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Demand for Contract Enforcement and Gains from Trade

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Abstract

We develop a dynamic exchange environment to analyze the value created by contracting institutions. In the model, a contract is a pre-agreed specification of behavior, which may be subsequently enforced by a third party. We study the effect of economic fundamentals on the demand for such an enforcement agency. We show that this demand may exist even when contracts are sometimes broken in equilibrium, and ask whether this demand is increasing in the potential gains from exchange. Surprisingly, this is not always the case – indeed, if the gains are sufficiently large, the demand may drop to zero. We identify the discount factor and the quality of enforcement as crucial factors behind this relationship.

JEL Codes: H11, H41, K42, 017

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1 Introduction

It is almost a truism that an opportunity for mutually beneficial exchange is also an occasion for opportunistic behavior, and an extensive literature is devoted to the

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effects of institutions that support exchange upon economic prosperity.[1] We explore the reverse. We ask under what conditions the existence of potential gains from trade can generate a demand for institutions that enforce contracts. In addition, we ask whether this demand is positively related to the potential gains from trade.

In our model individuals face *quid-pro-quo* exchange opportunities. We define a *contract* as a pre-agreed specification of behavior that may be used as a basis for enforcement:[2] thus, this environment, a contract is an agreement between agents on the procedure of exchange. However, each agent may independently choose not to fulfill her side of a contract. Following Dixit (2004), we assume a large, anonymous market in which agents do not meet twice. In such an environment, any exchange agreement will be broken in the absence of enforcement, and the equilibrium value of resources is low as a result. Thus, there is a natural role for an enforcement agency that sanctions contracts and punishes infractors. Our goal is to understand whether such an agency is sustainable as the gains from trade increase.

As noted, we focus on the *demand* for contract enforcement, envisioning the supplier as a centralized agency that can register contracts, impose a punishment on infractors and re-instate the victims. However, detection is imperfect, and – in the eyes of the agents – it occurs randomly, with a certain probability. We refer to this probability as *the quality of institutions*. We take the quality of institutions and the level of punishment as given, and derive the demand for enforcement for all possible levels thereof.

Individuals in our economy are ex-ante identical: their willingness to sign contracts and subsequently fulfill their obligations depends upon institutional parameters, as well as their expectation of how their trading partner is to behave. If punishment is sufficiently severe, agents are willing to pay for their contracts to be notarized. Interestingly, an agent's willingness to pay for enforcement may be independent of whether or not she actually does follow the terms of any given contract. This is so in spite of the fact that contracting is costlier for infractors, who may be subject to punishment if caught. However, their alternative is not to register their agreements — in which case their partners would not find it optimal to attempt to trade with them either. In effect, the enforcement agency introduces a commitment mechanism that helps individuals assure their partners that the likelihood of them behaving as traders is the same as the equilibrium fraction of traders in the population. This ability to commit increases the value of exchange goods for all individuals (not just traders) and, as a result, both trading behavior and breaches of contract may coexist in equilibrium.

[1] See North (1984), Alston and Mueller (2004), Besley (1995) and Shleifer (1998) *inter alia.*

[2] Contract: A mutual agreement between two or more parties that something shall be done or forborne by one or both esp. such as has legal effects [...]

Oxford English Dictionary (Second Edition, 1989).

Having established the conditions under which there exists a willingness to pay for contract enforcement in equilibrium, we examine how it is affected by what we call *gains from trade* – the potential value to an agent from a successful interaction with another agent. Perhaps surprisingly, we find that this effect is ambiguous. On the one hand, increased gains from trade directly augment the value of a successfully traded good. On the other hand, the lure to break the contract strengthens, promoting contract infringement. We show that the second effect dominates if the quality of institutions is low or the discount rate is high: in that case, the demand for enforcement falls to zero if the gains to trade are sufficiently large.

Section 2 provides an overview of the related literature. Section 3 develops the model environment, and Section 4 characterizes equilibrium behavior. Section 5 studies the demand for contract enforcement. Section 6 concludes by discussing further implications and extensions.

2 Motivation

We are motivated by the observation that, even in the absence of a formal government, enforcement institutions may emerge whenever there is an economic rationale for their existence. Stigler (1992) argues that, under a strong interpretation of the Coase (1960) Theorem, if there is value to be created by the provision (or the improved provision) of a certain service then some arrangement should arise that provides it, be it state-based or otherwise, whether motivated by benevolence or by personal gain. Along the same lines, Dixit (2004) suggests that institutions of economic governance, generally taken for granted in economic models, should be modeled explicitly.

> "Of course economic activity does not grind to a halt because the government cannot or does not provide an adequate underpinning of law. Too much potential value would go unrealized; therefore groups and societies have much to gain if they can create alternative institutions to provide the necessary economic governance." (Dixit (2004)).

This suggests that one approach to thinking about institutions of economic governance is to consider the determinants of the underlying value that they can create. This is the approach that we follow.[3]

There is significant breadth of form that these institutions have taken on historically. For example, the analysis of Greif (2005) spans from communes governing

[3] An independent question concerns the mechanisms whereby these institutions actually do arise, and we do not address this question herein. For example, Greif et al. (1994) and Greif (1992) provide historical case studies of the processes that lead to the supply of enforcement, suggesting that they are complex though amenable to economic analysis.

'impersonal exchange' in pre-modern Europe to the emergence of Genoa as a state. Dixit (2004) surveys some more contemporary cases of economic governance. In addition, there is a literature that interprets certain types of organized crime as "a form of governance of the illegal marketplace."[4] More recently, the example of Somalia[5] illustrates the variety of possible modes of economic governance that can arise in the absence of a well-functioning state.

In spite of this variety, North (1984) and more recently Acemoglu and Johnson (2003) offer a way to distinguish between two broad classes of economic governance: property rights institutions, and contracting institutions. They define "property rights" as protection from predation, and contracting institutions as the enforcement of private agreements. We focus squarely upon the latter.

Recent work on the economics of enforcement has largely concentrated upon property rights. The conventional economic rationale for property rights institutions to arise and provide welfare improvements hinges upon benefits from the centralization of force. If there are increasing returns to scale in defence, centralization eliminates over-investment in the "arms race" that would obtain in a decentralized (anarchic) society, as in Skaperdas (1992). Grossman (2001) suggests that effective property rights might result from an interplay between centrally and privately provided protection. Bös and Kolmar (2003) analyze the redistributive norms that might underlie the stability of an environment in which expropriation is possible.

We concentrate, instead, on contracting institutions. We develop a simple exchange environment in which a contract is a well-defined concept. Our benchmark model is one of *anarchy* in the sense of Hirshleifer (1995), characterized by the absence of "higher authorities or social pressures." Into this environment we introduce a "third party", contract enforcement agency. This mode of economic governance encompasses formal government, and is widely prevalent across time and place. Social norms could, under certain circumstances, serve as a contract enforcement device in an otherwise anarchic environment, in essentially the same way that cooperation can be supported in a repeated Prisoners' Dilemma (PD).[6] Crucial then are assumptions about the ability of agents to identify defectors, communicate this information to each other, and coordinate punishments. For example, Dixit (2003) develops a model in which both the gains from trade and the anonymity of interaction increase with the "distance" between traders. As a result, bigger markets are plagued by robbery in the absence of an external enforcer. He shows that, if effective enforcement is available at a fixed price, economies will only be able to afford it if they are sufficiently large. Thus, even in the presence of norms, third-party involvement can be

[4] Beare and Naylor (1999); see also Gambetta (1993).

[5] See Little (2003); an anecdotal overview is available at *http://news.bbc.co.uk/1/hi/world/africa/1615258.stm.*

[6] See for instance Kandori (1995).

beneficial. Moreover, in Dixit (2003), interactions involve a PD with fixed payoffs, so that the outcome of a given match bears no direct consequence for the future payoffs of either agent. By contrast, in our setup the value of bringing a tradeable good to a marketplace is determined in equilibrium, depending on the frequency of contract violations, which, in turn, depends on the enforcement technology available to the third-party. Consequently, interactions may or may not have the payoff structure of a PD in our model in equilibrium.

In related work, Moselle and Polak (2001) develop a model in which welfare depends on the level of property rights, which in turn is determined by the behavior of the (potentially predatory) state. Unlike their model, in our model all interaction between agents is voluntary: thus, following the terminology of North (1984) and Acemoglu and Johnson (2003), their paper is about property rights whereas ours is about contract enforcement. Moreover, our model is dynamic. The possibility that the agents' goods may have a future use is critical to the results, and the discount factor turns out to play a major role in the relationship between contract enforcement and the gains from trade.

3 Economic Environment

We build the model in stages. First, we describe a contracting environment in the absence of an enforcement agency in Section 3. Then in Section 4, we introduce the agency but assume that the forms of contracts are exogenous, in that they always stipulate trade. This is necessary to characterize the equilibria before we reach our main results. Finally, we endogenize the structure of contracts in Section 5, which allows us to derive the demand for contract enforcement services.

3.1 Basic Model

There is a continuum of infinitely-lived, risk-neutral agents who maximize the discounted sum of expected future payoffs using a discount factor $\delta \in (0, 1)$. Each agent holds one unit of an indivisible durable good. As in the related matching literature (for example Kiyotaki and Wright (1993)), we assume that the good comes in many varieties and that the agent herself does not directly enjoy her possession, but that other agents do. Thus, individuals must interact and obtain the goods of other agents to earn positive utility. The model of interaction is open to several interpretations – for instance, the goods could be interpreted as entrepreneurial ideas that require development or financing for their execution. What is important is that agents require other agents to obtain any gains, and that their partners may abscond.

Agents live in Market town, where there is a bilateral matching technology that pairs agents each period. Meetings are random and anonymous. Each matched pair then receives an opportunity for a mutually beneficial project as described below. When an agent uses (or loses) her good, she leaves the market and is replaced by another agent. If after a match an agent retains her good, she is matched anew the following period.

Matches have two stages. In the first stage, the pair may sign a *contract*, specifying actions to be taken by the pair in the second stage. Agents may also write a "null" contract that allows any action to be taken by either trader in the second stage. One agent (randomly assigned with equal probability) may write, sign, and offer a contract to the other, who either signs the contract or abandons the match. Thus, in accordance with the definition of a "contract" – requiring voluntary participation – agents cannot be worse off under the contract in expected terms than if they were to wait until the following period for another match.

In the second stage, agents choose from the set of actions $\{trade, rob\}$. If an agent chooses *trade*, her good is used up and her partner receives $G > 0$ units of utility. However, if she chooses *rob*, then she obtains her partner's good and receives utility G herself, provided her partner chose *trade*. If both agents choose *rob*, then each one succeeds in capturing the good of the partner with equal probability, giving nothing in return.[7] The winner consumes the good of the loser, retaining her own resources for future transactions. We refer to parameter G as potential gains from trade.

Next period agents having goods to trade are re-matched.

Agents view the value of their possession as the expected stream of utility for which they can exchange it in the Market town. Let V denote the equilibrium value of being in Market town with a unit of a good, or, simply, the value of a tradeable to its holder. The payoff matrix in a given match under *anarchy* is:

		Agent 1	
		Trade	Rob
Agent 2	Trade	G, G	$0, G+\delta V$
	Rob	$G+\delta V, 0$	$(G+\delta V)/2, (G+\delta V)/2.$

Notice that, so long as $V > 0$, this problem has the structure of a Prisoner's Dilemma. The value of the tradeable, V, depends on the probability γ of meeting a trader in the market place, and both of these values are endogenous. For example, if the everyone refuses to sign a contract offered by the partner at every match, this value is zero.

We assume that during each match (within one period), the agents are playing a subgame perfect Nash equilibrium. Hence if a contract is signed by both parties in an equilibrium, the expected payoff to each must be bounded below by δV, the value of waiting until the next period for another partner: recall, the agreement to interact

[7] One can imagine that having verified that the partner has the desired good, each of the individuals "fight" to get it, and both are equally skilled at fighting. The winner, naturally, pays nothing to the loser.

is voluntary. This rules out asymmetric contracts prescribing one of the agents to rob and another to trade, i.e., none of such contracts will be signed at an equilibrium. Out of the three symmetric contracts (including the 'null' one) we will be especially interested in the trading one.

Definition 1 *A trading contract stipulates an action profile* $\langle trade, trade \rangle$.

Determining when will trading contracts be written, signed and followed is the key to understanding the role of the enforcement agency in this model, which will be introduced in the next section.

We say that a *breach of contract* occurs if there is a discrepancy between the actions specified in the contract and those that are actually taken in the second stage.

We are looking for the stationary subgame perfect Nash equilibria of the infinitely repeated game. For all the specifications of payoffs that we will consider below, interacting (signing at least the 'null' contract) at least in one match brings a strictly positive payoff, while a refusal to ever sign (or write) a contract generates a zero value of the tradeable. Thus, *some* contracts will be signed by every matched pair.

For most of what follows the choice of the contract between the agents is simple, given their beliefs about behavior in the second stage. As a result, we limit our definition of equilibrium to profiles of actions at the second stage of each interaction.

Thus, $\gamma \in [0, 1]$ is an *equilibrium* if it describes the proportion of agents in the economy who choose to trade in a stationary subgame perfect equilibrium of the infinitely repeated game with random matching.

We focus on pure strategies. If we restrict attention to pure actions in each period, we can also view γ as the expectation held by any agent that his matched partner is going to trade. Mixed strategies (at each stage) might also be allowed in our environment as,[8] for any given set of parameters of the model, our dynamic game can be reduced (using stationarity) to a static one in which (1) agents have only two actions, *trade* and *rob*; (2) the payoff of the agents is V, the value of the tradeable, which depends on the "aggregate" value of γ and the action chosen by this agent; and (3) each agent uses an independent randomization device. However, we focus on pure actions to simplify the exposition and to avoid introducing additional notation. Note that, under both interpretations, if $\gamma \in (0, 1)$ is an equilibrium, any agent is indifferent between trading and robbing in every period, whereas if $\gamma \in \{0, 1\}$ in equilibrium then all agents adopt the same pure strategy each period,[9] and these are the conditions used to calculate equilibria of the model.

[8] See Al-Najjar (2004). That paper also contains an overview of the recent literature articulating and resolving the related measurability problems.

[9] Working with mixed strategies has an advantage of providing another rationale for "history independent" punishments that we introduce later, and we thank the anonymous referee for this insight.

Equilibrium under anarchy[10] is straightforward. Taking the proportion of traders γ as given, an agent has to choose her best action. If she opts to trade, the payoff is γG. On the other hand, if the agent robs and the partner chooses to trade, she earns G and retains her good for continuation payoff V in the following period. Thus, the latter encounter yields the value of $W = G + \delta V$. Finally, if both agents simultaneously attempt to rob, she expects to receive $\frac{1}{2}W$, as she has a chance of a half to capture the possession of the other, while retaining her own. In this case, the payoff conditional on the match is $\gamma W + (1-\gamma)\frac{1}{2}W$. Thus,

$$V = \max\left\{\gamma G, \gamma W + (1-\gamma)\frac{1}{2}W\right\} \tag{1}$$

Due to the stationarity of the problem, if an action is optimal in a given period, it is optimal in any subsequent period.

It is immediate that V is strictly positive for any γ, so that the only equilibrium under anarchy is $\gamma = 0$, and therefore value of the tradeable is $V = \frac{G}{2-\delta}$. Individuals come to Market town only to engage in contests, and although contracts are written and signed, their stipulations are ignored. Any attempt to trade eliminates the chance of consumption altogether.[11]

3.2 Enforcement Agency

Now we introduce an agency that enforces contracts. The agency endorses contracts that are signed by any pair of partners in the first stage of a match, but before agents have chosen their action $\{trade, rob\}$. Then, with probability $\omega \in (0,1)$, the second stage of any given match is observed by the agency. If a contract is broken and this is detected, the agency inflicts a cost c upon the defector and reinstates any stolen items to the injured party. We assume that, in the event that *both* partners attempt to rob and this is observed, only the successful robber is punished. The presumption is that it is impossible to verify an unsuccessful robbery attempt or an intent to rob.

Cost c can be thought of as physical punishment, ignominy, or a claim towards a stream of goods to be produced in the future. We allow c to be a function of the gains from trade, G. We analyze two cases. The first is when punishment is proportional to the crime, i.e. $c(G) = cG$. This is congruent with the observed penalties (fines, or even physical punishments, say, in the time of the Incas) increasing with the severity

[10] as mentioned in the introduction, we follow Hirshleifer (1995) in defining anarchy as a "social arrangement, in which contenders struggle to conquer and defend durable resources, without effective regulation by either higher authorities or social pressures."

[11] An interpretation of this equilibrium is that in order to capture the desired resource of a partner, one has to lure her into transaction first by demonstrating the good wanted by the other. If both select 'rob' in the second stage and even if both expect this outcome, each might still prefer to initiate the transaction in a hope of outwitting the other and capturing her good.

of the transgression, particularly in the case of economic crimes. The second is when $c(G) = c$ is a constant. This better describes cases in which the punishments are bounded by cultural norms or technological constraints.[12]

Parameter ω may be interpreted as reflecting limitations in the technology of surveillance and forensics. Probability of a successful enforcement might also depend on the structure of the internal organization of the enforcement agency, which we take as given. As mentioned in the introduction, we call ω the *quality of institutions*, and say that the agency is characterized by a (c, ω) pair. We will describe equilibria under all possible combinations (c, ω), which we denote the supply of enforcement, and determine economic value generated by each combination.[13] To rephrase, we take "production technology" as given and determine enforcement demand for each level of output.

In the presence of the enforcement agency the payoffs change. Let V^g be the value of her tradeable to an agent who has signed a trading contract. Now, an agent who chooses *trade* and is matched with another who chooses *rob* earns $\omega\delta V^g$ in expectation, as her good is reinstated if the violation (by her partner) is detected. If she meets a fair trader, the payoff is G, as before. Hence, the expected payoff to trading is

$$u(\text{trade}) = \gamma G + (1-\gamma)\,\omega\delta V^g.$$

The payoff to *rob* now takes into account that theft may be observed. If detected, an agent must pay the cost c. Thus, if her partner trades, she earns $W^g \equiv (1-\omega)(G + \delta V^g) + \omega(-c(G) + \delta V^g)$. If both agents *rob*, the expected payoff to each is:

[12] The model of punishment merits some discussion. First, punishment is limited, or else set to fit the crime. This is reasonable for most criminal codes, which do not stipulate for example death for stealing a loaf of bread. Second, punishment c is neither history-dependent nor modeled as a term of imprisonment. Immediate, history-independent punishments characterize most past cultures and judicial systems, except where slavery was used for purposes of retribution or redress. For example, in Europe, jurisprudence ignored individual characteristics (save for political power) until the 18th Century. Deeper in history, an important example is the Code of Hammurabi of the 20th Century BC, which punishes theft with a fine – or death if payment is beyond the ability of the perpetrator. Again, punishment is not history-dependent – except in the case in which the perpetrator is incapable of fulfilling the punishment. See Jastrow (1980). Jewish and Islamic criminal codes are related to the Babylonian codes.

As for imprisonment, according to Foucault (1975) in Europe, a term of service and fines were the common forms of punishment through the early Middle Ages, being replaced by a system of corporeal and capital punishment later on. Imprisonment as punishment did not appear until the 17th Century, and was the lot of few until the early 19th Century, when an elaborate prison system developed. See also Kirchheimer and Rusche (1939).

[13] Equivalently, ω may also be interpreted as the ex-post effectiveness of *guarding* by the enforcement agency. One might be concerned that the role of the enforcer is closer to guarding than to punishment: the "pure guarding" case considers $\omega > 0$, $c = 0$. As follows from Proposition 5, provided $\omega > \delta$, there will still be demand for a "pure guarding" agency.

$$u\,(\text{rob}) = \gamma W^g + (1-\gamma)\left[\frac{1}{2}W^g + \frac{1}{2}\omega\delta V^g\right] \tag{2}$$

Finally,

$$V^g = \max\left\{u\,(\text{trade})\,, u\,(\text{rob})\right\}. \tag{3}$$

4 Exogenous Trading Contracts

Before discussing the demand for contracting institutions, we must first characterize the equilibria of the model. To this aim, we start by describing the equilibrium payoffs that would result if agents were to write only *trading* contracts. To put it another way, in this section, the agency acts as a *trade* enforcement agency and contracts are exogenously given. This will help us to analyze the optimal contract choice in the subsequent section.

Figure (1) illustrate the way equilibria are determined. When the fraction of fair traders, $\gamma \in [0,1]$ is such that an agent is indifferent between the actions $(trade, rob)$; this value of γ is an equilibrium. This corresponds to the zeros of the difference between $u\,(\text{trade})$ and $u\,(\text{rob})$. If this difference is negative (positive) at a corner of the interval, then an equilibrium is $\gamma = 0$ (or unity).

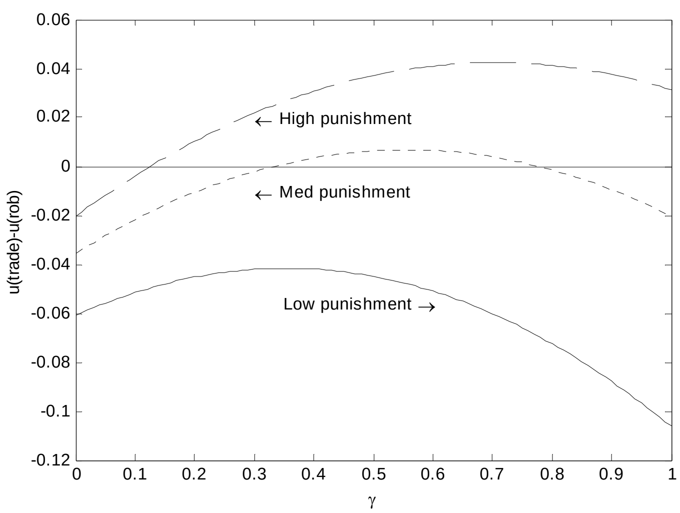

Figure 1: Difference in values of a tradeable accruing to a perpetual fair trader and that to a chronic robber as a function of γ, the fraction of fair traders. The difference is uniformly higher for higher c.

The following proposition describes the structure of equilibria in this model. For extreme values of punishment, either sufficiently high or sufficiently low, an equilibrium is always unique: all agents trade or all of them choose to rob. For intermediate values of punishments, there are interior equilibria, in which some agents rob and the rest choose to trade. Importantly, the boundaries that describe the range of punishments for which interior equilibria exist vary with the parameters. It is this dependence that we will exploit later. Denote the parameters of the model by $\phi = (\omega, \delta, G)$.

Proposition 1 1. *Suppose* $\delta \leq \frac{1}{2}$. *Let* $\underline{c}(\phi) \equiv \frac{G(\delta-\omega)}{\omega}$, $\bar{c}(\phi) \equiv \frac{G(1-\omega)}{\omega}$. *If* $c < \underline{c}(\phi)$, *the only equilibrium is* $\gamma = 0$. *If* $c > \bar{c}(\phi)$, *the only equilibrium is* $\gamma = 1$. *Finally, if* $c \in [\underline{c}(\phi), \bar{c}(\phi)]$, *then there are three equilibria: two on the boundaries,* $\gamma = 0$, $\gamma = 1$ *and one in the interior,* $\gamma_L(\phi, c)$.

2. *Suppose* $\delta > \frac{1}{2}$. *There exists* $\underline{\underline{c}}(\phi) < \underline{c}(\phi)$ *satisfying the following. If* $c < \underline{\underline{c}}(\phi)$, *the only equilibrium is* $\gamma = 0$. *If* $c > \bar{c}(\phi)$, *the only equilibrium is* $\gamma = 1$. *If* $c \in [\underline{\underline{c}}(\phi), \underline{c}(\phi))$, *then there are three equilibria, a corner one,* $\gamma = 0$, *and two interior ones,* $\gamma_L(\phi, c)$ *and* $\gamma_H(\phi, c)$, *where* $\gamma_L(\phi, c) < \gamma_H(\phi, c) < 1$. *Finally, if* $c \in [\underline{c}(\phi), \bar{c}(\phi)]$ *then there are three equilibria: two on the boundaries,* $\gamma = 0$, $\gamma = 1$ *and one in the interior,* $\gamma_L(\phi, c)$.

To clarify Proposition 1, Figure 2 depicts the equilibrium set for an environment in which $\delta > \frac{1}{2}$. Observe that $\gamma_H(\phi, c)$ is increasing in c, whereas the reverse is true of $\gamma_L(\phi, c)$. This is always the case.

Proposition 2 *Suppose* $c(G) = c$, *and that* $\gamma_H(\phi, c)$ *is well defined, so that* $\delta > 1/2$ *and* $c \in \left(\underline{\underline{c}}(\phi), \underline{c}(\phi)\right)$, *Then,* $\gamma_H(\phi, c)$ *is decreasing with* G *and is increasing with* c.

Interestingly, the existence of multiple equilibria is consistent with the empirical findings of Glaeser et al. (1996) that crime rates appear to vary significantly more than can be accounted for by observables.

Why might there be multiple equilibria in this environment? An increase in the proportion of traders γ benefits all agents, independently of whether they decide to rob or to trade. Consequently, the payoff to a robber and to a trader may in principle be equalized at more than one value of $\gamma \in (0, 1)$.

To understand the role of the discount factor, recall that it has an effect on the payoff to both actions, because a violation of the contract may be detected by the enforcement agency. It has a distinct effect on the value of the tradeable by those who rob as, in case of a successful undetected robbery, the agent retains her valued good. Thus, the effect of an increase in the proportion of traders γ is more sensitive to δ for those who choose to rob, than those who choose to trade.

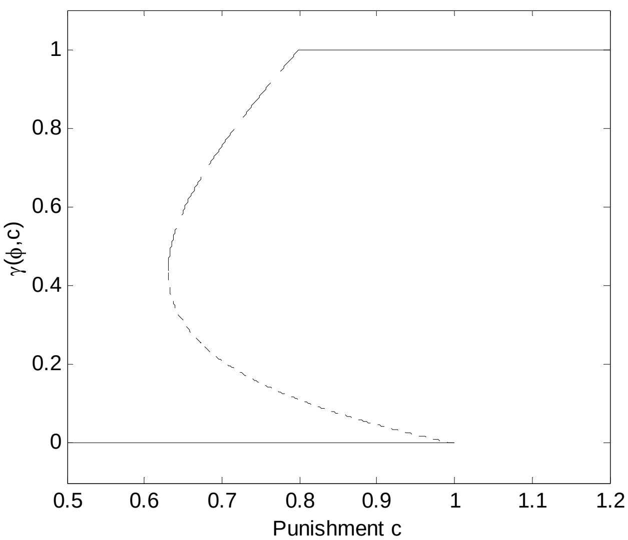

Figure 2: Equilibrium Structure when $\delta > \frac{1}{2}$. Note that γ_L is depicted as a dotted line and γ_H is depicted as a dashed line.

For purposes of our investigation, we rely on a refinement to proceed. As one might expect, the middle equilibrium $\gamma_L(\phi, c)$ is unstable in the sense suggested by DeMichelis and Germano (2000) that small perturbations of payoffs lead to equilibria that are not in its neighborhood. This is not true of other equilibria. We focus henceforth upon stable equilibria in which $\gamma = \gamma_H(\phi, c)$ – which is strictly increasing in c until it reaches unity – or else is constant, $\gamma = 1$.

5 Endogenous Contracts

So far we have assumed that trading behavior is enforced. However, if agents do choose the contracts they write, they will only agree to trading contracts, if this is to their advantage. They could, instead, select a different kind of contract, including a null contract in which behavior is not prescribed.

This allows us to address the question that motivates the paper: under what conditions is there an economic basis for an enforcement agency?

Definition 2 *The* equilibrium demand for enforcement $D^*(\phi, c)$ *is the most an agent is willing to pay for the third-party agency to endorse a contract, assuming that* γ *takes on its highest stable equilibrium value.*

$D^*(\phi, c)$ represents an upper bound on the economic value created by endorsing contracts, provided this value is positive.[14]

5.1 Existence

Consider a parameter profile (ϕ, c) such that a stable trading equilibrium exists, either $\gamma = 1$ or $\gamma_H(\phi, c) < 1$. According to the analysis in the previous subsection, a proportion $\gamma_H(\phi, c)$ of those agents who sign trading contracts would trade. In equilibrium, any two individuals who have to decide whether to sign a contract know that they should expect their partners to trade with probability $\gamma_H(\phi, c)$, *if they sign a trading contract.* Thus, the value of their tradeable in this case is $V^g(\phi, c) = \frac{G\gamma}{1-\delta(1-\gamma)\omega}$.

If they opt for a null contract, the enforcement agency does not supervise their interaction and their payoffs are $\frac{1}{2}(G + \delta V^g)$. When the value to a supervised interaction exceeds that of an unsupervised one, we say that an *equilibrium with contracting* exists.[15]

Note that, if $D^*(\phi, c) = V^g(\phi, c) - \frac{1}{2}(G + \delta V^g(\phi, c)) > 0$, then *all* agents write trading contracts in equilibrium, even if $\gamma_H(\phi, c) < 1$. Hence, breaches of contracts can occur on the equilibrium path. Enforcement benefits both agents who follow contracts and those who break them. Running the risk of punishment if caught, infringers enjoy being surrounded by more traders whom they might defraud. In this case the only way to lure the potential victims into a transaction is to agree to the contract, which thus functions as an imperfect commitment mechanism. This is true, of course, if those intending to rob expect their partners to trade with high enough probability, which, in turn, depends on technology of enforcement and gains from trade. That is why D^* is a function of parameters (ϕ, c).

Proposition 3 *An equilibrium with contracting exists if and only if c is larger than a threshold $c^D(\phi)$, where $c^D(\phi) \leq \underline{c}(\phi)$.*

Notably, if the inequality $c^D(\phi) \leq \underline{c}(\phi)$ is strict, the lower bound on the equilibrium fraction of traders γ consistent with an equilibrium with contracting is strictly below unity, thus, allowing for $\gamma_H(\phi, c) < 1$. Note that breaches of contract occur in equilibrium with contracting for a wide range of parameters.

[14] In an earlier version, we assumed that agents had a choice between two locations, one anarchic and the other with enforced trading contracts, defining D^* as the difference in payoffs between locations. The results turn out to be identical.

[15] The demand for the validation of contracts that do not stipulate trade is zero. Also, profile of actions $< rob, trade >$ gives $\omega\delta V < \delta V$ to the trader, so as mentioned in section 3.1, asymmetric contracts will never be signed in an equilibrium.

5.2 Properties of the Demand

In the introduction we asked: how does the willingness to pay for contract enforcement services depend on the potential gains from trade? Provided the punishment fits the crime, the relationship is positive.

Proposition 4 *Suppose punishment is proportional:* $c(G) = cG$. *If an equilibrium with contracting exists, then* $D^*(\phi, c)$ *is increasing in the gains from trade* G.

The reason for the result is that, when $c(G) = cG$, then the fraction of traders on the market γ is constant in G. As potential gains from trade increase, value of the tradeable under enforced trading contracts increases proportionally and, as a result, $D^*(\phi, c)$ is increasing in the gains from trade.

By contrast, if the punishment does *not* grow proportionally with the crime, larger gains from trade lead to a decrease in the proportion of traders on the market by Proposition 2. As a result, the effect of gains from trade, G, on the demand, $D^*(\phi, c)$, is not necessarily positive.

Proposition 5 *Suppose punishment is constant:* $c(G) = c$. *Then, if an equilibrium with contracting exists,* $D^*(\phi, c)$ *increases with* G *if and only if*

1. $\omega > \delta$; *or*

2. $\delta \geq \omega > \underline{\omega}$ *and* $\delta < \bar{\delta}$.

Otherwise, there exists $\bar{G} > 0$ *so that* $D^*(\phi, c) = 0$ *for all* $G \geq \bar{G}$.

There are two effects that stem from an increase in gains from trade. Its direct effect is to boost the value of the tradeable, holding the equilibrium fraction of fair traders constant. However, there is also an indirect effect: the equilibrium fraction of traders decreases. This lowers the equilibrium value of goods, and decreases the willingness to pay for contract enforcement. If the first effect dominates, the demand for enforcement increases with gains from trade for all possible values of G. Otherwise, the demand for contract enforcement will decrease to zero for large enough values of G. See Figure (3) for an illustration.

For the direct effect to dominate, i.e., for the relative value of a tradeable to increase in potential gains from trade, the equilibrium fraction of traders can not decrease too fast with G. This can happen if a deterrent to breaking the contracts is strong enough. As the potential gain G grows, the punishment c becomes negligible in relative terms. Thus, the direct effect will dominate only if parameters are such that some traders exist in equilibrium, even when the punishment is negligibly small. This occurs precisely when δ is low. Recall the dynamic aspect of market interaction

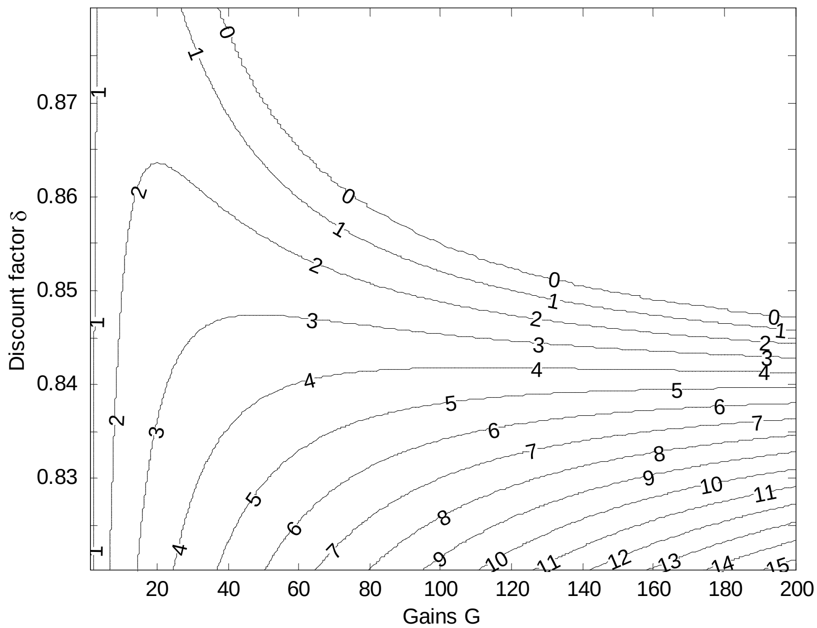

Figure 3: Demand $D^*(\phi, c)$, contour plot. The key to Proposition 5 is that $D^*(\phi, c)$ is concave in G. The demand D^* is increasing in G for lower values of δ, whereas for larger values of δ (above about 0.844) D^* is not monotonic in G and eventually drops to zero.

discussed in the previous subsection: if a violation of a contract is detected, robbers incur the punishment c along with the cost of waiting until another opportunity to interact arises in the future. If δ is low then this delay is more severe. This is when the direct effect of an increase in G dominates, and the enforcement agency can enjoy a growing willingness to pay for its services by more prosperous traders. Otherwise, the lower boundary on punishments that can support trading in equilibrium increases with G, and so demand for enforcement eventually drops to zero. Thus, "low quality" institutions in patient societies require more severe punishments when potential gains from trade are large, if there is to be an economic basis for their existence.

From an empirical perspective, if one envisions the gains from trade to be reflected in the size of GDP, then the proposition above might shed some light as to why the level of GDP has not been conclusively related to the level of various forms of crime, including breaches of contract.[16] Our results are consistent with this ambiguity, as the existence of a willingness to use contracting institutions may not be monotonic

[16] See Fafchamps and Minten (2006) for a survey.

in G, particularly where those institutions are not highly effective.

6 Implications and Extensions

6.1 Other Determinants of Demand for Contracting Institutions

The dependence of $D^*(\phi, c)$ upon parameters turns out to have further interesting features.

Proposition 6 *The demand for enforcement $D^*(\phi, c)$ increases with the quality of institutions ω, and falls with the discount factor δ.*

While the first result may seem intuitive, the second merits further comment. As noted in the introduction, third party enforcement is not the only solution to the problem of anonymous exchange. Aside from various monitoring and punishment systems, certain rituals or social relations may serve as mechanisms to resolve this problem. We do not pursue a detailed examination of the potentially interesting interaction between social pressures and a central authority. Even in a repeated random matching environment, Kandori (1995) shows that decentralized punishments may sustain cooperation. Their effectiveness is limited, however. First, in that class of models, agents are required to display a summary of their past behavior before a new encounter, so that there is a need for a truthful "book-keeping" agency. Second, if punishments involve partial or complete banishment from future trading opportunities, the discount factor restricts the severity of punishment that a community can impose upon deviators and, consequently, the level of cooperation that will be achieved. By contrast, we find that it *facilitates* the operation of an enforcement agency.

6.2 Interaction structure

To focus on contract enforcement institutions, we have made certain assumptions regarding manner in which the agents, and the enforcement agency, interact. It is worth discussing what might happen if we were to depart from some of these assumptions.

First, one might wish to consider alternative punishment schemes. In particular, our punishment setup involves an element of guarding, since stolen items are reinstated to their initial owners. An alternative would be to assume that the agency is unable to reinstate goods: instead, it obliges the thief to give her good to the victim. Note in that case the punishment no longer involves delayed consumption. It is easy to verify that the structure of the equilibria (under symmetric contracts) does not change in that case, although more severe punishment is needed to induce trading behavior. This is intuitive as, under this alternative punishment scenario, theft leads to consumption by both even when it is detected, which tips the balance in favor of theft and decreases the equilibrium value of γ.

Second, our model assumes the existence of *property rights*, in the sense that all agreements between private agents are reached voluntarily, even if these agreements are not necessarily followed in equilibrium. If property rights do not hold, however, it may be that agents can be *coerced* into signing enforceable documents. In this case, asymmetric deals of the form $< trade, rob >$ might not be ruled out and, in the presence of an enforcement agency, all agents would want to impose such a relationship upon their partners. The likely winner of such a contest would be willing to pay the enforcement agency for its services: however, in equilibrium, there might no longer be any *exchange*, extortion ruling the marketplace. This is consistent with the empirical findings of Acemoglu and Johnson (2003) that contracting institutions do not appear to function in the absence of property rights institutions.

There is another way of relating our work to the property rights literature. Suppose that V is a *parameter*, the value from personal use of the good by the owner. If trade is not successful, agents may derive utility V from the use of the good. An enforcement agency in such an environment could then impose a punishment on agents caught robbing, and the willingness to pay for protection would depend on both G and V as primitives. Naturally, the higher is V, the more there is a willingness to pay. In our model, however, the outside option V is endogenous and depends upon the safety of the marketplace itself. This endogeneity is consistent with our interpretation of the model in terms of contracts, and is central to the results of the paper.

Third, we have assumed that the third party is not *corrupt*, in the sense that it does not collude with defectors of a contract. Assume instead that, when caught robbing, agents may "bribe" the enforcement agency in exchange for not being punished. In Rubinchik-Pessach and Samaniego (2003) we show that if the enforcers can extract all that an infractor is willing to pay to avoid the punishment and reinstatement of the good, the structure of the equilibria remains the same, via an argument similar to that of Proposition 1. The qualitative nature of the main results is robust also.

6.3 Comments on the Provision of Enforcement

We have shown that an equilibrium exists in which contracts are written and to some extent followed so long as enforcement is sufficiently effective – even though contracts need not be followed to the letter by all agents. Moreover, in this event, there is a positive surplus that the enforcement agency may extract.

As noted, we do not model the supply of protection explicitly, taking the enforcement technology as given. Still, Proposition 3 suggests that, provided the gains from trade are high enough, there might be profits to be earned from the provision of enforcement services. The fees that might be charged for notarization would depend on the structure of the market for enforcement itself. Nonetheless, since participation is voluntary, there may still be surplus for the agents in the interval $[0, D^* (\phi, c)]$. For example, if the enforcement market is contestable and entry costs are low, then the profits to the agency may be driven down to zero.

Interestingly our results suggest that, in the presence of exogenous technical change that affects gains from trade, there might eventually be sufficient potential revenues from contracting services that an enforcement agency will arise to provide them. To see this, suppose that there exists a cost χ that the enforcer must incur each time it endorses a contract. Assume that the gains from trade increase exogenously over time due to technical change. Then, as the next proposition shows, such growth is consistent with a stationary equilibrium. This statement enables us to make a further claim about possible emergence of a sustainable enforcement agency.

Proposition 7 *Suppose that $c(G_t) = cG_t$, $G_t = G_0 g^t$ for some $g > 1$. Assume $\delta g < 1$.Then, there exist stationary equilibria with the same values of γ as in an environment without growth where gains from trade equal G_0 in each period, and the discount factor equals δg.*

Defining D_t^* as the maximum an agent would pay for the verification of a contract at time t, we have the following

Corollary 1 *Assume that punishment is proportional, $c(G_t) = cG_t$, and that an equilibrium with contracting exists with parameters $\phi = (\omega, \delta g, G_0)$. Then demand D_t^* increases by a factor g over time. Hence, there exists some finite T so that $D_T^* > \chi$ for all $t > T$.*

The result illustrates that the emergence of an enforcement agency can in fact be driven by economic fundamentals. However, a crucial condition for the emergence of the agency is that punishments increase in tandem with the gains from trade. Although a full inquiry into this issue would require a study of the structure of non-stationary equilibria, Proposition 5 suggests that, if $c(G_t) = c$, the demand for contract enforcement may not necessarily increase over time in the presence of an exogenous growth.

Recall, we focus on the demand for enforcement given any level of punishment, c. Our findings can be easily used to study the equilibrium on this market. Let us illustrate a possible such application.

First, observe the following.

Proposition 8 *Assume that $\delta > 1/2$, $c > 0, c < \underline{c}$, so that γ_H is well defined. Assume $c > c^D(\phi)$, so that the demand for enforcement is positive. Then D^* is concave in G and in c.*

Now suppose that the agency is a monopolist and is capable of selecting c, subject to a standard strictly convex cost function. Note that if $c \geq \underline{c}$, then no contracts are breached, so that $\gamma_H = 1$, in which case D^* is constant with c. It implies that if punishment is, indeed, costly, its severity should be bounded above by $\underline{c}$. Thus, there is a unique c that solves the enforcer's problem. Moreover, if the marginal cost is high enough, and if $\delta > 1/2$, then by Proposition 8 the punishment will be chosen 'in the interior' so that $c < \underline{c}$, and, therefore, breaches of contracts will occur, $\gamma_H < 1$. For this case, it is easy to show that the cross-derivative of D^* with respect to c and G is positive,[17] implying that an increase in gains from trade, G, encourages more punishment. Thus, for the range of parameters over which the demand D^* is increasing in G, (see proposition 5), 'market equilibrium' willingness to pay for enforcement is also growing with gains from trade. For the complementary set of parameters (over which D^* decreases in G), the result would depend upon the technology of enforcement. However, as in practice the level of available punishments might be bounded or constrained, even the 'market equilibrium' willingness to pay might decrease with gains from trade in that range of parameters. In short, even if the provision of enforcement depends on the gains from trade, the resulting willingness to pay might follow the pattern which is consistent with our results.

Second, recall, the paper centers on the relationship between gains from trade G and the demand D^*. G may be interpreted as productivity in an exchange economy. Given (c, ω), would the agency provide any *public goods* that might increase productivity G?

Suppose that G may be increased, at a convex cost to the agency. This might be interpreted as the provision of infrastructure or some network externality (roads, standardization, etc.). In the case of proportional punishment ($c(G) = cG$), the demand D^* is proportional to G so there would be a unique solution to the problem of providing public goods. If the public good becomes cheaper to provide, the quantity provided would increase accordingly. In the case of constant punishment ($c(G) = c$), however, this may not be the case. For parameters such that the equilibrium value of γ is interior, D^* is concave in G, and may be decreasing in G after a certain point. In this case, a "cheapening" of the public good may not increase public goods provision. Indeed, the agency may choose not to provide the public good even if it is free, as it could reduce the equilibrium "tax base."

[17] $\frac{d^2}{dGdc} D^*(\cdot) = \frac{1}{2} z_\gamma \gamma_C > 0$, where the subscripts denote partial derivatives and $D(\gamma, \phi, c) = \frac{1}{2} z(\gamma, \phi) G$, where $z(\gamma, \phi) = \frac{2\gamma - \gamma\delta + \delta\omega - \gamma\delta\omega - 1}{(1 - \delta\omega(1-\gamma))}$, as defined in the Appendix.

6.4 Concluding Remarks

Models of institutions resulting from agent interaction tend to concentrate on property rights institutions. In this paper, we define and study the value created by *contracting institutions*. To this end, we develop a model of contracting and exchange, in which agent interactions are subject to a voluntary participation constraint. Agents choose whether to notarize their contracts in order to commit themselves to trade, even though they may decide to break their promises later. As a result, trade may be facilitated by contracting institutions, and the exchange value of goods may rise as a result. We then use the model to ask whether the presence of potential gains from trade may generate an economic basis for contract enforcement institutions.[18]

Perhaps surprisingly, larger gains from trade do not necessarily provide this basis. In particular, the demand for "low quality" institutions decreases in the potential gains from trade. Although these gains increase the benefit to the contracting parties when contracts are followed, it also increases the equilibrium rate of contract violations, and this second effect may dominate for large enough gains from trade unless the severity of punishment is increased in tandem.

In future work, it could be interesting to extend the model to allow for the gains from trade or the enforcement technology to change over time. This would allow a more careful study not just of the effects of economic growth on institutions but of institutional *reform*, so that agents' behavior is not just a function of current conditions but also of their expectations of the future institutional environment. This would require a richer dynamic framework for which current model can serve as a basis. It would also be interesting to extend the model to allow for competing enforcement agencies, or for other institutional structures. Moselle and Polak (2001) study different scenarios in a model of property rights: the environment developed in the current paper could serve as a useful framework to address the same question for the case of contracting institutions.

A Appendix

A.1 Notation and Basic Results

Since we are concentrating upon stationary equilibria, a strategy is optimal in each period iff it is optimal in every period.

It is simple to show that the value of a good held by traders and robbers is

[18] This is in the spirit of the work by Nozick (1974), who analyzes the rise of the state that grows "by an invisible-hand process," thus relying on economic benefits it generates by preserving individual rights.

respectively

$$V_t^g(\gamma; c) = \frac{G\gamma}{1 - \delta(1-\gamma)\omega} \tag{4}$$

$$V_r^g(\gamma; c) = \frac{(\gamma+1)(G(1-\omega) - c\omega)}{\delta(1-\omega)(1-\gamma) + 2(1-\delta)}, \tag{5}$$

so that the value of a tradeable good introduced in the text, see (3), is then

$$V^g = \begin{cases} V_t^g(1; c), \text{if } \gamma = 1 \\ V_r^g(0; c), \text{if } \gamma = 0 \\ V_t^g(\gamma; c) = V_r^g(\gamma; c), \text{ otherwise} \end{cases}.$$

Let

$$F(\gamma; c) = \kappa(\gamma)\left[V_t^g(\gamma; c) - V_r^g(\gamma; c)\right]; \tag{6}$$
$$\kappa(\gamma) \equiv (\delta(1-\omega)(1-\gamma) + 2(1-\delta))(1 - \delta(1-\gamma)\omega).$$

Note that $\kappa > 0$, so the sign of $F(\gamma; c)$ coincides with the sign of the difference $V^g(\gamma; c) - V^g(\gamma; c)$: hence we concentrate upon finding roots of F.

$$F(\gamma; c) = \gamma^2 a_F(c) + \gamma b_F(c) + k_F(c), \tag{7}$$
$$k_F(c) = (c\omega - G(1-\omega))(1 - \delta\omega);$$
$$b_F(c) = G(1-\delta) + G\omega(1-\delta) + c\omega(1-\delta) + c\delta\omega;$$
$$a_F(c) = -\delta\left(G(1-\omega^2) - c\omega^2\right)$$

Remark 1 *All results concerning c extend mutatis mutandis to the case $c = c(G) = \kappa G$ by virtue of the fact that F is homogeneous of degree zero in G and c. Hence equilibrium values of γ will be the same for (c, G) as for $(\alpha c, \alpha G)$ for any $\alpha > 0$.*

Proof of Proposition 1. Recall, that the roots (γ) of polynomial $F(\gamma; c)$ defined in (6) correspond to the equilibria. Moreover, $F(1; c) > 0$ indicates that $\gamma = 1$ is an equilibrium and $F(0; c) < 0$ implies $\gamma = 0$ is an equilibrium. Observe that if $c > \underline{c}(\phi) \equiv \frac{G(\delta-\omega)}{\omega}$ then $F(1; c) > 0$, and if $c \leq \bar{c}(\phi) \equiv \frac{G(1-\omega)}{\omega}$ then $F(0; c) \leq 0$. Quadratic polynomial F is maximized at $\gamma = \gamma^*$, where

$$\gamma^* = \frac{1}{2}\frac{(G(1-\delta) + G\omega(1-\delta) + c\omega)}{(G(1-\omega^2) - c\omega^2)\,\delta}. \tag{8}$$

If $\gamma^* > 1$, then $F(1; c) > 0$, as the upper root should be above unity. $\gamma^* > 1$ if and only if

$$c > c^* \equiv \frac{(2\delta(1-\omega) - (1-\delta))}{(2\beta\delta\omega + 1)}\frac{(\omega+1)\,G}{\omega}. \tag{9}$$

We proceed by deriving the lower bound on punishments consistent with an interior equilibrium, $\gamma > 0$. Note that there are two possible cases that can lead the polynomial $F(\gamma; c)$ to be negative for all $\gamma \in [0, 1]$. The first case occurs when γ^*, at which F is maximized, is above unity and $F(1; c) < 0$. Secondly, if $\gamma^* < 1$ and $F(\gamma^*; c) < 0$, then, $F(\gamma; c) < 0$ for any γ. We start with the first case, as it generates a higher lower bound on c, given that γ^* strictly increases in c (which can be verified directly from (8)).

Lemma 1 *If $\delta \leq 1/2$ and $c < \underline{c}(\phi)$, then there is a unique equilibrium $\gamma = 0$.*

■

Proof. When $\delta \leq 1/2$ it is simple to show that $c^* < \underline{c}(\phi)$, so $c \in [c^*, \underline{c}(\phi)]$ implies $F(\gamma; c) \leq F(1; c) < 0$ for any $\gamma \in [0, 1]$, as $\gamma^* \geq 1$ in this case. So, there is a unique equilibrium $\gamma = 0$. It is left to show that this is also true if $c < c^*$. Consider $c = c^*$. As $c^* < \underline{c}(\phi)$, and $\gamma^* = 1$, it implies $F(\gamma^*(c^*); c^*) < 0$. As γ^* is the maximand of F, it follows that $F(\gamma; c^*) < 0$ for any γ. Now consider $c_0 < c^*$. It can be easily shown that F decreases in c for any γ. Therefore, $F(\gamma; c_0) < 0$.

Lemma 2 *If $\delta > 1/2$ and $c < \underline{\underline{c}}(\phi)$, then there is a unique equilibrium $\gamma = 0$.*

■

Proof. If $\delta > 1/2$ then $c^* > \underline{c}(\phi)$, therefore, for $c < \underline{c}(\phi) < c^*$ we have first, $F(1; c) < 0$ and, second, $\gamma^* < 1$. Moreover, if $c < \tilde{c} = \omega^{-1}(\omega + 1)(\delta - 1)G$, then $b_F(c)$ is negative and so are the rest of the coefficients, $a_F(c)$ and $k_F(c)$, which implies F has only negative roots in γ, supporting $\gamma = 0$. We will show that there exists $\underline{\underline{c}}(\phi) > \tilde{c}$, such that for c between $\tilde{c}$ and $\underline{\underline{c}}$, $\gamma = 0$ is the only equilibrium. The parabola $F(\gamma; c)$ can cross zero twice if the discriminant

$$H(c, \phi) \equiv b_F^2(c) - 4a_F(c)\, k_F(c) \tag{10}$$

is positive. $H(c; \phi)$ is quadratic in c:

$$H(c, \phi) = c^2 a_H + c b_H + k_H, \tag{11}$$

where $a_H > 0, b_H > 0$. This implies that the extremum (c) of this parabola (H) is negative, therefore so is one of the potential roots. H is negative between those roots and it is positive otherwise. Notice also that $\tilde{c}$ lies between the roots, as $H(\tilde{c}, \phi) < 0$. This implies that the upper root of H, which we will denote by $\underline{\underline{c}}(\phi)$, is strictly above $\tilde{c}$. H is negative between $\tilde{c}$ and $\underline{\underline{c}}(\phi)$, which implies F has no real roots, and is negative for any gamma. The result follows.

Notice also that H is strictly increasing for any $c \geq \underline{\underline{c}}(\phi)$. Thus,

$$H(\underline{c}(\phi), \phi) = G^2(2\delta - 1)^2(\delta\omega - 1)^2 > 0 = H\left(\underline{\underline{c}}(\phi), \phi\right) \tag{12}$$

implies $\underline{c}(\phi) > \underline{\underline{c}}(\phi)$.

Remark 2 *Since $a_H > 0, b_H > 0$, the upper root of H is positive, $\underline{\underline{c}}(\phi) > 0$, iff $k_H < 0$, which is true whenever $\delta > \delta_L$, where δ_L is the lower root of the quadratic polynomial*

$$P(\delta) = \omega + 1 + \delta\left(6\omega - 4\omega^2 - 6\right) + \delta^2\left(5\omega - 8\omega^2 + 4\omega^3 + 1\right);$$

It is easy to check that $\delta_L \in (0,1)$, provided $\omega < 1$, as $P(1) < 0$ and $P(0) > 0$.

Lemma 3 *Assume that $\delta > 1/2$ and $\underline{\underline{c}}(\phi) < c < \underline{c}(\phi)$. Then there are three equilibria: $\gamma = 0$, and a couple $\gamma_L < \gamma_H < 1$.*

■

Proof. The two roots of the polynomial $F(\gamma;c)$, are

$$\gamma_L(\phi,c) \equiv \frac{-b_F(c) + \sqrt{H(c,\phi)}}{2a_F(c)},\ \gamma_H(\phi,c) \equiv \frac{-b_F(c) - \sqrt{H(c,\phi)}}{2a_F(c)}, \tag{13}$$

where $H(c,\phi)$ is as defined in (10). Condition $c > \underline{\underline{c}}$ assures that $H(c,\phi)$ is strictly positive. Therefore γ_L, γ_H are real. As $a_F < 0$ for $c < \bar{c}(\phi)$ and $k_F < 0$, we have $0 < \gamma_L < \gamma_H$.

Since $\delta > 1/2$, $c^* > \underline{c}$, thus any $c < \underline{c}(\phi)$ is also below c^*, which implies that the maximand of F, γ^*, is less than one. Moreover, as $c < \underline{c}$, $F(1,c) < 0$, this, along with the fact that $a_F(c) < 0$ and that the discriminant H is positive guarantees that $\gamma_H(c) < 1$. Finally, $F(0,c) < 0$, as $k_F(c) < 0$, which justifies the first equilibrium $(\gamma = 0)$. ■

Proof of Propositions 3 and 4 . Define $D(\gamma,\phi,c)$ to be the willingness to pay for protection given γ, i.e., the difference between signing the contract that will subsequently be enforced (thus expecting a partner to be a fair trader with probability γ) and a sure fight between the two (in case neither signs the contract). In the environment with endogenous contracts, if agents agree to a trading contract and $\gamma > 0$ in equilibrium, their payoff is equal to that of a trader,$G\gamma + \omega(1-\gamma)\delta V^g$, even if they choose to renege. The alternative is to agree on a contest, in which case they get $1/2\left(G + \delta V^g\right)$. Thus,

$$\begin{aligned} D(\gamma,\phi,c) &\equiv G\gamma + \omega(1-\gamma)\delta V^g - 1/2\left(G + \delta V^g\right) \\ &= \frac{1}{2}\frac{2\gamma - \gamma\delta + \delta\omega - \gamma\delta\omega - 1}{(1 - \delta\omega(1-\gamma))}G \end{aligned} \tag{14}$$

where the last step follows from replacing V^g using equation (4). Then, $D^*(\phi,c)$ is defined as $D(\gamma(\phi,c),\phi,c)$, where $\gamma(\phi,c)$ is the highest stable equilibrium value of γ.

In the environment with endogenous contracts, if agents agree to a trading contract, then $D(\gamma, \phi, c) \geq 0$ which reduces to

$$\gamma > \underline{\gamma} \equiv \frac{1-\delta\omega}{2-\delta\omega-\delta}. \tag{15}$$

In this event, D is increasing in the gains from trade because it is linear in G (for a fixed γ).

Observe that $\underline{\gamma} < 1$. Moreover, provided $\gamma > \underline{\gamma}$, the demand $D(\gamma, \phi, c)$ is positively related to the gains from trade, G, keeping γ constant, $D_G^*(\phi, c) > 0$. Then proposition (4) stems from homogeneity of F, see Remark 1. Lastly, we have to demonstrate that condition $\gamma > \underline{\gamma}$ is equivalent to requiring $c > c^D(\phi)$. As we restrict attention to stable equilibria with the highest proportion of fair traders, the case $\delta < 1/2$ trivially reduces to requiring $c > \underline{c}(\phi)$, when $\gamma = 1$ is an equilibrium and therefore, condition (15) holds. So, in this case let $c^D(\phi) = \underline{c}(\phi)$. Next, for the case $\delta > 1/2$, notice that $\underline{\gamma}$ is not a function of c and the upper root of quadratic polynomial F, γ_H, strictly increases in c, as F strictly increases with c. Also, $\gamma_H(\phi, \underline{c}(\phi)) = 1 > \underline{\gamma}$. Two cases are possible. First, if $\gamma_H\left(\phi, \underline{\underline{c}}(\phi)\right) < \underline{\gamma}$, then let $c^D(\phi)$ be implicitly defined by $\gamma_H\left(\phi, c^D(\phi)\right) = \underline{\gamma}$, the existence of which is assured by the intermediate value theorem (besides, $c^D(\phi)$ is unique by strict monotonicity of γ_H in c). Second, if $\gamma_H\left(\phi, \underline{\underline{c}}(\phi)\right) = \gamma^* > \underline{\gamma}$, then let $c^D(\phi) = \underline{\underline{c}}(\phi)$. ■

A.2 Determinants of the Demand for Enforcement

According to the definition, $D(\gamma, \phi, c) = \frac{1}{2} z(\gamma, \phi) G$, where $z(\gamma, \phi) = \frac{2\gamma-\gamma\delta+\delta\omega-\gamma\delta\omega-1}{(1-\delta\omega(1-\gamma))}$.

Proof of Proposition 2. Recall, $F(\gamma; c) = \kappa(\gamma)\left[V_t^g(\gamma; c) - V_r^g(\gamma; c)\right]$. It also admits the following representation:

$$F(G, c, \gamma) = G f_G(\gamma) + c f_c(\gamma) \tag{16}$$
$$f_G(\gamma) \equiv k_G + \gamma b_G + \gamma^2 a_G \tag{17}$$
$$k_G \equiv (1-\omega)(\delta\omega - 1) < 0 \tag{18}$$
$$b_G \equiv (1-\delta)(\omega+1) > 0 \tag{19}$$
$$a_G \equiv \delta\left(\omega^2 - 1\right) < 1 \tag{20}$$

and

$$f_c(\gamma) \equiv \omega(1-\delta\omega) + \gamma\omega + \gamma^2\delta\omega^2 > 0$$

It is then evident that if $c \geq 0$, then $F(\gamma, \phi) > G f_G(\gamma, \phi)$ for any positive γ. It implies, that if $G > 0$

$$f_G(\gamma_H, \phi) < 0 = F(\gamma_H, \phi)/G. \tag{21}$$

It is easy to check that the derivative of F with respect to γ_H is $-\sqrt{H_F}$, where $H_F = b_F^2 - 4a_F k_F$. Therefore, γ_H is decreasing with G. Moreover,

$$\frac{\partial \gamma_H(\phi, c)}{\partial G} = \frac{f_G(\gamma_H, \phi)}{\sqrt{H_F}} < 0, \tag{22}$$

similarly,

$$\frac{\partial \gamma_H(\phi, c)}{\partial c} = \frac{f_c(\gamma_H, \phi)}{\sqrt{H_F}} > 0. \tag{23}$$

■

Proof of proposition 8. Recall that $c > c^D(\phi)$ implies $\gamma_H > \underline{\gamma} \equiv \frac{(\delta\omega-1)}{(\delta+\delta\omega-2)}$ if $\delta > 1/2$. Recall also that in this case $D^*(\gamma, \phi, c) = \frac{1}{2} z(\gamma, \phi) G$, where $z(\gamma, \phi) = \frac{2\gamma-\gamma\delta+\delta\omega-\gamma\delta\omega-1}{(1-\delta\omega(1-\gamma))}$. Consider equilibrium $\gamma_H(\phi, c)$. Then

$$2\frac{d^2 D^*}{(dG)^2}(\phi, c) = G\frac{\partial z(\gamma_H, \phi)}{\partial \gamma}\frac{\partial^2 \gamma_H(\phi, c)}{(\partial G)^2} \tag{24}$$

$$+ 2\frac{\partial z(\gamma_H, \phi)}{\partial \gamma}\frac{\partial \gamma_H(\phi, c)}{\partial G} + G\left(\frac{\partial \gamma_H(\phi, c)}{\partial G}\right)^2 \frac{\partial^2 z(\gamma_H, \phi)}{(\partial \gamma)^2} \tag{25}$$

Since $\frac{\partial^2 z(\gamma_H, \phi)}{(\partial\gamma)^2} = 2\frac{(\delta\omega-1)(2-\delta)\delta\omega}{(1-(1-\gamma)\delta\omega)^3} < 0$, $\frac{\partial \gamma_H(\phi, c)}{\partial G} = \frac{f_G}{\sqrt{H_F}} < 0$ (by identity (22)) and $\frac{\partial z(\gamma_H, \phi)}{\partial \gamma} > 0$, it is sufficient to show that

$$\Gamma(G) = G\frac{\partial^2 \gamma_H(\phi, c)}{(\partial G)^2} + \frac{\partial \gamma_H(\phi, c)}{\partial G} < 0 \tag{26}$$

Note that f_G does not depend on G, so

$$\frac{\partial^2 \gamma_H}{(\partial G)^2} = f_G\left(-\frac{\frac{\partial H_F}{\partial G}}{2H_F\sqrt{H_F}}\right). \tag{27}$$

Thus,

$$\Gamma(G) = \frac{f_G}{2H_F\sqrt{H_F}}\left(2H_F - G\frac{\partial H_F}{\partial G}\right). \tag{28}$$

Note that $f_G < 0$. So, to prove the claim we need to show that $2H_F - G\frac{\partial H_F}{\partial G} > 0$. Recall that

$$H_F(c, \omega, \delta, G) = (G(1-\delta) + G\omega(1-\delta) + c\omega(1-\delta) + c\delta\omega)^2 - \tag{29}$$

$$- 4\left((c\omega - G(1-\omega))(\delta(1-\omega) + (1-\delta))\right)\left(-\delta\left(G(1-\omega^2) - c\omega^2\right)\right),$$

so it is homogenous of degree two in c, G, i.e., $H_F(\alpha c, \omega, \delta, \alpha G) = \alpha^2 H_F(c, \omega, \delta, G)$ for any $\alpha > 0$, provided $c > 0$. By Euler's formula,

$$2H_F(c,\omega,\delta,G) = c\frac{\partial H_F}{\partial c}(c,\omega,\delta,G) + G\frac{\partial H_F}{\partial G}(c,\omega,\delta,G), \tag{30}$$

so we are left to show that

$$\frac{\partial H_F}{\partial c}(c,\omega,\delta,G) > 0. \tag{31}$$

but this is true, as by the argument in lemma 2, H_F is quadratic in c attaining its minimum at a negative value of c, so it should be strictly increasing for any $c > 0$.

As for concavity in c, note that

$$2\frac{d^2 D^*}{(dc)^2}(\phi, c) = G\left(\frac{\partial z(\gamma_H,\phi)}{\partial \gamma}\frac{\partial^2 \gamma_H(\phi,c)}{(\partial c)^2} + \left(\frac{\partial \gamma_H(\phi,c)}{\partial c}\right)^2 \frac{\partial^2 z(\gamma_H,\phi)}{(\partial\gamma)^2}\right) \tag{32}$$

Since $\frac{\partial^2 z(\gamma_H,\phi)}{(\partial\gamma)^2} = 2\frac{(\delta\omega-1)(2-\delta)\delta\omega}{(1-(1-\gamma)\delta\omega)^3} < 0$, $\frac{\partial\gamma_H(\phi,c)}{\partial c} = \frac{f_c}{\sqrt{H_F}} > 0$ (by identity (23)) and $\frac{\partial z(\gamma_H,\phi)}{\partial\gamma} > 0$, it is sufficient to show that

$$\frac{\partial^2\gamma_H}{(\partial c)^2} = f_c\left(-\frac{\frac{\partial H_F}{\partial c}}{2H_F\sqrt{H_F}}\right) < 0, \tag{33}$$

which is, indeed, true as $\frac{\partial H_F}{\partial c} > 0$ as follows from above. ■

Proof of proposition 5. Let us start with two simple cases. If $\delta < \omega$ then $\underline{c}(\phi) < 0$, so for any $c > 0$ $\gamma = 1$ is an equilibrium. Then, no matter how high is G, $\underline{c}$ stays negative and the demand for enforcement is proportional to G, $D(\gamma,\phi,c) = \frac{1}{2}(1-\delta)G$ so the conclusion follows. Second, if $\omega < \delta, \delta < 1/2$, we have $\underline{c}(\phi) = \frac{G(\delta-\omega)}{\omega} > 0$. Then as G increases, any $c > 0$ will fall below $\underline{c}(\phi)$, leaving the only equilibrium $\gamma = 0$, so the demand for enforcement drops to zero for G high enough.

Now consider the rest of the cases: $\omega < \delta, \delta > 1/2$. It is without loss of generality to assume $c < \underline{c}(\phi)$, as if it is not, for G big enough it will. Also, the case $c < \underline{\underline{c}}(\phi)$ is trivial, as there is no equilibrium with $\gamma > 0$ in that range, so no demand for enforcement exists. Start with some $G > 0$ and $c \in \left[\max\left\{\underline{\underline{c}}, 0\right\}, \underline{c}\right]$ at which the demand for enforcement is strictly positive, $D(\gamma_H, \delta, \omega, G) > 0$. As G increases two scenarios are possible. First, we will show that if $\underline{\underline{c}}(\phi) > 0$, then for G large enough any fixed $c > 0$ falls below $\underline{\underline{c}}(\phi)$, which leaves the only sustainable equilibrium, $\gamma = 0$, thus eliminating desire to pay for a useless enforcement. Second, in the complementary case, $\underline{\underline{c}}(\phi) \leq 0$, the equilibrium $\gamma_H(c,\phi)$ might fall to $\underline{\gamma}$, at which the demand for enforcement is zero. In case γ_H is bounded away from $\underline{\gamma}$ for any G, the demand should be strictly positive, which, coupled with the fact that it is concave in G (by proposition 8) implies it is monotonically increasing in G.

To support the first claim notice, that $\underline{\underline{c}}(\phi) = Gx_H(\delta, \omega)$, where $x_H(\delta, \omega)$ is the upper root of $T(x)$,

$$T(x, \delta, \omega) \equiv \frac{H_F(c, \omega, \delta, G)}{G^2}, x \equiv c/G. \tag{34}$$

Thus

$$T(x, \delta, \omega) = x^2 a_H + x\frac{b_H}{G} + \frac{k_H}{G^2}, \tag{35}$$

$$\begin{aligned} k_H &= G^2(\omega+1)\left(2(1-\delta)^2 - (\delta - 2\delta\omega + 1)^2(1-\omega)\right) \\ b_H &= G\left(2\omega(1-\delta)(\omega+1) + 4\delta(1-\delta\omega)\omega(2\omega+1)(1-\omega)\right) > 0 \\ a_H &= \omega^2(2\delta\omega - 1)^2 > 0 \end{aligned} \tag{36}$$

Notice that the expressions a_H, $\frac{b_H}{G}$, $\frac{k_H}{G^2}$ and are independent of G. T has real roots in the relevant range of parameters, which are either both negative or of the opposite sign. Clearly, $\underline{\underline{c}}(\phi)$ is positive iff $x_H(\delta, \omega) > 0$. Therefore, as $\underline{\underline{c}}(\phi)$ is proportional to G, any fixed $c > 0$ will fall short of $\underline{\underline{c}}(\phi)$ for big enough (finite) G.

As for the second claim, $\underline{\underline{c}}(\phi) < 0$ is equivalent to $\delta < \delta_L$, δ_L being the lower root of $P(\delta)$,

$$P(\delta) = \omega + 1 + 6\delta\omega - 4\delta\omega^2 - 6\delta + 5\delta^2\omega - 8\delta^2\omega^2 + 4\delta^2\omega^3 + \delta^2, \tag{37}$$

as follows from remark (2),

$$\delta_L = \frac{3 - 3\omega + 2\omega^2 - 2\sqrt{2(1-\omega)^3}}{1 + 5\omega - 8\omega^2 + 4\omega^3}. \tag{38}$$

Note that for γ_H to exist, we restrict attention to $\delta > 1/2$. It is easy to verify that (1) $\delta_L(\omega) > 1/2$ iff $\omega > 1/2$, (2) $\delta_L(\omega) > \omega$ for any $\omega \in \left(\frac{1}{2}, 1\right)$. Clearly, then if ω is below $\frac{1}{2}$, the previous case applies, so that $\underline{\underline{c}}(\phi) > 0$. Now assume $\delta \in [\omega, \delta_L]$. As G approaches infinity, F/G approaches

$$f_G(\gamma) = \left(\omega + \delta\omega - \delta\omega^2 - 1 + \gamma(\omega - \delta - \delta\omega + 1) + \gamma^2\left(\delta\omega^2 - \delta\right)\right), \tag{39}$$

which also implies that the upper root of F approaches the upper root of $f_G(\gamma)$.[19] Recall also that γ_H is decreasing in G. In order to find out whether γ_H will ever approach $\underline{\gamma}$, we only need to check the sign of $f_G(\underline{\gamma})$. If it is positive, γ_H will be

[19] For that we just have to assure that the real roots are well defined, which is true because $\underline{\underline{c}}(\phi) < 0$ for any G, thus making the discriminant, $H_F > 0$ for any $c > 0$. Besides, as we consider $c < \underline{c}$, a_F is bounded away from zero, so the roots are continuous functions of the coefficients, a_F, b_F, k_F.

always above $\underline{\gamma}$. In the other case, for G big enough γ_H will reach $\underline{\gamma}$, at which the demand is zero. Observe that the sign of $f_G\left(\underline{\gamma}\right)$ depends on δ and ω :

$$f_G\left(\underline{\gamma};\delta,\omega\right) = 2\left(1-\delta\omega\right)\frac{W\left(\delta,\omega\right)}{\left(\delta+\delta\omega-2\right)^2} \tag{40}$$

$$W\left(\delta,\omega\right) \equiv 3\omega - 1 + \delta^2\left(\omega+\omega^2\right) + \delta\left(-2\omega-2\omega^2\right). \tag{41}$$

Observe that W has two (δ) roots, both positive, the lower one being between zero and unity, as $W(1,\omega) = -(1-\omega)^2 < 0$ and $W(0,\omega) = 3\omega - 1 > 0$.Let us denote this root by $\bar{\delta}$,

$$\bar{\delta} \equiv 1 - \frac{(1-\omega)}{\left(\omega\left(\omega+1\right)\right)^{1/2}} \tag{42}$$

As we are interested in $\delta > \omega > 1/2$, and $\delta < \delta_L$ one has to check how $\bar{\delta}$ compares with these bounds. It is easy to verify that $\bar{\delta} < \delta_L$, which means that the interval $\left(\bar{\delta},\delta_L\right)$ is non-empty. Next, $\bar{\delta} > \frac{1}{2}$ is true iff $\omega > \frac{3}{2} - \frac{1}{6}\sqrt{33}$. Finally, $\bar{\delta} > \omega$ iff

$$\omega > \frac{1}{2}\sqrt{5} - \frac{1}{2}\left(> \frac{3}{2} - \frac{1}{6}\sqrt{33}\right). \tag{43}$$

Note that for any fixed ω, $W(\delta,\omega) > 0$ for $\delta \in \left[0,\bar{\delta}\right]$ and $W(\delta,\omega) < 0$ for $\delta \in \left[\bar{\delta},1\right]$. Besides, $f_G\left(\underline{\gamma};\delta,\omega\right)$ has the same sign as $W(\delta,\omega)$.

To summarize, if

$$\omega > \underline{\omega} \equiv \frac{1}{2}\sqrt{5} - \frac{1}{2}$$

and $\delta \in \left(\omega,\bar{\delta}\right)$, then $f_G\left(\underline{\gamma};\delta,\omega\right) > 0$, which implies $\gamma_H > \underline{\gamma}$ in this parameter range for any G and any fixed c. Otherwise, (if $\bar{\delta} \leq \omega$) by considering $\delta > \omega$, we necessarily have $\delta > \bar{\delta}$, implying $f_G\left(\underline{\gamma};\delta,\omega\right) < 0$, thus for G big enough γ_H reaches $\underline{\gamma}$. ■

Proof of proposition 6. First, we consider the case that where $c \in \left(\underline{c},\underline{c}\right)$, and $c > c^D(\phi)$. The assumption ensures that $\gamma_H(\phi,c) < 1$ is well-defined and that the demand for enforcement $D^*(\phi,c)$ is positive. In what follows we will use the following observations.

$$z_\gamma\left(\gamma,\phi\right) = \frac{\left(\delta\omega-1\right)\left(\delta-2\right)}{\left(\gamma\delta\omega-\delta\omega+1\right)^2} > 0 \tag{44}$$

$$z_\omega\left(\gamma,\phi\right) = \frac{\left(\delta-2\right)\left(\gamma-1\right)\gamma\delta}{\left(\gamma\delta\omega-\delta\omega+1\right)^2} > 0 \tag{45}$$

$$z_\delta\left(\gamma,\phi\right) = \frac{\left(2\gamma\omega-2\omega+1\right)\gamma}{-\left(\gamma\delta\omega-\delta\omega+1\right)^2} < 0 \text{ if } \gamma > \underline{\gamma} > \frac{2\omega-1}{2\omega} \tag{46}$$

$$\frac{dD^*}{(d\delta)}(\phi,c) = \frac{G}{2}z_\delta\left(\gamma_H,\phi\right) + \frac{\partial D(\gamma_H,\phi,c)}{\partial\gamma}\frac{\partial\gamma_H(\phi,c)}{\partial\delta} \tag{47}$$

First, $\frac{d}{d\delta}z(\phi,\gamma)$ is negative by (46). Next,

$$\frac{\partial D(\gamma_H,\phi,c)}{\partial\gamma}\frac{\partial\gamma_H(\phi,c)}{\partial\delta}<0 \tag{48}$$

Indeed, given that $D_\gamma(\gamma_H,\phi,c)>0$, by (44), it is enough to show that the polynomial, $F(\gamma,c,\phi)$ is decreasing in δ, then its upper root, γ_H, will be decreasing with δ as well.[20] In the relevant range of c, the derivative $\frac{\partial}{\partial\delta}F(\gamma,c,\phi)$ can be shown to be negative provided $(\gamma+\gamma\omega-\omega)>0$, which holds because $\frac{\omega}{1+\omega}<\underline{\gamma}$.

Second, we have to show that

$$\frac{dD^*}{(d\omega)}(\phi,c)=\frac{G}{2}z_\omega(\phi,\gamma_H)+\frac{\partial D(\gamma_H,\phi,c)}{\partial\gamma}\frac{\partial\gamma_H(\phi,c)}{\partial\omega}>0 \tag{49}$$

In view of (45), we are left with demonstrating

$$\frac{\partial D(\gamma_H,\phi,c)}{\partial\gamma}\frac{\partial\gamma_H(\phi,c)}{\partial\omega}>0. \tag{50}$$

Again, $\frac{\partial D(\gamma_H,\phi,c)}{\partial\gamma}>0$, so we have to show that $\frac{\partial\gamma_H(\phi,c)}{\partial\omega}>0$. For that, note that the value of a good held by a chronic robber decreases with ω:

$$\frac{\partial}{\partial\omega}V_r^g=\frac{((G+c)(\delta-1)-c(1-\gamma\delta)-G)(\gamma+1)}{(\gamma\delta\omega-\gamma\delta-\delta\omega-\delta+2)^2}<0 \tag{51}$$

That value for a trader, however, is, clearly, increasing, as $\delta(1-\gamma)\geq 0$ and

$$V_t^g=G\frac{\gamma}{1-\omega\delta(1-\gamma)}. \tag{52}$$

Therefore, γ_H increases with ω, in the case that where $c\in(\underline{\underline{c}},\underline{c})$, and $c>c^D(\phi)$.

The alternative is that $c\geq\underline{c}(\phi)$. In this case, the stable equilibrium involves $\gamma=1$, so that the same proof can be applied except that $\frac{\partial\gamma}{\partial\delta}=0$ and $\frac{\partial\gamma}{\partial\omega}=0$. ■

Proof of Proposition 7. Write the agent's expected payoffs as

$$\sum_{t=0}^{\infty}\delta^t(\pi_tG_t-\eta_tcG_t)$$

where π_t is the probability that the agent consumes in period t, and η_t is the probability she is punished,[21] both probabilities depend on equilibrium γ and on the rest of the

[20] We have dropped the dependence of F on parameters ϕ for simplicity beforehand.

[21] Note that in no event an agent is both consuming and is punished in the same period.

parameters, as well as on the action that agent chooses in period t. This dependence, however, is not important for the result. Indeed, with constant growth,

$$\sum_{t=0}^{\infty} \delta^t \left(\pi_t - \eta_t c\right) G_0 g^t = \sum_{t=0}^{\infty} \left(g\delta\right)^t \left(\pi_t - \eta_t c\right) G_0.$$

Hence the structure of the model is identical to one without growth when gains to trade are G_0 every period and the discount factor δ is replaced with δg, and hence the condition $\delta g < 1$ must be satisfied. ■

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