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## 'Wave riding' or 'Owning the issue': How do candidates determine campaign agendas?

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# Wave riding or Owning the issue: How do candidates determine campaign agendas? 

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#### Abstract

In this paper I address the question of how the agendas for political campaigns are determined, which issues candidates discuss, and whether or not candidates discuss similar issues. Two candidates compete for the votes of four groups of voters by choosing how to allocate their time across two different issues. Candidates' positions are fixed, and their most preferred policies will be implemented after the candidate is elected. Each candidate has a unit of time to clarify his position on both issues. The amount of time spent by a candidate discussing an issue will affect the level of uncertainty regarding a candidate's policy on that issue among the voters.

Both voter distribution and issue importance affect the outcome of the election. Voter distribution determines which candidate will have an advantage in the election, and issue importance determines the minimum amount of time that a candidate with the advantage has to devote to the most important issue in order to win the election. I find that in most cases, candidates are willing to discuss both issues to a certain degree, and dialogue between candidates is possible. Only when candidates disagree on both issues, which are equally important to the voters, each candidate discusses the issue upon which he agrees with the decisive group of voters.


## 1 Introduction

Political campaign agendas substantially differ from election to election, though some issues are discussed in almost every election. The state of the economy, taxes, and national defense were widedly debated during all presidential races in recent decades, but their importance substantially differed. For example, Sigelman and Buell (2004) showed that national defense was an important issue during the 1960 presidential election, while the economy and social security were rarely mentioned. On the other hand, in the 2000 presidential race between Al Gore and George W. Bush, health care, social security, and taxes were the issues of most importance, while national defense was rarely mentioned. Souley and Wicks (2005) found that war and terrorism received great attention in the 2004 campaign, and education, social security and medicare received less attention. Also, new issues arise from time to time, while some issues are dropped. Currently the abortion and gay marriage are getting more and more attention, and farm policy, are no longer discussed by candidates.

The purpose of this study is to develop a new theoretical model that explains how agendas for political campaigns are being determined, which issues are being mentioned by the competing candidates the most, and whether or not candidates discuss similar issues. First, I argue that candidates respond to the issues with which voters are concerned the most by devoting a certain amount of resources to the discussion of those issues. Second, I investigate what effect voter distribution has on candidates' strategies and on the outcome of the election.

While some papers (Aragones, 1999, Berliant, 2005) argue that candidates choose to deliver ambiguous messages to the voters, I assume that ambiguity is the result of time limitations faced by the candidates. I model the election with two issues and two policy motivated candidates who cannot lie about their preferences. Initially, voters are unaware of the candidates' most preferred policies and learn about them from the candidates' speeches. Both candidates have enough time to fully clarify their position on only one issue, or they can choose to discuss both issues to a certain extent. Depending on the time devoted to each issue, voters update their beliefs regarding candidates' preferences. Regardless of the candidates' strategies, at election time voters do not know with certainty candidates' preferences for every issue and they place their vote for one of the candidates based on their beliefs regarding the candidate's policies.

I find that candidates' strategies and election outcomes depend on both voter distribution and issue importance. Voter distribution determines whether or not one of the candidates will have an advantage against his rival. If candidates disagree on both issues, then the candidate with the advantage has to spend some minimum time discussing the issue that the public considers the most important in order to win the election. When candidates disagree on only one issue, the candidate with the advantage always mentions the issue upon which the candidates disagree. The time spend discussing that issue is determined by the level of issue importance.

## 2 Related literature

A broad range of existing literature on elections and campaigns is mainly concerned with the position of the candidate once the platform is established and the issues to be discussed are selected. This paper investigates how candidates select the issues for discussion and which factors affect their choice.

In the last few decades, two different theories of issue selection have emerged. The theory of issue ownership states that candidates will only discuss the issues which are better handled by their own party in the public opinion (Budge and Farlie 1983; Petrocik 1996, Sellers 1998, Holian 2004, Kaufmann 2004). The other theory, the so called "wave riding", theory, proposed by Ansolabehere and Iyengar 1994, posits that candidates will pay more attention to the issues salient to the public, disregarding their ability to handle that issue. Both theories were tested empirically, and both theories found their supporters and opponents. Few studies suggested that both issue salience and issue ownership influences voters' choice (RePass 1971, Mayer and Tiberj 2004, Belanger and Meguid 2004), yet, most scholars defend either one theory or another. This paper unifies both theories and identifies the conditions that give support to each.

Budge and Fairlie (1983) explained the differences in the elections in terms of the issues of interest. They argued that the issues and the posture of the parties have a long term and stable relationship, and thus vote-maximizing candidates will choose the issues for discussion that are salient to the public. At the same time, their parties are considered to be the most competent regarding those issues. This theory was further investigated by Petrocik (1996), who showed that candidates tend to emphasize the issues owned by their party much more than the issues owned by the competing party. Both analyses also predict some dialogue in the campaign, on the issues that are not "owned" by the party, such as performance issues (e.g. government performance, foreign affairs, national economy, and national security). Also, an analysis of candidates' television advertisements and nomination speeches in the presidential elections from 1952 through 2000 showed that both parties tend to discuss more "Republican"-owned issues in any campaign, with exception of the Bush - Gore campaign where both candidates talked more about issues owned by the Democrats (Petrocik et. al. 2003). The same was true in 2004 campaign by Bush and Kerry, where war and terrorism (both issues "owned" by Republican party) were discussed by both candidates the most, while traditional issues like education, social security and Medicare recieved little attention (Souley and Wicks, 2005).

Simon (2002) presented an empirical analysis and a theoretical model that analyzed issue selection by the candidates. He used the concept of dialogue to investigate the candidates' behavior. To engage in a dialogue, a candidate must respond to the claims made by the opponent, not ignore them. In the game, two candidates compete in the multidimensional policy space for a majority of votes. A candidate's position on particular issue is fixed, known to the electorate, and depends on his party affiliates. The time devoted to discussion of the particular issue is proportional to the amount of money spent to make that discussion possible. In such a framework, no dialogue exists. Each candidate will choose
themes that increase his advantage by informing voters on his position, instead of defending himself on the losing position. The possibility of dialogue arises when one of the candidates is lying or has close to unlimited amount of resources. One of the major assumptions in such a framework is that the importance of each issue is determined by the total spending on that issue. In this paper, instead of assuming that candidates' budget allocation is the determinant of issue importance, I assume that issue importance is an exogenous variable, which affects the time spent by the candidate on each issue.

The second theory, also known as the "wave riding" theory states that instead of focusing on the issues traditionally "owned" by their party, candidates concentrate on the issues that voters consider to be of the greatest importance. Sides (2006) analyzed 1998 House and Senate campaigns and argued that issues identified by voters as the most important influence candidates' agendas, but do not fully explain the differences in the campaigns of the two candidates within a given election. Sigelman and Buell (2004) in their study showed the existence of issue convergence in political campaigns. Kaplan et. al., (2006) examined the issue convergence in candidates' television advertising and found that competing candidates adopt similar campaign agendas, and when more money is allocated for the campaign, more similar issues are being discussed by competing parties. Another finding of the study showed that regardless of issue ownership, both candidates devote more resources to the issues that are more important to the public. This finding coincides with that of Ansolabehere and Iyengar (1994), who argue that during the campaign, candidates address the issues with which the public is concerned the most. The authors show that candidates gain by addressing the issues of the most concern, and are penalized if they fail to do so.

RePass showed that voters Belanger and Meguid 2004, argued that issue ownership plays and important role in voters' decision making process, but only for the voters who think that issue is important. had similar

Finally, this paper relates to the literature on ambiguity in electoral competition. Most authors assume that ambiguity is created by candidates in order to appeal to a broader range of voters by using a one-dimensional framework. Alesina and Cukierman (1990) assumed that candidates are office and policy motivated and might take an ambiguous policy in order to hide their true preferred policy. Aragones and Postlewaite (2002) analyzed how candidates use ambiguity to their advantage in an election with rational voters. The authors consider a one issue election with several alternatives, where voters' beliefs affected by the campaign statements. They define the conditions under which candidates choose to deliver ambiguous statements and by doing so increase the number of voters to whom they appeal. Laslier (2003) proposed a model that explains why ambiguity is present in the elections with voters that dislike ambiguity. Berliant and Konishi (2005) moved away from the one dimensional election and developed a model where office motivated candidates freely choose their positions on any of the issues and simultaneously announce them. Candidates are not aware of voter preferences at the stage of platform announcement, and voters are not aware of the candidates' positions on the issues that were
not discussed in their policy announcement. In such a framework, candidates will announce their policy on every issue.

I take a different approach and assume that ambiguity is a result of resource limitations faced by the candidates. Instead of determining whether ambiguity will exist in the election and why, I investigate what shapes candidates' agendas, which issues candidates will discuss the most and which issues will get little attention under the time constrains. Candidates are policy motivated and declare their preferred policies, but because of the time limitations cannot fully clarify them, though they choose what time to devote to discussion of each issues. Each candidate can focus on one of the issues, or equally discuss both issues, which will affect voters' beliefs about the policies each candidate will implement. A two dimensional framework is used in order to identify the conditions which determine which issues will be mentioned in election and to which extent.

## 3 The model

There are two candidates, $C_{k}, k=1,2$, who compete in the election. There are two issues, $Z_{i}, i=1,2$, and two policies for each issue. The position of each candidate and each voter can be represented as a pair $\left(X_{1}, X_{2}\right), X_{i} \in$ $Z_{i}$, where $Z_{1}=\{A, B\}, Z_{2}=\{E, D\}$. Candidates' positions are fixed, and their most preferred policies will be implemented after the candidate is elected. Each candidate has a unit of time to declare his position on both policies. The fraction of time spent by each candidate on discussion of issue $Z_{1}$ is denoted by $p_{k} \in[0,1]$.

There are four groups of voters, and the fraction of voters in the group $j=1, \ldots 4$, is denoted by $v_{j}$, and the total mass of voters is normalized to unity: $\sum_{j=1}^{4} v_{j}=1$.

Each voter has a single most preferred policy set, and preferences over issues are independent. I refer to a voter as partisan if his most preferred policy is the same as the policy set proposed by one of the candidates.

The voter's utility function, $U_{j}$, is the sum of utilities he gets from each implemented policy. The specifications of utility function are based on one used by Aragones and Postlewaite (1999). The utility function is normalized in such way that it assigns a value one to the most preferred alternative and a value of $\alpha \in(0,1)$ to the least preferred alternative. The total utility function of each voter can be represented as:

$$
U_{j}\left(x_{1}, x_{2}\right)=V_{1}\left(x_{1}\right)+V_{2}\left(x_{2}\right), \text { and } V_{i}: Z_{i} \longrightarrow\left\{\alpha_{i}, 1\right\}, \text { where } \alpha_{i} \in(0,1)
$$ represents the intensity of the voter's preferences.

Each group of voters has following preferences:
Group 1: $V_{1}(A)>V_{1}(B), V_{2}(E)>V_{2}(D)$
Group 2: $V_{1}(B)>V_{1}(A), V_{2}(E)>V_{2}(D)$
Group 3: $V_{1}(B)>V_{1}(A), V_{2}(D)>V_{2}(E)$
Group 4: $V_{1}(A)>V_{1}(B), V_{2}(D)>V_{2}(E)$.

The issue intensity can be treated as the indicator of relative issue importance to the voters. As $\alpha_{i} \longrightarrow 1$ the difference between utility from best alternative and worst alternative $\left(1-\alpha_{i}\right)$ is minimal, and as a result the voter might not care about the issue as much, as both alternatives are equally satisfying to him. If $\alpha_{i} \longrightarrow 0$, the difference between the best and next best alternative increases and voter will care more about his most preferred policy to be implemented.

The issue $Z_{i}$ is more important than the issue $Z_{-i}$ if $\alpha_{i}<\alpha_{-i}$. Parameter $\frac{1-\alpha_{i}}{1-\alpha_{-i}}$ represents relative issue importance. It shows by how much one issue is more important than the other. When $\frac{1-\alpha_{i}}{1-\alpha_{-i}} \longrightarrow 1$, issues become more equal in their importance to the voters.

The specification of the utility function might seem restrictive, but even when the upper bound of utility function is not limited by 1 the model produces exactly the same results. This states that the absolute importance of the issue is not important, it is the relative issue importance that drives the results of this paper.

I assume that all voters are alike in terms of issue intensities, or in other words all voters agree on which issue is more important and which issue is not, or everyone agrees that two issues are similarly important.

Candidates know the voters' most preferred policies, but the voters are not aware of candidates' positions and learn about candidates' most preferred policies from candidates' speeches. By discussing issues candidates clarify their position on those issues and reduce the uncertainty observed by the voters. If candidate spends $p$ time discussing issue $Z_{1}$ voters believe that that candidate will implement his most preferred policy with probability $f(p)>0.5$.

Assumption 1 The belief function $f(p)$ is strictly increasing function with $f(0)=0.5$ and $f(1)=1$.

Assumption 2 The belief function $f(p)$ is concave, $f^{\prime \prime}(p)<0$.
The first assumption is a standing assumption for the rest of the paper. It says that if a candidate spends more time discussing one of the issues, voters learn more about candidate's true position on that issue and update their beliefs accordingly. If candidate spends no time at all discussing the issue, voters have no information regarding the candidate's position on the issue and believe either policy can be implemented with equal probability. The second assumption implies that each unit of time spent by the candidate will bring less clarification to the policy than the previous. One interpretation of this result is that explaining last details of proposed policy might require more time than explaining which direction the policy is heading.

A candidate wins the election if he obtains more votes than his rival. Voters are expected utility maximizers and vote for a candidate based on their beliefs.

Definition 1 An equilibrium is a set of candidates' strategies $\left(p_{1}^{*}, p_{2}^{*}\right)$ where $p_{k}^{*} \in[0,1]$.

The game is divided into two stages. In the first stage, both candidates simultaneously decide how much time to devote to each issue. In stage two voters update their beliefs and vote for their most preferred candidate.

The first part of the analysis is devoted to the elections where candidates have completely different issue preferences. This assumption contradicts the classical Downsian model of political competition (Hotelling 1929, Downs1957, Black 1958), but some scholars (Glaeser et.al. 2005,) have showed that candidate's convergence is not guaranteed under assumptions different from median voter theorem.

In the second part, candidates agree on one of the issues, but disagree on the other one. I refer to the issues upon which candidates agree as a common issue. Each part is further divided into two cases with different voter distribution.

### 3.1 Candidates have opposite issue preferences

Assume that candidates' positions are different in every dimension. More specifically, if elected, candidate $C_{1}$ will implement policy set $(A, E)$ and candidate $C_{2}$ will implement policy set $(B, D)$. When candidate $C_{1}$ spends $p_{1}$ time discussing issue $Z_{1}$, the voters believe that policy $A$ will be implemented with probability $f\left(p_{1}\right)$ and that policy $B$ will be implemented with probability $1-f\left(p_{1}\right)$. At the same time, candidate $C_{1}$ has $1-p_{1}$ time left to discuss the issue $Z_{2}$, and thus voters believe that policy $E$ will be implemented with probability $f\left(1-p_{1}\right)$, and policy $D$ will be implemented with probability $1-f\left(1-p_{1}\right)$. Candidates' and voters' locations are presented in Figure 1.

Voters in group 1 have exactly the same preferences as candidate $C_{1}$ and voters in group 3 have exactly the same preferences as candidate $C_{2}$, thus each candidate has a partisan group of voters. Voters in group 2 have their most preferred policy on issue $Z_{1}$ matched with the policy proposed by candidate $C_{1}$, while their most preferred policy on issue $Z_{2}$ matches the policy proposed by candidate $C_{2}$. Voters in group 4's most preferred policy on issue $Z_{1}$ matches the policy proposed by candidate $C_{2}$, while their most preferred policy on issue $Z_{2}$ matches the policy proposed by candidate $C_{1}$.

Figure 1. Candidates' and voters' location


Note that voters from groups 1 and 3 never vote for the same candidate, unless they get exactly the same utility from voting for candidates $C_{1}$ and $C_{2}$. Then the voters from those groups are indifferent between candidates. The same holds for voters from group 2 and group 4.

There is a continuum of possible voter distributions, but all those cases can divided into four groups. First, I consider the distribution in which one group of voters decides the outcome of the election. Consider the following example. In an election with two competing candidates (a Democrat and a Republican), and four groups of voters, candidates disagree on whether or not taxes should be increased and whether or not gay couples should be allowed to marry. Each candidate has a partisan group of voters, and each one of those groups has a total mass of $15 \%$. Now, suppose that the group which agrees with a Democrat on the tax issue and disagrees on the gay marriage issue has a total mass of $34 \%$ and the group that agrees with republican on the tax issue and disagrees on the gay marriage issue has a mass of $36 \%$. Whichever candidate obtains the votes from the later group of voters will win the election.

What if the preferences of some Democrat partisans change with time? Suppose now $2 \%$ of voters who supported Democrat on both issues, now agree with republican on the gay marriage issue. Now, Democrat has $13 \%$ of partisans, Republican still has $15 \%$ of partisans, and each non partisan groups now has total mass equal to $36 \%$. If the Republican candidate receives the votes from either non partisan group of voters, he will win the election. This is an example of the second distribution I discuss in this paper. Voters are distributed in a way that one of the candidates can win the election if he obtains the votes from either one of non partisan groups of voters.

By symmetry, the other two distributions are similar to the first or the second distributions due to the symmetry. The first distribution is described in cases 1 and 3 and the second distribution is described in cases 2 and 4.

Case 1 If elected, candidate $C_{1}$ implements the policy set $(A, E)$ and candidate
$C_{2}$ implements the policy set $(B, D)$. Voters are distributed in such way that $\nu_{1}+\nu_{2}>0.5$ and $\nu_{2}+\nu_{3}>0.5$.

To win the election, a candidate must obtain more than a half of all the votes. Candidates have no preferences over voters, thus they do not care which group of voters vote for them. All they care about is winning the election so they can implement their most preferred policies. As previously stated, group 1 always votes for candidate $C_{1}$, and voters from group 3 unconditionally vote for candidate $C_{2}$. Thus, whichever candidate obtains the votes from group 2 wins the election. For example, if fraction of voters from group 1 and group 2 is larger than one half (which means total voter fraction from groups 3 and 4 is smaller than one half) and at the same time fraction of voters from group 2 and group 3 is larger than one half (which means total voter fraction from groups 1 and 4 is smaller than one half), each candidate can win the election by obtaining the votes from group 2 .

Define $\bar{p}$, such that $\frac{f(1-\bar{p})-0.5}{f(\tilde{p})}=\frac{1-\alpha_{1}}{1-\alpha_{2}}$ as the maximum time spent on issue $Z_{1}$ and $\hat{p}$, such that $\frac{f(1-\hat{p})}{f(\hat{p})-0.5}=\frac{1-\alpha_{1}}{1-\alpha_{2}}$ as the minimum time spent on issue $Z_{1}$.

Propostion 1 Assume Case 1. In the equilibrium:
(a) if $\frac{1-\alpha_{1}}{1-\alpha_{2}}<1$, then $p_{1}^{*}<\bar{p}, p_{2}^{*} \in[0,1]$, and voters from groups 1 and 2 vote for candidate $C_{1}$, who wins the election;
(b) if $\frac{1-\alpha_{1}}{1-\alpha_{2}}>1$, then $p_{2}^{*}>\hat{p}, p_{1}^{*} \in[0,1]$, and voters from groups 2 and 3 vote for candidate $C_{2}$, who wins the election;
(c) if $\frac{1-\alpha_{1}}{1-\alpha_{2}}=1$, then $p_{1}^{*}=0, p_{2}^{*}=1$, and voters from group 1 vote for candidate $C_{1}$, voters from group 3 vote for candidate $C_{2}$ and voters from groups 2 and 4 are indifferent between candidates and each candidates wins the election with a certain probability.

Recall that voters from group 2 determine the outcome of this election, and their most preferred policy set is $(E, B)$. Thus those voters have the same preferences over issue $Z_{2}$ as candidate $C_{1}$ and same preferences over issue $Z_{1}$ as candidate $C_{2}$.

In the set up of Case 1, a single determinant of candidates' strategies is the relative importance of the issues for the voters in group 2. It does not matter which issue is more important to voters from groups 1 or 3 , as those voters are partisans and their dominant strategy is to vote for the candidate they affiliate with. In fact, even if issue $Z_{1}$ is of most importance to groups 1,3 , and 4 , and issue $Z_{2}$ is more important to voters in group 2 , issue $Z_{1}$ might never be brought up by candidate $C_{1}$, but he will still win the election.

The issue intensities determine which candidate will have an advantage in the election. Henceforth, I will refer to a candidate as a favorite candidate if in the equilibrium that candidate wins the election. If both candidates can win the election with equal probability, that election does not have a favorite candidate. Candidate $C_{1}$ is a favorite candidate in election where $\alpha_{1}>\alpha_{2}$, candidate $C_{2}$ is
a favorite in the elections where $\alpha_{1}<\alpha_{2}$, and in election where $\alpha_{1}=\alpha_{2}$ there are no favorite candidates.

If issue $Z_{2}$ is more important than issue $Z_{1}$, it is in candidate's $C_{1}$ power to convince the voters that he will implement policy $E$ on the issue $Z_{2}$, which is more important to the voters than the implementation of policy $B$ on the issue $Z_{1}$, that they might learn candidate $C_{2}$ will implement after the election. In order for candidate $C_{1}$ to convince voters that policy $E$ will indeed be implemented and win the election, he needs to spend certain amount of time discussing issue $Z_{2}$. The time candidate $C_{2}$ spends discussing the issues is irrelevant in this case, as even if he spends all his time discussing first issue, and convince second group of voters that he will implement policy $B$ on the issue $Z_{1}$, candidate $C_{1}$ would still look more attractive to voters in group 2 simply because second issue is more important.

If issue $Z_{1}$ is more important, candidate $C_{2}$ wins the election if he spends a certain amount of time discussing issue $Z_{1}$. In this case, the voters from group 2 learn that candidate $C_{2}$ will implement policy $B$ on the issue $Z_{1}$ which is more important to voters than implementation of policy $E$. The time candidate $C_{1}$ spends discussing the issues is irrelevant in this case.

When the issues are equally important to the voters $\left(\alpha_{1}=\alpha_{2}\right)$, candidate $C_{1}$ spends all his time discussing the issue $Z_{2}$, and candidate $C_{2}$ spends all his time discussing the issue $Z_{1}$. Now, the non-partisan voter knows for sure that candidate $C_{1}$ will implement policy $E$, and policy $B$ will be implemented with $50 \%$ chance. He further knows with certainty that candidate $C_{2}$ will implement policy $B$, and policy $E$ will be implemented with probability 0.5 .

If any of the candidates spends at least some time discussing the other issue, the voter will learn that his most preferred policy set will be implemented with smaller probability, and vote for the other candidate.

The results of proposition 1c are consistent with the issue ownership theory, which states that no dialogue should exist between candidates. If one of the issues is even slightly more important than the other, dialogue is possible, but it is not guaranteed by the equilibrium.

Proposition 1 states that the favorite candidate's strategy depends on the relative issue importance parameter $\frac{1-\alpha_{1}}{1-\alpha_{2}}$. Given Assumption 1, it is easy to show that the minimum time a favorite candidate has to spend discussing the most important issue decreases when the difference between issue intensities increases, and vice versa. This means that if voters are concerned with one of the issues, a favorite candidate spending a little time discussing that issue would be enough to convince voters that their most preferred policy on their most important issue will be implemented by the favorite candidate. If one of the issues is slightly more important, the favorite candidate needs to spend almost all of his time discussing that issue in order to win the election because voters are almost indifferent between issues and thus will be almost indifferent between candidates. This means that in the election where voters are treating the issues as almost equally important, we should see candidates devoting most of their time to a single issue. In the election where one of the issues is much more important to the voters than the other issues, candidates could split their
time more evenly and discuss different issues.
Now assume that voters are distributed in such manner that if either voters from group 2 or group 4 vote for candidate $C_{1}$, he wins the election. The voter distribution is described in the following case.

Case 2 If elected, candidate $C_{1}$ implements the policy set $(A, E)$ and candidate $C_{2}$ implements the policy set $(B, D)$. There are four groups of voters who determine the outcome of the election. Voters are distributed in such way that $\nu_{1}+\nu_{2}>0.5$ and $\nu_{1}+\nu_{4}>0.5$.

As in previous case, in order for the candidate to win the election, he needs to obtain more votes than his rival. With this voter distribution, candidate $C_{1}$ will have an advantage as he can win the election by obtaining votes from either group 2 or group 4 (recall that voters from group 1 always vote for candidate $C_{1}$ ). Candidate $C_{2}$ wins the election if and only if voters from both groups, 2 and 4 , vote for him. But this is not possible, unless they get exactly the same utility from voting for candidate $C_{1}$ and $C_{2}$, in which case the voters from those groups are indifferent between candidates. The following proposition describes the strategies of candidates in this case.

Proposition 2. Assume Case 2. In the equilibrium:
(a) if $\frac{1-\alpha_{1}}{1-\alpha_{2}}<1$, then $p_{1}^{*}<\bar{p}, p_{2}^{*} \in[0,1]$, and voters from groups 1 and 4 vote for candidate $C_{1}$, who wins the election;
(b) if $\frac{1-\alpha_{1}}{1-\alpha_{2}}>1$, then $p_{1}^{*}>\hat{p}, p_{2}^{*} \in[0,1]$, and voters from groups 1 and 2 vote for candidate $C_{1}$, who wins the election;
(c) if $\frac{1-\alpha_{1}}{1-\alpha_{2}}=1$, no pure strategy equilibrium exists.

The proof of Proposition 2 is in Appendix B.
Candidate $C_{1}$ is the favorite candidate if one of the issues is more important than the other. He wins the election by spending a certain amount of time discussing the issue that is more salient to the public. The actions of the other candidate are not relevant in this case.

As in the previous case, issue intensities determine the minimum amount of time the favorite candidate has to spend on the issue that is salient to the public in order to win the election. This result is somewhat consistent with a wave riding theory, which states that candidate has to address the issue with which public is most concerned. Proposition 2 states that the favorite candidate in order to win the election has to address the most important issue. But as in the previous case, the minimum amount of time a favorite candidate devotes to most important issue decreases when the relevant issue importance increases.

Proposition 2 also shows that a favorite candidate does not always want to obtain the votes from the group with the highest number of voters. The following example illustrates such possibility.

Example 1 Suppose that issue $Z_{2}$ is more important to voters than issue $Z_{1}$ and also assume that voters are distributed in such manner where $\nu_{1}=0.29$, $\nu_{2}=0.33, \nu_{3}=16$, and $\nu_{4}=0.22$. Thus, candidate $C_{1}$ wins the election if either voters from group 2 or group 4 vote for him. According to Proposition 2(a) the time candidate $C_{1}$ spends discussing issue $Z_{1}$ is $p_{1}^{*}<\bar{p}$ and wins the election as voters from group 1 and group 4 vote for him. Even though, the total number of voters in group 2 is much greater than the number of voters in group 4, the candidate prefers to obtain the votes from group 4. This holds because issue $Z_{2}$ is more important and if candidate $C_{1}$ spends enough time discussing this issue, voters from group 4 always vote for him, which cannot be said about voters from group 2. This result shows that in some elections a candidate can choose to spend more time discussing the issue upon which he agrees with a smaller group of voters.

### 3.2 Candidates have same issue preferences for one of the issues

In the cases described below I assume that candidates agree on one of the issues proposed for discussion. Candidates' positions are fixed, and their most preferred policies will be implemented after the candidate is elected. Assume that candidate $C_{1}$ 's most preferred position is $(A, E)$ and candidate $C_{2}$ most preferred position is $(B, E)$.

Several possible outcomes are derived from different voter distributions.
Case 3 If elected, candidate $C_{1}$ implements policy set $(A, E)$ and candidate $C_{2}$ implements the policy set $(B, E)$. There are four groups of voters who determine the outcome of the election. Voters are distributed in such way that $\nu_{1}+\nu_{2}>0.5$ and $\nu_{1}+\nu_{4}>0.5$

In this scenario, the partisan voters are less aligned with their candidate and given certain candidate's strategies might vote for the other candidate, as he has the same most preferred policy on one of the issues. Also, in contrast to the previous cases, voters with completely opposite preferences from a certain candidate might still vote for him, depending on the level of ambiguity introduced by the candidate.

For example if both candidates spend all their time discussing the issue they agree upon, then the issue on which they disagree is not discussed at all, and thus, all voters believe that policy $E$ will be implemented by both candidates if they were elected, and policy $A$ or policy $B$ will be implemented with probability $50 \%$. So, candidates look exactly the same to all voters which means that all voters vote for each candidate with equal probability.

Another example illustrates the possibility of voters voting for the candidate with preferences opposite to their group. Suppose that issue $Z_{2}$ upon which the candidates agree is more important. Also, assume that candidate $C_{1}$ spends most of his time on issue $Z_{2}$ and candidate $C_{2}$ spends most of his time on issue $Z_{1}$. Recall that voters from group 4 agree with candidate $C_{1}$ on issue $Z_{1}$ and
disagree with candidate $C_{2}$ on both issues. Now, voters from group 4 believe that issue that they care the most about will not be implemented by candidate $C_{1}$ with greater probability than by candidate $C_{2}$, as position of candidate $C_{2}$ on the issue $Z_{2}$ is more ambiguous. Thus, voters from group 4 will vote for candidate $C_{2}$ rather than for candidate $C_{1}$, even though they do not agree with that candidate on any of the issues.

Even though the possibility of partisan voters voting for another candidate or possibility of voters voting for the candidate with opposite preferences exists, such strategies are not the equilibrium strategies. If a candidate spends certain minimum time discussing the issue that both candidates disagree upon, a partisan voter will realize which candidate he is aligned with and vote for that candidate. Thus, it is a weakly dominated strategy for both candidates to obtain the vote from their partisans, and in equilibrium partisans always vote for their candidate.

Define $\dot{p}$ such that $f\left(p^{\prime}\right)+\frac{1-a_{2}}{1-a_{1}} f\left(1-p^{\prime}\right)=\frac{1}{2}+\frac{1-a_{2}}{1-a_{1}}$, and $\hat{p}$, such that $\frac{f(1-\hat{p})}{f(\hat{p})-0.5}=$ $\frac{1-\alpha_{1}}{1-\alpha_{2}}$.

Proposition 3 Assume Case 3. Given Assumption 2, in the equilibrium:
(a) If $\frac{1-\alpha_{1}}{1-\alpha_{2}}<1$, then:
if $\hat{p}<\dot{p}$ then $\hat{p}<p_{1}^{*}<\dot{p}, p_{2}^{*} \in[0,1]$ and voters from groups 1 and 4 vote for candidate $C_{1}$, who wins the election;
if $\hat{p}>\not{ }^{p}$ then the equilibrium does not exist;
(b) If $\frac{1-\alpha_{1}}{1-\alpha_{2}}>1$, then $p_{1}^{*}>0, p_{2}^{*} \in[0,1]$ and and voters from groups 1 and 4 vote for candidate $C_{1}$, who wins the election;
(c) If $\frac{1-\alpha_{1}}{1-\alpha_{2}}=1$, then $0<p_{1}^{*}<1, p_{2}^{*} \in[0,1]$ and voters from groups 1 and 4 vote for candidate $C_{1}$, who wins the election;

The proof of Proposition 3 is in Appendix C.
Proposition 3 shows that when in election with a common issue, one of the candidates will have an advantage and win the election. Note, that in equilibrium the winning candidate never spends all of his time discussing the common issue.

When the common issue is more important to the public, two outcomes are possible. If common issue is just slightly more important than non common issue, candidate $C_{1}$ will discuss both issues and win the election. By discussing the common issue, $\left(p_{1}^{*}<\dot{p}\right)$ candidate $C_{1}$ makes voters from group 1 realize that he will implement the policy that is most important to them, but at the same time he needs to discuss the non common issue ( $\hat{p}<p_{1}^{*}$ ) in order to show voters from group 4 that he is different from the other candidate, and will implement their most preferred policy on the other issue. When the common issue is much more important than the non common issue, it is harder for candidates $C_{1}$ to convince his partisan voters that he has the same preferred policy on the issue that is more important to them, thus he needs to spend a lot of time discussing
that common issue. At the same time, he needs to devote a lot of time convincing voters from group 4 that the they will get higher utility from candidate $C_{1}$ as they agree with him on at least one of the issues, and thus needs to spend a lot of time discussing the non common issue.

When the non common issue is more important to the public, candidate $C_{1}$ wins the election by spending at least some time discussing that issue in order to obtain the votes from group 4. Partisan voters will vote for their candidate as well.

When the issues are equally important, candidate $C_{1}$ wins the election by spending at least some time discussing first issue in order to obtain the votes from group 4, and spends some time discussing second issue in order to obtain the votes from his partisan voters.

Case 4 If elected, candidate $C_{1}$ implements policy set $(A, E)$ and candidate $C_{2}$ implements the policy set $(B, E)$. There are four groups of voters who determine the outcome of the election. Voters are distributed in such way that $\nu_{1}+\nu_{4}>0.5$ and $\nu_{3}+\nu_{4}>0.5$

Note that candidate $C_{1}$ again has an advantage over candidate $C_{2}$. Voters from group 1 have exactly the same preferences as candidate $C_{1}$ and thus, in most cases would rather vote for him than candidate $C_{2}$. Also, whoever wins the election must obtain the votes from group 4 , whose position on one of the issues matches position of candidate $C_{1}$ and does not match position of candidate $C_{2}$ on any issues.

Proposition 4 Assume Case 4. Given Assumption 2, in the equilibrium:
(a) If $\frac{1-\alpha_{1}}{1-\alpha_{2}}<1$, then:
if $\hat{p}<\dot{p}$, then $\hat{p}<p_{1}^{*}<\dot{p}, p_{2}^{*} \in[0,1]$ and voters from groups 1 and 4 vote for candidate $C_{1}$, who wins the election;
if $\hat{p}>\dot{p}$, then the equilibrium does not exist;
(b) If $\frac{1-\alpha_{1}}{1-\alpha_{2}}>1$, then $p_{1}^{*}>0, p_{2}^{*} \in[0,1]$ and and voters from groups 1 and 4 vote for candidate $C_{1}$, who wins the election;
(c) If $\frac{1-\alpha_{1}}{1-\alpha_{2}}=1$, then $0<p_{1}^{*}<1, p_{2}^{*} \in[0,1]$ and voters from groups 1 and 4 vote for candidate $C_{1}$, who wins the election;

Note that candidates' strategies are identical to the previous case. If equilibrium exists, candidate $C_{1}$ obtains the votes from his partisans and the group of voters that have preferences opposite to the other candidate. Issue intensities determine the strategies of the favorite candidate, but voter distribution determines which candidate is the favorite candidate.

Similar to the Case 2, neither group 1 or group 4 has to be the largest group of voters. As long as the total number of voters exceeds the number of voters in other two groups, candidate $C_{1}$ will be the favorite candidate and win the election.

## 4 Conclusion and discussion

The purpose of this paper is both to develop a model that explains the behavior of candidates in a political campaign and to characterize the conditions which determine the focus of campaign participants. Candidates cannot reveal their true positions on every issue, and thus they have to choose how much time to devote to each issue proposed for the discussion.

When candidates disagree on both issues I find support for both issue ownership and wave riding theories. If issues are equally important to the public, candidates will spend all their time discussing different issues, and no dialogue between candidates will exist. Both candidate will devote all of their time to the issue upon which they agree with a group of voters that decides the outcome of the election. If one issue is more important than the other, one candidate will be a favorite in the election but he cannot win the election, unless he spends certain amount of time discussing the issue that is more important to the group of voters that determine the outcome of the election. The minimum amount of time a candidate with the advantage would have to spend on the issue will depend on how important that issue is to the deciding group of voters.

In the case where candidates agree on one of the issues, voter distribution determines which candidate has an advantage and can win the election. In most cases the candidate with advantage has to devote some of his time to both issues in order to win the election and the dialogue between candidates will exist. In all cases, the winning candidate has to spend some minimum time discussing the non common issue. Issue intensities determine candidates' strategies, but it is not possible to conclude that candidates will either devote most of their time to the salient issues or to the other issues.

Taken together, the results demonstrate that both, issue importance and voter distribution play an important role in determining the equilibrium strategies and the winner of the election. The mass of a single group of voters is not as important as its mass combined with the other groups, that have similar preferences over one of the issues, thus in order to win the election, favorite candidate might not always try to obtain the votes from the biggest group of voter, but rather from the group of voter that can committed to that candidate.

There are several limitations to this work. First, it was assumed that all voters share the same issue preferences, which is probably not the case in the real life. Some people might believe that economic issues are more important, and some people think that religious issues are of greatest importance. Thus, future work can investigate how the equilibrium changes if voters do not share the same issue intensities. Second, the paper investigates the cases where each issue is at least somewhat important to the voter ( $\alpha_{i}<1$ ), and even if the candidate with opposite issue preferences is elected, the voter still gets some utility out of it $\left(\alpha_{i}>0\right)$. If issue intensity bounds were extended, candidates' strategies might be quite different. Finally, it was assumed that each issue has only two alternatives, which is rarely seen in real life. Most issues require more complex thoughts than simply 'yes' or 'no' answers. Allowing candidates and voters to locate anywhere in between extreme alternatives will help answer the
questions raised by Fiorina (2005) regarding voters and candidates polarization.

## 5 Appendix A

Proof of Proposition 1. Voters from group 1 vote for candidate $C_{1}$ if $E_{1} U_{1}>$ $E_{2} U_{1}$ :
$f\left(p_{1}\right)+\left(1-f\left(p_{1}\right)\right) \alpha_{1}+f\left(1-p_{1}\right)+\left(1-f\left(1-p_{1}\right)\right) \alpha_{2}>f\left(p_{2}\right) \alpha_{1}+\left(1-f\left(p_{2}\right)\right)+$ $f\left(1-p_{2}\right) \alpha_{2}+\left(1-f\left(1-p_{2}\right)\right)$

Voters from group 2 vote for candidate $C_{1}$ if $E_{1} U_{2}>E_{2} U_{2}$ :
$f\left(p_{1}\right) \alpha_{1}+\left(1-f\left(p_{1}\right)\right)+f\left(1-p_{1}\right)+\left(1-f\left(1-p_{1}\right)\right) \alpha_{2}>f\left(p_{2}\right)+\left(1-f\left(p_{2}\right)\right) \alpha_{1}+$ $f\left(1-p_{2}\right) \alpha_{2}+\left(1-f\left(1-p_{2}\right)\right)$

Voters from group 3 vote for candidate $C_{1}$ if $E_{1} U_{3}>E_{2} U_{3}$ :
$f\left(p_{1}\right) \alpha_{1}+\left(1-f\left(p_{1}\right)\right)+f\left(1-p_{1}\right) \alpha_{2}+\left(1-f\left(1-p_{1}\right)\right)>f\left(p_{2}\right)+\left(1-f\left(p_{2}\right)\right) \alpha_{1}+$ $f\left(1-p_{2}\right)+\left(1-f\left(1-p_{2}\right)\right) \alpha_{2}$

Voters from group 4 vote for candidate $C_{1}$ if $E_{1} U_{4}>E_{2} U_{4}$ :
$f\left(p_{1}\right)+\left(1-f\left(p_{1}\right)\right) \alpha_{1}+f\left(1-p_{1}\right) \alpha_{2}+\left(1-f\left(1-p_{1}\right)\right)>f\left(p_{2}\right) \alpha_{1}+\left(1-f\left(p_{2}\right)\right)+$ $f\left(1-p_{2}\right)+\left(1-f\left(1-p_{2}\right)\right) \alpha_{2}$

First I show that voters from group 1 always vote for candidate $C_{1}$. It is true if $E_{1} U_{1}>E_{2} U_{1}$ holds for any set $\left(p_{1}, p_{2}\right)$, or
$f\left(p_{1}\right)+\left(1-f\left(p_{1}\right)\right) \alpha_{1}+f\left(1-p_{1}\right)+\left(f\left(1-p_{1}\right)\right) \alpha_{2}>f\left(p_{2}\right) \alpha_{1}+\left(1-f\left(p_{2}\right)\right)+$ $f\left(1-p_{2}\right) \alpha_{2}+\left(1-f\left(1-p_{2}\right)\right)$, or
$\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{2}\right)-f\left(1-p_{1}\right)\right)+\left(1-\alpha_{1}\right)\left(1-f\left(p_{1}\right)-f\left(p_{2}\right)\right)<0$
From assumption $1, f\left(p_{1}\right) \in[0.5,1]$ thus $1-f\left(1-p_{2}\right)-f\left(1-p_{1}\right) \leq 0$ and $1-f\left(p_{1}\right)-f\left(p_{2}\right) \leq 0$, and there exist no $p_{1}$ and $p_{2}$, s.t. $1-f\left(1-p_{2}\right)-$ $f\left(1-p_{1}\right)=1-f\left(p_{1}\right)-f\left(p_{2}\right)=0$.

In the same way voters from group 3 always vote for candidate $C_{2}$ as $E_{1} U_{3}<$ $E_{2} U_{3}$ holds for any set $\left(p_{1}, p_{2}\right)$, or $f\left(p_{1}\right) \alpha_{1}+\left(1-f\left(p_{1}\right)\right)+f\left(1-p_{1}\right) \alpha_{2}+$ $\left(f\left(1-p_{1}\right)\right)<f\left(p_{2}\right)+\left(1-f\left(p_{2}\right)\right) \alpha_{1}+f\left(1-p_{2}\right)+\left(1-f\left(1-p_{2}\right)\right) \alpha_{2}$, or
$\left(\alpha_{1}-1\right)\left(1-f\left(p_{2}\right)-f\left(p_{1}\right)\right)+\left(\alpha_{2}-1\right)\left(1-f\left(1-p_{2}\right)-f\left(1-p_{1}\right)\right)>0$.
This inequality always holds, as $1-f\left(1-p_{2}\right)-f\left(1-p_{1}\right) \leq 0,1-f\left(p_{1}\right)-$ $f\left(p_{2}\right) \leq 0$, and there exist no $p_{1}$ and $p_{2}$, s.t. $1-f\left(1-p_{2}\right)-f\left(1-p_{1}\right)=$ $1-f\left(p_{1}\right)-f\left(p_{2}\right)=0$.

Thus, regardless of the values of $\alpha_{1}$ and $\alpha_{2}$ voters in group 1 always vote for the candidate $C_{1}$ and voters in group 3 always vote for candidate $C_{2}$. The candidate who obtains votes from voters in group 2 wins the election.

Voters in group 2 vote for candidate $C_{1}$ if $E_{1} U_{2}>E_{2} U_{2}$ or $f\left(p_{1}\right) \alpha_{1}+(1-$ $\left.f\left(p_{1}\right)\right)+f\left(1-p_{1}\right)+\left(1-f\left(1-p_{1}\right)\right) \alpha_{2}>f\left(p_{2}\right)+\left(1-f\left(p_{2}\right)\right) \alpha_{1}+f\left(1-p_{2}\right) \alpha_{2}+$ $\left(1-f\left(1-p_{2}\right)\right)$, or after simplification: $\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{2}\right)-f\left(1-p_{1}\right)\right)<(1-$ $\left.\alpha_{1}\right)\left(1-f\left(p_{2}\right)-f\left(p_{1}\right)\right)$

Part (a). Let $\alpha_{1}>\alpha_{2}$. Candidate $C_{1}$ wins the election if and only if ( $1-$ $\left.\alpha_{1}\right) f\left(p_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-p_{1}\right)<\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)\right)-\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{2}\right)\right)$. Note that function $\frac{f(1-p)-0.5}{f(p)} \in[0,1]$ and is monotone, strictly decreasing function. Also, $\frac{1-\alpha_{1}}{1-\alpha_{2}} \in(0,1)$, and thus there exist $\bar{p} \in(0,1)$ s.t. $\frac{1-\alpha_{1}}{1-\alpha_{2}}=\frac{f(1-\bar{p})-0.5}{f(\bar{p})}$. Also
note that $\left(1-\alpha_{1}\right)(1-f(p))-\left(1-\alpha_{2}\right)(1-f(1-p)) \in\left[-0.5\left(1-\alpha_{2}\right), 0.5\left(1-\alpha_{1}\right)\right]$ and is strictly decreasing function. Now assume that $\left(p_{1}, p_{2}\right)$ are candidates' equilibrium strategies, and $p_{1}>\bar{p}$, where $\bar{p}$ is s.t. $\frac{1-\alpha_{1}}{1-\alpha_{2}}=\frac{f(1-\bar{p})-0.5}{f(\bar{p})}$ and further assume that under strategies $\left(p_{1}, p_{2}\right)$ candidate $C_{1}$ wins the election. In this case $E_{1} U_{2}>E_{2} U_{2}$ or $\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{2}\right)-f\left(1-p_{1}\right)\right)<\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)-f\left(p_{1}\right)\right)$ or $\left(1-\alpha_{1}\right) f\left(p_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-p_{1}\right)<\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)\right)-\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{2}\right)\right)$. But if $p_{1}>\bar{p}$, then $\frac{f\left(1-p_{1}\right)-0.5}{f\left(p_{1}\right)}<\frac{f(1-\bar{p})-0.5}{f(\bar{p})}=\frac{1-\alpha_{1}}{1-\alpha_{2}}$, which means $f\left(p_{1}\right)(1-$ $\left.\alpha_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-p_{1}\right)>-0.5\left(1-\alpha_{2}\right)$, and thus there exist $p_{2}=\dot{p}_{2}<1$, s.t. $\left(1-\alpha_{1}\right) f\left(p_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-p_{1}\right)>\left(1-\alpha_{1}\right)\left(1-f\left(\dot{p}_{2}\right)\right)-\left(1-\alpha_{2}\right)\left(1-f\left(1-\dot{p}_{2}\right)\right)$. Then candidate $C_{2}$ would want to deviate from $p_{2}$ to $\dot{p}_{2}$ in order to win the election. But $\left(p_{1}, \dot{p}_{2}\right)$ cannot be an equilibrium, because candidate $C_{1}$ could win the election by deviating to $p_{1}=\bar{p}$, as $f(\bar{p})\left(1-\alpha_{1}\right)-\left(1-\alpha_{2}\right) f(1-\bar{p})=$ $-0.5\left(1-\alpha_{2}\right)$, and thus $f(\bar{p})\left(1-\alpha_{1}\right)-\left(1-\alpha_{2}\right) f(1-\bar{p})<\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)\right)-$ $\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{2}\right)\right)$ is true for any $p_{2}<1$. In this case, candidate $C_{2}$ cannot win the election, unless $p_{2}=1$ and voters from group 2 are indifferent between candidates. So, there exist no equilibrium strategies $\left(p_{1}, p_{2}\right)$ where either $p_{1}>\bar{p}$, or $p_{2}<1$. Now, assume that $\left(p_{1}, p_{2}\right)$ are candidates' equilibrium strategies, where $p_{1}=\bar{p}$, and $p_{2}=1$. In this case voters are indifferent between candidates and each candidate wins election with certain probability (depending on the number of partisan voters). But candidate $C_{1}$ could deviate to $p_{1}=\ddot{p}_{1}<$ $\bar{p}$, then $\left(1-\alpha_{1}\right) f\left(\ddot{p}_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-\ddot{p}_{1}\right)<-0.5\left(1-\alpha_{2}\right)$ which means that $f\left(\ddot{p}_{1}\right)\left(1-\alpha_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-\ddot{p}_{1}\right)<\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)\right)-\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{2}\right)\right)$ holds for any $p_{2}$, and candidate $C_{1}$ wins the election. Thus, in equilibrium, candidate $C_{1}$ strategy is $p_{1}^{*}<\bar{p}$, and candidate $C_{2}$ strategy is $p_{2}^{*} \in[0,1]$.

Part (b). Let $\alpha_{1}<\alpha_{2}$. Candidate $C_{2}$ wins the election if and only if $\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{1}\right)\right)-\left(1-\alpha_{1}\right)\left(1-f\left(p_{1}\right)\right)>\left(1-\alpha_{2}\right) f\left(1-p_{2}\right)-(1-$ $\left.\alpha_{1}\right) f\left(p_{2}\right)$. The function $\frac{f(p)-0.5}{f(1-p)}$ is monotone, strictly increasing function s.t. $\frac{f(p)-0.5}{f(1-p)} \in[0,1]$. Also, $\frac{1-\alpha_{2}}{1-\alpha_{1}} \in[0,1]$, thus there exist $\hat{p}$ s.t. $\frac{1-\alpha_{2}}{1-\alpha_{1}}=\frac{f(\hat{p})-0.5}{f(1-\hat{p})}$. Note that $\left(1-\alpha_{2}\right)(1-f(1-p))-\left(1-\alpha_{1}\right)(1-f(p)) \in\left[-0.5\left(1-\alpha_{1}\right), 0.5(1-\right.$ $\left.\alpha_{2}\right)$ ] and is strictly increasing in $p$. Now assume that $\left(p_{1}, p_{2}\right)$ are candidates' equilibrium strategies, and $p_{2}<\hat{p}$, where $\hat{p}$ is s.t. $\frac{1-\alpha_{2}}{1-\alpha_{1}}=\frac{f(\hat{p})-0.5}{f(1-\hat{p})}$ and further assume that under strategies $\left(p_{1}, p_{2}\right)$ candidate $C_{2}$ wins the election. In this case $E_{1} U_{2}<E_{2} U_{2}$ or $\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{2}\right)-f\left(1-p_{1}\right)\right)>\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)-f\left(p_{1}\right)\right)$ or $\left(1-\alpha_{1}\right) f\left(p_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-p_{1}\right)>\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)\right)-\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{2}\right)\right)$. But if $p_{2}<\hat{p}$ then $\frac{f\left(p_{2}\right)-0.5}{f\left(1-p_{2}\right)}<\frac{f(\hat{p})-0.5}{f(1-\hat{p})}=\frac{1-\alpha_{2}}{1-\alpha_{1}}$ and thus $\left(1-\alpha_{2}\right) f\left(1-p_{2}\right)-(1-$ $\left.\alpha_{1}\right) f\left(p_{2}\right)>-0.5\left(1-\alpha_{1}\right)$, and thus there exist $p_{1}=\dot{p}_{1}<1$, s.t. $\left(1-\alpha_{2}\right)(1-$ $\left.f\left(1-\dot{p}_{1}\right)\right)-\left(1-\alpha_{1}\right)\left(1-f\left(\dot{p}_{1}\right)\right)<\left(1-\alpha_{2}\right) f\left(1-p_{2}\right)-\left(1-\alpha_{1}\right) f\left(p_{2}\right)$. Then candidate $C_{1}$ would want to deviate from $p_{1}$ to $\dot{p}_{1}$ in order to win the election. But ( $\dot{p}_{1}, p_{2}$ ) cannot be an equilibrium, because candidate $C_{2}$ could win the election by deviating to $p_{2}=\hat{p}$, as $\left(1-\alpha_{2}\right) f(1-\hat{p})-\left(1-\alpha_{1}\right) f(\hat{p})=-0.5\left(1-\alpha_{1}\right)$, and thus $\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{1}\right)\right)-\left(1-\alpha_{1}\right)\left(1-f\left(p_{1}\right)\right)>\left(1-\alpha_{2}\right) f(1-\hat{p})-(1-$ $\left.\alpha_{1}\right) f(\hat{p})$ is true for any $p_{1}>0$. In this case, candidate $C_{1}$ cannot win the election, unless $p_{1}=0$ and the second group of voters is indifferent between candidates. So, there exist no equilibrium strategies $\left(p_{1}, p_{2}\right)$ where either $p_{1}>\hat{p}$,
or $p_{2}<1$. Now, assume that $\left(p_{1}, p_{2}\right)$ are candidates' equilibrium strategies, where $p_{1}=0$, and $p_{2}=\hat{p}_{2}$. In this case voter is indifferent between candidates and each candidate wins election with certain probability. But candidate $C_{2}$ could deviate to $p_{2}=\ddot{p}_{2}>\hat{p}$, then $\left(1-\alpha_{2}\right) f\left(1-\hat{p}_{2}\right)-\left(1-\alpha_{1}\right)\left(f\left(\hat{p}_{2}\right)<-0.5(1-\right.$ $\left.\alpha_{1}\right)$ which means that $\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{1}\right)\right)-\left(1-\alpha_{1}\right)\left(1-f\left(p_{1}\right)\right)>(1-$ $\left.\alpha_{2}\right) f\left(1-\ddot{p}_{2}\right)-\left(1-\alpha_{1}\right) f\left(\ddot{p}_{2}\right)$ holds for any $p_{1}$, so candidate $C_{2}$ wins the election. Thus, in equilibrium, candidate $C_{1}$ strategy is $p_{1}^{*} \in[0,1]$, and candidate $C_{2}$ strategy is $p_{2}^{*}>\hat{p}$.

Part (c). Let $\alpha_{1}=\alpha_{2}$. Suppose that $\left(p_{1}, p_{2}\right)$, where $p_{k} \in(0,1)$ are candidates' equilibrium strategies. Also, assume that candidate $C_{1}$ wins and candidate $C_{2}$ looses the election. Thus for voters in group $2 f\left(p_{1}\right)-f\left(1-p_{1}\right)<$ $f\left(1-p_{2}\right)-f\left(p_{2}\right)$. Candidate $C_{2}$ would want to deviate in order to reverse the sign of inequality and win the election. If candidate $C_{2}$ chooses $p_{2}=\dot{p}_{2}=$ $1-p_{1}+\varepsilon$, s.t $\varepsilon \in\left(0, \bar{p}_{1}\right)$, then $f\left(p_{1}\right)-f\left(1-p_{1}\right)>f\left(1-\dot{p}_{2}\right)-f\left(\dot{p}_{2}\right)$ and candidate $C_{2}$ wins the election. But $\left(p_{1}, \dot{p}_{2}\right)$ cannot be equilibrium strategy, because if candidate $C_{1}$ can deviate from $p_{1}$ to $\dot{p}_{1}=1-\dot{p}_{2}-\eta$, s.t. $\eta \in\left(0,1-\dot{p}_{2}\right)$, then $f\left(\dot{p}_{1}\right)-f\left(1-\dot{p}_{1}\right)<f\left(1-\dot{p}_{2}\right)-f\left(\dot{p}_{2}\right)$ which makes candidate $C_{1}$ a winner of the election. But then again, candidate $C_{2}$ can depart from $\dot{p}_{2}$ to $\ddot{p}_{2}$ where $\ddot{p}_{2}=1-\dot{p}_{1}+\zeta$, s.t $\zeta \in\left(0, \dot{p}_{1}\right)$, and win the election. Candidate $C_{1}$ in this case would be better of by selecting $\ddot{p}_{1}=1-\ddot{p}_{2}-\theta$, s.t. $\theta \in\left(0,1-\ddot{p}_{2}\right)$. Thus, set of strategies $\left(p_{1}, p_{2}\right)$, where $p_{k} \in(0,1)$ cannot be an equilibrium, as loosing candidate always has an opportunity to win the election by deviating from the equilibrium. Now, suppose $\left(p_{1}, 1\right)$, where $p_{1} \in[0,1]$ are candidates' equilibrium strategies, and candidate $C_{2}$ wins the election, as $f\left(\bar{p}_{1}\right)-f\left(1-\bar{p}_{1}\right)>f(0)-f(1)=-0.5$. Now candidate $C_{1}$ would want to deviate from $p_{1}$, reverse the sign of inequality, and win the election. But if $p_{1} \in[0,1]$ then $f\left(p_{1}\right)-f\left(1-p_{1}\right) \in[-0.5,0.5]$, and thus $f\left(p_{1}\right)-f\left(1-p_{1}\right)<-0.5$ is not possible. Thus the best candidate $C_{1}$ can do is to select $p_{1}=0$, which, given strategy of candidate $C_{2}$, makes voters indifferent between candidates, as $f\left(p_{1}\right)-f\left(1-p_{1}\right)=f\left(1-p_{2}\right)-f\left(p_{2}\right)$. Thus, given candidate's $C_{2}$ strategy $p_{2}=1$, candidate's $C_{1}$ strategy is $p_{1}=0$, the voter is indifferent between candidates, and each candidate wins the election with certain probability. Candidates' equilibrium strategies are $p_{1}^{*}=0$ and $p_{2}^{*}=1$. Each candidate looses the election with probability 1 when deviating from the equilibrium.

## 6 Appendix B

Proof of proposition 2. Each candidate wins the election if he gets more votes than his rival. It was previously shown that voters in group 1 always vote fore candidate $C_{1}$, thus if either voters in group 2 or voters in group 4 vote for candidate $C_{1}$, he wins the election. Voters from groups 2 and 4 never vote for the same candidate unless both groups are indifferent between candidates and vote for each candidate with equal probability. Thus, candidate $C_{2}$ can never win the election, the best he can do is to win with some probability (which depends on the size of his partisan group), when voters from group 2 and voters
from group 4 are indifferent between candidates.
Voters in group 2 vote for candidate $C_{1}$ if: $f\left(p_{1}\right) \alpha_{1}+\left(1-f\left(p_{1}\right)\right)+f\left(1-p_{1}\right)+$ $\left(1-f\left(1-p_{1}\right)\right) \alpha_{2}>f\left(p_{2}\right)+\left(1-f\left(p_{2}\right)\right) \alpha_{1}+f\left(1-p_{2}\right) \alpha_{2}+\left(1-f\left(1-p_{2}\right)\right)$ or $\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)\right)-\left(1-\alpha_{2}\right)\left(\left(1-f\left(1-p_{2}\right)\right)>\left(1-\alpha_{1}\right) f\left(p_{1}\right)-(1-\right.$ $\left.\alpha_{2}\right) f\left(1-p_{1}\right)$.

Voters in group 4 vote for candidate $C_{1}$ if: $f\left(p_{1}\right)+\left(1-f\left(p_{1}\right)\right) \alpha_{1}+f\left(1-p_{1}\right) \alpha_{2}+$ $\left(1-f\left(1-p_{1}\right)\right)>f\left(p_{2}\right) \alpha_{1}+\left(1-f\left(p_{2}\right)\right)+f\left(1-p_{2}\right)+\left(1-f\left(1-p_{2}\right)\right) \alpha_{2}$ or $\left(1-\alpha_{2}\right)\left(\left(1-f\left(1-p_{2}\right)\right)-\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)\right)>\left(1-\alpha_{2}\right) f\left(1-p_{1}\right)-\left(1-\alpha_{1}\right) f\left(p_{1}\right)\right.$.

Part (a). Let $\alpha_{1}>\alpha_{2}$. And assume that $\left(p_{1}, p_{2}\right)$ are candidates' equilibrium strategies, and $p_{1} \geq \bar{p}_{1}$, where $\bar{p}_{1}$ is s.t. $\frac{1-\alpha_{1}}{1-\alpha_{2}}=\frac{f\left(1-\bar{p}_{1}\right)-0.5}{f\left(\bar{p}_{1}\right)}$ and further assume that candidate $C_{1}$ wins the election. Note that $\left(1-\alpha_{1}\right)(1-f(p))-(1-$ $\left.\alpha_{2}\right)(1-f(1-p)) \in\left[-0.5\left(1-\alpha_{2}\right), 0.5\left(1-\alpha_{1}\right)\right]$ and is strictly decreasing in $p$.

But then $f\left(p_{1}\right)\left(1-\alpha_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-p_{1}\right) \geq-0.5\left(1-\alpha_{2}\right)$, and thus there exist $p_{2}=\dot{p}_{2} \in[0,1]$, s.t. $\left(1-\alpha_{1}\right) f\left(p_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-p_{1}\right)=\left(1-\alpha_{1}\right)(1-$ $\left.f\left(\dot{p}_{2}\right)\right)-\left(1-\alpha_{2}\right)\left(1-f\left(1-\dot{p}_{2}\right)\right)$. Then candidate $C_{2}$ would want to deviate from $p_{2}$ to $\dot{p}_{2}$ in order to win the election with some probability. But ( $p_{1}, \dot{p}_{2}$ ) cannot be an equilibrium, because candidate $C_{1}$ could win the election by deviating to $\dot{p}_{1}<\bar{p}$, as $f\left(\dot{p}_{1}\right)\left(1-\alpha_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-\dot{p}_{1}\right)<-0.5\left(1-\alpha_{2}\right)$, and thus $f\left(\dot{p}_{1}\right)\left(1-\alpha_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-\dot{p}_{1}\right)<\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)\right)-\left(1-\alpha_{2}\right)\left(1-f\left(1-p_{2}\right)\right)$ holds for any $p_{2}$, and voters from group 2 vote for candidate $C_{1}$, who wins the election. Thus, in equilibrium, candidate $C_{1}$ strategy is $p_{1}^{*}<\bar{p}$, and candidate $C_{2}$ strategy is $p_{2}^{*} \in[0,1]$.

Part (b). Let $\alpha_{1}<\alpha_{2}$.And assume that $\left(p_{1}, p_{2}\right)$ are candidates' equilibrium strategies, and $p_{1} \leq \hat{p}$, where $\hat{p}$ is s.t. $\frac{1-\alpha_{2}}{1-\alpha_{1}}=\frac{f(\hat{p})-0.5}{f(1-\hat{p})}$ and further assume that candidate $C_{1}$ wins the election. Note that $\left(1-\alpha_{2}\right)((1-f(1-p))-(1-$ $\left.\alpha_{1}\right)(1-f(p)) \in\left[-0.5\left(1-\alpha_{1}\right), 0.5\left(1-\alpha_{2}\right)\right]$ and is strictly increasing in $p$. But then $\left(1-\alpha_{2}\right) f\left(1-p_{1}\right)-f\left(p_{1}\right)\left(1-\alpha_{1}\right) \geq-0.5\left(1-\alpha_{1}\right)$, and thus there exist $p_{2}=\dot{p}_{2} \in[0,1]$, s.t. $\left(1-\alpha_{2}\right)\left(\left(1-f\left(1-\dot{p}_{2}\right)\right)-\left(1-\alpha_{1}\right)\left(1-f\left(\dot{p}_{2}\right)\right)=(1-\right.$ $\left.\alpha_{2}\right) f\left(1-p_{1}\right)-\left(1-\alpha_{1}\right) f\left(p_{1}\right)$. Then candidate $C_{2}$ would want to deviate from $p_{2}$ to $\dot{p}_{2}$ in order to win the election with some probability. But ( $p_{1}, \dot{p}_{2}$ ) cannot be an equilibrium, because candidate $C_{1}$ could win the election by deviating to $\dot{p}_{1}>\hat{p}$, as $f\left(\dot{p}_{1}\right)\left(1-\alpha_{1}\right)-\left(1-\alpha_{2}\right) f\left(1-\dot{p}_{1}\right)<-0.5\left(1-\alpha_{1}\right)$, and thus $\left(1-\alpha_{2}\right)$ $\left(1-\alpha_{2}\right)\left(\left(1-f\left(1-p_{2}\right)\right)-\left(1-\alpha_{1}\right)\left(1-f\left(p_{2}\right)\right)>\left(1-\alpha_{2}\right) f\left(1-\dot{p}_{1}\right)-\left(1-\alpha_{1}\right) f\left(\dot{p}_{1}\right)\right.$ holds for any $p_{2}$, voters from group 4 vote for candidate $C_{1}$, who wins the election. Thus, in equilibrium, candidate $C_{1}$ strategy is $p_{1}^{*}>\hat{p}_{1}$, and candidate $C_{2}$ strategy is $p_{2}^{*} \in[0,1]$.

Part (c). Let $\alpha_{1}=\alpha_{2}$. Voters in group 2 vote for candidate $C_{1}$ if: $f\left(1-p_{2}\right)-f\left(p_{2}\right)>f\left(p_{1}\right)-f\left(1-p_{1}\right)$. Voters in $v_{4}$ vote for candidate $C_{1}$ if: $f\left(1-p_{2}\right)-f\left(p_{2}\right)<f\left(p_{1}\right)-f\left(1-p_{1}\right)$. And assume that $\left(p_{1}, p_{2}\right)$ are candidates' equilibrium strategies. Further assume that candidate $C_{1}$ wins the
election, and thus either voters from group 2 or group 4 voted for candidate $C_{1}$. But now candidate $C_{2}$ can deviate to $\dot{p}_{2}=1-p_{1}$, which will make voters from groups 2 and 4 indifferent between candidates, and candidate $C_{1}$ no longer wins the election with probability 1 . Now assume that ( $p_{1}, \dot{p}_{2}$ ) are candidates' equilibrium strategies, and each candidate wins the election with certain probability. But candidate $C_{1}$ can deviate to $\dot{p}_{1}=p+\xi, \xi \neq 0$, and win the election, as $f\left(1-\dot{p}_{2}\right)-f\left(\dot{p}_{2}\right)=f\left(\dot{p}_{1}\right)-f\left(1-\dot{p}_{1}\right)$ no longer holds and either voters from group 2 or group 4 vote for candidate $C_{1}$. But $\left(\dot{p}_{1}, \dot{p}_{2}\right)$ cannot be an equilibrium strategy either as candidate $C_{2}$ can deviate to $\ddot{p}_{2}=1-\dot{p}_{1}$, which will make voters from groups 2 and 4 indifferent between candidates. In this case candidate $C_{1}$ is better off with his original strategy $p_{1}$ as $f\left(1-\dot{p}_{2}\right)-f\left(\dot{p}_{2}\right)=f\left(p_{1}\right)-f\left(1-p_{1}\right)$ no longer holds. But now, candidate $C_{2}$ would be better off with $\dot{p}_{2}=1-p_{1}$, but it was already proved that $\left(p_{1}, \dot{p}_{2}\right)$ is not an equilibrium. Thus, the equilibrium does not exist.

## 7 Appendix C

Proof of proposition 3. Voters from group $j$ vote for candidate $C_{1}$ if $E_{1} U_{j}>$ $E_{2} U_{j}$. When candidate $C_{1}$ spends $p_{1}$ time discussing issue $Z_{1}$, the voters believe that policy $A$ will be implemented with probability $f\left(p_{1}\right)$ and policy $B$ will be implemented with probability $1-f\left(p_{1}\right)$. At the same time, he has $\left(1-p_{1}\right)$ time left to discuss issue $Z_{2}$, and thus voters believe that policy $E$ will be implemented with probability $f\left(1-p_{1}\right)$, and policy $D$ will be implemented with probability $1-f\left(p_{1}\right)$.

Each voter maximizes his utility and vote for candidate $C_{1}$ if $E_{1} U_{j}>E_{2} U_{j}$.
Voters from group 1 vote for candidate $C_{1}$ if:
$f\left(p_{1}\right)+\left(1-f\left(p_{1}\right)\right) \alpha_{1}+f\left(1-p_{1}\right)+\left(1-f\left(1-p_{1}\right)\right) \alpha_{2}>f\left(p_{2}\right) \alpha_{1}+\left(1-f\left(p_{2}\right)\right)+$ $f\left(1-p_{2}\right)+\left(1-f\left(1-p_{2}\right)\right) \alpha_{2}$.

Voters from group 2 vote for candidate $C_{1}$ if:
$f\left(p_{1}\right) \alpha_{1}+\left(1-f\left(p_{1}\right)\right)+f\left(1-p_{1}\right)+\left(1-f\left(1-p_{1}\right)\right) \alpha_{2}>f\left(p_{2}\right)+\left(1-f\left(p_{2}\right)\right) \alpha_{1}+$ $f\left(1-p_{2}\right)+\left(1-f\left(1-p_{2}\right)\right) \alpha_{2}$.

Voters from group 3 vote for candidate $C_{1}$ if:
$f\left(p_{1}\right) \alpha_{1}+\left(1-f\left(p_{1}\right)\right)+f\left(1-p_{1}\right) \alpha_{2}+\left(1-f\left(1-p_{1}\right)\right)>f\left(p_{2}\right)+\left(1-f\left(p_{2}\right)\right) \alpha_{1}+$ $f\left(1-p_{2}\right) \alpha_{2}+\left(1-f\left(1-p_{2}\right)\right)$.

Voters from group 4 vote for candidate $C_{1}$ if:
$f\left(p_{1}\right)+\left(1-f\left(p_{1}\right)\right) \alpha_{1}+f\left(1-p_{1}\right) \alpha_{2}+\left(1-f\left(1-p_{1}\right)\right)>f\left(p_{2}\right) \alpha_{1}+\left(1-f\left(p_{2}\right)\right)+$ $f\left(1-p_{2}\right) \alpha_{2}+\left(1-f\left(1-p_{2}\right)\right)$.

After simplification of voters' expected utility function was obtained that:
Voters from group 1 vote for candidate $C_{1}$ if:
$1-f\left(p_{1}\right)-\frac{1-\alpha_{2}}{1-\alpha_{1}} f\left(1-p_{1}\right)<f\left(p_{2}\right)-\frac{1-\alpha_{2}}{1-\alpha_{1}} f\left(1-p_{2}\right)$;
Voters from group 2 vote for candidate $C_{1}$ if:
$1-f\left(p_{2}\right)-\frac{1-\alpha_{2}}{1-\alpha_{1}} f\left(1-p_{2}\right)>f\left(p_{1}\right)-\frac{1-\alpha_{2}}{1-\alpha_{1}} f\left(1-p_{1}\right)$;
Voters from group 3 vote for candidate $C_{1}$ if:
$1-f\left(p_{1}\right)-\frac{1-\alpha_{2}}{1-\alpha_{1}} f\left(1-p_{1}\right)>f\left(p_{2}\right)-\frac{1-\alpha_{2}}{1-\alpha_{1}} f\left(1-p_{2}\right)$;
Voters from group 4 vote for candidate $C_{1}$ if:
$1-f\left(p_{2}\right)-\frac{1-\alpha_{2}}{1-\alpha_{1}} f\left(1-p_{2}\right)<f\left(p_{1}\right)-\frac{1-\alpha_{2}}{1-\alpha_{1}} f\left(1-p_{1}\right)$.
From now on I will refer to the function $1-f(p)-\frac{1-\alpha_{2}}{1-\alpha_{1}} f(1-p)$ as $g(p)$, and to the function $f(p)-\frac{1-\alpha_{2}}{1-\alpha_{1}} f(1-p)$ as $h(p)$.

Part (a). Let $\alpha_{1}>\alpha_{2}$, and thus $\frac{1-\alpha_{2}}{1-\alpha_{1}}>1$. Function $f(p)$ is concave and $f(p) \in[0.5,1]$ which implies that $g(p)$ is convex with maximum at $p=1$ where $g_{\max }(p)=-0.5 \frac{1-\alpha_{2}}{1-\alpha_{1}}$, and $p \in[0,1]$. Function $h(p) \in\left[0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}, 1-0.5 \frac{1-\alpha_{2}}{1-\alpha_{1}}\right]$ and is strictly increasing.

Note that $0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}<-0.5 \frac{1-\alpha_{2}}{1-\alpha_{1}}<1-0.5 \frac{1-\alpha_{2}}{1-\alpha_{1}}$ and thus, there exist $\tilde{p} \in[0,1]$ such that $h(\tilde{p})=-0.5 \frac{1-\alpha_{2}}{1-\alpha_{1}}=g_{\max }(p)$.

Also because $g(p)$ is a convex function, there exist $p \in R$ such that $g(p)=$ $g(0)=0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}$. But then $g\left(p^{\prime}\right)<g_{\max }(p)$ which implies that $p<1$.

Finally, observe that $g(0)=h(0)=0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}$ and for any $p \in(0,1]$, $g(p)<h(p)$.

Let $\hat{p}<\dot{p}$. Then $\dot{p}>0$ and for any $p \in(0, \dot{p}), g(p)<0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}$. Suppose that $(0,0)$ is an equilibrium, which implies that all voters are indifferent between candidates and each candidate wins the election with equal probability. But candidate $C_{1}$ can deviate to $p_{1} \leq \hat{p}$, then $p_{1}<\hat{p}$, which means $g\left(p_{1}\right)<0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}$ and voters from group 1 vote for candidate $C_{1}$ and also $0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}<h\left(p_{1}\right)$ and voters from group 4 vote for candidate $C_{1}$ and he wins the election. Now assume that $\left(p_{1}, 0\right)$ is an equilibrium, and as was shown, voters from group 1 and 4 vote for candidate $C_{1}$. But $p_{1} \leq \hat{p}$ and thus $h\left(p_{1}\right) \leq h(\tilde{p})$ which implies that there exist $p_{2}>\dot{p}$ such that $g\left(p_{2}\right)=h\left(p_{1}\right)$ and candidate $C_{2}$ can deviate to $p_{2}$, and voters from groups 2 and 4 will be indifferent between candidates, group 1 votes for candidate $C_{1}$ and group 3 votes for candidate $C_{2}$ and each candidate could win the election with certain probability.

Now, assume that $\left(p_{1}, p_{2}\right)$ is an equilibrium, but in this case candidate $C_{1}$ could deviate to $\dot{p}_{1}>\hat{p}$ which implies that $h\left(\dot{p}_{1}\right)>h(\hat{p})>g\left(p_{2}\right)$ and groups 4 and 1 vote for candidate $C_{1}$. Now, if $\dot{p}_{1}>\dot{p}$, then $g\left(\dot{p}_{1}\right)>0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}$ and thus, there exist $\dot{p}_{2}<\hat{p}$ such that $g\left(\dot{p}_{1}\right)>h\left(\dot{p}_{2}\right)$ and candidate $C_{2}$ could deviate to $\dot{p}_{2}$ and win the election as now voters from groups 1 and 2 vote for him. But then $\left(\dot{p}_{1}, \dot{p}_{2}\right)$ cannot be an equilibrium either, as candidate $C_{1}$ can deviate to $\ddot{p}_{1} \in(\hat{p}, \not p)$ and thus $g\left(\ddot{p}_{1}\right)<0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}$ and also $-0.5 \frac{1-\alpha_{2}}{1-\alpha_{1}}<h\left(\ddot{p}_{1}\right)$ and candidate $C_{1}$ wins the election. Recall that $g(p) \leq-0.5 \frac{1-\alpha_{2}}{1-\alpha_{1}}$ and $h(p) \geq$ $0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}$ for any $p \in[0,1]$ and thus there exists no such value $\ddot{p}_{2}$ that will improve the position of candidate $C_{2}$. Thus, in the equilibrium candidate $C_{1}$ selects $p_{1}^{*} \in(\hat{p}, p)$ and candidate $C_{2}$ strategy is $p_{2}^{*} \in[0,1]$.

Now, let $\hat{p} \geq \hat{p}$.There are two possible scenarios. First let's assume that $\dot{p}>0$. Suppose that $(0,0)$ is an equilibrium, which implies that all voters are indifferent between candidates and each candidate wins the election with equal probability. But candidate $C_{1}$ can deviate to $p_{1}<\dot{p}$, which means $g\left(p_{1}\right)<0.5-$ $\frac{1-\alpha_{2}}{1-\alpha_{1}}$ and voters from group 1 vote for candidate $C_{1}$ and also $0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}<h\left(p_{1}\right)$ and voters from group 4 vote for candidate $C_{1}$ and he wins the election. Now assume that $\left(p_{1}, 0\right)$ is an equilibrium, and as was shown, voters from group 1
and 4 vote for candidate $C_{1}$. But $p_{1}<\hat{p}$ and thus $h\left(p_{1}\right)<h(\hat{p})$ which implies that there exist $p_{2} \in(\dot{p}, 1]$ such that $g\left(p_{2}\right)=h\left(p_{1}\right)$ and candidate $C_{2}$ can deviate to $p_{2}$, and voters from groups 2 and 4 will be indifferent between candidates, group 1 still votes for candidate $C_{1}$ and group 3 votes for candidate $C_{2}$ and each candidate could win the election with certain probability. Now, assume that $\left(p_{1}, p_{2}\right)$ is an equilibrium, but in this case candidate $C_{1}$ could deviate to $\dot{p}_{1} \in(\hat{p}, 1]$ which implies that $h\left(\dot{p}_{1}\right)>g\left(p_{2}\right)$ group 4 votes for candidate $C_{1}$. But now, if $\dot{p}_{1}>\dot{p}$, then $g\left(\dot{p}_{1}\right)>0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}$ and thus, there exist $\dot{p}_{2}<\tilde{p}$ such that $g\left(\dot{p}_{1}\right)>h\left(\dot{p}_{2}\right)$ and candidate $C_{2}$ could deviate to $\dot{p}_{2}$ and win the election as now voters from groups 1 and 2 vote for him. But then $\left(\dot{p}_{1}, \dot{p}_{2}\right)$ cannot be an equilibrium either, as candidate $C_{1}$ can deviate to $\ddot{p}_{1}<\dot{p}$ thus $g\left(\ddot{p}_{1}\right)<0.5-\frac{1-\alpha_{2}}{1-\alpha_{1}}$ and such that $g\left(\dot{p}_{2}\right) \neq h\left(\ddot{p}_{1}\right)$ which makes voters from group 1 and either group 2 or 4 vote candidate $C_{1}$ who in this case wins the election. But then again, $\ddot{p}_{1}<\hat{p}$ and thus $h\left(\ddot{p}_{1}\right)<h(\hat{p})$ which implies that there exist $\ddot{p}_{2} \in(\dot{p}, 1]$ such that $g\left(\ddot{p}_{2}\right)=h\left(p_{1}\right)$ and candidate $C_{2}$ can deviate to $p_{2}$, and voters from groups 2 and 4 will be indifferent between candidates, group 1 still votes for candidate $C_{1}$ and group 3 votes for candidate $C_{2}$ and each candidate could win the election with certain probability. And we already showed that $\left(\ddot{p}_{1}, \ddot{p}_{2}\right)$ where $\ddot{p}_{1}<\dot{p}$ and $\ddot{p}_{2} \in(\dot{p}, 1]$ cannot be an equilibrium, thus, the equilibrium does not exist. The proof where $\dot{p}<0$ is very similar to the proof where $\dot{p}>0$, as there is no strategy for any of the candidates that will guarantee them votes from two groups of voters with mass greater than $50 \%$.

Part (b). Let $\alpha_{1}>\alpha_{2}$, and thus $\frac{1-\alpha_{2}}{1-\alpha_{1}}>1$. Function $f(p)$ is concave and $f(p) \in[0.5,1]$ which implies that $g(p)$ is convex and $g(1)<g(0)$. Thus, for any $p \in(0,1]$ and $p^{\prime \prime} \in(0,1], g(p)<h\left(p^{\prime \prime}\right)$. This implies that $g\left(p_{1}\right)<h\left(p_{2}\right)$ holds for any $p_{k}$ unless $p_{1}=p_{2}=0$, and voters from group 1 always vote for candidate $C_{1}$ unless $p_{1}=p_{2}=0$. Also, it implies that $g\left(p_{2}\right)<h\left(p_{1}\right)$ holds for any $p_{k}$ unless $p_{1}=p_{2}=0$ and voters from group 4 always vote for candidate $C_{1}$ unless $p_{1}=p_{2}=0$. Thus, $p_{1}^{*}>0$ is candidate's $C_{1}$ dominant strategy, and both groups 1 and 4 vote for candidate $C_{1}$. In this case, candidate $C_{2}$ cannot change the outcome of the election and loses when $p_{2} \in[0,1]$.

Part (c). Let $\alpha_{1}=\alpha_{2}$, and thus $\frac{1-\alpha_{2}}{1-\alpha_{1}}=1$. Function $f(p)$ is concave and $f(p) \in[0.5,1]$ which implies that $g(p)$ is convex and $g(1)=g(0)$. Thus, for any $p \in(0,1)$ and $p^{\prime \prime} \in(0,1], g(p)<h\left(p^{\prime \prime}\right)$. This implies that $g\left(p_{1}\right)<h\left(p_{2}\right)$ holds for any $p_{k}$ unless $p_{1}=p_{2}=0$, or $p_{1}=1$ and $p_{2}=0$, and voters from group 1 always vote for candidate $C_{1}$ unless $p_{1}=p_{2}=0$, or $p_{1}=1$ and $p_{2}=0$, Also, it implies that $g\left(p_{2}\right)<h\left(p_{1}\right)$ holds for any $p_{k}$ unless $p_{1}=p_{2}=0$, or $p_{1}=1$ and $p_{2}=0$, and voters from group 4 always vote for candidate $C_{1}$ unless $p_{1}=p_{2}=0$, or $p_{1}=1$ and $p_{2}=0$, Thus, $0<p_{1}^{*}<1$ is candidate's $C_{1}$ dominant strategy, and both groups 1 and 4 vote for candidate $C_{1}$. In this case, candidate $C_{2}$ cannot change the outcome of the election and loses when $p_{2} \in[0,1]$.

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