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Abstract

When a firm sells tickets in advance under capacity constraints, as with airlines, concerts, and sports tickets, one observes that in some cases advanced sales are made at a discount while other times a premium is charged. Previous research into this intertemporal price discrimination has focused on either premium or discount pricing but never both. Given that we observer both types of intertemporal price discrimination in markets characterized by advanced sales and capacity constraints, it is important to understand the conditions that determine the nature of the optimal pricing scheme. This paper is the first to shows that the nature of the profit maximizing intertemporal pricing scheme depends on the interaction of consumer preference intensity, preference certainty, and firm capacity. We then examine optimal capacity choice as a function of consumer preferences showing that the choice of capacity will be a negatively related to its cost for a given level of preference intensity and equal to the ex-post demand for one of the goods.

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1 Introduction

When purchasing a ticket for a flight, typically a lower price is paid if the purchase is made well in advance of the departure date. When attending a concert or sporting event those who purchase in advance of the event pay a higher price than those who purchase the day of the show. Gale and Holmes (1992) and Gale and Holmes (1993) examined the causes for advance purchase discounts in the airlines markets. DeGrabba and Mohammed (1999) looked at the prevalence of premium pricing for bundled concert tickets. As yet, no unified setting capable of addressing the question of what conditions lead to premium pricing verses discount pricing in capacity constrained markets exists.

This paper develops a model in which a capacity constrained monopolist sells two horizontally differentiated goods in two time periods. Consumers will vary in both their preference certainty and their preference intensity. These consumer characteristics in conjunction with the firms capacity will give rise to premium, discount, and uniform pricing. Consumers who purchase early do so to ensure they receive a good which may sell out. Consumers who are uncertain as to which good they will prefer later may wait to purchase. When a firm has a low capacity relative to the market size, high prices will prevail in both purchase periods. For medium capacities, a premium can be charged to those who purchase in advance because they are willing to pay a higher price to ensure they receive their preferred good. The firm has an incentive to price below the market clearing level if it knows it can charge a high price in advance and gain on these early sales. When capacity is large, consumers who are uncertain of their future preferences can be induced to purchase early via a discount. The firm benefits by smoothing demand for the goods because those that purchase in advance do so more evenly than those who wait. If most consumers already know their preferences, low prices in both time periods will prevail.

Ex-ante, a firm may be able to choose their capacity. Knowing which pricing scheme will be profit maximizing for each level of capacity reduces the capacity choice problem to a function of consumer preference certainty. This capacity choice will be discontinuous as the choice of pricing plans is discrete. As consumers become more preference uncertain, uniform pricing and premium pricing becomes less profitable relative to discount pricing. The choice of capacity will always be that which is profit maximizing for the resulting pricing scheme and thus has discontinuities when additional preference uncertainty causes a change in the pricing plan. We will show when capacity cost is low and consumers are generally uncertain as to their future preferences, capacity sufficient to satisfy all demand under discount pricing is optimal. When some consumers know their preferences in advance and capacity is relatively costly, the profit maximizing capacity is just sufficient to meet demand for the less popular good. Finally, when capacity is very expensive, relatively low capacity should be purchased so that only the highest valuation consumers purchase.

Section 2 develops this model in detail by defining the firm's profit maximizing problem and the consumers' surplus maximizing problem. Section 3 defines the possible pricing schedules and then proves the conditions under which a pricing schedule may be profit maximizing. Because the regions where each pricing plan is optimal are non-trivial, Section 4 examines the effect of changing parameter values on the optimal pricing plan. The optimal pricing schedule as a function of consumer characteristics and capacity is derived and the intuition behind these pricing plans is developed. Section 5 makes the firm capacity an ex-ante choice variable. It is then shown how capacity choice depends on consumer characteristics and the cost of capacity. The final section applies the model results to ticket pricing in the airline and sporting event markets and then summarizes the results.

2 The Model

Our model of intertemporal pricing uses a monopolist selling two horizontally differentiated goods in two time periods. Consumers maximize their expected surplus by choosing when to purchase. Consumers differ in both their preference certainty and preference intensity. First, the monopolist's profit maximization problem will be defined. Next, consumers' characteristics will be described. Finally, the consumers' surplus function will be made explicit. This will lead into Section 3 which uses the consumer behavior to determine what prices a firm may choose to maximize profits.

A monopolist produces and sells two horizontally differentiated goods in two time periods. The goods, good-A and good-B, can be produced at a fixed marginal cost of zero[1]. There is a common capacity of K for both goods[2]. The goods are sold in two time periods: in advance at $t = 0$ and at the day-of-consumption at $t = 1$. The monopolist maximizes profits by choosing prices for each good and at each time period, committed to advance. Prices for both goods will be the same due to the ex-ante symmetry of consumers. There is no discounting between the periods. This follows the treatment of advance purchase discounts in the Gale and Holmes (1992) model. Denote the quantity of good-g demanded at time t as Q^D_{gt} and its price as p_{gt}. Additionally, let $Q^S_{g1} = \max\{K - Q^D_{g0}, 0\}$ be the supply of good-g remaining at $t = 1$. The monopolist's profit maximization problem is then,

$$\begin{aligned} \max_{p_{A0},p_{B0},p_{A1},p_{B1}} \quad & p_{A0} \min\{Q^D_{A0}, K\} + p_{B0} \min\{Q^D_{B0}, K\} + \\ & p_{A1} \min\{Q^D_{A1}, Q^S_{A1}\} + p_{B1} \min\{Q^D_{B1}, Q^S_{B1}\}. \end{aligned} \tag{1}$$

There is a unit mass of risk-neutral consumers that vary in their preference certainty

[1] Alternatively, both prices and consumer valuations can be thought of as net with respect to a constant non-zero marginal cost.

[2] Separate capacities for each good are possible but imposed symmetry of the goods will lead to identical capacities.

and preference intensity. Preference intensity is the valuation consumers place on each good. Let γ be the proportion of high valuation consumers. They value their preferred good at 100 and their non-preferred good at v_{HN}. There are $1-\gamma$ low valuation consumers who value their preferred good at $v_{PL} < 100$ and there non-preferred good at v_{LN}. Define the expected valuation of a randomly chosen good as μ_j for $j \in \{L, H\}$. As proven in Gale and Holmes (1992), for discounts to be profit maximizing, $\mu_H < \mu_L$. This will be taken as given. Each consumer has unit demand for the goods and desires at most one of either good-A or good-B. All consumption occurs at $t = 1$.

Independent of preference intensity, consumers will either be preference certain, knowing their future preferences over good-A and good-B in advance, or be preference uncertain, not knowing their future preference in advance. Let their be a proportion λ of preference uncertain consumers. Preference uncertain consumers, denoted type-U, are equally likely to prefer each good at $t = 0$ and learn their preference at $t = 1$. Ex-post, a proportion $\alpha > \frac{1}{2}$ of the preference uncertain consumers will prefer the peak good. The peak good is the good with the higher ex-post demand. $1-\alpha$ consumers will prefer the non-peak good. Ex-ante each good is equally likely to be peak. The remaining $1-\lambda$ consumers are preference certain. For tractability of the model, half of these consumers will prefer good-A and half will prefer good-B.[3] Denote these consumers as type-A and type-B respectively. Preference certain consumers know their preferred good at $t = 0$. At times it will be useful to refer to a consumer group by both their preference certainty and their preference intensity. A type-C_v consumer will be of preference certainty $C \in \{A, B, U\}$ and of preference intensity $v \in \{L, H\}$. Because of the presumption that these consumer characteristics are independent within the unit mass of consumers, the number of type-C_v consumers is the product of the two proportions. For example, there are $\frac{1-\lambda}{2}\gamma$ type-A_H consumers.

[3] Relaxing this symmetry assumption will result the possibility that the firm may price the goods differently. This complicates the model without sufficiently changing the qualitative results.

The consumers' problem is to maximize expected surplus by choosing which good to purchase and when. If the expected surplus is identical between purchasing at $t = 0$, $t = 1$, or not purchasing it will be assumed that the consumer will purchase and will do so at the earlier of the two periods to ensure that the good is obtained. Defining $\chi_{gt} = 1$ if the consumer tries to purchase good-g at time t and $\chi_{gt} = 0$ otherwise and the valuation of good-g by v_g, the consumer problem is defined as,

$$\begin{aligned} \max_{\chi_{gt}} \quad & \max_{(g,t)\in\{A,B\}\times\{0,1\}} \chi_{gt} E[v_g - p_t] \\ & s.t. \sum_{(g,t)\in\{A,B\}\times\{0,1\}} \chi_{gt} \leq 1 \end{aligned} \tag{2}$$

Given prices (p_0, p_1), demand for each good and at each time period can be derived. The firm will then maximize profits for a given capacity K and proportion of preference uncertain consumers λ knowing these demand functions.

3 Monopolist Pricing

The monopolist commits to a pricing schedule for each good. This can be uniform pricing in which the prices in both periods are the same or it could be a form of intertemporal price discrimination. Intertemporal price discrimination is either a discount where prices are lower in advance or a premium where prices are higher in advance. In this section we will prove that there is only a limited set of pricing plans that can be profit maximizing for the monopolist. This will be done by systematically eliminating pricing plans which are always suboptimal leaving a finite list of potentially profit maximizing pricing schedules.

Due to the ex-ante symmetry of consumers preferences over good-A and good-B, the prices for each good in a given time period are the same. A pricing plan is then a two-tuple (p_0, p_1). There are three possible profit maximizing pricing schemes: uniform pricing where $p_0 = p_1$, premium pricing where $p_0 > p_1$, and discount pricing where $p_0 < p_1$. Via the lemmas in this section, we will show that there are only two potential profit maximizing uniform prices, two potentially profit maximizing discount pricing schemes, and a single class of premium prices.

3.1 Uniform Pricing

There are only two potentially profit maximizing uniform pricing strategies. When capacity is low it may be optimal to charge a high price in both periods serving only high valuation consumers. When capacity is high it may be optimal to charge a low price in both periods and sell to all consumers. In both cases preference certain consumers will purchase in advance at $t = 0$ and preference uncertain consumers will wait and purchase at $t = 1$.

When capacity is sufficiently low to serve only high valuation consumers, a uniform high price of $p_0 = p_1 = 100$ will be profit maximizing.

Lemma 1. *For any given K¿0, any pricing schedule* (p_0, p_1) *such that* $p_0 = p_1 \neq 100$ *and* $p_0 = p_1 > v_{PL}$ *is not profit maximizing.*

Proof. If $K = 0$, profits from any pricing plan are zero, so restrict to $K > 0$. If $100 > p_0 = p_1 > v_{PL}$ no low valuation consumers will purchase. All high valuation consumers will demand the good. Increasing $p_0 = p_1$ does not change demand in this range, thus increasing profits. So any price $100 > p_0 = p_1 > v_{PL}$ is not profit maximizing. If $p_0 = p_1 > 100$ then no one purchases. In this case profits are zero and any price $p_0 = p_1 \leq 100$ yields higher profits because high valuation consumers will purchase. Therefore, any price $p_0 = p_1 > 100$ is not profit maximizing. Thus, any pricing schedule (p_0, p_1) such that $p_0 = p_1 \neq 100$ and $p_0 = p_1 > v_{PL}$ is not profit maximizing. □

When capacity is sufficiently large that the monopolist can lower his price and gain enough additional customers to offset the loss in profits from the lower prices, a low uniform price may prevail. In this case, all preference certain consumers will purchase in advance at $t = 0$ and all preference uncertain consumer will wait and purchase day-of at $t = 1$.

Lemma 2. *For any given $K > 0$, any pricing schedule (p_0, p_1) such that $p_0 = p_1 < v_{PL}$ and is not profit maximizing.*

Proof. If $K = 0$, profits from any pricing plan are zero, so restrict to $K > 0$. Given $p_0 = p_1 < v_{PL}$ both high and low valuation consumers will demand. For $p_0 = p_1 < v_{PL}$ price can be increased without reducing demand and thus increasing profits. Any price $p_0 = p_1 < v_{PL}$ therefore cannot be profit maximizing. □

Any uniform price that is not $p_0 = p_1 = 100$ or $p_0 = p_1 = v_{PL}$ cannot be profit maximizing. For a given proportion of type-U consumers, λ, and a restricting to uniform pricing $p_0 = p_1$, high pricing has higher profits than low pricing at low capacities. When K is sufficiently low, the firm is unable to increase sales enough by lowering prices to compensate for the profits lost by the price effect. At high capacities, the firm will increase sales sufficiently by reducing prices to v_{LP} increasing profits. Low prices may be profit maximizing at high capacities.

Proposition 1. *For any $K > 0$, any pricing plan $(p_0, p_1) \notin \{(100, 100), (v_{PL}, v_{PL})\}$ such that $p_0 = p_1$ is not profit maximizing.*

Proof. Follows directly from Lemma 1 and Lemma 2. □

3.2 Intertemporal Price Discrimination

A second option is to charge different prices in each period. A premium occurs when consumers are charged a higher price for purchases at $t = 0$, $p_0 > p_1$. A discount occurs when consumers pay a lower prices at time $t = 0$, $p_0 < p_1$. These are each separating equilibria that will result in some consumer types purchasing in advance and others purchasing at the time of consumption.

In the case of a discount $p_0 < p_1$. The lower price in advance will induce all the preference certain consumers with valuations above p_0 to purchase in advance. In addition, this will induce all the uncertain consumers with an expected valuation below the price to purchase in advance. These uncertain consumers who purchase in advance will be evenly split between the two goods. If they had waited, α of them would have purchased the peak good. Because $\alpha > \frac{1}{2}$, inducing these consumers to purchase in advance causes them to be more evenly split between the two goods. This frees up additional capacity from the peak good by shifting some of it to the non-peak good. The freed up capacity can then be sold resulting in an increase in total quantity. This potentially increases profits.

Lemma 3. *For any capacity $K > 0$, any pricing strategy $(p_0, p_1) \notin \{(\mu_L, 100), (v_{PL}, 100)\}$ such that $p_0 < p_1$ is not profit maximizing.*

Proof. For $p_0 < p_1$, the discrete nature of the consumer space means that it is optimal to raise a price up to the point at which some set of consumers change their behavior. Thus any pricing plan such that $p_0 < p_1$ and

$$(p_0, p_1) \notin \{(\mu_H, \mu_L), (\mu_H, v_{PL}), (\mu_H, 100), (\mu_L, v_{PL}), (\mu_L, 100), (v_{PL}, 100)\}$$

is not profit maximizing.

For pricing schedules (μ_H, μ_L) and (μ_H, v_{PL}), everyone except type-U_H consumers purchase in advance. In either case, the monopolist can raise p_0 to $\mu_L > \mu_H$ and not lose consumers thus increasing profits. Thus (μ_H, μ_L) and (μ_H, v_{PL}) are not profit maximizing.

For pricing schedule $(\mu_H, 100)$, type-U_H consumers are indifferent between when they purchase as they receive zero consumer surplus either way. Increasing p_0 to μ_L will result in type-U_H consumers paying a higher price by waiting and not lose any other consumers. As this increases profits, pricing plan $(\mu_H, 100)$ is not profit maximizing.

For pricing plan (μ_L, v_{PL}) all Type-A and Type-B consumers purchase in advance. Type-U_H consumers wait to purchase and Type-U_L consumers are indifferent. Presuming that Type-U_L consumers purchase at the lower price at $t = 0$ when they are indifferent, the monopolist can increase p_1 to 100 without changing any consumer behavior. Type-U_H consumers pay a higher price increasing profits. Pricing plan (μ_L, v_{PL}) is thus not profit maximizing.

Removing these non-optimal pricing plans, any pricing plan $(p_0, p_1) \notin \{(\mu_L, 100), (v_{PL}, 100)\}$ with $p_0 < p_1$ is not profit maximizing. $\square$

Intertemporal price discrimination can also be achieved by charging a higher price in advance and a below market clearing price at time period $t = 1$. A premium can increase profits over uniform pricing by creating excess demand for the good at $t = 1$. The goods will then need to be rationed at $t = 1$. Preference certain consumers may be willing to pay a higher price in advance to ensure they receive the good and avoid it having sold out. The size of this premium is negatively related to the expected consumer surplus from waiting to purchase. The more rationing occurs, the less their expected surplus from waiting is and the higher is the premium they are willing to pay to ensure they obtain their preferred good. This is similar to the buying frenzy effect advanced bundle sales in DeGrabba and Mohammed (1999). As capacity increases, the need for rationing shrinks. This reduces the size of the premium to zero. Because the size of the premium depends both on the level of capacity and on the number of preference uncertain consumers, there is a full class of different potentially optimal advanced prices. The higher price paid by those who purchase in advance can increase profits over that of uniform low pricing. The increased demand by at lower prices than in the uniform high pricing plan can increase profits above those for uniform high pricing. Premium pricing can thus be optimal for intermediate capacities.

Lemma 4. *For any capacity* $0 < K < \frac{1}{2}$ *and prices* $p_0 > p_1$ *such that* $(p_0, p_1) \notin \{(v_{PL} + \delta, v_{PL})\}$ *where*

$$\delta = \frac{\frac{1}{2} - K}{(1-\gamma)(\frac{1}{2} - \frac{1}{2}\lambda) + \frac{1}{2}\lambda}(100 - v_{PL}). \tag{3}$$

(p_0, p_1) *is not profit maximizing.*

For any $K \geq \frac{1}{2}$ *there are no prices such that* $p_0 > p_1$ *are profit maximizing.*

Proof. If $p_0 > p_1 > 100$ no one will purchase and profits are zero. If $p_0 > 100 \geq p_1$, noone purchase at $t = 0$ and thus profits are at least as high if $100 \geq p_0 \geq p_1$. Given $p_0 > p_1$ we have that $100 \geq p_0 > p_1$. If $p_1 > v_{PL}$ then only high valuation consumers purchase and p_1 can be increased to increase profits. Therefore $p_1 \leq v_{PL}$. All consumers will demand one of the goods at $t = 1$ when $p_1 < v_{PL}$. The monopolist can then increase p_1 without reducing profits and thus any $p_1 \neq v_{PL}$ cannot be profit maximizing.

Given $p_0 > p_1 = v_{PL}$, all type-A_H and type-B_H consumers will purchase in advance if and only if

$$100 - p_0 \geq Pr[\text{Receive Rationed Good}](100 - p_1) = R(100 - p_1).$$

where

$$R = Pr[\text{Receive Rationed Good}] = \frac{K - \gamma(\frac{1}{2} - \frac{1}{2}\lambda)}{(1-\gamma)(\frac{1}{2} - \frac{1}{2}\lambda) + \frac{1}{2}\lambda}. \tag{4}$$

Thus a type-A_H and type-B_H consumers will purchase in advance if and only if

$$p_0 \leq 100 - 100R + Rv_{PL}$$

Any price above this reduces profits because noone pays the higher advanced price. Any price below this prices below the highest price these consumers are willing to pay. Denoting the size of the premium as $\delta = p_0 - p_1$,

$$\delta = (1-R)(100 - v_{PL}) = \frac{\frac{1}{2} - K}{(1-\gamma)(\frac{1}{2} - \frac{1}{2}\lambda) + \frac{1}{2}\lambda}(100 - v_{PL}).$$

Finally, at $R > 1$, $p_1 < p_0$. This corresponds to the restriction that $K < \frac{1}{2}$. □

Intertemporal price discrimination can only take the form of two discount pricing plans and the one premium pricing plan.

Proposition 2. *For any $K > 0$, any pricing plan*

$$(p_0, p_1) \notin \{(\mu_L, 100), (v_{PL}, 100), (v_{PL}, (v_{PL} + \delta, v_{PL})\}$$

such that $p_0 \neq p_1$ is not profit maximizing where $\delta > 0$ is the size of the premium defined in Equation 3.

Proposition 1 and Proposition 2 restrict the potentially profit maximizing prices to one of five pricing plans. Because any pricing plan is either uniform, a premium, or a discount, this exhausts all potentially optimal prices.

4 Profit Maximizing Pricing

The profit maximizing prices can be solved by examining the profits from each the pricing plans from Section 3. This section divides the parameter space into regions defined by the demands for the peak and non-peak goods under each of the five potentially profit maximizing pricing plans. Within each of these regions the profits of the relevant pricing plans are calculated and the conditions on which pricing plan is optimal is derived.

Because the regions within the parameter space where each pricing plan is optimal are complicated, the pricing space will be shown for a few specific parameter values. The conditions that are derived for when each pricing schedule is optimal is general, the graphs are drawn for the parameters in Table 4. Half the consumers are preference uncertain. Half the consumers are of high preference intensity. Three-quarters of the preference uncertain consumers will prefer the ex-post peak good.

Parameter	Value	Meaning
v_{PH}	100	Valuation for the preferred good by high valuation consumers
v_{NH}	40	Valuation for the non-preferred good by high valuation consumers
μ_H	60	Expected valuation for the high valuation consumers
v_{PL}	80	Valuation for the preferred good by low valuation consumers
v_{NL}	70	Valuation for the non-preferred good by low valuation consumers
μ_L	75	Expected valuation for the low valuation consumers
α	0.75	Percentage of uncertain consumers who will prefer the peak good
γ	0.5	Percentage of high valuation consumers
K	varies	Capacity for good-A and good-B
λ	varies	Percentage of preference uncertain consumers

Table 1: Parameter Values for Graphs

Using these parameters, the profits from the five possibly profit maximizing pricing strategies in Proposition 1 and Proposition 2 are calculated for each $\lambda \in [0, 1]$ and $K \in [0, 0.75]$ [4]. The maximum capacity ever required to fulfill all demand for the peak good is α. This occurs when all consumers are preference uncertain, $\lambda = 1$. Figure 1 shows the contour plot of p_0. The regions where each pricing plan is profit maximizing are labeled. Table 2 summarizes the time period in which each consumer type will attempt to purchase a good. Using these purchase times, ex-post total demand for each good is calculated in Table 4. Understanding how and when consumers change their demand is crucial to understanding the profit maximizing pricing regions. The critical points in the pricing regions are determined by a number of these ex-post demand lines as can be seen on Figure 4. By comparing

[4] These were calculated using a resolution of .000025. This gave us 400 λ values and 300 K values for a total of 120,000 points graphed. Any jagged lines are the result of the discrete nature of this graphing process and not inherent to the problem itself.

Pricing Plan (p_0, p_1)	Uniform Low (v_{PL}, v_{PL})	Uniform High $(100, 100)$	Premium $(v_{PL} + \delta, 100)$	Discount 1 $(\mu_L, 100)$	Discount 2 $(v_{PL}, 100)$
Types	Purchase time				
A_H, B_H	0	0	0	0	0
A_L, B_L	0	Never	1	0	0
U_H	1	1	1	1	1
U_L	1	Never	1	0	Never

Table 2: Consumer Purchase Times by Pricing Plan

Pricing Plan	Demand Non-Peak	Demand Peak
Uniform Low	$\frac{1}{2} - (\alpha - \frac{1}{2})\lambda$	$\frac{1}{2} + (\alpha - \frac{1}{2})\lambda$
Uniform High	$\frac{1}{2}\gamma - (\alpha - \frac{1}{2})\gamma\lambda$	$\frac{1}{2}\gamma + (\alpha - \frac{1}{2})\gamma\lambda$
Premium	$\frac{1}{2} - (\alpha - \frac{1}{2})\lambda$	$\frac{1}{2} + (\alpha - \frac{1}{2})\lambda$
Primary Discount	$\frac{1}{2} - (\alpha - \frac{1}{2})\gamma\lambda$	$\frac{1}{2} + (\alpha - \frac{1}{2})\gamma\lambda$
Alternative Discount	$\frac{1}{2} - (\frac{1}{2} - (1 - \alpha)\gamma)\lambda$	$\frac{1}{2} - (\frac{1}{2} - \alpha\gamma)\lambda$

Table 3: Demand

the profits from each pricing plan in between these demand lines we derive the conditions for the optimality of each the pricing plan in Lemma 5 to Lemma 15.

We start by examining the monopolist's choice between Uniform-High pricing and Premium pricing. When capacity is sufficiently low that both goods will sell out at the high prices then uniform high pricing must be profit maximizing.

Lemma 5. *For $K \leq \frac{1}{2}\gamma - (\alpha - \frac{1}{2})\gamma\lambda$, $p_0 = p_1 = 100$ is profit maximizing.*

Proof. $\frac{1}{2}\gamma - (\alpha - \frac{1}{2})\gamma\lambda \leq$ is the demand for the non-peak good at $p_0 = p_1 = 100$. Below this capacity, both goods sell out. Because lowering the prices does not increase sales and raising prices causes demand to be zero, $p_0 = p_1 = 100$ is the profit maximizing. □

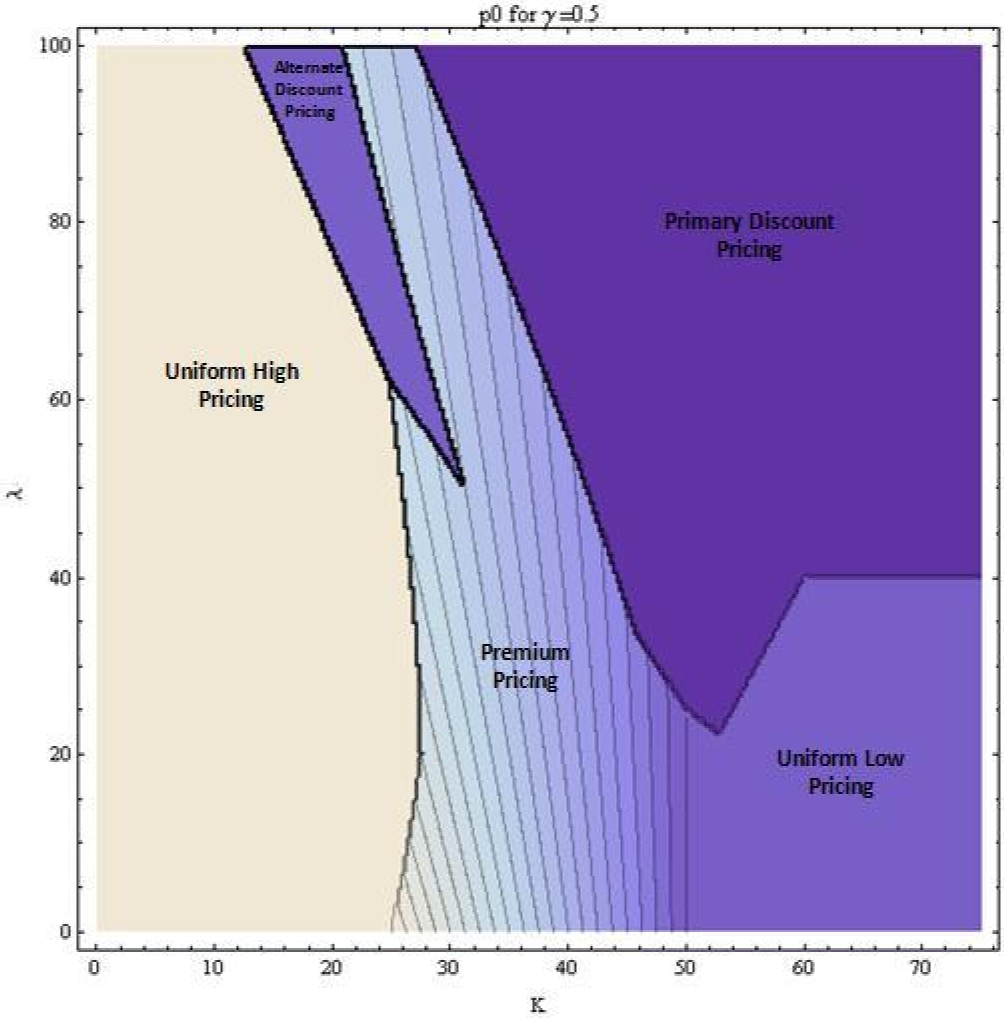

Figure 1: Pricing Regions for $\gamma = 0.5$

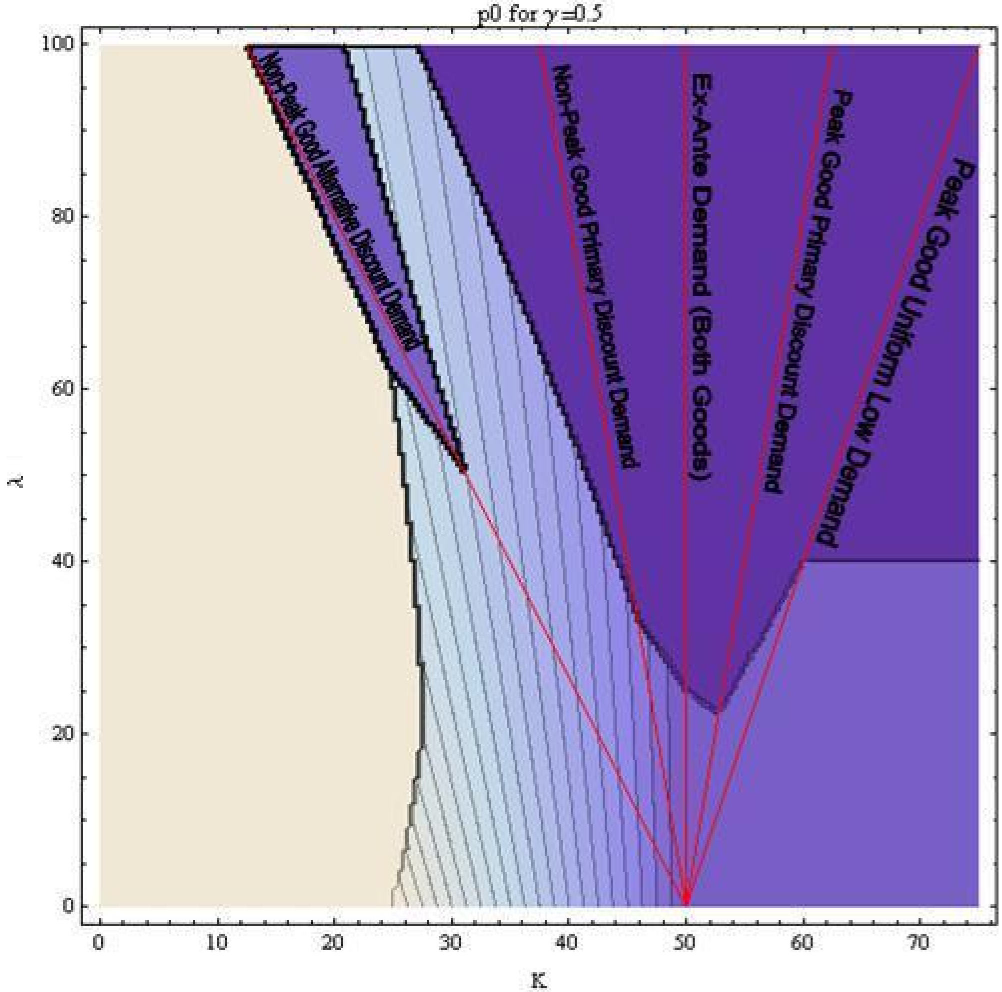

Figure 2: Critical Demand Rays for $\gamma = 0.5$

Once the capacity is such that uniform high pricing does not sell out both goods, the firm should consider changing to premium pricing.

Lemma 6. *For* $\frac{1}{2}\gamma - (\alpha - \frac{1}{2})\gamma\lambda < K \leq \frac{1}{2}\gamma + (\alpha - \frac{1}{2})\gamma\lambda$, *profits are higher for uniform high pricing than for premium pricing when*

$$K < \frac{50\gamma - \gamma\delta + 50\gamma\lambda - 100\alpha\gamma\lambda + \gamma\delta\lambda}{2(v_{PL} - 50)}.$$

Proof. For $\frac{1}{2}\gamma - (\alpha - \frac{1}{2})\gamma\lambda < K \leq \frac{1}{2}\gamma + (\alpha - \frac{1}{2})\gamma\lambda$, both goods sell out under premium pricing and the peak good sells out under uniform high pricing. Premium profits are then

$$\pi_{Prem} = (v_{PL} + \delta)(\gamma(1 - \lambda)) + v_{PL}(2K - \gamma(1 - \lambda)).$$

Uniform high profits are then

$$\pi_{UnifHigh} = 100(\gamma(1 - \lambda) + (1 - \alpha)\gamma\lambda + K - \gamma\frac{1 - \lambda}{2}).$$

Solving $\pi_{UnifHigh} > \pi_{Prem}$ for K yields

$$K < \frac{50\gamma - \gamma\delta + 50\gamma\lambda - 100\alpha\gamma\lambda + \gamma\delta\lambda}{2(v_{PL} - 50)}.$$

□

When there are only preference certain consumers in the market, $\lambda = 0$, and the monopolist prices at $p_0 = p_1 = 100$, only Type-A_H and Type-B_H consumers will purchase. They do so in advance and the demand for each good is $Q^d_{A0} = Q^d_{B0} = \gamma(\frac{1}{2} - \frac{1}{2}\lambda)$. Once there is additional capacity, $K = \gamma(\frac{1}{2} - \frac{1}{2}\lambda)$, the monopolist can increase the quantity sold via Premium pricing. Low valuation consumers will purchase at $t = 1$. This has two effects. The quantity sold will increase and the price that can be charged at $t = 1$ will fall. The size of the premium that can be charged in advance is less as capacity increases because there is less rationing and a higher chance that a high valuation consumer who waits to purchase will obtain his preferred good. In net the increased sales at $t = 0$ more than offset the lower price at $t = 0$ and profits increase via premium pricing.

Now as consumers become more preference uncertain, increasing λ, fewer consumers are purchasing in advance as preference uncertain consumers will not pay $p_0 = 100$. When λ is low the additional consumers who wait to purchase do not increase the need for rationing sufficiently and thus the lower price in $t = 0$ for the Premium pricing verses the Uniform-High pricing must be compensated by higher capacity. When there are many preference uncertain consumers, the quantity effect of gaining consumers at $t = 1$ by lowering p_1 in the premium case verse the high pricing case outweighs lower price charged at $t = 0$. This makes capacity where the monopolist is indifferent between the Uniform-High and Premium pricing lower as λ increase. These two effects cause the boundary between these pricing regions to be curved as is seen in Figure 1. In Lemma 6 this can be seen by incorporating the effects of both preference uncertainty, λ, and premium size, δ, on the critical capacity.

Next, examine the boundary between Premium pricing and Uniform-Low pricing.

Lemma 7. *Profits are higher for premium pricing than for uniform low pricing when* $K < \frac{1}{2}$.

Proof. Because the total demand for both goods is the same under premium and uniform low pricing, and p_1 is the same in both pricing schemes, premium pricing yields higher profits as long as there is rationing at $t = 1$. From equation 4, rationing will no longer occur when $R = 1$. Solving this for K yields $K = \frac{1}{2}$. Thus when $K < \frac{1}{2}$, premiums occur and when $K \geq \frac{1}{2}$ uniform low pricing occurs. □

Second period pricing is the same for both premium and uniform low pricing: $p_1 = v_{PL}$. The first period premium price is larger than the uniform low price by the size of the premium. As capacity increases, uniform low pricing is then just the limiting case of premium pricing.

Setting the advanced price premium, δ, from Equation 3 to be zero and solving for the capacity we find that the point at which premium pricing degenerates into Uniform-Low pricing is at $K = \frac{1}{2}$ independent of other parameter values. This occurs because of the ex-ante symmetry of demand.

By Lemma 6 and Lemma 7 the parameter space has been partitioned into three sections. When capacity is low, high pricing dominates premium and low pricing. When capacity is high, low pricing dominates premium and high pricing. For intermediate capacities, premium pricing dominates low and high pricing. The rest of the section compares these pricing schemes with the two discount pricing schemes.

Consider the monopolist's choice between primary discount pricing and the premium and uniform low pricing regions. Primary discount pricing is optimal when there are many preference uncertain consumers and capacity is sufficiently high. The boundary line for the primary discount pricing region changes slopes four times. Each time corresponds capacity reaching either the peak good or non-peak good demand for the competing pricing schemes.

For the primary discount, the monopolist prices lower in advance, $p_0 = \mu_L$, and higher at $t = 0$, $p_1 = 100$. This causes all preference certain consumers and the low valuation, preference uncertain consumers to purchase in advance. Only the type-U_H consumers wait and purchase at $t = 1$. Starting with only preference uncertain consumers, $\lambda = 1$, decreasing preference uncertainty results in selling more in advance at the lower price p_0. This reduces the profitability of the discount relative to the premium. To offset this effect, capacity needs to be increased. This will lower the size of the premium that can be charged and lowers premium profits relative to discount profits. The boundary is thus downward sloping as preference certain consumers are added to the market.

Lemma 8. *For* $\frac{1}{2} - (\alpha - \frac{1}{2})\lambda < K \leq \frac{1}{2} - (\alpha - \frac{1}{2})\gamma\lambda$*, premium pricing profits are higher than* *primary discount pricing profits when*

$$\lambda < \frac{100 + v_{PL} + 2\gamma\delta - 2(100 - v_{PL})K - 2\mu_L}{2v_{PL} + 300\gamma + 2\gamma\delta - v_{PL} - 200\alpha\gamma - 2\gamma\mu_L}$$

Proof. For $\frac{1}{2} - (\alpha - \frac{1}{2})\lambda < K \leq \frac{1}{2} - (\alpha - \frac{1}{2})\gamma\lambda$, both goods sell out under primary discount pricing and the peak good sells out under premium pricing. Premium profits are then

$$\pi_{Prem} = (v_{PL} + \delta)(\gamma(1 - \lambda)) + v_{PL}((1 - \alpha)\lambda + (1 - \gamma)(1 - \lambda) + (K - \frac{1 - \lambda}{2})).$$

Discount profits are then

$$\pi_{Disc} = \mu_L((1 - \lambda) + (1 - \gamma)\lambda) + 100((1 - \alpha)\gamma\lambda + (K - \frac{1 - \lambda}{2} - \frac{1}{2}(1 - \gamma)\lambda).$$

Solving $\pi_{Prem} > \pi_{Disc}$ yields

$$\lambda < \frac{100 + v_{PL} + 2\gamma\delta - 2(100 - v_{PL})K - 2\mu_L}{2v_{PL} + 300\gamma + 2\gamma\delta - v_{PL} - 200\alpha\gamma - 2\gamma\mu_L}$$

□

The downward sloping portion of the Primary Discount-Premium boundary changes slopes twice. At lower levels of capacity, the effect is only a relative price effect as both goods are selling out in the discount case. As capacity increases to the point at which only the peak good sells out, there is additional quantity that can be sold by switching to premium pricing. Additionally capacity needed to offset the relative loss of profit from having fewer preference uncertain consumers is larger. This occurs at,

$$Q^d_{Non-Peak,PrimDisc} = \frac{1}{2} - (\alpha - \frac{1}{2})\gamma\lambda. \tag{5}$$

Lemma 9. *For $\frac{1}{2} - (\alpha - \frac{1}{2})\gamma\lambda < K \leq \frac{1}{2}$, premium pricing profits are higher than primary discount pricing profits when*

$$\lambda < \frac{100 + v_{PL} + 2\gamma\delta - 2(100 - v_{PL})K - 2\mu_L}{2v_{PL}\alpha + 300\gamma + 2\gamma\delta - v_{PL} - 200\alpha\gamma - 2\gamma\mu_L}.$$

Proof. For $\frac{1}{2} - (\alpha - \frac{1}{2})\gamma\lambda < K \leq \frac{1}{2}$, the peak good sells out under both premium and primary discount pricing. Premium profits are then

$$\pi_{Prem} = (v_{PL} + \delta)(\gamma(1-\lambda)) + v_{PL}((1-\alpha)\lambda + (1-\gamma)(1-\lambda) + (K - \frac{1-\lambda}{2})).$$

Discount profits are then

$$\pi_{Disc} = \mu_L((1-\lambda) + (1-\gamma)\lambda) + 100((1-\alpha)\gamma\lambda + (K - \frac{1-\lambda}{2} - \frac{1}{2}(1-\gamma)\lambda).$$

Solving $\pi_{Prem} > \pi_{Disc}$ yields

$$\lambda < \frac{100 + v_{PL} + 2\gamma\delta - 2(100 - v_{PL})K - 2\mu_L}{2v_{PL}\alpha + 300\gamma + 2\gamma\delta - v_{PL} - 200\alpha\gamma - 2\gamma\mu_L}.$$

□

The second change in slope happens at $K = \frac{1}{2}$. Once capacity exceeds half the market, the premium pricing degenerates into uniform-low pricing. Having more preference certain consumers no longer lowers p_1. This results in a higher tradeoff between λ and K.

Lemma 10. *For $\frac{1}{2} < K \leq \frac{1}{2} + (\alpha - \frac{1}{2})\gamma\lambda$, uniform low profits are higher than primary discount pricing profits when*

$$\lambda < \frac{100 + v_{PL} - 2(100 - v_{PL})K - 2\mu_L}{2v_{PL}\alpha + 300\gamma - v_{PL} - 200\alpha\gamma - 2\gamma\mu_L}.$$

Proof. For $\frac{1}{2} < K \leq \frac{1}{2} + (\alpha - \frac{1}{2})\gamma\lambda$, the peak good sells out under both uniform low and primary discount pricing. Uniform low profits are then

$$\pi_{UnifLow} = v_{PL}(\frac{1-\lambda}{2} + (1-\alpha)\lambda + K).$$

Discount profits are then

$$\pi_{Disc} = \mu_L((1-\lambda) + (1-\gamma)\lambda) + 100((1-\alpha)\gamma\lambda + (K - \frac{1-\lambda}{2} - \frac{1}{2}(1-\gamma)\lambda).$$

Solving $\pi_{UnifLow} > \pi_{Disc}$ yields

$$\lambda < \frac{100 + v_{PL} - 2(100 - v_{PL})K - 2\mu_L}{2v_{PL}\alpha + 300\gamma - v_{PL} - 200\alpha\gamma - 2\gamma\mu_L}.$$

□

Once the capacity is large enough to sell out both the peak and the non-peak goods, increasing capacity does nothing for the discount profits but will increase profits from the uniform low pricing. The boundary then becomes increasing, leading to the telltale lower point for the discount pricing region which occurs at,

$$Q^d_{Peak,PrimDisc} = \frac{1}{2} + (\alpha - \frac{1}{2})\gamma\lambda. \tag{6}$$

Lemma 11. *For $\frac{1}{2} + (\alpha - \frac{1}{2})\gamma\lambda < K \leq \frac{1}{2} + (\alpha - \frac{1}{2})\lambda$, uniform low profits are higher than primary discount pricing profits when*

$$\lambda < \frac{v_{PL} + 2v_{PL}K - 2\mu_L}{2v_{PL}\alpha + 200\gamma - v_{PL} - 2\gamma\mu_L}.$$

Proof. For $\frac{1}{2} + (\alpha - \frac{1}{2})\gamma\lambda < K \leq \frac{1}{2} + (\alpha - \frac{1}{2})\lambda$, the peak good sells out under uniform low pricing but neither good sells out under primary discount pricing. Uniform low profits are then

$$\pi_{UnifLow} = v_{PL}(\frac{1-\lambda}{2} + (1-\alpha)\lambda + K).$$

Discount profits are then

$$\pi_{Disc} = \mu_L((1-\lambda) + (1-\gamma)\lambda) + 100\gamma\lambda).$$

Solving $\pi_{UnifLow} > \pi_{Disc}$ yields

$$\lambda < \frac{v_{PL} + 2v_{PL}K - 2\mu_L}{2v_{PL}\alpha + 200\gamma - v_{PL} - 2\gamma\mu_L}.$$

□

Finally, for uniform low pricing the highest capacity ever needed for the peak good is

$$Q^d_{Peak,UnifLow} = \frac{1}{2} + (\alpha - \frac{1}{2})\lambda. \tag{7}$$

Above this capacity there can be no change in the profits for any pricing plan. Whichever pricing plan is optimal at this capacity is optimal for all capacities above this line.

Lemma 12. *For $\frac{1}{2} + (\alpha - \frac{1}{2})\lambda < K$, uniform low profits are higher than primary discount pricing profits when*

$$\lambda < \frac{v_{PL} - \mu_L}{\gamma(100 - mu_L)}.$$

Proof. For $\frac{1}{2} + (\alpha - \frac{1}{2})\lambda < K$, neither good sells out under either uniform low pricing or primary discount pricing. Uniform low profits are then

$$\pi_{UnifLow} = v_{PL}.$$

Discount profits are then

$$\pi_{Disc} = \mu_L((1-\lambda) + (1-\gamma)\lambda) + 100\gamma\lambda).$$

Solving $\pi_{UnifLow} > \pi_{Disc}$ yields

$$\lambda < \frac{v_{PL} - \mu_L}{\gamma(100 - mu_L)}.$$

□

Finally, we must examine the alternate discount pricing region. The key to understanding this pricing plan is the demand for the non-peak good. When there are only preference uncertain certain consumers, $\lambda = 1$, discount pricing will be optimal if there is sufficient capacity that the non-peak good no longer sells out. For those purchasing the good in advance and taking advantage of the discount, there is demand smoothing between the two goods. This allows a higher quantity to be sold increasing profits. As we replace some preference uncertain consumers with preference certain consumers, lowering λ, less demand smoothing occurs because demand from preference certain consumers is already balanced. This makes the discount less profitable relative to uniform pricing. Increasing the capacity offsets this effect resulting in a downward sloping border.

Lemma 13. *For $\frac{1}{2}\gamma - (\alpha - \frac{1}{2})\gamma\lambda < K \leq \frac{1}{2} - (\frac{1}{2} - (1-\alpha)\gamma)\lambda$, uniform high profits are higher than alternate discount pricing profits when*

$$\lambda < \frac{100(1-K) + 50\gamma - v_{PL}}{100(1-\alpha\gamma) - 50\gamma - v_{PL}}.$$

Proof. For $\frac{1}{2}\gamma - (\alpha - \frac{1}{2})\gamma\lambda < K \leq \frac{1}{2} - (\frac{1}{2} - (1-\alpha)\gamma)\lambda$, both goods sell out under alternate discount pricing while only the peak good sells out under uniform high pricing. Uniform high profits are then

$$\pi_{UnifHigh} = 100((1-\lambda)\gamma + (1-\alpha)\gamma\lambda + K - \frac{1-\lambda}{2}\gamma).$$

Alternate discount profits are then

$$\pi_{AltDisc} = v_{PL}(1-\lambda) + 100(2K - (1-\lambda)).$$

Solving $\pi_{UnifHigh} > \pi_{AltDisc}$ yields

$$\lambda < \frac{100(1-K) + 50\gamma - v_{PL}}{100(1-\alpha\gamma) - 50\gamma - v_{PL}}.$$

□

Once we reach the boundary between the premium pricing and high pricing region from Lemma 6, there is additional demand from the low valuation consumers. They did not demand either good under the uniform high pricing plan. This makes the premium even more attractive than high pricing was. At higher capacities this flattens the slope of the border further.

Lemma 14. *For $\frac{1}{2}\gamma - (\alpha - \frac{1}{2})\gamma\lambda < K \leq \frac{1}{2} - (\frac{1}{2} - (1-\alpha)\gamma)\lambda$, premium profits are higher than alternate discount pricing profits when*

$$\lambda < \frac{v_{PL} - 100 - \gamma\delta + 2(100 - v_{PL})K}{v_{PL} - \gamma\delta - 100}.$$

Proof. For $\frac{1}{2}\gamma - (\alpha - \frac{1}{2})\gamma\lambda < K \leq \frac{1}{2} - (\frac{1}{2} - (1-\alpha)\gamma)\lambda$, both goods sell out under both alternate discount pricing and premium pricing. Premium pricing profits are then

$$\pi_{Prem} = (v_{PL} + \delta)(\gamma(1-\lambda)) + v_{PL}(2K - \gamma(1-\lambda)).$$

Alternate discount profits are then

$$\pi_{AltDisc} = v_{PL}(1-\lambda) + 100(2K - (1-\lambda)).$$

Solving $\pi_{Prem} > \pi_{AltDisc}$ yields

$$\lambda < \frac{v_{PL} - 100 - \gamma\delta + 2(100 - v_{PL})K}{v_{PL} - \gamma\delta - 100}.$$

□

Once capacity is such that the alternative discount pricing will no longer sell out the non-peak good, increasing the proportion of preference uncertain consumers will reduce the capacity at which the monopolist is indifferent between premium and alternate discount pricing. This occurs when

$$Q^d_{Non-Peak,AltDisc} = \frac{1}{2} + (\frac{1}{2} - (1-\alpha)\gamma)\lambda. \tag{8}$$

This results in the sharp lower point of the alternative discount region.

Lemma 15. *For $\frac{1}{2} - (\frac{1}{2} - (1-\alpha)\gamma)\lambda < K \leq \frac{1}{2} - (\alpha - \frac{1}{2})\lambda$, premium profits are higher than alternate discount pricing profits when*

$$\lambda > \frac{(2v_{PL} - 100)K + \gamma\delta + 50 - v_{PL}}{100(1-\alpha)\gamma + \gamma\delta - vPL}.$$

Proof. For $\frac{1}{2} - (\frac{1}{2} - (1-\alpha)\gamma)\lambda < K \leq \frac{1}{2} - (\alpha - \frac{1}{2})\lambda$, both goods sell out under premium pricing but only the peak good sells out under alternative discount pricing. Premium pricing profits are then

$$\pi_{Prem} = (v_{PL} + \delta)(\gamma(1-\lambda)) + v_{PL}(2K - \gamma(1-\lambda)).$$

Alternate discount profits are then

$$\pi_{AltDisc} = v_{PL}(1-\lambda) + 100((1-\alpha)\gamma\lambda + K - \frac{1-\lambda}{2}).$$

Solving $\pi_{Prem} > \pi_{AltDisc}$ yields

$$\lambda > \frac{(2v_{PL} - 100)K + \gamma\delta + 50 - v_{PL}}{100(1-\alpha)\gamma + \gamma\delta - vPL}.$$

□

If we increase the number of preference uncertain consumers when the capacity is large enough that the non-peak good fails to sell out, the discount pricing loses consumers relative to the premium pricing. This is because all preference certain consumers purchase while only the high valuation preference certain consumers will purchase. To offset this relative effect, capacity must be lowered toward the non-peak sell out line. This creates an upward sloping border to the right of this line and thus a sharply pointed region where the alternative discount pricing is profit maximizing.

Figure 4 shows each of the relevant lines for the maximum capacity needed. These lines, as defined in equations 5 through 8, move with changing parameter values and define the optimal pricing regions. Understanding when the peak and non-peak goods sell out is critical to understanding why each pricing plan is optimal where it is. Once the non-peak good fails to sell out there is an incentive consider discount pricing to begin demand shifting to increase profits. In general, when capacity is low, profits are maximized via uniform high pricing. Once capacity is large enough to exceed high valuation demand premium pricing will become a viable option. As capacity increases further, the size of the premium shrinks to zero resulting in uniform low pricing. When most consumers are preference uncertain, discount pricing can smooth demand for the goods and increase profits given sufficient capacity. When most consumers are preference certain, insufficient demand smoothing occurs under discount pricing and uniform low pricing will prevail.

5 Capacity Choice

Until now capacity has been treated as an exogenous variable. Firms, however, choose their capacity. Once capacity is chosen, a pricing plan for the goods is selected. Capacity choice is thus an exercise in backward induction. The choice of capacity will then depend on the cost of capacity, consumers' preference certainty and consumers' preference intensity. Once we understand how firms choose their capacity, we will be able to understand which pricing plans will be observed in markets with differing costs of capacity. This section develops the firm's problem after incorporating the cost of capacity and then examines the relationship between capacity cost and the profit maximizing pricing plan.

Here it will be assumed that capacity can be chosen separately for both good-A and good-B, K_A and K_B respectively. This is done to match with a model of airlines where different flights have different capacities. If capacity is ex-ante required to be the same for both goods, for example a concert hall or sports arena, the $K_A = K_B$ can be assumed prior to the maximization problem without altering the results. We first show that the profit maximizing capacity must be on the demand line for either the peak or non-peak good for the optimal ex-post pricing plan. Then we examine the optimal pricing plan that results from a given marginal cost of capacity. As cost of capacity increases, the firm uses less capacity. This will result a movement away from discount and low pricing toward premium and high pricing.

The monopolist's problem is now to maximize profits by choosing capacities and a pricing plan:

$$\max_{(p_0,p_1,K_A,K_B)} p_0(\min\{Q^D_{A0},K_A\}+\min\{Q^D_{B0},K_B\})+p_1(\min\{Q^D_{A1},Q^S_{A1}\}+\min\{Q^D_{B1},Q^S_{B1}\})-c(K_A,K_B). \tag{9}$$

We assume that the marginal cost of capacity for each good is symmetric. This along with the symmetry of consumer preferences over good-A and good-B results in the profit maximizing capacities being identical. The choice of capacities is then just a choice of a single K. Denote the marginal cost of an additional unit of capacity for both goods as $mc_K(K) = \frac{\partial c(K,K)}{\partial K_A}+\frac{\partial c(K,K)}{\partial K_B} = 2\frac{\partial c}{\partial K_A}$. For tractability mc_K is assumed to be constant. The optimal choice of capacity the depends on the marginal cost of capacity, consumers' preference intensity, consumers' preference certainty, and the ex-post peak demand parameter: $K^*(mc_K,\gamma,\lambda,\alpha)$. For compactness, the arguments will be dropped and optimal capacity will be denoted as K^*. It will be profit maximizing to choose a capacity equal to the demand for either the peak or non-peak goods at the ex-post optimal pricing plan.

Lemma 16. *Given a constant marginal cost $mc_K < 200$, any capacity K such that $K \neq Q^d_{Peak}$ and $K \neq Q^d_{Non-peak}$ cannot be profit maximizing.*

Proof. For all pricing plans except Premium pricing p_0 and p_1 are constant. Thus,

$$\frac{\partial \pi}{\partial K} = \begin{cases} 2p_0 - mc_K & \text{if } 0 \le K < Q^d_{0NP} \\ p_0 - mc_K & \text{if } Q^d_{0NP} \le K < Q^d_{0P} \\ p_0 + p_1 - mc_K & \text{if } K \le Q^d_{0NP} \text{ and } Q^d_{0P} \le K < Q^d_{0P} + Q^d_{1P} \\ 2p_1 - mc_K & \text{if } Q^d_{0P} \le K < Q^d_{0NP} + Q^d_{1NP} \\ p_1 - mc_K & \text{if } Q^d_{0NP} + Q^d_{1NP} \le K < Q^d_{0P} + Q^d_{1P} \\ -mc_K & \text{if } Q^d_{0P} + Q^d_{1P} \le K \end{cases}$$

Note that each piece of $\frac{\partial \pi}{\partial K}$ is constant for all pricing plans except premium pricing. If $\frac{\partial \pi}{\partial K} > 0$

then it is optimal for the monopolist to increase capacity to meet demand for either the peak or non-peak good. If $2p_0 \leq c'(K)$ then profit cannot be positive ever and $K^* = 0$. $\square$

Profit maximizing capacity is the demand for either the peak or non-peak goods. However, not all pricing plans are optimal at these levels of capacity. The lemmas that follow eliminate the capacities form Lemma 16 that can never be profit maximizing.

Lemma 17. *When K^* results in Uniform Low Pricing, $K^* = \frac{1}{2} + (\alpha - \frac{1}{2})\lambda$.*

Proof. By Lemma 16, $K^* = \frac{1}{2} - (\alpha - \frac{1}{2})\lambda$ or $K^* = \frac{1}{2} + (\alpha - \frac{1}{2})\lambda$. At $K = \frac{1}{2} - (\alpha - \frac{1}{2})\lambda$, premium pricing dominates uniform low pricing because demand and supply are the same but p_1 is higher under premium pricing. Thus $K^* = \frac{1}{2} + (\alpha - \frac{1}{2})\lambda$ whenever low pricing is profit maximizing. $\square$

Lemma 18. *When K^* results in Premium Pricing, $K^* = \frac{1}{2} - (\alpha - \frac{1}{2})\lambda$.*

Proof. By Lemma 16, $K^* = \frac{1}{2} - (\alpha - \frac{1}{2})\lambda$ or $K^* = \frac{1}{2} + (\alpha - \frac{1}{2})\lambda$. At $K = \frac{1}{2} + (\alpha - \frac{1}{2})\lambda$, the premium (see Lemma 4) that can be charged is zero and pricing degenerates into uniform low pricing. The optimal capacity is thus $K^* = \frac{1}{2} - (\alpha - \frac{1}{2})\gamma\lambda$. $\square$

Lemma 19. *When K^* results in Primary Discount Pricing, $K^* = \frac{1}{2} - (\alpha - \frac{1}{2})\gamma\lambda$ or $K^* = \frac{1}{2} + (\alpha - \frac{1}{2})\gamma\lambda$.*

Proof. This follows directly from Lemma 16. $\square$

Lemma 20. *When K^* results in Primary Discount Pricing, $K^* = \frac{1}{2}\gamma - (\alpha - \frac{1}{2})\gamma\lambda$ or $K^* = \frac{1}{2}\gamma + (\alpha - \frac{1}{2})\gamma\lambda$.*

Proof. This follows directly from lemma 16. $\square$

Figure 5 shows how the optimal capacity changes as the marginal cost of capacity increases. The white points are the optimal levels of capacity for $\gamma = \frac{1}{2}$. The other lines are

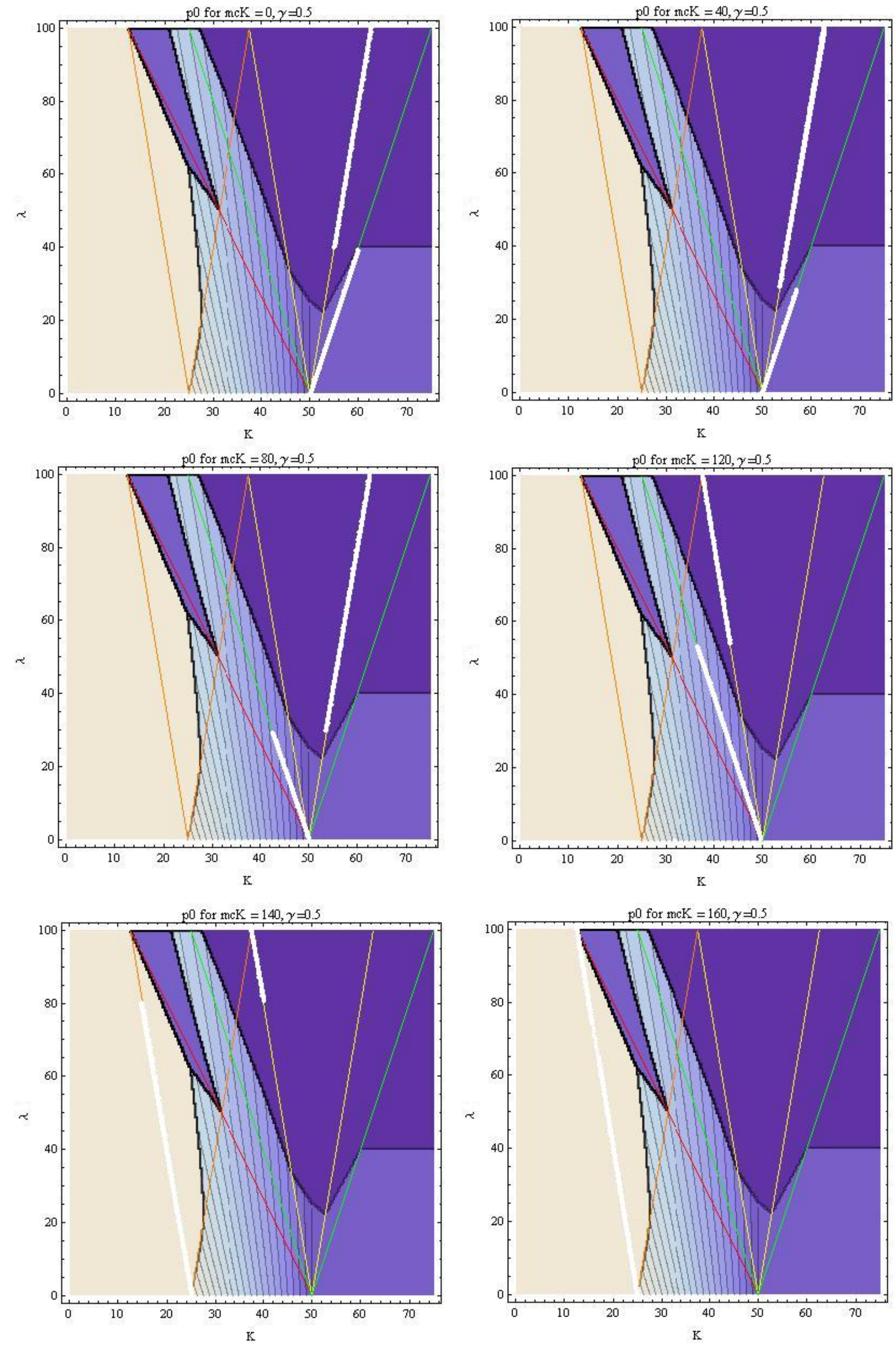

Figure 3: Profit Maximizing Capacity (in White) for Monopolist with Demand Lines

the demand lines given in Table 4. As shown in Lemma 16, the optimal capacity does follow the demand lines.[5] For low marginal cost of capacity, optimal capacity will be large enough to sell to all consumers. The firm will then choose uniform low pricing when most consumers are preference certain and discount pricing otherwise. As the cost of capacity rises, for low amounts of preference uncertain consumers, it becomes optimal to charge a premium and chose a capacity that just sells out non-peak good. At high proportions of preference uncertain consumers the discount will still be optimal. As the cost of capacity rises above v_{PH}, it is no longer profitable to only sell one more unit of the peak good. In order to recover the cost of the capacity, the monopolist must sell both the peak and non-peak goods. As a result, the optimal capacity must be on the non-peak demand lines. Finally, as capacity becomes very expensive, it is optimal to charge a uniform high price and keep capacity low.

6 Applications and Conclusions

Thus far we have shown how premiums, discounts, and uniform pricing arise in markets with constrained capacity, preferences uncertain consumers, and multiple sales periods. A firm that charges a premium to purchase in advance takes advantage of high valuation consumers who are concerned that the good may be rationed. A firm which offers a discount for advanced purchases does so to induce low valuation consumers to purchase before they know their preferences. This results in demand shifting away from the ex-post more popular good and increasing the overall quantity of goods sold.

From these observations we can deduce that industries for which advanced purchase discounts prevail will be characterized by many consumers who do not know their preferences well in advance. Airlines fit this description. Consumers often choose between very similar flight times and may not know until closer to the departure date which time they prefer.

[5] And deviation is just due to the discreteness of the problem.

Low valuation consumers, like many vacation travelers, will purchase in advance and often have chosen their ex-post less preferred flight. Consumers with high valuations, like many business travelers, will wait to purchase. They have a higher cost of ending up on the wrong flight and thus must wait until their preferences are known to purchase.

Advanced purchase premiums will prevail in industries where some consumers know their preferences well in advance and are concerned about potential rationing. These consumers are willing to pay the higher prices in advance rather than wait and risk not being able to purchase their preferred good. Capacity must be low enough to sell out the peak good and force rationing. We observe this at sporting events and concerts. Most consumers for these events will know their preferences over competing events well in advance and thus will be willing to pay the increased price in advance. This premium often takes the form of a service charge on tickets by an intermediary seller.

If the firm can choose its capacity, the profit maximizing prices are determined by the consumer characteristics and the cost of capacity. When capacity is relatively cheap the firm will sell to all consumers choosing low prices or advanced pricing discounts. In this case there will be excess capacity for the less preferred good and the peak good will just sell out. When capacity is expensive the firm will choose a capacity just large enough to meet demand for the less preferred good. This results in excess demand for the peak good and rationing. Premiums and high prices will generally prevail in such markets. From this we can infer that in markets where premiums prevail the cost of additional capacity is relatively high compared to valuations of the goods. When discounts are observed, additional capacity must be relatively cheap. Comparing airlines to concerts, airlines have the ability to add capacity by adding additional flights or flying larger planes. A concert or sporting event is typically constrained by its venue and adding additional seats comes at an extremely high cost relative to consumers' valuations. This coincides with the observation that discounts are more likely to prevail when capacity costs are relatively low. Premiums are more likely when additional capacity costs are high.

The model developed here was designed to unify the discount pricing found in Gale and Holmes (1992) and the premium pricing found in DeGrabba and Mohammed (1999). Both forms of intertemporal price discrimination can depend on the level of capacity, consumer preference certainty, and consumer preference intensity. We have shown that markets which are similar in terms being capacity constrained and sell their tickets intertemporally depend on consumer preference certainty and intensity for their pricing strategies.

To be more applicable to the applications of airline pricing and event ticket sales this model can be placed into a competitive framework. Using the intuition developed by Dana (1992), advanced purchase discounts would still prevail due to the demand shifting process. The ability of firms to charge premiums will disappear as market power erodes the ability of firms to charge above marginal cost. Introducing additional degrees of product differentiation between goods sold by different firm would allow for premiums to persist. This is left for future work. For now, it is clear that both discounts and premiums can arise in markets where a monopolist chooses capacity and then sets prices in multiple time periods. This choice is determined by the intensity of consumer preferences, the uncertainty of consumer preferences, and the cost of additional capacity.

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