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Companion Paper to “Tax Competition and the Creation of Redundant Products”: Proof of Proposition 2

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COMPANION PAPER TO “ TAX COMPETITION AND THE CREATION OF REDUNDANT PRODUCTS”: PROOF OF PROPOSITION 2

by

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This working paper provides the full proof of Proposition 2 in "Tax Competition and the Creation of Redundant Products" (*University of Colorado Working Paper* #05-3).

This paper shows that, if Region 2's strategy is $(t_k^2 = \delta\bar{w},\ \ t_a^2 = (\delta + T/2)\bar{w})$, the tax revenue collected by Region 1 using the proposed strategy $(t_k^1 = \delta\bar{w},\ \ t_a^1 = (\delta + T/2)\bar{w})$ is no less than the tax revenue collected by Region 1 using all possible alternative combinations of t_k^1 and t_a^1. Due to the large number of cases, this is laborious. The plan is (1) to establish the tax revenue R^{1*} collected under the proposed strategy, (2) show that a necessary condition for the proposed strategy to be a best response is $\delta > T$ and then (3) with $\delta > T$ show that the revenue collected under all possible alternative combinations of t_k^1 and t_a^1 is no greater than R^{1*}.

1. TAX REVENUE OF REGION 1 USING THE STRATEGY $(t_k^1 = \delta\bar{w},\ t_a^1 = (\delta + T/2)\bar{w})$.

Individuals in Region 1 buy kits if $p + t_k^1 + wT \le p + t_a^1$ or if $w \le (t_a^1 - t_k^1)/T$. Region 1 makes no exports; its tax revenue is

$$R^{1*} = \int_0^{\frac{t_a^1 - t_k^1}{T}} t_k^1\, Q f(w)\, dw + \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1\, Q f(w)\, dw \ ;$$

Using the uniform distribution, *f(w) = 1/*$\bar{w}$, and integrating

$$R^{1*} = \frac{Q}{\bar{w}}\left[t_k^1\,\frac{t_a^1 - t_k^1}{T} + t_a^1\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right)\right].$$

Setting $t_k^1 = \delta\bar{w},\ \ t_a^1 = (\delta + \frac{T}{2})\bar{w},$

$$R^{1*} = \left(\delta + \frac{T}{4} \right) Q \bar{w} .$$

2. A NECESSARY CONDITION FOR THE PROPOSED NASH EQUILIBRIUM IS $\delta \geq T$.

Suppose $\delta < T$ and Region 2 sets tax rates as $t_k^2 = \delta\bar{w}$ and $t_a^2 = (\delta + T/2)\bar{w}$. I find a strategy for Region 1 which creates strictly more tax revenue than R^{1*}.

Consider the tax revenue which Region 1 could generate if it set tax rates as $t_k^1 = t_a^1 \leq t_k^2 = \delta\bar{w} < t_a^2 = (\delta + T/2)\,\bar{w}$. Region 1 produces no kits and in Region 1

$$t_a^1 \leq \min\,[t_k^2 + wT + w\delta\;,\, t_a^2 + w\delta]$$

or Region 1 does not import cigarettes. However, Region 1 may export to Region 2.

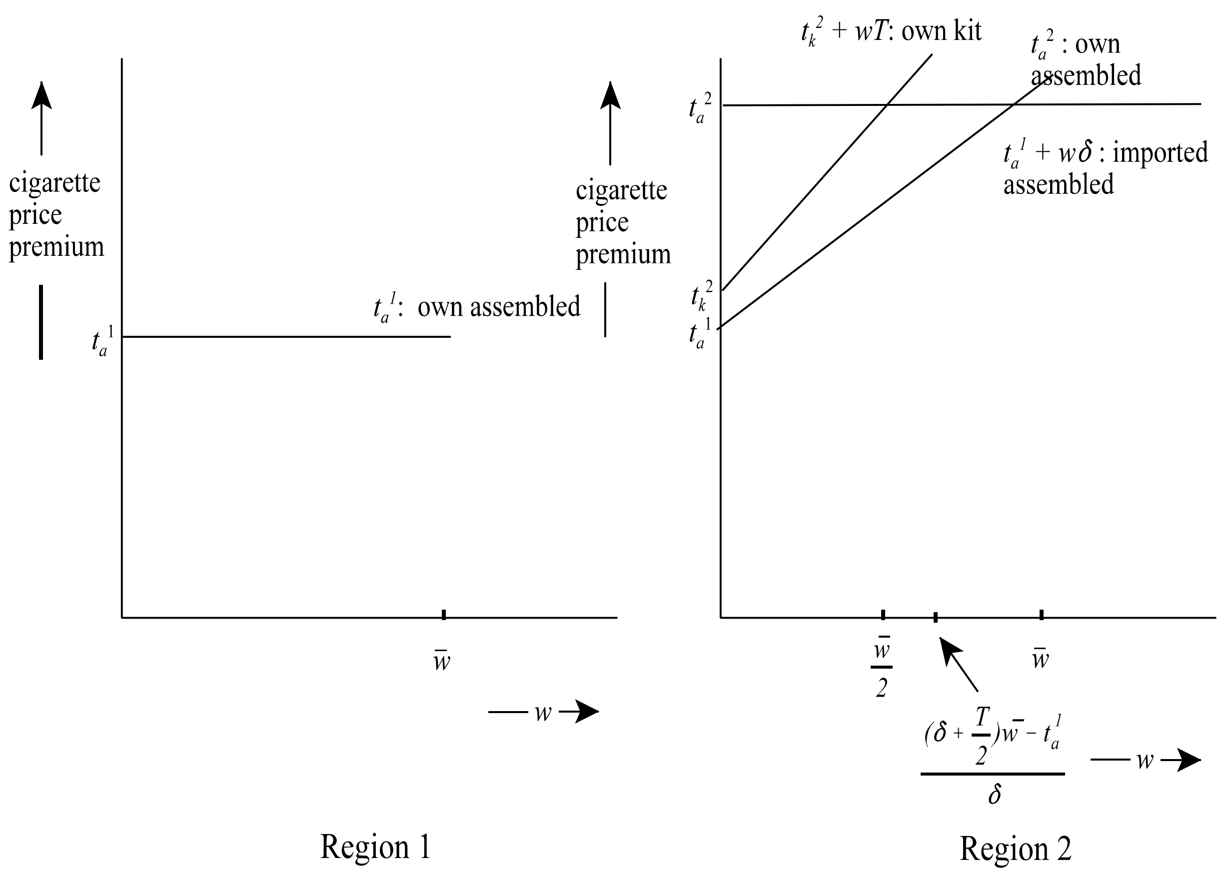


Figure: inclusive prices with $t_a^1 \le t_k^2 \le t_a^2$

An individual in Region 2 faces the same inclusive price for an own assembled product and an imported assembled product if $p + t_a^2 = p + t_a^1 + w\delta$ or, setting $t_a^2 = (\delta + T/2)\bar{w}$, if $w = ((\delta + T/2)\bar{w} - t_a^1)/\delta$. Region 1's problem is:

$$\max_{t_a^1} R^1 = \int_0^{\bar{w}} t_a^1 \, Q f(w) \, dw + \int_0^{\frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta}} t_a^1 \, Q f(w) \, dw$$

$$\text{s.t.} \qquad t_a^1 \le t_k^2 = \delta \bar{w}.$$

Setting $f(w) = 1/\bar{w}$ and integrating, tax revenue is:

$$R^1 = t_a^1 \bar{w} + t_a^1 \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta}.$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}} \left[t_a^1 \bar{w} + t_a^1 \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta} \right] + A(\delta \bar{w} - t_a^1) .$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}} \left[\bar{w} - \frac{t_a^1}{\delta} + \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta} \right] - A = 0 ; \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - t_a^1 \ge 0 \qquad CS \quad A \ge 0; \qquad (2)$$

where *CS* denotes “complementary slackness”. Try A > 0. From Equation (2):

$$t_a^1 = \delta \bar{w}.$$

From Equation (1):

$$A = \frac{Q}{\bar{w}}\left[\bar{w} - \bar{w} + \frac{(\delta + \frac{T}{2})\bar{w} - \delta\bar{w}}{\delta}\right] = \frac{T}{2\delta}Q > 0 \quad \text{as required} .$$

And tax revenue:

$$R^1 = \frac{Q}{\bar{w}}\left[\delta\bar{w}\bar{w} + \delta\bar{w}\frac{(\delta + \frac{T}{2})\bar{w} - \delta\bar{w}}{\delta}\right] = (\delta + \frac{T}{2})\, Q\, \bar{w} > R^{1*}.$$

Summarizing, if $\delta \leq T$ and $t_k^2 = \delta\bar{w}$ and $t_a^2 = (\delta + T/2)\bar{w}$, the best response of Region 1 is not to set $t_k^1 = \delta\bar{w}$ and $t_a^1 = (\delta + T/2)\bar{w}$. Therefore attention is henceforth restricted to $\delta > T$.

3. TAX REVENUE UNDER ALL ALTERNATIVE STRATEGIES.

To show that $t_k^1 = \delta \bar{w}$ and $t_a^1 = (\delta + T/2)\bar{w}$ are the best response, I must consider the tax revenue which can be generated by Region 1 under all possible alternative strategies. If Region 1 sets $t_k^1 \geq t_a^1$ it sells no kits. Therefore if any best response has $t_k^1 > t_a^1$, there is another best response with $t_k^1 = t_a^1$ and no generality is lost to restricting attention to strategies for which $t_k^1 \leq t_a^1$. Given $t_k^1 \leq t_a^1$ and $t_k^2 < t_a^2$, there are 6 possible orderings between (t_k^1, t_a^1) and (t_k^2, t_a^2):

CASE A: $t_k^1 \leq t_a^1 \leq t_k^2 < t_a^2$;

CASE B: $t_k^1 \leq t_k^2 \leq t_a^1 \leq t_a^2$;

CASE C: $t_k^1 \leq t_k^2 < t_a^2 \leq t_a^1$;

CASE D: $t_k^2 \leq t_k^1 \leq t_a^1 \leq t_a^2$;

CASE E: $t_k^2 \leq t_k^1 \leq t_a^2 < t_a^1$;

CASE F: $t_k^2 < t_a^2 \leq t_k^1 < t_a^1$;

These are illustrated in the figure below and I consider each in turn:

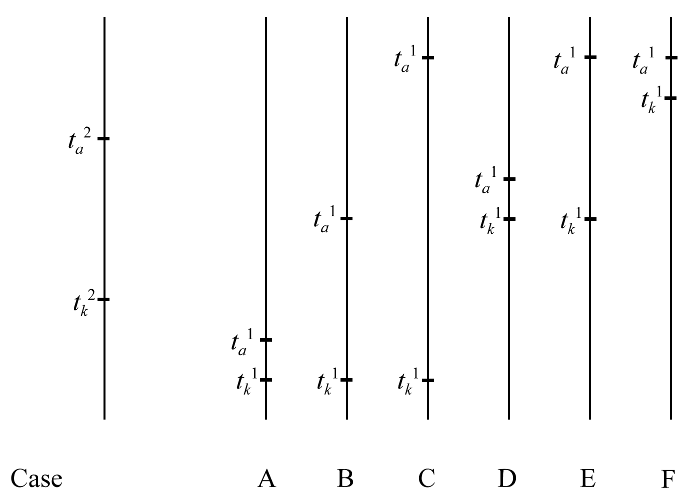


Figure: possible tax rates combinations

3.1 CASE A: $t_k^1 \le t_a^1 \le t_k^2 < t_a^2$.

In Region 1, $\min[\,t_k^1 + wT\,,\, t_a^1\,] < \min[\,t_k^2 + wT + w\delta\,,\, t_a^2 + w\delta\,]$ or Region 1 does not import. The individual in Region 1 who faces the same inclusive price for an own kit and own assembled cigarette has income such that $p + t_k^1 + wT = p + t_a^1$, or $w = (t_a^1 - t_k^1)/T$.

An individual in Region 2 faces the same inclusive price for an own kit and an own assembled product if $p + t_k^2 + wT = p + t_a^2$ or, setting $t_k^2 = \delta\,\bar{w}$ and $t_a^2 = (\delta + T/2)\bar{w}$, if

$$w = \frac{\bar{w}}{2}.$$

The inclusive price for an own cigarette in Region 2 varies with an individual's wage as the bold line ABC in the figure below:

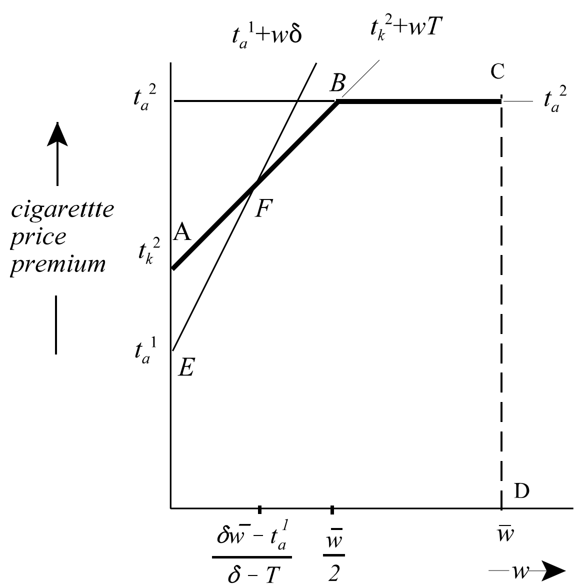


Figure: inclusive price schedules in Region 2

An individual in Region 2 faces the same price for an imported assembled and an own kit if $p + t_a^1 + w\delta = p + t_k^2 + wT$ or, setting $t_k^2 = \delta\,\bar{w}$, if

$$w = \frac{\delta\bar{w} - t_a^1}{\delta - T}\ .$$

An individual in Region 2 faces the same price for an imported assembled and an own assembled if $p + t_a^1 + w\delta = p + t_a^2$ or, setting $t_a^2 = (\delta + T/2)\bar{w}$, if

$$w = \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta}\ .$$

The price line of an imported assembled cigarette in Region 2 can intersect the envelope ABC

Case A1: the intersection is on AB. This occurs if $(\delta\bar{w} - t_a^1)/(\delta - T) \le \bar{w}/2$.

This case is illustrated in the figure above.

Case A2: the intersection is on BC. This occurs if $\bar{w}/2 \le ((\delta + T/2)\bar{w} - t_a^1)/\delta \le \bar{w}$.

Note that, if the inclusive price line of an imported assembled cigarette intersects ABC at C, Region 1 is selling assembled product to the whole market of Region 2. Further lowering of t_a^1 lowers tax revenue. Hence it can never be an optimal response of Region 1 to set t_a^1 so that the inclusive price line of an imported assembled cigarette intersects $w = \bar{w}$ below C, and this case is not considered further.

CASES A1: *the inclusive price line of an imported assembled cigarette intersects ABC on AB.*

There are three possible subcases which are characterized by the intersection of the inclusive price line of imported kits in Region 2 with the envelope EFBC:

Case A1.(i) the intersection is on EF;

Case A1.(ii) the intersection is on FB;

Case A1.(iii) the intersection is on BC.

Note that, if the inclusive price line of an imported kit lies below C at $w = \bar{w}$, $p + t_k^1 + \bar{w}T + \bar{w}\delta \leq p + t_a^2$. Setting $t_a^2 = (\delta + T/2)\bar{w}$, this becomes $t_k^1 = -T\bar{w}/2$. Region 1 is selling kits in Region 2 by subsidizing them: this cannot be a best response of Region1 and this case is not further considered.

CASE A1.(i)*: the inclusive price line of an imported kit intersects EFBC on EF*

This case establishes that $\delta \geq 2T$. The inclusive prices in the two regions are shown in the figure below:

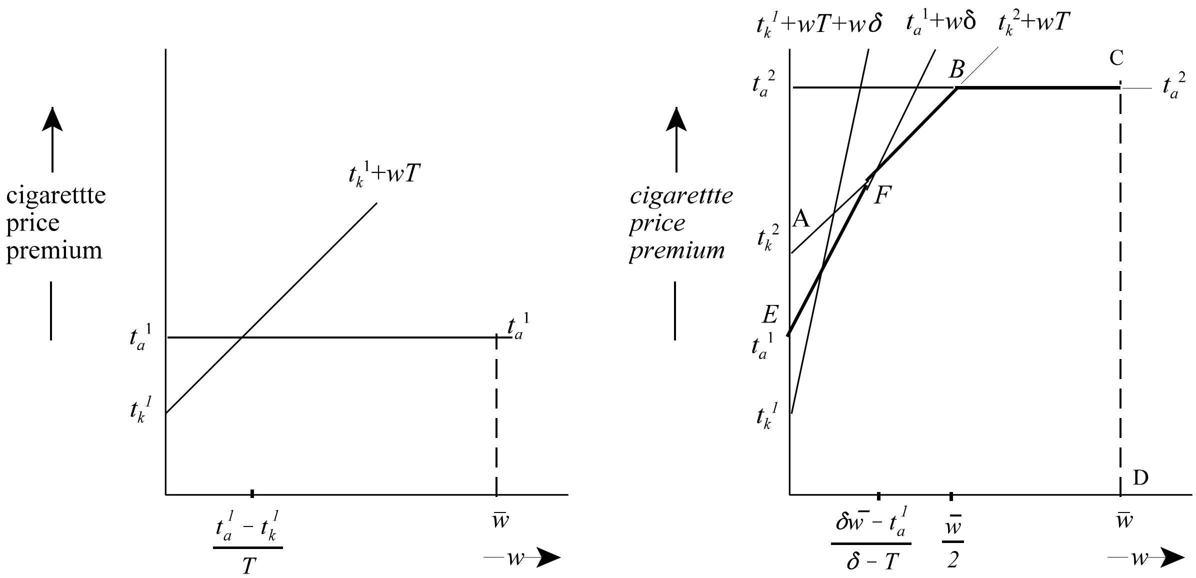


Figure: inclusive price lines for Case A1.(i)

If $t_k^1 < t_a^1$ (as drawn in the figure above), by raising t_k^1 to t_a^1 Region 1 does not change its total cigarette sales in either region but it does cause some individuals in both regions to substitute out of its low-tax kits into its high-tax assembled product. Tax revenue increases. Therefore, for this case Region 1 maximizes its tax revenue by setting $t_k^1 = t_a^1$, or there is a single tax rate to be chosen. Region 1's problem becomes:

$$\max_{t_a^1} \; R^1 = \int_0^{\bar{w}} t_a^1 \, Q f(w) \, dw + \int_0^{\frac{\bar{w}\delta - t_a^1}{\delta - T}} t_a^1 \, Q f(w) \, dw$$

s.t. $\quad t_a^1 \leq t_k^2 = \delta \bar{w};$

$$\frac{\delta \bar{w} - t_a^1}{\delta - T} \leq \frac{\bar{w}}{2} .$$

Using the uniform distribution, $f(w) = 1 / \bar{w}$ and integrating, R^1 becomes

$$R^1 = \frac{Q}{\bar{w}} \left[t_a^1 \bar{w} + t_a^1 \frac{\delta \bar{w} - t_a^1}{\delta - T} \right].$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}} \left[t_a^1 \bar{w} + t_a^1 \frac{\delta \bar{w} - t_a^1}{\delta - T} \right] + A\,(\delta \bar{w} - t_a^1) + B\,(\frac{\bar{w}}{2} - \frac{\delta \bar{w} - t_a^1}{\delta - T}) .$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}} \left[\bar{w} - \frac{t_a^1}{\delta - T} + \frac{\delta \bar{w} - t_a^1}{\delta - T} \right] - A + \frac{B}{\delta - T} = 0; \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - t_a^1 \geq 0 \qquad \text{CS} \quad A \geq 0; \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\bar{w}}{2} - \frac{\delta \bar{w} - t_a^1}{\delta - T} \geq 0 \qquad \text{CS} \quad B \geq 0. \tag{3}$$

Try $A = B = 0$. From Equation (1):

$$t_a^1 = \frac{2\delta - T}{2}\bar{w}.$$

And

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - \frac{2\delta - T}{2}\bar{w} = \frac{T}{2}\bar{w} \geq 0 \qquad \text{as required ;}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\bar{w}}{2} - \frac{\delta \bar{w} - \frac{2\delta - T}{2}\bar{w}}{\delta - T} = \frac{(\delta - 2T)}{2(\delta - T)}\bar{w}.$$

If $\delta \geq 2T$, $\partial\mathcal{L}/\partial B \geq 0$ as required. In this case:

$$R^1 = \frac{Q}{\bar{w}}\left[\frac{2\delta - T}{2}\bar{w}\,\bar{w} + \frac{2\delta - T}{2}\bar{w}\,\frac{\delta \bar{w} - \frac{2\delta - T}{2}\bar{w}}{\delta - T}\right] = \frac{2\delta - T}{2}\,\frac{2\delta - T}{2(\delta - T)}\,Q\bar{w} < R^{1*}.$$

If $T < \delta < 2T$: Try $A = 0$ and $B > 0$; From Equation (3):

$$t_a^1 = \frac{\delta + T}{2}\bar{w};$$

And

$$\frac{\partial \mathcal{L}}{\partial A} = \delta\bar{w} - \frac{\delta + T}{2}\bar{w} = \frac{\delta - T}{2}\bar{w} > 0 \text{ as required.}$$

From Equation (1):

$$B = Q(2T - \delta) > 0 \quad \text{as required}$$

In this case,

$$R^1 = \frac{Q}{\bar{w}}\left[\frac{\delta + T}{2}\bar{w}\,\bar{w} + \frac{\delta + T}{2}\bar{w}\;\frac{\delta\,\bar{w} - \frac{\delta + T}{2}\bar{w}}{\delta - T}\right] = \frac{3}{4}\,(\delta + T)\,Q\,\bar{w} \;\; > R^{1*}\,,$$

or $t_k^1 = \delta\bar{w}$, $t_a^1 = (\delta + T/2)\bar{w}$ is not the best response of Region 1.

CASE A1.(ii): *the inclusive price line of an imported kit intersects EFBC on FB*

The inclusive prices in the two regions are shown in the figure below:

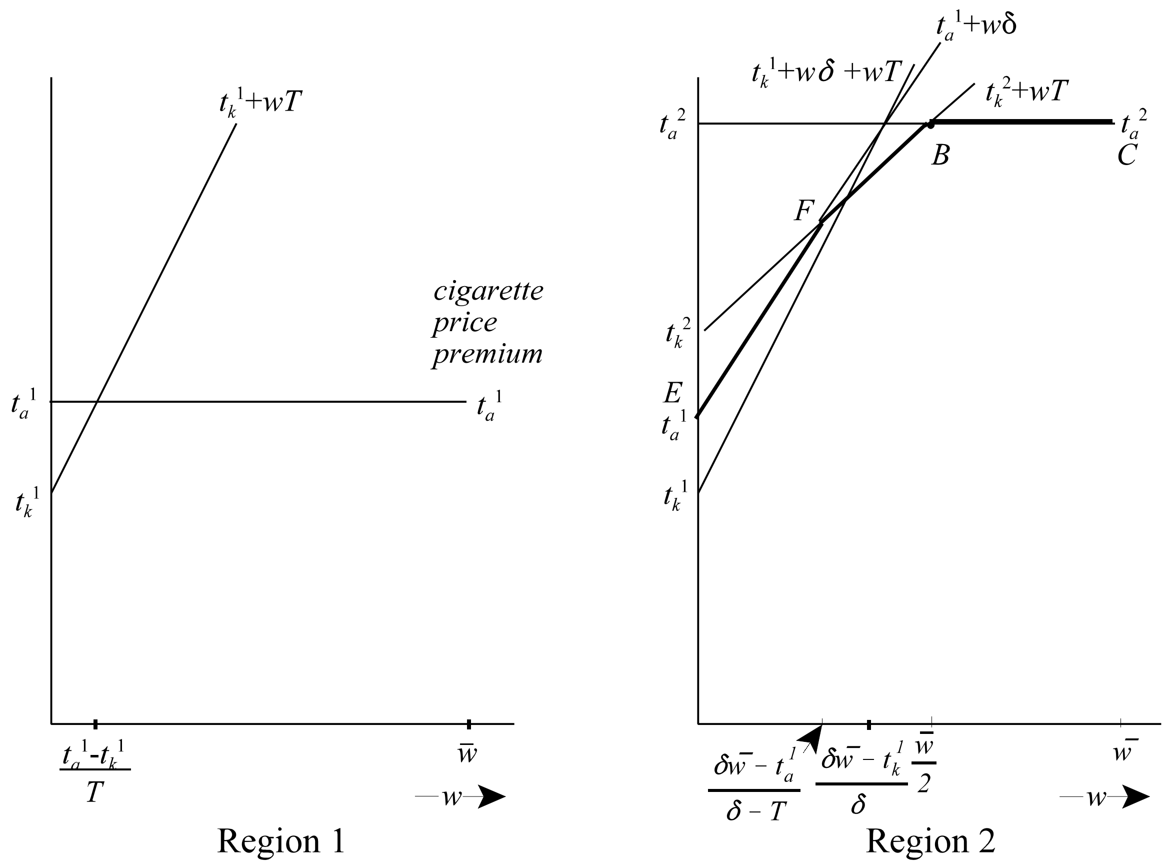


Figure : inclusive price lines for Case A1.(ii)

Region 1 exports kits to individuals in Region 2 for whom $p + t_k^1 + wT + w\delta \leq p + t_k^2 + wT$ or for whom $w \leq (t_k^2 - t_k^1)/\delta = (\delta\bar{w} - t_k^1)/\delta$.

The tax revenue function of Region 1 depends on whether kits and assembled product are both sold in Region 1:

$$\frac{t_a^1 - t_k^1}{T} \leq \bar{w};$$

OR whether only kits are sold in Region 1:

$$\bar{w} \leq \frac{t_a^1 - t_k^1}{T}.$$

I do the former subcase first. Region 1's problem is:

$$\max_{t_k^1,\, t_a^1} \quad R^1 = \int_0^{\frac{t_a^1 - t_k^1}{T}} t_k^1\, Qf(w)\,dw + \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1\, Qf(w)\,dw + \int_0^{\frac{\delta\bar{w} - t_k^1}{\delta}} t_k^1\, Qf(w)\,dw$$

$$\text{s.t.} \qquad t_a^1 \leq t_k^2 = \delta\bar{w};$$

$$\frac{\delta\bar{w} - t_a^1}{\delta - T} \leq \frac{\delta\bar{w} - t_k^1}{\delta};$$

$$\frac{\delta\bar{w} - t_k^1}{\delta} \leq \frac{\bar{w}}{2};$$

$$\frac{t_a^1 - t_k^1}{T} \leq \bar{w};$$

Note that the $(\delta\bar{w} - t_a^1)/(\delta - T) \leq \bar{w}/2$ is implied by the second and third restrictions, and can be omitted. Similarly the restriction $t_k^1 \leq t_a^1$ is implied by $(\delta\bar{w} - t_a^1)/(\delta - T) \leq (\delta\bar{w} - t_k^1)/\delta$ and can be omitted. Setting $f(w) = 1/\bar{w}$ and integrating, R^1 becomes

$$R^1 = \frac{Q}{\bar{w}}\left[t_k^1\,\frac{t_a^1 - t_k^1}{T} + t_a^1\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right) + t_k^1\,\frac{\delta\,\bar{w} - t_k^1}{\delta}\right].$$

The Lagrangean becomes

$$\mathcal{L} = \frac{Q}{\bar{w}}\left[t_k^1\,\frac{t_a^1 - t_k^1}{T} + t_a^1\,\bar{w} - t_a^1\,\frac{t_a^1 - t_k^1}{T} + t_k^1\,\frac{\delta\,\bar{w} - t_k^1}{\delta}\right]$$

$$+ A(\delta\,\bar{w} - t_a^1) + B\left(\frac{\delta\,\bar{w} - t_k^1}{\delta} - \frac{\delta\,\bar{w} - t_a^1}{\delta - T}\right) + C\left(\frac{\bar{w}}{2} - \frac{\delta\,\bar{w} - t_k^1}{\delta}\right) + D\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right).$$

The Kuhn-Tucker conditions are:

$$\frac{\partial\mathcal{L}}{\partial t_k^1} = \frac{Q}{\bar{w}}\left[-\frac{t_k^1}{T} + \frac{t_a^1 - t_k^1}{T} + \frac{t_a^1}{T} - \frac{t_k^1}{\delta} + \frac{\delta\,\bar{w} - t_k^1}{\delta}\right] - \frac{B}{\delta} + \frac{C}{\delta} + \frac{D}{T} = 0; \tag{1}$$

$$\frac{\partial\mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}}\left[\frac{t_k^1}{T} + \bar{w} - \frac{t_a^1}{T} - \frac{t_a^1 - t_k^1}{T}\right] - A + \frac{B}{\delta - T} - \frac{D}{T} = 0; \tag{2}$$

$$\frac{\partial\mathcal{L}}{\partial A} = \delta\,\bar{w} - t_a^1 \geq 0 \qquad \text{CS} \quad A \geq 0; \tag{3}$$

$$\frac{\partial\mathcal{L}}{\partial B} = \frac{\delta\,\bar{w} - t_k^1}{\delta} - \frac{\delta\,\bar{w} - t_a^1}{\delta - T} \geq 0 \qquad \text{CS} \quad B \geq 0; \tag{4}$$

$$\frac{\partial\mathcal{L}}{\partial C} = \frac{\bar{w}}{2} - \frac{\delta\,\bar{w} - t_k^1}{\delta} \geq 0 \qquad \text{CS} \quad C \geq 0; \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial D} = \bar{w} - \frac{t_a^1 - t_k^1}{T} \geq 0 \qquad \text{CS} \quad D \geq 0. \tag{6}$$

Try $A > 0, \quad B = 0, \quad C = 0, \quad D = 0$. Then from Equation (3):

$$t_a^1 = \delta \bar{w};$$

From Equation (1):

$$t_k^1 = \frac{\delta(2\delta + T)}{2(\delta + T)} \bar{w};$$

From (2):

$$A = Q \frac{T}{\delta + T} > 0 \text{ as required;}$$

And

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\delta \bar{w} - \frac{\delta(2\delta + T)}{2(\delta + T)}\bar{w}}{\delta} = \frac{T}{2(\delta + T)}\bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{\bar{w}}{2} - \frac{\delta \bar{w} - \frac{\delta(2\delta + T)}{2(\delta + T)}\bar{w}}{\delta} = \frac{\delta}{2(\delta + T)}\bar{w} \geq 0 \qquad \text{as required ;}$$

$$\frac{\partial \mathcal{L}}{\partial D} = \bar{w} - \frac{\delta \bar{w} - \frac{\delta(2\delta + T)}{2(\delta + T)}\bar{w}}{T} = \frac{\delta T + 2T^2}{2T(\delta + T)} \geq 0 \qquad \text{as required.}$$

In this case

$$R^1 = \frac{\delta\ (4\delta + 5T)}{4(\delta + T)} Q \bar{w} \le R^{1*}.$$

I now consider the alternative subcase when Region 1 sells only kits domestically:

$$\bar{w} \leq \frac{t_a^1 - t_k^1}{T}$$

In addition, because the inclusive price line of imported kits is intersecting EFBC on FB, Region 1 exports only kits. t_a^1 is not therefore relevant and Region 1's problem is:

$$\max_{t_k^1} \quad R^1 = t_k^1 Q + \int_0^{\frac{\delta \bar{w} - t_k^1}{\delta}} t_k^1 \, Q f(w) dw$$

s.t. $\quad t_k^1 \leq t_k^2 = \delta \bar{w}$;

$$\frac{\delta \bar{w} - t_k^1}{\delta} \leq \frac{\bar{w}}{2};$$

Setting $f(w) = 1 / \bar{w}$, Region 1's revenue becomes

$$R^1 = \frac{Q}{\bar{w}} \left[t_k^1 \; \bar{w} + t_k^1 \frac{\delta \bar{w} - t_k^1}{\delta} \right].$$

The Lagrangean becomes

$$\mathfrak{L} = \frac{Q}{\bar{w}} \left[t_k^1 \; \bar{w} + t_k^1 \frac{\delta \bar{w} - t_k^1}{\delta} \right] + A(\delta \bar{w} - t_k^1) + B(\frac{\bar{w}}{2} - \frac{\delta \bar{w} - t_k^1}{\delta}) .$$

The Kuhn-Tucker conditions become

$$\frac{\partial \mathfrak{L}}{\partial t_k^1} = \frac{Q}{\bar{w}} \left[\bar{w} - \frac{t_k^1}{\delta} + \frac{\delta - t_k^1}{\delta} \right] - A + \frac{B}{\delta} = 0; \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - t_k^1 \geq 0 \qquad \text{CS} \quad A \geq 0; \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\bar{w}}{2} - \frac{\delta \bar{w} - t_k^1}{\delta} \geq 0 \qquad \text{CS} \quad B \geq 0. \tag{3}$$

Try *A = 0; B = 0;*

From Equation (1):

$$t_k^1 = \delta \bar{w};$$

And

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - \delta \bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\delta}{2}\bar{w} - \frac{\delta \bar{w} - \delta \bar{w}}{\delta} \geq 0 \qquad \text{as required.}$$

And

$$R^1 = \delta Q \bar{w} < R^{1*}.$$

CASE A1.(iii): *the inclusive price line of an imported kit intersects EFBC on BC*

The inclusive prices in the two regions are shown in the figure below:

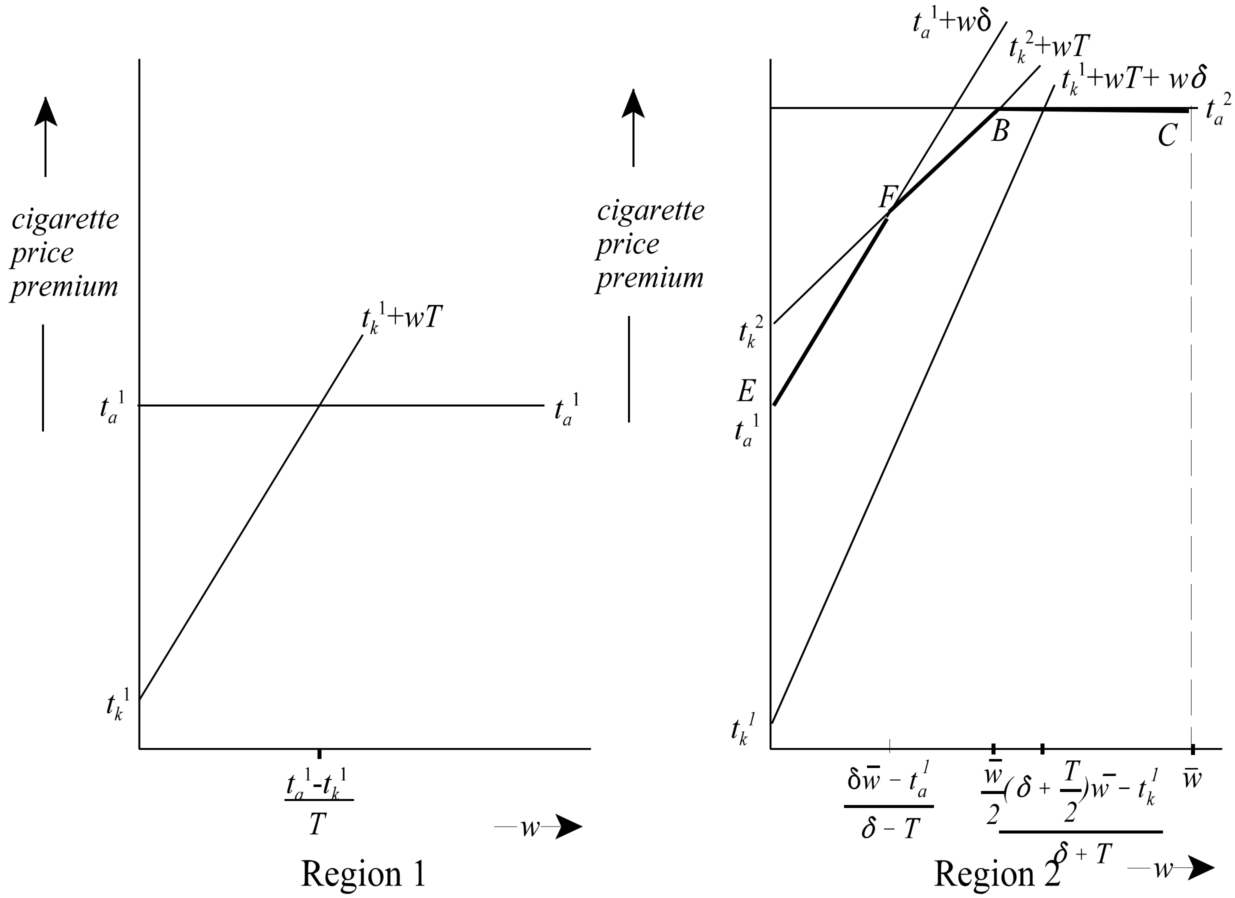


Figure: inclusive prices in Case A1.(iii)

Region 1 sells kits to individuals in Region 2 for whom $p + t_k^1 + wT + w\delta \le p + t_a^2$ or,

setting $t_a^2 = (\delta + T/2)\bar{w}$, $w \le ((\delta + T/2)\bar{w} - t_k^1) / (T + \delta)$.

The tax revenue function of Region 1 depends on whether kits and assembled product are both sold in Region 1:

$$\frac{t_a^1 - t_k^1}{T} \le \bar{w};$$

OR whether only kits are sold in Region 1:

$$\bar{w} \le \frac{t_a^1 - t_k^1}{T} .$$

I do the former subcase first. Region 1's problem is:

$$\max_{t_k^1,\, t_a^1} \; R^1 = \int_0^{\frac{t_a^1 - t_k^1}{T}} t_k^1 \, Qf(w)\,dw + \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1 \, Qf(w)\,dw + \int_0^{\frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T+\delta}} t_k^1 \, Qf(w)\,dw$$

s.t. $\quad t_a^1 \le t_k^2 = \delta \bar{w}$;

$$\frac{\delta \bar{w} - t_a^1}{\delta - T} \le \frac{\bar{w}}{2} ;$$

$$\frac{\bar{w}}{2} \le \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{\delta + T} ;$$

$$\frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{\delta + T} \le \bar{w} ;$$

$$\frac{t_a^1 - t_k^1}{T} \le \bar{w} .$$

Note that the restriction $t_k^1 \le t_k^2$ is implied by the other restrictions and is therefore omitted.

Setting $f(w) = 1 / \bar{w}$, Region 1's revenue becomes

$$R^1 = \frac{Q}{\bar{w}} \left[t_k^1 \, \frac{t_a^1 - t_k^1}{T} + t_a^1 \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) + t_k^1 \, \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} \right].$$

The Lagrangean becomes

$$\mathcal{L} = \frac{Q}{\bar{w}} \left[t_k^1 \, \frac{t_a^1 - t_k^1}{T} + t_a^1 \, \bar{w} - t_a^1 \, \frac{t_a^1 - t_k^1}{T} + t_k^1 \, \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} \right] + A(\delta \bar{w} - t_a^1)$$

$$+ B\left(\frac{\bar{w}}{2} - \frac{\delta \bar{w} - t_a^1}{\delta - T}\right) + C\left(\frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} - \frac{\bar{w}}{2}\right) + D\left(\bar{w} - \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta}\right) + E\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right)$$

.

The Kuhn-Tucker conditions become

$$\frac{\partial \mathcal{L}}{\partial t_k^1} = \frac{Q}{\bar{w}} \left[-\frac{t_k^1}{T} + \frac{t_a^1 - t_k^1}{T} + \frac{t_a^1}{T} - \frac{t_k^1}{T + \delta} + \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} \right] - \frac{C}{T + \delta} + \frac{D}{T + \delta} + \frac{E}{T} = 0; \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}} \left[\frac{t_k^1}{T} + \bar{w} - \frac{t_a^1}{T} - \frac{t_a^1 - t_k^1}{T} \right] - A + \frac{B}{\delta - T} - \frac{E}{T} = 0; \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - t_a^1 \geq 0 \qquad \text{CS} \quad A \geq 0; \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\bar{w}}{2} - \frac{\delta\,\bar{w} - t_a^1}{\delta - T} \geq 0 \qquad \text{CS} \quad B \geq 0\,; \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} - \frac{\bar{w}}{2} \geq 0 \qquad \text{CS} \quad C \geq 0\,; \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial D} = \bar{w} - \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} \geq 0 \qquad \text{CS} \quad D \geq 0\,; \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial E} = \bar{w} - \frac{t_a^1 - t_k^1}{T} \geq 0 \qquad \text{CS} \quad E \geq 0. \tag{7}$$

Try $A = 0, B = 0; C > 0; D = 0;$ and $E = 0$. From Equation (5)

$$t_k^1 = \frac{\delta}{2}\bar{w}\ .$$

In Equation (2):

$$t_a^1 = \frac{T + \delta}{2}\bar{w}\ ;$$

Hence

$$\frac{\partial \mathcal{L}}{\partial A} = \delta\,\bar{w} - \frac{T + \delta}{2}\bar{w} = \frac{\delta - T}{2}\bar{w} \geq 0 \quad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\bar{w}}{2} - \frac{\delta\,\bar{w} - \frac{T + \delta}{2}\bar{w}}{\delta - T} \geq 0 \qquad \text{as required.}$$

And from Equation (1):

$$\frac{C}{T + \delta} = \frac{Q}{\bar{w}}\left[-\frac{\delta}{2T}\bar{w} + \frac{\bar{w}}{2} + \frac{(T + \delta)}{2T}\bar{w} - \frac{\delta}{2(T + \delta)}\bar{w} + \frac{\bar{w}}{2}\right] = \frac{2\delta + 3T}{2(T + \delta)}Q > 0 \text{ as}$$

required;

and

$$\frac{\partial \mathcal{L}}{\partial D} = \bar{w} - \frac{(\delta + \frac{T}{2})\bar{w} - \frac{\delta \bar{w}}{2}}{\delta + T} = \frac{\bar{w}}{2} \geq 0 \quad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial E} = \bar{w} - \frac{\frac{T + \delta}{2}\bar{w} - \frac{\delta}{2}\bar{w}}{T} = \frac{\bar{w}}{2} \geq 0 \qquad \text{as required.}$$

In this case, the tax revenue of Region 1 is:

$$R^1 = \frac{Q}{\bar{w}}\left[\frac{\delta}{2}\bar{w}\frac{\bar{w}}{2} + \frac{(T + \delta)}{2}\bar{w}\frac{\bar{w}}{2} + \frac{\delta}{2}\bar{w}\frac{\bar{w}}{2}\right] = \frac{3\delta + T}{4}\bar{w}Q < R^{1*}.$$

I now consider the alternative subcase when Region 1 sells only kits domestically:

$$\bar{w} \leq \frac{t_a^1 - t_k^1}{T}.$$

In addition, because the inclusive price line of imported assembled product intersects ABC at E, Region 1 exports only kits. t_a^1 is not therefore relevant and Region 1's problem is:

$$\max_{t_k^1} \quad R^1 = t_k^1 Q + \int_0^{\frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta}} t_k^1 \, Q f(w) \, dw$$

s.t. $\quad t_k^1 \leq t_k^2 = \delta \bar{w};$

$$\frac{\bar{w}}{2} \leq \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta};$$

$$\frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} \leq \bar{w} .$$

The first constraint is implied by the second constraint and can be omitted. Setting $f(w) = 1/\bar{w}$ and integrating, tax revenue is:

$$R^1 = \frac{Q}{\bar{w}}\left[t_k^1\bar{w} + t_k^1\frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta}\right].$$

The Lagrangean is:

$$\mathfrak{L} = \frac{Q}{\bar{w}}\left[t_k^1\,\bar{w} + t_k^1\,\frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta}\right] + A\left(\frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} - \frac{\bar{w}}{2}\right) + B\left(\bar{w} - \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta}\right)$$

.

The Kuhn-Tucker conditions become

$$\frac{\partial \mathfrak{L}}{\partial t_k^1} = \frac{Q}{\bar{w}}\left[\bar{w} - \frac{t_k^1}{T + \delta} + \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta}\right] - \frac{A}{T + \delta} + \frac{B}{T + \delta} = 0; \tag{1}$$

$$\frac{\partial \mathfrak{L}}{\partial A} = \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} - \frac{\bar{w}}{2} \geq 0 \qquad \text{CS} \quad A \geq 0; \tag{2}$$

$$\frac{\partial \mathfrak{L}}{\partial B} = \bar{w} - \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} \geq 0 \qquad \text{CS} \quad B \geq 0. \tag{3}$$

Try $A > 0$ and $B = 0$.

From Equation (2):

$$t_k^1 = \frac{\delta}{2}\bar{w};$$

In Equation (1):

$$\frac{A}{T + \delta} = \frac{2\delta + 3T}{2} Q > 0 \qquad \text{as required.}$$

And

$$\frac{\partial \mathcal{L}}{\partial B} = \bar{w} - \frac{(\delta + \frac{T}{2})\bar{w} - \frac{\delta}{2}\bar{w}}{T + \delta} = \frac{\bar{w}}{2} = 0 \geq 0 \qquad \text{as required.}$$

And tax revenue is:

$$R^1 = \frac{3\delta}{4} Q\bar{w} < R^{1*}.$$

CASES A2: *the inclusive price line of an imported assembled cigarette intersects ABC on BC*

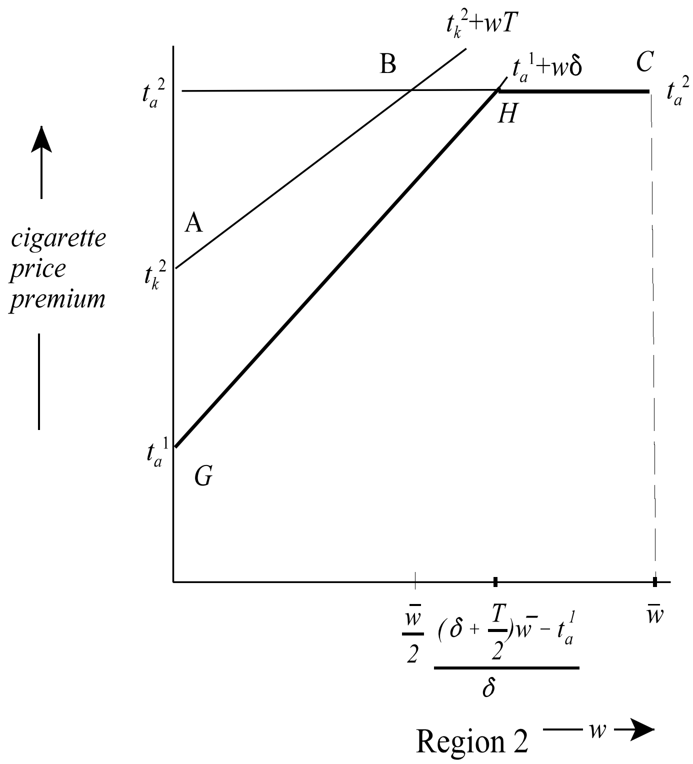


Figure: inclusive price lines in Region 2 in Cases A2

Figure: inclusive price lines in Region 2 for own kit, own assembled and imported kit.

The envelope of the inclusive price in Region 2 of an own assembled cigarette, an own kit and an imported assembled cigarette is shown as GHC in the figure above. The inclusive price of an own assembled cigarette and an imported assembled cigarette are the same for an individual for whom $p + t_a^2 = p + t_a^1 + \delta w$. Setting $t_a^2 = (\delta + T/2)\bar{w}$, his wage becomes:

$$w = \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta}$$

Cases (2) correspond to

$$\frac{\bar{w}}{2} \le \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta} \le \bar{w} .$$

There are two possible subcases which are characterized by the intersection of the inclusive price line of imported kits with GHC:

Case A2.(i) the intersection is on GH ;

Case A2.(ii) the intersection is on HC .

As in Case 1, if the inclusive price line of an imported kit lies below C at $w = \bar{w}$, $p + t_k^1 + \bar{w}T + \bar{w}\delta \le p + t_a^2$. Setting $t_a^2 = (\delta + T/2)\bar{w}$, $t_k^1 \le -T\bar{w}/2$. Region 1 is selling kits in Region 2 by subsidizing them: this cannot be a best response of Region 1 and this case is not considered further.

CASE A2.(i): *the inclusive price line an imported kits intersects GHC on GH.*

The inclusive price lines are shown below:

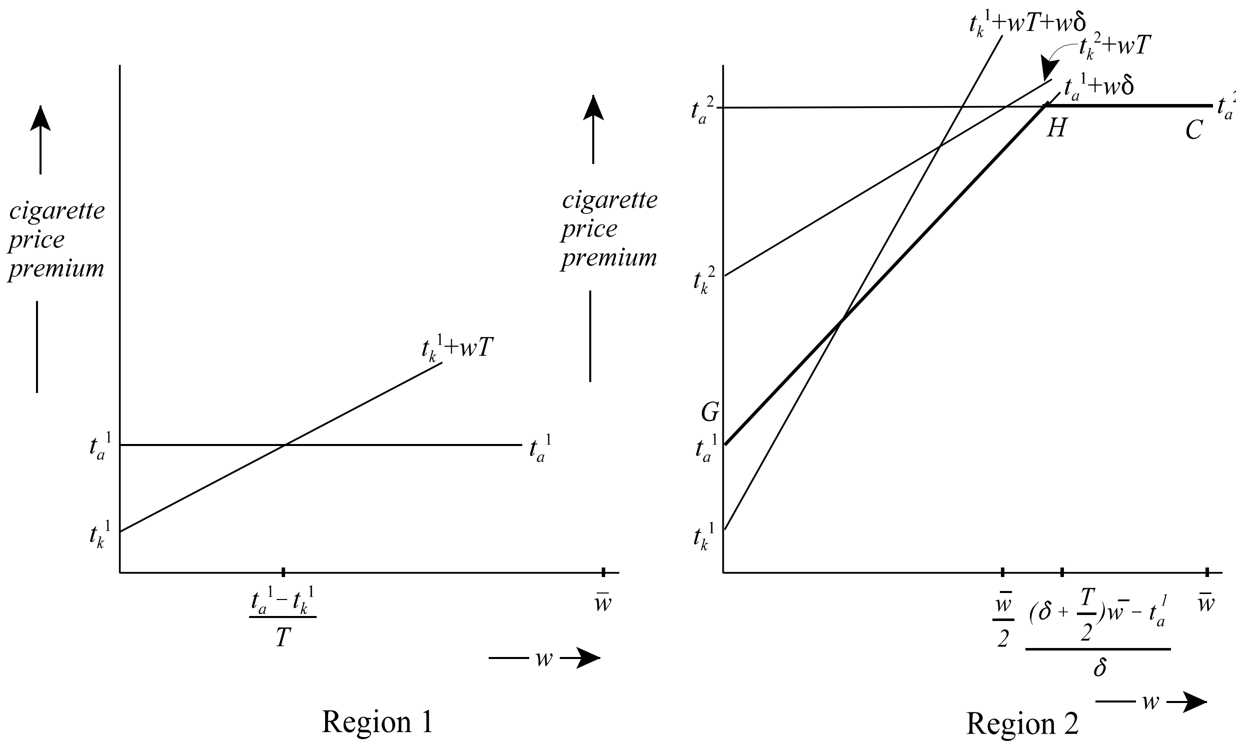


Figure: inclusive price lines for Case A2.(i)

Consider the case $t_k^1 < t_a^1$. If Region 1 increases t_k^1, it would not change its total cigarette sales in either region, but it would cause some households to substitute out of its kits into its pre-assembled cigarettes. With $t_k^1 < t_a^1$, tax revenue increases. Therefore, for this case, Region 1 maximizes tax revenue by setting $t_k^1 = t_a^1$. Region 1's problem is:

$$\max_{t_a^1} R^1 = \int_0^{\bar{w}} t_a^1 \, Q f(w) \, dw + \int_0^{\frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta}} t_a^1 \, Q f(w) \, dw$$

s.t. $$\frac{\bar{w}}{2} \leq \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta};$$

$$\frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta} \leq \bar{w} .$$

Note that the restriction $t_a^1 \leq t_k^2 = \delta\,\bar{w}$ is implied by $\bar{w}/2 \leq ((\delta + T/2)\bar{w} - t_a^1)/\delta$ and is therefore omitted.

Setting $f(w) = 1/\bar{w}$, Region 1's revenue becomes

$$R^1 = \frac{Q}{\bar{w}}\left[t_a^1\bar{w} + t_a^1\,\frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta}\right].$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}}\left[t_a^1\,\bar{w} + t_a^1\,\frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1}{\delta}\right]$$
$$+ A\left(\frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1}{\delta} - \frac{\bar{w}}{2}\right) + B\left(\bar{w} - \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1}{\delta}\right).$$

The Kuhn-Tucker conditions are:

$$\frac{\partial\mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}}\left[\bar{w} - \frac{t_a^1}{\delta} + \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1}{\delta}\right] - \frac{A}{\delta} + \frac{B}{\delta} = 0; \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial A} = \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1}{\delta} - \frac{\bar{w}}{2} \geq 0 \qquad \text{CS} \qquad A \geq 0\ ; \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \bar{w} - \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1}{\delta} \geq 0 \qquad \text{CS} \qquad B \geq 0. \tag{3}$$

Try $A > 0$, $B = 0$. From Equation (2)

$$t_a^1 = \frac{\delta + T}{2}\bar{w}\ .$$

From Equation (1):

$$\frac{A}{\delta} = \frac{Q}{\bar{w}}\left[\bar{w} - \frac{(\delta + T)}{2\delta}\bar{w} + \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - \left(\frac{\delta + T}{2}\right)\bar{w}}{\delta}\right] = \frac{2\delta - T}{2\delta}Q > 0 \text{ as required;}$$

and

$$\frac{\partial \mathcal{L}}{\partial B} = \bar{w} - \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - \frac{\delta + T}{2}\bar{w}}{\delta} = \frac{\bar{w}}{2} \geq 0 \qquad \text{as required.}$$

And tax revenue becomes:

$$R^1 = \frac{Q}{\bar{w}}\left[\frac{\delta + T}{2}\bar{w}\,\bar{w} + \frac{\delta + T}{2}\bar{w}\ \frac{(\delta + \frac{T}{2})\bar{w} - \left(\frac{\delta + T}{2}\right)\bar{w}}{\delta}\right] = \frac{3}{4}(\delta + T)\,Q\,\bar{w}.$$

So $R^1 \leq R^{1*}$ provided $\delta \geq 2T$.

If $\delta < 2T$, $R^1 > R^{1*}$ and $t_k^1 = \delta\,\bar{w}$, $t_a^1 = (\delta + T/2)\bar{w}$ is not a best response for Region 1.

CASE A2.(ii): *the inclusive price line of an imported kit intersects GHC on HC*

The inclusive price lines are shown in the figure below:

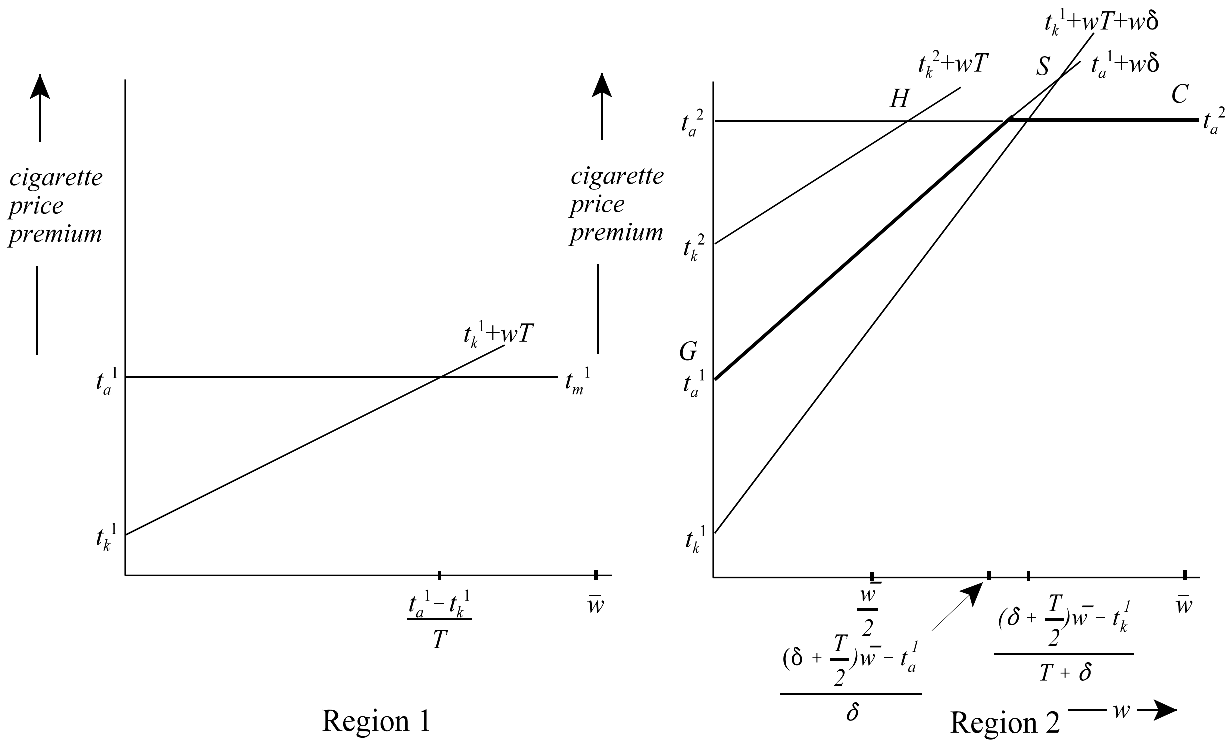


Figure: inclusive price lines for Case A2.(ii)

The inclusive price of an own assembled cigarette and an imported kit is the same for an individual in Region 2 for whom $p + t_a^2 = p + t_k^1 + wT + w\delta$, or , setting $t_a^2 = (\delta + T/2)\bar{w}$, for whom $w = ((\delta + T/2)\bar{w} - t_k^1) / (T + \delta)$.

The tax revenue function of Region 1 depends on kits and assembled product are both sold in Region 1:

$$\frac{t_a^1 - t_k^1}{T} \le \bar{w};$$

OR whether only kits are sold in Region 1:

$$\bar{w} \le \frac{t_a^1 - t_k^1}{T} .$$

I do the former subcase first. Region 1's problem is:

$$\max_{t_a^1,\, t_k^1} R^1 = \int_0^{\frac{t_a^1 - t_k^1}{T}} t_k^1\, Qf(w)\,dw + \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1\, Qf(w)\,dw + \int_0^{\frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta}} t_k^1\, Qf(w)\,dw$$

$$\text{s.t.} \qquad \frac{\bar{w}}{2} \le \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta};$$

$$\frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta} \le \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta};$$

$$\frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_k^1}{T + \delta} \le \bar{w};$$

$$\frac{t_a^1 - t_k^1}{T} \le \bar{w}.$$

Note that the restriction $t_a^1 \le t_k^2$ is implied by the first constraint and $t_k^1 \le t_k^2$ is implied by the second constraint. These restrictions are therefore omitted.

Setting $f(w) = 1/\bar{w}$, Region 1's tax revenue is:

$$R^1 = \frac{Q}{\bar{w}}\left[t_k^1\left(\frac{t_a^1 - t_k^1}{T}\right) + t_a^1\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right) + t_k^1\left(\frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_k^1}{T + \delta}\right)\right].$$

The Lagrangean becomes:

$$\mathfrak{L} = \frac{Q}{\bar{w}}\left[t_k^1\left(\frac{t_a^1 - t_k^1}{T}\right) + t_a^1\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right) + t_k^1\left(\frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_k^1}{T + \delta}\right)\right]$$

$$+ A\left(\frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1}{\delta} - \frac{\bar{w}}{2}\right) + B\left(\frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_k^1}{T + \delta} - \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1}{\delta}\right)$$

$$+ C\left(\bar{w} - \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_k^1}{T + \delta}\right) + D\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right).$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathfrak{L}}{\partial t_k^1} = \frac{Q}{\bar{w}}\left[-\frac{t_k^1}{T} + \frac{t_a^1 - t_k^1}{T} + \frac{t_a^1}{T} - \frac{t_k^1}{T + \delta} + \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_k^1}{T + \delta}\right] - \frac{B}{T + \delta} + \frac{C}{T + \delta} + \frac{D}{T} = 0; \quad (1)$$

$$\frac{\partial \mathfrak{L}}{\partial t_a^1} = \frac{Q}{\bar{w}}\left[\frac{t_k^1}{T} - \frac{t_a^1}{T} + \bar{w} - \frac{t_a^1 - t_k^1}{T}\right] - \frac{A}{\delta} + \frac{B}{\delta} - \frac{D}{T} = 0; \quad (2)$$

$$\frac{\partial \mathfrak{L}}{\partial A} = \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1}{\delta} - \frac{\bar{w}}{2} \geq 0 \qquad \text{CS} \qquad A \geq 0; \quad (3)$$

$$\frac{\partial\mathcal{L}}{\partial B} = \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_k^1}{T + \delta} - \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1}{\delta} \geq 0 \text{ CS} \qquad B \geq 0; \tag{4}$$

$$\frac{\partial\mathcal{L}}{\partial C} = \bar{w} - \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - t_k^1}{T + \delta} \geq 0 \qquad \text{CS} \qquad C \geq 0; \tag{5}$$

$$\frac{\partial\mathcal{L}}{\partial D} = \bar{w} - \frac{t_a^1 - t_k^1}{T} \geq 0 \qquad \text{CS} \qquad D \geq 0. \tag{6}$$

Try $A > 0$; $B > 0$; $C = 0$; $D = 0$. From Equation (3)

$$t_a^1 = \frac{\delta + T}{2}\,\bar{w}.$$

From Equation (4)

$$t_k^1 = \frac{\delta}{2}\,\bar{w}.$$

From Equation (2):

$$\frac{A}{\delta} = \frac{Q}{\bar{w}}\left[\frac{\delta}{2T}\bar{w} - \frac{\delta + T}{2T}\bar{w} + \bar{w} - \frac{\bar{w}}{2}\right] + \frac{B}{\delta} = \frac{B}{\delta} > 0 \qquad \text{as required;}$$

From Equation (1):

$$\frac{B}{T + \delta} = \frac{Q}{\bar{w}}\left[-\frac{\delta}{2T}\bar{w} + \frac{\bar{w}}{2} + \frac{\delta + T}{2T}\bar{w} - \frac{\delta}{2(T + \delta)}\bar{w} + \frac{\bar{w}}{2}\right] = \frac{2\delta + 3T}{2(T + \delta)}\,Q > 0 \text{ as}$$

required;

And

$$\frac{\partial \mathcal{L}}{\partial C} = \bar{w} - \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - \frac{\delta}{2}\bar{w}}{T + \delta} = \frac{\bar{w}}{2} \geq 0 \text{ as required;}$$

And

$$\frac{\partial \mathcal{L}}{\partial D} = \bar{w} - \frac{\frac{\delta + T}{2}\bar{w} - \frac{\delta}{2}\bar{w}}{T} = \frac{\bar{w}}{2} \geq 0 \qquad \text{as required.}$$

And tax revenue is:

$$R^1 = \frac{Q}{\bar{w}}\left[\frac{\delta}{2}\bar{w}\,\frac{\bar{w}}{2} + \frac{\delta + T}{2}\,\bar{w}\,\frac{\bar{w}}{2} + \frac{\delta}{2}\bar{w}\,\frac{\bar{w}}{2}\right] = \frac{3\delta + T}{4}\bar{w}Q < R^{1*}.$$

The alternative subcase has only own kits being sold in Region 1, or

$$\bar{w} \leq \frac{t_a^1 - t_k^1}{T}.$$

The analysis is identical to that of Case A.1.(iii) with the second formulation of $\bar{w} \leq (t_a^1 - t_k^1)/T$.

3.2 CASE B: $t_k^1 \le t_k^2 \le t_a^1 < t_a^2$.

In Region 1: $\min[t_k^1 + wT, t_a^1] \le \min[t_k^2 + wT + w\delta, t_a^2 + w\delta]$, or Region 1 imports no cigarettes. In addition,

$$t_k^2 + wT \le t_a^1 + w\delta:$$

Region 2 does not import assembled cigarettes (but may import kits). In the figure below, ABC is the line showing the lowest inclusive price of own cigarettes in Region 2.

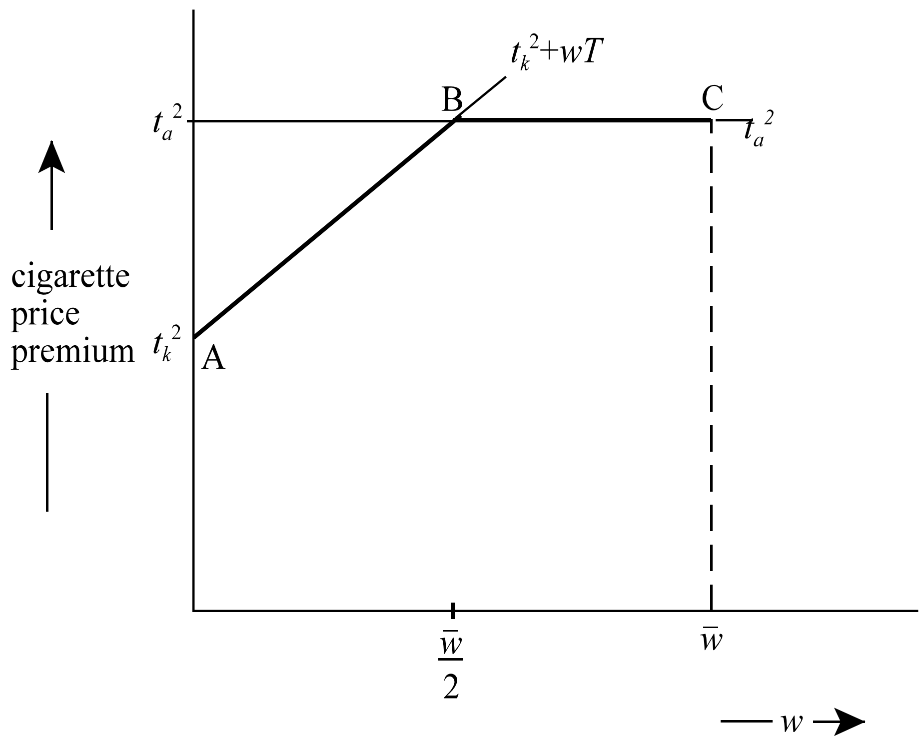


Figure: own inclusive price schedules in Region 2

In Region 2: an individual in Region 2 faces the same inclusive price for an imported kit and an own kit if $p + t_k^1 + wT + w\delta = p + t_k^2 + wT$ or, setting $t_k^2 = \delta\,\bar{w}$, if $w = (\delta\bar{w} - t_k^1)/\delta$. An individual faces the same inclusive price for an imported kit and an own assembled if $p + t_k^1 + wT + w\delta = p + t_a^2$ or, setting $t_a^2 = (\delta + T/2)\bar{w}$, if $w = (\delta + T/2)\bar{w} - t_k^1)/(T + \delta)$.

The price line of an imported kit in Region 2 can intersect ABC as

Case B1: the intersection is on AB. This occurs if $(\delta\bar{w} - t_1^k)/\delta \le \bar{w}/2$;

Case B2: the intersection is on BC. This occurs if $\bar{w}/2 \le (\delta + T/2)\bar{w} - t_k^1)/(T + \delta) \le \bar{w}$.

If the price line of an imported kit lies below C at $w = \bar{w}, p + t_k^1 + \bar{w}T + w\delta \le p + t_a^2$ or, setting $t_a^2 = (\delta + T/2)\bar{w}$, $t_k^1 \le -T\bar{w}/2$. Region 1 is selling kits in Region 2 by subsidizing them: this cannot be a best response of Region 1 and this case is not considered further.

CASE B1: *the inclusive price line of an imported kit intersects ABC on AB.*

The inclusive price lines are shown in the figure below:

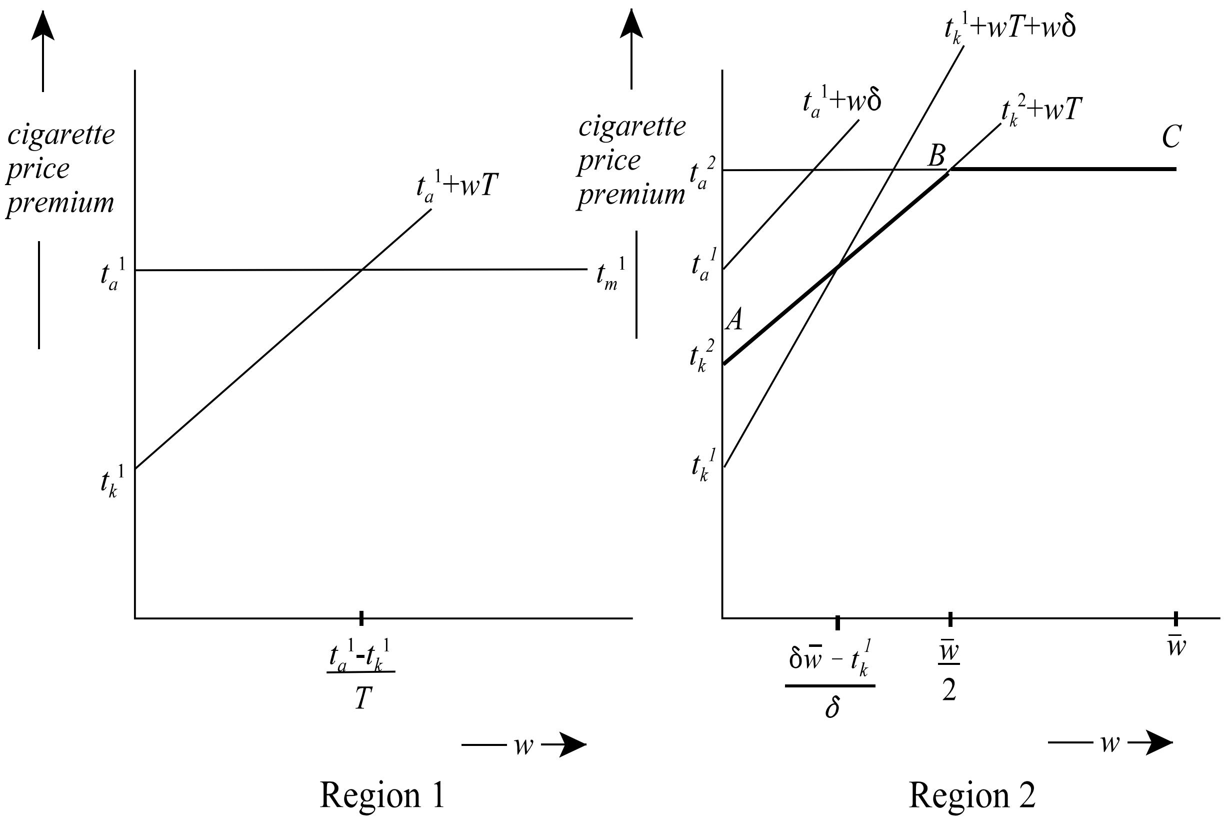


Figure: inclusive price lines in Case B.1

The tax revenue function of Region 1 depends on whether kits and assembled product are both sold in Region 1:

$$\frac{t_a^1 - t_k^1}{T} \leq \bar{w};$$

OR whether only kits are sold in Region 1:

$$\bar{w} \leq \frac{t_a^1 - t_k^1}{T}.$$

I do the first case first. Region 1's problem is:

$$\max_{t_k^1,\, t_a^1} R^1 = \int_0^{\frac{t_a^1 - t_k^1}{T}} t_k^1 \, Q f(w) \, dw + \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1 \, Q f(w) \, dw + \int_0^{\frac{\delta \bar{w} - t_k^1}{\delta}} t_k^1 \, Q f(w) \, dw$$

s.t $\quad t_k^1 \le t_k^2 = \delta \bar{w}$;

$$\delta \bar{w} = t_k^2 \le t_a^1 \; ;$$

$$t_a^1 \le t_a^2 = (\delta + \frac{T}{2}) \bar{w} \; ;$$

$$\frac{\delta \bar{w} - t_k^1}{\delta} \le \frac{\bar{w}}{2} \; ;$$

$$\frac{t_a^1 - t_k^1}{T} \le \bar{w} \, .$$

Setting $f(w) = 1/\bar{w}$, Region 1's tax revenue becomes:

$$R^1 = \frac{Q}{\bar{w}} \left[t_k^1 \left(\frac{t_a^1 - t_k^1}{T} \right) + t_a^1 \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) + t_k^1 \left(\frac{\delta \bar{w} - t_k^1}{\delta} \right) \right].$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}} \left[t_k^1 \left(\frac{t_a^1 - t_k^1}{T} \right) + t_a^1 \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) + t_k^1 \left(\frac{\delta \bar{w} - t_k^1}{\delta} \right) \right]$$

$$+ A \left(\delta \bar{w} - t_k^1 \right) + B \, (t_a^1 - \delta \bar{w}) + C \left((\delta + \frac{T}{2}) \bar{w} - t_a^1 \right) + D \left(\frac{\bar{w}}{2} - \frac{\delta \bar{w} - t_k^1}{\delta} \right)$$

$$+ E\left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right).$$

The Kuhn-Tucker conditions become:

$$\frac{\partial \mathfrak{L}}{\partial t_k^1} = \frac{Q}{\bar{w}} \left[- \frac{t_k^1}{T} + \frac{t_a^1 - t_k^1}{T} + \frac{t_a^1}{T} - \frac{t_k^1}{\delta} + \frac{\delta \bar{w} - t_k^1}{\delta} \right] - A + \frac{D}{\delta} + \frac{E}{T} = 0 ; \qquad (1)$$

$$\frac{\partial \mathfrak{L}}{\partial t_a^1} = \frac{Q}{\bar{w}} \left[\frac{t_k^1}{T} - \frac{t_a^1}{T} + \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) \right] + B - C - \frac{E}{T} = 0 ; \qquad (2)$$

$$\frac{\partial \mathfrak{L}}{\partial A} = \delta \bar{w} - t_k^1 \geq 0 \qquad CS \quad A \geq 0 ; \qquad (3)$$

$$\frac{\partial \mathfrak{L}}{\partial B} = t_a^1 - \delta \bar{w} \geq 0 \qquad CS \quad B \geq 0 ; \qquad (4)$$

$$\frac{\partial \mathfrak{L}}{\partial C} = \left(\delta + \frac{T}{2} \right) \bar{w} - t_a^1 \geq 0 \qquad CS \quad C \geq 0 ; \qquad (5)$$

$$\frac{\partial \mathfrak{L}}{\partial D} = \frac{\bar{w}}{2} - \frac{\delta \bar{w} - t_k^1}{\delta} \geq 0 \qquad CS \quad D \geq 0 ; \qquad (6)$$

$$\frac{\partial \mathfrak{L}}{\partial E} = \bar{w} - \frac{t_a^1 - t_k^1}{T} \geq 0 \qquad CS \quad E \geq 0 . \qquad (7)$$

Try $A = 0$, $B = 0$, $C = 0$, $D = 0$; $E = 0$.

From Equations (1) and (2):

$$t_k^1 = \delta \bar{w} \ ;$$

$$t_a^1 = \left(\delta + \frac{T}{2} \right) \bar{w} \ .$$

And

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - \delta \bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \left(\delta + \frac{T}{2} \right) \bar{w} - \delta \bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial C} = \left(\delta + \frac{T}{2} \right) \bar{w} - \left(\delta + \frac{T}{2} \right) \bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial D} = \frac{\bar{w}}{2} - \frac{\delta \bar{w} - \delta \bar{w}}{\delta} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial E} = \bar{w} - \frac{(\delta + \frac{T}{2})\bar{w} - \delta \bar{w}}{T} \geq 0 \qquad \text{as required.}$$

This is the posited response.

The alternative subcase is that no assembled product is sold in Region 1, $\bar{w} \leq (t_a^1 - t_k^1)/T$.

This case is identical to the second subcase of Case A.1.(ii).

CASE B2: *the inclusive price line of an imported kit intersects ABC on BC*

The inclusive price lines for this case are shown below:

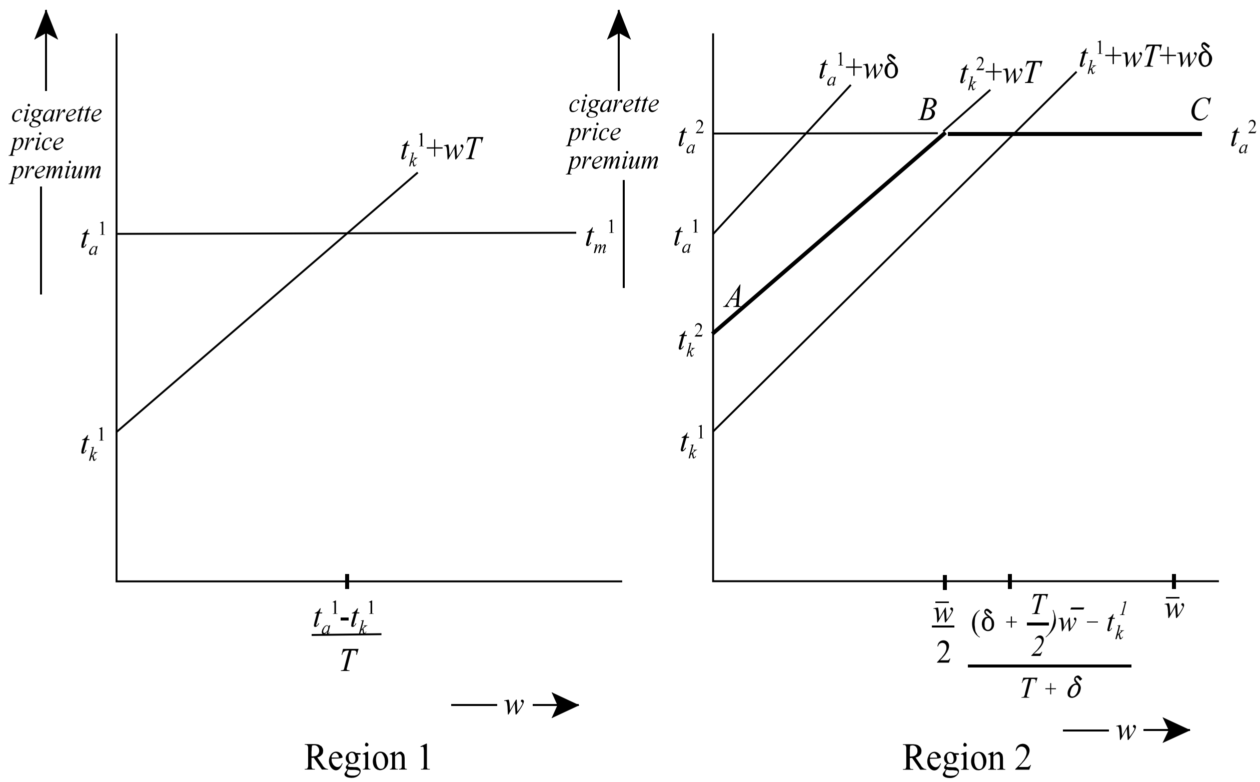


Figure: inclusive price lines for Case B.2

The individual for whom the price of own assembled and imported kits are the same has a wage such that $p + t_a^2 = p + t_k^1 + wT + w\delta$, or (setting $t_a^2 = (\delta + T/2)\bar{w}$) has a wage

$$w = ((\delta + T/2)\bar{w} - t_k^1) / (T + \delta).$$

The tax revenue function of Region 1 depends on whether kits and assembled product are both sold in Region 1:

$$\frac{t_a^1 - t_k^1}{T} \leq \bar{w};$$

OR whether only kits are sold in Region 1:

$$\bar{w} \le \frac{t_a^1 - t_k^1}{T}.$$

The former case is not a possible case if $\delta > 2T$. The case requires

$$\delta \bar{w} = t_k^2 \le t_a^1$$

and

$$\frac{\bar{w}}{2} \le \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} \quad \text{or} \quad t_k^1 \le \frac{\delta}{2}\bar{w}$$

These two inequalities imply

$$\frac{t_a^1 - t_k^1}{T} > \frac{\delta \bar{w} - \frac{\delta}{2}\bar{w}}{T} = \frac{\delta}{2T}\bar{w} > \bar{w}$$

which contradicts the assumption that $(t_a^1 - t_k^1)/T \le \bar{w}$.

The alternative subcase is that no assembled product is sold in Region 1, $\bar{w} \le (t_a^1 - t_k^1)/T$. This case is identical to the second subcase of Case A.1.(ii).

3.3 CASE C: $t_k^1 \le t_k^2 \le t_a^2 < t_a^1$.

In Region 1: $t_k^1 + wT \le t_k^2 + wT + w\delta$ and (with $\delta \ge T$) $t_k^1 + wT \le t_a^2 + w\delta$, so Region 1 imports no cigarettes. However, Region 1 may export kits to Region 2.

The inclusive price of an own cigarette in Region 2 varies with an individual's wage as the bold line ABC in the figure below:

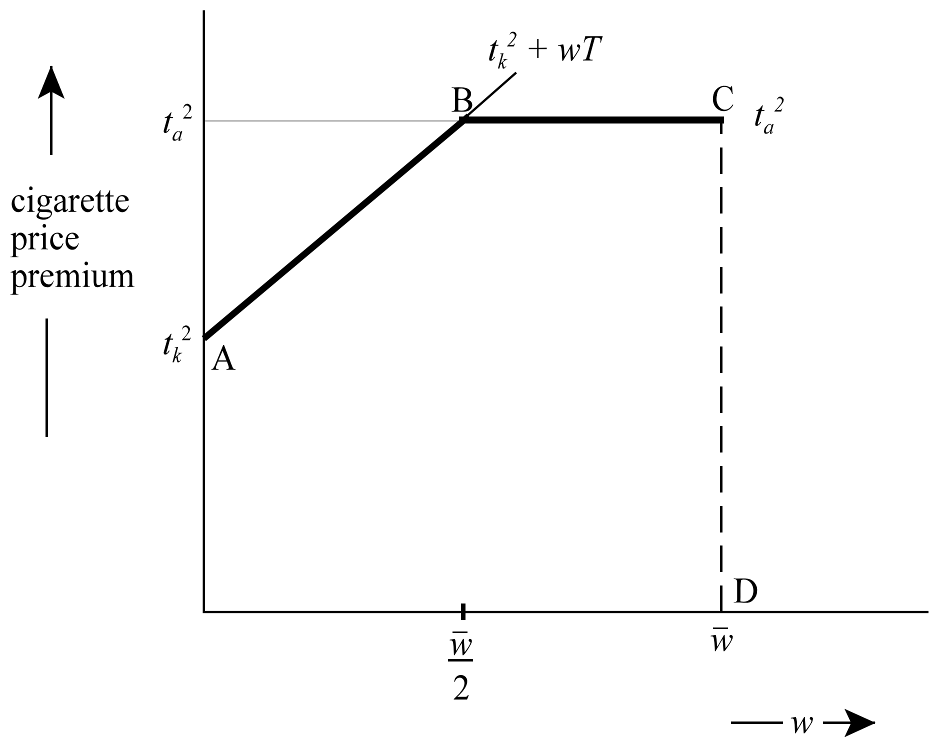


Figure: inclusive price of own cigarettes in Region 2

In Region 2: an individual faces the same inclusive price for an imported kit and an own kit if $p + t_k^1 + wT + w\delta = p + t_k^2 + wT$ or, setting $t_k^2 = \delta\bar{w}$, if $w = (\delta\bar{w} - t_k^1)/\delta$. An individual in Region 2 faces the same inclusive price for an imported kit and an own assembled if $p + t_k^1 + wT + w\delta = p + t_a^2$ or, setting $t_a^2 = (\delta + T/2)\bar{w}$, if $w = (\delta + T/2)\bar{w} - t_k^1)/(T + \delta)$.

The price line of an imported kit in Region 2 can intersect ABC as

Case C1: the intersection is on AB. This occurs if $(\delta\bar{w} - t_1^k)/\delta \leq \bar{w}/2$;

Case C2: the intersection is on BC . This occurs if

$\bar{w}/2 \leq (\delta + T/2)\bar{w} - t_k^1)/(T + \delta) \leq \bar{w}$.

If the price line of an imported kit lies below C at $w = \bar{w}, p + t_k^1 + \bar{w}T + w\delta \leq p + t_a^2$ or, setting $t_a^2 = (\delta + T/2)\bar{w}$, $t_k^1 \leq -T\bar{w}/2$. Region 1 is selling kits in Region 2 by subsidizing them: this cannot be a best response of Region 1 and this case is not considered further.

CASE C1: *the inclusive price line of an imported kit intersects ABC on AB*

The inclusive price lines for this case are shown in the figure below:

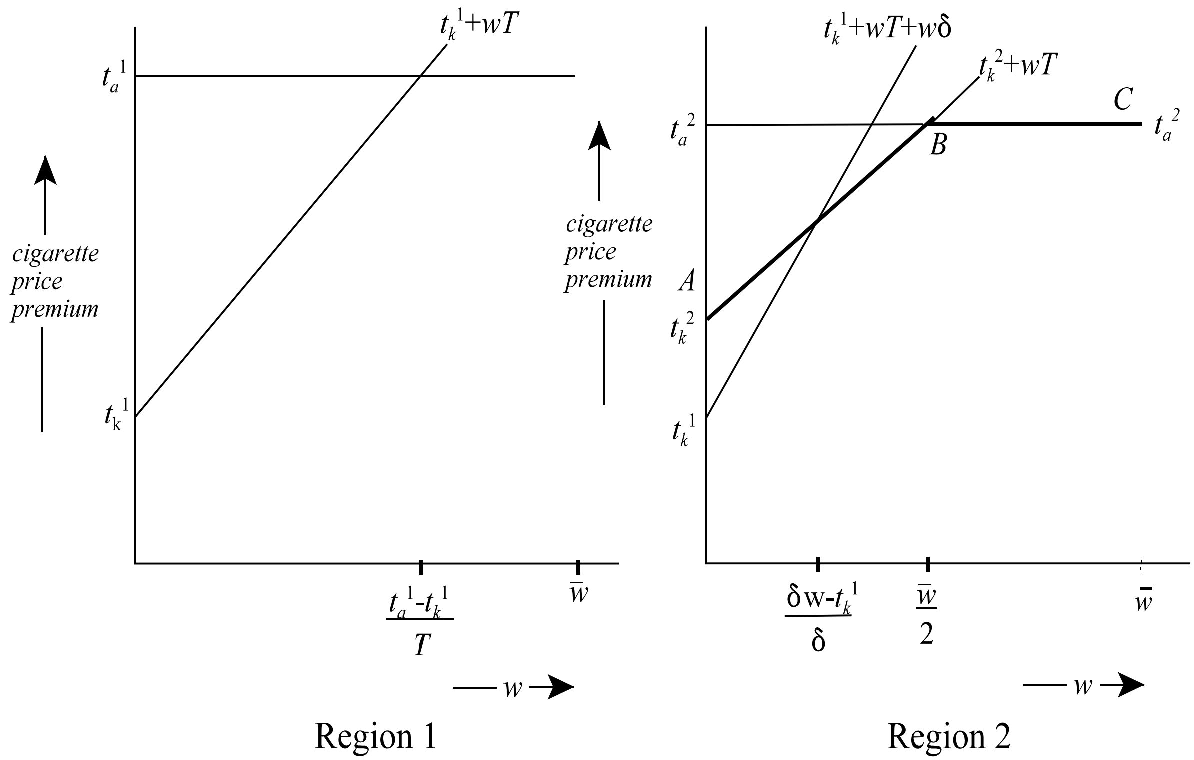


Figure: inclusive price lines for Case C1

The tax revenue function of Region 1 depends on whether kits and assembled product are both sold in Region 1:

$$\frac{t_a^1 - t_k^1}{T} \le \bar{w};$$

OR whether only kits are sold in Region 1:

$$\bar{w} \le \frac{t_a^1 - t_k^1}{T}.$$

I do the first case first. Region 1's problem is:

$$\max_{t_k^1,\, t_a^1} R^1 = \int_0^{\frac{t_a^1 - t_k^1}{T}} t_k^1\, Qf(w)\,dw + \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1\, Qf(w)\,dw + \int_0^{\frac{\delta\bar{w} - t_k^1}{\delta}} t_k^1\, Qf(w)\,dw$$

$$\text{s. t.} \qquad t_k^1 \le t_k^2 = \delta\bar{w}\ ;$$

$$(\delta + \frac{T}{2})\bar{w} = t_a^2 \le t_a^1\ ;$$

$$\frac{\delta\bar{w} - t_k^1}{\delta} \le \frac{\bar{w}}{2}\ ;$$

$$\frac{t_a^1 - t_k^1}{T} \le \bar{w}\ .$$

Setting $f(w) = 1/\bar{w}$, tax revenue is

$$R^1 = \frac{Q}{\bar{w}}\left[t_k^1\left(\frac{t_a^1 - t_k^1}{T}\right) + t_a^1\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right) + t_k^1\left(\frac{\delta\bar{w} - t_k^1}{\delta}\right)\right].$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}}\left[t_k^1\left(\frac{t_a^1 - t_k^1}{T}\right) + t_a^1\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right) + t_k^1\left(\frac{\delta\bar{w} - t_k^1}{\delta}\right)\right]$$

$$+ A\left(\delta\bar{w} - t_k^1\right) + B\left(t_a^1 - \left(\delta + \frac{T}{2}\right)\bar{w}\right) + C\left(\frac{\bar{w}}{2} - \frac{\delta\bar{w} - t_k^1}{\delta}\right) + D\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right).$$

The Kuhn-Tucker conditions are:

$$\frac{\partial\mathcal{L}}{\partial t_k^1} = \frac{Q}{\bar{w}}\left[-\frac{t_k^1}{T} + \frac{t_a^1 - t_k^1}{T} + \frac{t_a^1}{T} - \frac{t_k^1}{\delta} + \frac{\delta\bar{w} - t_k^1}{\delta}\right] - A + \frac{C}{\delta} + \frac{D}{T} = 0\ ; \qquad (1)$$

$$\frac{\partial \mathfrak{L}}{\partial t_a^1} = \frac{Q}{\bar{w}} \left[\frac{t_k^1}{T} - \frac{t_a^1}{T} + \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) \right] + B - \frac{D}{T} = 0 \; ; \tag{2}$$

$$\frac{\partial \mathfrak{L}}{\partial A} = \bar{w} - t_k^1 \geq 0 \qquad CS \quad A \geq 0 \; ; \tag{3}$$

$$\frac{\partial \mathfrak{L}}{\partial B} = t_a^1 - \left(\delta + \frac{T}{2} \right) \bar{w} \geq 0 \qquad CS \quad B \geq 0 \; ; \tag{4}$$

$$\frac{\partial \mathfrak{L}}{\partial C} = \frac{\bar{w}}{2} - \frac{\delta \bar{w} - t_k^1}{\delta} \geq 0 \qquad CS \quad C \geq 0 \; ; \tag{5}$$

$$\frac{\partial \mathfrak{L}}{\partial D} = \frac{\bar{w}}{2} - \frac{t_a^1 - t_k^1}{T} \geq 0 \qquad CS \quad D \geq 0 \; . \tag{6}$$

Try *A=0; B = 0; C = 0;* and *D= 0.*

Equations (1) and (2) imply:

$$t_k^1 = \delta \bar{w} \; ;$$

$$t_k^1 = (\delta + \frac{T}{2}) \bar{w} \; .$$

And

$$\frac{\partial \mathfrak{L}}{\partial A} = \delta \bar{w} - \delta \bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathfrak{L}}{\partial B} = (\delta + \frac{T}{2}) \bar{w} - (\delta + \frac{T}{2}) \bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathfrak{L}}{\partial C} = \frac{\bar{w}}{2} - 0 \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathfrak{L}}{\partial D} = \bar{w} - \frac{\bar{w}}{2} \geq 0 \qquad \text{as required.}$$

This is the same response as the proposed best response. $R^1 = R^{1*}$.

In the alternative subcase in which $\bar{w} \leq (t_a^1 - t_k^1)/T$, Region 1 is selling in assembled product and the setting of t_a^1 is irrelevant. The second subcase of Case A.1.(iii) refers

CASE C2: *the inclusive price line of an imported kit intersects ABC on BC*

The inclusive price lines for this case are shown in the figure below:

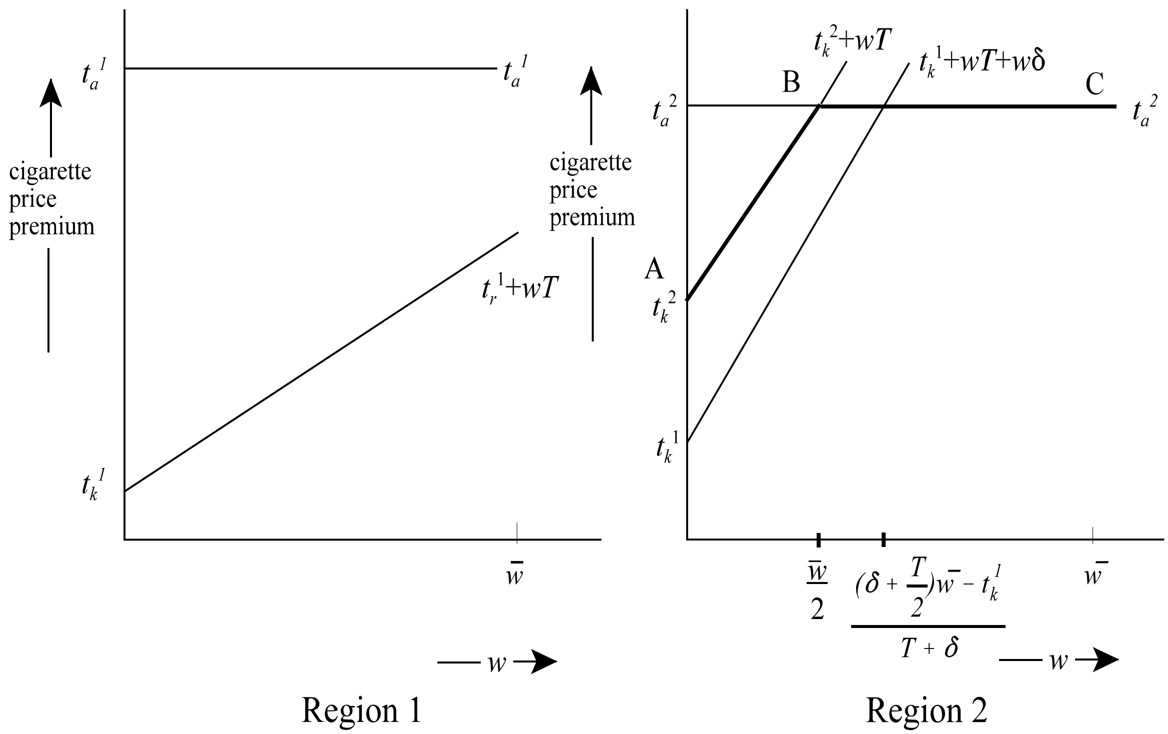


Figure: inclusive price lines for Case C2

The tax revenue function of Region 1 depends on whether kits and assembled product are both sold in Region 1:

$$\frac{t_a^1 - t_k^1}{T} \le \bar{w} \, ;$$

OR whether only kits are sold in Region 1:

$$\bar{w} \le \frac{t_a^1 - t_k^1}{T} \, .$$

The former case is not a possible case if $\delta > T$. The case requires that

$$(\delta + \frac{T}{2})\bar{w} = t_a^2 \le t_a^1$$

and that the inclusive price of the imported kit intersects the envelope on BC, or

$$\frac{\bar{w}}{2} \le \frac{(\delta + \frac{T}{2})\bar{w} - t_k^1}{T + \delta} \quad \text{or} \quad t_k^1 \le \frac{\delta}{2}\bar{w} .$$

These two inequalities imply

$$\frac{t_a^1 - t_k^1}{T} > \frac{(\delta + \frac{T}{2})\bar{w} - \frac{\delta}{2}\bar{w}}{T} = \frac{\delta + T}{2T}\bar{w} > \bar{w}$$

which contradicts the assumption that $(t_a^1 - t_k^1) / T \le \bar{w}$.

In the alternative case that $\bar{w} \le (t_a^1 - t_k^1) / T$, Region 1 is selling no assembled product and the setting of t_k^1 is irrelevant. The second subcase of Case A.1.(iii) applies.

3.4 CASE D: $t_k^2 \le t_k^1 \le t_a^1 < t_a^2$.

In Region 2, $t_k^2 + wT \le \min[t_k^1 + wT + w\delta, t_a^1 + w\delta]$ and Region 1 does not export any cigarettes. However, Region 1 may import kit sales. The bold line ABC in the figure below shows how the inclusive price of a kit in Region 1 varies with an individual's wage:

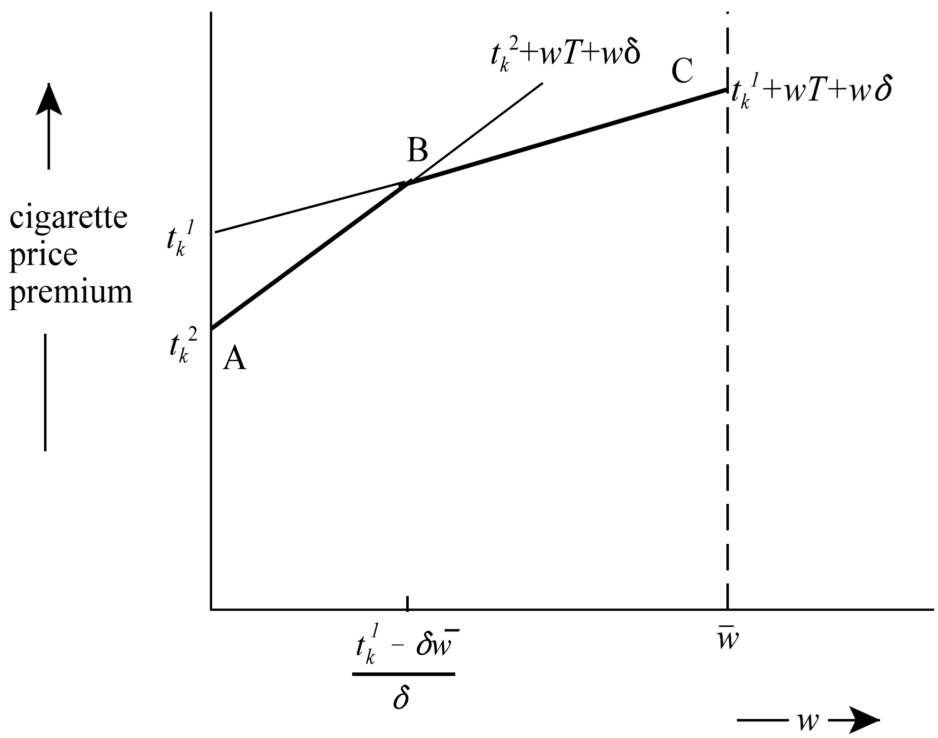


Figure: inclusive price lines for imported kits and own kits in Region 1.

In Region 1: an individual faces the same price for an own kit and an imported kit if $p + t_k^1 + wT = p + t_k^2 + wT + w\delta$ or, setting $t_k^2 = \delta\bar{w}$, if $w = (t_k^1 - \delta\bar{w})/\delta$. I note that

$t_k^1 \le t_a^2 = (\delta + T/2)\bar{w}$ ensures that $(t_k^1 - \delta\bar{w})/\delta < \bar{w}$ (as drawn).

An individual faces the same price for an own assembled and an imported kit if $p + t_a^1 = p + t_k^2 + wT + w\delta$ or, setting $t_k^2 = \delta\bar{w}$, if $w = (t_a^1 - \delta\bar{w})/(T + \delta)$. An individual faces the same price for an own assembled and an own kit $p + t_a^1 = p + t_k^1 + wT$ or if

$w = (t_a^1 - t_k^1)/T$. The price line of an own assembled can intersect ABC as:

Case D1: the intersection is on AB. This occurs if: $(t_a^1 - \delta\bar{w})/(T + \delta) \le (t_k^1 - \delta\bar{w})/\delta$.

Case D2: the intersection is on BC. This occurs if $(t_k^1 - \delta\bar{w})/\delta \le (t_a^1 - t_k^1)/T$.

Note that $t_a^1 \le t_a^2 = (\delta + T/2)\bar{w}$ and $\delta\bar{w} = t_k^2 \le t_k^1$ implies that at $w = \bar{w}$

$$t_a^1 \le (\delta + \frac{T}{2})\bar{w} < \delta\bar{w} + \bar{w}T \le t_k^1 + \bar{w}T$$

or at $w = \bar{w}$, the inclusive price line for own assembled product must lie below the inclusive price line for own kits so that the intersection must lie to the left of C.

The cases are considered in turn.

CASE D1: *the inclusive price line of an own assembled intersects ABC on AB*

The inclusive price lines in Region 1 are shown in the figure below

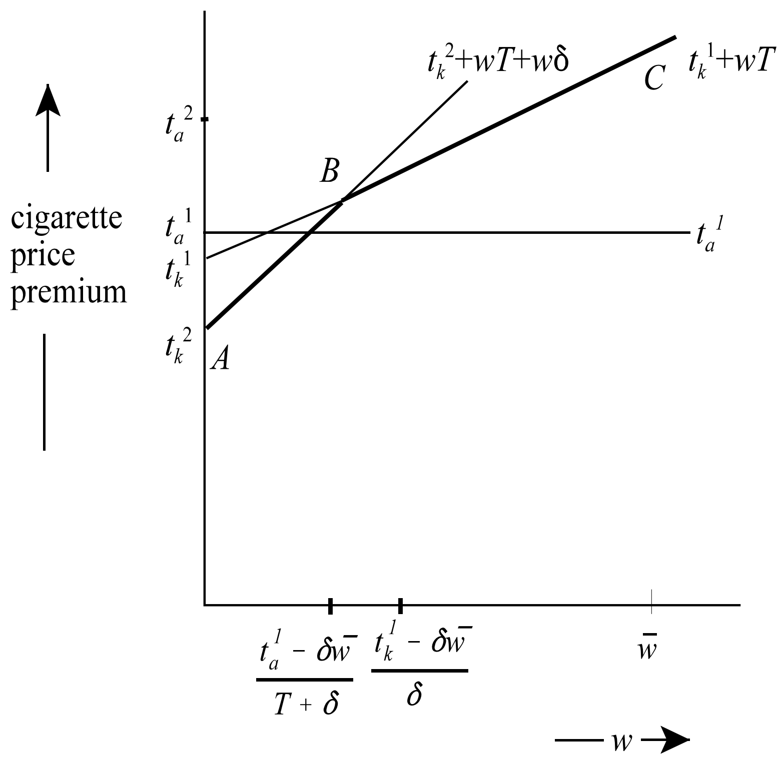


Figure : inclusive price lines in Region 1 in Case D.1

No kits taxed in Region 1 are sold in Region 1. Therefore t_k^1 is not a relevant choice variable and can be ignored. Region 1's problem is:

$$\max_{t_a^1} R^1 = \int_{\frac{t_a^1-\delta\bar{w}}{T+\delta}}^{\bar{w}} t_a^1 \, Q f(w) dw .$$

s.t. $\quad \delta\bar{w} = t_k^2 \leq t_a^1$;

$$t_a^1 \le (\delta + \frac{T}{2})\bar{w} .$$

Setting $f(w) = 1/\bar{w}$ and integrating, the tax revenue in Region 1 is:

$$R^1 = \frac{Q}{\bar{w}} t_a^1 \left(\bar{w} - \frac{t_a^1 - \delta\bar{w}}{T+\delta} \right).$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}} t_a^1 \left(\bar{w} - \frac{t_a^1 - \delta\,\bar{w}}{T+\delta} \right) + A \left(t_a^1 - \delta\,\bar{w} \right) + B \left(\left(\delta + \frac{T}{2} \right) \bar{w} - t_a^1 \right).$$

The Kuhn-Tucker conditions are:

$$\frac{\partial\mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}} \left[- \frac{t_a^1}{T+\delta} - \left(\bar{w} - \frac{t_a^1 - \delta\bar{w}}{T+\delta} \right) \right] + A - B = 0;$$

$$\frac{\partial\mathcal{L}}{\partial A} = t_a^1 - \delta\,\bar{w} \ge 0 \qquad\qquad CS \quad A \ge 0;$$

$$\frac{\partial\mathcal{L}}{\partial B} = \left(\delta + \frac{T}{2} \right) \bar{w} - t_a^1 \ge 0 \qquad\qquad CS \quad B \ge 0.$$

Try $A = 0$ and $B = 0$.

From Equation (1):

$$t_a^1 = \left(\delta + \frac{T}{2} \right) \bar{w}.$$

And

$$\frac{\partial \mathcal{L}}{\partial A} = \left(\delta + \frac{T}{2}\right)\bar{w} - \delta\bar{w} = \frac{T}{2}\bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \left(\delta + \frac{T}{2}\right)\bar{w} - \left(\delta + \frac{T}{2}\right)\bar{w} = 0 \geq 0 \qquad \text{as required;}$$

And tax revenue:

$$R^1 = \frac{Q}{\bar{w}}\left(\delta + \frac{T}{2}\right)\bar{w}\left(\bar{w} - \frac{\left(\delta + \frac{T}{2}\right)\bar{w} - \delta\bar{w}}{T+\delta}\right) = Q\bar{w}\,\frac{(T+2\delta)^2}{4(T+\delta)} < R^{1*}\,.$$

CASE D2: *the inclusive price line of an own assembled ABC on BC*

The inclusive price lines are shown in the figure below:

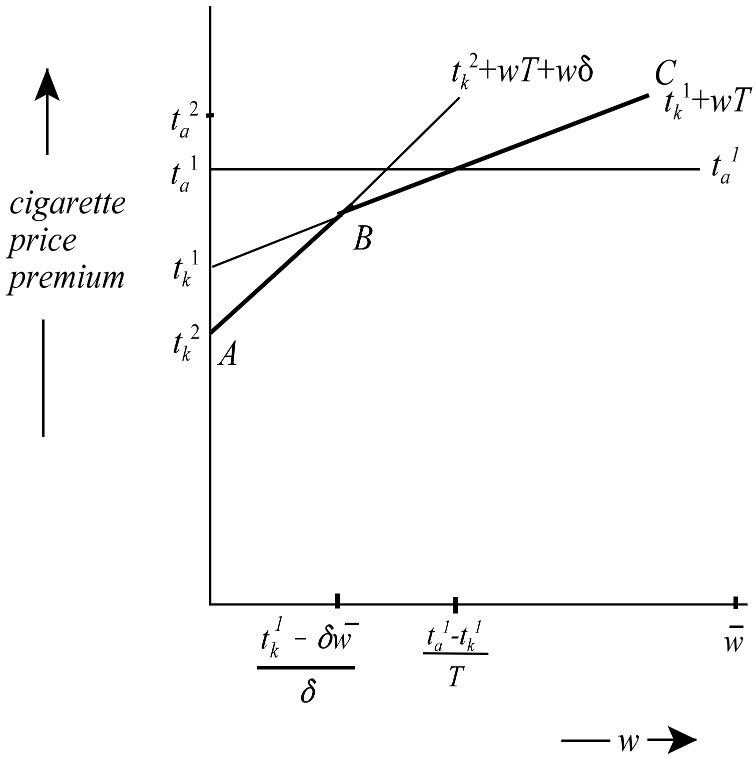


Figure: the inclusive price lines in Region 1 in Case D.2

Region 1's problem is:

$$\max_{t_k^1,\, t_a^1} R^1 = \int_{\frac{t_k^1-\delta\bar{w}}{\delta}}^{\frac{t_a^1-t_k^1}{T}} t_k^1\, Q f(w)\, dw + \int_{\frac{t_a^1-t_k^1}{T}}^{\bar{w}} t_a^1\, Q f(w)\, dw$$

s.t. $\delta\bar{w} = t_k^2 \le t_k^1$;

$t_a^1 \le t_a^2 = (\delta + \frac{T}{2})\bar{w}$;

$$\frac{t_k^1 - \delta\bar{w}}{\delta} \leq \frac{t_a^1 - t_k^1}{T} .$$

Setting $f(w) = 1/\bar{w}$, and integrating, tax revenue in Region 1 is:

$$R^1 = \frac{Q}{\bar{w}} \left[t_k^1 \left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\,\bar{w}}{\delta} \right) + t_a^1 \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) \right] .$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}} \left[t_k^1 \left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\,\bar{w}}{\delta} \right) + t_a^1 \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) \right]$$

$$+ A \left(t_k^1 - \delta\bar{w} \right) + B \left(\left(\delta + \frac{T}{2} \right) \bar{w} - t_a^1 \right) + C \left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\bar{w}}{\delta} \right) .$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial t_k^1} = \frac{Q}{\bar{w}} \left[t_k^1 \left(-\frac{1}{T} - \frac{1}{\delta} \right) + \left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\bar{w}}{\delta} \right) + \frac{t_a^1}{T} \right] + A - \frac{C}{T} - \frac{C}{\delta} = 0 \,; \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}} \left[\frac{t_k^1}{T} - \frac{t_a^1}{T} + \bar{w} - \frac{t_a^1 - t_k^1}{T} \right] - B + \frac{C}{T} = 0 \,; \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial A} = t_k^1 - \delta\bar{w} \geq 0 \qquad CS \quad A \geq 0 \,; \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial B} = \left(\delta + \frac{T}{2}\right)\bar{w} - t_a^1 \geq 0 \qquad CS \quad B \geq 0\ ; \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\bar{w}}{\delta} \geq 0 \qquad CS \quad C \geq 0\ . \tag{5}$$

Try $A = 0, \quad B = 0$ and $C = 0$.

From Equations (1) and (2):

$$t_k^1 = \delta\bar{w}$$

and

$$t_a^1 = \left(\delta + \frac{T}{2}\right)\bar{w}.$$

And

$$\frac{\partial \mathcal{L}}{\partial A} = 0 \geq 0 \qquad \text{as required.}$$

From (2):

$$\frac{\partial \mathcal{L}}{\partial B} = 0 \geq 0 \qquad \text{as required}$$

And

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{(\delta + \frac{T}{2})\bar{w} - \delta\,\bar{w}}{T} - \frac{\delta\,\bar{w} - \delta\bar{w}}{\delta} = \frac{\bar{w}}{2} \geq 0 \text{ as required.}$$

This is the posited response, and

$$R^1 = R^{1*}\ .$$

3.5 CASE E: $t_k^2 \le t_k^1 \le t_a^2 \le t_a^1$.

In Region 2: $t_k^2 + wT \le t_k^1 + wT + w\delta$ and $t_a^2 \le t_a^1 + w\delta$ and hence Region 1 does not export to Region 2. In Region 1: $t_k^1 + wT \le t_a^2 + w\delta$, or Region 1 may import kits but not assembled product. The bold line ABC in the figure below shows how the inclusive price of a kit in Region 1 varies with an individual's wage:

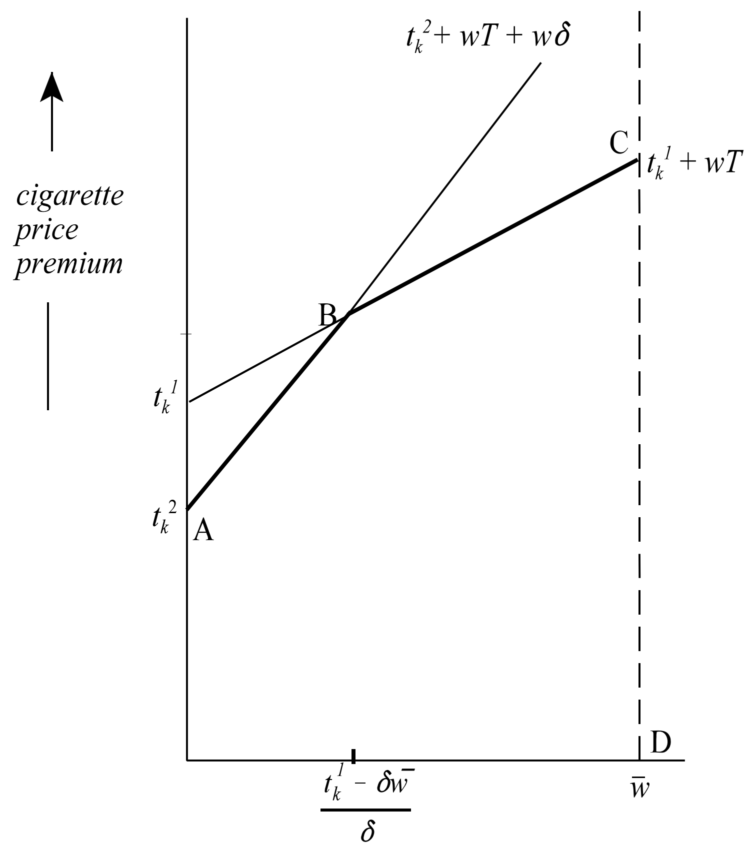


Figure: inclusive price lines for imported kits, own kits and imported assembled in Region 1

In Region 1: an individual faces the same inclusive price for an own kit and an imported kit if $p + t_k^1 + wT = p + t_k^2 + wT + w\delta$ or, setting $t_k^2 = \delta\bar{w}$, if $w = (t_k^1 - \delta\bar{w})/\delta$. Note that

$t_k^1 \le t_a^2 = (\delta + T/2)\bar{w}$ ensures that $(t_k^1 - \delta\bar{w})/\delta < \bar{w}$ (as drawn).

An individual faces the same price for an own assembled and an imported kit if $p + t_a^1 = p + t_k^2 + wT + w\delta$ or, setting $t_k^2 = \delta\bar{w}$, if $w = (t_a^1 - \delta\bar{w})/(T + \delta)$. An individual faces the same price for an own assembled and an own kit $p + t_a^1 = p + t_k^1 + wT$ or if $w = (t_a^1 - t_k^1)/T$. The price line of an own assembled cigarette in Region 1 can intersect ABC as:

Case E1: the intersection is on AB. This occurs if $(t_a^1 - \delta\bar{w})/(T + \delta) \le (t_k^1 - \delta\bar{w})/\delta$.

Case E2: the intersection is on BC. This occurs if $(t_k^1 - \delta\bar{w})/\delta \le (t_k^1 - t_k^1)/T$.

I note that if the price line of an own assembled cigarette in Region1 lies above C at $w = \bar{w}$, $t_a^1 > t_k^1 + \bar{w}T$, no assembled cigarettes are sold in Region 1. This setting of t_a^1 cannot be a best response: if t_a^1 is lowered so that high wage individuals shift from own low-tax kits to high-tax assembled product, tax revenue increases.

The cases are considered in turn.

CASE E1: *the inclusive price line of an own assembled cigarette intersects ABC on AB*

The inclusive price lines for this case are shown in the figure below:

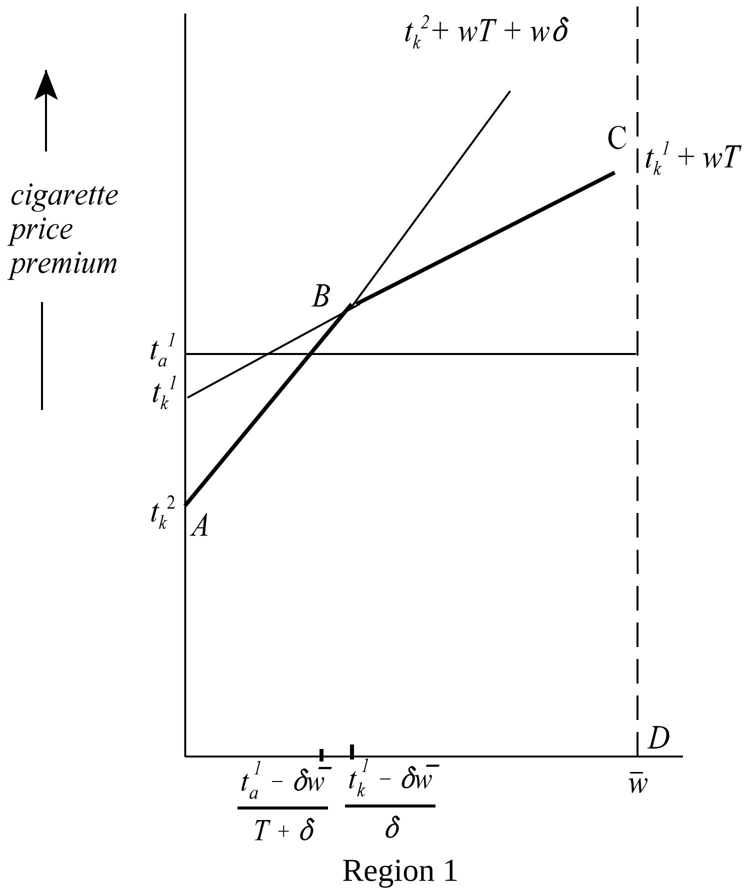


Figure: inclusive price lines in Region 1 in Case E1.

No own kits are sold in Region 1. Region 1's problem is:

$$\max_{t_k^1,\, t_a^1} R^1 = \int_{\frac{t_a^1 - \delta\bar{w}}{T+\delta}}^{\bar{w}} t_a^1 \, Q f(w)\, dw$$

s.t. $\delta\bar{w} = t_k^2 \le t_k^1$;

$$t_k^1 \;\le\; t_a^2 \;=\; (\delta \;+\; \frac{T}{2})\,\bar{w} \;;$$

$$(\delta \;+\; \frac{T}{2})\bar{w} \;=\; t_a^2 \;\le\; t_a^1 \;;$$

$$\frac{t_a^1 - \delta\bar{w}}{T + \delta} \;\le\; \frac{t_k^1 - \delta\bar{w}}{\delta} \;.$$

Setting $f(w) \;=\; 1/\bar{w}$, tax revenue in Region 1 is

$$R^1 \;=\; \frac{Q}{\bar{w}}\; t_a^1 \left(\bar{w} \;-\; \frac{t_a^1 - \delta\bar{w}}{T + \delta} \right) .$$

The Lagrangean is:

$$\mathcal{L} \;=\; \frac{Q}{\bar{w}}\; t_a^1 \left(\bar{w} \;-\; \frac{t_a^1 - \delta\bar{w}}{T + \delta} \right) \;+\; A(t_k^1 \;-\; \delta\bar{w}) \;+\; B((\delta \;+\; \frac{T}{2})\bar{w} - t_k^1)$$

$$+\; C\left(t_a^1 - (\delta \;+\; \frac{T}{2})\bar{w} \right) \;+\; D\left(\frac{t_k^1 \;-\; \delta\bar{w}}{\delta} \;-\; \frac{t_a^1 \;-\; \delta\bar{w}}{T \;+\; \delta} \right) .$$

The Kuhn-Tucker conditions are:

$$\frac{\partial\mathcal{L}}{\partial t_k^1} \;=\; A \;-\; B \;+\; \frac{D}{\delta} \;=\; 0 \;; \tag{1}$$

$$\frac{\partial\mathcal{L}}{\partial t_a^1} \;=\; \frac{Q}{\bar{w}} \left[-\; \frac{t_a^1}{T + \delta} \;+\; \bar{w} \;-\; \frac{t_a^1 \;-\; \delta\bar{w}}{T + \delta} \right] \;+\; C \;-\; \frac{D}{T \;+\; \delta} \;=\; 0 \;; \tag{2}$$

$$\frac{\partial\mathcal{L}}{\partial A} \;=\; t_k^1 \;-\; \delta\bar{w} \;\ge\; 0 \qquad\qquad CS \quad A \;\ge\; 0 \;; \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \bar{w}\left(\delta + \frac{T}{2}\right) - t_k^1 \geq 0 \qquad CS \quad B \geq 0 \; ; \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial C} = t_a^1 - (\delta + \frac{T}{2})\,\bar{w} \geq 0 \qquad CS \quad C \geq 0 \; ; \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial D} = \frac{t_k^1 - \delta\bar{w}}{\delta} - \frac{t_a^1 - \delta\bar{w}}{T + \delta} \geq 0 \qquad CS \quad D \geq 0 \; . \tag{6}$$

Try $A = 0, \quad B = 0, \quad C = 0$ and $D = 0$.

From Equation (2):

$$t_a^1 = (\delta + \frac{T}{2})\bar{w}.$$

Choose t_k^1 such that

$$\frac{\delta\,(2\delta + 3T)}{2(T + \delta)} \leq t_k^1 \leq (\delta + \frac{T}{2})\bar{w}.$$

Equation (1) becomes:

$$\frac{\partial \mathcal{L}}{\partial t_k^1} = 0 \qquad \text{as required .}$$

And

$$\frac{\partial \mathcal{L}}{\partial A} \geq 0 \qquad \text{as required ;}$$

$$\frac{\partial \mathcal{L}}{\partial B} \geq 0 \qquad \text{as required ;}$$

$$\frac{\partial \mathcal{L}}{\partial C} \geq 0 \qquad \text{as required ;}$$

$$\frac{\partial \mathcal{L}}{\partial D} \geq 0 \qquad \text{as required .}$$

And the tax revenue of Region 1 is:

$$R^1 = \frac{Q}{\bar{w}}\left(\delta + \frac{T}{2}\right)\bar{w}\left[\bar{w} - \frac{(\delta + \frac{T}{2})\bar{w} - \delta\bar{w}}{T + \delta}\right] = \left(\delta + \frac{T}{2}\right)^2 \frac{1}{T + \delta} Q\bar{w} < R^{1*}.$$

CASE E2: *the inclusive price line of own assembled cigarette intersects ABC on BC*

The inclusive price lines in Region 1 are shown in the figure below:

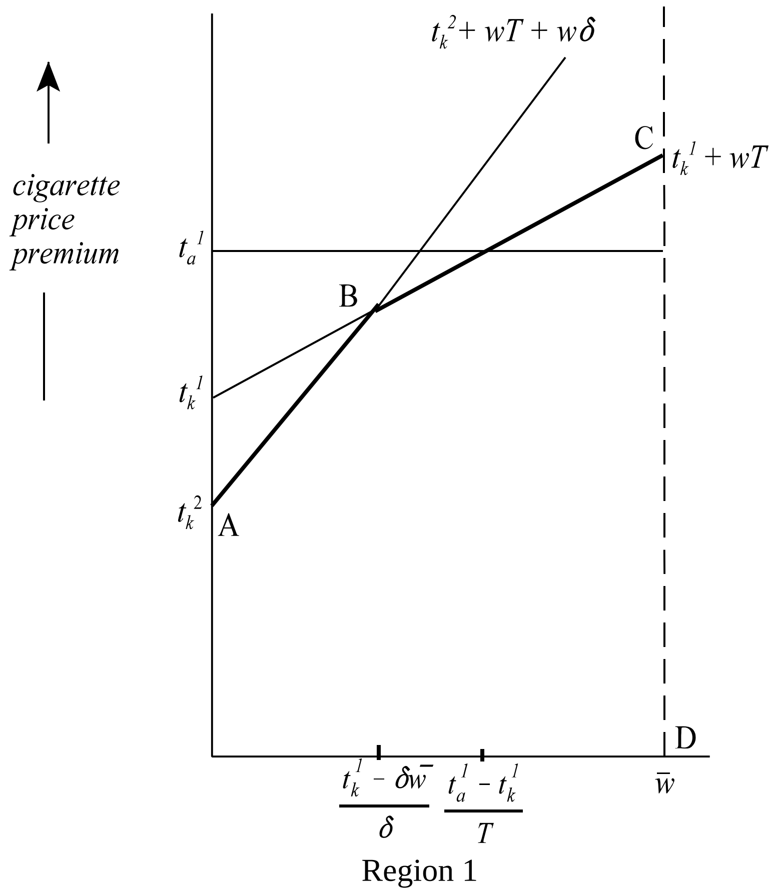


Figure: inclusive price lines in Region 1 for Case E2

The discussion at the bottom of the introduction to Case E informs that Region 1 does not want to set t_a^1 so high that only kits are sold in Region 1 or so that $(t_a^1 - t_k^1)/T \geq \bar{w}$. Therefore the potential restriction $(t_a^1 - t_k^1)/T \leq \bar{w}$ is not tight and can be ignored. Region 1's problem is:

$$\max_{t_k^1,\, t_a^1} R^1 = \int_{\frac{t_k^1 - \delta\bar{w}}{\delta}}^{\frac{t_a^1 - t_k^1}{T}} t_k^1\, Q f(w)\, dw + \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1\, Q f(w)\, dw$$

$$\text{s.t.} \qquad \delta \bar{w} \;=\; t_k^2 \;\le\; t_k^1 \; ;$$

$$t_k^1 \;\le\; t_a^2 \;=\; (\delta + \frac{T}{2})\bar{w} \; ;$$

$$(\delta + \frac{T}{2})\bar{w} \;=\; t_a^2 \;\le\; t_a^1 \; ;$$

$$\frac{t_k^1 - \delta \bar{w}}{\delta} \;\le\; \frac{t_a^1 - t_k^1}{T} \; ;$$

Writing $f(w) = 1/\bar{w}$ and integrating, the tax revenue becomes:

$$R^1 \;=\; \frac{Q}{\bar{w}} \left[t_k^1 \left(\frac{t_a^1 - t_k^1}{T} \;-\; \frac{t_k^1 - \delta \bar{w}}{\delta} \right) \;+\; t_a^1 \left(\bar{w} \;-\; \frac{t_a^1 - t_k^1}{T} \right) \right].$$

The Lagrangean is:

$$\mathcal{L} \;=\; \frac{Q}{\bar{w}} \left[t_k^1 \left(\frac{t_a^1 - t_k^1}{T} \;-\; \frac{t_k^1 - \delta \bar{w}}{\delta} \right) \;+\; t_a^1 \left(\bar{w} \;-\; \frac{t_a^1 - t_k^1}{T} \right) \right]$$

$$+\; A \left(t_k^1 - \delta \bar{w} \right) \;+\; B \left((\delta + \frac{T}{2})\bar{w} \;-\; t_k^1 \right) \;+\; C \left(t_a^1 - \left(\delta + \frac{T}{2} \right) \bar{w} \right)$$

$$+\; D \left(\frac{t_a^1 - t_k^1}{T} \;-\; \frac{t_k^1 - \delta \bar{w}}{\delta} \right).$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial t_k^1} \;=\; \frac{Q}{\bar{w}} \left[t_k^1 \left(-\frac{1}{T} - \frac{1}{\delta} \right) + \left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta \bar{w}}{\delta} \right) + \frac{t_a^1}{T} \right] + A - B + D \left(-\frac{1}{T} - \frac{1}{\delta} \right) \;=\; 0 \; ; \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}} \left[\frac{t_k^1}{T} - \frac{t_a^1}{T} + \bar{w} - \frac{t_a^1 - t_k^1}{T} \right] + C + \frac{D}{T} = 0; \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial A} = t_k^1 - \delta \bar{w} \geq 0 \qquad CS \quad A \geq 0; \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \left(\delta + \frac{T}{2} \right) \bar{w} - t_k^1 \geq 0 \qquad CS \quad B \geq 0; \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial C} = t_a^1 - \left(\delta + \frac{T}{2} \right) \bar{w} \geq 0 \qquad CS \quad C \geq 0; \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial D} = \frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta \bar{w}}{\delta} \geq 0 \qquad CS \quad D \geq 0. \tag{6}$$

Try $A = 0$, $B = 0$, $C = 0$ and $D = 0$.

From Equations (1) and (2):

$$t_k^1 = \delta \ \bar{w}$$

and

$$t_a^1 = \left(\frac{T}{2} + \delta \right) \bar{w}.$$

And:

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - \delta \bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \left(\delta + \frac{T}{2} \right) \bar{w} - \delta \bar{w} = \frac{T}{2} \bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial C} = \left(\delta + \frac{T}{2} \right) \bar{w} - \left(\delta + \frac{T}{2} \right) \bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial D} \;=\; \frac{\bar{w}\left(\dfrac{T}{2} + \delta\right) \;-\; \bar{w}\delta}{T} \;-\; \frac{\bar{w}\delta - \bar{w}\delta}{\delta} \;=\; \frac{\bar{w}}{2} \;\geq\; 0 \qquad \text{as required.}$$

This is the posited response and tax revenue in Region 1 is

$$R^1 \;=\; R^{1*} \;.$$

3.6 CASE F: $t_k^2 \le t_a^2 \le t_k^1 \le t_a^1$.

In Region 2, $t_k^2 + wT \le t_k^1 + wT + w\delta$ and $t_a^2 \le t_a^1 + w\delta$, or Region 1 does not export. But Region 1 does import product from Region 2. An individual in Region 1 faces the same inclusive price for imported kits and imported pre-assembled product if

$p + t_k^2 + wT + w\delta = p + t_a^2 + w\delta$ or, setting $t_k^2 = \delta\bar{w}$ and $t_a^2 = (\delta + T/2)\bar{w}$, if $w = \bar{w}/2$. The bold line ABC in the figure below shows the inclusive price for an imported cigarette in Region 1:

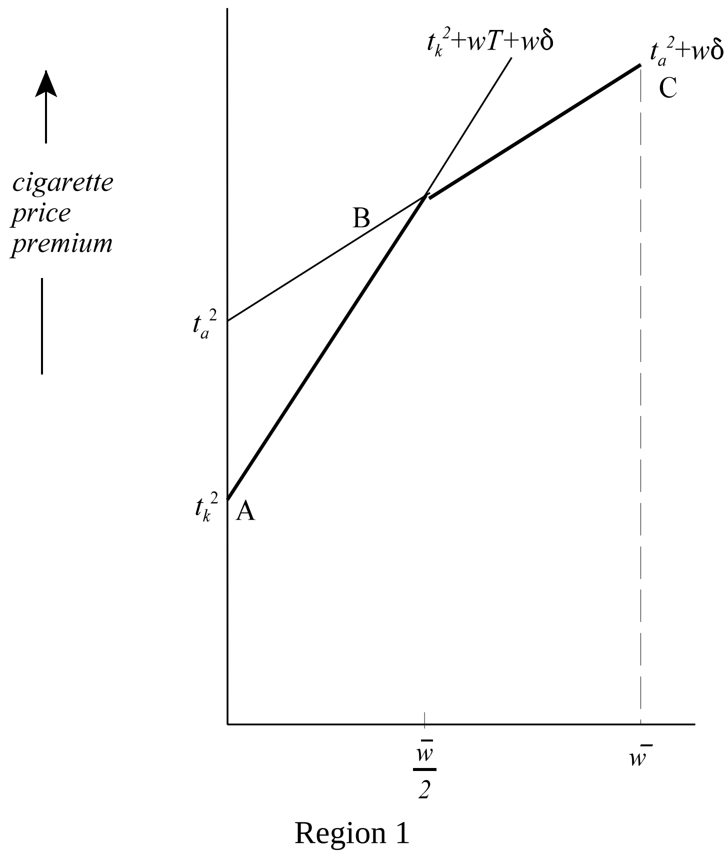


Figure: inclusive price of imported product in Region 1

In Region 1: an individual faces the same inclusive price for an own assembled and an imported kit if $p + t_a^1 = p + t_k^2 + wT + w\delta$ or, setting $t_k^2 = \delta\bar{w}$, if

$$w = \frac{t_a^1 - \delta\bar{w}}{T + \delta}.$$

An individual faces the same inclusive price for an own assembled and an imported assembled if $p + t_a^1 = p + t_a^2 + w\delta$ or, setting $t_a^2 = (\delta + T/2)\bar{w}$, if

$$w = \frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta}$$

The price line of an own assembled cigarette can intersect the envelope ABC as

Case F1: the intersection is on AB. This occurs if $(t_a^1 - \delta\bar{w}) / (T + \delta) \leq \bar{w}/2$;

Case F2: the intersection is on BC. This occurs if $\bar{w}/2 \leq (t_a^1 - (\delta + T/2)\bar{w}) / \delta$

Note that if the inclusive price of an own assembled cigarette intersects ABC at C or lies above ABC at C, Region 1 is selling no own assembled cigarettes. This setting of t_a^1 cannot be a best response: if t_a^1 is lowered so that some high-wage individuals shift from lower-tax own kits to higher-tax assembled product (or from imported product to own assembled product), tax revenue must increase.

CASES F1: *inclusive price line of an own assembled cigarette intersects ABC on AB*

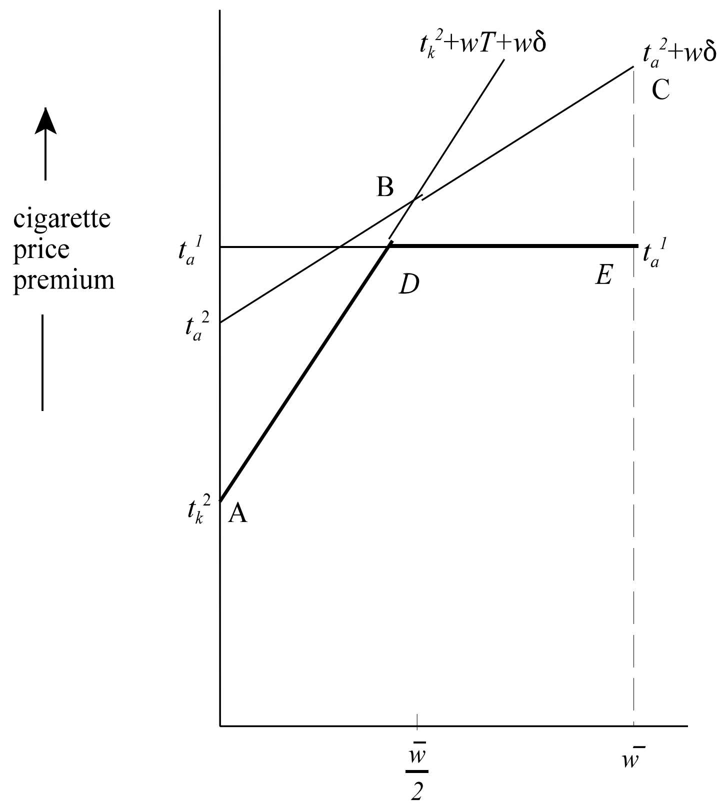


Figure: some inclusive price lines in Region 1 in Case F1

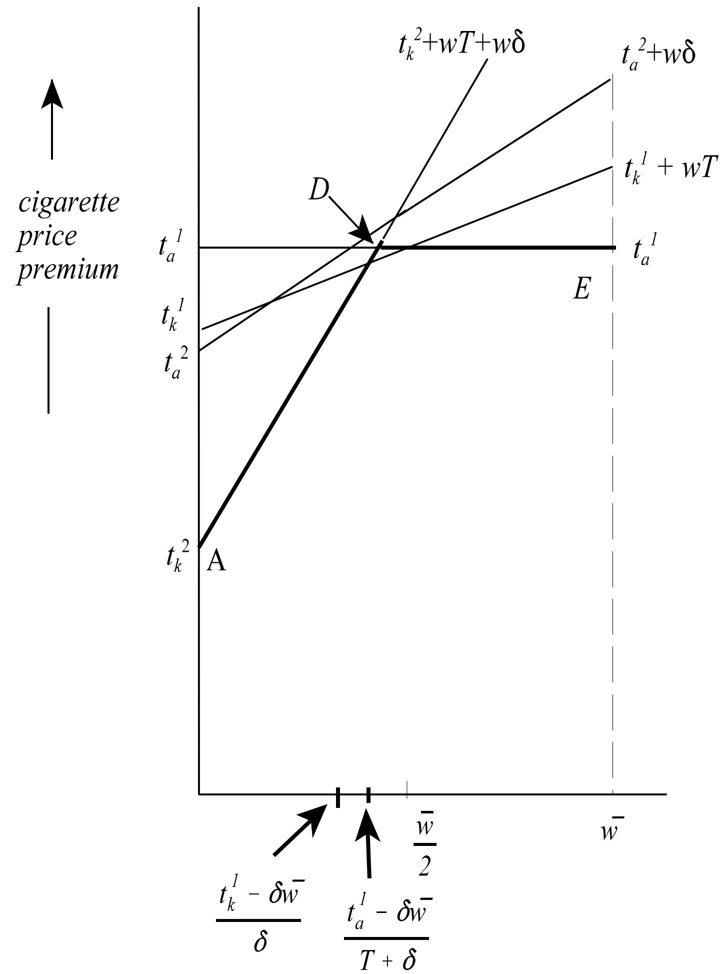
The lower envelope of the inclusive price lines for an imported kits, an imported assembled product and an own assembled product is ADE. There are two possible subcases which are characterized by the intersection of the inclusive price line of an own kit with ADE:

Case F1.(i) the intersection is on AD.

Case F1.(ii) the intersection lies above ADE[1]

CASE F1.(i): *the inclusive price line of an own kit intersects ADE on AD* .

The inclusive price lines for this case are shown in the figure below:



Region 1

Figure: inclusive price lines in region1 in Case F1.(i)

In Region 1: an individual faces the same inclusive price for own kits and imported kits if $p + t_k^1 + wT = t_k^2 + wT + w\delta$ or, setting $t_k^2 = \delta\bar{w}$, if $w = (t_k^1 - \delta\bar{w}) / \delta$. An individual faces the same price of an own assembled and an imported kit if $p + t_a^1 = p + t_k^2 + wT + w\delta$ or, setting

$t_k^2 = \delta\bar{w}$, if $w = (t_a^1 - \delta\bar{w}) / (T + \delta)$. An individual in Region 1 faces the same inclusive price of an own kit and of an own assembled product if $p + t_k^1 + wT = p + t_a^1$ or if $w = (t_a^1 - t_k^1) / T$. Conceptually I should include the constraint $(t_a^1 - t_k^1)/T \leq \bar{w}$ but this constraint does not bind: if Region 1 lowers the tax rate t_a^1 to relax the constraint, some high-wage individuals shift from lower-tax kits to higher-tax assembled product and tax revenue increases.

Region 1's problem is:

$$\max_{t_k^1,\, t_a^1} R^1 = \int_{\frac{t_k^1 - \delta\bar{w}}{\delta}}^{\frac{t_a^1 - t_k^1}{T}} t_k^1\, Q f(w)\, dw + \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1\, Q f(w)\, dw$$

s.t. $$(\delta + \frac{T}{2})\bar{w} = t_a^2 \leq t_k^1 ;$$

$$t_k^1 \leq t_a^1 ;$$

$$\frac{t_k^1 - \delta\bar{w}}{\delta} \leq \frac{t_a^1 - \delta\bar{w}}{T + \delta} ;$$

$$\frac{t_a^1 - \delta\bar{w}}{T + \delta} \leq \frac{\bar{w}}{2} .$$

Setting $f(w) = 1/\bar{w}$ and integrating, tax revenue in Region 1 is:

$$R^1 = \frac{Q}{\bar{w}}\left[t_k^1 \left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\bar{w}}{\delta} \right) + t_a^1 \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) \right] .$$

The Lagrangean is:

$$\mathfrak{L} = \frac{Q}{\bar{w}}\left[t_k^1\left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\bar{w}}{\delta}\right) + t_a^1\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right)\right] + A\left(t_k^1 - (\delta + \frac{T}{2})\bar{w}\right)$$

$$+ B(t_a^1 - t_k^1) + C\left(\frac{t_a^1 - \delta\bar{w}}{T+\delta} - \frac{t_k^1 - \delta\bar{w}}{\delta}\right) + D\left(\frac{\bar{w}}{2} - \frac{t_a^1 - \delta\bar{w}}{T+\delta}\right) .$$

The Kuhn-Tucker Conditions are:

$$\frac{\partial\mathfrak{L}}{\partial t_k^1} = \frac{Q}{\bar{w}}\left[-\frac{t_k^1}{T} - \frac{t_k^1}{\delta} + \frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\bar{w}}{\delta} + \frac{t_a^1}{T}\right] + A - B - \frac{C}{\delta} = 0 ; \tag{1}$$

$$\frac{\partial\mathfrak{L}}{\partial t_a^1} = \frac{Q}{\bar{w}}\left[\frac{t_k^1}{T} - \frac{t_a^1}{T} + \bar{w} - \frac{t_a^1 - t_k^1}{T}\right] + B + \frac{C}{T+\delta} - \frac{D}{T+\delta} = 0; \tag{2}$$

$$\frac{\partial\mathfrak{L}}{\partial A} = t_k^1 - (\delta + \frac{T}{2})\bar{w} \geq 0 \qquad CS \quad A \geq 0; \tag{3}$$

$$\frac{\partial\mathfrak{L}}{\partial B} = t_a^1 - t_k^1 \geq 0 \qquad CS \quad B \geq 0; \tag{4}$$

$$\frac{\partial\mathfrak{L}}{\partial C} = \frac{t_a^1 - \delta\bar{w}}{T+\delta} - \frac{t_k^1 - \delta\bar{w}}{\delta} \geq 0 \qquad CS \quad C \geq 0; \tag{5}$$

$$\frac{\partial\mathfrak{L}}{\partial D} = \frac{\bar{w}}{2} - \frac{t_a^1 - \delta\bar{w}}{T+\delta} \geq 0 \qquad CS \quad D \geq 0. \tag{6}$$

Try $A > 0$, $B = 0$, $C = 0$ and $D = 0$.

From Equation (3):

$$t_k^1 = (\delta + \frac{T}{2})\bar{w}.$$

From Equation (2):

$$t_a^1 = (\delta + T)\bar{w}$$

From Equation (1):

$$A = \frac{T}{\delta}Q > 0 \quad \text{as required}$$

And

$$\frac{\partial \mathcal{L}}{\partial B} = (\delta + T)\bar{w} - (\delta + \frac{T}{2})\bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{(\delta + T)\bar{w} - \delta\bar{w}}{T + \delta} - \frac{(\delta + \frac{T}{2})\bar{w} - \delta\bar{w}}{\delta} = \frac{T(\delta - T)}{2\delta(T + \delta)}\bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial D} = \frac{\bar{w}}{2} - \frac{(T + \delta)\bar{w} - \delta\bar{w}}{T + \delta} = \frac{\delta - T}{2(T + \delta)}\bar{w} \geq 0 \qquad \text{as required.}$$

And tax revenue in Region 1 is:

$$R^1 = \frac{4\delta^2 + \delta T - T^2}{4\delta}Q\bar{w} < R^{1*}$$

CASE F1.(ii): *the inclusive price line of own kit lies above ADE* .

The inclusive price lines for this case are shown in the figure below:

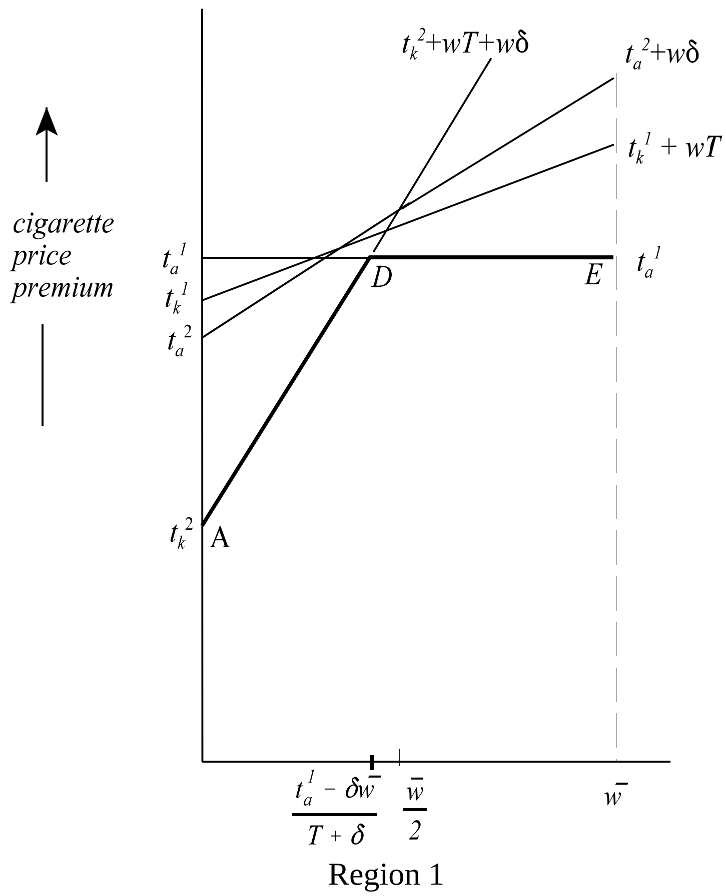


Figure: inclusive price lines in Region 1 in Case F1.(ii)

No own kits are sold in Region 1 so that the setting of t_k^1 is not relevant. Region 1's problem is:

$$\max_{t_a^1} R^1 = \int_{\frac{t_a^1-\delta\bar{w}}{T+\delta}}^{\bar{w}} t_a^1 \, Q f(w) dw$$

$$\text{s.t.} \qquad (\delta + \frac{T}{2}) \bar{w} = t_a^2 \leq t_a^1 ;$$

$$\frac{t_a^1 - \delta \bar{w}}{T + \delta} \leq \frac{\bar{w}}{2} .$$

Setting $f(w) = 1/\bar{w}$, tax revenue in Region 1 is

$$R^1 = \frac{Q}{\bar{w}} t_a^1 \left(\bar{w} - \frac{t_a^1 - \delta \bar{w}}{T + \delta} \right) .$$

The Lagrangean is

$$\mathcal{L} = \frac{Q}{\bar{w}} t_a^1 \left(\bar{w} - \frac{t_a^1 - \delta \bar{w}}{T + \delta} \right) + A \left(t_a^1 - (\delta + \frac{T}{2})\bar{w} \right) + B \left(\frac{\bar{w}}{2} - \frac{t_a^1 - \delta \bar{w}}{T + \delta} \right) .$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}} \left[- \frac{t_a^1}{T + \delta} + \bar{w} - \frac{t_a^1 - \delta \bar{w}}{T + \delta} \right] + A - \frac{B}{T + \delta} = 0 ; \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial A} = t_k^1 - (\delta + \frac{T}{2})\bar{w} \geq 0 \qquad CS \quad A \geq 0 ; \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\bar{w}}{2} - \frac{t_a^1 - \delta \bar{w}}{T + \delta} \geq 0 \qquad CS \quad B \geq 0 . \tag{3}$$

Try $A = 0$ and $B = 0$.

From Equation (1):

$$t_a^1 = (\delta + \frac{T}{2})\bar{w};$$

And

$$\frac{\partial \mathcal{L}}{\partial A} = 0 \geq 0 \qquad \text{as required;}$$

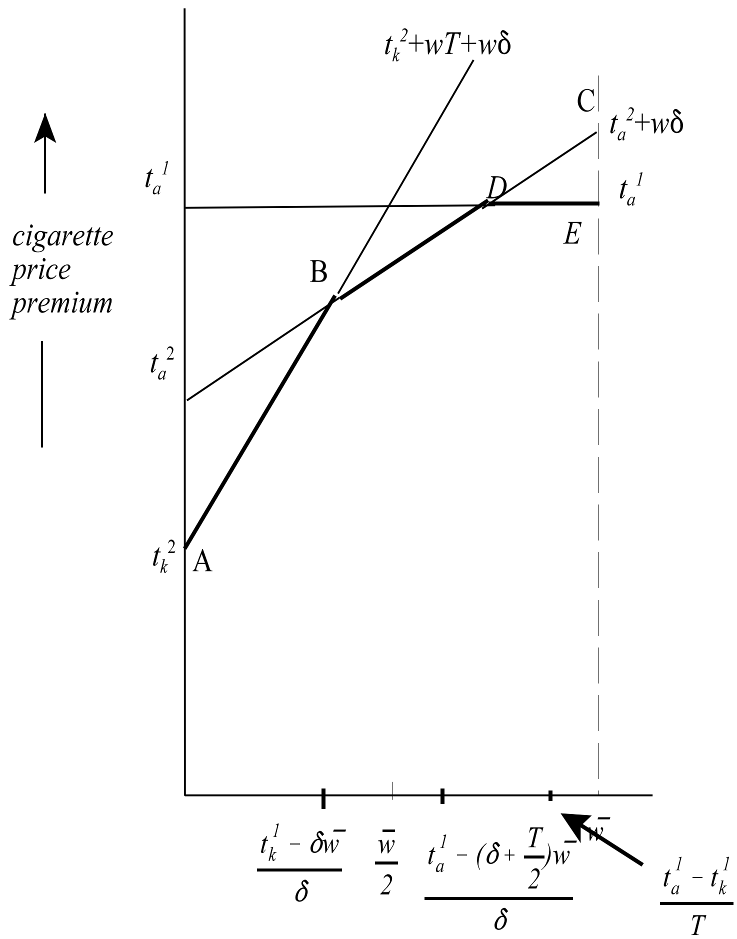
$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\bar{w}}{2} - \frac{(\delta + \frac{T}{2})\bar{w} - \delta \bar{w}}{T + \delta} = \frac{\delta}{2(T + \delta)} \bar{w} \geq 0 \text{ as required;}$$

And tax revenue in Region 1 is:

$$R^{1} = \frac{Q}{\bar{w}}\left(\delta + \frac{T}{2}\right)\bar{w}\left[\bar{w} - \frac{(\delta + \frac{T}{2})\bar{w} - \delta\bar{w}}{T + \delta}\right] = \left(\delta + \frac{T}{2}\right)^{2}\frac{1}{T + \delta}Q\bar{w} < R^{1*}.$$

CASES F2: *the inclusive price line of an own assembled cigarette intersects ABC on BC*

The inclusive price lines for imported kits, imported assembled and own assembled are shown in the figure below:



Region 1

Figure: inclusive price lines in Region 1 for Cases F

The envelope of the inclusive price of imported kits, imported assembled and own assembled is shown as the bold line ABDE in the figure above. There are three subcases which are characterized by the intersection of the inclusive price line of the own kit with ABDE:

Case F2.(i): the intersection is on AB;

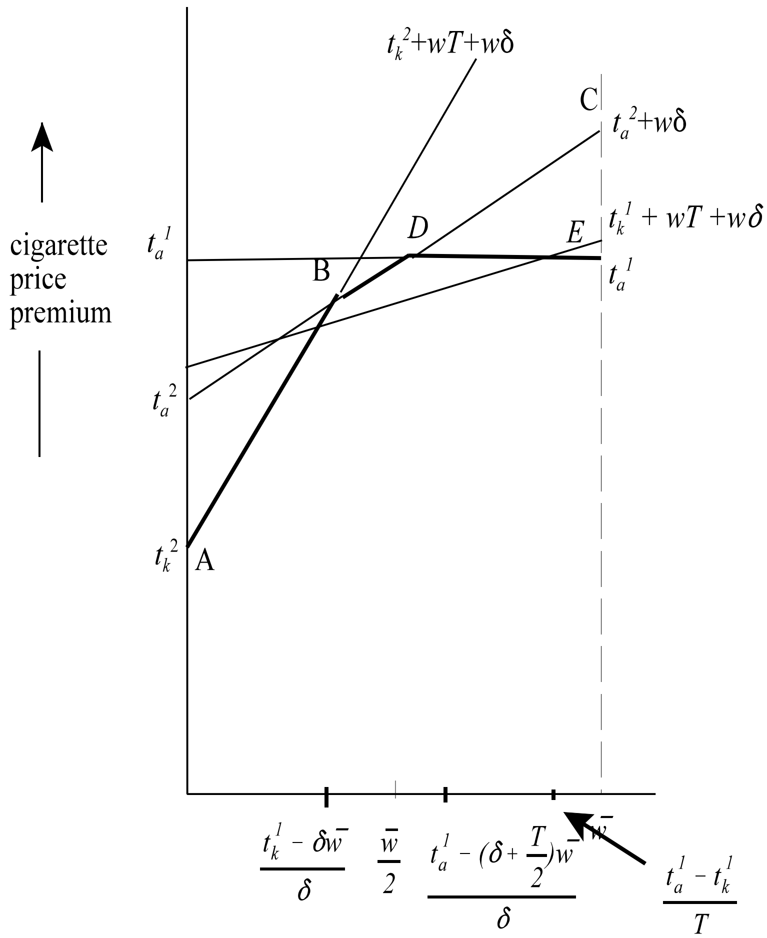
Case F2.(ii): the intersection is on BD

Case F2.(iii): the intersection lies above ABDE.[2]

These are now considered in turn

CASE F2.(i) *the inclusive price line of an own kits intersects ABDE on AB*

The inclusive prices in Region 1 are shown in the figure below:



Region 1

Figure: inclusive price lines in Region 1 in Case F2.(i)

In Region 1: An individual in Region 1 faces the same inclusive price for an own kit and an own assembled product if $p + t_k^1 + wT = p + t_a^1$ or if $w = (t_a^1 - t_k^1) / T$. Assuming that $(t_a^1 - t_k^1) / T \le \bar{w}$, Region 1's tax problem is:

$$\max_{t_k^1,\, t_a^1} \quad R^1 = \int_{\frac{t_k^1 - \delta\bar{w}}{\delta}}^{\frac{t_a^1 - t_k^1}{T}} t_k^1 \, Q f(w)\, dw \; + \; \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1 \, Q f(w)\, dw$$

$$\text{s.t.} \qquad (\delta + \frac{T}{2})\bar{w} = t_a^2 \le t_k^1 \; ;$$

$$t_k^1 \le t_a^1 \; ;$$

$$\frac{t_k^1 - \delta\bar{w}}{\delta} \le \frac{\bar{w}}{2} \; ;$$

$$\frac{\bar{w}}{2} \le \frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} \; ;$$

$$\frac{t_a^1 - t_k^1}{T} \le \bar{w} \, .$$

The constraint $t_k^1 \le t_a^1$ is implied by the inequalities $(t_k^1 - \delta\bar{w})/\delta \le \bar{w}/2$ and $\bar{w}/2 \le (t_a^1 - (\delta + T/2)\bar{w})/\delta$, and so is ignored.

Setting $f(w) = 1/\bar{w}$ and integrating, tax revenue in Region 1 is:

$$R^1 = \frac{Q}{\bar{w}}\left[t_k^1 \left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\bar{w}}{\delta} \right) + t_a^1 \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) \right] .$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}}\left[t_k^1 \left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\bar{w}}{\delta} \right) + t_a^1 \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) \right] + A \left(t_k^1 - \left(\delta + \frac{T}{2} \right) \bar{w} \right)$$

$$+ \; B\left(\frac{\bar{w}}{2} - \frac{t_k^1 - \bar{w}\delta}{\delta}\right) + C\left(\frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} - \frac{\bar{w}}{2}\right) + D\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right).$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial t_k^1} = \frac{Q}{\bar{w}}\left[-\frac{t_k^1}{T} - \frac{t_k^1}{\delta} + \frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - \delta\bar{w}}{\delta} + \frac{t_a^1}{T}\right] + A - \frac{B}{\delta} + \frac{D}{T} = 0\,; \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}}\left[\frac{t_k^1}{T} - \frac{t_a^1}{T} + \bar{w} - \frac{t_a^1 - t_k^1}{T}\right] + \frac{C}{\delta} - \frac{D}{T} = 0\,; \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial A} = t_k^1 - \left(\delta + \frac{T}{2}\right)\bar{w} \geq 0 \qquad CS \quad A \geq 0\,; \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\bar{w}}{2} - \frac{t_k^1 - \delta\bar{w}}{\delta} \geq 0 \qquad CS \quad B \geq 0\,; \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} - \frac{\bar{w}}{2} \geq 0 \qquad CS \quad C \geq 0\,; \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial D} = \bar{w} - \frac{t_a^1 - t_k^1}{T} \geq 0 \qquad CS \quad D \geq 0. \tag{6}$$

Try $A = 0$, $B = 0$, $C > 0$ and $D = 0$.

From Equation (5):

$$t_a^1 = (\delta + \frac{T}{2})\bar{w} + \delta\frac{\bar{w}}{2} = \frac{3\delta + T}{2}\bar{w}.$$

From Equation (1):

$$t_k^1 = \frac{\delta(3\delta + 2T)}{2(\delta + T)}\bar{w}.$$

In Equation (2):

$$\frac{C}{\delta} = \frac{Q}{\bar{w}}\left(2\frac{t_a^1 - t_k^1}{T} - \bar{w}\right) = Q\left(\frac{2}{T}\left(\frac{3\delta + T}{2} - \frac{\delta(3\delta + 2T)}{\delta + T}\right) - 1\right) = \frac{\delta T}{T(\delta + T)}Q > 0 \text{ as}$$

required;

And

$$\frac{\partial \mathcal{L}}{\partial A} = \left(\frac{\delta(3\delta + 2T)}{2(\delta + T)} - \frac{2\delta + T}{2}\right)\bar{w} = \frac{\delta^2 - \delta T - T^2}{2(\delta + T)}\bar{w}$$

which is positive provided $\delta > 2T$;

$$\frac{\partial \mathcal{L}}{\partial B} = \left(\frac{1}{2} - \frac{3\delta + 2T}{2(\delta + T)} + 1\right)\bar{w} = \frac{T}{2(\delta + T)}\bar{w} \geq 0 \qquad \text{as required;}$$

$$\frac{\partial \mathcal{L}}{\partial D} = \left(\bar{w} - \frac{3\delta + T}{2T}\bar{w} + \frac{\delta(3\delta + 2T)}{2(\delta + T)T}\bar{w}\right) = \frac{T}{2(\delta + T)}\bar{w} \geq 0 \qquad \text{as required;}$$

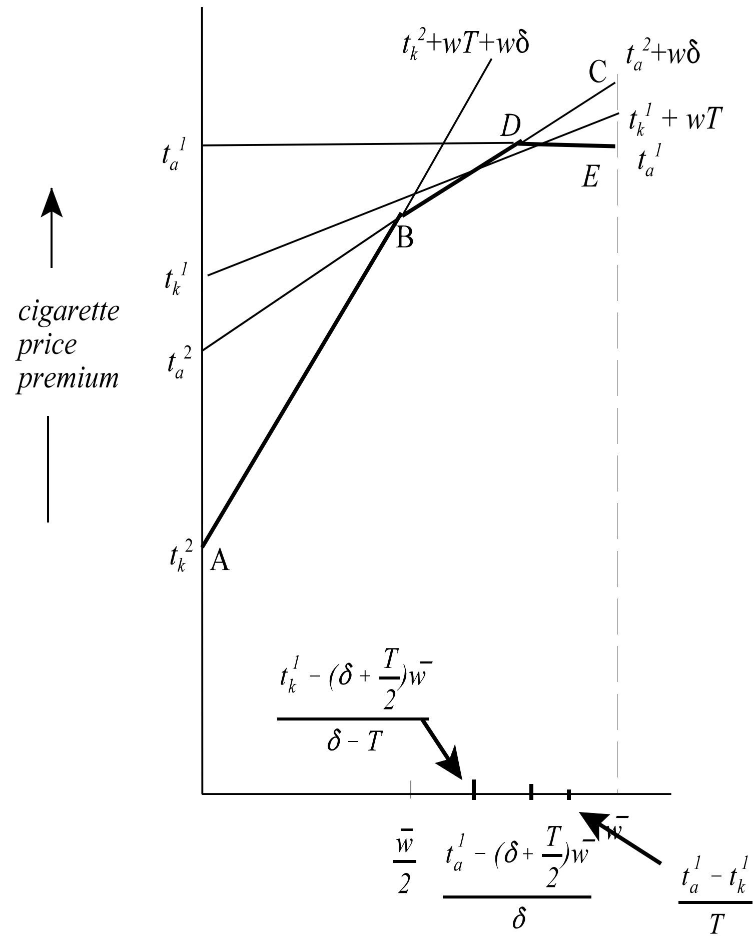
And

$$R^1 = \frac{3\delta^2 + 5\delta T + T^2}{4(\delta + T)}Q\bar{w} < R^{1*} .$$

The alternative case is that t_a^1 is set so high that no own assembled product is sold in Region 1, or $\bar{w} \le (t_a^1 - t_k^1) / T$. However, given that this constraint does not bind in the case above, it could never increase tax revenue by raising t_a^1 till it does: by lowering t_a^1 to relax the constraint, some high-wage individuals can be induced to shift from lower-taxed kits to higher-taxed assembled product, and tax revenue increases.

CASE F2.(ii): the *inclusive price line of an own kit intersects ABDE on BD.*

The inclusive price lines in Region 1 are shown in the figure below:



Region 1

Figure: inclusive price lines in Region 1 in Case F2.(ii)

In Region 1: an individual in Region 1 faces the same price for an own kit and an imported pre-assembled product if $p + t_k^1 + wT = p + t_a^2 + w\delta$ or, setting $t_a^2 = (\delta + T/2)\bar{w}$, if

$w = (t_k^1 - (\delta + T/2)\bar{w})/(\delta - T)$.

In this case, revenue maximization by Region 1 requires that $(t_a^1 - t_k^1)/T \le \bar{w}$. If this constraint were to bind, no other constraint prevents Region 1 from lowering t_a^1. By doing so, it could induce some high-wage individuals to shift from lower-tax kits to higher-tax assembled product, raising tax revenue.

Region 1's problem is:

$$\max_{t_k^1,\, t_a^1} R^1 = \int_{\frac{t_k^1 - (\delta + \frac{T}{2})\bar{w}}{\delta - T}}^{\frac{t_a^1 - t_k^1}{T}} t_k^1\, Q f(w)\, dw + \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1\, Q f(w) dw$$

s.t. $$(\delta + \frac{T}{2})\bar{w} = t_a^2 \le t_k^1 ;$$

$$t_k^1 \le t_a^1 ;$$

$$\frac{\bar{w}}{2} \le \frac{t_k^1 - (\delta + \frac{T}{2})\bar{w}}{\delta - T} ;$$

$$\frac{t_k^1 - (\delta + \frac{T}{2})\bar{w}}{\delta - T} \le \frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} .$$

$(\delta + T/2)\,\bar{w} \le t_k^1$ is implied by the third constraint and $t_k^1 \le t_a^1$ is implied by the fourth constraint. These two constraints are therefore ignored.

Setting $f(w) = 1/\bar{w}$ and integrating, tax revenue is:

$$R^1 = \frac{Q}{\bar{w}}\left[t_k^1 \left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - (\delta + \frac{T}{2})\bar{w}}{\delta - T} \right) + t_a^1 \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) \right].$$

The Lagranean is:

$$\mathfrak{L} = \frac{Q}{\bar{w}}\left[t_k^1\left(\frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - (\delta + \frac{T}{2})\bar{w}}{\delta - T}\right) + t_a^1\left(\bar{w} - \frac{t_a^1 - t_k^1}{T}\right)\right]$$

$$+ A\left(\frac{t_k^1 - (\delta + \frac{T}{2})\bar{w}}{\delta - T} - \frac{\bar{w}}{2}\right) + B\left(\frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} - \frac{t_k^1 - (\delta + \frac{T}{2})\bar{w}}{\delta - T}\right).$$

The Kuhn Tucker Conditions are:

$$\frac{\partial \mathfrak{L}}{\partial t_k^1} = \frac{Q}{\bar{w}}\left[-\frac{t_k^1}{T} - \frac{t_k^1}{\delta - T} + \frac{t_a^1 - t_k^1}{T} - \frac{t_k^1 - (\delta + \frac{T}{2})\bar{w}}{\delta - T} + \frac{t_a^1}{T}\right] + \frac{A}{\delta - T} - \frac{B}{\delta - T} = 0; \quad (1)$$

$$\frac{\partial \mathfrak{L}}{\partial t_a^1} = \frac{Q}{\bar{w}}\left[\frac{t_k^1}{T} - \frac{t_a^1}{T} + \bar{w} - \frac{t_a^1 - t_k^1}{T}\right] + \frac{B}{\delta} - \frac{C}{T} = 0; \quad (2)$$

$$\frac{\partial \mathfrak{L}}{\partial A} = \frac{t_k^1 - (\delta + \frac{T}{2})\bar{w}}{\delta - T} - \frac{\bar{w}}{2} \geq 0 \qquad CS \quad A \geq 0; \quad (3)$$

$$\frac{\partial \mathfrak{L}}{\partial B} = \frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} - \frac{t_k^1 - (\delta + \frac{T}{2})\bar{w}}{\delta - T} \geq 0 \qquad CS \quad B \geq 0. \quad (4)$$

Try $A > 0$ and $B = 0$:

From Equation (3):

$$t_k^1 = \frac{3\,\delta}{2}\bar{w} \; .$$

From Equation (2)

$$t_a^1 = \frac{3\delta + T}{2}\,\bar{w} \; .$$

Equation (1) gives:

$$A = (\delta + \frac{T}{2})\,Q > 0 \qquad\qquad \text{as required ;}$$

And

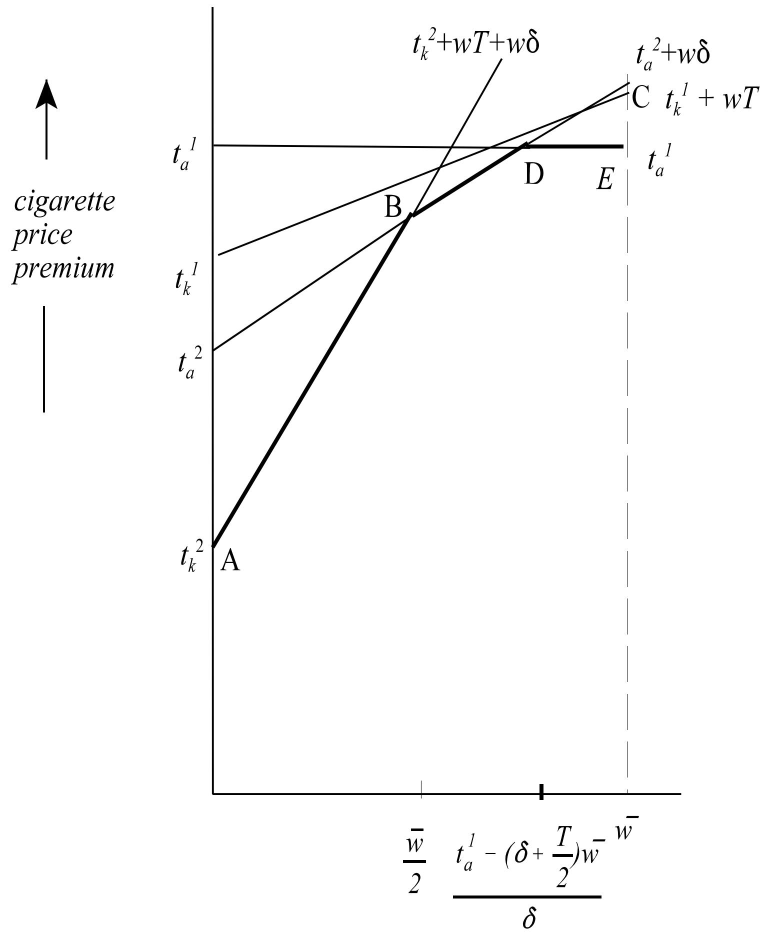
$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\frac{3\delta + T}{2}\bar{w} - (\delta + \frac{T}{2})\bar{w}}{\delta} - \frac{\frac{3\delta}{2}\bar{w} - (\delta + \frac{T}{2})\bar{w}}{\delta - T} = 0 \geq 0 \qquad\qquad \text{as required.}$$

Tax revenue in Region 1 is:

$$R^1 = \frac{Q}{\bar{w}}\left[\frac{3\delta}{2}\bar{w}\left(\frac{\bar{w}}{2} - \frac{\frac{3\delta}{2}\bar{w} - (\delta + \frac{T}{2})\bar{w}}{\delta - t}\right) + \frac{3\delta + T}{2}\bar{w}\left(\bar{w} - \frac{\bar{w}}{2}\right)\right] = \frac{3\delta + T}{4}Q\bar{w} < R^{1*} \; .$$

CASE F2.(iii): *the inclusive price line of an own kit lies above ABDE*.

The inclusive price lines in Region 1 are shown in the figure below:



Region 1

Figure: inclusive price lines in Region 1 in Case F2.(iii)

Region 1 sells no own kits so that effectively t_a^1 is the only choice variable for Region 1. Region 1's problem is:

$$\max_{t_a^1} R^1 = \int_{\frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta}}^{\bar{w}} t_a^1 \, Q f(w) \, dw$$

$$\text{s.t.} \qquad (\delta + \frac{T}{2})\bar{w} = t_a^2 \leq t_a^1 \; ;$$

$$\frac{\bar{w}}{2} \leq \frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} \; .$$

The first constraint is implied by the second constraint and can be omitted.

Setting $f(w) = 1/\bar{w}$ and integrating, tax revenue in Region 1 is:

$$R^1 = \frac{Q}{\bar{w}} t_a^1 \left[\bar{w} - \frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} \right] .$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}} t_a^1 \left[\bar{w} - \frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} \right] + A \left(\frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} - \frac{\bar{w}}{2} \right) .$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}} \left[-\frac{t_a^1}{\delta} + \bar{w} - \frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} \right] + \frac{A}{\delta} = 0; \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial A} = \frac{t_a^1 - (\delta + \frac{T}{2})\bar{w}}{\delta} - \frac{\bar{w}}{2} \geq 0 \qquad CS \quad A \geq 0 . \tag{2}$$

Try A > 0.

From Equation (2):

$$t_a^1 = \frac{3\delta + T}{2}\bar{w}.$$

In Equation (1):

$$\frac{A}{\delta} = \frac{2\delta + T}{2\delta}Q > 0 \qquad \text{as required.}$$

By construction:

$$\frac{\partial \mathcal{L}}{\partial A} = 0 \qquad \text{as required.}$$

Tax revenue in Region 1 is:

$$R^1 = \frac{Q}{\bar{w}}\frac{3\delta + T}{2}\bar{w}\left[\bar{w} - \frac{\frac{3\delta + T}{2}\bar{w} - (\delta + \frac{T}{2})\bar{w}}{\delta}\right] = \frac{3\delta + T}{4}Q\bar{w} < R^{1*}.$$

4. SUMMARY

Given the strategy of Region 2 of $t_k^2 = \delta\bar{w}$ and $t_a^2 = (\delta + T/2)\bar{w}$, I have considered the tax revenue achieved by Region 1 under all possible strategies of t_k^1 and t_a^1. Provided $\delta \geq 2T$, no alternative strategy gives more revenue than tax revenue R^{1*} achieved by the strategy of $t_k^1 = \delta\bar{w}$ and $t_a^1 = (\delta + T/2)\bar{w}$. Therefore $t_k^1 = \delta\bar{w}$ and $t_a^1 = (\delta + T/2)\bar{w}$ is a best response of Region 1, and by symmetry $t_k^1 = t_k^2 = \delta\bar{w}$ and $t_a^1 = t_a^2 = (\delta + T/2)\bar{w}$ is a Nash Equilibrium.

ENDNOTES

1. Note that the inclusive price line of an own kit is upward sloping but the price line of an assembled product is horizontal. Theorefore if the inclusive price line of an own kit does not intersect on AB, it does not intersect ABC

2. Note that the inclusive price line of an own kit is upward sloping but the price line of an assembled product is horizontal. Therefore if the inclusive price line of own kits does not intersect on AB or BD, it does not intersect ABDE