DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 08-05

Estimating Discount Factors within a Random Utility Theoretic Framework

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October 2008

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October 30, 2008

*This work was supported by the STC program of the National Science Foundation via the National Center for Earth-surface Dynamics under the agreement Number EAR-0120914.

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Abstract

Choices involving tradeoffs of benefits and costs over time are pervasive in our everyday lives. The observation of declining discount rates in experimental settings has led many to promote hyperbolic discounting over standard exponential discounting as the preferred descriptive model of intertemporal choice. In this paper, I develop a new framework that directly models the intertemporal utility function associated with an intertemporal outcome. This random utility model produces explicit maximum likelihood estimates of the discounting parameters. The main benefit of this approach is that I am able to perform formal statistical tests of quasi-hyperbolic and hyperbolic discounting, which has not been done previously in the economics literature. I apply this estimation method to two original data sets, a stated-preference survey of cleanup options for the Minnesota River Basin and revealed-preference choices of lottery payment options, in addition to one published data set. Formal statistical tests fail to find evidence in support of hyperbolic or quasi-hyperbolic discounting. Constant (exponential) annual discount rates range from eight to eleven percent over the three data sets, which are lower than those usually found in experimental studies but consistent with interest rates found in capital markets. I propose that confounding experimental artifacts may be responsible for previous evidence in favor of hyperbolic discounting. Specifically, uncertainty in future rewards, perceived future transaction costs, and subadditive discounting may confound estimates of rates of time preference (discount rates) from previous experimental designs.

JEL Codes: D90, Q25, Q53, H43

Keywords: discounting, hyperbolic, random utility, intertemporal choice

1 Introduction

Every day we make decisions involving tradeoffs of benefits and costs over time. Would I rather spread my workload evenly over the next few days and distribute the pain or procrastinate and have an extremely painful task several days from now? Should I exercise regularly while I'm younger so that I can enjoy the health benefits when I'm older? Will I invest time and money in my education today so that I can have a better lifestyle later? Am I willing to give up some consumption today so that I and others can enjoy a better environment in the future? These intertemporal choices penetrate nearly every aspect of our behavior. Such decisions require weighing benefits and costs that are realized with differing temporal patterns. Typically, individuals discount future outcomes, but how much, and in what way?

To answer this question, I develop a new approach that facilitates estimating discount factors for monetary choices or for other choices that can be presented in a stated-preference framework. I directly model the intertemporal utility function associated with an intertemporal outcome, which produces explicit estimates of discounting parameters within a random utility framework. This empirical strategy allows direct testing of competing hypotheses of how people discount future benefits and costs in a unified statistical framework.

Recent evidence suggests that individuals may discount the future hyperbolically or quasi-hyperbolically. That is, some studies find that inferred discount rates decline over time. While several studies have observed that discount rates appear to decline with the length of delay and conducted some indirect testing, there are no instances in the published economics literature where researchers provide direct statistical tests of hyperbolic or quasi-hyperbolic discounting using microeconomic data.[1] I address this gap in the literature with this new empirical methodology.

After presenting the statistical model for estimating discount factors, I estimate

[1] Quasi-hyperbolic discounting has been recently tested using macroeconomic financial data.[1]

the model on three data sets. Two of the data sources are original; one comes from a stated-preference survey on river basin improvements and one comes from choices that individuals make when they win state lottery jackpots. The former data set represents a public good choice, and the other two represent private good choices. These three distinct sources provide a comprehensive picture of discounting at the individual level. All data sets produce similar discounting results. There is no statistical evidence supporting hyperbolic or quasi-hyperbolic models over the standard exponential model. Constant annual discount rates range from eight to eleven percent over the three data sets. I propose that the prior experimental evidence in favor of hyperbolic and quasi-hyperbolic discounting may be due to confounding factors from the experimental designs, rather than true rates of time preference.

2 Existing Literature

2.1 Historical Development of the Discounted Utility Model

Paul Samuelson first developed the discounted utility (DU) model in 1937 in an attempt to provide a general model of intertemporal choice. Commonly referred to as the exponential discounting model, the DU model simplified all discounting into a single parameter, the discount rate. A consumer's preferences over consumption bundles, $(c_o, c_1, ..., c_T)$ are represented by an intertemporal utility function, $U(c_o, c_1, ..., c_T)$. Furthermore, the DU model assumes that the intertemporal utility function is described by

$$U(c_0, c_1, ..., c_T) = \sum_{t=0}^{T} \psi_t u(c_t), \tag{1}$$

where the discount factor for year t is $\psi_t = \left[\frac{1}{1+\rho}\right]^t$ and ρ is the discount rate.

Samuelson's DU model was accepted almost immediately because of its analytic simplicity and elegance. Interestingly, Samuelson did not endorse the DU model as a normative model of intertemporal choice or as a valid descriptive model. The DU model was never empirically verified but still became the standard model for intertemporal utility. [24]

2.2 Departures from the Discounted Utility Model

In the past several decades, research has uncovered many situations in which the DU model does not fit behavior.[2] One major departure from the DU model is that inferred discount rates often decline over time in experimental settings. This phenomenon is commonly termed hyperbolic discounting. This discounting gets its name because a hyperbolic functional form fits the data better than the traditional exponential functional form. Several functional forms have been suggested for hyperbolic discounting. The most popular of these takes the form of

$$\psi_t = (1 + \alpha t)^{-\beta/\alpha}, \text{where } \alpha, \beta > 0 \text{ [19]}. \tag{2}$$

As α goes to 0, this hyperbolic discounting function becomes the exponential discounting function. To facilitate estimation, researchers typically simplify equation 2 to have only one parameter. Constraining α to be equal to one produces the model suggested by Harvey [10]. Harvey's single-parameter hyperbolic structure is given by

$$\psi_t^{Harvey} = (1 + t)^{-\mu}. \tag{3}$$

Alternatively, constraining the ratio of β/α to be equal to one results in the single-parameter model suggested by Herrnstein [11] and Mazur [21] (HM);

$$\psi_t^{HM} = (1 + \omega t)^{-1}. \tag{4}$$

[2] See, for example, Cairns and van der Pol.[5][4]

In recent years, an alternative model of discounting that has received much attention is the quasi-hyperbolic (β, δ) discounting model. This model, developed by David Laibson, is also motivated by the observation of declining discount rates [18]. The functional form was first introduced by Phelps and Pollak in the context of intergenerational altruism [22]. The form of the quasi-hyperbolic discounting function is very simple and its contrast with the standard exponential discounting model is readily apparent. The functional form is given by

$$\psi_t = \left\{ \begin{array}{c} 1 \text{ if } t = 0 \text{ and} \\ \beta\delta^t \text{ if } t > 0 \end{array} \right\}, \text{ where } 0 < \beta < 1, \text{ and } \delta < 1. \tag{5}$$

Thus, the only difference between discount factors in the quasi-hyperbolic formulation and the exponential formulation is that all future time periods are discounted by the additional β factor in the quasi-hyperbolic model. Especially large importance is placed on immediate utility as compared to deferred utility. The (β, δ) discounting model is much easier to analyze than the true hyperbolic model, yet it retains many of the qualitative aspects of the more complicated model.

As shown in Figure 1 [3], both hyperbolic and the quasi-hyperbolic discounting functions weight the near future less heavily than exponential discounting. However, for time periods far in the future, exponential discounters place less weight on the deferred utility than hyperbolic or quasi-hyperbolic discounters. Figure 2 shows the corresponding marginal discount rates for all four discounting functions. The point plotted for time period t is the marginal discount rate between time period $t - 1$ and time period t.

[3] The parameter values used for the exponential, Harvey hyperbolic, and HM hyperbolic models in these figures are consistent with those that I find from the data sets employed in this paper. The β chosen for the quasi-hyperbolic model is in the range of values discussed in the literature.

Figure 1: Comparison of Discount Factors: Exponential (δ^t) with $\delta = .9$, Harvey Hyperbolic ($(1+t)^{-\mu}$) with $\mu = .4$, Quasi-hyperbolic ($1,\ \beta\delta^t$) with $\beta = .75$, $\delta = .92$, and HM Hyperbolic ($(1+\omega t)^{-1}$) with $\omega = .15$.

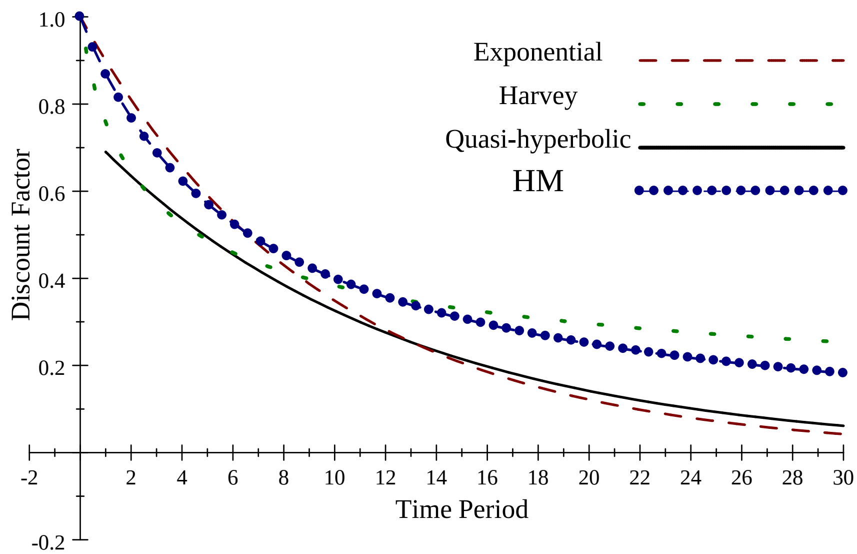
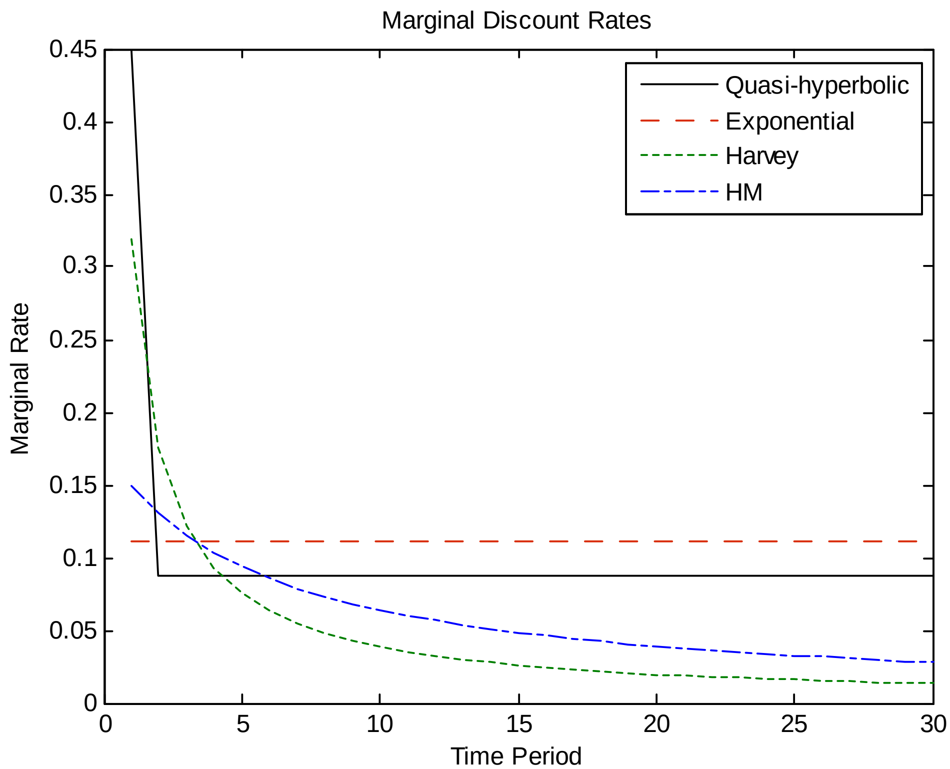


Figure 2: Comparison of Marginal Discount Rates: Exponential (δ^t) with $\delta = .9$, Harvey Hyperbolic ($(1+t)^{-\mu}$) with $\mu = .4$, Quasi-hyperbolic ($1,\ \beta\delta^t$) with $\beta = .75$, $\delta = .92$, and HM Hyperbolic ($(1+\omega t)^{-1}$) with $\omega = .15$.



Exponential discounters will always display time consistency because their marginal discount rate is constant over all time periods. Quasi-hyperbolic discounters have a large marginal discount rate between time period 0 (now) and time period 1 and a constant marginal discount rate thereafter. Thus, quasi-hyperbolic discounters are dynamically consistent for any choice that does not involve the present. Regardless, most interesting economic choices involve the present. Finally, hyperbolic discounters always have declining discount rates. Therefore, a hyperbolic discounter is subject to dynamic inconsistency for any time period. However, hyperbolic marginal discount rates change less for time periods farther in the future. That is, they will be less likely to be dynamically inconsistent for tradeoffs that occur far in the future than for tradeoffs that occur near to the present. Hyperbolic discounting makes individuals appear to be impatient for immediate tradeoffs, but sufficiently patient for tradeoffs occurring far enough in the future.

A simple example highlights the time inconsistency inherent in hyperbolic and quasi-hyperbolic discounting. Assuming parameter values that are in the range of those found in the literature, I analyze the choice between \$100 now and \$120 a year from now and compare this with the choice between \$100 five years from now and \$120 six years from now. The interval length between the options for each choice is one year so a dynamically consistent discounter should choose either the more proximate reward in both scenarios or the more distant reward in both scenarios. Table 1 presents the discounted values of \$100 now and \$120 one year from now for all four discounting models. Table 2 shows the discounted values of \$100 five years from now and \$120 six years from now. The exponential discounter remains consistent in their choice to take the deferred payoff. However, the hyperbolic and quasi-hyperbolic discounters choose the early reward for the immediate tradeoff and choose the more distant payoff for the future tradeoff.

It is desirable to be dynamically consistent from a normative standpoint. With

Table 1: Present Discounted Values of 100 Dollars Now vs. 120 Dollars 1 Year from Now for Exponential with $\delta = .9$, Harvey Hyperbolic with $\mu = .4$, HM Hyperbolic with $\omega = .15$, and Quasi-hyperbolic with $\beta = .75, \delta = .92$

Model	**Discounted Value of $100 Now**	**Discounted Value of $120 1 Year from Now**	**Choice**
Exponential	$100.00	$108.00	$120 in 1 Year
Harvey Hyperbolic	$100.00	$90.95	$100 Now
HM Hyperbolic	$100.00	$92.30	$100 Now
Quasi-hyperbolic	$100.00	$82.80	$100 Now

Table 2: Present Discounted Values of 100 Dollars 5 Years from Now vs. 120 Dollars 6 Years from Now for Exponential with $\delta = .9$, Harvey Hyperbolic with $\mu = .4$, HM Hyperbolic with $\omega = .15$, and Quasi-hyperbolic with $\beta = .75, \delta = .92$

Model	**Discounted Value of $100 5 Years from Now**	**Discounted Value of $120 6 Years from Now**	**Choice**
Exponential	$59.05	$63.77	$120 6 Years from Now
Harvey Hyperbolic	$48.84	$55.10	$120 6 Years from Now
HM Hyperbolic	$57.14	$63.16	$120 6 Years from Now
Quasi-hyperbolic	$49.43	$54.58	$120 6 Years from Now

free access to capital markets, individuals should equate the marginal rate of substitution between two time periods to one plus the interest rate. A third party planner could improve a hyperbolic or quasi-hyperbolic discounter's intertemporal utility by rearranging consumption between time periods. In contrast, the welfare of an exponential discounter that is trading off consumption between time periods at one plus the interest rate cannot be improved upon by a third party planner.[4]

[4]Note that in this paper I abstract from the notion of discounting the utility of others. While that is a fundamental question in itself, I only examine the behavior of an individual concerned with their own utility. I do not attempt to derive the socially optimal effective discount rate, as in Weitzman [31].

2.3 Discounting Studies

I concentrate on several of the more recent contributions and note that a more extensive literature review on discounting is provided by Frederick, Loewenstein, and O'Donoghue [8]. Table 3 summarizes several of the discounting studies related to public goods. Table 4 provides examples of the more common money discounting studies. While three recent working papers use utility-theoretic models incorporating goods other than money, the majority of previous studies examine monetary trade-offs over time. Table 5 summarizes some of the indirect tests on various discounting models. I point out that much of the evidence supporting hyperbolic discounting can be recast in terms of confounding factors. I am not aware of any previous research that has performed direct nested testing like I propose in this research.

Table 3: Empirical Discounting Studies (Health and Public Goods)

Author	**Type**	**Discounting**	**Methodology**	**Good**	**Time Frame**
Cameron, Gerdes[6]	experimental	exponential, hyperbolic	RUM, money lottery combined with conjoint health policy, individual level parameters, ordered logit	public health outcomes	0, 20, 30, or 40 years
Bosworth, Cameron, DeShazo[3]	experimental	exponential, hyperbolic	RUM, money lottery combined with conjoint health policy, individual level parameters, conditional logit	public health outcomes	2 to 30 years
van der Pol, Cairns[28]	experimental	exponential	Conjoint, dichotomous choice, ordered probit	own and others' health outcomes	0 to 15 years
Viscusi, Huber[29]	experimental	exponential, quasi-hyperbolic	Reduced form RUM, conjoint, 2 stage-estimation, conditional logit, mixed logit	local water quality	0, 2, 4 or 6 years

Table 4: Empirical Discounting Studies (Money)

Author	Type	Discounting	Methodology	Good	Time Frame
Warner, Pleeter[30]	revealed	exponential	Lump-sum or annuity choice during downsizing, reduced form probit	money	0 to 2 times years of service
Coller, Williams[7]	experimental	exponential	choice between payoff now and later, censored data, maximum likelihood	money	0 to 3 months
Harrison, Lau, Williams[9]	experimental	exponential	choice between payoff now and later, individual explanatory variables	money	0 to 36 months
Alberini, Chiabai[2]	experimental	exponential	choice between lump-sum and annuity payment, reduced form maximum likelihood	money	0 to 10 years

Table 5: Indirect Tests of Discounting Models

Author	Type	Discounting	Methodology	Good	Time Frame
Kirby and Marakovic[17]	experimental	exponential and hyperbolic	matching task, fit exponential and hyperbolic parameters for each subject	money	3 to 29 days
Slonim et al.[25]	experimental	Informally tests whether discount rates are different for longer front-end delays. Compares patterns to exponential, hyperbolic, and quasi-hyperbolic models.	Discrete choices between earlier and later payoff. Reduced form regression on decision to wait.	money	0 to 6 months
Cairns and van der Pol[4]	experimental	Compare 3 hyperbolic models with exponential	Choice between benefit 1 year from now or delayed benefit. 2-stage indirect test of discounting models.	Private and Social Financial Benefits	1 to 19 years
Keller and Strazzera[16]	simulated data from experiment	exponential, single parameter hyperbolic	Comparison of simulated matching values with actual matching values for hyperbolic and exponential models	money	0 to 120 months

2.3.1 Estimation Methods

The most common method for gathering data on discounting is to elicit experimental responses to hypothetical or real monetary rewards. Two approaches are most widely utilized. Respondents are either asked to choose between two different sized rewards realized at different times in the future or to state the payoff today that would make them indifferent to a larger payoff in the future (or the payoff in the future that would make them indifferent to a smaller payoff today). Harrison et al. [9] represents the former approach and Coller and Williams [7] falls into the latter category. Harrison et al. find an overall individual discount rate in Denmark of 28.1 percent using money data and they observe significant heterogeneity in the data. One notable exception to the experimental emphasis is the revealed-preference study by Warner and Pleeter [30]. They examine the decisions of military personnel when faced with a downsizing. Personnel choices of whether to take a lump-sum payment or an annuity reveal information about their intertemporal preferences.

Various studies have examined discounting for health outcomes. This branch of the discounting literature appears to begin with Horowitz and Carson [12]. In these studies, respondents state how many lives saved in the future is equivalent to saving a certain number of lives today, or respondents choose between varying durations of illness experienced at different times in the future. van der Pol and Cairns [28] use this second method and provide the first example of discrete choice experiments to address discounting for health outcomes.[5]

Two recent related papers use an empirical model that is similar to the model I propose. Bosworth, Cameron, and DeShazo [3] jointly estimate individual-specific discount rates and the demand for preventative public health policies. They utilize a conjoint survey design in which respondents make choices between policies that

[5] Multiple other health discounting studies exist. For example, see two papers by Johannesson and Johansson[15][14].

reduce the number of illnesses and deaths in their community. At the same time, they have individuals choose between a hypothetical lottery that provides a series of payments over several years and a lottery that provides a lump sum payment. This method is based on the identification strategy developed by Cameron and Gerdes [6]. The authors of both papers argue that the two distinct data sources allow improved joint estimation of the utility parameters and discount rates and that it is often not possible to identify discounting parameters out of a public goods choice.

I show that discounting parameters for public goods are identified in a stated-preference framework if the policy options are designed correctly. Bosworth, Cameron, and DeShazo's empirical model uses a utility-theoretic structure for preferences, much like I propose in this study. They assume i.i.d. extreme value errors for the policy choices, which is inconsistent with the structural model that they propose. As I show, the structure imposed on the model implies heteroskedastic errors at the policy level. Furthermore, Bosworth, Cameron, and DeShazo do not allow for discount rates to take forms other than the standard exponential and single parameter hyperbolic models. I extend the model to test for quasi-hyperbolic preferences.

W. Kip Viscusi and Joel Huber designed a study to infer discount rates for a publicly provided good [29]. They utilize a stated preference survey concerning improvements in local water quality to identify individual rates of time preference. In this paper, I build upon the survey design from Viscusi and Huber. Using a random utility model, they find that the data fit better with the quasi-hyperbolic discounting model than with the exponential discounting model. However, they employ a two-stage reduced form empirical approach. They are unable to provide confidence intervals or do any hypothesis testing about the quasi-hyperbolic discounting parameters. In contrast, my approach produces explicit standard errors for all discounting parameters and formal hypothesis tests are straightforward in my random utility theoretic framework.

2.3.2 Confounding Factors in Discounting Studies

Although evidence in the literature suggests that individuals have hyperbolic discounting preferences, I propose that much of this evidence can be explained by confounding factors. As emphasized in the review article by Frederick, Loewenstein, and O'Donoghue [8], it is important to differentiate between pure rates of time preference and other reasons that cause individuals to care less about future outcomes. Pure time preference refers to "the preference for immediate utility over delayed utility" [8]. Confounding factors that cause individuals to care less about the future but should be considered separately from pure time preference include uncertainty about a future outcome, perceived future transaction costs, and the phenomenon of subadditive discounting. In this section, I show how experimental designs that do not address these three confounding factors could make an exponential discounter appear as though they are a hyperbolic discounter.

Imagine an experimental setting in which an individual is choosing between a smaller immediate reward and a larger delayed reward. Uncertainty in the receipt of the future reward can be problematic for estimating discount rates in this scenario. Suppose that this individual is truly an exponential discounter but perceives only a 70 percent chance that the researcher will actually deliver the delayed reward at any time in the future and a 100 percent chance that the immediate reward will be delivered. Then, the results from the experiment would look exactly like the individual is a quasi-hyperbolic discounter with a β value of 0.7. Or, suppose that this individual is truly an exponential discounter with a constant discount factor of $\delta < 1$ but believes with probability $p_0 = 1$ that they will receive an immediate reward, with probability $p_1 < 1$ that they will receive a delayed reward at $t = 1$, and with probability p_t, such that $p_{t+1} < p_t$ and $p_{t+1} - p_t > p_{t+2} - p_{t+1}$, that they will receive a delayed reward at time t. That is, the perceived probability of receiving a future reward declines at a decreasing rate. Then, observed discount factors including the confounding effect of uncertainty are given by $\{1, p_1\delta, p_2\delta^2, p_3\delta^3, p_4\delta^4, ...\}$. Marginal observed discount rates are given by $\{1/p_1\delta - 1, p_1/\delta p_2 - 1, p_2/\delta p_3 - 1, p_3/\delta p_4 - 1, ...\}$. These resulting observed discount rates are consistent with a hyperbolic functional form. To further illustrate with a numerical example, assume $\delta = .9, p_1 = .8, p_2 = .7, p_3 = .65, p_4 = .61$. This gives marginal discount rates of $\{38.9\%, 26.9\%, 19.7\%, 18.4\%\}$. However, when abstracting from the effects of uncertainty, true marginal rates of time preference are given by $\{1/\delta - 1, 1/\delta - 1, 1/\delta - 1, ...\}$. Thus, it is important to minimize the effects of future uncertainty in a discounting study.

Next, suppose that within an experimental setting an individual perceives a transaction cost of c_t in order to collect a payment at time t in the future. Also suppose that this individual is an exponential discounter with a discount factor of δ^t. Then, in order to be indifferent between an immediate payment of $\$x_0$ and a delayed payment of $\$x_t$, it must be that $x_0 = \delta^t(x_t + c_t)$. If $c_{t+1} = c_t$ for all $t > 0$, observed marginal discount rates look like quasi-hyperbolic discount rates. If $c_{t+1} > c_t$ for all $t > 0$ observed marginal discount rates can look like hyperbolic discount rates.

To make ideas more concrete, consider the following example. Consider this individual indicating their indifference point between an immediate reward of \$100 and a larger delayed reward. Let the perceived future transaction costs $c = \{c_0, c_1, c_2, c_3, c_4\} = \{0, 10, 20, 30, 40\}$. Assume $\delta = .9$. Denote the marginal discount rate between time periods t and $t+1$ as $r_{t,t+1}$. Let superscripts of *true* and *obs* denote the true (exponential) and observed values. Then $r^{true}_{t,t+1} = 11.1\%$ for all t. Denote the delayed reward at time period t as x_t. Next, ignoring the transaction cost, c_0, it holds that $100 = .9 * x_1$. Solving, $x_1 = 111.11$ would make this individual indifferent in absence of transaction costs. Taking into account the effect of the transaction cost, $100 = \delta_1^{obs}(111.11 + 10)$. Solving, $\delta_1^{obs} = .8257$. Then, $r^{obs}_{0,1} = 1/\delta^{obs} - 1 = 21\%$. Again ignoring the transaction cost, c_1, $100 = .81 * x_2$. Solving, $x_2 = 123.46$ would make this individual indifferent in absence of transaction costs. Taking into account the effect of the transaction cost, $100 = \delta_2^{obs^2}(123.46 + 20)$. Solving, $\delta_2^{obs^2} = .6971$. This implies $r_{1,2}^{obs} = \delta_2^{obs^2}/\delta_1^{obs} - 1 = 18.45\%$. Continuing with this pattern, I find $r_{2,3}^{obs} = 16.53\%$ and $r_{3,4}^{obs} = 15.10\%$. I observe declining marginal discount rates even though the true marginal discount rates are constant. The larger the transaction cost relative to the size of the reward, the more pronounced this effect will be.

One other explanation for the observation of declining discount rates is the idea of subadditive discounting. That is, "discounting over a delay is greater when the delay is divided into subintervals than when it is left undivided" [23]. Most laboratory experiments look over days or months and confound the length of the delay with the length of the interval between choices. For example, a researcher will compare the discount rate inferred from a choice involving zero to six month delays to that from a choice involving zero to twelve month delays. When annualized, the discount rate will look larger from the choice involving zero to six month delays. Therefore, the discount rate looks like it declines over time. However, discount rates are declining because the length of the interval is increasing. Most experiments anchor all choices to a particular time and do not design choices to have interval length independent of the length of delay. Typically, a shorter interval length necessarily means a shorter delay until the delayed outcome. Read [23] uses experiments to verify the presence of subadditive discounting but finds no evidence of hyperbolic discounting.

2.3.3 Indirect Tests of Hyperbolic Discounting

Several studies have attempted to determine whether exponential or hyperbolic discounting is preferred. In this section, I summarize the studies that have indirectly tested for hyperbolic discounting. Also, I analyze how each study addresses uncertainty in a delayed reward, perceived future transaction costs, and subadditive discounting.

Kirby and Marakovic [17] fit hyperbolic and exponential discount functions for each subject. They utilize nonlinear regression techniques on the continuous time equations for exponential and hyperbolic discounting. They find that, while both do a good job explaining subjects' responses, the hyperbolic model fits better in terms of R^2 for almost all of the subjects. Uncertainty in the payment of the delayed reward is present since delayed rewards were not to be delivered until the evening on the day that it came due. Transaction costs are especially relevant because the rewards are small ($14.75-$28.50 for delayed rewards). This study confounds length of delay until the delayed reward is received with the length of the interval between options since all choices are anchored to the present.

Slonim et al. [25] conduct an experimental study in which they examine whether or not possession of the delayed reward affects subjects' discounting patterns. They find that discount rates decline over time in all cases. Possession of rewards supports quasi-hyperbolic discounting and no possession supports hyperbolic discounting. They do not find any evidence of exponential discounting. This study attempts to control for transaction costs in the best way possible by using possession of the reward as a control variable. Also, this study uses a common interval length of two months for all choices so interval length is not confounded with the length of delay until the receipt of the future reward. Uncertainty in future rewards is nullified in the cases where individuals choose between two future rewards if the perceived probability of receipt of the reward is constant over time. However, uncertainty in future rewards is still an issue if the probability of receipt of the reward declines with longer time delays. Also, for the choices anchored to the present, uncertainty in future rewards remains a confounding factor.

Cairns and van der Pol [4] compare three hyperbolic models with the exponential model. For each individual and discounting model, they first estimate optimal parameter values using non-linear least squares. Second, they regress these parameter values on the period in years for which the benefit is delayed, claiming that delay should be insignificant for a correctly specified discounting model. Delay is insignificant only in the Loewenstein and Prelec model (2 parameter hyperbolic). They also note that the first stage regressions have the highest R^2 for the hyperbolic models. Since all choices are anchored to one year in the future, uncertainty in rewards is controlled for if the perceived probability of receiving the reward is constant over all time periods but not if the perceived probability of receipt declines with time. Transaction costs are minimized in the case of social financial benefits since the receipt of the reward does not require any work on part of the survey respondent. For private financial benefits, transaction costs likely get larger as the delayed reward moves farther into the future. If transaction costs are constant over all future time periods, they will have no influence in this study since all choices are anchored to one year from the present. However, because of this common anchor, the length of delay and length of interval are confounded. Subadditive discounting may explain any evidence for hyperbolic discounting.

Keller and Strazzera [16] examine the predictive accuracy of the exponential and hyperbolic models in a simulated data set. Using Thaler's [26] 1981 experimental data to calculate implicit monthly discount rates, the authors generate a simulated data set of predicted matching values, m_t, that would make a respondent indifferent to an immediate reward, m_0. Comparing these predicted values with the actual matching values from Thaler's data set, they find that the hyperbolic model does a better job than the exponential model. Thus, indirect tests suggest that hyperbolic discounting is preferred to exponential discounting. All choices are anchored to the present. This leaves open the possibility of confounding effects from uncertainty in future rewards, future transaction costs, and subadditive discounting.

I build on these previous discounting studies by more closely considering potential confounding factors. I select data sets that minimize uncertainty in delayed rewards, decision-maker transaction costs, and subadditive discounting. Through jointly addressing these experimental concerns and developing a new empirical model that directly estimates the discounting parameters, I am able to isolate pure rates of time preference for various models and test to find the statistically preferred model.

3 Empirical Strategy

3.1 Derivation of the General Model

Here I present the random utility model to analyze discrete choice data. This model analyzes choices over goods that are intertemporal in nature. In general, let the instantaneous utility for an individual i for choice j in year t be given by

$$u_{ijt} = v_{ijt} + \eta_{ijt}. \tag{6}$$

Here, v_{ijt} is the deterministic portion of utility and η_{ijt} is the instantaneous error draw. It is important to note at this point that instantaneous utility is not at all observable. That is, the researcher only observes behavior at the choice level.

I make the usual assumption that intertemporal utility is additively separable over time periods. Then the utility for individual i that is associated with choice j defined through time period T_j is given by

$$U_{ij}(u_{ijt}, \psi_t) = \psi_0 u_{ij0} + \psi_1 u_{ij1} + ... + \psi_{T_j} u_{ijT_j}, \tag{7}$$

where ψ_t is the discount factor for year t . Substituting equation 6 into equation 7 and rewriting in summation notation produces

$$U_{ij} = \sum_{t=0}^{T_j} \psi_t v_{ijt} + \epsilon_{ij}, \tag{8}$$

where $\epsilon_{ij} = \sum_{t=0}^{T_j} \psi_t \eta_{ijt}$ is the error for individual i associated with choice j. Thus, the intertemporal utility from a choice is essentially the weighted sum of all instantaneous utilities. Discount factors determine the weight placed on each time period. The specification of v_{ijt} will depend on the type of intertemporal choice that is being analyzed.

3.2 Structure of the Error Terms

Since a rational individual makes utility evaluations at the instantaneous level and discounts them back to the present, it is appropriate to assume the distribution of the instantaneous errors (η_{ijt}). However, the researcher observes choices at the alternative level so it is necessary to use the model structure to determine the alternative level error structure. This approach contrasts the Bosworth et al. assumption that alternative errors are i.i.d. extreme value. I show in this section that even i.i.d. error assumptions at the instantaneous level imply heteroskedastic errors at the alternative level.

I first examine the expectation of the alternative error terms and then explore the alternative error variance structure.

Proposition 1 *The alternative error terms (ϵ_{ij}) have zero expectation as long as the instantaneous errors have zero expectation.*

Proof. $E(\epsilon_{ij}) = E(\sum_{t=0}^{T_j} \psi_t \eta_{ijt})$

$= E(\psi_0 \eta_{ij0} + ... + \psi_{T_j} \eta_{ijT_j})$

$= \psi_0 E(\eta_{ij0}) + ... + \psi_{T_j} E(\eta_{ijT_j})$

$= 0$ if $E(\eta_{ijt}) = 0\ \forall t.$ ■

Case 1 *Assume that instantaneous errors are independently, identically distributed. Assume that $\eta_{ijt}\tilde{}N(0, \sigma_\eta)$.*

Proposition 2 *In Case 1, $V(\epsilon_{ij}) = \sigma_\eta \sum_{t=0}^{T_j} \psi_t^2$.*

Proof. $V(\epsilon_{ij}) = V(\sum_{t=0}^{T_j} \psi_t \eta_{ijt})$

$= V(\psi_0 \eta_{ij0} + ... + \eta_{ijT_j})$

$= \psi_o^2 V(\eta_{ij0}) + ... + \psi_{T_j}^2 V(\eta_{ijT_j})$ since the η_{ijt} are independent.

$= \sigma_\eta \sum_{t=0}^{T_j} \psi_t^2$ since the η_{ijt} are identically distributed with variance σ. ■

Proposition 3 *In Case 1,* $Cov(\epsilon_{ij}, \epsilon_{ik} \; \forall j \neq k) = 0$.

Proof. $Cov(\epsilon_{ij}, \epsilon_{ik} \; \forall j \neq k) = Cov(\sum_{t=0}^{T_j} \psi_t \eta_{ijt}, \sum_{t=0}^{T_k} \psi_t \eta_{ikt})$

$= 0$ since the η_{ijt} are independent. ■

Thus, errors from alternatives with longer durations have larger variances. That is, the alternative errors are heteroskedastic because of the different time dimensions of the alternatives. This error structure is intuitively appealing. As much as the researcher can try to minimize the confounding effects, intertemporal decisions often involve some degree of uncertainty. The model essentially controls for this potential decision-maker uncertainty in delayed outcomes even though there is no uncertainty explicitly incorporated into the formulation of the model. As the time dimension of an alternative increases, the researcher expects decision-maker uncertainty to also increase. As this uncertainty increases, the variance of the error term increases. In the next section, I show how the variance of an alternative relates the variance of an observation and how the likelihood function minimizes the effects of decision-maker uncertainty.

3.3 Variance of the Alternative Error-Difference Terms

For each choice set A, an individual chooses the public good policy that provides the most utility. Therefore, the probability that individual i chooses alternative j from choice set A is

$$P_{ij} = \Pr(U_{ij} > U_{ik} \; \forall k \neq j \in A). \tag{9}$$

The task is to determine the form of P_{ij}. Begin by substituting equation 8 into equation 9 to get

$$\begin{aligned} P_{ij} &= \Pr(\sum_{t=0}^{T_j} \psi_t v_{ijt} + \epsilon_{ij} > \sum_{t=0}^{T_k} \psi_t v_{ikt} + \epsilon_{ik}) && (10) \\ &= \Pr(\epsilon_{ik} - \epsilon_{ij} < \sum_{t=0}^{T_j} \psi_t v_{ijt} - \sum_{t=0}^{T_k} \psi_t v_{ikt}) && (11) \end{aligned}$$

Next, denote the alternative error-difference term as $\tilde{\epsilon_{ikj}} = \epsilon_{ik} - \epsilon_{ij}$. Recalling that $\epsilon_{ij} = \sum_{t=0}^{T_j} \psi_t \eta_{ijt}$, I have

$$\tilde{\epsilon_{ikj}} = \sum_{t=0}^{T_k} \psi_t \eta_{ikt} - \sum_{t=0}^{T_j} \psi_t \eta_{ijt}. \tag{12}$$

For any decision maker, i, and time period, t, assume that if $v_{ijt} = v_{ikt}$, then $\eta_{ijt} = \eta_{ikt}$. That is, within a time period, if the observable components of utility associated with two choices for a given decision maker are equal, then the instantaneous error draws are equal also. For this analysis, assume that there are no time periods for which the observable components of utility are exactly the same. Then, note that $\tilde{\epsilon_{ikj}}$ is heteroskedastic because the number of terms in the summations is determined by the length of the intertemporal alternative. $\tilde{\epsilon_{ikj}}$ is a normal error term with mean zero and variance given by

$$V(\tilde{\epsilon_{ikj}}) = V(\sum_{t=0}^{T_k} \psi_t \eta_{ikt} - \sum_{t=0}^{T_j} \psi_t \eta_{ijt}). \tag{13}$$

$$= \psi_0^2 V(\eta_{ik0}) + \psi_1^2 V(\eta_{ik1}) + ... + \psi_{T_k}^2 V(\eta_{ikT_k}) + \psi_0^2 V(\eta_{ij0}) + \psi_1^2 V(\eta_{ij1}) + ... + \psi_{T_j}^2 V(\eta_{ijT_j}) \tag{14}$$

since the instantaneous errors are independent. With the assumption that η_{ijt} $i.i.d$ $N(0, \sigma_\eta)$. This leads to

$$V(\tilde{\epsilon_{ikj}}) = \sum_{t=0}^{T_k} \psi_t^2 \sigma_\eta + \sum_{t=0}^{T_j} \psi_t^2 \sigma_\eta. \tag{15}$$

It is well known that a probit model needs to be normalized for scale so set $\sigma_\eta = 1$ and I have

$$V(\tilde{\epsilon_{ikj}}) = \sum_{t=0}^{T_k} \psi_t^2 + \sum_{t=0}^{T_j} \psi_t^2 = V(\epsilon_{ik}) + V(\epsilon_{ij}). \tag{16}$$

Therefore, for any choice set, the variance of the alternative error-difference term will be larger when both policies have longer durations. Ignoring this in the likelihood function will lead to inconsistent parameter estimates and biased standard error estimates. Returning to equation 10 and using the definition of the c.d.f. (F) of a normal random variable, I have

$$P_{ij} = F\left(\frac{\sum_{t=0}^{T_j} \psi_t v_{ijt} - \sum_{t=0}^{T_k} \psi_t v_{ikt}}{\sqrt{\sum_{t=0}^{T_k} \psi_t^2 + \sum_{t=0}^{T_j} \psi_t^2}} \right). \tag{17}$$

The log-likelihood equation is then

$$LL = \sum_i \sum_j y_{ij} \ln P_{ij}, \tag{18}$$

where $y_{ij} = 1$ if i chose alternative j and zero otherwise.

Note that observations from choice sets with alternatives having longer durations are weighted less heavily than observations from choice sets with alternatives having shorter durations. Again, this serves a control for potential decision-maker uncertainty. Observations associated with longer time dimensions likely have more confounding effects from uncertainty so they receive less weight in the likelihood function.

3.4 Application to a Public Good Choice

This model is particularly well suited to analyze attribute based stated-preference data. Attribute based (conjoint) survey designs allow the researcher to specify several attribute dimensions of the intertemporal choices. Thus, the researcher can specify when the benefits and costs of an intertemporal choice are to be realized so that it is possible to identify the discount factors from respondents' choices. Public goods policies are a good example of choices that receive benefits and costs at differing points in times. For example, it is common to pay taxes today for a public good that will deliver benefits years into the future. In this section I develop the model for conjoint data in the context of public goods choices.[6]

At any time the utility an individual receives from a simple public good policy depends on the level of benefit provided and the cost incurred. Specify the deterministic portion of instantaneous utility as

$$v_{ijt} = \alpha q_{ijt} + \gamma(Y_{it} - c_{ijt}), \tag{19}$$

where q_{ijt} is the level of benefits from the public good, Y_{it} is income, and c_{ijt} is the cost of the public good for individual i for policy j in year t. In this specification, α is the marginal utility of the public good benefit and γ is the marginal utility of money. Let T_j denote the last year for which there are non-zero costs or benefits for policy j. Substituting equation 19 into equation 8 results in

$$U_{ij} = \sum_{t=0}^{T_j} \psi_t[\alpha q_{ijt} + \gamma(Y_{it} - c_{ijt})] + \epsilon_{ij}. \tag{20}$$

This equation is the foundation of my econometric model.

Because only differences in utility matter in the RUM, any personal characteristic

[6]Viscusi and Huber [29] provide the first example of a study designed to infer discount rates for public goods.

on its own such as Y_{it} drops out of the analysis. Personal characteristics can enter through interactions with policy characteristics. Since $\psi_0 = 1$ by economic theory, there are $T_j + 2$ parameters to estimate in this model. The α parameter is identified through contemporaneous variation in the level of the public good benefit. Similarly, the γ parameter is identified through contemporaneous variation in the level of cost of the policy. That is, α and γ can be identified without considering the discounting. Then, the discounting parameters (ψ_t) are identified through variation over time. If there is not enough variation in the data to identify each ψ_t individually, structure can be placed on the type of discounting.[7] For example, with quasi-hyperbolic discounting, there are only two discounting parameters (β, δ). Exponential discounting imposes the restriction that $\beta = 1$ in equation 5. A likelihood ratio test on the constrained and unconstrained models determines whether I reject the null hypothesis that $\beta = 1$. This is an improvement over previous studies which tend to just assume a specific functional form for discounting. I apply this test to the Minnesota River Basin data since it has sufficient intertemporal variation to identify quasi-hyperbolic discount factors.

3.5 Application to a Monetary Choice

As long as the money choices have sufficient intertemporal variation, this framework can be easily applied. In this section I develop the model for discrete choice data on monetary choices.

Suppose that an intertemporal monetary choice, j, describes a real or hypothetical amount of money, m_{ijt}, that will be paid to or collected from individual i, in time period t. During any time period, an individual receives utility from their non-experimental income, Y_{it}, and the money from the experiment, m_{ijt}. Specify the

[7] It is difficult to imagine a data source that would provide sufficient intertemporal variation to identify each discount factor separately. Instead, I restrict attention to exponential, hyperbolic, and quasi-hyperbolic functional forms.

deterministic portion of instantaneous utility as

$$v_{ijt} = \gamma(Y_{it} + m_{ijt}), \tag{21}$$

where γ is the marginal utility of money. Substituting equation 21 into equation 8 results in

$$U_{ij} = \sum_{t=0}^{T_j} \psi_t[\gamma(Y_{it} + m_{ijt})] + \epsilon_{ij}. \tag{22}$$

It is clear that γ is not identified in this model. As γ gets larger, all monetary choices get more appealing at the same rate. Therefore, I normalize the model with $\gamma = 1$. I utilize equation 22 for the two lottery data sets.

4 Data

4.1 The Minnesota River Basin Survey

I administered a survey to approximately 250 Minnesota residents in January of 2008. Survey participants from Survey Sampling International (SSI) completed the questionnaire on the internet. After removing partial responses from some individuals that did not complete all required questions, I was left with a sample of 237 individuals. Each respondent faced a series of eight attribute-based stated preference questions. All together, this yielded a total of 1803 choice occasions.

4.1.1 Survey Design

In order to identify the discounting parameters in this model it is essential that there is enough intertemporal variation. The survey design must provide enough variation while still remaining plausible and comprehensible to the survey respondents. That is, one must consider the real world decision so that policy options make sense. In this section I explore how the survey design can affect the ability to identify discount factors.

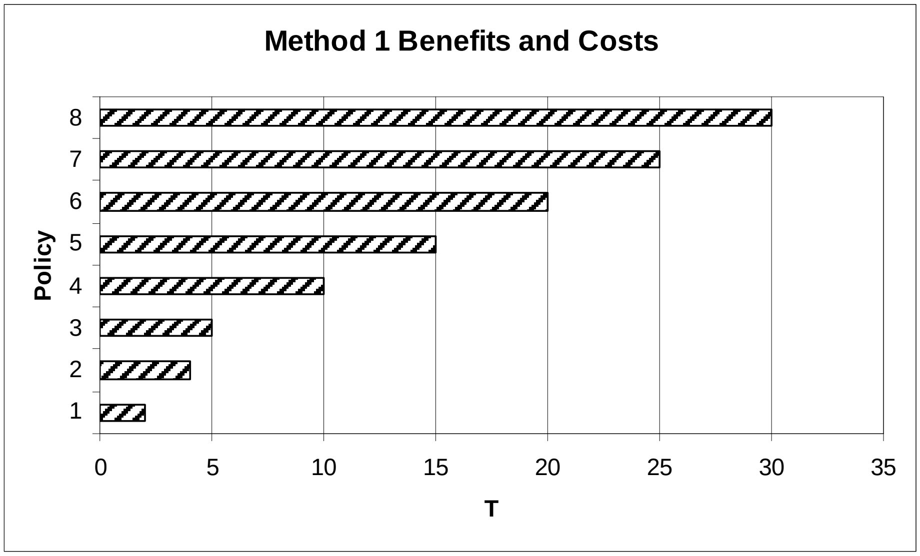
For illustrative purposes, first consider the two extremes of intertemporal variation. On one extreme, if benefits and costs vary across policies, individuals, and all time periods, model parameters are overidentified. However, it is not possible to have this type of variation with conjoint data. Respondents would not be able to comprehend the complexity of the policy. And, as a practical matter, the number of choice profiles would be prohibitively large. At the other extreme, if all policies have the same time horizon and costs and benefits do not vary across time periods within policies, discounting parameters are not identified. This follows from the general property of the RUM that parameters that only affect scale of utility are not identified. As the discounting parameters get larger the scale of utility increases but choice behavior is not at all impacted because all policies get more attractive at the same rate.

In between these two extremes exist many alternatives on the survey design. The task is to determine how much intertemporal variation is needed to still be able to identify the discounting parameters through choice-based surveys. Some guidance is provided in the literature.

In one possible survey design, all policies have costs and benefits that run for the entire length of the policy. To identify discount rates, policies have several different time horizons. For example, Bosworth, Cameron, and DeShazo design their survey to include health policies that run for 2, 4, 5, 10, 15, 20, 25, or 30 years [3]. The per-year attributes (cost, lives saved, etc.) of the policy are constant throughout the duration of the policy. (See Figure 3.)

In the second possible survey design, all policies have costs that start immediately and run for a specified time common to each policy. Benefits are timed differently. Some policies have immediate benefits and some have delayed benefits. All benefits run for a specified time common to each policy. Following this approach, Viscusi and

Figure 3: Survey Design 1



Huber design their survey such that costs uniformly begin immediately and run for five years [29]. Benefits (improvements to local water quality) begin with a delay of 0, 2, 4, or 6 years and run for five years. After five years, the water quality returns to the status quo at the beginning of the policy. In this design, per-year costs and benefits are also constant throughout the duration of the policy. (See Figures 4,5.) The following figures show eight different hypothetical policies for each survey design. The shaded boxes represent the duration of the various policies.

Figure 4: Survey Design 2 Benefits

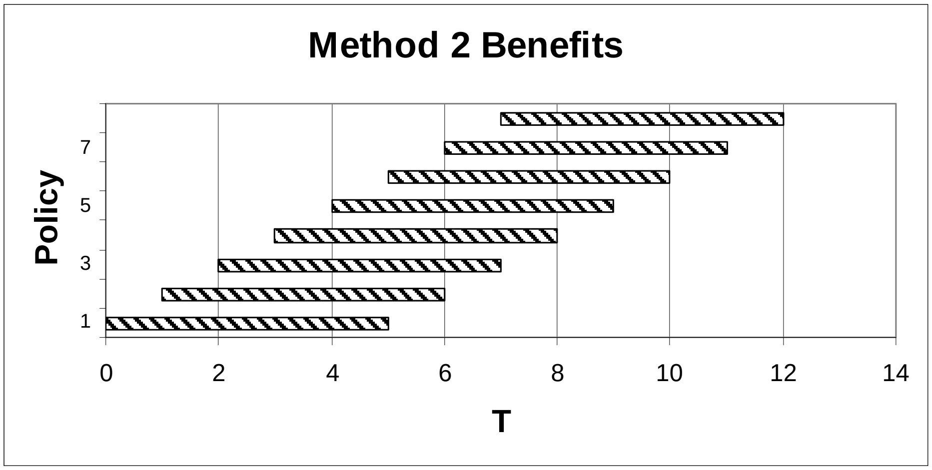
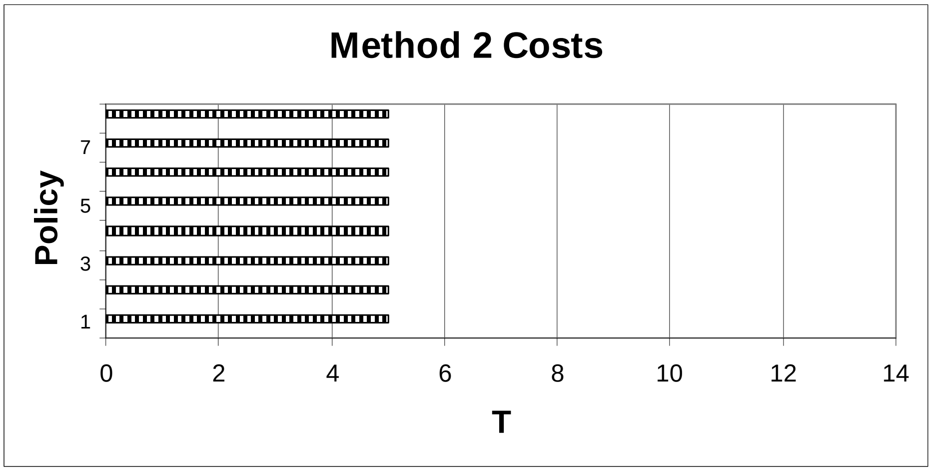


Figure 5: Survey Design 2 Costs



When considering the design of the survey it is important to make the policy choices as close to a real life situation as possible. In the case of public goods, I believe that it is most realistic to have costs uniformly start today and benefits start with a delay of zero to Y years, with Y selected such that respondents still believe that the policy will affect them. It is common for taxes to begin now and continue with a specific duration at the same cost per year and benefits to arrive at different times in the future at the same level of benefits per year. Therefore, I design my survey like "Method 2."

There are four principles identified as important in the literature when designing survey questions for choice experiments (conjoint questions). Level balance means that each level of an attribute should occur an equal number of times in the survey. Orthogonality essentially means that estimable effects should not be correlated. Minimal overlap stipulates that attribute levels should be repeated within choice sets as little as possible. Utility balance attempts to balance the utility of the alternatives within a choice set. It is not generally possible to simultaneously uphold all four of these design principles. One popular quantitative measure of design efficiency is D-error $= |\Sigma|^{1/k}$, where Σ is the covariance matrix of the maximum likelihood estimator in the conditional logit model and k is the number of parameters in the model. By minimizing D-error, the researcher can approximately satisfy the four design principles.

Clearly, utility balance can only be achieved when the researcher has some a priori information about the parameters to be estimated. Huber and Zwerina [13] show that even when the parameter estimates are incorrect there are efficiency gains from using them in the survey design. The SAS *choiceff* macro directly minimizes D-error to generate efficient choice designs for the conditional logit model, allowing the researcher to use a priori estimates on model parameters. No research exists on design efficiency for more complicated models, like the one proposed in this paper.

However, meeting the design principles for the simple conditional logit model should provide a good design for my more complicated model. Applying this reasoning, I use the conditional logit results from Viscusi and Huber as the parameter estimate inputs for the *choiceff* macro to create my survey design.

There is little consensus on how many choice sets to create or how many choice sets each individual should face. Respondents can become fatigued when faced with too many choice sets. Not including enough choice sets can lead to an inability to estimate the desired parameters. In my design, I generate 32 choice sets and divide them into four versions so that each respondent answers eight choice questions. Each choice set contains two alternatives. Each alternative is defined by three attributes: "percentage of basin cleaned", "cost of the policy per year", and "time when cleanup is fulfilled". The first two attributes each have three levels, while the third attribute has six levels. The percentage of basin cleaned ranges from fifty to seventy percent, costs range from \$100 to \$300 per year, and delays range from zero to five years. I identify discount factors by varying the level of the River Basin cleanup and the number of years until the cleanup is fulfilled. The cost attribute facilitates estimation of the per-year willingness to pay but doesn't not affect the discount rate because costs have the same time dimension over all alternatives. Simulation analysis confirms that this survey design is sufficient to identify the discounting parameters.[8]

Importantly, note that this survey design keeps the length of delay before the more delayed cleanup independent from the length of the interval between cleanup alternatives. Also, since this is a public good choice, transaction costs are not a factor. Once a respondent indicates their preferred policy, they no longer have a role in the execution of the policy. The effort required of the respondent is no different whether the cleanup happens today or years from now. Finally, uncertainty in the receipt of a future reward is also minimized in this survey. Explicit instructions are

[8] Simulation results are not presented in this paper but are available from the author upon request.

repeated in the survey that there is no difference in the probability of cleanup for a policy with immediate benefits versus one with delayed benefits.

4.2 Italian Money Data

I also have money choice data from Alberini and Chiabai's 2004 survey of 776 Italian residents [2]. In this survey, respondents choose between a hypothetical immediate lump-sum payment and a hypothetical stream of constant payments over 10 years. The lump-sum payment option is always €10,000 received now. The stream of constant payments option is varied with annual payments of €1150, €1500, or €1650. The respondents also have a third option of being indifferent between the lump-sum and the annuity. In this analysis, I throw out the observations for which the respondent is indifferent (64 observations lost) since an individual cannot be indifferent between alternatives in the random utility model. This leaves 712 observations.

4.3 State Lottery Lump Sum vs. Annuity Choice Data

In addition to the two stated-preference data sources already presented, I introduce a data set containing choices that lottery jackpot winners have made between lump sum and annuity payment options. Many states offer winners the option between a smaller lump sum payout and a larger sum of annual payments–the annuity option. Winners make choices over huge sums of money, providing a rich source of revealed-preference data. I have gathered data from three different state lotteries: Colorado Lotto, Texas Lotto, and Florida Lotto. These three states have open records laws which facilitated collection of the data.[9]

Annuity options are defined by two variables; the number of annual payments and the dollar amount of each payment. Comparing the stream of payments option

[9] Colorado's lotto data is publicly available on the internet. Texas and Florida's lottery agencies responded to my requests for data. Unfortunately, no information on personal characteristics of winners is available.

to the lump sum option, one can calculate the implicit interest rate of the annuity. The implicit interest rate is the rate that equates the present value of the annuity stream to the lump sum option. An individual prefers the lump sum payment over the annuity payments if the lump sum value exceeds their own internal present value of the annuity. Equivalently, an individual prefers the lump sum payment over the annuity payments if their internal (exponential) discount rate is higher than the implicit interest rate offered in the annuity. The less patient the individual, the more likely they will be to take the lump sum option. By observing the choices that winners make between the two options at multiple implicit interest rates, I am able to identify the average discount rate for lottery winners.

All three of these state lotteries advertise the dollar amount of the annuity option. Colorado and Florida allow winners to choose whether they want the lump sum or the annuity option after winning when claiming the prize. However, Texas requires winners to select their payment option when purchasing the ticket. Texas provides information to lottery players about the estimated lump sum payment for a given drawing. Therefore, I use the actual lump sum and annuity options offered to winners for Colorado and Florida but rely on the advertised lump sum and annuity options available to Texas lottery winners at the time of ticket purchase. Currently, the Colorado Lotto stipulates that the lump sum option is 50 percent of the annuity option. Prior to November of 2003, Colorado's lump sum option was equal to 40 percent of the annuity option. Alternatively, Texas and Florida's lump sum option varies as a percentage of the annuity option. Therefore, I get variation in the implicit annuity interest rate over the lotteries, which aids identification of the discount rate. Florida Lotto 30(20)-yr offers 30(20) annual payments for the annuity option. Federal and State tax rates are equivalent for lump sum and annuity payments so they do not bias the results.

Table 6 summarizes the data for the three state lotteries. As expected, the lottery

Table 6: Summary of State Lotteries

Lottery	**Date Range**	**N**	**Lump Sum / Annuity**	**Implicit Annuity Interest Rate**	**% Choosing Lump Sum**
Colorado Lotto 40%	08/20/1994–10/25/2003	177	40%	9.98%	60.45%
Colorado Lotto 50%	11/12/2003–1/05/2008	37	50%	6.97%	86.49%
Texas Lotto	10/27/2001–12/08/2007	74	54.7% to 64%	5.89% to 4.16%	82.43%
Florida Lotto 30-yr	11/28/1998–12/22/2007	343	42.5% to 70.3%	7.45% to 2.64%	91.80%
Florida Lotto 20-yr	10/24/1998–11/14/1998	5	64.5% to 64.7%	5.2% to 5.15%	60.00%
Total		636			81.43%

Note: The implicit annual interest rate is the interest rate that equates the present value of the sum of annuity payments to the lump sum option.

with the highest implicit annuity interest rate (Colorado Lotto 40 percent) has the lowest percentage of winners choosing the lump sum option. The lotteries with the lowest implicit annuity interest rates (Texas and Florida 30-yr) have the lowest percentage of winners choosing the lump sum option. Within Colorado, moving from an implicit annuity interest rate of 9.98 percent to 6.97 percent results in a jump in the percentage of winners choosing the lump sum option from 60.45 percent to 86.49 percent.

Table 7 illustrates the distribution of the Texas and Florida lotteries. For Texas, higher implicit annuity interest rates correlate with a lower percentage of winners choosing the lump sum option. Almost half of the Texas winners faced with implicit annuity interest rates higher than 5.43 percent choose the annuity option whereas only about 5.5 percent of the Texas winners facing implicit annuity interest rates lower than 4.75 percent choose the annuity option. There is less variation in the percentage of winners choosing the lump sum option in the Florida Lotto. However, it holds that fewer winners choose the lump sum option with a higher implicit annuity interest rate. Clearly, winners are considering the implicit annuity interest rate.

One expects decision makers to perceive more credibility in the receipt of a future reward for an official state lottery than for a laboratory experiment. Transaction costs are likely to be minimal for the receipt of future lottery payments because payments are spelled out explicitly in the annuity agreement. Also, transaction costs will be much less significant as a percentage of the huge sums of money at stake here compared to the small rewards in laboratory experiments. As shown in Table 8, the average size of the lump sum option throughout the data set is almost six million dollars. Finally, the nature of the lottery choice is different from most laboratory choices. Instead of comparing various one time payments to an anchor time period, a stream of annual payments is compared to a lump sum. Time interval is not defined in the same sense as Read's concept of subadditive discounting.

Table 7: Distribution of Texas and Florida Lottery Winners

Lottery	**N**	**Lump Sum/Annuity**	**Implicit Annuity Interest Rate**	**% Choosing Lump Sum**
Texas Lotto	17	54.7% to 56.99%	5.89% to 5.43%	58.82%
	16	57% to 58.99%	5.4% to 5.04%	81.25%
	23	59% to 60.49%	5.02% to 4.76%	91.30%
	18	60.5% to 64%	4.75% to 4.16%	94.44%
Florida Lotto 30-yr	86	42.5% to 50.64%	7.45% to 5.6%	84.90%
	84	50.68% to 54.64%	5.59% to 4.85%	94.00%
	85	54.7% to 57.5%	4.83% to 4.38%	94.10%
	88	57.49% to 70.3%	4.37% to 2.6%	94.30%

Table 8: Magnitude of the State Lottery Lump Sum Options

Lottery	**Date Range**	**N**	**Mean Lump Sum Option**	**Median Lump Sum Option**	**Standard Deviation Lump Sum Option**
Colorado Lotto 40%	08/20/1994–10/25/2003	177	1,996,676.76	1,600,000.00	1,401,166.07
Colorado Lotto 50%	11/12/2003–1/05/2008	37	1,691,205.00	1,416,067.00	1,121,074.35
Texas Lotto	10/27/2001–12/08/2007	74	12,366,727.35	7,993,148.73	13,263,747.56
Florida Lotto 30-yr	11/28/1998–12/22/2007	343	6,813,203.19	4,374,972.36	6,457,104.32
Florida Lotto 20-yr	10/24/1998–11/14/1998	5	4,567,324.61	4,866,004.40	408,984.15
All Data	08/20/1994–12/22/2007	636	5,803,285.40	3,483,569.56	7,341,957.70

5 Estimation Results

Since the parameters enter choice utility in a nonlinear fashion it is necessary to write my own estimation code. I utilize the unconstrained minimization routine in Matlab's Optimization Toolbox V3.0.4 to minimize the negative of the log likelihood function as in Equations 17 and 18. The asymptotic standard errors for the maximum likelihood parameter estimates, $\widehat{\beta}$, are estimated with the diagonal entries of $\sqrt{H^{-1}}$, where H is the Hessian matrix of second derivatives $= \frac{\partial^2 LL(\widehat{\beta})}{\partial\widehat{\beta}\partial\widehat{\beta}'}$. The Hessian is calculated by the BFGS method. [20]

5.1 Results for the Minnesota River Basin Survey

Three specifications are explored for the deterministic portion of utility. In Specification I, no interactions are assumed and I have $U_{ij} = \sum_{t=0}^{T_j} \psi_t[\alpha q_{ijt} + \gamma(Y_{it} - c_{ijt})] + \epsilon_{ij}$. Results for Specification I are shown in Table 9. In I.a. I assume Harvey hyperbolic discounting. I.b. assumes HM hyperbolic discounting, I.c. assumes exponential discounting, and I.d. assumes quasi-hyperbolic discounting. All coefficients are highly significant for each estimation. In I.a., for the Harvey model, $\widehat{\mu} = 0.389$. This is in line with estimates from previous studies. In I.b., for the HM Model, $\widehat{\omega} = 0.148$. Recall that as ω goes to zero the HM discounting model becomes the exponential discounting model. Thus, $\widehat{\omega} = 0.148$ suggests that the best fitting HM hyperbolic model is close to an exponential model. In I.c. I estimate a constant discount factor of $\widehat{\delta} = .9074$. This is equivalent to an estimated discount rate of $\widehat{r} = 10.2$ percent. In other words, individuals discount the future at a constant rate of 10.2 percent per year. In I.d., for quasi-hyperbolic discounting, $\widehat{\beta} = .9287$. Recall that the β parameter measures the extent of the departure from the exponential discounting model. Quantitatively, $\widehat{\beta} = .9287$ does not represent a large deviation from the exponential assumption.

Table 9: Minnesota River Basin Maximum Likelihood Results: Specification I

Variable	**I.a.**	**I.b.**	**I.c.**	**I.d.**
Basin Improvement	0.02476***	0.02496***	0.02462***	0.02493***
	(0.00261)	(0.00259)	(0.00260)	(0.00267)
Cost	-0.00298***	- 0.00302***	-0.00303***	-0.00304***
	(0.00022)	(0.00022)	(0.00022)	(0.00023)
Harvey (μ) Parameter	0.38926***			
	(0.03762)			
HM (ω) Parameter		0.14756***		
		(0.02156)		
Exponential (δ) Parameter			0.90740***	
			(0.00905)	
Quasi-Hyperbolic (β) Parameter				0.92871***
				(0.12708)
Quasi-Hyperbolic (δ) Parameter				0.91071***
				(0.01079)
Log L	-1144.579	-1141.172	-1140.104	-1139.964

Note: Asymptotic Standard Errors are given in parenthesis.
* significant at 10%, **significant at 5%, *** significant at 1%

In Specification I, the exponential discounting model has the second highest log likelihood value. Thus, exponential discounting is preferred to either of the single parameter hyperbolic models. Models I.c. and I.d. are nested; I.c. is a special case of I.d. with the restriction that $\beta = 1$. Therefore, I can perform a likelihood ratio test with the null hypothesis that $\beta = 1$. The test statistic is equal to twice the difference of the log likelihoods and is distributed chi-square with one degree of freedom. From Table 9, the likelihood ratio test statistic is equal to 0.2796. Hence, I fail to reject the null hypothesis that $\beta = 1$. There is no evidence in this first specification to support quasi-hyperbolic discounting over the standard exponential model.

In Specification II, I assume a full set of personal interactions: $U_{ij} = \sum_{t=0}^{T_j} \psi_t[\alpha q_{ijt} +$

Table 10: Minnesota River Basin Maximum Likelihood Results: Specification II

Variable	**II.a.**	**II.b.**	**II.c.**	**II.d.**
Basin Improvement	0.03216*** (0.01116)	0.03532*** (0.01134)	0.03652*** (0.01176)	0.03579*** (0.01165)
Cost	-0.00283*** (0.00078)	- 0.00301*** (0.00079)	-0.00310*** (0.00079)	-0.00305*** (0.00079)
Improvement X Age	-0.00023 (0.00014)	-0.00024 (0.00014)	-0.00024 (0.00015)	-0.00024 (0.00016)
Improvement X Income/10000	0.00276*** (0.00081)	0.00259*** (0.00079)	0.00246*** (0.00078)	0.00258*** (0.00083)
Improvement X Male	-0.00247 (0.00590)	-0.00340 (0.00573)	-0.00368 (0.00563)	-0.00332 (0.00581)
Improvement X Resident	-0.00776* (0.00466)	-0.00674 (0.00455)	-0.00602 (0.00449)	-0.00659 (0.00465)
Improvement X Education	-0.00097 (0.00161)	-0.00148 (0.00158)	-0.00172 (0.00156)	-0.00152 (0.00158)
Cost X Income/10000	-0.00019*** (0.00007)	-0.00019*** (0.00007)	-0.00019*** (0.00007))	-0.00019*** (0.00007)
Cost X Male	0.00021 (0.00053)	0.00027 (0.00053)	0.00029 (0.00053)	0.00027 (0.00053)
Cost X Resident	0.00060 (0.00043)	0.00055 (0.00043)	0.00051 (0.00043)	0.00055 (0.00044)
Cost X Education/100	0.00781 (0.01531)	0.01096 (0.01537)	0.01275 (0.01538)	0.01140 (0.01540)
Harvey (μ) Parameter	0.39486*** (0.04111)			
HM (ω) Parameter		0.15102*** (0.02525)		
Exponential (δ) Parameter			0.90658*** (0.01123)	
Quasi-Hyperbolic (β) Parameter				0.88830*** (0.11928)
Quasi-Hyperbolic (δ) Parameter				0.91197*** (0.01330)
Log L	-1140.263	-1137.214	-1136.473	-1136.077

Note: Asymptotic Standard Errors are given in parenthesis.
* significant at 10%, **significant at 5%, *** significant at 1%

$\gamma(Y_{it} - c_{ijt}) + \lambda q_{ijt} x_{it} + \xi c_{ijt} x_{it}] + \epsilon_{ij}$, where x_{it} is a vector of personal characteristics for individual i at time t. Personal characteristics that could potentially influence utility include age, income, sex, education level, and whether the respondent resides within the Minnesota River Basin. Attempts to estimate the model including the variable "Cost X Age" fail to converge. Therefore, I drop "Cost X Age" from the model and estimate Specification II with the remaining variables. Table 10 reports results for this interactions specification.

Results for the discounting parameters in Specifications II.a.-d. are similar to results from Specifications I.a.-d. Again, the exponential discounting model fits the data better than the two single-parameter hyperbolic models. Viewing II.c. as a restricted model of II.d. I can again perform a likelihood ratio test. The test statistic is equal to .792 so I fail to reject the null hypothesis that $\beta = 1$. There is no evidence in this interactions model in support of quasi-hyperbolic discounting.

In specification III, I assume that discounting parameters are random coefficients. Specifically, I assume that discounting parameters vary over people but are constant over choice situations for each person. In III.a., I assume hyperbolic discounting with the single parameter, μ_i, being distributed normally with mean $\overline{\mu}$ and variance z_μ^2. In III.b., I assume that the single parameter for HM hyperbolic discounting, ω_i, is distributed normally with mean $\overline{\omega}$ and variance z_ω^2. III.c. assumes exponential discounting with a discount factor, δ_i, that is distributed normally with mean $\overline{\delta}$ and variance z_δ^2. Finally, III.d. assumes quasi-hyperbolic discounting with a constant β factor and a δ_{Hi} factor that is distributed normally with mean $\overline{\delta_H}$ and variance $z_{\delta_H}^2$. I derive the Simulated Log Likelihood equation in appendix A. Attempts to treat both the β factor and the δ_H factors as random fail to converge.

Table 11 gives results for the random coefficients specifications. The maximized value of the simulated log likelihood equation is greater in the exponential specification (III.c.) than in either of the single parameter hyperbolic specifications (III.a. and

Table 11: Minnesota River Basin Simulated Maximum Likelihood Results: Specification III

Variable	**III.a.**	**III.b.**	**III.c.**	**III.d.**
Basin Improvement	0.52481***	0.41877***	0.45925***	0.44873***
	(0.08040)	(0.05772)	(0.05906)	(0.09564)
Cost	-0.07194***	- 0.05439***	-0.06271***	-0.06170***
	(0.00813)	(0.00638)	(0.00680)	(0.00993)
Harvey (μ) Parameter Mean	0.50596***			
	(0.09250)			
Harvey (μ) Parameter S.D.	0.45465***			
	(0.09364)			
HM (ω) Parameter Mean		0.32920***		
		(0.04742)		
HM (ω) Parameter S.D.		0.14180***		
		(0.01616)		
Exponential (δ) Mean			0.87976***	
			(0.02154)	
Exponential (δ) S.D.			0.10377***	
			(0.02616)	
Quasi-Hyperbolic (β) Parameter				1.02834***
				(0.20711)
Quasi-Hyperbolic (δ) Mean				0.87761***
				(0.02727)
Quasi-Hyperbolic (δ) S.D.				0.10558***
				(0.02941)
Simulated Log L	-1222.1843	-1229.4897	-1218.8206	-1218.8106

Note: Asymptotic Standard Errors are given in parenthesis.
* significant at 10%, **significant at 5%, *** significant at 1%

III.b.). Freeing up the additional β parameter in III.d. leads to only a miniscule improvement in the simulated log likelihood at convergence compared to III.c. with β restricted to one. Once again, I fail to reject the null hypothesis of exponential discounting.

As seen in Table 11, significant heterogeneity exists in the discounting parameters throughout all four discounting models. There is an especially wide distribution in the Harvey hyperbolic discounting parameter, μ, for which the standard deviation is almost as large as the parameter estimate. By theory, exponential discount factors should be less than or equal to one. Estimates in III.c. imply that approximately 88 percent of respondents have an exponential discount factor that is less than one so it is encouraging that a large percentage of people fit with the theory. When accounting for heterogeneity in discount factors, the average exponential discount factor is 0.87976, which corresponds to an average annual exponential discount rate of 13.67 percent.

In summary, all specifications lead us to fail to reject the null hypothesis that $\beta = 1$. In other words, freeing up the additional β parameter in the quasi-hyperbolic framework does not significantly improve model fit over the standard exponential model. Finally, the maximized value of the log likelihood function is greater in the exponential cases (c.) than in the hyperbolic cases (a. and b.) for all specifications. I conclude that standard exponential is the preferred discounting model for all utility specifications.

5.2 Results for the Italian Money Data

With the two money data sets, I can estimate the discounting model assuming either standard exponential or single parameter hyperbolic functional forms. It is not possible to uniquely identify quasi-hyperbolic discount factors because I never observe choices between two future outcomes. That is, annuity and lump sum options are anchored to the present in all choices.

I apply equations 17, 18 and 22 to the Italian money data set. Assuming exponential discounting, maximum likelihood estimation gives $\widehat{\delta} = .8999$ with an estimated standard error of .0013. Since $\delta = 1/(1+r)$, this implies $\widehat{r} = 0.111$. That is, individuals discount with a constant rate of 11.1%. This is slightly higher than the results from Alberini and Chiabai, as they found $\widehat{r} = 0.087$. Recall that I did throw out the observations for which the respondent is indifferent between the lump-sum and the annuity, which may explain the difference. For the Harvey hyperbolic model, I find $\widehat{\mu} = .2992$ with an estimated standard error of 0.0038. Finally, for the HM hyperbolic model, I estimate $\widehat{\omega} = 0.1431$ with an estimated standard error of 0.0024. The maximized value of the log-likelihood function is identical under all three specifications. This data set does not provide enough information to prefer one discounting specification over the others.

5.3 Results for the State Lottery Data

Here, I apply equations 17, 18 and 22 to the state lottery data. Table 12 summarizes maximum likelihood results for exponential and hyperbolic specifications. Assuming an exponential discounting form leads to an estimate of 0.927 for the constant discount factor. This is equivalent to a constant discount rate of 7.84%. I also assume the Harvey hyperbolic functional form and estimate the single parameter at 0.375. For the HM hyperbolic model, I estimate the single parameter at 0.134. These point estimates on the discounting parameters are similar to those from the other two data sources. Discounting parameters are highly significant in each model. Comparing the maximized values of the log likelihood functions, the HM hyperbolic model fits the data better than the Harvey hyperbolic model. Consistent with the Minnesota River Basin results, the exponential specification is preferred to both of the hyperbolic specifications. Thus, this data set also supports constant discount rates over declining

Table 12: Results for State Lottery Data: N=636

Discounting Model	**Parameter Estimate**	**Log L**
Harvey Hyperbolic	0.375*** (0.001623)	-865.708
HM Hyperbolic	0.134*** (0.000945)	-837.749
Exponential	0.927*** (0.000342)	-822.686

Note: Asymptotic Standard Errors are given in parenthesis.
* significant at 10%, **significant at 5%, *** significant at 1%

discount rates. The magnitude of the discount rate is within the range of interest rates found in capital markets, which implies that individuals do equate the marginal rate of substitution between two years to one plus the interest rate.

6 Concluding Remarks

The empirical strategy introduced in this paper provides a method of estimating discounting factors that is consistent with utility-maximization theory. I structurally model intertemporal choices to produce explicit estimates of discounting parameters. This is an improvement over previous work because different forms of discounting functions can be formally tested. This general estimation framework can be applied to private or public goods choices.

I apply the empirical model to three data sources which represent private and public goods, and stated-preference and revealed-preference choices. Estimation results from two of the data sources suggest that the standard exponential discounting model is preferred to single-parameter specifications of the hyperbolic discounting model. Likelihood ratio tests of the quasi-hyperbolic model for the public goods data fail to reject the null hypothesis of standard exponential discounting. Estimates of the constant exponential discount rates range from approximately eight to eleven percent throughout the three data sets.

I find evidence that individuals do behave rationally when making intertemporal decisions. They are dynamically consistent in their choices and do not appear to be present-biased. The range of discount rates estimated here falls below the discount rates commonly found in the experimental literature but is consistent with interest rates that we see in capital markets, as we would expect from theory. From a policy perspective, these results have implications for a variety of contexts including personal savings decisions, participation in preventative health programs, the formation of human capital, and environmental sustainability.

Because of the nature of the original data sets employed in this paper, confounding factors that are commonly part of experimental studies are minimized. Specifically, the data sets minimize perceived uncertainty in the receipt of future rewards, perceived future transaction costs, and the correlation between the length of delay before a future outcome and the length of the interval between two outcomes. I propose that much of the prior evidence for hyperbolic discounting may be questionable when these confounding factors are considered.

A Random Coefficients Simulated Log Likelihood Equation

Here, I develop the simulated log likelihood equation for the random coefficients specification. For clarity, I present the exponential discounting case. All other discounting models are easily derived with a few substitutions. This section loosely follows the exposition of Train. [27]

Recall the probability of a single choice for the non-stochastic discounting parameters case, $P_{ij} = F\left(\frac{\sum_{t=0}^{T_j}\delta^t v_{ijt} - \sum_{t=0}^{T_k}\delta^t v_{ikt}}{\sqrt{\sum_{t=0}^{T_k}\delta^{2t} + \sum_{t=0}^{T_j}\delta^{2t}}}\right)$. In the case of random discounting parameters, I focus on the sequence of choices by individual i. Denote the choice situation as s and a sequence of alternatives as $\mathbf{j} = \{j_1, ..., j_S\}$ Then, conditional on δ, the probability that individual i makes a sequence of choices is the product over all s of the single choice probabilities. I have

$$\mathbf{P}_{i\mathbf{j}}(\delta) = \prod_{s=1}^{S} F\left(\frac{\sum_{t=0}^{T_{j,s}}\delta_i^t v_{ijts} - \sum_{t=0}^{T_{k,s}}\delta_i^t v_{ikts}}{\sqrt{\sum_{t=0}^{T_{k,s}}\delta_i^{2t} + \sum_{t=0}^{T_{j,s}}\delta_i^{2t}}}\right). \tag{23}$$

Since the δ are random, I integrate out over all values of δ to get the unconditional choice probability

$$P_{i\mathbf{j}} = \int \mathbf{P}_{i\mathbf{j}}(\delta) f(\delta) d\delta. \tag{24}$$

I draw R values of δ from $f(\delta)$ and denote them δ_r. The simulated choice probability is $\widetilde{P}_{i\mathbf{j}} = \frac{1}{R}\sum_{r=1}^{R}\mathbf{P}_{i\mathbf{j}}(\delta_r)$. In this application, I set $R = 200$. Finally, I insert these simulated choice probabilities into the log-likelihood function to get the simulated log likelihood (SLL)

$$\text{SLL} = \sum_i \sum_{\mathbf{j}} y_{i\mathbf{j}} \ln \widetilde{P}_{i\mathbf{j}}, \tag{25}$$

where $y_{i\mathbf{j}} = 1$ if i chose sequence $\mathbf{j}$ and zero otherwise.

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