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Matching in Auctions with an Uninformed Seller*

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Abstract

In many auctions, matching between the bidder and seller raises the value of the contract for both parties although information about the matching may be incomplete. We consider the case in which each bidder observes the quality of his match with the seller but the seller does not observe the quality of the matches. Our objective is to determine whether it is in the seller's interest to (1) account for matching in his allocation decision and (2) observe the matches prior to the auction.

It is shown that irrespective of how important matching may be to the seller, the optimal mechanism can be implemented without using matching as a factor. If the seller has commitment power, he can raise his expected utility further by observing the matches ex ante. However, if the seller cannot commit, his value for the information may be negative: the seller's knowledge of the matches generates an asymmetry across bidders which depresses bids. The more matching matters, the greater the penalty associated with observing the matches in advance.

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1 Introduction

The real value of a contract lies beyond its financial components: the degree to which the parties are compatible also matters. According to a KPMG study, "83% of all mergers and acquisitions (M&As) failed to produce any benefit for the shareholders and over half actually destroyed value." Interviews of over 100 senior executives revealed that the overwhelming cause of failure "is the people and the cultural differences" (Gitelson et al., 2001).

Compatibility between a buyer and seller matters for more than just mergers and acquisitions. It is an issue in publishing and team sports as well: a good match between an author and his editor may generate a better book, and an athletic team is more likely to win games if the players have compatible skills. Given the impact of matching on contract value, it is not surprising that we observe sellers using matching as a factor in their choice of buyer. During the 2002 auction for the rights to his second novel, Charles Frazier, author of *Cold Mountain*, asserted that "money was not the only consideration and that he was keen to choose the right editor to help him shape the book from the beginning" (Gumbel, 2002). Venezuela's state-owned oil company, Petróleos de Venezuela (PDVSA), accounted for technological compatibility when it selected private partners for the development of marginal fields in the early 1990s (Chalot, 1996).

The purpose of this paper is to develop a theory of auctions in which matching raises the value of the contract for both the seller and bidder but information about the matching is incomplete. We assume each bidder is better informed about his match with the seller than the seller is. Our objective is twofold: we first examine whether it is in the seller's interest to use matching as a factor in his allocation decision and then determine the extent to which obtaining information about the quality of the matches before the auction is of value to the seller.

We pay special attention to the role of commitment. A seller is said to have *commitment power* if he can allocate the contract in accordance with the allocation rule announced at the outset of the auction even when he prefers a different allocation after observing the bids. In contrast, a seller without commitment power cannot refrain from allocating the contract to the bidder whose combination of bid payment and expected match maximizes the seller's expected utility.

Our results indicate that the seller need not consider matching as a factor in his allocation decision. That is, the optimal mechanism can be implemented without using matching irrespective of how important matching may be to the seller. We also show that if the seller has commitment power, his value for observing the matches before the auction is positive. However, without commitment, his value for the information may be negative. Moreover, the more matching matters, the greater the penalty associated with observing the matches in advance.

We develop a model in which a single seller seeks to contract with one of several bidders. Each pairing of seller and bidder is characterized by a match. The better the match, the more each party values the contract. Each bidder observes his match with the seller (but not the matches of his opponents with the seller). In contrast, the seller does not observe his matches with the bidders.

The reader will note that the notion of matching advanced here is different from the notion advanced in the two-sided matching literature.[1] While the latter use the term to refer to the pairing of agents in a two-sided market, our paper uses the term to refer to the compatibility between a bidder and seller. Since this compatibility induces a positive correlation between the valuations of the bidder and seller, our notion of matching is more closely related to the literatures on affiliated values (e.g., Milgrom and Weber, 1982a) and interdependent valuations (e.g., Jehiel and Moldovanu, 2001), but while these literatures are more concerned with linking the bidders' valuations, our paper focuses on linking the valuations of the bidder and seller.

We solve for the optimal mechanism and find that it can be implemented via a standard first-price auction with an appropriate choice of reserve price. Since a better match implies a higher contract value, well matched bidders face a higher opportunity cost of not raising their bids. As a result, bids increase in match. By awarding the contract to the highest bidder, the seller finds himself automatically contracting with the best matched bidder. Since a first-price auction does not (directly) account for matching, we conclude that the seller need not consider matching as a factor in his allocation decision – no matter how important matching may be to the seller.

[1]See Gale and Shapley (1962), Shapley and Shubik (1972), Crawford and Knoer (1981), Kelso and Crawford (1982), Kamecke (1998), Hatfield and Milgrom (2005), and Bulow and Levin (forthcoming).

Implementing the first-price auction may require commitment power. Since the seller's utility is determined by both price and match, the seller may prefer a lower price and higher match to a higher price and lower match. Hence, if after observing the bids the seller believes the winning bidder is a poor match, he may be inclined to deviate from the "high bid wins" rule and offer the contract to another bidder instead.

The question naturally arises: can the optimal mechanism be implemented without commitment? In order to answer this question, we examine a *first-score* auction, in which each bidder bids on price alone but the seller selects the bidder whose combination of price and expected match maximizes his expected utility.[2] Since this allocation rule reflects the seller's true preferences, there should be no incentive for the seller to deviate from the rule ex post.

We find that the equilibrium bidding strategies in the first-score auction are identical to those in the first-price auction. In a first-score auction, well matched bidders have an incentive to convey their information to the seller in order to raise their probability of winning. Since well matched bidders have a higher value for the contract, they can credibly signal their favorable matches by raising their bids beyond the point at which it is profitable for poorly matched bidders to mimic them.[3] Given that higher bids signal better matches,

[2]Che (1993), Branco (1997), Zheng (2000), and Asker and Cantillon (2004) analyze a similar auction format in which the winning bidder is selected on the basis of price and quality. Our mechanism differs in that bidders bid only on price and the seller is left to estimate match on his own. Our mechanism is more closely linked to the biased procurement problem studied by Rezende (2004), in which bidders bid only on price but the allocation rule incorporates a bias determined by the seller. Our environment differs in that the bias is linked to the bidder's private information about his match.

[3]Bikhchandani and Huang (1989), Katzman and Rhodes-Kropf (2002), Das Varma (2003), Goeree (2003), Haile (2003), and Molnár and Virág (2004) examine signaling in auctions, but these papers are concerned with bidders signaling their private information to other bidders so as to affect future strategic interactions. In contrast, the signaling behavior in our paper is motivated by the structure of the auction game itself: bidders are interested in signaling their private information to the seller in order to influence the seller's choice of winner. In this sense, our paper is more similar to Avery (1998), which addresses the use of jump bids to signal a high valuation and encourage competing bidders to withdraw.

the contract goes to the bidder submitting the highest bid – which is precisely the allocation rule in a first-price auction.

Although the absence of commitment power does not change the equilibrium bidding strategies, it may affect the feasibility of adhering to the prescribed reserve price. Burguet and Sákovics (1996), McAfee and Vincent (1997), and Skreta (2004) examine auctions in which the seller can commit to a reserve price in the current period but cannot commit not to resell the object in a future period. In this environment, the seller minimizes his losses by holding the initial auction with an elevated reserve and reducing the reserve with every subsequent auction.[4]

In contrast, we find that a seller without commitment power can sustain an elevated reserve price provided that the quality of the match is sufficiently important to him. If the seller associates bids which fall below the reserve with a poor match and the adverse effect of a poor match is sufficiently large, then the seller will prefer retaining the contract to accepting a bid which does not meet the reserve. We conclude that if matching is important, the optimal mechanism can be implemented without commitment.

In sum, we find that the seller need not account for matching in his allocation decision since the optimal outcome can be achieved via a first-price auction. If matching is sufficiently important to the seller, commitment power is not required to implement the optimal mechanism. However, if the seller's utility does not vary enough with the quality of the match, commitment is needed in order to adhere to the prescribed reserve price.

[4]Menezes and Ryan (2005) also address the seller's inability to adhere to a reserve price, but rather than characterizing the path of reserve prices, they establish an equivalence between commitment and bargaining power.

We then introduce an opportunity for the seller to observe the matches before the auction. Under commitment, the value of the information is positive since the seller can extract the entire surplus by making a take-it-or-leave-it offer to the best-matched bidder. However, in the absence of commitment, the seller is unable to appropriate all the rent because he cannot reject offers which exceed his reservation value but are unaffordable for any other bidder.

Moreover, when the seller lacks commitment power, his knowledge of the matches may depress bids. If the seller has observed the matches prior to the submission of bids, a well matched bidder knows that the auction is biased in his favor irrespective of the offer he makes. As a result, he need not bid as aggressively to win. In fact, the bias permits a well matched bidder to bid less than a poorly matched counterpart and still win the auction. Thus, there is an incentive for well matched bidders to capitalize on their advantage by reducing their bids. We call this effect the *asymmetry effect*.[5]

Since the asymmetry effect reduces bids, the value of the information under no commitment is not only lower than it is under commitment but may actually be negative. The greater the effect of matching on the seller's utility, the greater the advantage enjoyed by a well matched bidder and the greater the incentive to reduce his bid. Therefore, we find

[5]There is a vast literature on the negative effect of asymmetries on price competition, the majority of which addresses asymmetries in the distributions from which valuations are drawn (see Milgrom (1981), Myerson (1981), Milgrom and Weber (1982b), Graham and Marshall (1987), McAfee and McMillan (1987), Bulow and Roberts (1989), McAfee and McMillan (1989), Mailath and Zemsky (1991), McAfee and McMillan (1992), Marshall, Meurer, Richard, and Stromquist (1994), Tschantz, Crooke, and Froeb (1997), Waehrer (1999), Dalkir, Logan, and Mason (2000), Kaplan and Zamir (2002), Waehrer and Perry (2003), Cantillon (2005), and Bulow and Levin (forthcoming)). The nature of the asymmetry in our paper is more closely related to that in Che (1993) and Rezende (2004), where the source of asymmetry is the seller's use of factors other than price to determine the auction winner.

that the more the seller cares about matching, the stronger his incentive not to observe the matches in advance.

This result lies in stark contrast to the conventional wisdom that bidders derive their profits from their private information (see Milgrom (1981), Milgrom and Weber (1982a), Milgrom and Weber (1982b), and McAfee and McMillan (1987)). The difference arises from the two modifications made to the standard independent private values model: relaxing the assumption that the seller has commitment power and augmenting the seller's utility function to account for matching. In this framework, well matched bidders can only gain from (verifiably) disclosing their private information: the lack of commitment power prevents the seller from driving the price above the competitive level, and the bias in favor of well matched bidders dampens price competition even further.[6]

The remainder of this paper is organized as follows. Section 2 introduces the model. In Section 3, we solve for the optimal mechanism and identify which allocation rules implement the optimal mechanism in the commitment and no commitment case, respectively. We also derive a set of conditions under which the optimal outcome can be achieved in the absence of commitment. Section 4 examines the extent to which obtaining information about the matches ex ante is of value to the seller. Special attention is paid to the no commitment case and the role of the asymmetry effect in reducing bids. Conclusions are offered in Section 5. All proofs are relegated to the Appendix.

[6]Benoît and Dubra (forthcoming) offer another exception: they modify the standard common values model by allowing bidders to verifiably disclose their private information to other bidders. They find that if bidders lack commitment power, they always disclose their private information even when doing so is harmful ex ante. Consequently, they prefer not to have any private information so as to avoid the compulsion to disclose it.

2 The Model

A seller is to auction off a contract to one of n risk-neutral bidders ($n \geq 2$).[7] Every potential pairing of seller and bidder has an associated match. We denote the match between the seller and bidder i by $\theta_i \in [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$, where $\theta_i > \theta_j$ indicates that bidder i has a better match than bidder j does. We assume the θ_i's are independently and identically distributed according to a commonly known cumulative distribution function (cdf) F with $F(\underline{\theta}) = 0$ and $F(\overline{\theta}) = 1$.

Assumption 1 *F is continuous over $[\underline{\theta}, \overline{\theta}]$ and has positive density f.*

Bidder i's utility from contracting with the seller is

$$\theta_i - b_i,$$

where θ_i is bidder i's value for the contract and $b_i \in \mathbb{R}$ is the bid submitted by bidder i. Bidder i's utility is zero if he does not win the contract.

The seller derives utility from both the bid payment and his match with the winning bidder. We assume the seller's utility from contracting with bidder i is

$$V(\theta_i) + b_i,$$

where $V(\theta_i)$ represents the seller's value for his match with bidder i. The following assumption captures the notion that a good match raises the value of the contract for both the seller and the bidder:

[7]We employ the term "auction" throughout, but the model is general enough to accommodate both auction and procurement settings.

Assumption 2 $V : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}$ *is continuous and strictly increasing over* $[\underline{\theta}, \overline{\theta}]$.

The seller's utility is zero if he does not contract with any bidder.[8]

The following assumption is imposed so as not to rule out the possibility of a mutually beneficial trade:

Assumption 3 (participation condition) $V(\overline{\theta}) + \overline{\theta}$ *is positive.*

Additionally, we impose the following regularity condition:

Assumption 4 (regularity condition) *The function*

$$V(\theta_i) + \theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)}$$

is strictly increasing over $[\underline{\theta}, \overline{\theta}]$.[9]

We assume bidder i is better informed about his match than the seller is. Bidder i observes his match θ_i but not the matches of his opponents.[10] The seller does not observe the matches directly, and therefore, his beliefs about the matches are determined by the prior, F, and the observed bids.

3 The Optimal Mechanism

Intuition suggests that if matching affects the seller's expected utility, the seller should account for matching in his allocation decision. In fact, we observe this behavior in a number

[8] When $V(\theta_i) = v\theta_i$, where v is a positive constant, our model can be mapped to the interdependent valuations framework outlined in Section 5 of Jehiel and Moldovanu (2001): simply let the agents be indexed by $i \in \{0, 1, 2, ..., n\}$, where agent i is the seller if $i = 0$ and bidder i otherwise; let p_k^i be the probability the contract is awarded to agent i in alternative k; and let $s^0 = 0$, $s^i = \theta_i$, $a_{k0}^i = vp_k^i$, $a_{ki}^i = p_k^i$, and $a_{ki}^j = 0$ for all $j \neq i$.

[9] Note that Assumption 4 is less restrictive than the monotone hazard rate condition.

[10] Bidder i's beliefs about θ_j, $j \neq i$, are determined by the prior, F.

of environments. For example, PDVSA accounted for technological complementarities in its selection of partners for the development of marginal oil fields while novelist Charles Frazier accounted for his compatibility with the editor in his choice of a publisher.

In this section, we ask whether the seller should account for matching in allocating the contract. Our approach will be to solve for the optimal mechanism and then identify which allocation rules implement it.

Special attention is paid to the role played by the seller's ability to commit. Under *commitment*, the seller can credibly announce any allocation rule before the auction. That is, the seller is committed to allocating the contract in accordance with the announced rule even if he prefers a different selection ex post. In contrast, when the seller lacks commitment power, the contract is simply allocated to the bidder whose combination of bid payment and expected match maximizes the seller's expected utility.

3.1 Implementation under Commitment

We begin by assuming the seller can commit to any allocation rule and deriving the optimal mechanism. By appealing to the revelation principle (Myerson, 1981), we restrict attention to direct revelation mechanisms $\{p(\cdot), t(\cdot)\}$, where $p_i : \left[\underline{\theta}, \overline{\theta}\right]^n \to [0,1]$ is the probability that bidder i is awarded the contract and $t_i : \left[\underline{\theta}, \overline{\theta}\right]^n \to \mathbb{R}$ is the expected transfer from bidder i to the seller.

If bidder i observes his type θ_i but announces that his type is x, his expected utility from the mechanism $\{p(\cdot), t(\cdot)\}$ is

$$U_i\left(x, \theta_i\right) \equiv E_{\theta_{-i}}\left[\theta_i p_i\left(x, \theta_{-i}\right) - t_i\left(x, \theta_{-i}\right)\right]. \tag{3.1}$$

Similarly, the seller's expected utility from the mechanism is

$$U_0 \equiv E_\theta \left[\sum_{i=1}^{n} V(\theta_i) p_i(\theta) + \sum_{i=1}^{n} t_i(\theta) \right]. \tag{3.2}$$

The mechanism is optimal if it maximizes U_0 subject to

Incentive compatibility (IC): $\quad U_i(\theta_i, \theta_i) \geq U_i(x, \theta_i) \quad \forall i, \forall \theta_i, \forall x;$

Individual rationality (IR): $\quad U_i(\theta_i, \theta_i) \geq 0 \quad \forall i, \forall \theta_i;$

and

$p_i(\theta) \geq 0 \quad$ and $\quad \sum_{i=1}^{n} p_i(\theta) \leq 1 \quad \forall i, \forall \theta.$

Proposition 1 *The optimal mechanism satisfies*

$$p_i(\theta) = \begin{cases} 1 & \text{if } \theta_i \geq \theta_j \ \forall j \text{ and } \theta_i \geq \theta_* \\ 0 & \text{otherwise} \end{cases}$$

and

$$E_{\theta_{-i}}[t_i(\theta_i, \theta_{-i})] = \begin{cases} \theta_i F^{n-1}(\theta_i) - \int_{\theta_*}^{\theta_i} F^{n-1}(x)dx & \text{if } \theta_i \geq \theta_* \\ 0 & \text{otherwise} \end{cases}$$

where

$$\theta_* = \begin{cases} \left\{ x \in \left(\underline{\theta}, \overline{\theta}\right) : V(x) + x - \frac{1-F(x)}{f(x)} = 0 \right\} & \text{if } V(\underline{\theta}) + \underline{\theta} - \frac{1}{f(\underline{\theta})} < 0 \\ \underline{\theta} & \text{otherwise.} \end{cases}$$

Proof: See Appendix. □

The proof develops a relaxed optimization program by reducing the number of choice variables from two to one. It then identifies the unique solution of the relaxed program, and demonstrates that it satisfies the constraints of the original program. The regularity condition plays a key role.

Note that the mechanism outlined in Proposition 1 can be implemented via a standard first-price sealed-bid auction with an appropriate choice of reserve price. Since a higher match implies that the bidder has a higher value for the contract, well matched bidders can afford to bid more than poorly matched bidders. Hence, a rule that allocates the contract to the highest bidder also allocates the contract to the bidder with the best match.

Since the allocation rule in a first-price auction does not (directly) incorporate matching, we conclude that the seller can maximize his expected utility without using matching as a factor in his allocation decision – no matter how important matching may be to the seller.

However, implementing a first-price auction may require commitment power. Since $V(\theta_*) + \theta_* > 0$, there exists a mutually beneficial trade between the seller and a bidder with type θ_*. As a result, we encounter the usual problem associated with an elevated reserve price: when the high bid falls just short of θ_*, the seller may prefer awarding the contract to retaining it.

The fact that the seller's utility is a function of both price and match generates an additional problem. In the absence of commitment, the seller is not bound by the "first price wins" rule announced at the auction's outset. Consequently, if the seller believes the quality of the winning bidder's match is sufficiently poor, he may elect to award the contract to another bidder, whose price offer is lower but whose match is believed to be higher. Hence, if the seller's inability to commit is common knowledge, bidders select their strategies assuming the contract goes to the bidder whose combination of price and expected match maximizes the seller's expected utility.

In the following section, we address these issues in investigating whether the optimal mechanism can be implemented without commitment.

3.2 Implementation under No Commitment

Consider the following auction game:

1. Each bidder submits a bid independently and simultaneously.

2. The seller contracts with the bidder whose offer maximizes the seller's expected utility provided that the offer is not less than the seller's reservation utility of zero. That is, bidder i wins the contract if

$$E\left[V(\theta_i) \mid b_i\right] + b_i \geq 0$$

   and

$$E\left[V(\theta_i) \mid b_i\right] + b_i > E\left[V(\theta_j) \mid b_j\right] + b_j \quad \forall j \neq i.$$

   Ties are resolved by a random draw with equal probability.

3. Once the contract is allocated to bidder i, θ_i is revealed. The seller's payoff is $V(\theta_i)+b_i$, bidder i's payoff is $\theta_i - b_i$, and all other bidders get zero. The auction game is then over.

We call this auction game a *first-score* auction, where the term "score" refers to the combination of bid and expected match. For instance, bidder i's score is given by $E\left[V(\theta_i) \mid b_i\right]+b_i$. As indicated in the timeline above, the contract is allocated to the bidder with the highest score provided that that score is nonnegative.

The first-score auction is much like a first-price auction in that the winning bidder pays his bid. However, unlike a first-price auction, the winner is the bidder who offers the most attractive combination of bid and expected match. This allocation rule allows the seller to reject an offer made by the highest bidder and allocate the contract to another bidder whose pairing of bid and expected match is more attractive than the pairing offered by the highest bidder. Moreover, the first-score auction does not require an elevated reserve price: the seller retains the contract only when every offer falls short of the seller's reservation utility. Since the allocation rule of the first-score auction reflects the seller's true preferences, there is no incentive for the seller to deviate from it after observing the bids.

Our use of a first-score auction to represent the seller's lack of commitment is consistent with the literature. Che (1993) states that in the absence of commitment "the only feasible scoring rule is one that reflects the seller's [true] preference ordering," and Rezende (2004) allows a seller without commitment power to renege on the announced allocation rule and select the auction winner arbitrarily. Our representation is also consistent with the principal-agent model in Bester and Strausz (2000): they define imperfect commitment in terms of a two-stage game, in which agents select messages in the first stage and the principal updates his beliefs and selects an allocation in the second stage.

Our approach will be to investigate the perfect Bayesian equilibria of the first-score auction to see whether the optimal mechanism outlined in Proposition 1 can be implemented in the absence of commitment.

Let $P_i(b)$ represent bidder i's probability of winning with a bid of b. Since the seller observes only the bids offered and not the vector of types, each bidder's probability of winning the auction is a function of his bid but not of his type. Let $B_i : \mathbb{R} \times [\underline{\theta}, \overline{\theta}] \to [0, 1]$

represent bidder i's equilibrium bidding strategy (possibly a mixed strategy), where $B_i(b, \theta_i)$ is the cdf from which bidder i draws a bid of b when his type is θ_i. Let β_i be the density function associated with B_i.

Lemma 1 *Suppose there exist $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right)$ and some b in the support of $\beta_i(\cdot, \theta_i)$ such that $P_i(b) > 0$. Then for all $\hat{\theta}_i > \theta_i$ and all $\hat{b}$ in the support of $\beta_i(\cdot, \hat{\theta}_i)$, it is the case that $P_i(\hat{b}) > 0$.*

Proof: See Appendix. □

Lemma 1 indicates that if the equilibrium strategies are such that bidder i's probability of winning is positive when he draws type θ_i, then it is also positive when he draws any type greater than θ_i. It establishes that for each bidder, there exists a threshold, $\tilde{\theta}_i$, such that if the type drawn exceeds this threshold, the bidder's probability of winning is positive but if the type drawn is below this threshold, the bidder's probability of winning is zero.

In the following lemma, we show that in any separating equilibrium, bids are strictly increasing in type when θ_i exceeds $\tilde{\theta}_i$.

Lemma 2 *Let B_i be a separating equilibrium bidding strategy. Suppose there exist $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right)$ and some b in the support of $\beta_i(\cdot, \theta_i)$ such that $P_i(b) > 0$. If $\hat{\theta}_i > \theta_i$ and $\hat{b}$ is in the support of $\beta_i(\cdot, \hat{\theta}_i)$, then $\hat{b} > b$.*

Proof: See Appendix. □

In a separating equilibrium, the seller can perfectly infer types from bids. Lemma 2 indicates that for each bidder, a higher bid signals a better match. Since a better match implies a higher value for the contract, the return on raising one's bid increases with the level of matching. As a result, bidders can credibly signal their better matches by offering higher bids.

The optimal mechanism requires that if a contract is awarded, it is awarded to the bidder with the best match. Lemma 2 implies that this condition is satisfied for any symmetric separating equilibrium. Therefore, we will proceed by solving for the symmetric separating equilibria of the first-score auction game.

Lemma 3 *Let B be a symmetric separating equilibrium bidding strategy. If $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ and there exists some b in the support of $\beta(\cdot, \theta)$ such that $P(b) > 0$, then $B(\cdot, \theta)$ is a pure strategy.*

Proof: See Appendix. □

Let

$$\tilde{\theta} \equiv \inf \left\{x \in \left[\underline{\theta}, \overline{\theta}\right] : P(b) > 0 \text{ for some } b \text{ in the support of } \beta(\cdot, x)\right\}. \tag{3.3}$$

Lemma 3 indicates that the bidding strategy is pure when the bidder's type exceeds $\tilde{\theta}$. Therefore, the equilibrium bidding strategy of any bidder with type $\theta \in (\tilde{\theta}, \overline{\theta}]$ can be represented by $b : (\tilde{\theta}, \overline{\theta}] \to \mathbb{R}$, where

$$b(\theta) \equiv \{b : B(x, \theta) = 0 \ \forall x < b \text{ and } B(x, \theta) = 1 \ \forall x \geq b\}. \tag{3.4}$$

The following two lemmas determine the functional form of b for an arbitrary $\tilde{\theta}$.

Lemma 4 *In any symmetric separating equilibrium, the bidding strategy $b : (\tilde{\theta}, \overline{\theta}] \to \mathbb{R}$ is continuous.*

Proof: See Appendix. □

Lemma 5 *In any symmetric separating equlibrium, the bidding strategy is*

$$b(\theta) = \theta - \frac{\int_{\tilde{\theta}}^{\theta} F^{n-1}(x)dx}{F^{n-1}(\theta)}$$

for $\theta \in (\tilde{\theta}, \overline{\theta}]$.

Proof: See Appendix. □

An interesting feature of the bidding function specified by Lemma 5 is that the seller's value for the match, $V(\cdot)$, drops out. In general, one would expect V to appear in the function since it enters into the allocation rule and, therefore, affects the bidder's probability of winning. However, since b and V are strictly increasing, each bidder's score,

$$V(b^{-1}[b(\theta)]) + b(\theta), \tag{3.5}$$

increases in his type, θ. As a result, his probability of winning reduces to $F^{n-1}(\theta)$. Note that the functional form of V is not relevant; all that is required is that V be nondecreasing.

Furthermore, the bidding function specified by Lemma 5 is identical to the bidding function in a first-price auction with a reserve price of $\tilde{\theta}$. The equivalency follows from the fact that a higher bid signals a higher type, which, in turn, generates a higher V and a higher score. Consequently, the contract goes to the bidder submitting the highest bid – which is precisely the allocation rule in a first-price auction.

Although the absence of commitment power does not change the form of the bidding function, it may affect the feasibility of adhering to a reserve price of θ_*. Let θ_0 be the lowest type for which there exists a mutually beneficial trade; that is, let

$$\theta_0 \equiv \begin{cases} \left\{x \in \left(\underline{\theta}, \overline{\theta}\right) : V(x) + x = 0\right\} & \text{if } V\left(\underline{\theta}\right) + \underline{\theta} < 0 \\ \underline{\theta} & \text{otherwise.} \end{cases} \tag{3.6}$$

Suppose $V(\underline{\theta})+\underline{\theta}<0$. In this case, $V(\theta_0)+\theta_0=0$ and $V(\theta_*)+\theta_*>0$. Since V is increasing, $\theta_0<\theta_*$. Now consider a scenario in which the seller announces a reserve price of θ_* but a bidder with type $\theta\in(\theta_0,\theta_*)$ has the highest type and bids his valuation, θ. In the absence of commitment, the seller will contract with this bidder unless the bid θ is off the equilibrium path and the seller's off-equilibrium-path beliefs are sufficiently punishing. In particular, the difference between $V(\theta)$ and $V(\underline{\theta})$ must be large enough.

Let $\mu:\mathbb{R}\times[\underline{\theta},\overline{\theta}]\rightarrow[0,1]$ represent the seller's posterior beliefs, where $\mu(\theta\mid b)$ is the probability the seller assigns to the event that a bidder's type is in $[\underline{\theta},\theta]$ when he offers a bid of b.

Proposition 2 *Let θ_* be defined as in Proposition 1. Suppose the seller's off-equilibrium-path beliefs are such that $\mu(\theta\mid b)=1$ for all $\theta\in[\underline{\theta},\overline{\theta}]$ when $b<\theta_*$.*[11] *If*

$$V(\theta_*)-V(\underline{\theta})\geq\frac{1-F(\theta_*)}{f(\theta_*)},$$

then there exists a symmetric separating equilibrium in which bidders bid according to

$$\begin{aligned} b(\theta) &= \theta-\frac{\int_{\theta_*}^{\theta}F^{n-1}(x)dx}{F^{n-1}(\theta)} && \text{if } \theta\in\left[\theta_*,\overline{\theta}\right] \\ b(\theta) &< -V(\theta_*) && \text{otherwise.} \end{aligned}$$

Proof: See Appendix. □

Proposition 2 indicates that if matching is sufficiently important to the seller (i.e., the range of V is sufficiently large), the optimal mechanism can be implemented in the absence of commitment. Hence, when matching is important, there is no loss associated with a lack of

[11] This equilibrium satisfies the Intuitive Criterion of Cho and Kreps (1987).

commitment power. This may explain why we observe agents, such as PDVSA, who are able to commit to a different allocation mechanism, simply implementing a first-score auction.

In the next section, we consider the value of information about the matches and ask whether the seller can do better if he observes the matches ex-ante.

4 The Value of Information

Suppose the seller could observe the vector of matches before selecting a mechanism for allocating the contract. Would it be in the seller's interest to do so? How does the seller's value for the information vary with his ability to commit to an allocation rule?

Under commitment, the seller can clearly do better by observing the matches in advance. With the information in hand, he simply makes a take-it-or-leave-it offer to the bidder with the best match in the amount of that bidder's valuation. Since the outcome is efficient and the seller extracts all the surplus, he necessarily improves upon the mechanism outlined in Proposition 1. Therefore, a seller with commitment power has a positive value for the information.

Suppose the seller cannot commit. In this case, the seller will not be able to capture the entire surplus. At best, the seller can extract the value-match combination, $V(\theta) + \theta$, of the bidder with the second-highest match since any package of greater value offered by the best-matched bidder goes uncontested by the other bidders. This suggests that the seller's value for the information is lower under the no commitment paradigm than under the commitment paradigm.

Our approach will be to revisit the first-score auction outlined in Section 3.2 under the assumption that the seller observes the matches in advance and compare equilibrium bids across the two information structures. In doing so, we will identify how the seller's ex ante knowledge of the matches alters the bidders' incentives.

In the first-score auction game with an informed seller, bidder i selects an equilibrium bidding strategy $B_i : \mathbb{R} \times \left[\underline{\theta}, \overline{\theta}\right] \to [0,1]$ such that for any type $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right]$ and any bid b_i in the support of $\beta_i(\cdot, \theta_i)$, the following two conditions are satisfied:

$$(\theta_i - b_i)P_i(b_i, \theta_i) \geq (\theta_i - b)P_i(b, \theta_i) \quad \forall b \in \mathbb{R} \tag{4.1}$$

and

$$(\theta_i - b_i)P_i(b_i, \theta_i) \geq 0, \tag{4.2}$$

where $P_i(b, \theta_i)$ represents bidder i's probability of winning the contract with a bid of b given that his type is θ_i. Note that because the seller observes the matches ex ante, each bidder's probability of winning is a function of both his bid and his type.

We proceed by reformulating the bidder's problem using bidder i's score as the choice variable. This reformulation permits us to directly apply the standard independent private values results. Let

$$s_i \equiv V(\theta_i) + b_i \tag{4.3}$$

denote the score offered by bidder i. Note that since both bidder i and the seller observe θ_i ex ante, bidder i's choice of bid unambiguously determines his score. Furthermore, let $Q_i(s)$ represent bidder i's probability of winning with a score of s. Since a first-score auction allocates the contract to the bidder offering the highest score, provided that score is not less than zero, $Q_i(s)$ is simply the probability that s is the highest score offered if $s \geq 0$ and zero otherwise. Finally, let $\Sigma_i : \mathbb{R} \times [\underline{\theta}, \overline{\theta}] \to [0,1]$ represent bidder i's equilibrium score strategy (possibly a mixed strategy), where $\Sigma_i(s, \theta_i)$ is the cdf from which bidder i draws a score of s when his type is θ_i, and let σ_i be the density function associated with Σ_i.

Using this notation, we can now reformulate the bidder's problem. Bidder i selects an equilibrium score strategy Σ_i such that for any type $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right]$ and any score s_i in the support of $\sigma_i(\cdot, \theta_i)$, the following two conditions are satisfied:

$$[V(\theta_i) + \theta_i - s_i]\, Q_i(s_i) \geq [V(\theta_i) + \theta_i - s]\, Q_i(s) \quad \forall s \in \mathbb{R} \tag{4.4}$$

and

$$[V(\theta_i) + \theta_i - s_i]\, Q_i(s_i) \geq 0. \tag{4.5}$$

Note that by interpreting $V(\theta_i) + \theta_i$ as the bidder's type, s_i as the bidder's bid, and zero as the seller's reserve price, we can map this formulation into the standard independent private values framework.[12] Therefore, we can invoke Maskin and Riley (1986) and Riley and Samuelson (1981) to obtain the following result:

Lemma 6 *Let θ_0 be defined by equation (3.6). There exists a unique equilibrium of the first-score auction game in which any bidder with type $\theta \in [\theta_0, \overline{\theta}]$ offers a score of*

$$s(\theta) = V(\theta) + \theta - \frac{\int_{\theta_0}^{\theta} F^{n-1}(x)dx}{F^{n-1}(\theta)} - \frac{\int_{\theta_0}^{\theta} V'(x)F^{n-1}(x)dx}{F^{n-1}(\theta)}$$

and any bidder with type $\theta \in [\underline{\theta}, \theta_0)$ offers a score less than zero.

[12]This technique is similar to that used in Asker and Cantillon (2004). In their terminology, $V(\theta_i) + \theta_i$ is bidder i's "pseudotype."

Lemma 6 indicates that for all $\theta > \theta_0$, higher types offer higher scores. However, this result does not imply that higher types offer higher bids. Using Lemma 6 and the fact that in equilibrium $s(\theta) = V(\theta) + b(\theta)$, we derive the equilibrium bidding function specified in the following proposition.

Proposition 3 *Let θ_0 be defined by equation (3.6). There exists a unique equilibrium of the first-score auction game in which any bidder with type $\theta \in [\theta_0, \overline{\theta}]$ bids according to*

$$b(\theta) = \theta - \frac{\int_{\theta_0}^{\theta} F^{n-1}(x)dx}{F^{n-1}(\theta)} - \frac{\int_{\theta_0}^{\theta} V'(x)F^{n-1}(x)dx}{F^{n-1}(\theta)}$$

and any bidder with type $\theta \in [\underline{\theta}, \theta_0)$ bids

$$b(\theta) < -V(\theta).$$[13]

The first two terms of the bidding function are identical to the bidding function outlined in Proposition 2 with the exception of the lowest participating type, θ_0. The third term, however, is a novel addition. The seller's value for matching enters the bidding function and depresses bids. The greater the importance of matching, V', the lower the bids. As a result, the bidding function need not be well behaved. In fact, for θ sufficiently close to θ_0, bids decrease in type.

Two conflicting effects are at play:

The value effect: A well matched bidder has a higher value for the contract, and therefore, the opportunity cost of not increasing his bid is higher;

[13]Despite the multiplicity of bids satisfying the condition $b(\theta) < -V(\theta)$, we assert that the equilibrium is unique in the sense that these bids can be classified as "non-participating": any bid satisfying this condition maps to a score less than the seller's reservation utility.

The asymmetry effect: A well matched bidder is preferred by the seller, and therefore, he need not bid as aggressively to win.[14]

The value effect induces bids to increase with match. It is the reason we observe monotonicity in bids in both the standard first-price auction and the first-score auction with an uninformed seller. It follows that the value effect is represented by the first two terms of the bidding function outlined in Proposition 3.

The asymmetry effect is represented by the third term. Since the seller's utility increases with match and the matches are known to the seller, well matched bidders have an advantage over poorly matched bidders: a bidder with a high match can bid less than a bidder with a low match and still win the contract. In other words, the asymmetry across bidders tends to dampen price competition. The greater the importance of matching, V', the greater the asymmetry and the lower the bids.

If the lowest participating type is fixed across information structures (i.e., if $\theta_* = \theta_0 = \underline{\theta}$), then bids are lower when the seller observes the matches in advance. In this case, the seller is clearly better off when he remains uninformed. But what if the lowest participating type differs across the two auctions (i.e., if $\theta_* > \theta_0$)?[15] In this case, the seller is still better off remaining uninformed since θ_* is the optimal reserve price. This observation delivers the following result:

[14]The asymmetry effect is similar to "the competition effect" in Rezende (2004).

[15]When the matches are observed, the lowest participating type must be θ_0 since the seller is unable to commit to excluding bidders with higher types and there is no mutually beneficial trade between the seller and a bidder whose type is less than θ_0. However, when the matches are not observed in advance, the seller may be able to exclude bidders with types greater than θ_0 if he believes their types are sufficiently low.

Corollary 1 *In a first-score auction, the seller can raise his expected utility by choosing not to observe the matches in advance.*

Therefore, the seller's value for information about the matches is not only lower in the absence of commitment but may actually be negative.

We conclude this section by offering a more intuitive interpretation of the first-score auction with an informed seller. Without commitment, the seller is not able to reject an offer which exceeds his reservation value but cannot be matched by any other bidder. That is, if Bertrand competition drives offers up to s, the seller requires commitment power to reject any offer which exceeds s. Therefore, the best the seller can do is allocate the contract to the bidder with the best match, who, in turn, delivers a score equal to $V(\theta_2) + \theta_2$, where θ_2 is the second-highest match. The selling price is given by

$$p = \theta_2 - [V(\theta_1) - V(\theta_2)], \tag{4.6}$$

where θ_1 is the highest match.[16]

This is precisely the outcome in a *second-score* auction. In a second-score auction, the object is allocated to the bidder offering the highest score but that bidder is required to deliver only the second-highest score. By invoking the revenue equivalence results in Riley and Samuelson (1981), we assert that the seller's expected utility in the second-score auction is the same as it is in the first-score auction.

[16] Vartiainen (2005) is the only paper we are aware of that examines the complete absence of commitment: at any point in the game, the seller can alter the allocation rule, and the bidders can renege on their offers and exit the game. Vartiainen (2005) finds that the English auction is the only feasible trading procedure in this environment. This result offers support to Bulow and Klemperer (1996), in which the no commitment paradigm is an English auction with no reserve price.

The second-score formulation yields a convenient interpretation of the asymmetry effect. In equation (4.6), the selling price p is less than the price offered by the bidder with the second-highest match, θ_2. The reduction is driven by the difference in the seller's values for the two matches, $V(\theta_1) - V(\theta_2)$. This term represents the advantage enjoyed by the best-matched bidder, or rather, the asymmetry effect. Once again, we see that the greater the importance of matching, V', the greater the asymmetry and the lower the selling price.

5 Conclusion

For a wide range of commercial arrangements, a good match between the buyer and seller raises the value of the contract for both parties. However, at the time the terms of the contract are set, the parties may not be fully informed about the degree to which they match. In this paper, we have addressed the case in which the quality of the match is the private information of the bidder.

The paper opened by asking whether the seller should account for matching in his allocation decision. It is shown that no matter how important matching is to the seller, he need not consider matching as a factor in order to implement the optimal mechanism. Since the bidder's value for the contract increases with match, allocating the contract to the highest bidder is equivalent to selecting the bidder with the best match.

However, it is also shown that if the seller cannot commit to allocating the contract on the basis of price alone, he can still implement the optimal mechanism. Since well matched bidders have a higher value for the contract, higher bids signal higher matches; as a result, the seller finds that he has no incentive to contract with anyone other than the highest bidder. Moreover, if matching is sufficiently important, the seller can credibly commit to the prescribed reserve price by associating bids which fall below the reserve with a poor match.

This paper has also provided answers to the natural questions regarding the effects of information asymmetry, namely (1) What is the seller's value for the bidder's information? and (2) How would the equilibrium behavior of the bidders change if the seller were to observe the matches in advance? The answers to these questions hinge on whether the seller can credibly commit to an allocation rule ex ante which he would prefer to violate ex post.

We find that if the seller can commit, observing the matches in advance allows him to appropriate all the rent. Therefore, the value of the information is positive. However, if the seller cannot commit, observing the matches introduces an asymmetry across bidders that depresses bids. Consequently, the seller's value for the information may be negative.

This result is surprising for two reasons. First, we observe that the bidders capture less rent when their information is private. This follows from the fact that the seller's knowledge of the matches eliminates the need for well matched bidders to signal their favorable matches through higher bids. Well matched bidders can, instead, capitalize on their preferred status, bid less than their poorly matched counterparts, and still win the auction. Second, we observe that the more the seller cares about matching, the less it pays for him to observe the matches in advance. The intuition is that the more matching matters, the greater the advantage enjoyed by a well-matched bidder, and the larger the margin by which he can reduce his bid and still win.

We conclude with a brief discussion of the relationship between our results and the Coase conjecture. In his seminal paper, Coase (1972) argued that if a durable-good monopolist cannot refrain from lowering his price in future periods and buyers are sufficiently patient, the monopoly price will converge to marginal cost almost instantaneously.[17] Our results are consistent with the conjecture when the seller observes the matches: under commitment, the seller captures the entire surplus, but under no commitment, the selling price falls to the competitive level. However, our results conflict with the conjecture when the seller does not observe the matches: if the quality of the match is sufficiently important to the seller, he can obtain the same outcome with or without commitment power.

Although a full treatment is outside the scope of this paper, the models we have presented appear to represent a class of environments for which the Coase conjecture does not hold. By introducing a factor the seller cares about but cannot directly observe, we have expanded the set of beliefs which can be used to sustain the monopoly outcome. Developing this idea further would be an interesting direction for future research.

A Appendix: Proofs

Proof of Proposition 1: The proof proceeds in four parts. We first show that if the mechanism $\{p(\cdot), t(\cdot)\}$ is optimal, then $U_i(\underline{\theta}, \underline{\theta}) = 0$. We then develop a relaxed optimization program and find its solution. After noting that the solution of the relaxed program coincides

[17]This argument is developed more formally in Stokey (1981), Bulow (1982), Gul, Sonnenschein, andWilson (1986), and Hart and Tirole (1988). McAfee and Vincent (1997) and Variainen (2005) develop analogous arguments in an auction setting.

with the mechanism outlined in Proposition 1, we demonstrate that the mechanism satisfies the constraints of the original program.

Suppose the mechanism $\{p(\cdot), t(\cdot)\}$ satisfies the IC and IR constraints for all $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right]$ but that $U_i(\underline{\theta}, \underline{\theta}) = \epsilon > 0$. Now consider a different mechanism $\left\{p(\cdot), \hat{t}(\cdot)\right\}$, where $\hat{t}_i(\cdot) \equiv t_i(\cdot) + \epsilon$. The mechanism $\left\{p(\cdot), \hat{t}(\cdot)\right\}$ satisfies the IR constraint for all $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right]$ since

$$\begin{aligned} \widehat{U}_i\left(\theta_i, \theta_i\right) &= E_{\theta_{-i}}\left[\theta_i p_i\left(\theta_i, \theta_{-i}\right) - \hat{t}_i\left(\theta_i, \theta_{-i}\right)\right] \\ &= E_{\theta_{-i}}\left[\theta_i p_i\left(\theta_i, \theta_{-i}\right) - t_i\left(\theta_i, \theta_{-i}\right)\right] - \epsilon \\ &\geq E_{\theta_{-i}}\left[\theta_i p_i\left(\underline{\theta}, \theta_{-i}\right) - t_i\left(\underline{\theta}, \theta_{-i}\right)\right] - \epsilon \\ &\geq E_{\theta_{-i}}\left[\underline{\theta} p_i\left(\underline{\theta}, \theta_{-i}\right) - t_i\left(\underline{\theta}, \theta_{-i}\right)\right] - \epsilon \\ &= 0. \end{aligned} \tag{A.1}$$

The first inequality holds because $\{p(\cdot), t(\cdot)\}$ satisfies the IC constraint; the second inequality holds because $p_i \in [0, 1]$. The mechanism $\left\{p(\cdot), \hat{t}(\cdot)\right\}$ also satisfies the IC constraint for all $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right]$ since

$$\begin{aligned} \widehat{U}_i\left(\theta_i, \theta_i\right) &= E_{\theta_{-i}}\left[\theta_i p_i\left(\theta_i, \theta_{-i}\right) - t_i\left(\theta_i, \theta_{-i}\right)\right] - \epsilon \\ &\geq E_{\theta_{-i}}\left[\theta_i p_i\left(x, \theta_{-i}\right) - t_i\left(x, \theta_{-i}\right)\right] - \epsilon \\ &= \hat{U}_i\left(x, \theta_i\right) \end{aligned} \tag{A.2}$$

for all $x \in \left[\underline{\theta}, \overline{\theta}\right]$. The inequality follows from the fact that $\{p(\cdot), t(\cdot)\}$ satisfies the IC constraint. Since $\epsilon > 0$ and $\hat{t}_i(\cdot) \equiv t_i(\cdot) + \epsilon$, the seller's expected utility is strictly greater under $\left\{p(\cdot), \hat{t}(\cdot)\right\}$ than it is under $\{p(\cdot), t(\cdot)\}$. Therefore, the original mechanism $\{p(\cdot), t(\cdot)\}$ cannot be optimal – a contradiction.

Having established that $U_i(\underline{\theta}, \underline{\theta}) = 0$, we turn our attention to developing a relaxed optimization program. Our approach is to modify the original program so as to reduce the number of choice variables from two, $p(\cdot)$ and $t(\cdot)$, to one, $p(\cdot)$. From equation (3.1), we have

$$E_{\theta_{-i}}\left[t_i\left(\theta_i, \theta_{-i}\right)\right] = E_{\theta_{-i}}\left[\theta_i p_i\left(\theta_i, \theta_{-i}\right)\right] - U_i\left(\theta_i, \theta_i\right), \tag{A.3}$$

and after substituting for $E_{\theta_{-i}}\left[t_i\left(\theta_i,\theta_{-i}\right)\right]$ in equation (3.2), we obtain

$$U_0 = E_\theta\left[\sum_{i=1}^{n}\left(V\left(\theta_i\right)+\theta_i\right)p_i(\theta)\right] - \sum_{i=1}^{n} E_{\theta_i}\left[U_i\left(\theta_i,\theta_i\right)\right]. \tag{A.4}$$

It remains to develop an expression for $E_{\theta_i}\left[U_i\left(\theta_i,\theta_i\right)\right]$ in terms of $p_i(\theta)$.

Incentive compatibility requires that $U_i\left(\theta_i,\theta_i\right) \geq U_i\left(x,\theta_i\right)$ for all $\theta_i \in \left[\underline{\theta},\overline{\theta}\right]$ and all $x \in \left[\underline{\theta},\overline{\theta}\right]$. Subtracting $U_i(x,x)$ from both sides and applying equation (3.1) yields

$$U_i\left(\theta_i,\theta_i\right) - U_i\left(x,x\right) \geq \left(\theta_i - x\right)E_{\theta_{-i}}\left[p_i\left(x,\theta_{-i}\right)\right]. \tag{A.5}$$

Since this inequality holds for all $\theta_i \in \left[\underline{\theta},\overline{\theta}\right]$ and all $x \in \left[\underline{\theta},\overline{\theta}\right]$, it must be the case that

$$\left(\theta_i - x\right)R_i(x) \leq \widetilde{U}_i\left(\theta_i\right) - \widetilde{U}_i\left(x\right) \leq \left(\theta_i - x\right)R_i(\theta_i), \tag{A.6}$$

where $R_i(x) \equiv E_{\theta_{-i}}\left[p_i\left(x,\theta_{-i}\right)\right]$ and $\widetilde{U}_i(x) \equiv U_i\left(x,x\right)$.

Suppose $\theta_i > x$ and let $\delta \equiv \theta_i - x$. Then inequality (A.6) can be rewritten as follows:

$$R_i(x) \leq \frac{\widetilde{U}_i\left(x+\delta\right) - \widetilde{U}_i\left(x\right)}{\delta} \leq R_i(x+\delta). \tag{A.7}$$

Since $R_i(\cdot)$ is nondecreasing, it is Riemann integrable. Hence,

$$\int_{\underline{\theta}}^{\theta_i} R_i\left(x\right)dx = \widetilde{U}_i\left(\theta_i\right) - \widetilde{U}_i\left(\underline{\theta}\right). \tag{A.8}$$

After substituting for $R_i(x)$, $\widetilde{U}_i\left(\theta_i\right)$, and $\widetilde{U}_i\left(\underline{\theta}\right)$ and applying our earlier result, $U_i(\underline{\theta},\underline{\theta}) = 0$, we obtain

$$U_i\left(\theta_i,\theta_i\right) = \int_{\underline{\theta}}^{\theta_i} E_{\theta_{-i}}\left[p_i\left(x,\theta_{-i}\right)\right]dx. \tag{A.9}$$

Taking the expectation of $U_i\left(\theta_i,\theta_i\right)$ and integrating by parts yields the expression we seek:

$$E_{\theta_i}\left[U_i\left(\theta_i,\theta_i\right)\right] = E_\theta\left[\frac{1-F\left(\theta_i\right)}{f\left(\theta_i\right)}p_i(\theta)\right]. \tag{A.10}$$

Using equation (A.10), we substitute for $E_{\theta_i}[U_i(\theta_i, \theta_i)]$ in equation (A.4) and obtain an expression for U_0 in terms of $p(\cdot)$. The resulting relaxed program is as follows:

$$\max_{\{p(\cdot)\}} E_\theta \left[\sum_{i=1}^{n} \left(V(\theta_i) + \theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)} \right) p_i(\theta) \right]$$

subject to

$p_i(\theta) \geq 0 \quad \text{and} \quad \sum_{i=1}^{n} p_i(\theta) \leq 1 \quad \forall i, \forall \theta.$

Since $V(\theta_i) + \theta_i - \frac{1-F(\theta_i)}{f(\theta_i)}$ is strictly increasing by Assumption 4, the unique solution of the relaxed program is

$$p_i(\theta) = \begin{cases} 1 & \text{if } \theta_i \geq \theta_j \ \forall j \text{ and } V(\theta_i) + \theta_i - \frac{1-F(\theta_i)}{f(\theta_i)} \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Note that the expression above coincides with the $p_i(\theta)$ outlined in Proposition 1. We proceed by deriving the associated $t(\cdot)$ and verifying that it too coincides with Proposition 1. From equation (A.3) we have

$$E_{\theta_{-i}}[t_i(\theta_i, \theta_{-i})] = \theta_i E_{\theta_{-i}}[p_i(\theta_i, \theta_{-i})] - U_i(\theta_i, \theta_i). \tag{A.11}$$

After substituting for $U_i(\theta_i, \theta_i)$ using equation (A.9) and replacing $p_i(\theta_i, \theta_{-i})$ with the solution of the relaxed program, we obtain

$$E_{\theta_{-i}}[t_i(\theta_i, \theta_{-i})] = \begin{cases} \theta_i F^{n-1}(\theta_i) - \int_{\theta_*}^{\theta_i} F^{n-1}(x)dx & \text{if } V(\theta_i) + \theta_i - \frac{1-F(\theta_i)}{f(\theta_i)} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

which clearly coincides with the corresponding expression in Proposition 1.

It remains to show that the mechanism $\{p(\cdot), t(\cdot)\}$ satisfies the constraints of the original program. The IR constraint, $p_i(\theta) \geq 0$, and $\sum_{i=1}^{n} p_i(\theta) \leq 1$ are satisfied trivially. To verify that the IC constraint holds, we must show that

$$U_i(\theta_i, \theta_i) \geq U_i(x, \theta_i) \tag{A.12}$$

for all $\theta_i \in [\underline{\theta}, \overline{\theta}]$ and all $x \in [\underline{\theta}, \overline{\theta}]$. Subtracting $U_i(x,x)$ from both sides and applying equation (3.1) yields

$$U_i(\theta_i, \theta_i) - U_i(x, x) \geq (\theta_i - x) E_{\theta_{-i}}[p_i(x, \theta_{-i})]. \tag{A.13}$$

If θ_i and x are less than θ_*, the inequality is trivially satisfied. If $\theta_i < \theta_*$ and $x \geq \theta_*$, the left-hand side is

$$\begin{aligned} U_i(\theta_i, \theta_i) - U_i(x, x) &= -\int_{\theta_*}^{x} F^{n-1}(y)dy \\ &\geq -(x - \theta_*) F^{n-1}(x) \\ &\geq -(x - \theta_i) F^{n-1}(x) \\ &= (\theta_i - x) E_{\theta_{-i}}[p_i(x, \theta_{-i})] \end{aligned} \tag{A.14}$$

and the inequality is again satisfied. If $\theta_i \geq \theta_*$ and $x < \theta_*$, the inequality reduces to $\int_{\theta_*}^{\theta_i} F^{n-1}(y)dy \geq 0$, which is trivially satisfied. If $\theta_i \geq x \geq \theta_*$,

$$\begin{aligned} U_i(\theta_i, \theta_i) - U_i(x, x) &= \int_{x}^{\theta_i} F^{n-1}(y)dy \\ &\geq (\theta_i - x) F^{n-1}(x) \\ &= (\theta_i - x) E_{\theta_{-i}}[p_i(x, \theta_{-i})] \end{aligned} \tag{A.15}$$

An analogous argument can be used to verify that the inequality holds if $x \geq \theta_i \geq \theta_*$. □

Proof of Lemma 1: Suppose $\hat{b}$ is in the support of $\beta_i(\cdot, \hat{\theta}_i)$ and b is in the support of $\beta_i(\cdot, \theta_i)$. If $\hat{\theta}_i > \theta_i$ and $P_i(b) > 0$, then by definition of equilibrium

$$\begin{aligned} (\hat{\theta}_i - \hat{b})P_i(\hat{b}) &\geq (\hat{\theta}_i - b)P_i(b) \\ &> (\theta_i - b)P_i(b) \\ &\geq 0. \end{aligned} \tag{A.16}$$

The first inequality follows from incentive compatibility while the third inequality follows from individual rationality. Since $(\hat{\theta}_i - \hat{b})P_i(\hat{b}) > 0$, it must be the case that $P_i(\hat{b}) > 0$. □

Proof of Lemma 2: Since we are constraining ourselves to separating equilibria, $\theta_i \neq \hat{\theta}_i$ implies $b \neq \hat{b}$. Hence, it is sufficient to show that $\hat{\theta}_i > \theta_i$ implies $\hat{b} \geq b$.

By definition of equilibrium,

$$(\theta_i - b)P_i(b) \geq (\theta_i - \hat{b})P_i(\hat{b}) \tag{A.17}$$

and

$$(\hat{\theta}_i - \hat{b})P_i(\hat{b}) \geq (\hat{\theta}_i - b)P_i(b). \tag{A.18}$$

Combining inequalities (A.17) and (A.18),

$$\begin{aligned} \theta_i \left[P_i(b) - P_i(\hat{b})\right] &\geq bP_i(b) - \hat{b}P_i(\hat{b}) \\ &\geq \hat{\theta}_i \left[P_i(b) - P_i(\hat{b})\right]. \end{aligned} \tag{A.19}$$

Since $\hat{\theta}_i > \theta_i$, it must be the case that $P_i(\hat{b}) \geq P_i(b) > 0$. Given this, inequality (A.17) can be rewritten as

$$\hat{b} - b \geq (\theta_i - b)\frac{P_i(\hat{b}) - P_i(b)}{P_i(\hat{b})}. \tag{A.20}$$

The right-hand side of the inequality above is nonnegative since by definition of equilibrium, $(\theta_i - b)P_i(b) \geq 0$. Therefore, it must be the case that $\hat{b} \geq b$. □

Proof of Lemma 3: By Lemma 1, there exists $\tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$ such that bidders with types in $(\tilde{\theta}, \overline{\theta}]$ win the auction with positive probability, while bidders with types in $[\underline{\theta}, \tilde{\theta})$ win with probability zero.

We begin by addressing the case in which a bidder with type $\tilde{\theta}$ wins the auction with positive probability.

Consider a bidder with type $\theta \in [\tilde{\theta}, \overline{\theta}]$. Given our definition of $\tilde{\theta}$, this bidder beats any bidder whose type is in $[\underline{\theta}, \tilde{\theta})$. Moreover, this bidder also beats any bidder with type $\hat{\theta} \in [\tilde{\theta}, \theta)$ since by Lemma 2, bids are strictly increasing in type and by Assumption 2, $V(\theta) > V(\hat{\theta})$.

By combining these two results, we find that a bidder with type $\theta \in [\tilde{\theta}, \overline{\theta}]$ wins the auction if the type of every other bidder is strictly less than θ and loses the auction if there exists a bidder whose type is strictly greater than θ; that is, a bidder with type $\theta \in [\tilde{\theta}, \overline{\theta}]$ wins the auction with probability $F^{n-1}(\theta)$.

Suppose a bidder with type θ plays the bidding strategy $B(\cdot, \theta)$ in equilibrium. Suppose further that $B(\cdot, \theta)$ is a mixed strategy and that the support of $\beta(\cdot, \theta)$ includes the bids b and b', where $b \neq b'$. Since the bidder should be indifferent among bids in the support of $\beta(\cdot, \theta)$,

$$(\theta - b)F^{n-1}(\theta) = (\theta - b')F^{n-1}(\theta). \tag{A.21}$$

Since $\theta \geq \tilde{\theta}$, $F^{n-1}(\theta)$ must be positive, which in turn implies that b and b' are equal – a contradiction. Hence, the bidding strategy $B(\cdot, \theta)$ must be a pure strategy.

An analogous argument can be used to prove the lemma for the case in which a bidder with type $\tilde{\theta}$ wins the auction with probability zero. □

Proof of Lemma 4: Suppose $b : (\tilde{\theta}, \overline{\theta}] \to \mathbb{R}$ is not continuous at some $\theta \in (\tilde{\theta}, \overline{\theta}]$. Then, for some $\epsilon > 0$, there is no $\delta > 0$ such that

$$\hat{\theta} \in (\tilde{\theta}, \overline{\theta}] \text{ and } \left|\hat{\theta} - \theta\right| < \delta \Rightarrow \left|b(\hat{\theta}) - b(\theta)\right| < \epsilon. \tag{A.22}$$

By Lemma 2, b is strictly increasing on $(\tilde{\theta}, \overline{\theta}]$. Therefore, we can restate the discontinuity condition as follows: there exists $\epsilon > 0$ such that either

$$b(\theta) - b(\hat{\theta}) \geq \epsilon, \quad \forall \hat{\theta} \in (\tilde{\theta}, \theta) \tag{A.23}$$

or

$$b(\hat{\theta}) - b(\theta) \geq \epsilon, \quad \forall \hat{\theta} \in \left(\theta, \overline{\theta}\right]. \tag{A.24}$$

Incentive compatibility for the type θ bidder requires that

$$[\theta - b(\theta)]F^{n-1}(\theta) \geq [\theta - b(\hat{\theta})]F^{n-1}(\hat{\theta}) \tag{A.25}$$

for all $\hat{\theta} \in (\tilde{\theta}, \overline{\theta}]$. If condition (A.23) holds, then

$$\left[\theta - b(\hat{\theta})\right]\left[F^{n-1}(\theta) - F^{n-1}(\hat{\theta})\right] \geq \epsilon F^{n-1}(\theta) \tag{A.26}$$

for all $\hat{\theta} \in (\tilde{\theta}, \theta)$. Since $\theta \in (\hat{\theta}, \overline{\theta}]$ and $\epsilon > 0$ are fixed, $\epsilon F^{n-1}(\theta)$ is both positive and fixed. However, since F is continuous on $\left[\underline{\theta}, \overline{\theta}\right]$, F^{n-1} is also continuous, which implies that for all $\epsilon' > 0$, there exists $\delta' > 0$ such that

$$\hat{\theta} \in (\tilde{\theta}, \theta) \text{ and } \theta - \hat{\theta} < \delta' \Rightarrow F^{n-1}(\theta) - F^{n-1}(\hat{\theta}) < \epsilon';$$

that is, $F^{n-1}(\theta) - F^{n-1}(\hat{\theta})$ can be brought arbitrarily close to zero by selecting a $\hat{\theta}$ sufficiently close to θ. Moreover, Lemma 2 indicates that b is increasing over $(\tilde{\theta}, \overline{\theta}]$, and since θ is fixed, $\theta - b(\hat{\theta})$ is decreasing as $\hat{\theta}$ approaches θ. Therefore, inequality (A.26) is violated for $\hat{\theta}$ sufficiently close to θ, and condition (A.23) cannot hold.

An analogous argument using incentive compatibility for the type $\hat{\theta}$ bidder can be used to show that condition (A.24) cannot hold either. □

Proof of Lemma 5: Our approach will be to first establish a boundary condition by showing that

$$\lim_{\theta \to \tilde{\theta}^+} b(\theta) = \tilde{\theta}$$

and then use this condition to derive the bid function.

By Lemma 4, b is continuous, and hence, the limit exists. Suppose $\lim_{\theta\to\tilde{\theta}^+} b(\theta) \neq \tilde{\theta}$. Then there exists a $\epsilon > 0$ such that either

$$\lim_{\theta\to\tilde{\theta}^+} b(\theta) = \tilde{\theta} + \epsilon \tag{A.27}$$

or

$$\lim_{\theta\to\tilde{\theta}^+} b(\theta) = \tilde{\theta} - \epsilon. \tag{A.28}$$

Suppose condition (A.27) holds and consider a bidder with type $\theta \in (\tilde{\theta}, \tilde{\theta}+\epsilon)$. By Lemma 2, the bidding function b is strictly increasing for all $\theta > \tilde{\theta}$, and hence, the bidder submits a bid $b(\theta) > \tilde{\theta}+\epsilon$. Furthermore, since $\theta > \tilde{\theta}$, the bidder's probability of winning is $F^{n-1}(\theta) > 0$. As a result, the bidder's expected utility is

$$[\theta - b(\theta)]\, F^{n-1}(\theta) < 0, \tag{A.29}$$

which violates individual rationality.

Now suppose condition (A.28) holds and consider a bidder with type $\theta \in (\tilde{\theta} - \epsilon, \tilde{\theta})$. Since $\theta < \tilde{\theta}$, the bidder does not win the auction and earns utility of zero. By Lemmas 2 and 4, b is continuous and strictly increasing. Therefore, there exists a $x > \tilde{\theta}$ such that $b(x) \in (\tilde{\theta} - \epsilon, \theta)$. If the bidder deviates to $b(x)$, his expected utility is

$$[\theta - b(x)]\, F^{n-1}(x) > 0, \tag{A.30}$$

a profitable deviation.

Having shown that $\lim_{\theta\to\tilde{\theta}^+} b(\theta) = \tilde{\theta}$, we proceed with our derivation of the bidding function, $b(\cdot)$.

Consider a bidder with type $\theta \in (\tilde{\theta}, \overline{\theta}]$. If the bidder offers a bid of $b(x)$, where $x \in (\tilde{\theta}, \overline{\theta}]$, then the bidder's expected utility is

$$U(x,\theta) \equiv [\theta - b(x)]\, F^{n-1}(x). \tag{A.31}$$

In equilibrium, $b(\cdot)$ must satisfy

$$\text{Global IC}: \quad U(\theta,\theta) \geq U(x,\theta), \quad \forall \theta \in (\tilde{\theta}, \overline{\theta}], \forall x \in (\tilde{\theta}, \overline{\theta}].$$

Since b and F are continuous, Global IC implies that $b(\cdot)$ satisfies

$$\text{Local IC}: \quad U_x(\theta,\theta) = 0 \quad \forall \theta \in (\tilde{\theta}, \overline{\theta}).$$

Taking the derivative of $U(x,\theta)$ with respect to x, substituting θ for x, and setting the resulting expression equal to zero yields

$$\frac{db(\theta)}{d\theta} F^{n-1}(\theta) + b(\theta)\frac{dF^{n-1}(\theta)}{d\theta} = \theta \frac{dF^{n-1}(\theta)}{d\theta}. \tag{A.32}$$

After integrating both sides, evaluating the integrals from $\tilde{\theta}$ to θ, applying the boundary condition ($\lim_{\theta \to \tilde{\theta}_+} b(\theta) = \tilde{\theta}$), and solving for $b(\theta)$, we obtain the bidding function

$$b(\theta) = \theta - \frac{\int_{\tilde{\theta}}^{\theta} F^{n-1}(x)dx}{F^{n-1}(\theta)} \tag{A.33}$$

for $\theta \in (\tilde{\theta}, \overline{\theta})$. Since $b(\cdot)$ is continuous over $(\tilde{\theta}, \overline{\theta}]$, the bidding function specified gives the equilibrium bid for type $\overline{\theta}$ as well. □

Proof of Proposition 2: Our approach will be to derive the equilibrium expected utility for a bidder with type $\theta \in [\underline{\theta}, \overline{\theta}]$ and then show that there is no profitable deviation available to that bidder.

Consider a bidder with type $\theta \in [\underline{\theta}, \theta_*)$. Since the equilibrium is separating, the seller can infer θ from $b(\theta)$, and the bidder's score is given by

$$\begin{aligned} V(\theta) + b(\theta) &< V(\theta_*) - V(\theta_*) \\ &= 0. \end{aligned} \tag{A.34}$$

Since the seller's reservation utility is zero, the bidder will not be awarded the contract. Hence, any bidder with type $\theta \in [\underline{\theta}, \theta_*)$ earns utility of zero in equilibrium.

Consider a bidder with type $\theta \in [\theta_*, \overline{\theta}]$. In this case, the bidder's score is given by

$$V(\theta) + \theta - \frac{\int_{\theta_*}^{\theta} F^{n-1}(x)dx}{F^{n-1}(\theta)}. \tag{A.35}$$

Since the score is strictly increasing in θ, the bidder's probability of winning the contract is $F^{n-1}(\theta)$. Hence, in equilibrium, any bidder with type $\theta \in [\theta_*, \overline{\theta}]$ earns expected utility of

$$[\theta - b(\theta)]\, F^{n-1}(\theta). \tag{A.36}$$

We now show that no bidder has an incentive to deviate to another bid on the equilibrium path. Suppose a bidder has type $\theta \in [\underline{\theta}, \overline{\theta}]$. If this bidder deviates to a bid $b(x)$, where $x \in [\underline{\theta}, \theta_*)$, then his resulting utility is zero, which does not improve upon his equilibrium expected utility. Similarly, if the bidder deviates to a bid $b(x)$, where $x \in [\theta_*, \overline{\theta}]$, then his expected utility is

$$[\theta - b(x)]\, F^{n-1}(x). \tag{A.37}$$

Substituting for $b(x)$ yields

$$(\theta - x)F^{n-1}(x) + \int_{\theta_*}^{x} F^{n-1}(y)dy, \tag{A.38}$$

which is increasing in x for $x < \theta$ and decreasing in x for $x > \theta$. Therefore, if $\theta \in [\theta_*, \overline{\theta}]$, bidding $b(x)$ delivers lower expected utility than bidding $b(\theta)$. Furthermore, if $\theta \in [\underline{\theta}, \theta_*)$, then θ is strictly less than $b(x)$, and $b(x)$ cannot be a profitable deviation.

Finally, we show that no bidder has an incentive to deviate to a bid off the equilibrium path. Consider the deviating bid $b > b(\overline{\theta})$. If the bidder's type, θ, is less than b, then the expected utility associated with b must be nonpositive, and b cannot be a profitable deviation. If, instead, $\theta \geq b$, then the expected utility associated with b is at best $\theta - b$, which is strictly less than the expected utility associated with bidding $b(\overline{\theta})$. Since $b(\overline{\theta})$ is not a profitable deviation for the bidder, then b is not a profitable deviation either.

Now consider the deviating bid $b < \theta_*$. The score associated with b is given by

$$\begin{aligned} V(\underline{\theta}) + b &< V(\underline{\theta}) + \theta_* \\ &= \tfrac{1-F(\theta_*)}{f(\theta_*)} - [V(\theta_*) - V(\underline{\theta})] \qquad \text{(A.39)} \\ &\leq 0. \end{aligned}$$

Since the seller's reservation utility is zero, the contract is never awarded to a bidder offering a bid of b, and b cannot be a profitable deviation. □

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