

# DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 05-04

## Community Income Distributions in a Metropolitan Area

Charles A. M. de Bartolome  
*University of Colorado at Boulder*

*and*

Stephen L. Ross  
*University of Connecticut*

July 2005

Center for Economic Analysis  
Department of Economics



University of Colorado at Boulder  
Boulder, Colorado 80309

© 2005 Charles A. M. de Bartolome and Stephen L. Ross

# COMMUNITY INCOME DISTRIBUTIONS IN A METROPOLITAN AREA

by

Charles A.M. de Bartolome  
*University of Colorado*

and

Stephen L. Ross  
*University of Connecticut*

## ABSTRACT

We extend de Bartolome and Ross (2003) to the case when the income distribution in the metropolitan area is a continuous distribution. In particular, we consider a circular central city surrounded by a suburban community. All households must commute to the metropolitan center and the commuting advantages of locations closer to the city center are capitalized into house prices. Our model has equilibria in which the income distributions of the central city and of the suburb *do* overlap. Our finding contrasts with the traditional finding of Tiebout-type models of fiscal sorting and of Alonso-Mills-Muth-type models of spatial sorting, both of which predict that the income distributions of the two communities do not overlap. In addition, it explains the fixedness in jurisdictional boundaries.

JEL Classification: H73, R12, R14.

Key words: Communities, income distribution.

Address for correspondence: Charles A.M. de Bartolome, Department of Economics, University of Colorado, 256 UCB, Boulder, CO 80300-0256, USA. Tel #: (303) 492-4464, Fax #: (303) 492-8960, e-mail address: [debartol@spot.colorado.edu](mailto:debartol@spot.colorado.edu)

## 1. INTRODUCTION

A long literature in local public economics discusses the way households of different incomes distribute themselves across a metropolitan area. In Tiebout's (1956) model of fiscal competition, the primary characteristic of a jurisdiction is the public service it provides. A household chooses the jurisdiction in which to reside by trading off the public service provided by a jurisdiction against the tax it levies. If a household's public service demand increases with his income, households with higher income choose jurisdictions which provide higher public service levels, or there is sorting by income of households between jurisdictions (McGuire (1974), Berglas (1976a and 1976b), Wooders (1978)). In Tiebout's original model, jurisdictions are costlessly formed on a featureless plain and their boundaries may be adjusted to match the land demand of the households choosing to live in the jurisdiction. The conclusion of sorting by income is robust if the assumption of adjustable jurisdictional boundaries is changed to exogenously fixed jurisdictional boundaries: households distribute themselves between jurisdictions so that the highest income in one jurisdiction is the lowest income in another jurisdiction (Elickson (1971), Yinger (1982), Epple et al. (1984)).

Although analytically convenient, such strict income sorting is not observed. We take as our motivation the observation by Epple and Platt (1998):

“...sorting by income is incomplete. The incomes of the wealthiest households in communities with low average income typically exceed those of the poorest households in communities with high average income. For example, when the 92 municipalities in the Boston Standard Metropolitan Statistical Area (SMSA) are

ranked by median income, Chelsea is the poorest and Weston is the wealthiest.

Chelsea and Weston had median incomes in 1980 of \$11 201 and \$46 646

respectively. Yet 19% of the households in Chelsea had incomes above \$22 500

in 1980, while 19% of the households in Weston had incomes below \$22 500 in

1980."

Although this observation applies to suburbs, examination of census data shows that it also describes the sorting between a city and its suburbs.

What is lacking in Teibout's model and its later derivatives is spatial structure. As noted above, jurisdictions are defined by their public service level and there is no suggestion as to which jurisdictions are located closer to the center of the metropolitan area and which are located closer to the outside. The Alonso (1964)-Mills (1967)-Muth (1961, 1969) model of a metropolitan area focuses on this spatial structure but overlooks issues of public service provision. When choosing where to live, households are now viewed as considering the trade-off of commuting costs and land prices. All households are viewed as commuting to the metropolitan center: because living further from the metropolitan center involves larger commuting costs, land prices fall as the location moves away from the metropolitan center. If the income elasticity of commuting costs exceeds the income elasticity of land demand, commuting cost considerations dominate and higher income households live closer to the metropolitan center: income decreases as the distance from the metropolitan center increases. In contrast, if the income elasticity of commuting costs is less than the income elasticity of land demand, land demand considerations dominate and higher income households (who buy much land) prefer to live on the outside of the metropolitan area where land is relatively cheap: income increases as

distance from the metropolitan center increases (Wheaton (1977)). In either case, there is a monotonic income gradient from the metropolitan center.

Like the Tiebout model, the Alonso-Mills-Muth model is a poor fit with the data. Firstly, the income elasticity of commuting costs is almost certain to exceed the income elasticity of land demand but the poor tend to live disproportionately in the central cities. As Glaeser, Kahn and Rappaport (©, 2000) report:

“Theory suggests that the income elasticity of commuting cost per mile should be close to one. The marginal cost of an extra mile spent commuting includes both time and cash costs, but generally cash costs per mile are small relative to time costs. Valuing time at either the wage rate...implies a unitary income elasticity of commuting costs. ...Our objective now is to estimate the income elasticity of demand for land...our results show a quite consistent pattern where the elasticity of the demand for space with respect to income lies between 0.1 and 0.4. If these elasticities are correct, then the Alonso-Mills-Muth theory can only explain [the observed] sorting if the income elasticity of commuting time were lower than 0.3, which seems implausible.”

Secondly, the income gradient is not monotonic: Glaeser, Kahn and Rappaport (©, 2000) write:

“[We discuss] the income -distance relationship for four older metropolitan areas (New York, Chicago, Philadelphia and Boston). In these cities (and in most other older cities) there is a clear U-shaped pattern. The census tracts closest to the city center are often among the

richest in the metropolitan area. The poorest census tracts come next with the bottom of the curve generally lying between three and five miles away from the central business district. After that point income rises again. In most cities, income begins to fall again in the outer suburbs.”<sup>1</sup>

Our view of the topic is that both the Tiebout model and the Alonso-Mills-Muth model contain parts of the truth and we therefore integrate their key features into a single model. In particular, we consider a metropolitan area in which a circular central city is surrounded by a suburb. At the center of the central city is the business district to which all households must commute. This is the spatial aspect of the model. Fiscal competition issues are captured because the public service level in each jurisdiction is determined by the median voter in each jurisdiction. At equilibrium, no household wishes to change its location. We find that two classes of equilibrium exist: one in which there is strict income sorting between jurisdictions and one in which the income distributions of the central city and the suburb overlap (“income mixing”). The paper extends the model of de Bartolome and Ross (2003). de Bartolome and Ross consider a model with two income classes whereas in this paper we consider a continuous income distribution.

Our qualitative predictions for the second class of equilibrium (“income mixing”) fit well with the earlier quotes. Although median income is higher in the suburb than in the central city, the income distributions of the two jurisdictions overlap so that the highest income in the central city exceeds the lowest income in the suburb. The U-shaped income distribution is created because, as the location moves away from the center, within the central city income decreases but at the jurisdictional boundary between the central city and the suburb income jumps up. For

the US this boundary typically occurs around 4 miles from the metropolitan center. Finally, income decreases in the suburb as the location moves away from the center.

Other authors have noted the failure of the canonical models to explain the empirically relevant case of income mixing between jurisdictions. Epple and Platt (1998) show that jurisdictional income distributions can overlap if households differ in two dimensions - in their income and in a parameter which reflects their preference for the public service. The preference parameter is distributed independently of income so that a jurisdiction providing a medium public service level is chosen both by a high-income household with a low preference for the public service and by a low-income household with a high preference for the public service. In our model households also differ in two ways - by their income and by their preference for the public service - but the preference parameter is perfectly correlated with income so that at a fundamental level households differ only in the single dimension of income or income is a sufficient statistic to describe a household.

LeRoy and Sonstelie (1983) use commuting considerations to explain why some rich households live outside of some poor households in a metropolitan area. They have two income classes each with its own commuting cost, and two modes of transport which are labeled “car” and “bus”. Car travel is faster but more expensive, and in consequence it is used only by households living further out. They therefore create an income profile as the locations moves away from the metropolitan center as: rich households using the bus live on the land closest to the metropolitan center, then poor households using the bus, then rich households using the car and then poor households using the car. Although the final income gradient in LeRoy and Sonstelie is similar to our income gradient, there are important differences. In LeRoy and

Sonstelie, there are no jurisdictions and no public services (except possibly the bus). In turn this implies that house prices are continuous in the metropolitan area whereas in our model house prices change discontinuously as the location moves across the jurisdictional boundary between the central city and the suburb.

Finally, Nechyba (2000) has a model in which there is income-mixing between jurisdictions as a consequence of jurisdictions having different housing stocks. Some high-income households choose to live in the jurisdiction of low average-income because the large houses in that jurisdiction are relatively cheap. In his model the housing stock in each jurisdiction is exogenous and the reason for the assortment of house sizes is unexplained. In contrast, in our model all households buy the same lot size.

Our model also explains the empirical findings of Epple and Romer (1989) that jurisdictional boundaries do not adjust very frequently and that the direction of movement is not consistent with the expansion of high-bid areas. Moving a boundary usually requires annexation of part of one jurisdiction by another. In our model the households who are indifferent between the two jurisdictions reside away from the jurisdictional boundary: the households who actually reside near the boundary are not indifferent and therefore oppose annexation.

The paper is organized as follows. Section 2 lays out the model. Section 3 presents an example of an equilibrium with income mixing. Section 4 examines the incentives for boundary movements through annexation. Section 5 concludes.

## 2. THE MODEL

### 2.1 Spatial overview

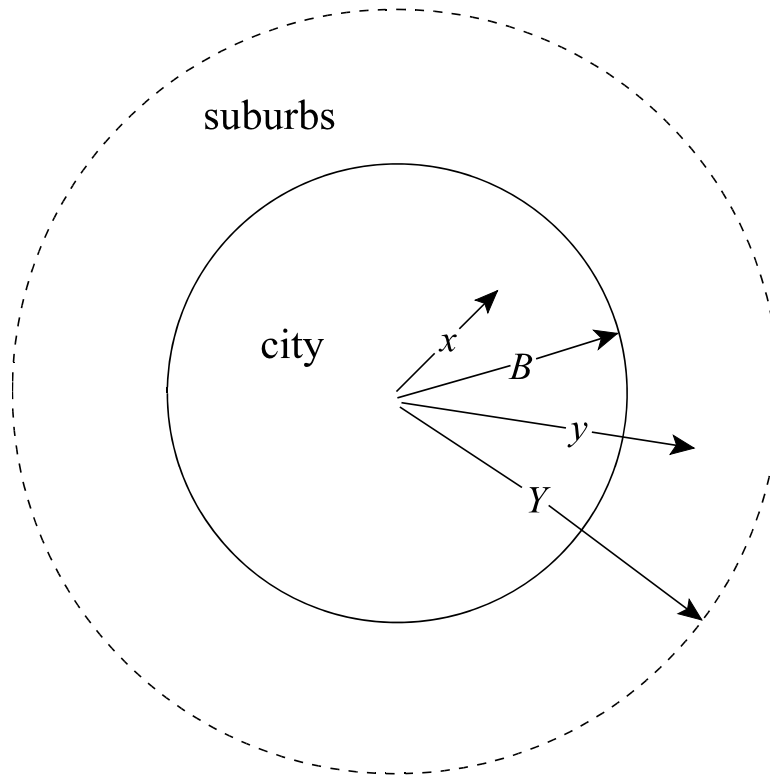


Figure 1: the metropolitan area

The spatial layout of the metropolitan area is illustrated by Figure 1. At the center of the metropolitan area is the business district to which all households must commute; for ease of presentation the business district is assumed to be a point with no area. The business district is surrounded by a circular central city, henceforth denoted as “the city” and labeled  $c$ . The city has an exogenous jurisdictional boundary of radius  $B$ . In the city there may be undeveloped land, so that the limit of development has radius  $X$ :

$X < B$ : there is undeveloped land at the edge of the city;

$X = B$ : there is no undeveloped land in the city.

The city is surrounded by a suburb, labeled  $s$ . The outer jurisdictional boundary of the suburb is sufficiently distant that all households live in the city or in the suburb; the outer limit of development in the suburb is a circle of radius  $Y$ . Our interest is in how households of differing incomes distribute themselves across the metropolitan area.

## 2.2 Basic Analytic Structure

A household has an endowed income  $M$  and obtains utility  $U$  from consuming a privately-provided good  $c$  and a public service  $g$ . The privately-provided good is the numeraire good. In this paper we make two simplifications. Firstly, the household's demand for lot size,  $a$ , is assumed to be exogenous so that housing *per se* does not enter the utility function<sup>2</sup>: the non-land components of housing are included as part of the private good. Secondly, we consider the utility function to have consumer surplus form. In particular, we assume that a household  $i$  of endowed income  $M_i$  has a utility function of form

$$U^i = c + \beta_i V(g)$$

where  $V(\cdot)$  is a strictly concave function. Because we want the public service to appear normal or to be more valued by households of higher income, we assume that  $\beta_i$  is a function of endowed income,

$$\beta_i \equiv \beta(M_i) ; \partial \beta / \partial M > 0.$$

In this formulation, households differ in their tastes for the public service and their tastes vary systematically with endowed income.<sup>3</sup> We make both of these simplifications for ease of computation; we believe that our results do not depend on these simplifications. Indeed we are currently computing using a more traditional utility function of Cobb-Douglas form,  $U = \log_e (c + \bar{c}) + \alpha \log_e h + \beta \log_e g$ , in which housing  $h$  is an endogenous variable.

Each household has a unit time endowment which he can use either for working or for commuting to the metropolitan center where all firms are located. The household's endowed income  $M$  is the income he earns if he spends no time commuting (by living at the metropolitan center). If he lives at distance  $s$  from the metropolitan center, he must spend time commuting instead of working, and his earned income is reduced by the opportunity cost of the commute. The time spent commuting is proportional to  $s$  and the opportunity cost of time is  $M$ , and hence the opportunity cost of the commute can be written as  $tMs$  where  $t$  is a constant. As noted in the Introduction, a household's taste for the public service and his commuting cost are both perfectly correlated with his income or income is a sufficient statistic to describe the household.

The household chooses to live in a jurisdiction  $j$  ( $j \in \{c, s\}$ ) providing public service  $g_j$ . Using his earned income and any government transfers, the household buys his consumption, pays for his lot and pays the tax levied by the jurisdiction to finance the public service. The cost of a lot at distance  $s$  from the metropolitan center is  $r(s)$ . The resource cost of one unit of the public service per household is one unit of numeraire.<sup>4</sup> Although property taxes are the main source of local government tax revenue in the U.S., we assume that the jurisdiction  $j$  uses a uniform residency tax  $g_j$  to finance the public service it provides. We make this assumption because the property tax provides an incentive for income mixing: the property tax reduces the

cost of the public service to low-income households and thereby encourages them to move into high-service jurisdictions which tend to be inhabited by high-income households (Wheaton (1975)) . The focus of this paper is to show how inter- and intra-jurisdictional capitalization facilitate income mixing between jurisdictions. Therefore we abstract from other institutional effects which promote income mixing, such as the property tax, and use a residency tax.

The central government may provide a lump-sum transfer  $T$ . The consumption of the private good by the household if he locates in jurisdiction  $j$  at distance  $s$  from the city center is therefore

$$c = M - tMs - r(s) - g_j + T.$$

Endowed income  $M$  is distributed between  $\underline{M}$  and  $\bar{M}$  with density function  $f(M)$ . Our focus is on how households distribute themselves across the two jurisdictions. The income distribution, the rent schedule and public service level in each community are determined endogenously. The model is summarized descriptively.

### 2.3 Sorting and rents within a jurisdiction

At equilibrium a household with endowed income  $M$  achieves utility  $W(M)$ . His bid-rent for a location at distance  $s$  from the metropolitan center in jurisdiction  $j$  is  $R(s, j; M)$  and is defined by:

$$W(M) \equiv M - tMs - R(s, j; M) - g_j + T + \beta(M) V(g_j).$$

Differentiating with respect to  $s$  within a jurisdiction and rearranging,

$$\frac{\partial R(s, j; M)}{\partial s} = -tM. \quad (1)$$

As the distance from the metropolitan center increases, the household lowers its bid-rent to reflect its increased commuting cost.

Bid-rent curves have the “single-crossing” property. In particular, differentiating Equation (1) with respect to  $M$ :

$$\frac{\partial}{\partial M} \frac{\partial R(s, j; M)}{\partial s} = -t < 0,$$

or the bid-rent curve steepens with increasing income reflecting the greater commuting cost to high-income households of living farther from the metropolitan center.

The rent paid at any location is the highest bid-rent of all households at that location or the rent schedule  $r(s)$  in the jurisdiction is the envelope of the bid-rent functions. A household locates at the point where his bid-rent curve touches the envelope, or

$$\frac{\partial R(s, j; M)}{\partial s} = \frac{dr(s)}{ds}. \quad (2)$$

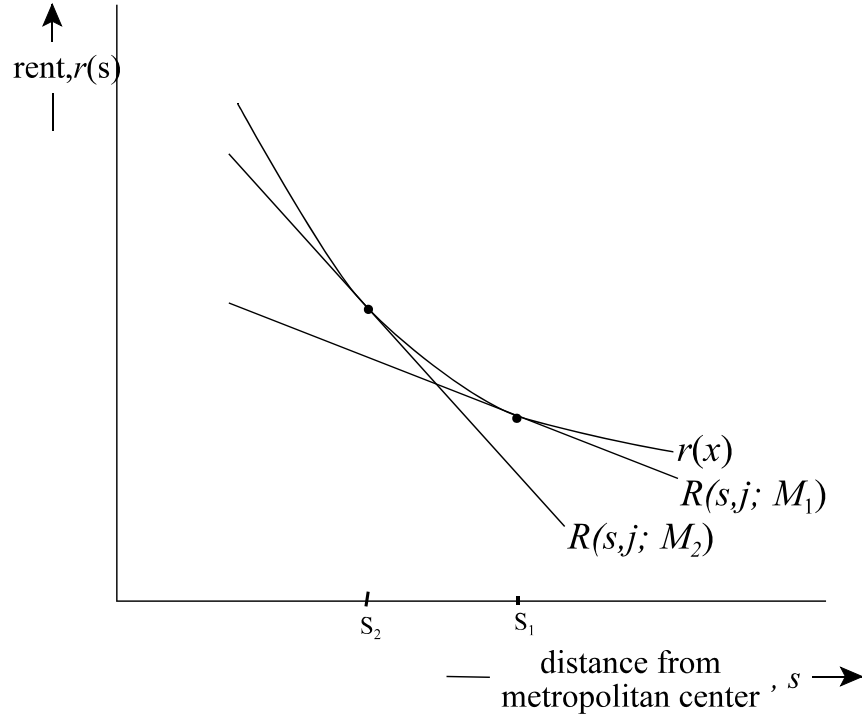


Figure 2: the rent schedule as the envelope of the bid-rent curves

Figure 2 illustrates the creation of the rent schedule in a jurisdiction. It shows the rent schedule  $r(s)$  and two bid-rent curves  $R(s,j; M_1)$  and  $R(s,j; M_2)$  of two households, with endowed income  $M_1$  and  $M_2$  respectively where  $M_1 < M_2$ . The rent schedule is the envelope of the bid-rent curves with each household locating where its bid-rent curve just touches the rent schedule. The household of income  $M_1$  locates at  $s_1$  and the household of income  $M_2$  locates at  $s_2$  :  $s_1 > s_2$ . Rich households in a jurisdiction are outbidding the poor households for the locations closer to the metropolitan center because the benefit to them of the saved commuting is greater, or income decreases in a jurisdiction as distance from the metropolitan center increases.

The household takes the rent schedule  $r(s)$  as given when choosing his location in the jurisdiction. Therefore, combining Equation (1) and (2), the household who resides at distance  $s$

from the city's center has income  $M$  which satisfies

$$\frac{dr(s)}{ds} = -tM \quad (3)$$

Equation (3) determines the location  $s$  chosen by a household of income  $M$ ,  $s(M)$ ; alternatively, it determines the income  $M$  of the household who locates at  $s$ ,  $M(s)$ . In the following discussion, we use both descriptions.

We note finally that the rent schedule in a jurisdiction must be continuous as otherwise a household who is located adjacent to the discontinuity on the side of high rent can increase his utility by moving across the discontinuity to the side of low rent: his rent would decrease by discrete amount but his commuting cost would increase only marginally.<sup>5</sup>

These results are summarized in Lemma A:

- LEMMA A:*
- (a) within a jurisdiction, rent decreases as distance from the metropolitan center increases.*
  - (b) within a jurisdiction, income decreases as distance from the metropolitan center increases.*
  - (c) within a jurisdiction, the rent schedule is continuous.*

## 2.4 Strict income sorting between jurisdictions

For the sake of completeness and in order to provide the contrast with the equilibrium with income-mixing, we state below that, in our structure which is laid out more fully in the following subsections, there is always an equilibrium with strict income sorting:

*LEMMA B: there is always a sorting equilibrium: households with endowed income in the range  $[\underline{M}, \tilde{M}]$  reside in the suburb and households with endowed income in the range  $[\tilde{M}, \bar{M}]$  reside in the city. The sorting equilibrium may involve undeveloped land in the city.*

PROOF: See Appendix A.

## 2.5 Income mixing between jurisdictions

Section 2.4 establishes the existence of an equilibrium in which households are sorted by income between jurisdictions. We consider this to be the “traditional” equilibrium predicted by the canonical models of Tiebout and Alonso-Mills-Muth. Our primary interest is in showing the existence of an alternative equilibrium in which households are distributed between the jurisdictions so that the income distributions of the two jurisdictions overlap or in which there is “income-mixing” between the jurisdictions. Formally, we are searching for an equilibrium with the form:

$$\begin{aligned}\underline{M} \leq M \leq M_1: & \quad \text{households live only in the city;} \\ M_1 \leq M \leq M_2: & \quad \text{households live in the city and in the suburb;} \\ M_2 \leq M \leq \bar{M}: & \quad \text{households live only in the city.}\end{aligned}$$

The required income profiles of the two jurisdictions are illustrated in Figure 3 where, for ease of presentation, we focus on the case in which there is no undeveloped city land.

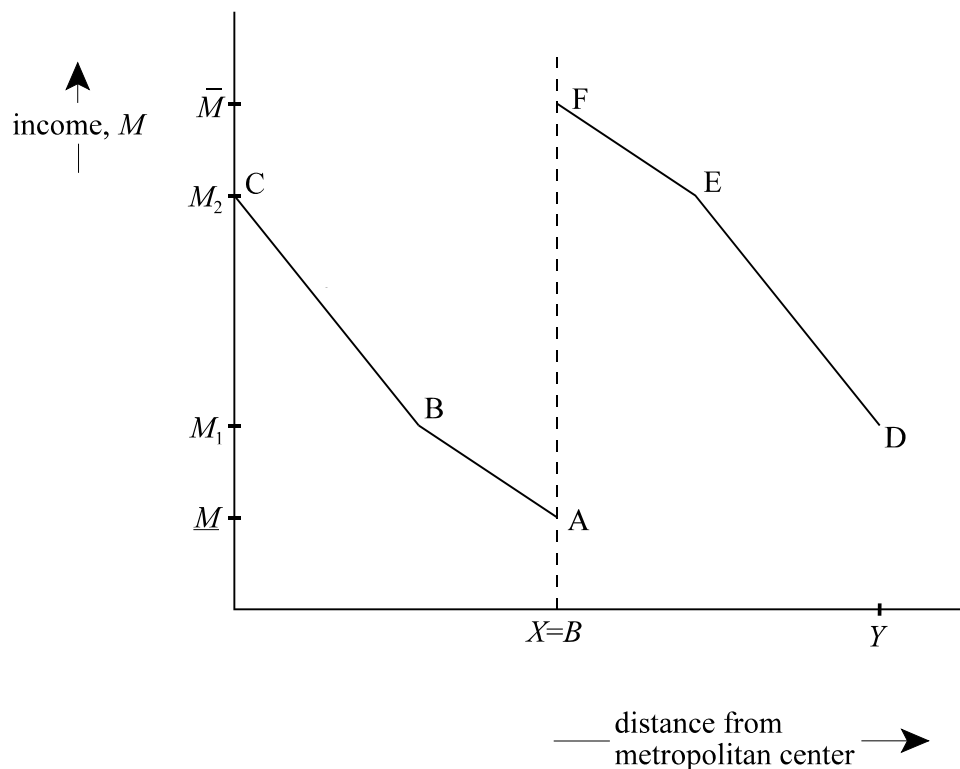


Figure 3: income distribution with income mixing between jurisdictions

In each jurisdiction, income increases as the location moves closer to the metropolitan center. Households with income below  $M_1$  live only in the city; their locations are shown by the line  $AB$ . Households with income  $M_1$  are to be found in the city and at the limit of suburban development. Households with incomes between  $M_1$  and  $M_2$  are found in the city *and* in the suburb: their city and suburban locations are shown by the lines  $BC$  and  $DE$ . Households with incomes above  $M_2$  live only in the suburb; their locations are shown by the line  $EF$ .

The model is now presented heuristically. The full set of equations which define

equilibrium are presented in the Appendix. For ease of presentation and because it seems the empirically relevant case, we restrict our attention to the case in which there is no undeveloped city land ( $X = B$ ).

If households of income  $M$  locate in the city at distance  $x(M)$  from the city's center and in the suburb at distance  $y(M)$  from the city's center, the rent level at each location must be such that the households achieve the same utility at either location, or (using the assumed utility function)

$$\begin{aligned} M_1 \leq M \leq M_2: \quad & M - tMx(M) - r(x(M)) - g_c + T + \beta(M)V(g_c) \\ & = M - tMy(M) - r(y(M)) - g_s + T + \beta(M)V(g_s) \end{aligned}$$

or

$$M_1 \leq M \leq M_2: \quad r(y(M)) - r(x(M)) = (\beta(M)V(g_s) - g_s) - (\beta(M)V(g_c) - g_c) - tM(y - x) \quad (4)$$

The suburban rent premium equals the net benefit of the higher suburban public service less the increased commuting cost. This is the equation typically estimated in a capitalization study. The relationship is shown in Figure 4.

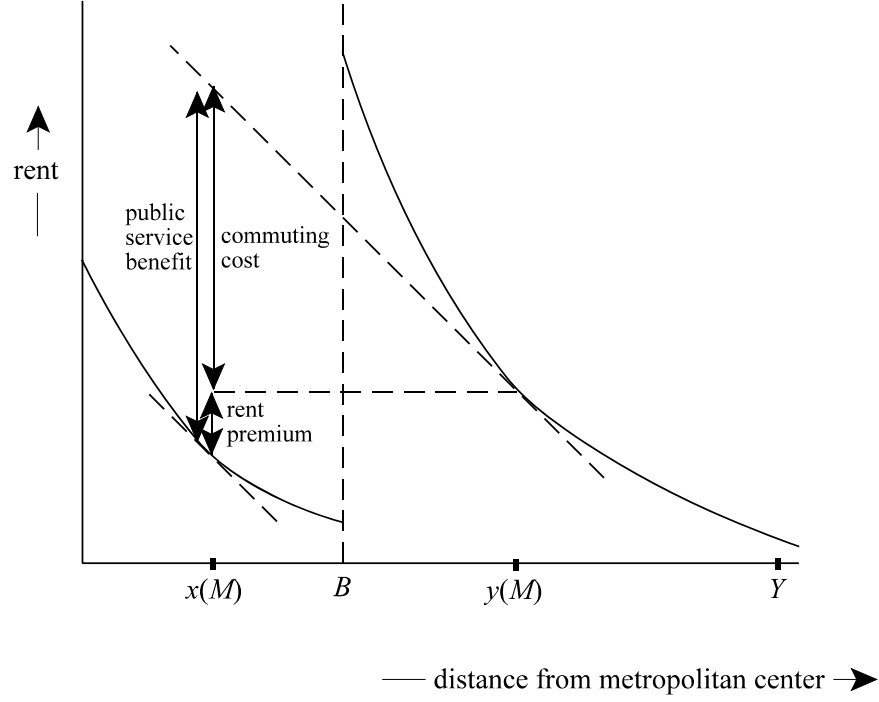


Figure 4: the suburban rent premium

The rent schedules of the city and the suburb underlie the equilibrium, ensuring that households at each income achieve the same utility in either jurisdiction. Using Equation (3), the rent schedules in the suburb and in the city must change as

$$M_1 \leq M \leq M_2 : \frac{d}{dM}(r(y(M)) - r(x(M))) = \frac{dr}{dy} \frac{dy}{dM} - \frac{dr}{dx} \frac{dx}{dM} = -tM \left( \frac{dy}{dM} - \frac{dx}{dM} \right) \quad (5)$$

The rent gradient at a point inside a jurisdiction is determined by the income of the households who live at that point (Equation (3)). As the location changes, household income changes and the rent gradient changes. What allows the rent gradients to change to satisfy Equation (5) is the variation in the fraction of households of income  $M$  who reside in the city: as the fraction of all households of income  $M$  who reside in the city increases, income rises less fast over space and the

rent schedule in the city flattens.

Formally, denote the fraction of households of income  $M$  who live in the city as  $\alpha(M)$ . Consider an annular element, of inner radius  $x$  and width  $dx$ ; households living at the inner surface have income  $M$  and movement across the element is associated with a fall  $|dM|$  in household income.<sup>6</sup> Equating land supply and demand in the annulus,

$$2\pi x dx = Na \alpha(M) f(M) |dM| .$$

In Section 2.3 we established that  $x$  is a monotonically decreasing function of  $M$  or  $dx/dM \leq 0$ , and hence

$$2\pi x dx = - Na \alpha(M) f(M) dM ,$$

or

$$\frac{dx}{dM} = - \frac{Na}{2\pi x} \alpha(M) f(M) . \quad (6)$$

In the suburb, households of income  $M$  reside at  $y$  and, conserving households, the fraction of the households of income  $M$  who reside in the suburb is  $1 - \alpha(M)$ . An identical argument establishes that

$$\frac{dy}{dM} = - \frac{Na}{2\pi y} (1 - \alpha(M)) f(M) .$$

As  $\alpha$  increases,  $dx/dM$  gets larger (in absolute value) and  $dy/dM$  gets smaller (in absolute value).

Hence, at equilibrium,  $\alpha$  adjusts so that the rent schedules in the city and suburb change to satisfy Equation (5).

Differentiating Equation (4) with respect to  $M$ , and using Equation (3):

$$M_1 \leq M \leq M_2 : \beta'(M)(V(g_s) - V(g_c)) = t(y(M) - x(M)). \quad (7)$$

Because a household locates so that his indifference curve is tangent to the rent schedule, his utility is unchanged by a small change in  $x$  or  $y$ . Therefore, if a household is indifferent between a city and a suburban location, a household of marginally larger income must also be indifferent between these two locations (to a first order approximation), or the increase in the benefit of the higher suburban public service is exactly offset by the higher cost of commuting from the suburb (from  $y$  instead of  $x$ ).

### *2.5 Intra-jurisdictional equilibrium - the public service level.*

The public service in each jurisdiction is determined by majority voting. We assume that households vote taking the rent as given or that voters are myopic concerning the influence of the public service on land market outcomes.<sup>7</sup> The concavity of the utility function  $U$  implies that preferences are single-peaked (Perrson and Tabellini (2000, pp 22)). The normality of  $g$  implies that, within a community, the desired public service level increases monotonically with income.<sup>8</sup> Hence the voter with median income is decisive. With all households having the same exogenous demand for land, the household of median income in the city is located at  $x^{med}$  where:

$$\pi x^{med\ 2} = \frac{1}{2} \pi B^2. \quad (8)$$

Writing the income of the household located at  $s$  as  $M(s)$ , the city public service is the value of  $g$  which satisfies

$$g_c = \underset{g}{argmax} \quad M(x^{med}) - tM(x^{med})x^{med} - r(x^{med}) - g + T + \beta(M(x^{med})) V(g)$$

or, taking the first-order condition,

$$\beta(M(x^{med})) V'(g_c) = 1. \quad (9)$$

Similarly in the suburb: the household of median income is located at  $y^{med}$  where

$$\pi y^{med\ 2} = \frac{1}{2}(\pi Y^2 + \pi B^2) \quad (10)$$

and the voted public service  $g_s$  is determined as

$$\beta(M(y^{med})) V'(g_s) = 1. \quad (11)$$

If the public service were higher in the city, higher-income households value the city more than lower-income households for both its commuting advantage and for its public service. In consequence, they outbid lower-income households for all locations in the city; this case is the case of income sorting considered in the previous section. Hence, if the jurisdictions are to have overlapping income distributions, higher-income households must be willing to live in the suburb because the cost of the longer commute is offset by a better match with the suburban public service, or the suburban public service must exceed the public service in the city. This is formalized in Lemma C.

*LEMMA C: If the income distributions of the two jurisdictions overlap in a range of the income distribution, the public service in the suburb is greater than the public service in the city, and the median income in the suburb exceeds the median income in the city.*

PROOF: See Appendix A.

## 2.6 Closing the model

The reservation price of land is  $r_0$ : at the limit of development,

$$r(Y) = r_0 . \quad (12)$$

The difference in the public service in the two jurisdictions suggests that the rent changes discontinuously across the jurisdictional boundary. In the case under consideration there is no undeveloped land in the city so that on the city side of the jurisdictional boundary the rent may exceed  $r_0$ , or<sup>9</sup>

$$X = B : \quad \lim_{x \rightarrow B^-} r(x) \geq r_0 . \quad (13)$$

There are  $N$  households and each household buys a house on a lot of fixed size  $a$ . With no undeveloped land in the city, equating land supply and demand implies:<sup>10</sup>

$$\pi X^2 + \pi (Y^2 - B^2) = Na . \quad (14)$$

We close the model by assuming that rents are returned as the lump-sum transfer  $T$ .

Remembering that the lot size is  $a$ ,

$$T = \frac{1}{N} \left[ \int_0^X \frac{2\pi x}{a} r(x) dx + \int_B^Y \frac{2\pi y}{a} r(y) dy \right] . \quad (15)$$

### 3. THE EXISTENCE OF AN EQUILIBRIUM WITH INCOME MIXING

Equilibrium requires that we solve Equations (3), (7) - (11), (14) and (15) for the variables  $x^{med}$ ,  $y^{med}$ ,  $Y$ ,  $g_c$ ,  $g_s$ ,  $M_1$ ,  $M_2$  and  $T$ , and for the functions  $M(s)$  and  $r(s)$ . In addition, we require the solution to satisfy the Self-Selection Inequalities that low-income households ( $M \leq M_1$ ) and high-income households ( $M \geq M_2$ ) cannot get more utility by moving, the Rent Inequality (13) and the feasibility constraints :  $0 \leq x(M_1) \leq B \leq y(M_2) \leq Y$  and  $\underline{M} \leq M_1 \leq M_2 \leq \bar{M}$ . We now state our central result:

**THEOREM:** *There exist equilibria in which the income ranges of the city and of the suburb overlap.*

**PROOF:** The proof is by the construction of an example. Income in the metropolitan area is uniformly distributed between  $\underline{M}$  and  $\bar{M}$ . The utility function has form  $U = c + bM\sqrt{g}$ .

Parameter values are:

$$Na = \pi 100 ; B = 7 ; \underline{M} = 0 ; \bar{M} = 1 ; f(M) = \frac{1}{\bar{M} - \underline{M}} ; b = .568 ; t = .01 .$$

Using the algorithm discussed in the Appendix B, we calculate

$$x(M_1) = 2 ; y(M_2) = 8 ; Y = 10 ;$$

$$M(x^{med}) = .245 ; M_1 = .45 ; M(y^{med}) = .742 ; M_2 = .85 ;$$

$$g_c = .0048 ; g_s = .0443 .$$

By calculation,  $r(B) > r_0$ . In addition, we confirm that the self-selection constraints are satisfied.

Because the inequalities are strictly satisfied, continuity implies that an equilibrium, in which the city and suburb income ranges overlap, continues to exist if there is a small change in the model's parameter values.

Our next observation concerns the distribution of the households between the city and suburb in the range of overlap. All households with incomes below  $M_1$  live in the city, or  $\alpha(M: M < M_1) = 1$ . Equation (6) shows that in general  $\alpha(M_1) < 1$ ; therefore  $\alpha(M)$  is in general discontinuous at  $M_1$ . In contrast, all households with incomes above  $M_2$  live in the suburb, or  $\alpha(M: M > M_2) = 0$ .  $x(M_2) = 0$  and hence Equation (6) shows  $\alpha(M_2) = 0$ , or  $\alpha(M)$  is continuous at  $M_2$ . The likely case is that  $\alpha(M)$  decreases monotonically from  $\alpha(M_1)$  to  $\alpha(M_2)$ , or the city contains a decreasing share of the higher income households in the range of overlap. This is a consequence of the geometry of a monocentric metropolitan area: as  $x$  decreases, circular elements in the city become smaller at a faster rate than circular elements in the suburb.

Having established the existence of an equilibrium with income-mixing, we now revisit the three quotations of the Introduction. Firstly, the equilibrium does have the property that median city income is less than median suburban income but the highest income in the city exceeds the lowest income in the suburb. Secondly, our model has an elasticity of land demand (zero) which is much less than the income elasticity of commuting cost (unity). Thirdly, referring to Figure 3, the income distribution predicted in the equilibrium is approximately U-shaped: higher income households live close to the city's center, income then decreases but it jumps up again at the city's boundary. Finally, income falls away in the outer suburb.

#### 4. BOUNDARY FIXEDNESS

In the equilibrium with income mixing, households adjacent to the jurisdictional boundary at  $B$  are *not* indifferent as to the jurisdiction in which they live. Referring to Figure 3, on the city side of the boundary households have income  $\underline{M}$  and strictly prefer the city to the suburb. On the suburban side of the boundary, households have income  $\bar{M}$  and strictly prefer the suburb to the city. Hence households on both sides of the boundary oppose a change in the boundary, and there is no particular reason why the boundary should shift into the side of lower rent. This is consistent with the findings of Epple and Romer (1989).

## 5. CONCLUSION

In this paper we have placed the model of fiscal competition inside a spatial model of a central city surrounded by a suburb. In doing so, we have produced a model in which there are equilibria with the property that, although the city has lower median income, the highest income in the city exceeds the lowest income in the suburb. The model thereby explains some stylized facts which the canonical models of Tiebout and Alonso-Mills-Muth are unable to explain. We are currently working on testing the model using Connecticut data.

In addition, the model explains the fixedness of jurisdiction boundaries and why there is no presumption that boundaries shift to expand jurisdictions with high rents. This explains the findings of Epple and Romer (1989).

The model has two limitations. Firstly, it assumes that all households have a fixed demand for land and the normality of the public service has been introduced by making a household's taste depend on its endowed income. We are currently addressing this limitation by reworking the model, making land demand endogenous and giving all households the same Cobb-Douglas utility function. Secondly, the model does not explain why the equilibrium with income-mixing has been selected by most U.S. cities. We are working on a model which makes this a consequence of the history of a city's development.

## APPENDIX A: PROOF OF LEMMAS A AND C

### PROOF OF LEMMA A:

The proof is sketched - a full proof is available from the authors on request. An equilibrium is constructed by considering an initial assignment as: The limit of city development is set at the jurisdictional boundary,  $X = B$ ; the associated value of  $Y$  is calculated by setting land demand equal to land supply, or  $N\alpha = \pi Y^2$ . Households are assigned to locations  $s$  so that households of income  $\underline{M}$  live at  $Y$  and household income increases steadily as the location moves closer to the metropolitan center: this assignment process sets the suburb and city locations  $y(M)$  and  $x(M)$ . The median income in each jurisdiction is calculated and the public service level in each jurisdiction is imputed. Set the rent at  $Y$  at  $r_0$  (Equation (12)): as the location moves inwards from the outer suburbs, rent rises (Equation (3)). As the location crosses the jurisdictional boundary at  $B$ , the rent must change to ensure that the boundary household at  $B$  is indifferent between the two jurisdictions (Equation (4)). If the required rent at the city's side of the boundary at  $B$  exceeds  $r_0$ , the allocation is an equilibrium. If the required rent at the city's side of the boundary is less than  $r_0$ , the assignment is adjusted to create an equilibrium as: steadily decrease  $X$  and at each  $X$  repeat the above assignment process. As  $X \rightarrow 0^+$ , the boundary household has income  $\bar{M}$  and strictly prefers the high public service level in the city, or  $\lim(X \rightarrow 0^+) r(X) > r_0$ . By continuity, there is some value of  $X$  for which  $r(X) = r_0$  and this is an equilibrium.

PROOF OF LEMMA C:

By assumption, households with incomes in the interval  $[M_1, M_2]$  reside in both jurisdictions. Therefore, these households must be indifferent between the two jurisdictions. In the region of overlap, Equation (7) implies that

$$\frac{d\beta}{dM} (V(g_s) - V(g_c)) = t(y(M) - x(M)).$$

The right-hand side is unambiguously positive. Therefore, the assumption that  $\beta$  increases with  $M$  implies that  $g_s$  must exceed  $g_c$ .

The public service in the city and in the suburb are the public services desired by voters with the median city income  $M_c^{med}$  and median suburb income  $M_s^{med}$ , or

$$\beta(M_c^{med}) V'(g_c) = 1;$$

$$\beta(M_s^{med}) V'(g_s) = 1.$$

$\beta' > 0$  and  $g_s > g_c$  implies  $M_s^{med} > M_c^{med}$ .

APPENDIX B: ALGORITHM USED TO CALCULATE EQUILIBRIUM  
FOR THE EXAMPLE USED TO ESTABLISH THE THEOREM

The metropolitan income distribution is uniform on the range  $[\underline{M}, \bar{M}]$  so that  $f(M) = 1 / (\bar{M} - \underline{M})$ . In addition,  $\beta(M) = bM$  and write  $V = V(g_s) - V(g_e)$ ,  $x_1 \equiv x(M_1)$  and  $y_2 \equiv y(M_2)$ . . Note that  $Na = \pi Y^2$ .

In the outer city,  $\underline{M} \leq M \leq M_1$  or  $x_1 \leq x \leq B$ : Land supply equals land demand, or

$$\pi (B^2 - x^2) = Na \int_{\underline{M}}^{M(x)} f(M) dM = Na \frac{M(x) - \underline{M}}{\bar{M} - \underline{M}} ;$$

or

$$M(x) = \underline{M} + \frac{\bar{M} - \underline{M}}{Y^2} B^2 - \frac{\bar{M} - \underline{M}}{Y^2} x^2 . \quad (B.1)$$

To determine  $M_1$  and  $x_1$  . From Equation (B.1):

$$M_1 = \underline{M} + \frac{\bar{M} - \underline{M}}{Y^2} B^2 - \frac{\bar{M} - \underline{M}}{Y^2} x_1^2 ;$$

From Equation (7):

$$bV = t(Y - x_1).$$

Solving simultaneously:

$$x_1 = Y - \frac{bV}{t} ; \quad (B.2)$$

$$M_1 = \underline{M} + \frac{\bar{M} - \underline{M}}{Y^2} B^2 - \frac{\bar{M} - \underline{M}}{Y^2} \left( Y - \frac{bV}{t} \right)^2 . \quad (B.3)$$

In the inner city and outer suburb,  $M_1 \leq M \leq M_2$ , or  $0 \leq x \leq x_1$  and  $y_2 \leq y \leq Y$ : Land supply equals land demand, or

$$\pi(B^2 - x^2) + \pi(Y^2 - y^2) = Na \int_{\underline{M}}^M f(M) dM = Na \frac{M - \underline{M}}{\bar{M} - \underline{M}}$$

Equation (7) implies:

$$bV = t(y - x).$$

These equations are solved simultaneously to give (noting that  $Na = \pi Y^2$ )

$$M = \underline{M} + \frac{\bar{M} - \underline{M}}{Y^2} \left( B^2 + Y^2 - \frac{b^2 V^2}{t^2} - 2 \frac{bV}{t} x - 2x^2 \right); \quad (\text{B.4})$$

$$M = \underline{M} + \frac{\bar{M} - \underline{M}}{Y^2} \left( B^2 + Y^2 - \frac{b^2 V^2}{t^2} + 2 \frac{bV}{t} y - 2y^2 \right). \quad (\text{B.5})$$

To determine  $M_2$  and  $y_2$ : From Equation (B.5)

$$M_2 = \underline{M} + \frac{\bar{M} - \underline{M}}{Y^2} \left( B^2 + Y^2 - \frac{b^2 V^2}{t^2} + 2 \frac{bV}{t} y_2 - 2y_2^2 \right).$$

Equation (7) implies

$$bV = ty_2.$$

Solving simultaneously:

$$y(M_2) = \frac{bV}{t}; \quad (\text{B.6})$$

$$M_2 = \underline{M} + \frac{\bar{M} - \underline{M}}{Y^2} \left( B^2 + Y^2 - \frac{b^2 V^2}{t^2} \right). \quad (\text{B.7})$$

In the inner suburb,  $M_2 \leq M \leq \bar{M}$  or  $B \leq y \leq y_2$ : land demand supply equals land demand, or

$$\pi(Y^2 - y^2) + \pi B^2 = Na \int_{\underline{M}}^M f(M) dM = Na \frac{M - \underline{M}}{\bar{M} - \underline{M}}$$

or

$$M = \underline{M} + \frac{1}{Y^2}(B^2 + Y^2 - y^2). \quad (\text{B.8})$$

Determining median income in the city,  $M_c^{med}$ . The household with median income locates at  $x^{med}$ ,

$$\pi x^{med\ 2} = \frac{1}{2}\pi B^2$$

or

$$x^{med} = \frac{B}{\sqrt{2}}.$$

Knowing  $x^{med}$ ,  $M_c^{med}$  is calculated using either Equation (B.1) if  $x^{med} \geq x_1$  or Equation (B.4) if  $x^{med} \leq x_1$ .

Determining median income in the suburb,  $M_s^{med}$ . The household of median income in the suburb locates at  $y^{med}$  such that:

$$\pi(Y^2 - y^{med\ 2}) = \frac{1}{2}\pi(Y^2 - B^2);$$

or

$$y^{med} = \sqrt{\frac{Y^2 + B^2}{2}}.$$

Knowing  $y^{med}$ ,  $M_s^{med}$  is calculated using either Equation (B.5) if  $y^{med} \geq y_2$  or Equation (B.8) if  $y^{med} \leq y_2$ .

Determining  $g_c$  and  $g_s$ . Substituting for  $M_c^{med}$  and  $M_s^{med}$ , the public service provided in the city and in the suburb is

$$bM_c^{med} V'(g_c) = 1 ; \quad (B.9)$$

$$bM_s^{med} V'(g_s) = 1 . \quad (B.10)$$

Noting that  $M_c^{med}$  and  $M_s^{med}$  are both functions of  $V \equiv V(g_s) - V(g_c)$ , they are solved simultaneously for  $g_c$  and  $g_s$ .

At the solution values, city rents must exceed the reservation rent, or

$$r(X) \geq r_0 .$$

Rent  $r(x_1)$  anchors the city's rent schedule. To calculate  $r(B)$ , first calculate  $r(x_1)$  using the indifference of the households of income  $M_1$  between locating at  $x_1$  and at  $Y$

$$M_1 - tM_1x_1 - r(x_1) - g_c + \beta(M_1)V(g_c) = M_1 - tM_1Y - r_0 - g_s + \beta(M_1)V(g_s) .$$

$r(B)$  is then calculated by integrating over  $dr/dM$  using Equation (3) and  $dx/dM$  calculated using

Equation (B.1)

$$r(B) = r(x_1) + \int_M^{M_1} tM \frac{dx}{dM} dM .$$

## REFERENCES

- Alonso, W., (1964), *Location and land use: toward a general theory of land rent*. Cambridge: Harvard University Press.
- Beckmann, M.J., (1969), "On the distribution of urban rent and residential density," *Journal of Economic Theory*, 1, 60-67.
- Berglas, E., (1976a), "On the theory of clubs," *Papers and Proceedings of the American Economic Association* 66, 116-121.
- Berglas, E., (1976b), "Distribution of tastes and skills, and the provision of local public goods," *Journal of Public Economics* 6, 409-423.
- de Bartolome, C.A.M., and S.L. Ross, (2003), "Equilibrium with local governments and commuting: income sorting v. income mixing," *Journal of Urban Economics* 54, 1-20.
- Elickson, B., (1971), "Jurisdictional fragmentation and residential choice," *American Economic Review* 61, 334-339.
- Epplé, D., and G.J. Platt, (1998), "Equilibrium and local redistribution in an urban economy when households differ in both preferences and income," *Journal of Urban Economics*, 43, 23-51.
- Epplé, D., R. Filimon and T. Romer, (1984), "Equilibrium among local jurisdictions: toward an integrated treatment of voting and residential choice," *Journal of Public Economics*, 24, 281-308.
- Fujita, M., (1989), *Urban Economic Theory*. Cambridge: Cambridge University Press.
- Glaeser, E.S., M.E. Kahn and J. Rappaport, (2000), "Why do the poor live in cities," *National Bureau of Economic Research Working Paper #7636*.

- LeRoy S.F., and J. Sonstelie, (1983), "Paradise lost and paradise regained: transportation, innovation, income and residential location," *Journal of Urban Economics* 13, 67-89
- McGuire, M.C., (1974), "Group segregation and optimal jurisdictions," *Journal of Political Economy* 82 ,112-132.
- McGuire , M.C., (1991), "Group Composition, Collective Consumption, and Collaborative Production," *American Economic Review* 81 1391-1407.
- Mills, E.S., (1967), "An aggregative model of resource allocation in a metropolitan area," *Papers and Proceedings of the American Economic Association* 57, 197-210.
- Montesano, A., (1972), "A restatement of Beckmann's model on the distribution of urban rent and residential density," *Journal of Economic Theory*, 4, 329-354.
- Muth, R., (1961), "The spatial structure of the housing market," *Papers and Proceedings of the Regional Science Association* 7, 207-220.
- Muth, R., (1969), *Cities and Housing*. Chicago: University of Chicago Press.
- Perrson, T., and G. Tabellini, (2000), *Political Economics: Explaining Economic Policy*. Cambridge, MA: MIT Press
- Rose-Ackerman, S., (1979), "Market models of local government: exit, voting and the land market", *Journal of Urban Economics* 6, 319-337.
- Ross, S.L., and J. Yinger, (1999), "Sorting and voting: a review of the literature on urban public finance," in *The Handbook of Urban and Regional Economics*, Vol 3, edited by P. Cheshire and E.S. Mills. Amsterdam: North Holland.
- Tiebout, C.M., "A pure theory of local expenditures," *Journal of Political Economy* 94, 416-424.
- Wheaton, W.C., (1975), "Consumer mobility and community tax bases", *Journal of Public*

*Economics* 4, 377-384.

Wheaton, W.C., (1977), "Income and urban residence: an analysis of consumer demand for location," *American Economic Review* 67, 620-631.

Wheaton, W.C., (1977), "Monocentric models of urban land use: contributions and criticisms,"

Wooders, M., (1978), "Equilibria, the core and jurisdictional structures in economies with a local public good," *Journal of Economic Theory* 18, 328-348.

Yinger, J., (1982), "Capitalization and the theory of local public finance," *Journal of Political Economy* 90, 917-943.

## ENDNOTES

1. A possible reason for the difference between the older cities and the new cities is that the newer cities have more decentralized employment (as noted by Glaeser et al. (2000)) .
2. The assumption of a fixed housing size greatly simplifies the analysis as it makes constant the population of each jurisdiction and ensures that, within each jurisdiction, the rich households live closer to the central business district. In addition, it avoids well-known existence problems for traditional stratified local public finance equilibria (as in Rose-Ackerman (1979) and Epple, Filimon and Romer (1984, 1993)) and it allows us to focus thereby on the existence of the equilibrium with income-mixing.
3. We want to stress that, because household  $i$ 's income is exogenous, its taste parameter  $\beta_i \equiv \beta(M_i)$  is exogenous.
4. For convenience of presentation, the public service is assumed to show constant returns to jurisdiction size. Because each jurisdiction contains a fixed number of households, no results change if the service is a local public good.
5. See Fujita (1989, Chap. 4) for a more general development. See Beckman (1969) and Montesano (1972) for the development of an urban model with an income distribution.
6. The importance of using the absolute value in this way is stressed by Montesano (1972).
7. We do not believe that the myopic assumption is important. What is important is that the suburb votes a different public service than the city. See Epple and Romer (1986) for non-myopic voting in a model with redistributive local governments.
8. A household with income  $M$  is located at  $s(M)$ . His desired public service is:

$$g = \underset{g}{\operatorname{argmax}} \quad M - tMs - r(s) - g + T + \beta(M)V(g),$$

or is defined by the first-order condition:

$$\beta(M)V'(g) = 1.$$

Differentiating with respect to  $M$  and rearranging,

$$\frac{dg}{dM} = - \frac{\beta'(M)V'(g)}{\beta V''(g)} > 0.$$

9. If there is undeveloped land at the periphery of the city, the rent at the limit of development is  $r_0$ :

$$X < B : r(X) = r_0 .$$

10. If there is undeveloped land in the city,

$$\pi X^2 + \pi(Y^2 - B^2) = Na .$$