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Price-Directed Consumer Search

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Abstract

This paper presents an oligopoly model in which consumers conduct sequential costly search for a desired product. In contrast to the specification of random or pre-determined search order in most current studies, I allow the order of consumer search to be endogenously determined. In the model, consumers observe product prices before searching among firms. Thus, price additionally affects profit by influencing the order in which consumers search. I find that the pattern of equilibrium price distribution depends on the size of search cost. In particular, for a medium search cost, a mixture price distribution prevails: firms randomly play two separate price distributions with a gap in between. Firms either price high, following a high price distribution, or price low, following a low price distribution, but always avoid prices at the intermediate interval. For low or high search costs, equilibrium price distribution is continuous with positive density on the entire support. Comparative statics show that equilibrium price is non-monotonic in search cost and firm profit can be higher when consumers have lower search costs.

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1 Introduction

An extensive literature has examined how consumers' costly sequential for information affects market performance. These models typically assume that consumers search randomly among firms. Stahl (1989) analyzes a model where firms sell homogenous goods, and consumers with positive search costs visit firms in a random order and buy from the first firm that offers a price below their reservation price. Wonlinsky (1986) and Anderson and Renault (1999) consider differentiated product markets in which consumers incur costs to obtain price as well as product information. Consumers examine each product with an equal probability and purchase if a product yields a sufficient match to their preference. In these models, since firms are ex-ante identical and consumers do not obtain any information before search, it is natural to assume that consumers visit firms in a random order.

Recently, several papers have studied the impact of non-random consumer search on market prices. Arbatskaya (2007) considers a model where consumers conduct sequential search among firms selling homogenous goods. In her model, consumers have to visit firms in an exogenous pre-determined order. In equilibrium, prices charged by firms decline in the order of consumers search. Consumers with higher (lower) search costs buy from the firms on the top (bottom) of the order paying higher (lower) prices. Armstrong, Vickers and Zhou (2009) investigate a model of differentiated products where consumers visit a prominent firm before turning to non-prominent rivals. They show that the prominent firm charges a lower price and the non-prominent firms charge a higher price, compared to the situation with random consumer search.

While it is reasonable to assume a pre-determined search order or a prominent firm, it would be interesting to examine a model where the order of consumer search is endogenously influenced by some strategic variable chosen by firms. This paper makes an attempt to construct a model where consumers search sequentially among differentiated products with an *endogenously* determined order. In the model, we assume that consumers can access price information before conducting costly search. This assumption is justified given the rapid development of Internet technology. For instance, with price search engines, consumers may easily obtain a list of prices of products by different sellers. However, the costs of time and effort for consumers to find out if a product matches their preferences remain significant. With observable prices, an interesting twist occurs. Prices, in addition to entering into firms' profit function directly, it may also influence the order in which consumers search, thus, indirectly affect firms' profits. By charging a relatively low price, a firm may become more "prominent" and increase sales. Surprisingly, the latter strategic consideration has not been explored in the literature. This paper attempts to fill in this gap.

To this end, we develop a stylized model where consumers have "needs" and firms provide products that may meet consumers' needs. We assume two groups of consumers: informed and uninformed. All consumers can observe prices. An informed consumer knows if a product satisfies her "need" while an uninformed consumer has to incur a positive search cost (examine cost) to find out. After observing prices, consumers determine optimal search rules. Firms simultaneously set prices taking into account consumers' search strategy.

We show that, in equilibrium, firms charge prices following a price distribution, informed consumers buy from the matched firm with the lowest price in the market, and uninformed consumers search sequentially starting from the firms charging lower prices to higher prices provided that the prices do not exceed a certain threshold which we shall call the reservation price. The nature of the equilibrium price distribution depends on the size of search cost. For a medium search cost, the equilibrium is a mixture distribution where firms randomize between a high-price distribution and a low-price distribution, placing zero probability on an interval of intermediate prices. By adopting the mixture pricing strategy, firms swing between targeting matched informed consumers and uninformed consumers. For a low or a high search cost, the price distribution is continuous which has positive density on the entire support. In particular, for a low search cost, no firm prices above uninformed consumers' reservation price, and both informed and uninformed consumers pay the same expected price. For a high search cost, the entire price distribution is above uninformed consumers' reservation price since firms target informed consumers.

There are also some interesting results from the comparative statics on search cost. We show that a decrease in search cost may increase, decreases or not affect the equilibrium market price. Armstrong, Vickers and Zhou (2009) also suggest that if consumers can observe prices in advance but have to incur a search cost to find out the match utility, it is possible that lower prices might result from higher search costs. This is because higher search costs, in their model, increase firms' profits of being prominent, and thus intensify the incentive to choose a low price. In contrast, I illustrate another mechanism leading to this somewhat counter-intuitive result. In my model, a decrease in search cost induces uninformed consumers to search more as their reservation price increases. For a low search cost, firms' price strategy is driven by uninformed consumers since their reservation price is relatively high – all firms charge prices below the reservation price in equilibrium. Thus, an increase in the reservation price allows firms to charge higher prices resulting in higher market prices. For a medium search cost, firms randomize between a high-price distribution and a low price distribution. As before, a higher reservation price allows firms to increase prices as long as the prices are below the reservation price (the low-price distribution). However, an additional effect exists in this equilibrium. Firms may focus more on uninformed consumers by placing a higher probability on the low-price distribution. In fact, we show that as search cost decreases prices in the mixture price distribution decrease stochastically. For a high search cost, firms only target informed consumers by pricing above the uninformed consumers' reservation price. Thus, market prices stay unchanged despite of a decrease in search cost.

In addition to the papers mentioned above, our paper is related to Chen and He (2006) who investigate paid advertisement with consumer search. In particular, They study a model where firms differing in relevance of matching consumers needs bid for the advertised position for their products. Consumers are initially uncertain about whether a product matches their needs but can learn through a costly search. They show that paid advertisement induces efficient consumer search: consumers optimally search in the order of the listed products. The result relies on the vertical difference of the firms. In contrast, our model consider ex-ante symmetric firms and the search order is determined by the prices that are chosen by firms.

The rest of the paper is organized as follows. Section 2 describes the model.

Equilibrium analysis is conducted in section 3. Section 4 shows comparative statics on search cost. Section 5 concludes.

2 The model

There is a continuum of consumers, each with a "need." A consumer derives utility V if her need is met and 0 otherwise. $N \geq 2$ firms exist in the market, each of which carries a product that meets a consumer's need with a certain probability.[1] The probability is assumed independent and identical for all firms and is denoted as $\theta \in [0, 1]$. The marginal cost is assumed constant and normalized to zero.

There are two types of consumers: *informed* and *uninformed.* In particular, $\mu \in [0, 1]$ of consumers are informed knowing if products meet their needs (without search). The remaining $1 - \mu$ of consumers are uninformed. An uninformed consumer initially does not know if a product meets her need but can learn it by incurring a search cost s per product. However, both informed and uninformed consumers can observe prices.[2] The structure of the model is similar to Stahl (1989), which assumes a proportion of consumers, called *shoppers*, are informed of prices and the other consumers search costly for price information. Here, we assume some consumers (informed consumers) know if products match their needs, perhaps from previous experiences, and the others are inexperienced consumers (uninformed consumers) who have to spend time and effort to review products.[3]

[1] This could arise when consumers have specific needs and the available products are so broad that consumers have to verify if a product can satisfy her need. Alternatively but related, one can think of a product as a bundle of characteristics. Each consumer only values a few product attributes and is satisfied as long as a product embeds the wanted characteristics. For instance, a consumer may only need one or few of functions in a (bundled) software. A software with these functions matches perfectly with the consumers' demand or gives no utility otherwise. Chen and He (2006) and Athey and Ellision (2009) also make an assumption that firms provide products that either perfectly match consumers' preferences or do not match.

[2] Armstrong and Chen (2009) assume all consumers observe price but inattentive consumers do not know product (vertical) quality. They do not consider consumer search in the model.

[3] Alternatively, one may interpret that shoppers in Stahl (1989)'s model are consumers who love to shop. Under this interpretation, the informed consumers in our model are consumers who are interested in knowing each product well.

Since an informed consumer observes all prices and knows if a product matches her preference she will purchase from the matched firm (if any) with the lowest price in the market. An uninformed consumer has to decide a search strategy to maximize her expected payoff. Given that a firm charges price p, an uninformed consumer will search the firm only if the expected payoff is non-negative, that is,

$$\theta(V-p)-s\geq 0$$

or

$$p\leq V-\frac{s}{\theta}.$$

Define

$$r=V-\frac{s}{\theta}. \tag{1}$$

We shall call r as reservation price. Thus, an uninformed consumer will only search a firm if the firm charges a price that is equal to or below the reservation price, that is, $p\leq r$. Next, we consider the order by which uninformed consumers search among firms charging prices below the reservation price. Given that each firm has a same probability θ to match a consumer's preference, uninformed consumers get highest payoff by searching the first with the lowest price in the market. If the firm's product matches her preference, the uninformed consumer will buy from the firm. If the firm's product does not match, the uninformed consumer searches the firm with the second to the lowest price. She purchases if the product matches and continues to search otherwise.

In summary, the uninformed consumer's optimal search strategy is that (1) search order: searching from the firms charging lower prices to the ones charging higher prices; (2) reservation price: deciding a reservation price r such that continue to search if $p\leq r$ and stop searching when $p>r$. (3) buying decision: buy from the first matched firm (the matched firm with the lowest price) if there is any during the search and quit otherwise.

As in the literature, we are interested in the symmetric Nash equilibrium of the model. An equilibrium is a price distribution function $\Phi\left(p\right)$ and a search strategy of an uninformed consumers. In equilibrium, given firms' pricing strategies, consumers adopting optimal search strategy, it is optimal for each firm to choose $\Phi(p)$; and given $\Phi(p)$, the search strategy is optimal for uninformed consumers.

We now turn to analyze firms' price strategy in equilibrium.

3 Equilibrium Price Distribution

Given consumers' strategies described as above we consider firms' strategies. Note that the model has no pure-strategy equilibrium and the equilibrium price distribution $\Phi(p)$ is atomless on its entire support. To see why, note that, in the symmetric equilibrium, if some price p were charged with positive probability, there would be a positive probability of a tie at p. It would be profitable for a firm to deviate to charge a slightly lower price, $p-\varepsilon$, with the same probability with which other firms charged p. This is because the deviant firm would lose profits on order ε, but gain a positive measure of informed consumers when other firms price at p.

In the following, we show that the nature of the equilibrium depends crucially on the magnitude of the search cost, s. We shall divide the possible value of s into three regions: (1) when s is medium $\left[1-\mu(1-\theta)^{N-1}\right]\theta V > s > (1-\mu)\theta V$; (2) when s is small: $s < (1-\mu)\theta V$; (3) when s is large: $s > \left[1-\mu(1-\theta)^{N-1}\right]\theta V$. We consider these cases in turn.

We first consider the case where s is medium.

Proposition 1 *When $\left[1-\mu(1-\theta)^{N-1}\right]\theta V > s > (1-\mu)\theta V$ there exists a symmetric equilibrium, in which each firm prices according to mixed strategy*

$$G(p) = \begin{cases} (1-\alpha)G_h(p) & if \quad r > p > (1-\theta)^{N-1}\mu V \\ 1-\alpha & if \quad \frac{r}{\mu} > p > r \\ 1-\alpha+\alpha G_l(p) & if \quad V \geq p > \frac{r}{\mu} \end{cases} \tag{2}$$

where

$$G_h = 1 - \frac{1-\theta}{\alpha\theta}\left[\left(\frac{V}{p}\right)^{\frac{1}{N-1}} - 1\right] \tag{3}$$

$$G_l = \frac{1}{1-\alpha} - \frac{1-\theta}{(1-\alpha)\,\theta}\left[\left(\frac{\mu V}{p}\right)^{\frac{1}{N-1}} - 1\right] \tag{4}$$

$$\alpha = \frac{1-\theta}{\theta}\left[\left(\frac{\mu V}{r}\right)^{\frac{1}{N-1}} - 1\right] \tag{5}$$

and r is defined as in (1).

Proof. We first verify that $G(p)$ is a c.d.f. Since $G(V) = 1$, $G\left((1-\theta)^{N-1}\mu V\right) = 0$, $G\left(V - \frac{s}{\theta}\right) = G\left(\frac{V-\frac{s}{\theta}}{\mu}\right) = 1 - \frac{1-\theta}{\theta}\left[\left(\frac{\mu V}{V-\frac{s}{\theta}}\right)^{\frac{1}{N-1}} - 1\right]$ and $G(p)$ weakly increases in p, it follows that $G(p)$ is a continuous c.d.f.

We next show that each firm is optimizing following $G(p)$, given that other firms choose prices according to $G(p)$ and uninformed consumers' reservation price is r. Note that a firm can only sell to an (informed and uninformed) consumer if its price is the lowest among the matched firms. In addition, selling to a uninformed consumers requires the price is lower than r to induce a search. The expected profit when a firm chooses p is:

(i) If $p = V$,

$$\pi = V\mu\theta\,(1-\theta)^{N-1} \tag{6}$$

because the firm only sells to the informed consumers who find the firm is the only match.

(ii) If $p = r$ a firm sell to matched consumers (informed or uninformed) who find that the firm has the lowest price. That is, firms with lower prices do not match. Thus, the profit is

$$\begin{aligned}
\pi &= r\theta[(1-\theta)^{N-1} + \tbinom{N-1}{1}\theta\,(1-\theta)^{N-2}\alpha + \tbinom{N-1}{2}\theta^2\,(1-\theta)^{N-3}\alpha^2 \\
&\quad +\ldots + \tbinom{N-1}{i}\theta^i\,(1-\theta)^{N-1-i}\alpha^i + \ldots + \theta^{N-1}\alpha^{N-1}] \\
&= r\theta\sum_{i=0}^{N-1}\tbinom{N}{i}\theta^i\,(1-\theta)^{N-1-i}\alpha^i \\
&= \left(V - \frac{s}{\theta}\right)\theta\,(\theta\alpha + 1 - \theta)^{N-1}
\end{aligned} \tag{7}$$

(iii) if $p \in G_h(p)$ or $p > r$,

$$\begin{aligned}
\pi &= p\mu\theta \sum_{i=0}^{N-1} \binom{N}{i} \theta^i (1-\theta)^{N-1-i} \alpha^i (1 - G_h(p))^i \\
&= p\mu\theta \left[\theta\alpha (1 - G_h(p)) + 1 - \theta\right]^{N-1} \qquad (8)
\end{aligned}$$

because the firm only sells to the informed consumers who find its price is lowest among the matched firms.

(iv) If $p \in G_l(p)$ or $p < r$,

$$\begin{aligned}
\pi &= p\theta \sum_{i=0}^{N-1} \binom{N}{i} \theta^i (1-\theta)^{N-1-i} \left[\alpha + (1-\alpha)(1 - G_l(p))\right]^i \\
&= p\theta \left\{\theta\left[\alpha + (1-\alpha)(1 - G_l(p))\right] + 1 - \theta\right\}^{N-1} \qquad (9)
\end{aligned}$$

because the firm can sell to both informed and uninformed consumers when the its price is lowest among the matched firms.

Equal profit from (i) and (ii) yield (5), Equal profit from (i) and (iii) yield (3). Equal profit from (i) and (iv) yield (4). Therefore, the firm optimizes choosing prices according to $G(p)$.

Finally, note that when $\left[1 - \mu(1-\theta)^{N-1}\right]\theta V > s > (1-\mu)\theta V$, by (5) and (1), we have

$$\begin{aligned}
0 &= \frac{1-\theta}{\theta}\left[\left(\frac{\mu V}{V - \frac{(1-\mu)\theta V}{\theta}}\right)^{\frac{1}{N-1}} - 1\right] \\
&< \alpha < \frac{1-\theta}{\theta}\left[\left(\frac{\mu V}{V - \frac{\left[1-\mu(1-\theta)^{N-1}\right]\theta V}{\theta}}\right)^{\frac{1}{N-1}} - 1\right] = 1.
\end{aligned}$$

This completes the proof. ■

Interestingly, in contrast to the existing literature where price distributions are unclustered, the price distribution here is clustered. In particular, it consists of two separate cumulative distributions, $G_h(p)$ and $G_l(p)$, playing respectively with probability α and $1-\alpha$. Moreover, a gap exists between the lower support of $G_h(p)$ and the upper support of $G_l(p)$. In equilibrium, with probability α, each firm will price above r according to c.d.f. G_h, and in doing so it targets the matched informed consumers. With probability $1-\alpha$, each firm will price below r according to c.d.f. G_l, trying to selling to both informed and uninformed consumers. Given the reservation price r of the uninformed consumers, a firm is guaranteed to sell to at least $\theta(1-\theta)^{N-1}$ of them if pricing at r whereas with a slight increase of the price above r it will lose sales to all the uninformed consumers. Thus, the lower limit of support for G_h, which achieves the same expected profit as r must be discretely higher than r: when raising its price above r, a firm's demand jumps down, which must be exactly offset by a jump-up of the price so that the firm's expected profit remains the same. Consequently, an interval of prices on the support of the equilibrium distribution G will be played with zero probability.[4]

By (5), α increases as r decreases. Firms play more frequently with the high price distribution, G_h, focusing on the informed consumers. Moreover, by (3), (4) and (5), we have

$$\frac{\partial G_h}{\partial r}=\frac{\partial G_h}{\partial \alpha}\cdot\frac{\partial \alpha}{\partial r}<0$$

and

$$\frac{\partial G_l}{\partial r}=\frac{\partial G_l}{\partial \alpha}\cdot\frac{\partial \alpha}{\partial r}<0$$

Thus, as r decreases, both price distributions, G_h and G_l increases stochastically. It follows that the average prices of both G_h and G_l are lower. However, from the above discussion, it is not clear how the overall market price are affected. We will give a definite answer to this question section 4.

[4] Chen and Zhang (2009) also find mixture equilibrium price distribution, but for a rather different reason. In their model, firms selling homogenous products to shoppers, local searchers who buy at first random search and global consumers who search optimally. Their mixture price distribution arises when the local searchers' valuation is sufficiently high.

Substituting (3), (4) and (5) into (2), the price distribution can be rewritten as

$$G(p)=\begin{cases} 1-\frac{1-\theta}{\theta}\left[\left(\frac{\mu V}{p}\right)^{\frac{1}{N-1}}-1\right] & if \quad V-\frac{s}{\theta}>p>(1-\theta)^{N-1}\mu V \\ 1-\frac{1-\theta}{\theta}\left[\left(\frac{\mu V}{V-\frac{s}{\theta}}\right)^{\frac{1}{N-1}}-1\right] & if \quad \frac{V-\frac{s}{\theta}}{\mu}>p>V-\frac{s}{\theta} \\ 1-\frac{1-\theta}{\theta}\left[\left(\frac{V}{p}\right)^{\frac{1}{N-1}}-1\right] & if \quad V\geq p>\frac{V-\frac{s}{\theta}}{\mu} \end{cases} \tag{10}$$

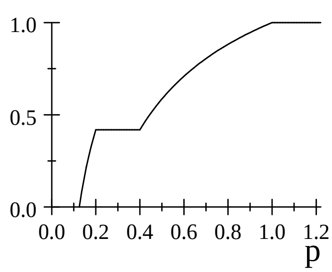
and its p.d.f. is :

$$g(p)=\begin{cases} \frac{1}{N-1}\frac{1-\theta}{\theta p}\left(\frac{\mu V}{p}\right)^{\frac{1}{N-1}} & if \quad V-\frac{s}{\theta}>p>(1-\theta)^{N-1}\mu V \\ 0 & if \quad \frac{V-\frac{s}{\theta}}{\mu}>p>V-\frac{s}{\theta} \\ \frac{1}{N-1}\frac{1-\theta}{\theta p}\left(\frac{V}{p}\right)^{\frac{1}{N-1}} & if \quad V\geq p>\frac{V-\frac{s}{\theta}}{\mu} \end{cases} \tag{11}$$

The example below illustrates the price distribution in Proposition 1.

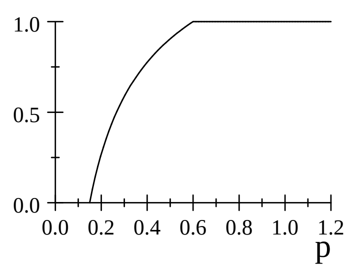
Example 1: Suppose that $N=3, V=1, \theta=0.5, \mu=0.5$. Then $\left[1-\mu(1-\theta)^{N-1}\right]\theta V=0.44$ and $(1-\mu)\theta V=0.25$. Let $s=0.4$. We find $\alpha=0.\,58$. Thus,

$$G_h(p)=2.\,720\,8-1.\,720\,8\sqrt{\frac{1}{p}} \qquad \text{and} \qquad G_l(p)=4.\,774\,9-2.\,387\,4\sqrt{\frac{0.5}{p}}$$



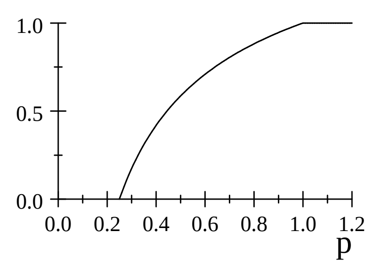
$G(p)$

Figure 1a



$F(p)$

Figure 1b



$H(p)$

Figure 1c

As shown in Figure 1a, the equilibrium price distribution function stays constant over some intermediate prices. This implies zero probability placed on these prices.

We next discuss the case where s is small.

Proposition 2 *When $s < (1-\mu)\,\theta V$ there exists a symmetric equilibrium, in which each firm prices according to mixed strategy*

$$F(p) = 1 - \frac{1-\theta}{\theta}\left[\left(\frac{r}{p}\right)^{\frac{1}{N-1}} - 1\right] \qquad if \qquad r \geq p \geq (1-\theta)^{N-1} r. \tag{12}$$

Proof. Given $F(p)$, the uninformed consumers search optimally. To show that the proposed is an equilibrium, we thus only need to show that given r and other firms choose $F(p)$, each firm optimizes choosing any $p \in \left[(1-\theta)^{N-1} r, r\right]$. For any such price, the firm's expected profit is

$$\begin{aligned}
& p\theta \sum_{i=0}^{N-1} \tbinom{N}{i} \theta^i (1-\theta)^{N-1-i} (1-G_l(p))^i \\
= \; & p\theta \left[\theta (1-F(p)) + 1 - \theta\right]^{N-1} \\
= \; & p\theta \left[\theta\left(1 - \frac{1-\theta}{\theta}\left[\left(\frac{r}{p}\right)^{\frac{1}{N-1}} - 1\right]\right) + 1 - \theta\right]^{N-1} \\
= \; & r\theta (1-\theta)^{N-1}
\end{aligned}$$

Note that $p > r$ would lead to zero sale and any $p < (1-\theta)^{N-1} r$ would result in the same amount of sales as $p = (1-\theta)^{N-1} r$ but at a lower price. Therefore, the firm is maximizing its profit by choosing its price from $F(p)$. ■

When the search cost is sufficiently small, no firm changes a price above r, the reservation price of uninformed consumers. The reason is quite simple. When uninformed consumers incur little costs to search they set the reservation price relatively higher. Accordingly, firms are able to price higher but still induce the uninformed consumers to search. When s is sufficiently low that r is sufficiently close to V, firms' price strategy is driven by the consideration of the reservation price of the uninformed consumers.

In this equilibrium, a uninformed consumer continues to search until she finds a match or oxalises searching all firms. Since the uninformed consumers optimally start searching from the firm with the lowest prices, uninformed consumers pay the same expected price as informed consumers do.

The example below illustrates the price distribution in Proposition 2.

Example 2: Everything is the same as in *Example 1* except letting $s = 0.2$. The equilibrium price distribution is

$$F(p) = 2.0 - 1.0\sqrt{\frac{0.6}{p}} \quad if \quad 0.6 \geq p \geq 0.15$$

shown in Figure 1b.

We finally turn to the case where s is large

Proposition 3 *When* $s > \left[1 - \mu(1-\theta)^{N-1}\right]\theta V$ *there exists a symmetric equilibrium, in which each firm prices according to mixed strategy*

$$H(p) = 1 - \frac{1-\theta}{\theta}\left[\left(\frac{V}{p}\right)^{\frac{1}{N-1}} - 1\right] \qquad if \qquad V \geq p \geq (1-\theta)^{N-1} V \tag{13}$$

Proof. For $\left[1 - \mu(1-\theta)^{N-1}\right]\theta V$, from (1),

$$r < V - \frac{\left[1 - \mu(1-\theta)^{N-1}\right]\theta V}{\theta} = \mu(1-\theta)^{N-1} V < (1-\theta)^{N-1} V.$$

Thus, given $H(p)$, the uninformed consumers do not search. To show that the proposed is an equilibrium, we thus only need to show that given r and other firms choose $H(p)$, each firm optimizes choosing any $p \in \left[(1-\theta)^{N-1} V, V\right]$. For any such price, the firm's expected profit is

$$
\begin{aligned}
& p\mu\theta \sum_{i=0}^{N-1} \binom{N}{i} \theta^i (1-\theta)^{N-1-i} (1-G_l(p))^i \\
= \;& p\mu\theta \left[\theta (1-H(p)) + 1 - \theta\right]^{N-1} \\
= \;& p\mu\theta \left[\theta \left(1 - \frac{1-\theta}{\theta}\left[\left(\frac{V}{p}\right)^{\frac{1}{N-1}} - 1\right]\right) + 1 - \theta\right]^{N-1} \\
= \;& V\mu\theta (1-\theta)^{N-1}
\end{aligned}
$$

Note that $p > V$ would lead to zero sale and any $p < (1-\theta)^{N-1} V$ would result in the same amount of sales as $p = (1-\theta)^{N-1} V$ but at a lower price. Therefore, the firm is maximizing its profit by choosing its price from $H(p)$. ■

When the search cost is significantly large, the entire price distribution is above the reservation price of uninformed consumers. In other words, firms set prices, expecting no search from the uninformed consumers, to sell to the informed consumers. The intuition is straightforward. The reservation price of uninformed consumers are low due to the high search cost. Thus, a firm has to lower the price to attract the search of the uninformed consumers. However, the profits is so lower that it is better for the firm to price high and sell to the informed consumers.

The example below illustrates the price distribution in Proposition 3.

Example 3: Everything is the same as in *Example 1* except letting $s = 0.5$. The equilibrium price distribution is

$$
H(p) = 2.0 - 1.0\sqrt{\tfrac{1}{p}} \quad if \quad 1 \geq p \geq 0.25
$$

shown in Figure 1c.

Remarkably, $F(p)$ and $H(p)$ can be seen as special cases of $G(p)$ where $\alpha = 0$ and $\alpha = 1$ respectively. Moreover, when $N = 2$, $F(p)$ $(H(p))$ is same in the Rosenthal (1980)'s model with the number of royal consumers is $\theta(1-\theta)$ $(\mu\theta(1-\theta))$ and switching consumer θ^2 $(\mu\theta^2)$.

We next move to comparative statics on search cost.

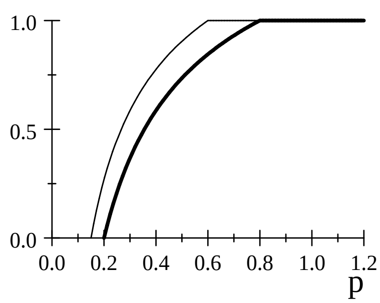
4 Comparative Statics on Search Cost

Our previous analysis shows how the equilibrium price strategies depend on if the search cost is large, medium or small. In this section, we further examine how a marginal change in search cost in different regions affects the equilibrium prices.

We start with the case when s is small. By (1) and (12),

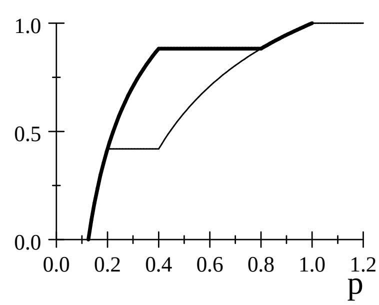
$$\begin{aligned} \frac{\partial F}{\partial s} &= \frac{\partial F}{\partial r} \cdot \frac{\partial r}{\partial s} \\ &= -\frac{1-\theta}{\theta} \left(\frac{r}{p}\right)^{\frac{1}{N-1}-1} \left(-\frac{s}{\theta p}\right) > 0. \end{aligned}$$

Moreover, both the limits of upper bound and lower bound of F decrease in s because of $\frac{\partial r}{\partial s} < 0$. Therefore, the price distribution $F(p)$ stochastically decreases, thus, the equilibrium prices are stochastically higher as s decreases. Figure 2a illustrates the change of $F(p)$ for a decrease in search cost.



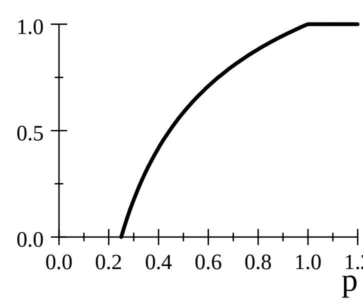
thin: $s = 0.2$; thick: $s = 0.1$

Figure 2a



thin: $s = 0.4$; thick: $s = 0.3$

Figure 2b



$s = 0.6, s = 0.5$: same curve

Figure 2c

Common Parameter Values: $N = 3, V = 1, \theta = 0.5, \mu = 0.5$

Proposition 4 *When s is small, that is $s < (1-\mu)\theta V$, a decrease in s (i) increases* *the equilibrium price stochastically; (ii) increases firms' profits; and (iii) increases total welfare.*

When s is small, firms choose a price strategy with all prices below the uninformed consumers reservation price, r. By (1), as s decreases, r increases. This allows firms charge higher prices resulting in a higher profit. Note that there is no welfare loss as long as consumers with at least one match will find her matched products and purchase. This is true in this equilibrium. Moreover, there is a reduction in search cost. Therefore, total welfare increases. Note that the change of consumer welfare is not clear. On one hand, the lower search cost benefits consumers. On the other, equilibrium prices increase which lowers consumer welfare.

When s is medium, by (10), $G(p)$ is weakly increasing in s. Moreover, the limits of upper bound, $\frac{V-\frac{s}{\theta}}{\mu}$ and lower bound, $V-\frac{s}{\theta}$, increase in as s decreases. Therefore, the price distribution $G(p)$ stochastically increases, thus, the equilibrium prices are stochastically lower as s decreases. Figure 2b illustrates the change of $G(p)$ for a decrease in search cost.

Proposition 5 *When s is medium, that is $\left[1-\mu(1-\theta)^{N-1}\right]\theta V > s > (1-\mu)\theta V$, a decrease in s (i) decreases the equilibrium prices stochastically; (ii) increases consumer surplus; (iii) does not affect firms' profits; and (iv) increases total welfare.*

When s is medium, firms adopt a clustered price distribution: randomly playing a high price distribution with prices above r and a low price distribution with price below r. As before, r increases as s decreases. However, in this case, firms will lower the played probability associated with the high price distribution as r increases. Interestingly, the shape of high and low price distributions remains the same. The reason is that some consumers will drop out of the market. However, those consumers are the ones who do not find a match from the firms with lower prices. Hence, their drop-outs do not affect the behaviors of the firms charging relatively lower prices. Moreover, the behavior of the firms charging high prices are also the same. As a consequence, prices stochastically decrease as search costs fall. Moreover, a firm can always guarantees $\theta(1-\theta)^{N-1}V$ of profit by selling to informed consumers who find it the only match. Thus, firms profit does not decrease. In addition, as search costs fall, there is a welfare gain which comes from the fact that firms play more frequently with the low price distribution allowing some informed consumers search and purchase who previously do not due to high prices.

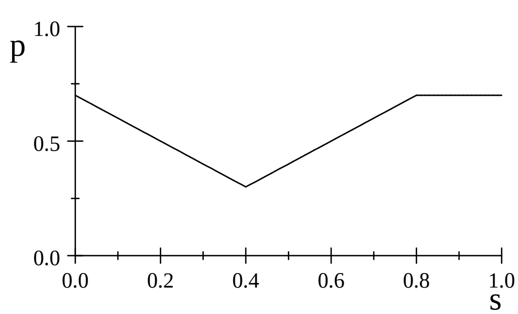
When s is large, by (13), it is straightforward to show the following.

Proposition 6 *When s is large, that is $s > \left[1-\mu(1-\theta)^{N-1}\right]\theta V$, a decrease in s does not affect the equilibrium prices, consumer surplus, firms' profits and total welfare.*

In this region, firms adopt high price strategies targeting informed consumers. The uninformed consumers do not search due to the high search cost. Thus, a small reduction in search cost does not affect the price distribution. Hence, consumer surplus, profits, and total welfare are invariant.

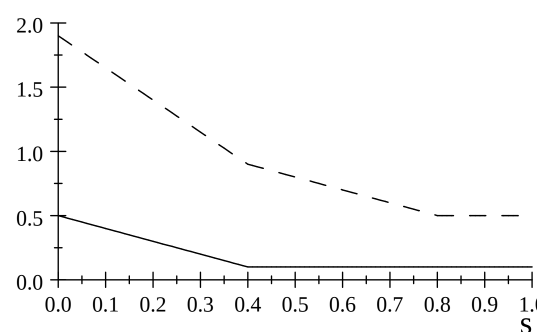
We now have a complete picture of how a change in search cost affects the equilibrium price, firms' profits and total welfare.

Proposition 7 *Starting with an arbitrarily small search cost, as search cost increases, (i) the equilibrium market price first decreases, then increases and finally stabilizes; (ii) firms' profit weakly decreases; and (iii) total welfare weakly decreases.*



Market price changes in search cost

Profit-solid line;Welfare-dotted line



Profit and Welfare change in search cost

Several points that are worth to notice. First, equilibrium prices are non-monotonic in search cost and reach their minimum at some intermediate search costs. Second, since the welfare of informed consumers is positively correlated with the expected minimum market price it follows that a decrease in uninformed consumers' search cost may exert positive, negative or none externality to informed consumers. Third, firms earn more profit with a lower consumer search cost. In fact, maximum profit is obtained when uninformed consumers have zero search cost. Equivalently, profit maximizes when consumers are all informed.

5 Conclusion

In many situations, consumers need to search costly for goods or services to meet their needs. The search often is conducted in a non-random order. This paper presents a model with endogenous consumer search order. The rapid development of Internet allows consumers to have easy access of price information before searching for the desired product. In the model, prices affect firms' profits directly and indirectly via influencing the order in which consumers search. Firms charging lower prices appear on the top of the consumers' search order, and thus, make more sales. We show that the pattern of equilibrium price distribution depends on the size of search cost. In particular, mixture price distribution occurs when the search cost is medium while standard continuous price distribution appears when the search costs are high or low. We find that a decrease in search cost may increase, decrease or have no impact on equilibrium prices depending on the initial size of search cost; and market price minimizes at some medium size of search cost. Moreover, a decrease in search cost weakly increases firms' profits.

For future research, it would be interesting to compare a price-directed search model as in this paper to a random search model where consumers search costly for both price and product information. In the comparison, one could examine how price patterns and the impact of search costs on welfare differ in these models.

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