DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 05-11

Patent Protection of Basic Research in Cumulative Innovation

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November 2005

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November 2005

Abstract

There is an ongoing debate on what is the optimal patent protection of basic technologies in cumulative innovations. I study the optimal patent protection of basic research in a two-stage patent race model with basic research at the first stage and commercial product development at the second stage. I find the following. Investment in basic research initially increases and then decreases in the degree of patent protection of the basic research, while investment in commercial development always decreases in the degree of protection of the basic research. The welfare-maximizing degree of protection of basic research decreases in the monopoly rent from the basic innovation, increases in the marginal cost of the basic innovation, increases in the consumer surplus from the basic innovation and decreases in the consumer surplus from the commercial product. Basic research firms overinvest (underinvest) relative to the social planner at intermediate degrees (low and high degrees) of protection of basic research. Commercial development firms overinvest (underinvest) relative to the social planner when the protection of basic research is weak (strong).

JEL Classification: D23, D45, L14

Keywords: cumulative innovation, basic research, patent protection, two-stage patent races.

[*]I thank my advisor Keith Maskus and my dissertation committee members Yongmin Chen, Terra McKinnish and Anna Rubinchik-Pessach for insightful comments. I also thank the participants in the Graduate Seminar at the Department of Economics, University of Colorado at Boulder, for valuable discussions. All remaining errors are mine.

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1 Introduction

Cumulative innovation is a common phenomenon. In high technology industries such as biotechnology, pharmaceuticals, computers and electronics, products are the result of cumulative innovation. In cumulative innovation, an initial discovery is used for a subsequent discovery. For example, the basic research for the diagnostic test for the BRCA1 breast cancer gene, the basic research for the synthetic Hepatitis B vaccine and the basic research for multiple drugs for cancer treatment were conducted in universities and research firms, while the diagnostic test, the vaccine and the drugs, respectively, were developed by commercial firms.

In this paper I focus on cumulative innovation, when the initial discovery is a basic research innovation and the subsequent discovery is commercial product development, and the two discoveries are performed by separate firms. It is common that research firms and universities specialize in the discovery of basic technologies and license them to commercial firms for the development of commercial products. For example, eight out of the top ten biotechnology products in 2002 were developed using licensed university technologies (Edwards et al. 2003). Another example, which emphasizes the scale of licensing of basic technologies is that in 2003 a total of 3,926 U.S. patents were issued to U.S. academic institutions participating in a survey conducted by the Association of University Technology Transfer Managers (AUTM)[1] and these same institutions executed 4,464 licenses and options, and received \$1.34 billion U.S. dollars in gross license income (AUTM 2004). The annual economic impact of the licenses of the above institutions on the U.S. economy is estimated to over \$21 billion U.S. dollars[2] and the creation of 180,000 jobs (AUTM 1995). Note that these figures include only licensings of academic institutions and do not include licensings of research firms and their respective contribution to economy, which would increase the impact significantly.

The patent system plays an important role in the process of innovation. It encourages innovation by ensuring that innovators can be remunerated for their R&D investment. Patents protect innovators from imitators, known as backward protec-

[1]The Association of University Technology Transfer Managers (AUTM) collects and reports survey data on technology licensing at U.S. universities, hospitals and research institutes. The 2003 AUTM Licensing Survey has data on 132 U.S. universities and 26 U.S. hospitals and research institutes.

[2]This estimate includes \$4 billion U.S. dollars of pre-production investment (made prior to the sales of licensed products) and \$17 billion U.S. dollars of sales of licensed products (AUTM 1995).

tion, and reward innovators for providing the basis for subsequent innovations, known as forward protection. Forward patent protection is provided by both the Patent Office and the courts through the patentability requirement and the patent breadth, which means that if a subsequent innovation is not sufficiently novel or if it falls within the claims of a previous patent, it is found to infringe on that patent. In cumulative innovation, especially when it does not take place within a single firm, the patent system has to ensure that all inventors have sufficient incentives to carry out their part of the research. The challenge is that the commercial value of the entire cumulative innovation is embodied in the application, while the basic innovation has no stand-alone value. The only way the innovator of the basic technology can be compensated for his contribution to the cumulative innovation is through the division of the profits from the commercial application. Strong patent protection of the basic innovation gives the basic innovator more bargaining power as to the division of these profits in the licensing agreement, and leaves the developer of the commercial application with less profit. Therefore, strong patent protection of the basic innovation has two opposing effects on the investment incentives in cumulative innovation. It stimulates the R&D investment in basic research, but discourages the R&D investment in the development of commercial products. The question arises: What is the optimal degree of patent protection of the basic innovation?

The importance of the above question has been recognized by the U.S. and foreign governments, which amend and fine-tune the degree of patent protection of basic research in their respective countries (see Mowery et al. 2004 for a detailed review). Since the 1980s the United States has supported a policy of strong patent protection of basic research. Two influential events marked the beginning of that policy. First, the Bayh-Dole Act of 1980 gave universities the right to patent and exclusively license the results of federally funded research, and encouraged universities to participate in technology transfer activities with commercial companies in order to promote the utilization of inventions arising from federal funding. Second, in 1980 in Diamond vs. Chakrabarty the US Supreme Court ruled that living, manmade microorganisms are patentable and this decision stimulated the patenting of fundamental biotechnology discoveries. Subsequently, in 1984 an amendment to Bayh-Dole removed the time limits on the length of exclusive licenses universities could offer to large businesses, making patented university research more attractive to businesses and strengthening its protection. The patent protection of basic research was further enhanced during the 1980s through a general change in the U.S. policy toward stronger intellectual property rights.[3] Recently, there has been a common trend across other OECD countries to allow universities and small businesses which perform government research to obtain patents and license their inventions (OECD 2002).[4]

The theoretical literature on optimal patent protection of the first-stage innovation in cumulative innovation is somewhat polarized. Chang (1995), Green and Scotchmer (1995), Scotchmer (1996), Matutes, Regibeau and Rockett (1996), O'Donoghue (1998), O'Donoghue, Scotchmer and Thisse (1998) argue for strong patent protection of the initial innovation because it enables subsequent innovations and is therefore entitled to a significant share of its profits. Merges and Nelson (1990), Heller and Eisenberg (1998), Denicolo (2000), and Nelson (2005) argue for weak patent protection of the first-stage innovation because strong patent protection stifles the development of second-stage products.

This paper contributes to the literature on optimal patent protection of the first stage in cumulative innovation in three ways. First, I present a game theoretical two-stage patent race model, which accommodates the specific features of cumulative innovation with basic research at the first stage and commercial product development at the second stage. In the previous literature, only Denicolo (2000) uses a patent race model, that is, a model with R&D competition. Unlike the model in this paper, however, his model is specific to product development at the first stage and product improvement at the second stage. Second, in this model the degree of patent protection of basic research is represented by a continuous parameter, which allows systematic analysis of the relationship between the degree of patent protection of basic research on one side and the investment at each innovation stage and the social welfare on the other. In its attempt to quantify the optimal degree of protection of the first-stage innovation, this paper is closest to those of Chang (1995) and Denicolo (2000). This paper departs from Chang's and Denicolo's, however, by assuming that the first stage of innovation is basic research, ad not first-generation product development. The paper also differs from Chang's, in which there is no R&D competition and differs from Denicolo's, in which three distinct degrees of protection

[3]According to Katz and Ordover (1990), at least fourteen congressional bills were passed in the 1980s, which aimed at strenthening domestic and international intellectual property rights protection. According to the same authors the Court of Appeals for the Federal Circuit, created in 1982 as the court of final appeal for federal patent cases, has upheld 80 percent of the cases argued before it, while on average only 30 percent of the federal cases were upheld before 1982.

[4]In Germany, Italy and Sweden researchers currently own the intellectual property rights from publicly funded research (OECD 2002).

of the first-stage innovation are considered. Third, the paper shows that there is no definite answer to the question whether the patent protection of basic research should be weak or strong, but the degree of protection depends on the parameters of the innovation races at the two stages.

The main findings in the paper are as follows. Investment in basic research initially increases and then decreases in the degree of patent protection of the basic research, while investment in product development always decreases in the degree of protection of the basic research. The welfare-maximizing degree of protection of basic research decreases in the monopoly rent from the basic innovation, increases in the marginal cost of the basic innovation, increases in the consumer surplus from the basic innovation and decreases in the consumer surplus from the commercial product. Basic research firms overinvest (underinvest) relative to the social planner at intermediate degrees (low and high degrees) of protection of basic research. Commercial development firms overinvest (underinvest) relative to the social planner when the protection of basic research is weak (strong).

The paper is organized as follows. Section 2 discusses the relevant patent policy tools for protection of basic research innovations. Section 3 reviews related literature. Section 4 presents the model and its solution, and examines the relationship between patent protection of basic research and investment at the two stages. Section 5 analyzes the welfare-maximizing degree of patent protection of basic research and describes the deviations of the equilibrium investments from the social optimum. Section 6 concludes.

2 Patent Policy Tools

In the U.S. the patent policy is determined by the Patent Office and by the courts. An inventor files a patent application with the Patent Office, in which he includes a description of the innovation and a set of claims as to what uses of the invention should be protected by the patent. The Patent Office reviews the application and decides whether the innovation is patentable and what claims to allow. The innovation is patentable if it meets the statutory requirements for patentability: novelty, utility and non-obviousness. The claims define the technological boundaries of the invention. If an inventor is granted a patent, during the 20 years of statutory life of the patent he can sue other patentholders or product-makers for infringement if he thinks that their respective inventions or products fall within the claims of his patent. The courts decide whether or not there is an infringement. If a product is found to infringe a previous patent, the maker of the product needs to license the initial patent in order to be able to continue to market the product legally.

The patent policy has the following tools in determining the degree of patent protection for an innovation: the length of the patent life, the statutory requirement for patentability and the patent claims. While the duration of the patent has been established, the Patent Office and the courts always make decisions using the last two policy tools. In addition, according to Merges and Nelson (1990), while following the law the Patent Office and the courts have significant room for discretion in making decisions. The Patent Office can exercise discretion when deciding what is patentable and what claims to allow on a patent. The courts have discretion in determining whether a patent or a product infringes on a previous patent, and whether a previously issued patent is valid or not.

I use the terminology developed by O'Donoghue (1998) and I follow his paper in placing these two policy tools within the vocabulary used in the R&D literature. The first tool, "the patentability requirement", includes the statutory requirements for novelty, non-obviousness and utility. The patentability requirement can be interpreted as a minimum innovation size needed to obtain a patent. In the R&D literature the patentability requirement is also referred to as "novelty requirement", "non-obviousness requirement" or "patentability". The second tool, "the patent breadth", coincides with the claims in a patent. In other words, the patent breadth is the set of products covered by the patent which would be found to infringe it. Alternative terminology for the patent breadth in the R&D literature is "patent scope" and simply "patent protection". The patent breadth can be one of two types "leading patent breadth" and "lagging patent breadth". The lagging breadth is the set of inferior products that infringe on the patent. The leading breadth is the set of superior products, which require further innovation, that infringe on the patent. In the R&D literature the combination of patentability requirement and lagging breadth is also called "backward patent protection" and the combination of patentability requirement and leading breadth is also called "forward patent protection".

In this paper I am concerned with the degree of patent protection of basic research in cumulative innovation. The relevant policy tools for determining that degree, are the patentability requirement and the leading patent breadth. I do not focus on any one of these two tools specifically. Instead I view the degree of patent protection of basic research as the result of the joint application of these tools. In the following model, the government decides the degree of patent protection of basic research, where the government represents the system of the Patent Office and the courts.

3 Related Literature

The literature on optimal patent protection of the first-stage in cumulative innovation recognizes that profit needs to be transferred from the second to the first innovator in order to provide the first innovator with sufficient incentives to invest. However, the literature is not unanimous as to how much protection and therefore how much of the second-stage profits the initial innovator should receive. On one side, Chang (1995), Green and Scotchmer (1995), Scotchmer (1996), Matutes, Regibeau and Rockett (1996), O'Donoghue (1998), O'Donoghue, Scotchmer and Thisse (1998) argue for strong protection of the first-stage innovation because the first innovation facilitates the second one. On the other side, Merges and Nelson (1990), Heller and Eisenberg (1998), Denicolo (2000), Nelson (2005) argue for weak protection of the first-stage innovation because strong protection will stifle innovations at the second-stage.

Only Chang (1995) and Denicolo (2000) attempt to quantify the optimal degree of patent protection of the first-stage innovation. The rest of the literature raises arguments either in favor or against protection of the first-stage innovation and proposes appropriate patent policy tools for achieving the respective goal. Below I summarize the patent policy recommendations in this literature. Then I discuss in more detail the papers of Chang (1995) and Denicolo (2000), which are closest in spirit to this paper.

The arguments in support of strong protection of the first-stage innovation and the corresponding policy recommendations are as follows. Green and Scotchmer (1995) argue that the social value of a basic innovation includes the net social value of the applications that it facilitates. They propose that the first-stage innovator's share of the profit from the second-stage product should be as large as possible, while leaving just sufficient incentives for the second-stage innovator to invest. They recommend that when the first innovation has no stand-alone value, the patent policy allows *ex ante* licensing (licensing before the second-stage investment has been sunk), which they argue is welfare-improving. Scotchmer (1996) argues that when *ex ante* licensing is possible, social welfare can be improved by denying patents on infringing second generation products (including applications of basic research). The idea is that denying patents on applications increases the profit share of the first-stage innovator, while the *ex ante* licensing makes sure that the second-stage innovator's profit share gives him sufficient incentives to invest. Matutes, Regibeau and Rockett (1996) are concerned about a delay in the diffusion of basic innovations in the absence of patent protection of the basic innovation. In particular, they are concerned that if the basic innovation is not protected, in fear of imitation of the basic technology, the basic innovator is tempted to wait and develop multiple applications of the basic technology before commercializing any of these applications. They recommend that basic innovations receive a patent "scope" protection, which they define as the reserved rights of the basic innovator to develop a certain set of applications of the basic technology, while applications outside of that set can be developed by rivals. In an infinite sequence of innovations, O'Donoghue, Scotchmer and Thisse (1998) propose that patents should provide protection from future innovators and stimulate R&D investment through the leading patent breadth, while O'Donoghue (1998) proposes that this should be done through the patentability requirement, which achieves the same goal without the undesired effect of consolidating market power.

The arguments for weak protection of the first-stage innovation are as follows. Heller and Eisenberg (1998) warn of a tragedy of the "anticommons" in the case of the "privatization" of biomedical research through the patentability of basic biomedical discoveries. They are concerned that the patentability of basic research is stifling downstream innovations in the course of research and product development. Merges and Nelson (1990) raise a concern that in science-based industries, such as the biotechnology industry, broad patents on basic discoveries have an undesired effect on market structure (consolidating market power in a few firms) and the rate of innovation. Nelson (2005) argues that for the same reasons there should be open access to scientific research results, that is basic innovations should receive no patent protection.

This paper is closest in spirit to the papers of Chang (1995) and Denicolo (2000), which attempt to quantify the optimal degree of patent protection of basic research. Chang (1995) shows that the optimal patent scope of the first-stage discovery is a non-monotonic function of its value. In particular, he proposes that broadest protection should be provided to basic inventions in two distinct situations: when a basic invention has a very small stand-alone value relative to subsequent improvements and when a basic invention has a very large stand-alone value relative to subsequent improvements. The idea is that in these two cases the initial innovation is particularly important because in the first case it enables a very valuable second-stage product, while in the second case the initial innovation itself is very valuable. The policy implication for basic research innovations with no stand-alone value is that they should receive strong patent protection. This paper differs from Chang's in two ways. First, I introduce a patent race at each innovation stage. In Chang's model a single firm has a novel idea and a second firm has an idea that improves on the first. Second, I focus on basic research at the first stage of innovation. In Chang's model there is product development at both stages of innovation and the second product is an improved substitute for the first product.

Denicolo (2000) analyzes the optimal patent protection of the first-stage innovation in three distinct regimes depending on whether or not the second-stage innovation is patentable and whether or not it infringes on the patent of the first-stage innovation. The three regimes: PN (patentable and not infringing), PI (patentable and infringing) and UI (unpatentable and infringing) in this order provide increasing protection of the first-stage innovation. Denicolo shows that with symmetric innovations, the three regimes in the above order induce decreasing social welfare. Thus he supports weak protection of the first-stage innovation. A critical assumption in the model is that the same firm can perform the innovation at both stages, which is typical for the case of product development and subsequent product improvement. Like Denicolo, I allow for competition, which is an important feature of the environment in which innovations are performed. However, I depart from Denicolo's paper in two ways. First, I assume that the same firm cannot participate in both innovation races. This assumption reflects the reality that typically research firms and institutions specialize in basic research, while commercial firms specialize in product development. Second, I assume that the degree of patent protection of the first-stage innovation can be measured continuously. I introduce a parameter α, with $\alpha \in [0, 1]$, which represents the degree of patent protection of the basic innovation. The value of α is chosen by the government in order to maximize expected social welfare. In contrast, Denicolo examines three discrete degrees of patent protection of the first stage innovation.

4 Model

The model in this paper is based on Denicolo's (2000) two-stage patent race model of cumulative innovation, which itself is based on the models of Loury (1979) and Dasgupta and Stiglitz (1980). Denicolo (2000) allows for repeated innovation by the same firm at the two innovation stages, which is the case when both stages of innovation are product development and the second product is an improvement of the first. Denicolo analyzes the patent protection of the first-stage innovation in three distinct regimes, which arise depending on whether the second-stage innovation (improvement) is patentable or unpatentable, and infringing or not infringing on the patent of the first innovation.

I modify this model to accommodate the features of cumulative innovation with basic research at the first stage of innovation and commercial product development at the second stage of innovation. In particular I make the following assumptions. First, firms are specialized in either basic research or product development but cannot do both. Second, I assume that the degree of patent protection of the first stage innovation can be measured continuously. I introduce a parameter α, with $\alpha \in [0, 1]$, which represents the degree of patent protection of the basic innovation. The value of α is chosen by the government in order to maximize expected social welfare. Using a continuous parameter to measure the degree of protection of the basic innovation, I can examine more systematically the effect of protection of the basic innovation on the R&D investments at the two stages and on the social welfare.

The setup of the model is as follows. There are two stages of innovation. Stage one is basic research (which I will also call "research") and stage two is commercial product development (which I will also call "development"). There is an innovation race at each stage. Each innovation race ends with the first success. The innovation race at the development stage starts only after success in the innovation race at the research stage. At each stage there is free entry in the innovation race and the number of active symmetric firms is endogenously determined. At the beginning of each innovation race, each active firm i chooses R&D investment effort x_{it} and pays cost $c_t x_{it}$, where c_t is the constant marginal cost of innovation. The innovation at each stage is patentable. The successful innovator at the t-th stage ($t = 1, 2$) obtains a patent, which for analytic simplicity is assumed to have infinite life. Over the life of its patent, the t-th innovation creates a constant per period flow of monopoly rent V_t for the innovator and a constant per period flow of consumer surplus S_t for the consumers. The respective present discounted values are $v_t = V_t/r$ and $s_t = S_t/r$, where r is the interest rate. Thus the present discounted value of the total social surplus from the t-th innovation is the sum $v_t + s_t$. Even though a basic research innovation typically has no stand-alone value v_1 and creates no consumer surplus s_1, I will keep the variables v_1 and s_1 in the model in order to obtain a more general solution which applies to both basic and non-basic research at the first stage of innovation.

After solving the model, at each step of the analysis that follows, I discuss the model solution with $v_1 = 0$ and $s_1 = 0$ in the typical case of basic research at the first stage, and emphasize how it differs from the case of non-basic research at the first stage, with $v_1 > 0$ and $s_1 > 0$.

The success date in each innovation race is random. The expected time to successful innovation depends on the firms' investment effort and is exponentially distributed according to a Poisson innovation process with hazard rate $\lambda(x_{it})$.[5] The active firms in each innovation race follow different research strategies, so that their instantaneous probabilities of success are independent of each other, and the aggregate instantaneous probability of success in the race is the sum of the individual probabilities. Specifically, if firm i invests an amount x_{it} at time $\tau = 0$ and assuming that there are n_t active symmetric firms in the t-th innovation race ($t = 1, 2$), the probability density function of firm i being the first firm to succeed at τ is $\lambda(x_{it})\, e^{-\left(\Sigma_{i=1}^{n_t}\lambda(x_{it})\right)\tau}$, where $\lambda(x_{it})$ is the firm i's instantaneous probability of success at τ. The probability density function of success in the the t-th innovation race at τ is $\left(\Sigma_{i=1}^{n_t}\lambda(x_{it})\right) e^{-\left(\Sigma_{i=1}^{n_t}\lambda(x_{it})\right)\tau}$. The expected date of success in the t-th race is $\left(\Sigma_{i=1}^{n_t}\lambda(x_{it})\right)^{-1}$. The Poisson innovation process implies that a firm's probability of success depends only on its current investment effort and its investment history is irrelevant, that is, there is no learning by doing. I assume that the hazard function has the functional form $\lambda(x_{it}) = x_{it}$, which is linear and implies constant returns to scale in R&D.

I assume that both innovations are patentable and the second innovation infringes on the patent of the first, because the second innovation is based on the first one. Therefore the second innovator has to license the basic innovation in order to be able to commercialize his innovation. The degree of patent protection of the basic

[5] A Poisson process with rate λ, $\lambda > 0$, is the collection of random variables $\{N(t), t \geq 0\}$, where $N(t)$ is the number of events that occur at random time points in the time interval $[0, t]$. A Poisson process assumes that the number of events that occur in disjoint time intervals are independent and the distribution of the number of events that occur in a given interval depends only on the length of the interval and not on its location. The probability of occurrence of n events by time t is $P\{N(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$. The inter-arrival times T_1, T_2, ..., where T_1 is the time of the first event and where T_n, for $n > 1$, is the time between the $(n-1)$-st and the n-th event, are independent exponential random variables each with mean $\frac{1}{\lambda}$ (Ross, 1994).

In the context of the model in this paper, if firm i invests an amount x_{it} at time $\tau = 0$, then the probability that firm i does not innnovate at or prior to date τ is $e^{-\lambda(x_{it})\tau}$ and the probability that firm i does innovate at or prior to date τ is $1 - e^{-\lambda(x_{it})\tau}$. Firm i's instantaneous probability of success, that is, the probabilty that firm i innovates in the interval $(\tau, \tau + d\tau)$ is $\lambda(x_{it})\, d\tau$. Firm i's expected success date is $(\lambda(x_{it}))^{-1}$.

innovation determines the bargaining power of the basic innovator in the licensing negotiations, in which the division of the second-stage profit is arranged. I assume that the licensing fees paid by the second innovator are equal to a share α of the monopoly rent from the second-stage innovation.

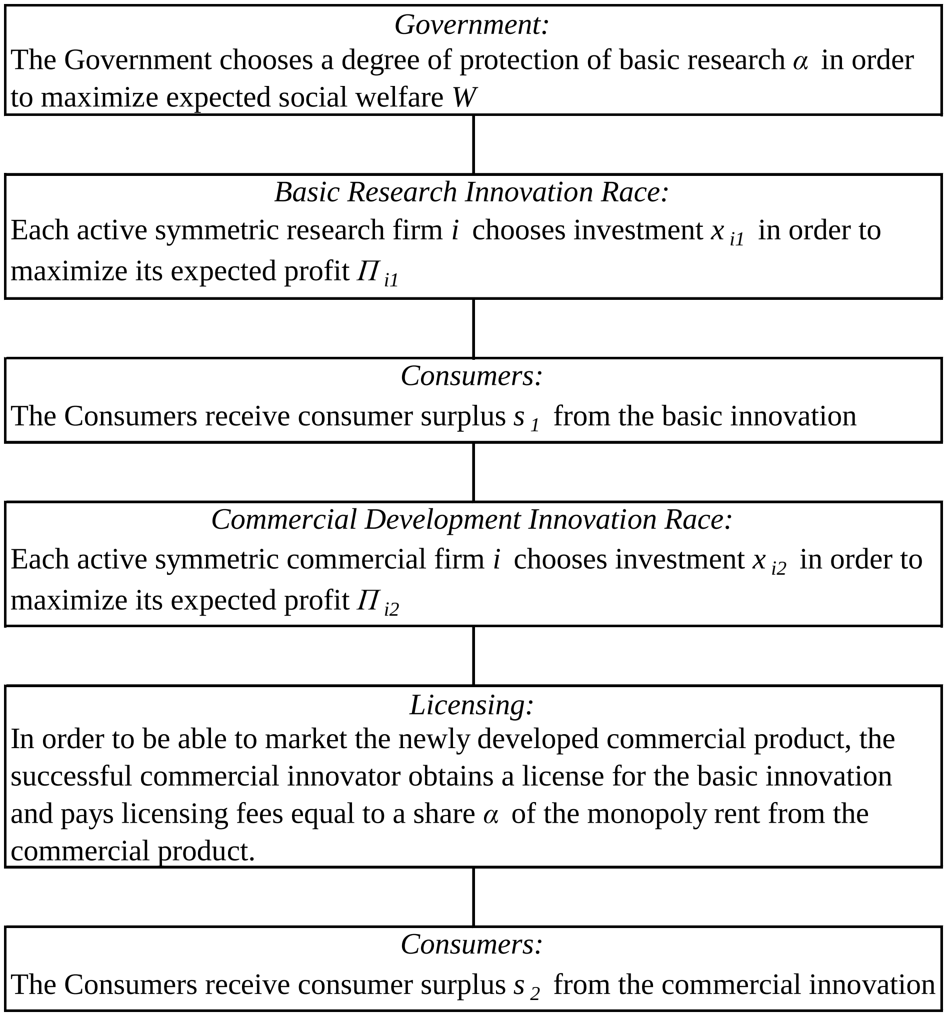

Figure 1: Timing of interactions.

The timing of the interactions in the game is presented in Figure 1. At the beginning of the game the government chooses a degree of patent protection of basic research α in order to maximize expected social welfare. Then each active symmetric basic research firm chooses its investment effort x_{i1} and competes in the research innovation race. The first research firm to innovate receives a patent and the basic research race ends. The consumers receive the consumer surplus from the basic innovation, s_1. Next, each active symmetric commercial development firm chooses its investment effort x_{i2} and competes in the product development race. The product development race ends with the first success and the successful commercial innovator obtains a patent. Because the second patent infringes on the first, the second innovator (product developer) has to license the basic technology from the first innovator (basic researcher) in order to be able to commercialize the newly developed product. The commercial innovator pays a licensing fee equal to a share α of the monopoly rent from the commercial product and markets the product. The consumers receive the consumer surplus from the commercial product, s_2. The solution of this game is a sub-game perfect Nash equilibrium and is found by backward induction.

4.1 Product Development Innovation Race

At the beginning of the product development innovation race, the problem of a symmetric commercial firm i is to choose its investment effort for that race x_{i2} in order to maximize its expected discounted profit:

$$\begin{aligned}
\Pi_{i2} &= \int_0^{\infty} e^{-rt} e^{-\left(\Sigma_{j=1}^{n2} x_{j2}\right)t} x_{i2} \left(1-\alpha\right) v_2 dt - c_2 x_{i2} \\
&= \frac{x_{i2}}{\Sigma_{j=1}^{n2} x_{j2} + r} \left(1-\alpha\right) v_2 - c_2 x_{i2}.
\end{aligned} \tag{1}$$

The term $e^{-\left(\Sigma_{j=1}^{n2} x_{j2}\right)t} x_{i2}$ is the probability density that firm i is the first firm to innovate in the product development race at time t, in which event at time t it starts receiving a share $(1-\alpha)$ of the constant flow of monopoly rent from the product innovation, which at time t has present discounted value v_2. At the beginning of the product development race, firm i commits cost $c_2 x_{i2}$. The term $\frac{x_{i2}}{\Sigma_{j=1}^{n2} x_{j2}+r}$, which emerges after integration, is the time discounted probability that firm i is the successful innovator in the development race.

The first order condition for expected profit maximization is:

$$\frac{d\Pi_{i2}}{dx_{i2}} = \frac{\left(X_2 - x_{i2} + r\right)\left(1-\alpha\right) v_2}{\left(X_2 + r\right)^2} - c_2 = 0, \tag{2}$$

where $X_2 = \Sigma_{j=1}^{n2} x_{j2}$ is the aggregate investment at the second stage. If the first order condition is satisfied firm i invests effort x_{i2} and participates in the product development race. Otherwise, it stays out of the race.

The second order condition is negative:

$$\frac{d^2\Pi_{i2}}{dx_{i2}^2} = \frac{-2\left(X_2+r\right)\left(X_2-x_{i2}+r\right)\left(1-\alpha\right)v_2}{\left(X_2+r\right)^4} < 0, \tag{3}$$

which guarantees that the solution to the first order condition in (2) indeed maximizes the expected profit function in (1).

The zero profit condition determines the aggregate investment in the product development race:

$$X_2 = \frac{\left(1-\alpha\right)v_2}{c_2} - r. \tag{4}$$

Let $\overline{\alpha_{X_2}} = 1 - \frac{rc_2}{v_2}$. The feasible degrees of patent protection of basic research, which guarantee that the aggregate investment in the product development race X_2 is positive, are $\alpha < \overline{\alpha_{X_2}}$. If the patent protection of basic research is greater than $\overline{\alpha_{X_2}}$, then the prize for the potential innovator in the product development race, $\left(1-\alpha\right)v_2$, will be too small to attract commercial firms to invest in product development and the product development race will not take place. The value $\overline{\alpha_{X_2}}$ is increasing in the monopoly rent from the commercial product v_2 and decreasing in the discount rate r and in the marginal cost of R&D in the product development race c_2. It is intuitive, that a more valuable commercial product will make it possible for the commercial innovator to share more of the monopoly rent from the product with the basic research innovator and still have sufficient incentives to invest in the product development. Conversely, more costly product development will make it unfeasible for the commercial innovator to share as much rent and still have sufficient incentives to invest in development. A higher discount rate, implies that a delayed product innovation is valued less and requires larger investment in product development to increase the probability of success in the development race. This calls for weaker protection of the basic research.

Assumption 1 below is necessary for $\overline{\alpha_{X_2}}$ to be positive and therefore for the interval of feasible degrees of patent protection of basic research to be of non-zero length. The assumption requires that the monopoly profits from the commercial product be sufficiently large to allow sharing of monopoly profits with the basic research innovator and still justify investment in product development.

Assumption 1 *I assume that $v_2 > rc_2$, which guarantees that $\overline{\alpha_{X_2}} > 0$.*

4.2 Basic Research Innovation Race

At the beginning of the basic research innovation race, the problem of a symmetric basic research firm i is to choose its investment effort x_{i2} in that race in order to maximize its expected discounted profit:

$$
\begin{aligned}
\Pi_{i1} &= \int_0^{\infty} e^{-rt} e^{-\left(\Sigma_{j=1}^{n_1} x_{j1}\right)t} x_{i1} \left(v_1 + \int_0^{\infty} e^{-r\tau} e^{-\left(\Sigma_{j=1}^{n_2} x_{j2}\right)\tau} \left(\Sigma_{j=1}^{n_2} x_{j2}\right) \alpha v_2 d\tau \right) dt - c_1 x_{i1} \\
&= \frac{x_{i1}}{\Sigma_{j=1}^{n_1} x_{j1} + r} \left(v_1 + \frac{\Sigma_{j=1}^{n_2} x_{j2}}{\Sigma_{j=1}^{n_2} x_{j2} + r} \alpha v_2 \right) - c_1 x_{i1}. \qquad (5)
\end{aligned}
$$

The term $e^{-\left(\Sigma_{j=1}^{n_1} x_{j1}\right)t} x_{i1}$ is the probability density of the event that firm i is the first firm to innovate in the basic research race at time t, and in that event it receives a share α of the expected monopoly rent from the innovation in the development race, which at the time the product innovation occurs has present discounted value v_2. At the beginning of the basic research race, firm i commits cost $c_1 x_{i1}$. The term $\frac{x_{i1}}{\Sigma_{j=1}^{n_1} x_{j1} + r}$, which emerges after integration, is the time discounted probability that firm i is the successful innovator in the basic research race and the term $\frac{X_2}{X_2 + r}$ is the time discounted probability that an innovation occurs in the product development race.

The first order condition for expected profit maximization in the product development race is:

$$
\frac{d\Pi_{i1}}{dx_{i1}} = \frac{\left(X_1 - x_{i1} + r\right)\left(v_1 + \frac{X_2}{X_2 + r}\alpha v_2\right)}{\left(X_1 + r\right)^2} - c_1 = 0, \qquad (6)
$$

where $X_1 = \sum_j x_{j1}$ is the aggregate investment in the basic research race. If the first order condition is satisfied firm i invests effort x_{i1} and participates in the basic research innovation race. Otherwise, it stays out of the race.

The second order condition is negative:

$$
\frac{d^2\Pi_{i1}}{dx_{i1}^2} = \frac{-2\left(X_1 + r\right)\left(X_1 - x_{i1} + r\right)\left(v_1 + \frac{X_2}{X_2 + r}\alpha v_2\right)}{\left(X_1 + r\right)^4} < 0, \qquad (7)
$$

which guarantees that the solution to the first order condition in (6) maximizes the expected profit function in (5)

The zero profit condition determines the aggregate investment in the basic research race:

$$
\begin{aligned}
X_1 &= \frac{v_1 + \frac{X_2}{X_2+r}\alpha v_2}{c_1} - r \\
&= \frac{v_1 + \frac{\alpha}{(1-\alpha)}\left((1-\alpha)\, v_2 - rc_2\right)}{c_1} - r. \qquad (8)
\end{aligned}
$$

The degrees of patent protection of basic research, which guarantee that the aggregate investment in basic research X_1 is positive, are $\underline{\alpha_{X_1}} < \alpha < \overline{\alpha_{X_1}}$, where $\underline{\alpha_{X_1}}$ and $\overline{\alpha_{X_1}}$ are defined in Appendix A.1. If the degree of patent protection of basic research is less than $\underline{\alpha_{X_1}}$, then the monopoly rent which the basic research innovator will collect from the commercial developer, αv_2, will be too small to warrant investment by research firms and the basic research race will not take place. If the degree of patent protection of basic research is greater than $\overline{\alpha_{X_1}}$, then the commercial innovator's rent, $(1-\alpha)\, v_2$, will be small and so will be the aggregate investment in product development X_2 and the probability of success in the development race $\frac{X_2}{X_2+r}$. Then the basic research innovator's expected rent from the commercial product, $\frac{X_2}{X_2+r}\alpha v_2$, will be too small to justify investment in the basic research race.

It follows from (4) and from (8) that the feasible degrees of patent protection of basic research, which guarantee that the aggregate investment in both innovation races is positive, are $\alpha < \overline{\alpha_{X_2}}$ and $\underline{\alpha_{X_1}} < \alpha < \overline{\alpha_{X_1}}$. Appendix A.2 derives the degrees of protection α which satisfy both of these inequalities and the result is shown in Assumption 2. In the analysis of the equilibrium investment and the social welfare which follows in this paper, I will focus on the degrees of patent protection, which support positive investment in both innovation races and therefore support a true cumulative innovation. Thus Assumption 2 is needed.

Assumption 2 *(Feasible degrees of patent protection of basic research) I assume that patent protection of basic research α satisfies the following conditions:*

$$
\begin{gathered}
0 \leq \alpha < \overline{\alpha_{X_2}} \textit{ when } v_1 > rc_1 \text{ and} \\
\underline{\alpha_{X_1}} < \alpha < \overline{\alpha_{X_1}} \textit{ when } v_1 \leq rc_1,
\end{gathered}
$$

which guarantee that the aggregate investment in both innovation races is positive.

Assumption 2 implies that for a typical basic innovation with no stand-alone value ($v_1 = 0$), the interval of feasible degrees of protection of basic research is $\underline{\alpha_{X_1}} < \alpha < \overline{\alpha_{X_1}}$. In that case the basic research is financed entirely through the second-stage product rent and the protection of basic research has to insure that the basic research firms' expected share of product rent is sufficiently large. As was discussed above, this is possible with intermediate degrees of protection α.

Assumption 3 below is sufficient for $\underline{\alpha_{X_1}}$ and $\overline{\alpha_{X_1}}$ to be real numbers (see Appendix A.1).

Assumption 3 *I assume that* $c_2 \geq \frac{-v_1+rc_1}{4r}$ and $v_2 \geq \left(\sqrt{rc_2} + \sqrt{-v_1+rc_1}\right)^2$, *which guarantees that* $\underline{\alpha_{X_1}}$ *and* $\overline{\alpha_{X_1}}$ *are real numbers.*

4.3 Equilibrium

I examine the relationship between the degree of patent protection of basic research α and the equilibrium aggregate investments in the two innovation races. The relationship between α and the aggregate investment in the product development race is intuitive. Stronger patent protection of basic research decreases the incentives for investment in product development because it leaves the commercial firms with a smaller share of the monopoly rent from the commercial product.

Proposition 1 *The aggregate investment in the product development race* X_2 *decreases in the degree of patent protection of basic research* α*.*

Proof. See Appendix A.3. ■

The relationship between the degree of protection of basic research α and the aggregate investment in the basic research race is more complex. Stronger patent protection of basic research has two opposing effects on the aggregate investment in basic research: a direct positive effect and an indirect negative effect. These effects occur through the term $\frac{X_2}{X_2+r}\alpha v_2$ in (8), which is the basic research innovator's share of the expected monopoly rent from the commercial product. The direct effect of stronger patent protection of basic research is to increase the basic innovator's expected rent from the commercial product by increasing his share α of the rent. The indirect effect of stronger patent protection of basic research is to decrease the basic innovator's expected rent by decreasing the aggregate investment in the product development race and thus decreasing the probability of success in the product development race $\frac{X_2}{X_2+r}$. Proposition 2 states that the positive direct effect dominates at small degrees of patent protection of basic research and the negative indirect effect dominates for large degrees of patent protection of basic research. Thus, aggregate investment in the basic research race increases for small degrees of patent protection of basic research and decreases for large degrees of patent protection of basic research.

Proposition 2 *Let* $\widehat{\alpha_{X_1}} = 1 - \sqrt{\frac{rc_2}{v_2}}$. *The aggregate investment in the basic research race* X_1 *increases in the degree of patent protection of basic research* α *for* $\alpha < \widehat{\alpha_{X_1}}$, *has a maximum at* $\alpha = \widehat{\alpha_{X_1}}$ *and decreases in* α *for* $\alpha > \widehat{\alpha_{X_1}}$.

Proof. See Appendix A.4. ■

Proposition 1 establishes that the investment incentives of commercial firms are monotonically decreasing in the degree of patent protection of basic research. Note that this is possible because the product development race starts after the completion of the basic research race and the commercial firms do not take into account the fact that their investment decision has an effect on the probability of success in the basic research race $\frac{X_1}{X_1+r}$, by affecting the basic research firms' expected product rent $\frac{X_2}{X_2+r}\alpha v_2$, and therefore affecting the basic research firms' investment X_1. In contrast, if the commercial firms had to commit investments at the beginning of the basic research race and therefore take into account the effect of their investment decision on the probability of success in the basic research race, the aggregate investment in the product development race would be non-monotonic and would have an inverse U-shape similar to that of the aggregate investment in the research race. The model setup in this paper is not unrealistic, however. It reflects the common case of *ex post* licensing of the basic innovation, that is, the basic innovation is being licensed after it has been completed. The case of *ex ante* technology licensing, in which the licensor sponsors the research of the basic technology and licenses the technology before its development, is less common. Anand and Khanna (2000) report that the incidence of *ex ante* licensing, is only 24% in chemicals and less than 6% in electronics and computers.

In contrast to the result in Proposition 2, Denicolo (2000) finds that when firms can innovate repeatedly, the aggregate investment at the first-stage always increases in the degree of protection of the first stage.

Figure 2 shows the equilibrium aggregate investment at the two stages for a typical basic innovation with $v_1 = 0$ and $s_1 = 0$.

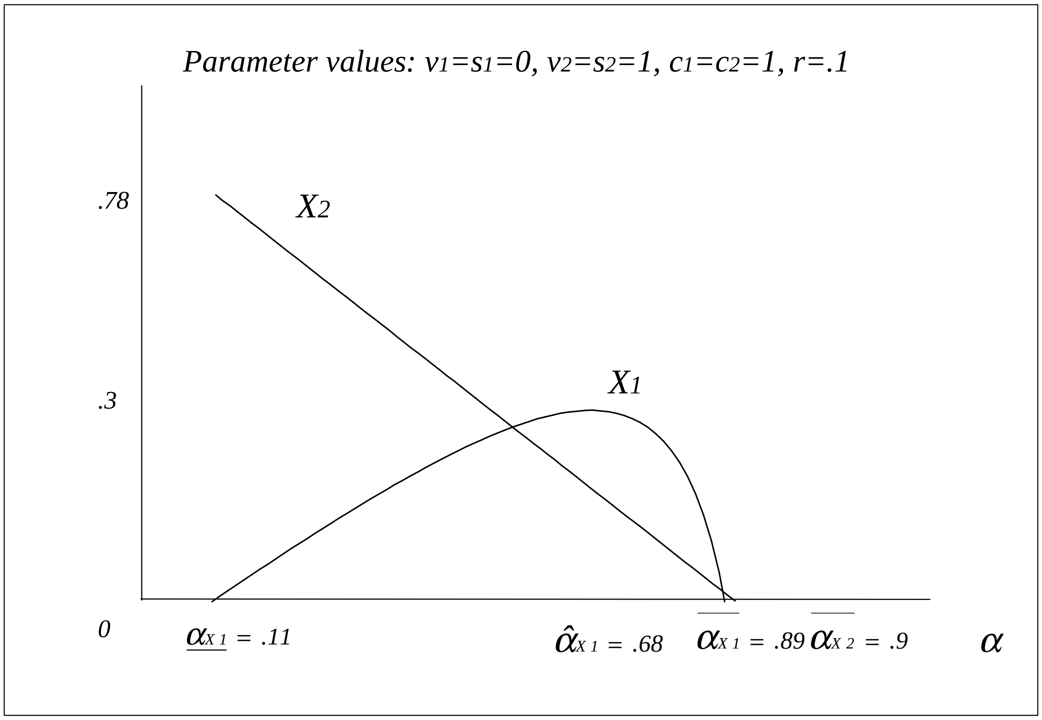

Figure 2: Aggregate investment in the two innovation races.

5 Welfare

Next, I define social welfare and compare the welfare in the competitive equilibrium with the welfare in the social planner's solution. In the competitive equilibrium, the government chooses the degree of patent protection of basic research in order to maximize the expected social welfare. This reflects the real-life patent policy decisions made by the Patent Office and the courts. Yet, not surprisingly, the welfare in the competitive equilibrium is second-best. The reason for that is that the patent protection of basic research cannot address all the externalities in cumulative innovation. I solve implicitly for the welfare-maximizing degree of patent protection of basic research and show how it varies with the parameters in the model.

Following Denicolo (2000), I define expected social welfare as the sum of expected firms' profits and expected consumer surpluses from the two innovations:

$$
\begin{aligned}
W &= \textstyle\sum_{i=1}^{n_1} \Pi_{i1} + \displaystyle\int_0^\infty e^{-rt} e^{-\left(\Sigma_{i=1}^{n_1} x_{i1}\right)t} \left(\Sigma_{i=1}^{n_1} x_{i1}\right) s_1 dt \\
&\quad + \int_0^\infty e^{-rt} e^{-\left(\Sigma_{i=1}^{n_1} x_{i1}\right)t} \left(\Sigma_{i=1}^{n_1} x_{i1}\right) \left(\textstyle\sum_{j=1}^{n_2} \Pi_{j2} + \displaystyle\int_0^\infty e^{-r\tau} e^{-\left(\Sigma_{j=1}^{n_2} x_{j2}\right)\tau} \left(\Sigma_{j=1}^{n_2} x_{j2}\right) s_2 d\tau \right) dt \\
&= \textstyle\sum_{i=1}^{n_1} \Pi_{i1} + \dfrac{X_1}{X_1 + r} s_1 + \dfrac{X_1}{X_1 + r} \left(\textstyle\sum_{j=1}^{n_2} \Pi_{j2} + \dfrac{X_2}{X_2 + r} s_2 \right). \qquad (9)
\end{aligned}
$$

As was discussed earlier, the term $e^{-rt} e^{-\left(\Sigma_{i=1}^{n_1} x_{i1}\right)t} \left(\Sigma_{i=1}^{n_1} x_{i1}\right)$, which after integration becomes $\frac{X_1}{X_1+r}$, is the time discounted probability of success in the basic research innovation race and the term $e^{-r\tau} e^{-\left(\Sigma_{j=1}^{n_2} x_{j2}\right)\tau} \left(\Sigma_{j=1}^{n_2} x_{j2}\right)$, which after integration becomes $\frac{X_2}{X_2+r}$, is the time discounted probability of success in the product development innovation race.

5.1 Social Planner

The social planner maximizes the expected social welfare function with respect to aggregate investment at the two innovation stages. Patent protection of basic research is irrelevant in his maximization problem because the social planner makes sure that optimal investments are being made in both innovations. In the social planner's problem firms' profits are not necessarily equal to zero and therefore using the definitions for Π_{i1} and Π_{j2} from (5) and (1), respectively, the social welfare function in (9) can be rewritten as:

$$W = \frac{X_1}{X_1 + r} (v_1 + s_1) - c_1 X_1 + \frac{X_1}{X_1 + r} \left(\frac{X_2}{X_2 + r} (v_2 + s_2) - c_2 X_2 \right) \qquad (10)$$

The first-order conditions for welfare maximization in the social planner's problem are:

$$\frac{dW}{dX_1} = \frac{r}{(X_1 + r)^2} \left((v_1 + s_1) + \left(\frac{X_2}{X_2 + r} (v_2 + s_2) - c_2 X_2 \right) \right) - c_1 = 0 \qquad (11)$$

$$\frac{dW}{dX_2} = \frac{X_1}{X_1 + r} \left(\frac{r}{(X_2 + r)^2} (v_2 + s_2) - c_2 \right) = 0. \qquad (12)$$

Let X_1^S denote the socially optimal investment in basic research and let X_2^S denote the socially optimal investment in product development. The first order conditions determine the socially optimal investments in the two innovations:

$$X_1^S = \sqrt{\frac{r}{c_1} \left((v_1 + s_1) + \left(\sqrt{(v_2 + s_2)} - \sqrt{r c_2} \right)^2 \right)} - r \qquad (13)$$

$$X_2^S = \sqrt{\frac{r}{c_2}\left(v_2 + s_2\right)} - r. \tag{14}$$

5.2 Competitive Equilibrium

In the free entry competitive equilibrium expected profits are zero and the social welfare function in (9) simplifies to the expected sum of the consumer surpluses from the two innovations:

$$W = \frac{X_1}{X_1 + r}\left(s_1 + \frac{X_2}{X_2 + r} s_2\right). \tag{15}$$

At the beginning of the game, before any investments are being made, the government chooses the degree of patent protection of basic research α that maximizes the expected social welfare in the competitive equilibrium in (15). This action of the government in the model corresponds to the patent policy decisions about patentability and leading patent breadth made by the Patent Office and the courts.

The first order condition for welfare maximization with respect to the degree of patent protection of basic research α is:

$$\frac{dW}{d\alpha} = \frac{r\left(s_1\left(X_2 + r\right) + s_2 X_2\right)}{\left(X_1 + r\right)^2\left(X_2 + r\right)}\frac{dX_1}{d\alpha} - \frac{-r s_2 X_1}{\left(X_1 + r\right)\left(X_2 + r\right)^2}\frac{dX_2}{d\alpha} = 0. \tag{16}$$

The first term in (16) is the indirect effect of α on W through X_1. It can be interpreted as the marginal social benefit from the protection of basic research, since the protection of basic research benefits first-stage investment X_1 and hence benefits welfare. Note that the marginal social benefit becomes negative when $\alpha > \widehat{\alpha_{X_1}}$. The second term in (16) is the indirect effect of α on W through X_2. It can be interpreted as the marginal social cost of the protection of basic research, because the protection of basic research decreases second-stage investment X_2 and ultimately decreases social welfare. Let MSB denote marginal social benefit and let $MSB = \frac{r(s_1(X_2+r)+s_2X_2)}{(X_1+r)^2(X_2+r)}\frac{dX_1}{d\alpha}$. Let MSC denote marginal social cost and let $MSC = \frac{-rs_2X_1}{(X_1+r)(X_2+r)^2}\frac{dX_2}{d\alpha}$.

Note that the marginal social benefit from patent protection of the basic research is positive (negative) when $\alpha < \widehat{\alpha_{X_1}}$ ($\alpha > \widehat{\alpha_{X_1}}$), and zero when $\alpha = \widehat{\alpha_{X_1}}$. This follows from the finding in Proposition 1 that $\frac{dX_1}{d\alpha} \geq 0$ $\left(\frac{dX_1}{d\alpha} < 0\right)$ when $\alpha \leq \widehat{\alpha_{X_1}}$ ($\alpha > \widehat{\alpha_{X_1}}$). Note also that the marginal social cost from patent protection of basic research is always positive. This follows from the finding in Proposition 2 that $\frac{dX_2}{d\alpha} < 0$.

In other words, at small degrees of protection α, such that $\alpha < \widehat{\alpha_{X_1}}$, a marginal increase in the protection of basic research has two opposing effects on social welfare: the social welfare benefits by the increase in investment in basic research but it is hurt by the decrease in investment in product development. Depending on which effect dominates, social welfare is increasing or decreasing in the degree of patent protection of basic research α. At large degrees of protection of basic research α, such that $\alpha > \widehat{\alpha_{X_1}}$, a marginal increase in the patent protection of basic research decreases welfare, because it decreases the investment in both innovation races.

Using the new notation, the first order condition in (16) can be rewritten as:

$$\frac{dW}{d\alpha} = MSB - MSC = 0. \tag{17}$$

Lemma 1 *If the social welfare function $W(\alpha)$ has a maximum, it occurs in the interval $\alpha < \widehat{\alpha_{X_1}}$.*

Proof. See Appendix A.5. ■

In view of Lemma 1, hereafter, the analysis of the welfare function and the welfare-maximizing degree of patent protection of basic research focuses on degrees of patent protection α, such that $\alpha < \widehat{\alpha_{X_1}}$.

Lemma 2 *The second order condition for welfare maximization with respect to α:*

$$\frac{d^2W}{d\alpha^2} = \frac{dMSB}{d\alpha} - \frac{dMSC}{d\alpha} \tag{18}$$

is negative when $\alpha < \widehat{\alpha_{X_1}}$, which implies that the welfare function W has a unique maximum in the interval $\alpha < \widehat{\alpha_{X_1}}$.

Proof. See Appendix A.6. ■

Let $\widehat{\alpha_W}$ be the implicit solution to (16). Assuming that an interior solution to (16) exists, then $\widehat{\alpha_W}$ is the welfare-maximizing degree of patent protection of basic research in the competitive equilibrium. If the first order condition in (16) does not have an interior solution, then the welfare-maximizing degree of protection of basic research is a corner solution. This is stated in Proposition 3. Let $\widetilde{v_1}$ be as defined in Appendix A.7 and note that $\widetilde{v_1} > rc_1$.

Proposition 3 *When $v_1 < \widetilde{v_1}$, the social welfare in the competitive equilibrium increases (decreases) in the degree of patent protection of basic research α for $\alpha < \widehat{\alpha_W}$ ($\alpha > \widehat{\alpha_W}$) and achieves a maximum at $\alpha = \widehat{\alpha_W}$, where $0 < \widehat{\alpha_W} < \widehat{\alpha_{X_1}}$. When $v_1 \geq \widetilde{v_1}$, the social welfare in the competitive equilibrium decreases in the degree of patent protection of basic research α and is maximized at $\alpha = 0$.*

Proof. See Appendix A.7. ■

In other words, when $v_1 < \widetilde{v_1}$ the social welfare function has an interior maximum at $\widehat{\alpha_W}$, and when $v_1 \geq \widetilde{v_1}$ the social welfare function is maximized at the corner point $\alpha = 0$. The meaning of Proposition 3 is that when the monopoly rent from the first innovation, v_1, is small relative to the monopoly rent from the second innovation, v_2, the first innovation should receive patent protection and when the rent from the first innovation, v_1, is large relative to the rent from the second innovation, v_2, the first innovation should not be protected.

Proposition 3 implies that a typical basic innovation with no stand-alone value ($v_1 = 0$) should receive some patent protection. Another implication of Proposition 3 is that, because the welfare-maximizing degree of protection lies in the interval $0 < \widehat{\alpha_W} < \widehat{\alpha_{X_1}}$ and in view of Propositions 1 and 2, basic research firms have an incentive to lobby the government to increase the protection of basic innovations to $\widehat{\alpha_{X_1}}$, and commercial development firms have an incentive to lobby the government that the protection of basic research should be as weak as possible. Figure 3 shows the social welfare function of a typical basic research innovation with $v_1 = 0$ and $s_1 = 0$.

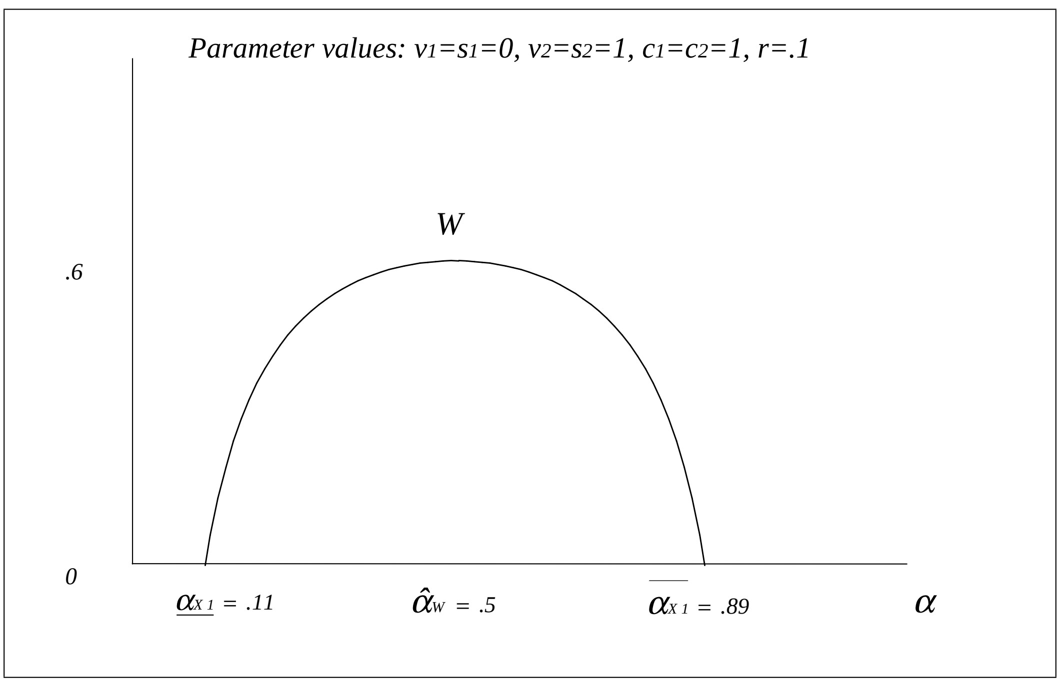

Figure 3: Social welfare in the competitive equilibrium.

Next, I examine how the welfare-maximizing degree of patent protection of basic research in the competitive equilibrium, $\widehat{\alpha_W}$, varies with the parameters of the two innovation races. I summarize the results in Proposition 4.

Proposition 4 *The welfare-maximizing degree of patent protection of basic research* $\widehat{\alpha_W}$ *decreases in the monopoly rent from the basic innovation* v_1*, increases in the consumer surplus from the basic innovation* s_1*, increases in the marginal cost of the basic research* c_1*, and decreases in the consumer surplus from the product innovation* s_2*.*

Proof. *See Appendix A.8.* ■

The results in this proposition are intuitive. If the basic innovation has some stand-alone value v_1, then this value provides incentives for investment in basic research in addition to the incentives provided by the transfer of a share of the product innovation's rent. When the stand-alone value of the basic innovation is small (large), then strong (weak) patent protection of basic research is needed to create sufficient incentives for investment in basic research and ultimately to maximize social welfare. Because a typical basic innovation has no stand-alone value ($v_1 = 0$), it requires stronger patent protection than if the basic innovation had some value of its own. An alternative interpretation is that a basic innovation with no stand-alone value requires stronger patent protection than a valuable non-basic innovation. Clearly, the cost of the basic innovation, c_1, has just the opposite effect on the welfare-maximizing degree of patent protection of basic research. When the basic innovation is costly, strong patent protection of basic research is needed to shift a sufficient share of the product innovation's rent to the basic innovator to cover the cost of research. If the basic innovation creates some consumer surplus s_1, when that surplus is large, the patent protection of basic research has to be strong to induce large probability of success in the research race and therefore large probability that the consumer surplus will be received by the consumers. Because a typical basic innovation creates no consumer surplus ($s_1 = 0$), it requires a weaker patent protection than a non-basic innovation which generates some consumer surplus. The size of the consumer surplus from the product innovation, s_2, has the opposite effect on the welfare-maximizing degree of protection of basic research.

The relationships between the welfare-maximizing degree of patent protection of basic research on one side, and the rent v_2 and the cost c_2 of the product innovation on the other side, is ambiguous. The reason is that both v_2 and c_2 have conflicting indirect effects on the welfare-maximizing degree of protection of basic research through the equilibrium investment in the two innovation races.

Example *In this numerical example I assume a typical basic innovation with no stand-alone value* ($v_1 = 0$)*, which creates no consumer surplus* ($s_1 = 0$)*. The welfare function in* (15) *simplifies to the expected discounted value of the consumer surplus from the product innovation,* $W = \frac{X_1}{X_1+r}\frac{X_2}{X_2+r}s_2$*. By solving the first order condition in (16) for* α*, I obtain the welfare-maximizing degree of protection of basic research* $\widehat{\alpha_W} = \frac{\sqrt{c_1}}{\sqrt{c_1}+\sqrt{c_2}}$ *(see Appendix A.9).*

Note that the welfare-maximizing degree of protection of basic research does not depend on the monopoly rent from the product innovation v_2*. This is unlike when the first-stage innovation has some value of its own* v_1 *and then the welfare-maximizing degree of protection of the first-stage innovation depends on both* v_1 *and* v_2 *(see Proposition 4 and the discussion following it). The reason is that there is only one source of monopoly rent* (v_2)*.*

Note also that the welfare-maximizing degree of protection of basic research does not depend on the consumer surplus from the product innovation s_2*. The reason is that the welfare function simplifies to the expected discounted value of the consumer surplus from the product innovation and then the government's welfare maximization problem is to maximize the probability that the two innovations occur,* $\frac{X_1}{(X_1+r)}\frac{X_2}{(X_2+r)}$*, which is independent of the actual size of the consumer surplus from the product innovation.*

The welfare-maximizing degree of protection of basic research depends only on the marginal costs of innovation at the two stages. In particular, it increases in the marginal cost of basic research c_1*, as was stated in Proposition 4. Proposition 4 could not derive analytically the relationship between* $\widehat{\alpha_W}$ *and* c_2*. In this example the welfare-maximizing degree of protection decreases in the marginal cost of product development* c_2*. The intuition is that when the product development is costly, the protection of basic research should be weak, so that the commercial innovator's share of the product rent can cover the cost of product development.*

5.3 Deviations of the Competitive Equilibrium from the Social Optimum

In cumulative innovation there are several known externalities which can cause the investment and the social welfare in the competitive equilibrium to deviate from the social optimum. The first externality arises when the cumulative innovation is not performed within the same firm. Then the first-stage innovation enables the second-stage innovation but cannot collect the entire value that it facilitates. In the competitive equilibrium the first innovator has insufficient incentives and underinvests, while the second innovator overinvests relative to the social optimum. This externality is especially severe when the first innovation is basic research with no stand-alone value.

The second externality arises because when the active firms in each innovation race maximize their expected profits, they consider only the monopoly rent from the potential innovation, v_t, but do not account for the consumer surplus from the innovation, s_t. Thus, in the competitive equilibrium the innovators at both stages underinvest relative to the social optimum.

The third externality arises from the assumption of free entry in each race, which causes wasteful competition and overinvestment at both innovation stages relative to the social optimum. The monopoly rents from the innovations at the two stages are dissipated through competition and this externality alone implies that social welfare in the competitive equilibrium is less than the social welfare achieved by the social planner.

Note that in the model, the monopoly situation granted to the successful innovator in each race by the patent protection does not cause a deviation from the social optimum, because the monopoly rents, v_t, enter the objective functions of both the firms in the competitive equilibrium and of the social planner in the social planner's problem.

The patent protection of the first-stage innovation directly alleviates the first externality through the transfer of second-stage rent to the first-stage innovator. The welfare-maximizing degree of protection of the first-stage innovation also lessens the second externality, because the consumer surpluses from the innovations at the two stages are part of the objective function of the government in the welfare maximization problem. However, the protection of the basic research does not address the last externality. This implies that even if the protection of basic research can fully correct the first two externalities, the welfare in the competitive equilibrium remains suboptimal. I state that in Proposition 5.

Proposition 5 *The welfare in the competitive equilibrium is a second-best solution.*

Proof. See previous paragraph. ■

Next, I compare the aggregate investment in the two innovations in the competitive equilibrium with that in the social planner's problem. The results are presented in Proposition 6 and Proposition 7. Let $\overline{\overline{\alpha_{X_2}}}$, $\underline{\underline{\alpha_{X_1}}}$ and $\overline{\overline{\alpha_{X_1}}}$ be as defined in Appendix A.10.

Proposition 6 *The aggregate investment in product development in the competitive equilibrium deviates from that in the social planner's problem in the following way:* $X_2 \geq X_2^S$ $(X_2 < X_2^S)$ *when* $\alpha \leq \overline{\overline{\alpha_{X_2}}}$ $(\alpha > \overline{\overline{\alpha_{X_2}}})$*.*

Proof. See Appendix A.11. ∎

Proposition 7 *The aggregate investment in basic research in the competitive equilibrium deviates from that in the social planner's problem in the following way:* $X_1 \geq X_1^S$ *when* $\underline{\underline{\alpha_{X_1}}} \leq \alpha \leq \overline{\overline{\alpha_{X_1}}}$ *and* $X_1 < X_1^S$ *when* $\alpha < \underline{\underline{\alpha_{X_1}}}$ *and* $\alpha > \overline{\overline{\alpha_{X_1}}}$*.*

Proof. See Appendix A.12. ∎

The finding in Proposition 6 is intuitive in view of Proposition 1, which states that investment in product development decreases in the degree of patent protection of basic research. Clearly then, overinvestment in product development relative to the social optimum can occur at small degrees of protection of basic research, and underinvestment can occur at large degrees of protection of basic research. Note that if the commercial product creates a very large consumer surplus s_2, then in the competitive equilibrium there is always underinvestment at the second stage relative to the social optimum.

The deviation of the investment in basic research in the competitive equilibrium from the social optimum is more intuitive in light of Proposition 2. Proposition 2 states that the investment in basic research initially increases and then decreases in the degree of protection of basic research. Consequently, at small and at large degrees of patent protection of basic research there is underinvestment in basic research relative to the social optimum, and at intermediate degrees of patent protection of basic research there is overinvestment in basic research in the competitive equilibrium.

Because in general it is not the case that $\overline{\overline{\alpha_{X_2}}} = \underline{\underline{\alpha_{X_1}}}$ or that $\overline{\overline{\alpha_{X_2}}} = \overline{\overline{\alpha_{X_1}}}$, the competitive equilibrium cannot yield the socially optimal solution, and therefore the equilibrium investment levels and welfare even under the welfare-maximizing degree of protection of basic research are suboptimal. This confirms the result in Proposition 5.

6 Conclusions

This paper studies the optimal patent protection of basic research in cumulative innovation. It assumes that the two stages of innovation are basic research and commercial product development, and they are performed by separate firms.

The novelties in this paper are as follows. First, the model in the paper accommodates the specific features of cumulative innovation with basic research at the first stage. Second, the degree of patent protection of basic research is represented by a continuous parameter, which allows systematic analysis of the relationship between the degree of protection of basic research on one side and the R&D investment and the social welfare on the other side. Third, the paper shows that there is no definite answer to the question whether the patent protection of basic research should be weak or strong. The degree of protection depends on the monopoly rents, the marginal costs and the consumer surpluses of the two innovations.

The paper finds that investment in basic research initially increases and then decreases in the degree of patent protection of the basic research, while investment in commercial development always decreases in the degree of protection of the basic research. The first finding differs from that in Denicolo's (2000) model, in which the first-stage investment always increases in the degree of protection of the first-stage innovation. The reason is that Denicolo allows for repeated innovation by the same firm, which is typical for cumulative innovation with product development at the first stage and subsequent product improvement at the second stage.

The paper also finds that the welfare-maximizing degree of protection of basic research decreases in the monopoly rent from the basic innovation, increases in the marginal cost of the basic innovation, increases in the consumer surplus from the basic innovation and decreases in the consumer surplus from the commercial product. The implication for a typical basic innovation with no value of its own is that it should receive stronger protection than a valuable non-basic innovation. Another implication, however, is that because a typical basic innovation does not generate any consumer surplus it should not be protected as much as a first-stage innovation which creates surplus for the consumers.

The paper compares the social welfare in the competitive equilibrium with that in the social planner's problem and finds that it is a second-best. The paper also compares the R&D investment in the competitive equilibrium with the social optimum and finds the following. Basic research firms overinvest (underinvest) relative to the social planner at intermediate degrees (low and high degrees) of protection of basic research. Commercial development firms overinvest (underinvest) relative to the social planner when the protection of basic research is weak (strong).

7 Appendix

Appendix A.1: Definitions of $\underline{\alpha_{X_1}}$ and $\overline{\alpha_{X_1}}$.

Let $\underline{\alpha_{X_1}} = \frac{1}{2v_2}\left(-v_1 + rc_1 + v_2 - rc_2 - \sqrt{(-v_1 + rc_1 + v_2 - rc_2)^2 + 4v_2(v_1 - rc_1)}\right)$.

Let $\overline{\alpha_{X_1}} = \frac{1}{2v_2}\left(-v_1 + rc_1 + v_2 - rc_2 + \sqrt{(-v_1 + rc_1 + v_2 - rc_2)^2 + 4v_2(v_1 - rc_1)}\right)$.

The cutoff values $\underline{\alpha_{X_1}}$ and $\overline{\alpha_{X_1}}$ are real numbers when the following is satisfied:

$$\begin{array}{ll} v_2 \in \left[\left(\sqrt{rc_2} + \sqrt{-v_1 + rc_1}\right)^2, \infty\right) & \text{if} \quad c_2 \geq \frac{-v_1 + rc_1}{4r} \\ v_2 \in \left(rc_2, \left(\sqrt{rc_2} - \sqrt{-v_1 + rc_1}\right)^2\right] \cup \left[\left(\sqrt{rc_2} + \sqrt{-v_1 + rc_1}\right)^2, \infty\right) & \text{if} \quad c_2 < \frac{-v_1 + rc_1}{4r}. \end{array}$$

To simplify the analysis in the paper I will assume that $c_2 \geq \frac{-v_1+rc_1}{4r}$ and $v_2 \in \left[\left(\sqrt{rc_2} + \sqrt{-v_1 + rc_1}\right)^2, \infty\right)$, which is sufficient for $\underline{\alpha_{X_1}}$ and $\overline{\alpha_{X_1}}$ to be real numbers. This is contained in Assumption 2. Note that the expression under the root sign in $\sqrt{-v_1 + rc_1}$ is non-negative when $v_1 \leq rc_1$. Appendix A.2 shows that the cutoff values $\underline{\alpha_{X_1}}$ and $\overline{\alpha_{X_1}}$ are binding, and therefore relevant to the analysis in the paper, only when $v_1 \leq rc_1$.

Appendix A.2: Degrees of patent protection of basic research for which $X_1 > 0$ and $X_2 > 0$.

It follows from (4) and from (8) that aggregate investment in the product development race X_2 is positive if $\alpha < \overline{\alpha_{X_2}}$ and aggregate investment in the basic research race X_1 is positive if $\underline{\alpha_{X_1}} < \alpha < \overline{\alpha_{X_1}}$. Aggregate investment in both races are positive if $\alpha < \overline{\alpha_{X_2}}$ and $\underline{\alpha_{X_1}} < \alpha < \overline{\alpha_{X_1}}$.

If $v_1 > rc_1$, then

$$\begin{aligned} \underline{\alpha_{X_1}} &= \frac{1}{2v_2}\left(-v_1 + rc_1 + v_2 - rc_2 - \sqrt{(-v_1 + rc_1 + v_2 - rc_2)^2 + 4v_2(v_1 - rc_1)}\right) \\ &< \frac{1}{2v_2}\left(-v_1 + rc_1 + v_2 - rc_2 - \sqrt{(-v_1 + rc_1 + v_2 - rc_2)^2}\right) = 0 \qquad \text{(A1)} \end{aligned}$$

and

$$\begin{aligned}
\overline{\alpha_{X_1}} &= \frac{1}{2v_2}\left(-v_1+rc_1+v_2-rc_2+\sqrt{\left(-v_1+rc_1+v_2-rc_2\right)^2+4v_2\left(v_1-rc_1\right)}\right) \\
&> \frac{1}{2v_2}\left(-v_1+rc_1+v_2-rc_2+\sqrt{\left(-v_1+rc_1+v_2-rc_2\right)^2+4\left(v_2-rc_2\right)\left(v_1-rc_1\right)}\right) \\
&= \frac{1}{2v_2}\left(-v_1+rc_1+v_2-rc_2+\sqrt{\left(v_1-rc_1+v_2-rc_2\right)^2}\right) \\
&= \frac{1}{v_2}\left(v_2-rc_2\right)=\overline{\alpha_{X_2}}. \qquad \text{(A2)}
\end{aligned}$$

Therefore if $v_1 > rc_1$, then $0 < \alpha < \overline{\alpha_{X_2}}$ is necessary and sufficient for aggregate investment in both races to be positive.

If $v_1 \leq rc_1$, then

$$\begin{aligned}
\underline{\alpha_{X_1}} &= \frac{1}{2v_2}\left(-v_1+rc_1+v_2-rc_2-\sqrt{\left(-v_1+rc_1+v_2-rc_2\right)^2+4v_2\left(v_1-rc_1\right)}\right) \\
&\geq \frac{1}{2v_2}\left(-v_1+rc_1+v_2-rc_2-\sqrt{\left(-v_1+rc_1+v_2-rc_2\right)^2}\right)=0 \qquad \text{(A3)}
\end{aligned}$$

and

$$\begin{aligned}
\overline{\alpha_{X_1}} &= \frac{1}{2v_2}\left(-v_1+rc_1+v_2-rc_2+\sqrt{\left(-v_1+rc_1+v_2-rc_2\right)^2+4v_2\left(v_1-rc_1\right)}\right) \\
&\leq \frac{1}{2v_2}\left(-v_1+rc_1+v_2-rc_2+\sqrt{\left(-v_1+rc_1+v_2-rc_2\right)^2+4\left(v_2-rc_2\right)\left(v_1-rc_1\right)}\right) \\
&= \frac{1}{2v_2}\left(-v_1+rc_1+v_2-rc_2+\sqrt{\left(v_1-rc_1+v_2-rc_2\right)^2}\right) \\
&= \frac{1}{v_2}\left(v_2-rc_2\right)=\overline{\alpha_{X_2}}. \qquad \text{(A4)}
\end{aligned}$$

Therefore if $v_1 \leq rc_1$, then $\underline{\alpha_{X_1}} < \alpha < \overline{\alpha_{X_1}}$ is necessary and sufficient for aggregate investment in both races to be positive.

Appendix A.3: Proof of Proposition 1.

Proof. Differentiating (4) with respect to α gives:

$$\frac{dX_2}{d\alpha} = -\frac{v_2}{c_2} < 0. \qquad \text{(A5)}$$

■

Appendix A.4: Proof of Proposition 2.

Proof. Differentiating (8) with respect to α gives:

$$\frac{dX_1}{d\alpha} = \frac{(1-\alpha)^2 v_2 - rc_2}{(1-\alpha)^2 c_1}. \tag{A6}$$

The derivative is positive (negative) when $\alpha < 1 - \sqrt{\frac{rc_2}{v_2}}$ ($\alpha > 1 - \sqrt{\frac{rc_2}{v_2}}$) and equal to zero when $\alpha = 1 - \sqrt{\frac{rc_2}{v_2}}$. ■

Appendix A.5: Proof of Lemma 1.

Proof. It follows directly from Proposition 1 and Proposition 2 that when $\alpha \geq \widehat{\alpha_{X_1}}$, then $MSB \leq 0$ and $MSC > 0$, and therefore $\frac{dW}{d\alpha} < 0$. Therefore if the social welfare function achieves a maximum, it has to occur for $\alpha < \widehat{\alpha_{X_1}}$. ■

Appendix A.6: Proof of Lemma 2.

Proof. The second order condition for welfare maximization with respect to α in (18) is:

$$\frac{d^2W}{d\alpha^2} = \frac{dMSB}{d\alpha} - \frac{dMSC}{d\alpha}.$$

To show that when $\alpha < \widehat{\alpha_{X_1}}$, then $\frac{d^2W}{d\alpha^2} < 0$, it is sufficient to show that when $\alpha < \widehat{\alpha_{X_1}}$, then $\frac{dMSB}{d\alpha} < 0$ and $\frac{dMSC}{d\alpha} > 0$.

The derivative of MSB with respect to α is :

$$\begin{aligned}\frac{dMSB}{d\alpha} &= \frac{r^2 s_2 \frac{dX_1}{d\alpha}\frac{dX_2}{d\alpha}}{(X_1+r)^2 (X_2+r)^2} \\ &\quad + \frac{r\left(s_1 (X_2+r) + s_2 X_2\right)\left((X_1+r)\frac{d}{d\alpha}\left(\frac{dX_1}{d\alpha}\right) - 2\left(\frac{dX_1}{d\alpha}\right)^2\right)}{(X_1+r)^3 (X_2+r)}.\end{aligned} \tag{A7}$$

When $\alpha < \widehat{\alpha_{X_1}}$, the first term in (A7) is negative because $\frac{dX_1}{d\alpha} > 0$ by Proposition 2 and $\frac{dX_2}{d\alpha} < 0$ by Proposition 1. When $\alpha < \widehat{\alpha_{X_1}}$, the second term is negative because $\frac{d}{d\alpha}\left(\frac{dX_1}{d\alpha}\right) = -\frac{2rc_2}{(1-\alpha)^3 c_1} < 0$. Therefore, when $\alpha < \widehat{\alpha_{X_1}}$, then $\frac{dMSB}{d\alpha} < 0$.

The derivative of MSC with respect to α is:

$$\begin{aligned}\frac{dMSC}{d\alpha} &= \frac{-r^2 s_2 \frac{dX_1}{d\alpha}\frac{dX_2}{d\alpha}}{(X_1+r)^2 (X_2+r)^2} \\ &\quad + \frac{-r s_2 X_1\left((X_2+r)\frac{d}{d\alpha}\left(\frac{dX_2}{d\alpha}\right) - 2\left(\frac{dX_2}{d\alpha}\right)^2\right)}{(X_1+r)(X_2+r)^3}.\end{aligned} \tag{A8}$$

When $\alpha < \widehat{\alpha_{X_1}}$, the first term in (A8) is positive because $\frac{dX_1}{d\alpha} > 0$ by Proposition 2 and $\frac{dX_2}{d\alpha} < 0$ by Proposition 1. When $\alpha < \widehat{\alpha_{X_1}}$, the second term is positive because $\frac{d}{d\alpha}\left(\frac{dX_2}{d\alpha}\right) = 0$. Therefore, when $\alpha < \widehat{\alpha_{X_1}}$, then $\frac{dMSC}{d\alpha} > 0$. ■

Appendix A.7: Proof of Proposition 3.

Proof. Recall that by Assumption 2 feasible degrees of protection of basic research are:

$$0 \leq \alpha < \overline{\alpha_{X_2}} \text{ when } v_1 > rc_1 \text{ and}$$
$$\underline{\alpha_{X_1}} < \alpha < \overline{\alpha_{X_1}} \text{ when } v_1 \leq rc_1.$$

Assuming that an interior solution to the first order condition of the government's welfare maximization problem in (16) exists, that is assuming that $\widehat{\alpha_W}$ is among the feasible degrees of protection of basic research defined in Assumption 2, it follows from Lemma 1 and from the negativity of the second order condition for welfare maximization when $\alpha < \widehat{\alpha_{X_1}}$ that social welfare increases (decreases) in the degree of patent protection of basic research α for $\alpha < \widehat{\alpha_W}$ ($\alpha > \widehat{\alpha_W}$) and achieves a maximum at $\alpha = \widehat{\alpha_W}$, where $\widehat{\alpha_W} < \widehat{\alpha_{X_1}}$.

Next, I examine whether the government's welfare maximization problem may have a corner solution. A corner solution would occur if the solution to the first order condition in (16), $\widehat{\alpha_W}$, lies outside the range of feasible degrees of protection of basic research α. It follows from Lemma 1, that the welfare-maximizing degree of protection of basic research $\widehat{\alpha_W}$ occurs in the interval $\alpha < \widehat{\alpha_{X_1}}$ and therefore a corner solution cannot occur at either of the upper bounds on the feasible degree of protection of basic research ($\overline{\alpha_{X_2}}$ when $v_1 > rc_1$ and $\overline{\alpha_{X_1}}$ when $v_1 \leq rc_1$). I then examine whether a corner solution can occur at the lower bounds on the feasible degree of protection of basic research (0 when $v_1 > rc_1$ and $\underline{\alpha_{X_1}}$ when $v_1 \leq rc_1$). A corner solution would occur at $\alpha = 0$ if $\widehat{\alpha_W} < 0$ when $v_1 > rc_1$ and then $\frac{dW}{d\alpha} \mid_{\alpha=0} \leq 0$. A corner solution would occur at $\alpha = \underline{\alpha_{X_1}}$ if $\widehat{\alpha_W} < \underline{\alpha_{X_1}}$ when $v_1 \leq rc_1$ and then $\frac{dW}{d\alpha} \mid_{\alpha=\underline{\alpha_{X_1}}} \leq 0$. Thus to find out whether corner solutions exist at the lower bounds it is sufficient to check whether $\frac{dW}{d\alpha} \mid_{\alpha=0} \leq 0$ or $\frac{dW}{d\alpha} \mid_{\alpha=\underline{\alpha_{X_1}}} \leq 0$.

First, when $v_1 > rc_1$, the first order condition in (16) evaluated at the lower bound on the feasible degree of protection of basic research, 0, is:

$$\frac{dW}{d\alpha} \mid_{\alpha=0} = \frac{r\frac{1}{c_1}\frac{v_2}{c_2}\left(\left(s_1\frac{v_2}{c_2} + s_2\left(\frac{v_2}{c_2} - r\right)\right)(v_2 - rc_2) - s_2\left(\frac{v_1}{c_1} - r\right)v_1\right)}{(X_1 + r)^2 (X_2 + r)^2}.$$

Let $\widetilde{v_1} = \frac{1}{2}\left(rc_1 + \sqrt{(rc_1)^2 + 4\frac{c_1}{s_2}\left(s_1\frac{v_2}{c_2} + s_2\left(\frac{v_2}{c_2} - r\right)\right)(v_2 - rc_2)}\right)$. The sign of the first order condition is:

$$\frac{dW}{d\alpha} \mid_{\alpha=0} \begin{cases} > 0 & \text{if} \quad rc_1 < v_1 < \widetilde{v_1} \\ \leq 0 & \text{if} \quad v_1 \geq \widetilde{v_1}. \end{cases}$$

Thus, when $v_1 \geq \widetilde{v_1}$, the government's welfare maximization problem has a corner solution at $\alpha = 0$.

Second, when $v_1 \leq rc_1$, the first order condition in (16) evaluated at the lower bound on the feasible degree of protection of basic research, $\underline{\alpha_{X_1}}$, and simplified is:

$$\frac{dW}{d\alpha}\,|_{\alpha=\underline{\alpha_{X_1}}}= \frac{r\frac{\left((1-\alpha)^2 v_2 - rc_2\right)}{c_1(1-\alpha)^2}\left(s_1\left(X_2+r\right)+s_2X_2\right)}{\left(X_1+r\right)^2\left(X_2+r\right)},$$

which is positive because $(1-\alpha)^2 v_2 - rc_2 > 0$ by Assumption 1. Thus, the government's welfare maximization problem does not have a corner solution at $\alpha = \underline{\alpha_{X_1}}$. ■

Appendix A.8: Proof of Proposition 4.

Proof. To determine the sign of $\frac{d\widehat{\alpha_W}}{dv_1}$, I apply the implicit function theorem to the first order condition for welfare maximization in the competitive equilibrium in (16):

$$\frac{d\widehat{\alpha_W}}{dv_1} = -\frac{\frac{d}{dv_1}\left(\frac{dW}{d\alpha}\,|_{\alpha=\widehat{\alpha_W}}\right)}{\frac{d}{d\alpha}\left(\frac{dW}{d\alpha}\,|_{\alpha=\widehat{\alpha_W}}\right)}. \tag{A9}$$

The denominator is negative at $\alpha = \widehat{\alpha_W}$ by the negativity of the second order condition for welfare maximization in the competitive equilibrium when $\alpha < \widehat{\alpha_{X_1}}$, which is shown in Appendix A.6, and because $\widehat{\alpha_W} < \widehat{\alpha_{X_1}}$ by Proposition 3. Therefore the sign of $\frac{d\widehat{\alpha_W}}{dv_1}$ is the same as the sign of the numerator in $(A9)$. The numerator is:

$$\begin{aligned}\frac{d}{dv_1}\left(\frac{dW}{d\alpha}\,|_{\alpha=\widehat{\alpha_W}}\right) &= \frac{r\left(X_2+r\right)\left(s_1\left(X_2+r\right)+s_2X_2\right)\frac{d}{dv_1}\left(\frac{dX_1}{d\alpha}\right)}{\left(X_1+r\right)^2\left(X_2+r\right)^2} \\ &\quad+\frac{rs_2\left(2X_1+r\right)\frac{dX_1}{dv_1}\frac{dX_2}{d\alpha}}{\left(X_1+r\right)^2\left(X_2+r\right)^2},\end{aligned} \tag{A10}$$

which is negative because $\frac{d}{dv_1}\left(\frac{dX_1}{d\alpha}\right) = 0$, $\frac{dX_1}{dv_1} = \frac{1}{c_1} > 0$ and $\frac{dX_2}{d\alpha} < 0$ by Proposition 1. Therefore $\frac{d\widehat{\alpha_W}}{dv_1} < 0$.

Similarly, to determine the sign of $\frac{d\widehat{\alpha_W}}{ds_1}$, I apply the implicit function theorem to (16):

$$\frac{d\widehat{\alpha_W}}{ds_1} = -\frac{\frac{d}{ds_1}\left(\frac{dW}{d\alpha}\,|_{\alpha=\widehat{\alpha_W}}\right)}{\frac{d}{d\alpha}\left(\frac{dW}{d\alpha}\,|_{\alpha=\widehat{\alpha_W}}\right)}, \tag{A11}$$

where the numerator is:

$$\frac{d}{ds_1}\left(\frac{dW}{d\alpha}\,|_{\alpha=\widehat{\alpha_W}}\right) = \frac{r\frac{dX_1}{d\alpha}}{\left(X_1+r\right)^2}, \tag{A12}$$

which is positive at $\alpha = \widehat{\alpha_W}$ because $\frac{dX_1}{d\alpha} > 0$ when $\alpha < \widehat{\alpha_{X_1}}$ by Proposition 2 and because $\widehat{\alpha_W} < \widehat{\alpha_{X_1}}$ by Proposition 3. Therefore $\frac{d\widehat{\alpha_W}}{ds_1} > 0$.

Next, to determine the sign of $\frac{d\widehat{\alpha_W}}{dc_1}$, I apply the implicit function theorem to (16):

$$\frac{d\widehat{\alpha_W}}{dc_1} = -\frac{\frac{d}{dc_1}\left(\frac{dW}{d\alpha}|_{\alpha=\widehat{\alpha_W}}\right)}{\frac{d}{d\alpha}\left(\frac{dW}{d\alpha}|_{\alpha=\widehat{\alpha_W}}\right)}, \tag{A13}$$

where the numerator is:

$$\begin{aligned}\frac{d}{dc_1}\left(\frac{dW}{d\alpha}|_{\alpha=\widehat{\alpha_W}}\right) &= \frac{r\left(s_1\left(X_2+r\right)+s_2X_2\right)\left(X_2+r\right)\frac{d}{dc_1}\left(\frac{dX_1}{d\alpha}\right)}{\left(X_1+r\right)^2\left(X_2+r\right)^2} \\ &\quad+\frac{rs_2\left(2X_1+r\right)\frac{dX_1}{dc_1}\frac{dX_2}{d\alpha}}{\left(X_1+r\right)^2\left(X_2+r\right)^2}.\end{aligned} \tag{A14}$$

Using the fact that $r\left(s_1\left(X_2+r\right)+s_2X_2\right)\left(X_2+r\right) = -rs_2X_1\left(X_1+r\right)\frac{dX_2}{d\alpha}\frac{1}{\frac{dX_1}{d\alpha}}$ when the the first-order condition for welfare maximization in (16) is satisfied and using $\frac{d}{dc_1}\left(\frac{dX_1}{d\alpha}\right) = -\frac{1}{c_1}\frac{dX_1}{d\alpha}$ and $\frac{dX_1}{dc_1} = -\frac{1}{c_1}\left(X_1+r\right)$, $(A14)$ simplifies to:

$$\frac{d}{dc_1}\left(\frac{dW}{d\alpha}|_{\alpha=\widehat{\alpha_W}}\right) = \frac{-rs_2\frac{1}{c_1}\frac{dX_2}{d\alpha}}{\left(X_2+r\right)^2}, \tag{A15}$$

which is positive because $\frac{dX_2}{d\alpha} < 0$ by Proposition 1. Therefore $\frac{d\widehat{\alpha_W}}{dc_1} > 0$.

Finally, to determine the sign of $\frac{d\widehat{\alpha_W}}{ds_2}$, I apply the implicit function theorem to (16):

$$\frac{d\widehat{\alpha_W}}{ds_2} = -\frac{\frac{d}{ds_2}\left(\frac{dW}{d\alpha}|_{\alpha=\widehat{\alpha_W}}\right)}{\frac{d}{d\alpha}\left(\frac{dW}{d\alpha}|_{\alpha=\widehat{\alpha_W}}\right)}, \tag{A16}$$

where the numerator is:

$$\frac{d}{ds_2}\left(\frac{dW}{d\alpha}|_{\alpha=\widehat{\alpha_W}}\right) = \frac{rX_2\left(X_2+r\right)\frac{dX_1}{d\alpha}+rX_1\left(X_1+r\right)\frac{dX_2}{d\alpha}}{\left(X_1+r\right)^2\left(X_2+r\right)^2}. \tag{A17}$$

Using the fact that $rX_1\left(X_1+r\right)\frac{dX_2}{d\alpha} = -r\left(\frac{s_1}{s_2}\left(X_2+r\right)+X_2\right)\left(X_2+r\right)\frac{dX_1}{d\alpha}$ when the the first-order condition for welfare maximization in (16) is satisfied, the numerator simplifies to:

$$\frac{d}{ds_2}\left(\frac{dW}{d\alpha}|_{\alpha=\widehat{\alpha_W}}\right) = \frac{-r\frac{s_1}{s_2}\frac{dX_1}{d\alpha}}{\left(X_1+r\right)^2}, \tag{A18}$$

which is negative at $\alpha = \widehat{\alpha_W}$ because $\frac{dX_1}{d\alpha} > 0$ when $\alpha < \widehat{\alpha_{X_1}}$ by Proposition 2 and because $\widehat{\alpha_W} < \widehat{\alpha_{X_1}}$ by Proposition 3. Therefore $\frac{d\widehat{\alpha_W}}{ds_2} < 0$. ■

Appendix A.9: Derivation of $\widehat{\alpha_W}$ when $v_1 = s_1 = 0$.

Substituting with the equilibrium values of X_1 and X_2 in the first order condition for welfare maximization in (16), and using the fact that $v_1 = s_1 = 0$, gives:

$$\frac{dW}{d\alpha} = \frac{rs_2\left((1-\alpha)^2 c_1 - \alpha^2 c_2\right)}{\alpha^2 (1-\alpha)^2 v_2} = 0.$$

The solution to that first order condition is $\widehat{\alpha_W} = \frac{\sqrt{c_1}}{(\sqrt{c_1}+\sqrt{c_2})}$.

Appendix A.10: Definitions of $\overline{\overline{\alpha_{X_2}}}$, $\underline{\underline{\alpha_{X_1}}}$ and $\overline{\overline{\alpha_{X_1}}}$.

Let $\overline{\overline{\alpha_{X_2}}} = 1 - \frac{\sqrt{rc_2(v_2+s_2)}}{v_2}$.

Let $\underline{\underline{\alpha_{X_1}}} = \frac{(v_2-rc_2+A)-\sqrt{(v_2-rc_2+A)^2-4v_2A}}{2v_2}$ and $\overline{\overline{\alpha_{X_1}}} = \frac{(v_2-rc_2+A)+\sqrt{(v_2-rc_2+A)^2-4v_2A}}{2v_2}$, with $A = \sqrt{rc_1\left(v_1+s_1+\left(\sqrt{v_2+s_2}-\sqrt{rc_2}\right)^2\right)} - v_1$.

Appendix A.11: Proof of Proposition 5.

Proof. The result in the proposition follows from comparing the aggregate investment in product development in (4) and (14). ■

Appendix A.12: Proof of Proposition 6.

Proof. The result in the proposition follows from comparing the aggregate investment in basic research in (8) and (13). ■

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