. They are willing to wait, risking their preferred good being sold-out, because they don't give up as much by getting their less preferred good. By charging a premium in advance the monopolist can price discriminate preference-certain consumers. This is similar to the "buying frenzy" in DeGrabba and Mohammed (1999). Unlike the

[1]If the premium is small, then under some restrictive conditions, a small number of preference-uncertain consumers with a medium cost of getting the wrong good may purchase in advance paying the higher price. This case will be examined in the consumer behavior section, Section 4.

preference-uncertain consumers, it is the high-cost preference-certain consumers that are now willing to purchase in advance paying the premium. The low-cost preference-certain consumers will wait. Having more high-cost, preference certain consumers makes charging a premium for advanced-purchases more effective. In the end, because of the dichotomy in how different consumer types can be discriminated against, the relative size of these types and the distribution of mismatch costs will determine whether a premium or discount is profit maximizing. If there are mostly preference-certain consumers with a high cost to consuming the less preferred good, a premium will be charged. If there are mostly preference-uncertain consumers with low costs of consuming the less preferred good, an advanced-purchase discount will prevail.

Others have examined various causes for discount and premium pricing for advanced sales. Dana (1992) showed that advanced-purchase discounts can persist in competitive markets for goods in which capacity is not storable. Gilbert and Klemperer (2000) examines the case where consumers have to make a sunk cost investment to participate in the market. Under this case committing to a price that leads to excess demand is optimal. Rosen and Rosenfield (1997) looks at intertemporal pricing issues when faced with the problem of managing capacity. They show that under constrained capacity for multiple substitute goods, like theater performances on different days, it can be optimal to queue the consumers for the early good and let prices decline over time. Finally, Spulber (1993) proves that there are multiple profit maximizing pricing strategies, from reference point pricing to priority service, for a capacity constrained monopolist selling two quality differentiated goods when faced with an unknown distribution of consumers who have multi-unit demand.

The paper proceeds in Section 2 by developing an example to give intuition for when an advanced-purchase premiums are possible. Section 3 expands the Gale and Holmes model by adding preference-certain consumers and sets up the key assumptions. Section 4 examines the behavior of consumers within the model showing how various consumer types can be discriminated against. Section 5 will develop the optimal pricing policy for the monopolist. Section 6 relates the model results back to the motivating examples of concert tickets and airlines. Section 7 summarizes and concludes.

2 An Example of Advanced-Purchase Premiums

To show how advanced-purchase premiums are possible, a motivating example is presented here. Consider a monopolist selling concert tickets to a unit mass of two consumer types: high valuation and low valuation consumers. The monopolist has .75 tickets and each set of consumer types account for half of the total consumer population. The high valuation consumers value the tickets at \$50, the low valuation consumers value them at \$40 each, and all consumers have unit demand. For simplicity the seller has a marginal cost of zero[2]. The monopolist can charge one price for day of concert sales and a second price for advanced-purchases but must commit to both prices in advance.

The monopolist considers three different potentially optimal strategies. First, she could set a price of \$50 selling only to the .5 high valuation consumers, netting a profit of $\pi = \$25$. Alternatively, she could sell all the tickets by pricing at the lower valuation. She would sell a quantity of .75 at a price of \$40 and yield a profit $\pi = \$30$. Finally, she could price discriminate charging \$45 in advance and \$40 the day of the concert resulting in all high-types purchasing in advance and half the low-types waiting to purchase yielding a profit of $\pi = .5 \cdot \$45 + .25 \cdot \$40 = \$32.50$. To see that this is an equilibrium first examine the high-type's strategy. In equilibrium, all of the high types purchase in advance, so if one high valuation customer were to wait, there are only .25 tickets remaining. He must then compete with the .5 low-types for a ticket and thus only gets a ticket half of the time[3]. His

[2] Alternatively, both prices and valuations can be thought of as the net with respect to a constant marginal cost.

[3] This assumes a rule of randomly distributing rationed tickets via a uniform distribution among all consumers that wish to purchase. If the tickets could be distributed to lower valuation consumers with a

expected surplus of waiting is $E[CS_H^{Wait}] = .5 \cdot (\$50 - \$40) = \$5$. If he purchases now he gets a surplus of $CS_H^{Now} = \$50 - \$45 = \$5$, so he is just as well off purchasing now. The low-types get zero surplus by waiting and negative surplus by purchasing now so they are happy to wait. Finally, the monopolist cannot charge a higher price in either period without losing customers. Lowering the price in either period will reduce revenue so her strategy is a best response to the consumer behavior. Charging an advanced-purchase premium of \$5 on top of the \$40 base price is the profit maximizing strategy.

To examine what determines the size of the premium that can be charged, the example is expanded here. Let there be λ_H high valuation customers who value a ticket at v_H and λ_L low valuation customers who value a ticket at v_L. Using a continuum of consumers, normalize λ_H and λ_L to be proportions of the total population so that $\lambda_L + \lambda_H = 1$. Also, without loss of generality, $v_L < v_H$. The monopolist has a capacity of $K \in (\lambda_H, 1)$. This ensures that all high valuation consumers can purchase a ticket in advance and that some tickets are rationed among the low valuation consumers. The monopolist will set the day of price to v_L. If all high-type consumers purchase in advance then there will be $K - \lambda_H$ tickets remaining for day of show sales to be purchased by λ_L low valuation customers. As such any high-type consumer who chooses to wait will get a surplus of $E[CS_H^{Wait}] = Pr(\text{Get Ticket}|\text{High Type \& Wait}) \cdot (v_H - p_{Wait}) = \frac{K-\lambda_H}{1-\lambda_H}(v_H - v_L)$. The monopolist must give the high-type consumers this much in surplus from the advance purchase. So $p_{Advance} = v_H - E[CS_H^{Wait}] = \frac{1-K}{1-\lambda_H}v_H + \frac{K-\lambda_H}{1-\lambda_H}v_L$. The absolute premium charged is then $p_{Advance} - p_{Wait} = \frac{1-K}{1-\lambda_H}(v_H - v_L)$, while the markup is given by $\frac{p_{Advance}-p_{Wait}}{p_{Wait}} = \frac{1-K}{1-\lambda_H}(\frac{v_H}{v_L} - 1)$[4].

From the solution it can be seen that a lower capacity results in more rationing, increasing the premium that high-types are willing to pay because they are less likely to receive a ticket

higher probability, a larger premium could be sustained during advanced sales.

[4]This solution is for the separating equilibrium that prevails under the given capacity restriction. The two pooling equilibria where a single price equal to v_H or v_L prevails can happen if the capacity is smaller than the number of high valuation consumers. Which one occurs is dependent on both the sizes of the consumer populations and their relative valuations.

if they wait. Likewise, having more high-type consumers reduces the day of show supply of tickets increasing the premium that can be charged. Finally, the larger the absolute spread between the high and low valuations the more that high-type customers are giving up by not receiving a ticket, and thus the higher premium they are willing to pay. The higher the relative spread, the higher the markup percentage on advanced ticket prices for the same reason. It is worth noting that unlike the DeGraba-Mohammed result bundling is not needed to get an advanced-purchase premium. While not captured in this example, risk aversion by the high valuation types would increase the premium they would be willing to pay because it increases the cost of not getting a ticket. Also, a rationing rule that lowered the probability of a high valuation consumer receiving a ticket if they waited would increase the premium that could be charged.

3 Model Description

This model extends the Gale and Holmes (1992) model of advanced purchase discounts by adding additional consumer types. Their model showed how advanced-purchase discounts arise with preference-uncertain consumers and a monopolist selling two goods. This model will extend that framework to include preference-certain consumers. There are two goods: good-A and good-B. Sales of the goods occur both in advance at $t = 0$ and immediately prior to consumption at $t = 1$. There is a continuum of measure one, risk-neutral consumers with unit demand and distribution of reservation valuations[5] of $r \sim f_r(\cdot)$ on $[0, r_{max}$ for their preferred good. Consumers vary in two dimensions: their preference over the goods and their cost of being mismatched with their preferred good. Expanding the Gale-Holmes model, there are continua of three consumer types rather than just the one. Type-A consumers have a strict preference for good-A; type-B consumers have a strict preference for good-B;

[5] The reservation value is the highest price that a consumer is willing to pay and still receive a non-negative surplus. This terminology is used to match with that of Gale and Holmes.

type-U consumers are initially uncertain as to their preferences at $t = 0$. For simplicity, equal numbers of type-A and type-B consumers is assumed. Denoting the number of consumers of each type by λ_i, $\lambda_A = \lambda_B$ and $\lambda_A + \lambda_B + \lambda_U = 1$. Henceforth, imposing the symmetry assumption, I will use λ_A for both the number of type-A and type-B consumers.

Assumption 1. *Ex-ante Probability that Type-U Prefers Peak Good*

(a) $Pr[\text{Type-U prefers peak good}|t = 0] = \alpha$

(b) $Pr[\text{Type-U prefers off-peak good}|t = 0] = 1 - \alpha$

(c) $\alpha > \frac{1}{2}$

At $t = 1$, $\alpha > \frac{1}{2}$ of these uncertain consumers will prefer the peak good[6] while the remaining $1 - \alpha$ uncertain consumers will prefer the off-peak good. Ex-ante each good is equally likely to be the high demand (peak) good.

Assumption 2. *Ex-ante Peak Demand*

Pr[Good-A is Peak$|t = 0]$=Pr[Good-B is Peak $|t = 0]$=$\frac{1}{2}$

Consumers also vary in their costs of mismatching denoted by y. A y-cost consumer has a reservation valuation of $r - y$ for the less preferred good. The distribution of costs y has a continuous density function $f_y(\cdot)$ and a differentiable cumulative distribution $F_y(\cdot)$ on $[0, r]$ which is stochastically independent of the consumer's preference type[7]. y may however be correlated with the reservation valuation r for a given consumer. Thus the reservation valuation and cost of mismatching have the joint distribution $(r, y) \sim f(r, y)$ on the support $[0, r_{max}] \times [0, r]$. This distribution is assumed to be continuous everywhere. This distribution ensures that the valuation for the less preferred good is non-negative and is weakly less than

[6]The peak good is the good which has higher ex-post demand.

[7]While different consumers types could have a different distribution of mismatch costs, this would complicate the analysis without sufficiently adding to the results.

the reserve valuation for the preferred good. Additionally, demand will be downward sloping in price. Finally, each consumer knows their own reservation value and cost of consuming the less preferred good in advance but the seller only knows the distribution of the valuations and costs.

Assumption 3. *Capacity Bounds*

$\alpha\lambda_U + \lambda_A > K \geq \frac{1}{2}$

This condition ensures that while the peak good will need to be rationed if all consumers were to wait to purchase, every consumer can always choose to wait and purchase one of the two goods. Additionally, the marginal cost of both goods is normalized to zero[8].

Finally, as in Gale and Holmes (1992), the monopolist firm will commit to prices for both periods in advance. Let p_0 be the price for advance sales and p_1 be the price for sales at time $t = 1$. All sales at $t = 0$ are final and there is no secondary market for these goods. Since the monopolist commits to the pricing of the goods in advance, the goods' prices cannot be conditional on whether the good is peak or off peak. Since this paper examines the optimality of advanced pricing discounts and premiums, define $p_0 = p_1 + \delta$ where $\delta < 0$ denotes a discount for purchasing in advance and $\delta > 0$ denotes having to pay a premium in advance.

4 Consumer Behavior

Consumers in this model make three choices in two time periods. Consumers will first decide whether to purchase in the advance period, $t = 0$, at price p_0 or postpone the purchase decision to until $t = 1$. Type-U consumers learn their preference between time $t = 0$ and time $t = 1$. If a consumer did not purchase in period zero he will try to purchase the preferred

[8]Alternatively, the reservation valuations and prices could be considered as the net with respect to a constant marginal costs.

good in period $t = 1$, given that $p_1 \leq r$. If the preferred good is sold out he will choose to purchase the less preferred good if and only if its price is less than his valuation for that good, $p_1 \leq r - y$. A consumer will purchase in advance if his expected surplus from buying now is larger than his expected surplus from waiting.

First, the behavior of each consumer type will be examined. Low-cost preference-uncertain consumers will accept a discount for purchasing in advance because they care less about getting their mismatched good. Likewise, high-cost, preference-certain consumers will be shown to be willing to pay a premium when purchasing in advance due to their concern of their preferred good selling out. Next, the aggregate demand for tickets will then be shown to be continuous across the relevant range of prices. Using a fixed point theorem, it will be proved that an equilibrium fraction of consumers are willing to wait for tickets exists. This will be used to ensure a solution to the monopolist's profit maximization problem.

Type-A and type-B consumer face the same problem. If they buy in advance they will purchase the correct good because they already know their preferences and gain a net surplus of $v_{0i}(r, y)$.

$$v_{0A}(r, y) = v_{0B}(r, y) = r - p_0 = r - (p_1 + \delta) \tag{1}$$

If a type-A or type-B consumer waits to purchase the good he risks his preferred good being the peak good (happens half the time) and it possibly being sold out. Let ϕ_p be the probability of getting the peak good given that a customer prefers the peak good. ϕ_p will be determined by the number of consumers who want the peak good but didn't purchase it in advance and the remaining supply of the peak good in the second period. If the preferred good is sold out, a consumer will be willing to purchase the less preferred good only if $r - y \geq p_1$. As such the expected surplus for preference-certain consumers of waiting until

$t = 1$ to purchase the good is,[9]

$$v_{1A}(r,y) = v_{1B}(r,y) = \begin{cases} \frac{1}{2}[\phi_p(r-p_1) + (1-\phi_p)(r-p_1-y)] + \frac{1}{2}(r-p_1) & \text{if } y \leq r-p_1 \\ \frac{1}{2}[\phi_p(r-p_1)] + \frac{1}{2}(r-p_1) & \text{if } y > r-p_1 \end{cases}$$

Simplifying yields,

$$v_{1A}(r,y) = v_{1B}(r,y) = \begin{cases} (r-p_1) - \frac{1-\phi_p}{2}y & \text{if } y \leq r-p_1 \\ \frac{1+\phi_p}{2}(r-p_1) & \text{if } y > r-p_1 \end{cases} \tag{2}$$

It is easily verified that $v_{1A}(r,y)$ and $v_{1B}(r,y)$ are continuous, weakly decreasing functions in y since both halves of the piecewise function take the same value at $y = r - p_1$. Since each section of the piecewise $v_{1A}(r,y)$ is linear, the only possible discontinuity occurs at $y = r-p_1$. Because both sides of the function take the same value, $\frac{1+\phi_p}{2}(r-p_1)$, at this point $v_{1A}(r,y)$ is continuous. For low values of y, $v_{1A}(r,y)$ is a negative sloped line and for high values of y the valuation function is constant. This ensure that $v_{1A}(r,y)$ is weakly decreasing.

Lemma 1. *$v_{1A}(r,y)$ and $v_{1B}(r,y)$ are continuous and weakly decreasing in y. $v_{1A}(r,y)$ and $v_{0A}(r,y)$ intersect at most once if $\delta \in [0, \frac{1-\phi_p}{2}(r-p_1))$ and never intersect if $\delta \notin [0, \frac{1-\phi_p}{2}(r-p_1))$*[10].

Proof. $v_{1A}(r,y)$ and $v_{1B}(r,y)$ are clearly continuous and weakly decreasing.

For the single crossing property, consider three cases: a discount, a small premium, and a large premium. (See Figure 1)

Case I, Discount: In the discount case the surplus for waiting is always below that for purchasing now because $\delta < 0$ gives us $v_{1A}(r,0) = r - p_1 \leq v_{0A}(r,0) = r - p_1 - \delta$. With

[9] This assumes that there is no excess demand for the off peak good. This is ensured by the capacity Assumption 3.

[10] In the case of $\delta = \frac{1-\phi_p}{2}(r-p_1)$, $v_{1A}(r,y) = v_{0A}(r,y) \forall y \geq r - p_1$. The curves intersect but never cross. This corresponds to high-y consumers being indifferent between purchasing in advance and waiting. All consumers can be considered wait.

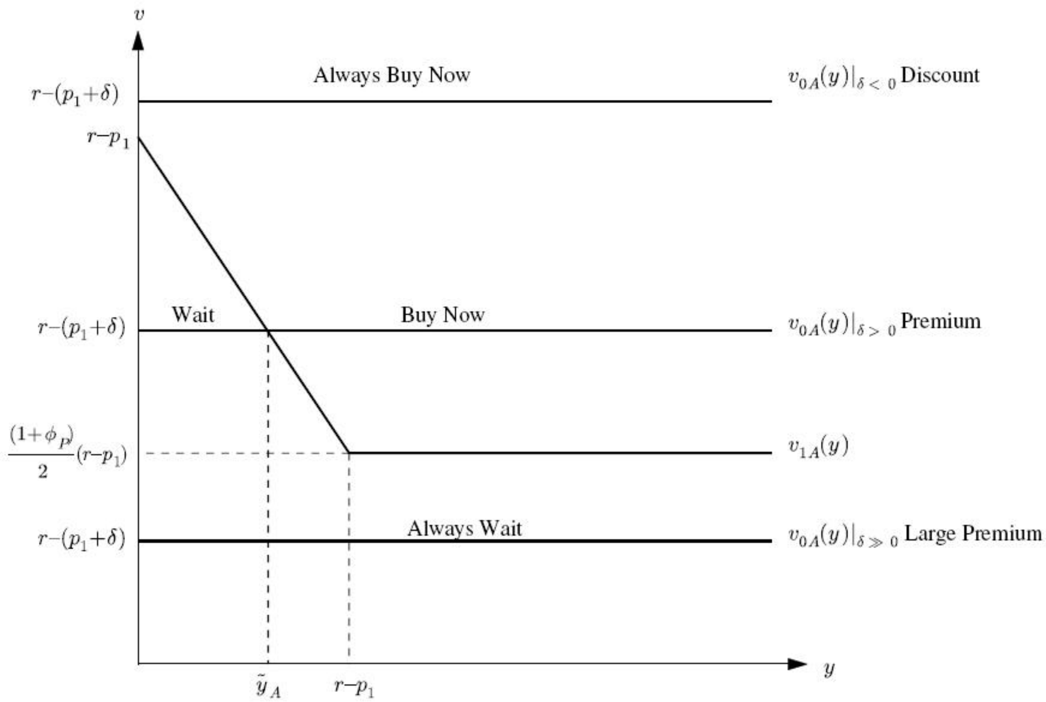

Figure 1: Expected Consumer Surplus for Type-A Consumers

$v_{1A}(r, y)$ being weakly decreasing in y, the surplus lines never cross.

Case II, Small Premium: In this case the consumer surplus from waiting starts above that for buying now but ends below. Examining the range $\delta \in [0, \frac{1-\phi_p}{2}(r-p_1))$. $v_{1A}(r, 0) = r-p_1 \geq v_{0A}(r, 0) = r - p_1 - \delta$ since δ is positive. $v_{1A}(r, r-p_1) = \frac{1-\phi_p}{2}(r-p_1) \geq v_{0A}(r, 0) = r - p_1 - \delta$ since $\delta \leq \frac{1-\phi_p}{2}(r-p_1)$. Since the surplus functions are continuous, the intermediate value theorem guarantees a crossing. Monotonicity ensures that it is a single crossing.

Case III, Large Premium: In this case the consumer surplus from waiting is always above that for purchasing now. For $\delta > \frac{1-\phi_p}{2}(r-p_1)$, $\min v_{1A}(y) = r - p_1 - \delta > v_{0A}(y) = r - p_1 - \delta$. The surplus functions will never cross. $\square$

Next we can verify that $v_{1A}(r, y)$ and $v_{0A}(r, y)$ cross at most once.

Let $\tilde{y}_A$ be the crossing point of $v_{1A}(r, y)$ and $v_{1B}(r, y)$ or 0 if they never cross. Solving

$v_{1A}(r, \tilde{y}_A) = v_{1B}(r, \tilde{y}_A)$ yields,

$$\tilde{y}_A(r) = \begin{cases} 0 & \text{if } \delta \in (-r, 0) \\ \frac{2\delta}{1-\phi_p} & \text{if } \delta \in [0, \frac{1-\phi_p}{2}(r - p_1)] \\ r & \text{if } \delta \in (\frac{1-\phi_p}{2}(r - p_1), r) \end{cases} \tag{3}$$

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If a discount is offered for advanced purchases, all type-A consumers will purchase in advance resulting in $\tilde{y}_A(r) = 0$. Alternatively, for a large premium, $\delta \geq \frac{1-\phi_p}{2}(r_{max} - p_1)$, all type-$A$ customers will wait resulting in $\tilde{y}_A(r) = r$. A type-A consumer will purchase in advance when his cost of getting the wrong good is too large. The quantity of type-A consumers who will be purchasing in advance can thus be defined by $Q_{0A} = \lambda_A \int_0^{r_{max}} \int_{\tilde{y}(r)}^{r} f(r, y) dy dr$. Likewise, for type-$B$ consumers $v_{0B}(r, y)$ crosses $v_{1B}(r, y)$ at most once defining a critical value of $\tilde{y}_B(r)$. For $y > \tilde{y}_B(r)$ the consumer will purchase in advance and for $y \leq \tilde{y}_B(r)$ the consumer will wait to purchase. The quantity of type-B consumers purchasing in advance is $Q_{0B} = \lambda_B \int_0^{r_{max}} \int_{\tilde{y}(r)}^{r} f(r, y) dy dr = Q_{0A}$ by symmetry of type-A and type-B consumers. Price discrimination of high-y and low-y preference-certain consumers (type-A and type-B) is thus possible. By charging a small premium and having some risk of selling out should they wait to purchase, high-y preference-certain consumers are willing to purchase in advance.

Lemma 2. *Q_{0A} is a function of δ and ϕ_p and is continuous everywhere except at $\delta = \frac{1-\phi_p}{2}(r - p_1)$.*

Proof. $\tilde{y}_A(r)$ is a continuous function of δ on $(-r, r) \sim \{\frac{1-\phi_p}{2}(r - p_1)\}$ and ϕ_p on $(0, 1)$. Since $f(r, y)$ is continuous, Q_{0A} is also a continuous function of δ and ϕ_p on the domain except at $\delta = \frac{1-\phi_p}{2}(r - p_1)$. □

Type-U consumers face a very different problem. Since they are unsure of their preferences they risk purchasing the wrong good if they buy in advance. Per Assumption 2, this happens half the time. Uncertain consumers are equally like to prefer either good[11]. This makes the expected surplus of a type-U consumer buying in advance v_{0U}.

$$v_{0U}(r,y) = r - p_0 - \frac{1}{2}y \tag{4}$$

Lemma 3. *$v_{0U}(r,y)$ is continuous and decreasing with respect to y.*

If a preference-uncertain consumer waits, he risk wanting the rationed peak good. Since each good is equally likely to be the peak good, half the time the uncertain consumers will end up wanting the peak good.

$$v_{1U}(r,y) = \begin{cases} \alpha\phi_p(r-p_1) + \alpha(1-\phi_p)(r-p_1-y) + (1-\alpha)(r-p_1) & \text{if } y \le r-p_1 \\ \alpha\phi_p(r-p_1) + (1-\alpha)(r-p_1) & \text{if } y > r-p_1 \end{cases}$$

Simplifying yields,

$$v_{1U}(r,y) = \begin{cases} (r-p_1) - \alpha(1-\phi_p)y & \text{if } y \le r-p_1 \\ ((1-\alpha(1-\phi_p))(r-p_1) & \text{if } y > r-p_1 \end{cases} \tag{5}$$

$v_{0U}(r,y)$ is downward sloping in y rather than constant as is $v_{0A}(r,y)$. In the case of a premium, this may lead to either a no crossing or double crossing depending on the slope of $v_{1U}(r,y)$. For an advanced-purchase discount a single crossing will occur. Regardless of the number of crossings in the premium case, the quantity of goods purchased in advance by type-U consumers will be decreasing and continuous with respect to δ (increasing as a larger discount is offered, see Figure 2 and Figure 3). To define this quantity it is useful to define the two potential crossing points as $\tilde{y}_U^H(r)$ and $\tilde{y}_U^L(r)$. If a discount is given then there is a single crossing. In this case define $\tilde{y}_U^H(r)$ as this crossing point and define $\tilde{y}_U^L(r) = 0$. If a

[11] If this were not the case then they would have a strict preference between the goods. Consumers who have a strict preference for one good in advance but may have that preference switch in the second time period are left out of the analysis.

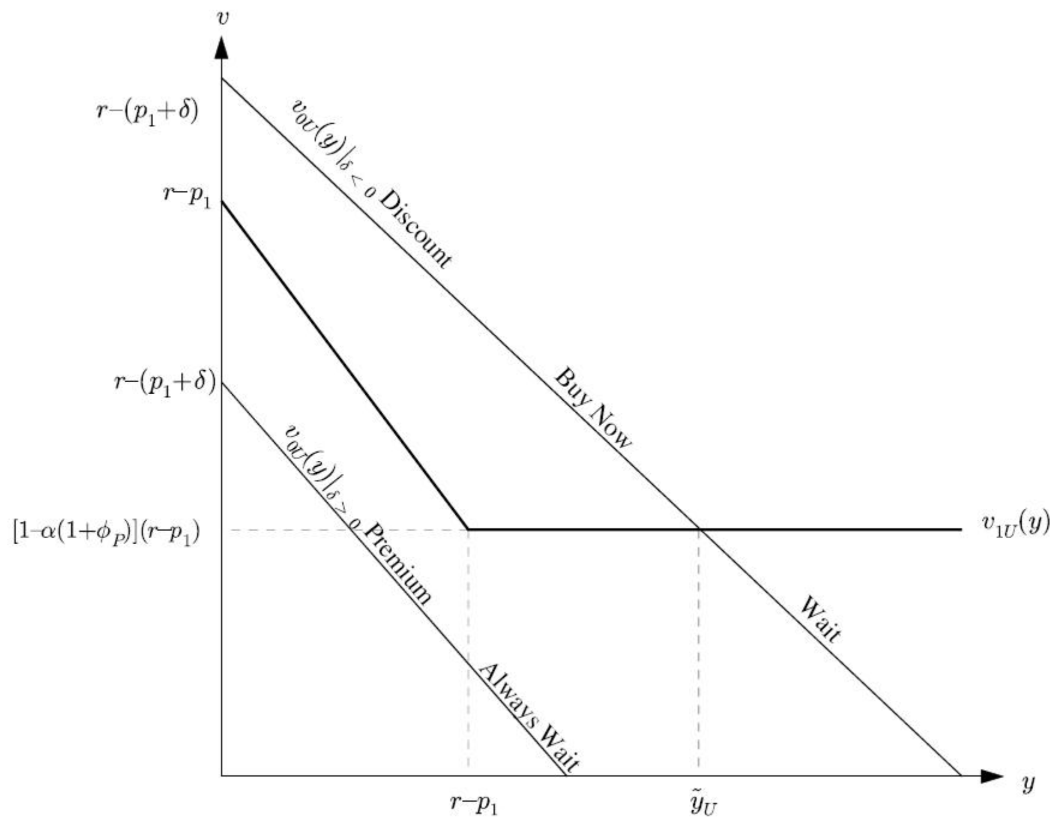

Figure 2: Expected Consumer Surplus for Type-U Consumers

premium is charged and there are two crossings, let $\tilde{y}_U^H(r)$ be the larger of the two and $\tilde{y}_U^L(r)$ be the smaller of the two. As the premium charged increases, $\tilde{y}_U^L(r)$ increases to $r - p_1$ while $\tilde{y}_U^H(r)$ falls to $r - p_1$. Alternatively there is no crossing if $v_{0U}(r, y)$ is steeper than $v_{1U}(r, y)$: $\frac{1}{2} > \alpha(1 - \phi_p)$. In this case, no type-$U$ consumers will purchase in advance and all will wait if there is a premium charged so $\tilde{y}_U^H(r) = \tilde{y}_U^L(r)$. This yields

$$\tilde{y}_U^L(r) = \begin{cases} 0 & \text{if } \frac{1}{2} > \alpha(1-\phi_p) \\ 0 & \text{if } \frac{1}{2} \leq \alpha(1-\phi_p),\ \delta \leq 0 \\ \frac{\delta}{\alpha(1-\phi_p)-\frac{1}{2}} & \text{if } \frac{1}{2} \leq \alpha(1-\phi_p),\ (\alpha(1-\phi_p)-\frac{1}{2})(r-p_1) > \delta > 0 \\ r - p_1 & \text{if } \frac{1}{2} \leq \alpha(1-\phi_p), \delta \geq (\alpha(1-\phi_p)-\frac{1}{2})(r-p_1) \end{cases} \tag{6}$$

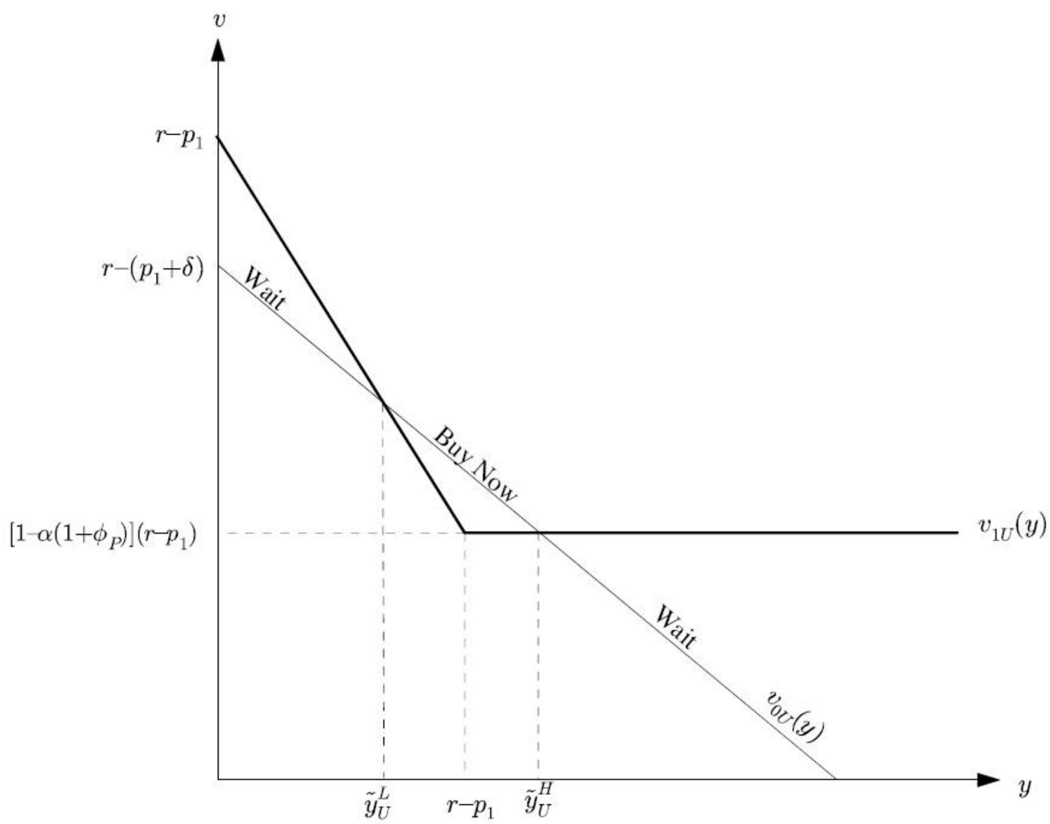

Figure 3: Expected Consumer Surplus for Type-U Consumers, Double Crossing

and,

$$\tilde{y}_U^H(r) = \begin{cases} 2\alpha(1-\phi_p)(r-p_1) - 2\delta & \text{if } \delta < (\alpha(1-\phi_p) - \frac{1}{2})(r-p_1) \\ r - p_1 & \text{if } \delta \geq (\alpha(1-\phi_p) - \frac{1}{2})(r-p_1),\ \alpha(1-\phi_p) \geq \frac{1}{2} \\ \frac{\delta}{\alpha(1-\phi_p)-\frac{1}{2}} & \text{if } 0 > \delta \geq (\alpha(1-\phi_p) - \frac{1}{2})(r-p_1),\ \alpha(1-\phi_p) < \frac{1}{2} \\ 0 & \text{if } \delta > 0,\ \alpha(1-\phi_p) \geq \frac{1}{2} \end{cases} \tag{7}$$

By defining the critical y values in this way they are continuous in δ. $\tilde{y}_U^H(r)$ is weakly decreasing and $\tilde{y}_U^L(r)$ is weakly increasing in δ. Finally, the quantity of type-U consumers purchasing in advance, $Q_{0U} = \lambda_U \int_0^{r_{max}} \int_{\tilde{y}_U^L(r)}^{\tilde{y}_U^H(r)} f(r,y)dydr$ is continuous and weakly decreasing in δ. Half of these uncertain consumers will purchase each product since they don't know which product they prefer.

Lemma 4. *Q_{0U} is a function of δ, ϕ_p, and p_1 and is continuous everywhere.*

Proof. $\tilde{y}_U^L$ is weakly increasing in δ and continuous in δ, ϕ_p, and p_1. $\tilde{y}_U^H$ is continuous in δ, ϕ_p, and p_1 and declines to $\tilde{y}_U^L$ as δ increases. Thus Q_{0U} is continuous everywhere since $f(r, y)$ is continuous everywhere. $\square$

In order to show that a solution to the producer profit maximization problem exists, it is necessary to prove that there is some equilibrium amount of tickets remaining to be rationed after the advanced sales: ϕ_p^*. The number of consumers that purchase a good in advance depends on ϕ_p, but as more consumers purchase tickets in advance there are fewer ticket remaining and less demand remaining changing ϕ_p. ϕ_p can be defined as the ratio of peak tickets remaining to the demand for the peak tickets. The supply of peak tickets in period $t = 1$ is the capacity less the tickets sold in advance: $S_1^P = K - Q_{0A} - \frac{1}{2}Q_{0U}$ [12]. All the preference-certain consumers who didn't purchase in advance will try to purchase their preferred good first. Additionally, α of the preference-uncertain consumers who didn't purchase in advance will try to purchase the peak good. As such, demand for the peak good in period $t = 1$ is $D_1^P = (\lambda_A - Q_{0A}) + \alpha(\lambda_U - Q_{0U})$.

Lemma 5. *D_1^P and S_1^P are continuous everywhere except at $\delta = \frac{1-\phi_p}{2}(r - p_1)$.*

Proof. $D_1^P = (\lambda_A - Q_{0A}) + \alpha(\lambda_U - Q_{0U})$ and $S_1^P = K - Q_{0A} - \frac{1}{2}Q_{0U}$. These are continuous functions of functions that are continuous everywhere (by lemmas 2 and 4) except at the Q_{0A} discontinuity. Thus D_1^P and S_1^P are continuous everywhere except at $\delta = \frac{1-\phi_p}{2}(r - p_1)$. $\square$

This defines the probability of getting the peak good in period $t = 1$ as $\phi_p = \frac{S_1^P}{D_1^P}$. Since Q_{0A} and Q_{0U} are functions of ϕ_p, ϕ_p is circularly defined. Since ϕ_p is the probability of getting to purchase a peak ticket in period $t = 1$, ϕ_p is bounded on the unit interval. Using this fact it can be shown that such a ϕ_p exists.

[12] I have stated this in terms of good-A being the peak good. Since type-A and type-B consumers are symmetric this expression doesn't change value if good-B is the peak good. For the rest of the paper most expressions will be in terms of good-A.

Proposition 1. *There exists a* ϕ_p^* *such that* $\phi_p^* = \frac{S_1^P(\phi_p^*)}{D_1^P(\phi_p^*)}$.

Proof. $\frac{S_1^P(\phi_p)}{D_1^P(\phi_p)}$ has only one discontinuity. It will be shown to be outside of the possible range of ϕ_p. Then on the continuous region, ϕ_p^* will be shown to exist.

S_1^P and D_1^P are continuous functions of ϕ_p on $\delta \in (-r, r) \sim \{\frac{1-\phi_p}{2}(r - p_1)\}$ by lemma 5. Solving for ϕ_p, the discontinuity occurs at $\phi_p = 1 - \frac{2\delta}{r-p_1}$. But $\phi_p \leq \frac{K}{\lambda_A+\alpha\lambda_U}$, so as long as the discontinuity is above this bound a fixed point theorem can still be applied. Rearranging the restriction that $1 - \frac{2\delta}{r-p_1} > \frac{K}{\lambda_A+\alpha\lambda_U}$ yields that $\delta < \frac{r-p_1}{2}(1 - \frac{K}{\lambda_A+\alpha\lambda_U})$. This is exactly the same condition for any type-A or type-B consumers choosing to purchase in advance[13]. If this restriction on δ doesn't hold everyone waits to purchase and $\phi_p^* = \frac{K}{\lambda_A+\alpha\lambda_U} < 1$ (by Assumption 3). This collapses to all sales being made in the second period. If the restriction does hold $\frac{S_1^P}{D_1^P}$ is continuous in ϕ_p within the compact bounds of $\phi_p \in [0, \frac{K}{\lambda_A+\alpha\lambda_U}]$. By Brouwer's Fixed Point Theorem, a ϕ_p^* exists such that $\phi_p^* = \frac{S_1^P}{D_1^P}$. □

The existence of an equilibrium rationing of peak tickets in period $t = 1$ guarantees that the consumer behavior will be stable for any choice of p_1 and δ. ϕ_p can thus be expressed as a indirect function of the parameter: $\phi_p(\delta, p_1)$. Type-U consumers can be separated into high-cost and low-cost types if there is an advanced-purchase discount with the low-cost consumers purchasing in advance. The type-A and type-B consumers can be separated into high-cost and low-cost types if a small premium is charged with the high-cost consumers purchasing in advance. Finally, changing the number of each consumer type or the distribution of the mismatch costs will affect the number of consumers who purchase in advance.

5 Optimal Pricing Policy

The monopolist producer of good-A and good-B will maximize profits by choosing a price and an advanced-purchase discount or premium. Here the focus will be on the determination

[13] This corresponds to the discount or small premium case.

of the size of the advanced-purchase discount or premium and show that either can occur depending on the relative size of each consumer type, the number of consumers who will prefer the peak good, and the capacity constraint. First, the monopolist's profit maximization problem will be defined. Next, properties of the demand function will be proven. Finally, using the demand properties, it will be shown that under different consumer characteristics a premium or a discount can prevail.

If all consumers are preference-uncertain, this model collapses to the Gale-Holmes model and an advanced-purchase discount will be optimal. Alternatively, if there are mostly preference-certain consumers in the market an advanced-purchase premium will prevail. The premium will be possible because the preference-uncertain consumers will have a lower expected valuation than the preference-certain consumers. This occurs because they have the risk of getting the wrong good when purchasing in advance. Preference-certain consumers always purchase the correct good in advance. The difference in valuations for advanced-purchases creates a risk of rationing in the second period. As in the simple example (Section 2) this gives the higher-valuation, preference-certain consumers an incentive to pay a premium and purchase in advance.

The monopolist's profit maximization problem is as follows:

$$\max_{(\delta,p_1)} \sum_{j=A,B,U} Q_{0j} \cdot (p_1+\delta) + \sum_{j=A,B,U} Q_{0j} \cdot p_1 \tag{8}$$

To simplify the future expressions define $Q_0 = Q_{0A} + Q_{0B} + Q_{0U}$ and $Q_1 = Q_{1A} + Q_{1B} + Q_{1U}$. Since all quantities are continuous within the possible range of δ and everywhere for p_1, a solution exists and is characterized by the following first order conditions.

$$(\frac{\partial Q_0}{\partial p_1})(p_1+\delta) + (\frac{\partial Q_1}{\partial p_1})p_1 + Q_0 + Q_1 = 0 \tag{9}$$

$$(\frac{\partial Q_0}{\partial \delta})(p_1+\delta)+(\frac{\partial Q_1}{\partial \delta})p_1+Q_0=0 \tag{10}$$

Solving the first order condition for δ yields:

$$\delta=\frac{-(\frac{\partial Q_0}{\partial \delta}+\frac{\partial Q_1}{\partial \delta})p_1-Q_0}{\frac{\partial Q_0}{\partial \delta}} \tag{11}$$

As the premium charged increases, the quantity of advanced purchase must fall so $\frac{\partial Q_0}{\partial \delta} \leq 0$[14].

Lemma 6. $\frac{\partial Q_0}{\partial \delta} \leq 0$

Proof. $\tilde{y}_A$ is the intersection of $(r-p_1)-\frac{1-\phi_p}{2}y$ and the constant $r-p_1-\delta$ when the intersection is at $y \leq r-p_1$. $\tilde{y}_A=r$ for $\delta \geq \frac{1-\phi_p}{2}(r-p_1)$. Thus

$$\tilde{y}_A(\delta)=\tilde{y}_B(\delta)=\begin{cases} \frac{2\delta}{1-\phi_p} & \text{if } \delta<\frac{1-\phi_p}{2}(r-p_1) \\ r & \text{if } \delta \geq \frac{1-\phi_p}{2}(r-p_1) \end{cases} \tag{12}$$

$\tilde{y}_A(\delta)$ is increasing in δ. $Q_{0A}=\lambda_A(1-F(\tilde{y}_A))$ is decreasing in δ because $F(\cdot)$ is an increasing function of $\tilde{y}_A(\delta)$.

As shown before, $\tilde{y}_U^L$ and $\tilde{y}_U^H$ converge to $r-p_1$ as $\delta \rightarrow (\frac{1}{2}-\alpha(1-\phi_p))(r-p_1)$. Thus $Q_{0U}=\lambda_U(F(\tilde{y}_U^H)-F(\tilde{y}_U^L)$ is decreasing in δ because $F(\cdot)$ is increasing, $\tilde{y}_U^H$ is decreasing, and $\tilde{y}_U^L$ is increasing.

Since $Q_0=Q_{0A}+Q_{0B}+Q_{0U}$ it must also be decreasing in δ. □

This means that for a premium to be charged for advanced purchases ($\delta>0$) it must be the case that $(\frac{\partial Q_0}{\partial \delta}+\frac{\partial Q_1}{\partial \delta})p_1+Q_0>0$.

First, in examining $\frac{\partial Q_0}{\partial \delta}+\frac{\partial Q_1}{\partial \delta}$ it is useful to decompose the expression into $\frac{\partial Q_{0A}}{\partial \delta}+\frac{\partial Q_{1A}}{\partial \delta}+\frac{\partial Q_{0B}}{\partial \delta}+\frac{\partial Q_{1B}}{\partial \delta}+\frac{\partial Q_{0U}}{\partial \delta}+\frac{\partial Q_{1U}}{\partial \delta}$. It is shown in Lemma 7 that $\frac{\partial Q_{0A}}{\partial \delta}+\frac{\partial Q_{1A}}{\partial \delta}=0$. The same is

[14] Intuitively as the premium charged increases fewer consumers will purchase in advance.

true for type-B consumers. The low-cost consumers are more likely to wait but they are the types that are willing to purchase their less preferred good.

Lemma 7. *As the premium charged increases, all preference-certain consumers that no longer purchase at* $t = 0$ *purchase at* $t = 1$*. Thus* $\frac{\partial Q_{0A}}{\partial \delta} + \frac{\partial Q_{1A}}{\partial \delta} = 0$.

Proof. As long as the premium is not too large[15], all type-A consumers who wait until $t = 1$ to purchase have $y < r - p_1$ since it is the low-y preference-certain consumers who wait (see Figure 1). For these relevant premiums ($\delta < \frac{1-\phi_p}{2}(r - p_1)$), the consumers who no longer purchase in advance are willing to purchase the less preferred good. As such, all preference-certain consumers purchase and any decrease in Q_{0A} is matched by an increase in Q_{1A}. Thus $\frac{\partial Q_{0A}}{\partial \delta} + \frac{\partial Q_{1A}}{\partial \delta} = 0$. □

For type-U consumers, however, a decrease in Q_{0U} corresponds to a less than one-for-one increase in Q_{1U} because it is high-y consumers waiting and they won't purchase their less preferred good if their preferred good is sold out. Therefore $\frac{\partial Q_0 U}{\partial \delta} + \frac{\partial Q_1 U}{\partial \delta} \leq 0$.

Lemma 8. *As the premium charged increases, only some preference-uncertain consumers that no longer purchase in advance choose to purchase in the second period. Thus* $\frac{\partial Q_{0U}}{\partial \delta} + \frac{\partial Q_{1U}}{\partial \delta} \leq 0$.

Proof. For a premium, either all uncertain consumers waited in which case changing δ has no effect: $\frac{\partial Q_0 U}{\partial \delta} + \frac{\partial Q_1 U}{\partial \delta} = 0$. Alternatively, it may be the case that some low-y consumers and some high-y consumers wait (see Figure 3). Here increasing δ leads to more of both consumer types waiting. The low-y types have $y < r - p_1$ and thus will purchase a less preferred good if need be. The high-y types have $y > r - p_1$ and thus will not purchase a less preferred good if need be. Since not all of the consumers who used to purchase in advance purchase now, $-\frac{\partial Q_0 U}{\partial \delta} > \frac{\partial Q_1 U}{\partial \delta}$. Thus, $\frac{\partial Q_0 U}{\partial \delta} + \frac{\partial Q_1 U}{\partial \delta} \leq 0$. □

[15] This is the same restriction as for the existence of ϕ^*.

Finally, using these two properties of the demand functions it can be shown that it is possible to get a premium or a discount in equilibrium. The discount case is the limiting case of no preference-certain consumers and was proved in Gale and Holmes (1993). Due to the continuity of the problem, this result must hold in the neighborhood of having no preference-certain consumers. The premium case is possible if there are sufficient numbers of preference-certain consumers and they have a high enough cost[16] of purchasing their less preferred good.

Proposition 2. *An advanced purchase premium is optimal if there are enough preference-certain consumers with high enough costs to consuming the less preferred good.*

Proof. A premium corresponds to $\delta = \frac{-(\frac{\partial Q_0}{\partial \delta} + \frac{\partial Q_1}{\partial \delta})p_1 - Q_0}{\frac{\partial Q_0}{\partial \delta}} > 0$ (Equation 11).

Since $\frac{\partial Q_{0A}}{\partial \delta} + \frac{\partial Q_{1A}}{\partial \delta} = 0$ (lemma 7) and $\frac{\partial Q_0}{\partial \delta} < 0$, for δ to be positive it must be the case that $Q_0 > -(\frac{\partial Q_{0U}}{\partial \delta} + \frac{\partial Q_{1U}}{\partial \delta})p_1$. By choosing λ_A and $F(\cdot)$ appropriately, Q_0 can be made large (approaching 1), and $-(\frac{\partial Q_{0U}}{\partial \delta} + \frac{\partial Q_{1U}}{\partial \delta})$ can be made small. Since all type-A and type-B consumers with $y > r - p_1 > \tilde{y}_A$ purchase in advance paying the premium, when $f(\cdot)$ is weighted heavily on $r - p_1 < y < r$ and λ_A is large, $Q_{0A} = Q_{0B} = \lambda_A(1 - F(\tilde{y}_A))$ approaches $\frac{1}{2}$. So $Q_0 = Q_{0A} + Q_{0B} + Q_{0U} \to 1$ as $Q_{0A} = Q_{0B} \to \frac{1}{2}$. On the other side of the equation, $-(\frac{\partial Q_{0U}}{\partial \delta} + \frac{\partial Q_{1U}}{\partial \delta})$ depends on both the number of consumers with high-y's and the number of preference-uncertain consumers, λ_U. As λ_A becomes large, $\lambda_U = 1 - 2\lambda_A$[17] becomes arbitrarily small and $-(\frac{\partial Q_{0U}}{\partial \delta} + \frac{\partial Q_{1U}}{\partial \delta}) \to 0$. Thus $Q_0 > -(\frac{\partial Q_{0U}}{\partial \delta} + \frac{\partial Q_{1U}}{\partial \delta})p_1$ holds and a premium will prevail. □

Having many preference-certain, high-cost consumers makes more consumers willing to purchase in advance while paying a premium. Additionally, having a small ratio of preference-uncertain consumers will make $\frac{\partial Q_{0U}}{\partial \delta} + \frac{\partial Q_{1U}}{\partial \delta}$ small. Thus as seen in Proposition 2, a premium will persist in markets that are characterized by mostly preference-certain consumers. From

[16] This cost is on average, relating to the distribution of costs.
[17] Because $\lambda_A = \lambda_B$, and $\lambda_A + \lambda_B + \lambda_U = 1$.

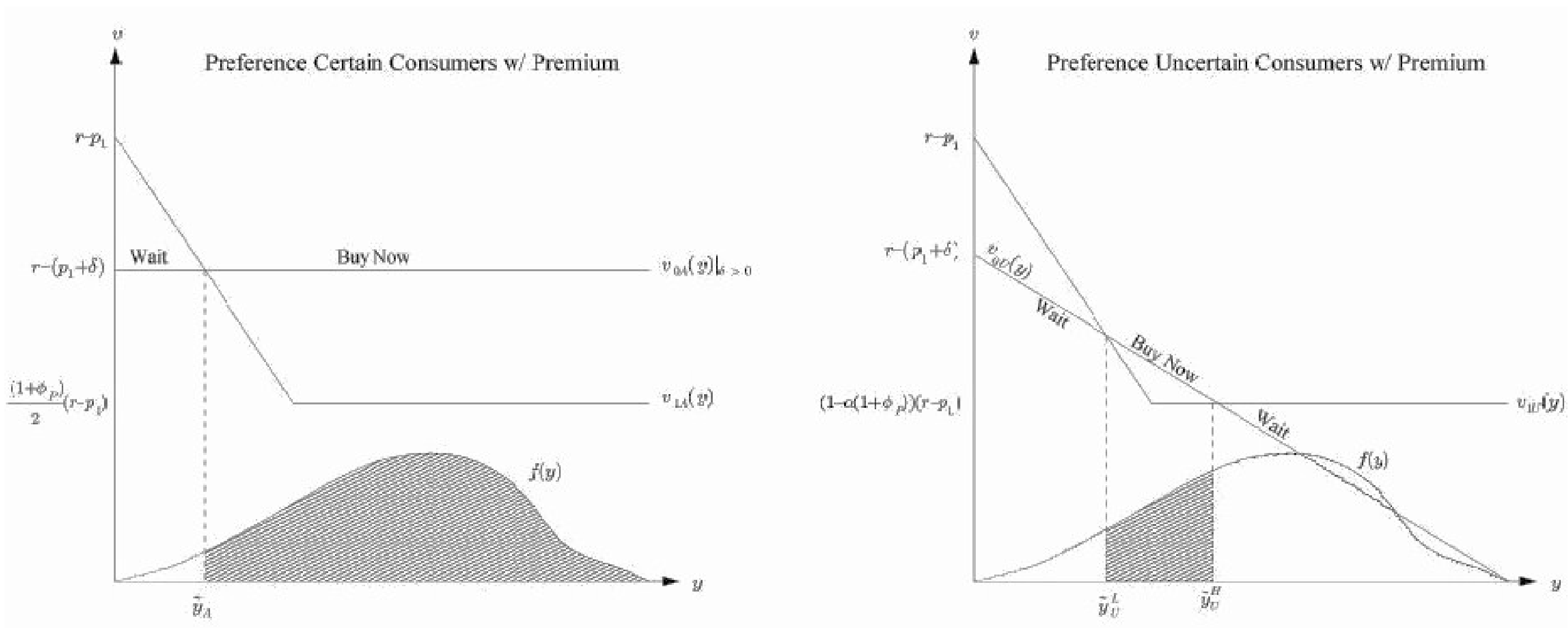

Figure 4: Effect of High-Cost Distribution

the proof it can be seen that two main factors will influence when a premium or a discount prevails. First, the ratio of preference-certain consumers to preference-uncertain consumers is a determining factor. Since it is mostly preference-certain consumers that are willing to pay a premium for advanced purchases, there must be a sufficient number of these consumers for a premium to prevail. Second, the distribution of the costs of purchasing the less preferred good must be skewed to the high cost end[18]. Changing the ratio of consumer types, distribution of mismatching costs, or a combination of both can switch the result between having an advanced-purchase discount and having an advanced-purchase premium prevail.

6 Analyzing Model Results

The model shows that if there are many preference-uncertain consumers in a market with a capacity constrained producer, then a discount for advanced-purchases will be offered. If there are mostly preference-certain consumers in the market, a premium for advanced-purchases is charged. To make sense of these results, let's apply them to a number of examples. First, in airline travel, customers looking to book a flight are often uncertain of the exact flight time that they will prefer until a few weeks in advance of the departure date. Customers with a low time cost of being on their less preferred flight, like recreational travelers, can be induced into purchasing in advance by offering a discount. Business travelers who have a higher time cost will wait until their exact preferences become known and pay a higher price for doing so. Both sets of consumers are uncertain of their preference but self-identify by choosing to either take the discount or not.

The premium case can be seen in the market for movie tickets on opening weekend. Consumers are given the choice of purchasing tickets in advance through an online site like

[18]While the same distribution was assumed for all consumer types, if different consumers had different cost distributions then only the preference-certain consumers would need to have their distribution skewed high as they are the ones willing to pay the premium.

Fandango or waiting until they arrive at the theater to purchase the tickets. Fandango charges a premium of $1.00 per ticket purchased in advance. Most consumers that buy in advance do so either the same day as the show or a few days prior choosing between different show times and locations. These consumers typically know which showtime and location they will prefer. For popular shows however there is a risk of selling out. As such consumers who have strong preferences (i.e. high cost to getting the less preferred good) for a particular movie or showtime are willing to pay the surcharge to guarantee themselves a ticket.

The difference in advanced sales pricing behavior between the airline and movie industries can be explained by the model in two different but complementary ways. First, as espoused in the preceding paragraphs, the preference-certainty of the consumers in these markets is different. Alternatively, the distribution of costs of getting a less preferred good may be different between different markets[19]. Getting a less preferred flight may not impose a large cost for most travelers if the day of the flight is already scheduled for travel and other plans can be rearranged as the purchase is made weeks in advance. This results in few preference-certain consumers being willing to pay extra in advance to guarantee a seat on their preferred flight and advanced-purchase discounts being offered. In the market for movie tickets, because the purchase is typically made close to the day of the show, ending up with a less preferred time could cause one to have to change other plans. As such, some consumers are willing to pay extra in advance to avoid the potential higher relative cost of getting the wrong showtime resulting in a premium being charged.

It is worth noting that typically the size of the premium that can be charged for an advanced-purchase is relatively small. Per the model, a large premium will cause all of the consumers to wait to purchase and thus eliminate the ability of the seller to price discriminate. A large discount, on the other hand, is possible based on the distribution of the costs of

[19] While I believe that the number of preference-certain consumers is more of a determining factor, at least one colleague who helped review the paper believes that the distribution of mismatching costs is the more important consideration. In the end it is an empirical question and open to debate.

consuming the wrong good. If most consumers have a high cost to consuming their less preferred good, a large discount is needed to induce advanced-purchases and thus price discriminate. In the premium case it is the high-cost, preference-certain consumers who purchase in advance, but there is a bound on the premium they are willing to pay. Therefore large premiums for advanced purchase do not typically occur.

Finally, this model can be used to help explain the typical movement of airline flight prices. As discussed above, prices a month or so in advance of a flight are typically lower than those closer to the departure date. However, as one gets within a few days of the flight's departure, if there are tickets remaining, the price on the tickets drops considerably. Per the advice of a travel website

> ...while it is potentially possible to get great deals at the very last minute, the safest option is to buy your ticket as early as possible[20].

For the very early customers, the market is characterized by preference-uncertain consumers and a discount relative to future prices is received. Once it is within three weeks or so of the departure date, most consumers now know their flight preferences and thus a premium over the expected day of flight price is paid. If there are seats still remaining within a few days of the flight the price will fall. One concern for this application of the model is that the airline is not truly committing to a price but can change prices dynamically through time. However, the consumer behavior is set by the expectation of future prices. This means that if consumer expectations are set by the historical observation of price changes then consumers will behave as described in the model and the airline faces the same problem. As a result, these price changes are stable with respect to both consumer behavior and profit maximization.

[20]Quote taken for the Travel Library's advice on airline consolidators: *http://www.travel-library.com/air-travel/consolidators.html*

7 Conclusions

Based on the model presented here, consumer's knowledge of their future preferences and the costs associated with consuming the less preferred good are determining factors in whether a premium or discount will be associated with advanced sales of a capacity constrained good. Preference-certain consumers risk not getting their preferred good if they wait to consume and will thus be willing to pay a premium to purchase in advance. Preference-uncertain consumers risk purchasing the wrong good if they buy in advance. They will only purchase in advance when offered a discount sufficient to compensate them for this risk. The size of the costs associated with consuming the less preferred good naturally affects the size of the discount or premium. The model explains why in some cases, like airlines, a discount pricing scheme is used, while in others, like concerts, a premium can be charged.

There is still room for additional analysis to be done. First, the market should be opened up to competition. While a concert or movie theater may be a local monopoly, most airline routes are open to competition. Because the pricing behavior is primarily dependent on the consumer types, as in Dana (1992), price discrimination may be robust to competition. Second, there is the question as to what effect secondary markets will have on the allocation of tickets and ability to charge a premium or discount. Allowing resale of tickets may eliminate the ability to offer a discount because a third party could purchase in advance and resell them, undercutting the higher second period price. A premium may still persist since resale is not profitable. Future work will look at the effects of secondary markets on ticket sales. Finally, in some markets firms do not commit to a price path. Relaxing the commitment assumption in the future will yield additional insights into the intertemporal movement of prices.

References

James D. Jr. Dana. Advanced-purchase discounts and price discrimination in competitive markets. *The Journal of Political Economy*, 10(4):413–37, 1992.

Patrick DeGrabba and Rafi Mohammed. Intertemporal mixed bundling and buying frenzies. *The RAND Journal of Economics*, 30(4):694–718, 1999.

Ian Gale and Thomas Holmes. The efficiency of advanced-purchase discounts in the presence of aggregate demand uncertainty. *International Journal of Industrial Organization*, 10(4): 413–37, 1992.

Ian Gale and Thomas Holmes. Advanced-purchase discounts and monopoly allocation of capacity. *The American Economic Review*, 83(1):135–46, 1993.

Richard J. Gilbert and Paul Klemperer. An equilibrium theory of rationing. *The RAND Journal of Economics*, 31(1):1–21, Spring 2000.

Sherwin Rosen and Andrew Rosenfield. Ticket pricing. *Journal of Law and Economics*, 40 (2):351–376, Oct. 1997.

Daniel F. Spulber. Monopoly pricing of capacity usage under asymmetric information. *The Journal of Industrial Economics*, 41(3):241–257, Sep. 1993.