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Tax Competition and the Creation of Redundant Products

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ABSTRACT

There are products which are assembled from kits but which, once assembled, are identical to other products. An example is the roll-your-own cigarette. Because the kit requires time to assemble, it is more costly than the assembled product; in the absence of taxation, the kit is not bought or is "redundant". Regions seek to maximize the tax revenue gained by excise taxes. We show that tax competition supports strategies which tax the "redundant" product at a lower tax rate than its assembled counterpart and it is bought. A welfare loss is thereby created.

Key Words: tax competition, new products, cigarettes

Suggested Running Title: Tax Competition and Redundant Products

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1. INTRODUCTION

It is now widely understood that the competition between tax authorities seeking to maximize tax revenue is similar in many respects to the competition between firms seeking to maximize profits: the ability of a tax authority to extract tax revenue is limited by the presence of other tax authorities and this limitation may be “efficiency enhancing”.[1] Most of the literature has focused on taxes associated with factors of production - namely capital and labor - although a few authors (e.g. Mintz and Tulkens (1986), de Crombrugghe and Tulkens (1990) and Kanbur and Keen (1993)) have discussed taxes associated with consumption. This paper shows that competition for excise tax revenue may lead to the creation of commodities which otherwise would not exist or would be “redundant” and discusses the welfare implications. I use the competition between Canadian provinces for tobacco tax revenue as my motivating example.

In the Canadian tobacco market there are two cigarette products - the traditional assembled cigarette and the cigarette kit (or stick). As the former product the cigarette is bought rolled and ready to smoke but as the latter product the cigarette is bought as a kit which contains tobacco and a sleeve and which must be assembled prior to smoking. Once assembled, the kit is (almost) identical to the traditional cigarette. Because their tobacco contents are the same, the firm's prices for the two products are (approximately) the same.[2] To the firm’s price the consumer adds the opportunity cost of the time he spends assembling the kit so that, if there were no taxes, the kit would have a higher “inclusive price” than the traditional cigarette and would not be bought. Therefore the cigarette kit is a redundant product in the sense that it is dominated by an identical product of lower cost. However, provinces levy tobacco taxes and the model suggests that competition for tax revenue leads provinces to choose to tax the cigarette kit at a lower rate than the traditional cigarette and the kit is bought. As such, tax competition creates a product which would otherwise not be produced.

The incentive to lower the tax on the cigarette kit arises because the province sets the legal tax rate on cigarettes smoked in the province (i.e., the tax is a destination-based tax). If Province A has a lower tax rate than Province B, there is the possibility of arbitrage: an entrepreneur can make a profit by buying cigarettes in Province A (paying the associated tax) and trucking the cigarettes across that province's boundary for sale in Province B (where the cigarettes are sold illegally and without tax being paid to Province B). In this way, by lowering its tax rate, Province A is able to gain tax revenue on the cigarettes "exported" to Province B. However, the lower tax rate must be applied to all cigarettes bought in Province A so that Province A gains less tax revenue from the cigarettes sold in Province A. The province therefore seeks to find a way to limit the tax cut to the products which are predominantly exported - these products are the kits.

In Province B a buyer of illegally-imported cigarettes may be caught and punished so that such cigarettes have a "psychological cost" or risk premium associated with their purchase. The risk premium is likely to be proportional to the income of the buyer. Therefore the individuals who buy the contraband cigarettes are likely to be of low income. However, as noted earlier, a kit must be assembled and the opportunity cost of time is a component of the inclusive price so that kits tend to be bought by low-income individuals. Therefore, by lowering its tax rate on kits, a province can target exports to individuals most likely to buy contraband cigarettes and limit the cut in its domestic tax base to the cigarettes bought by its low-income residents.

In our model the demand for cigarettes is inelastic so that, in the absence of tax competition, a province can raise almost unlimited tax revenue by raising the tax rate on cigarettes. Tax competition limits the ability of the provincial government to collect tax revenue - "Leviathan is tamed" - but the control of tax revenue comes at a cost. Some kits are bought which are of higher inclusive cost than their more-highly taxed assembled counterparts.

The cigarette kit was created originally to take advantage of the difference in tax rates between pipe-tobacco and cigarettes: entrepreneurs realized the advantage of designing a cigarette which would be taxed at the lower rate associated with pipe-tobacco. Cars with three wheels were similarly developed in the 1960s in the United Kingdom to take advantage of the lower tax rate on motor-cycles relative to cars: they were designed to be taxed as a motor-cycle and marketed as a car. Like the cigarette kit, use of the three-wheeled car required additional time as it traveled relatively slowly and its buyers tended in consequence to be of relatively low income - people for whom the lower after-tax price mattered more than the opportunity cost of the slower travel. However, unlike the cigarette kit, the three-wheeled car's life was short: the U.K. government quickly redefined the appropriate tax rates and production of the car was effectively abandoned. I attribute the difference in the two cases to the importance of tax competition. In Canada the provinces have taxing authority and the strong competition between provinces for tax revenue maintains the separate tax rate on the kits. In contrast, the U.K.'s tax policy is set centrally with no competition between regions so that there is no incentive for the state to maintain a lower tax rate on the redundant product. In general, the argument presented in this paper applies whenever there are portable products which are taxed using excise taxes - so that similar products can be taxed at different rates - and tax competition. Other possible examples include beer - brewed beer and home-brew beer kits are often taxed at different rates -

and the different tax rates levied by U.S. states on chewing tobacco and cigarettes.

The model can be considered an extension of the models of Mintz and Tulkens (1986) and de Crombrugghe and Tulkens (1990). Both of these models have destination-based taxes so that residents in the high-tax region may reduce their tax payments by crossing the border and buying the product in the low-tax region. Such cross-border shopping takes time and so it tends to be done by low-income households. The time cost of traveling across the border in their model functions in a way similar to the psychological cost or risk-premium of buying an illegal product in my model. However, in these models, there is one exogenous product which is taxed. This paper extends this to show that the tax competition between the regions may endogenously create a new product - similar to the old product except that it is marketed as a kit and requires time to assemble - and that this new product is taxed at a lower rate.

The paper is organized as follows. Section 2 presents the model and Section 3 solves the model for the case when the regions are symmetric. Section 4 considers the case when the regions are asymmetric in population and in their wage distributions. *Ceteris paribus* a less-populated region is more aggressive in trying to create sales in the other regions. In consequence it has a lower tax rate and exports the product. These results are similar to the results of Kanbur and Keen (1993). Section 5 discusses the welfare implications. Section 6 concludes.

2. THE MODEL

2.1 Model structure

There are two regions denoted by the subscript i, $i = 1, 2$. The population of Region i is N^i. An individual earns a wage w and the wage is distributed over the interval $[0, \bar{w}]$ with density $f(w)$. As noted in the Introduction, I use the Canadian cigarette market as the motivating example but the model is readily extended to other markets. There are two types of cigarettes, the *a*ssembled cigarette (denoted by the subscript a) and the cigarette *k*it (denoted by the subscript k). The products are assumed to have the same production cost: I assume that the market is competitive so that competition forces firms to sell the two products at the same producer price p.[3] An individual living in Region i and earning wage w can potentially buy cigarettes in four forms:

(1) *assembled cigarettes taxed by his own Region i* : the region levies an excise tax t_a^i on assembled cigarettes so that, if he buys this product, he pays the price $p + t_a^i$.

(2) *assembled cigarettes taxed by the other Region j* ($j \neq i$): the excise tax levied on assembled cigarettes by Region j is t_a^j . However, such a cigarette must be smuggled into Region i and buying an illegal product imposes a "psychological cost" on the individual. This "psychological cost" is associated with the risk of being "caught", or is similar to a risk premium; I assume that it increases with income so that the monetary value of this cost is $w\delta$.[4] Hence the inclusive price to the individual in Region i of an assembled cigarette smuggled from Region j is $p + t_a^j + w\delta$.

(3) *cigarette kits taxed by his own Region i*: the region levies an excise tax t_k^i on cigarette kits. It takes time T for an individual to assemble the kit so that the inclusive price to an individual of the cigarette bought as a kit is $p + t_k^i + wT$.

(4) *cigarette kits taxed by the other Region j* ($j \neq i$): the excise tax levied on cigarette kits by

Region *j* is t_k^j. In addition to the cost of assembling, *wT*, these cigarettes are smuggled and buying an illegal product imposes the psychological cost of $w\delta$. Hence the inclusive price to the individual in Region *i* of a kit taxed in Region *j* is: $p + t_k^j + wT + w\delta$.

The individual chooses the cigarette with the lowest price, or the relevant price of a cigarette for the individual in Region *i* is

$$q = \min[p + t_a^i , p + t_a^j + w\delta , p + t_k^i + wT , p + t_k^j + wT + w\delta].$$

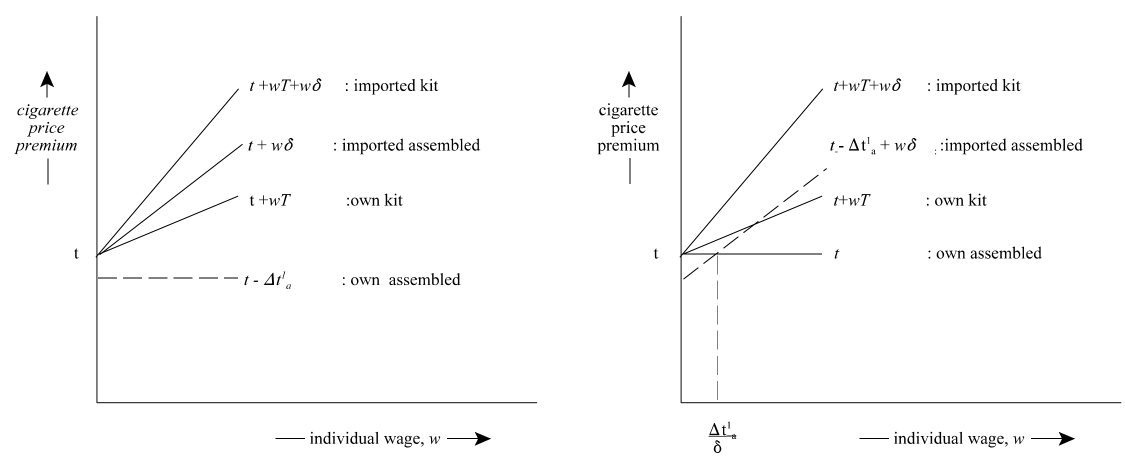
Notice that the price to the individual may depend on his wage *w*. For ease of presentation, an individual's demand for cigarettes is assumed to be inelastic at quantity *Q*. Using a price-sensitive demand structure would make the structure more realistic but would not change any of the qualitative results.[5]

Each region is a Leviathan and its objective is to maximize its tax revenue; its strategy is its choice of tax rates. The assumption that each government is a Leviathan allows issues associated with the use of the tax revenue to be ignored. Each region is constrained by the choice of the other region - if it sets its tax rate too high, there is smuggling and its tax base shrinks. At the Nash Equilibrium Region *i* chooses its strategy (t_k^i , t_a^i) such that, given the strategy (t_k^j , t_a^j) of the other region, (t_a^i , t_k^i) maximizes its tax revenue; and *vice versa* for

Region j.

2.2 Intuitive analysis for the lower tax rate on kits

This section provides the intuition for the reduced tax rate on kits. For ease of presentation I focus on the case of identical regions. Initially I suppose that regions are required to set the same tax rate on all cigarette products so that at equilibrium $t_k^1 = t_k^2 = t_a^1 = t_a^2 = t$. Figure 1 shows how the prices of the different cigarette products vary in Region 1 and Region 2 as the wage *w* of the smoker varies. The vertical axis is the price premium - the excess of the cigarette's inclusive price above the producer price *p*. An "own" cigarette is a cigarette taxed by the region in which the smoker resides and an "imported" cigarette is a cigarette taxed by the other region. The individual with income *w = 0* faces the same inclusive price for all products and all other individuals face strictly lower prices for assembled product taxed in the home region.



(a) in Region 1 (b) in Region 2

Figure 1: effect if Region 1 lowers its tax on assembled product

Suppose that Region 1 lowers the tax rate on the assembled product to $t - \Delta t_a^1$. In

Figure 1(a) the price line for an own assembled cigarette and in Figure 1(b) the price line for imported assembled cigarette shifts down . Region 1 would sell assembled cigarettes to individuals in Region 2 for whom the price of the imported assembled product is less than the price of the assembled product

taxed in Region 2, or for whom $p + (t - \Delta t_a^1) + w\delta \leq p + t$, or for whom $w \leq \Delta t_a^1 / \delta$. The local wage

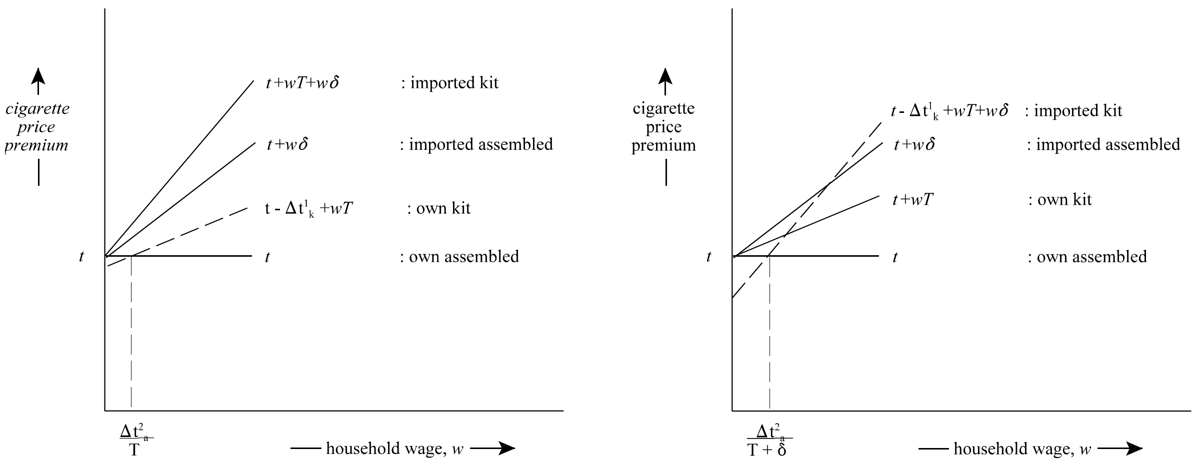
density is *f(0)* so that total exports would be $N\,Q\,f(0)\,\Delta t_a^1 / \delta$. Region 1 would gain additional tax revenue from these exports of

$$N\,Q\,f(0)\,\frac{\Delta t_a^1}{\delta}\,(t - \Delta t_a^1).$$

Because (approximately) tax revenue *t* is generated on each new sale, the tax revenue gain is first-order. However, the lower tax rate lowers the price of assembled cigarettes sold in Region 1 so that Region 1 loses tax revenue on all own sales of

$$N\,Q\,\Delta t_a^1 \ .$$

Because the tax rate decrease is being applied to all domestic sales, the tax revenue loss from domestic sales is of first-order. Competition between the regions sets the tax rate *t* so that the first-order gain equals the first-order loss; neither region gains from changing its tax rate.



In Region 1 In Region 2

Figure 2: effect if Region 1 lowers its tax on kits

Suppose now that kits may be taxed at a different rate to the assembled product. Does Region 1 have an incentive to lower the tax rate on kits? If Region 1 lowers the tax rate on kits to $t - \Delta t_k^1$, the relevant price lines are shifted down and become the dashed lines in Figure 2. Region 1 sells cigarette kits to individuals in Region 2 for whom $p + (t - \Delta t_k^1) + wT + w\delta \leq p + t$, or for whom $w \leq \Delta t_k^1 / (T + \delta)$, and it gains tax revenue from these new sales as

$$N Q f(0) \frac{\Delta t_k^1}{T + \delta} (t - \Delta t_k^1)$$

Again, because (approximately) tax revenue t is generated on each new sale, the tax revenue gain is of first order. In its own region, the lower tax rate on kits causes individuals to shift from assembled product into kits if $p + (t - \Delta t_k^1) + wT \leq p + t$ or if $w \leq \Delta t_k^1 / T$. Tax revenue falls by

$$N\,Q\,f(0)\,\frac{\Delta t_k^1}{T}\,\Delta t_k^1$$

Because only a few individuals are switching to the lower-taxed kit and each cigarette switched gives only slightly less tax revenue, the tax revenue loss on domestic sales is of second-order. Hence if $t_a^1 = t_a^2 = t_k^2$, by lowering its tax rate on kits, Region 1 has a first-order revenue gain but

only a second-order revenue loss, or Region 1 has an incentive to lower the tax rate on kits.

2.3 Formal analysis for the lower tax rate on kits

Proposition 1 below establishes formally that, at the symmetric Nash Equilibrium, kits are taxed at a lower rate than the assembled product.

PROPOSITION 1: *At the symmetric Nash equilibrium (if it exists):* $t_k^1 \;=\; t_k^2 \;<\; t_a^1 \;=\; t_a^2$.

PROOF: Note that if a region *i* sets $t_k^i \;\geq\; t_a^i, \quad (i = 1, 2)$ it sells no kits. Therefore if any equilibrium strategy has $t_k^i > t_a^i$, there is another equilibrium strategy for which $t_k^i \;=\; t_a^i$ and no generality is lost by restricting attention to strategies for which $t_k^i \;\leq\; t_a^i, \quad (i = 1, 2)$.

The proof is by contradiction. I assume that $t_k^1 = t_a^1 = t_k^2 = t_a^2$ is a Nash Equilibrium and then show that $t_k^1 = t_a^1 = t$ is not a best response to $t_k^2 = t_a^2 = t$. Suppose Region 2 uses a strategy $t_k^2 \;=\; t_a^2 \;=\; t$ and suppose Region 1 sets $t_a^1 \;=\; t_k^2 \;=\; t_a^2$, but sets $t_k^1 \;<\; t_a^1$. In its own region it sells kits to individuals for whom $t_k^1 + wT \leq t_a^1$ or for whom $w \leq (t_a^1 - t_k^1)\,/\,T$ and assembled product otherwise. In Region 2 it sells kits to individuals for whom $t_k^1 + wT + w\delta \leq t_a^2$ or for whom $w \leq (t_a^2 - t_k^1)/(T+\delta)$. Its tax revenue is

$$R^1 = N^1 \int_0^{\frac{t_a^1 - t_k^1}{T}} t_k^1 Q f(w) dw + N^1 \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1 Q f(w) dw + N^2 \int_0^{\frac{t_a^2 - t_k^1}{T+\delta}} t_k^1 Q f(w) dw .$$

The first term is the tax revenue from kits sold in Region 1, the second term is the tax revenue from assembled product sold in Region 1 and the third term is the tax revenue on kits exported to Region 2. By inspection, tax revenue R^1 changes continuously as t_k^1 approaches $t_a^1 = t_a^2$ from below. Differentiating,

$$\frac{dR^1}{dt_k^1} = N^1 \int_0^{\frac{t_a^1 - t_k^1}{T}} Q f(w) dw - N^1 t_k^1 Q f\left(\frac{t_a^1 - t_k^1}{T}\right) \frac{1}{T}$$

$$+ N^1 t_a^1 Q f\left(\frac{t_a^1 - t_k^1}{T}\right) \frac{1}{T} + N^2 \int_0^{\frac{t_a^2 - t_k^1}{T+\delta}} Q f(w) dw - N^2 t_k^1 Q f\left(\frac{t_a^2 - t_k^1}{T+\delta}\right) \frac{1}{T+\delta} .$$

In the limit

$$\lim_{t_k^1 \to t_a^{1-}} \frac{dR^1}{dt_k^1} = -N^2 t_k^1 Q f(0) \frac{1}{T+\delta} < 0.$$

Summarizing, if Region 2 uses a strategy for which $t_k^2 = t_a^2$ and Region 1 sets $t_a^1 = t_k^2 = t_a^2$, it would gain tax revenue by setting $t_k^1 < t_a^1$. Therefore a strategy with $t_k^1 = t_a^1 = t_k^2 = t_a^2$ is not a best response of Region 1 and $t_k^1 = t_a^1 = t_k^2 = t_a^2$ cannot be a Nash Equilibrium. Because symmetry requires $t_k^1 = t_k^2$ and $t_a^1 = t_a^2$, and strategies have the property $t_k^i \leq t_a^i$, any symmetric Nash equilibrium must have $t_k^1 = t_k^2 < t_a^1 = t_a^2$.

3. THE SYMMETRIC NASH EQUILIBRIUM

I proceed heuristically to characterize the symmetric Nash Equilibrium by considering the tax rates from which small perturbations give no tax revenue gain. Knowing that at the symmetric Nash Equilibrium $t_k^1 = t_k^2 = t_k$, $t_a^1 = t_a^2 = t_a$ and $t_k < t_a$, I consider the tax revenue change if Region 1 were to marginally change its tax rate structure.

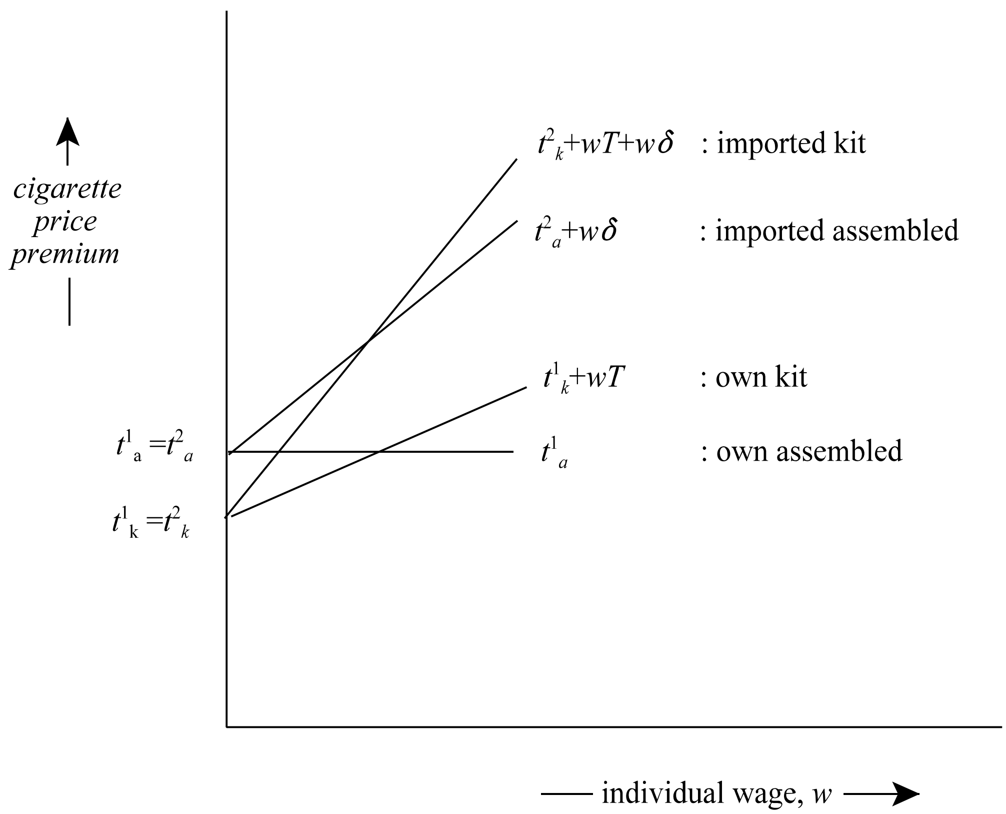


Figure 3 : excess of inclusive prices above producer prices in Region 2 if $t_k^1 = t_k^2 < t_a^1 = t_a^2$

Referring to Figure 3, when $t_k^1 = t_k^2 < t_a^1 = t_a^2$, all residents in Region 1 buy cigarettes taxed

in Region 1; residents buy kits for whom $p + t_k^1 + wT \leq p + t_a^1$ or for whom $w \leq (t_a^1 - t_k^1) / T$,

and

buy assembled product otherwise. The region has an external and an internal margin. Because the risk premium increases with the individual's wage *w*, the only individuals who are indifferent between own and imported cigarettes are the individuals with wage $w = 0$. These individuals face a lower inclusive price for kits than assembled product. Hence by lowering the tax it charges on kits, Region 1 can seek to gain tax revenue from sales to these individuals in Region 2; these individuals constitute the external margin of a region. In contrast, a marginal change in the tax rate on assembled product changes the quantity of kits and assembled product sold domestically,

but does not generate external sales. Individuals with wage $w = (t_a^1 - t_k^1) / T$ therefore constitute the internal margin for a region.

If $t_k^1 = t_k^2$, no cigarettes taxed in Region 1 are sold in Region 2. If Region 1 lowers t_k^1 below t_k^2, it changes the internal and the external margin by selling kits to its own residents for whom $w \leq (t_a^1 - t_k^1) / T$ and by selling kits in Region 2 to individuals for

whom $p + t_k^1 + wT + w\delta \leq p + t_k^2 + wT$ or for whom $w \leq (t_k^2 - t_k^1) / \delta$. Hence its revenue is:

$$t_k^1 \leq t_k^2 : \quad R^1 = N^1 \int_0^{\frac{t_a^1 - t_k^1}{T}} t_k^1 Q f(w)\, dw + N^1 \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1 Q f(w)\, dw + N^2 \int_0^{\frac{t_k^2 - t_k^1}{\delta}} t_k^1 Q f(w)\, dw \ . \qquad (1)$$

The firsts term is the tax revenue from domestic sales of kits, the second term is the tax revenue from domestic sales of assembled product and the third term is tax revenue from sales of kits exported to Region 2.

Similarly, if Region 1 raises its tax rate on kits above t_k^2, its residents switch to imported kits if $p + t_k^2 + wT + w\delta \leq p + t_k^1 + wT$ or if $w \leq (t_k^1 - t_k^2)/\delta$; residents with wages w such that $(t_k^1 - t_k^2)/\delta < w \leq (t_a^1 - t_k^1)/T$ buy own kits and residents for whom $w > (t_a^1 - t_k^1)/T$ buy own assembled product. Region 1 makes no sales in Region 2 and its tax revenue is

$$t_k^1 > t_k^2 : \quad R^1 = N^1 \int_{\frac{t_k^1 - t_k^2}{\delta}}^{\frac{t_a^1 - t_k^1}{T}} t_k^1 Q f(w)\, dw + N^1 \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1 Q f(w)\, dw. \tag{2}$$

Comparison of Equations (1) and (2) shows that the revenue function is continuous at $t_k^1 = t_k^2$.

The marginal tax revenue function is potentially discontinuous as the region switches from importing to exporting. To consider its properties, consider first the case $t_k^1 < t_k^2$.

Differentiating Equation (1)

$$t_k^1 < t_k^2 : \quad \frac{dR^1}{dt_k^1} = N^1 \int_0^{\frac{t_a^1 - t_k^1}{T}} Q f(w)\, dw - N^1 t_k^1 Q f\left(\frac{t_a^1 - t_k^1}{T}\right)\frac{1}{T} + N^1 t_a^1 Q f\left(\frac{t_a^1 - t_k^1}{T}\right)\frac{1}{T}$$

$$+ N^2 \int_0^{\frac{t_k^2 - t_k^1}{\delta}} Q f(w)\, dw - N^2 t_k^1 Q f(\frac{t_k^2 - t_k^1}{\delta})\frac{1}{\delta};$$

or $(t_k^1 < t_k^2)$

$$\frac{dR^1}{dt_k^1} = N^1 \int_0^{\frac{t_a^1 - t_k^1}{T}} Qf(w)\,dw + N^1 (t_a^1 - t_k^1) Qf\left(\frac{t_a^1 - t_k^1}{T} \right) \frac{1}{T} + N^2 \int_0^{\frac{t_k^2 - t_k^1}{\delta}} Qf(w)\,dw - N^2 t_k^1 Qf(\frac{t_k^2 - t_k^1}{\delta}) \frac{1}{\delta} \,. \tag{3}$$

The first term is the gain in tax revenue from the higher tax rate on pre-existing own sales of kits, the second term is the gain in tax revenue from the sales shifted from low-tax own kits to high-tax own assembled product, the third term is the gain in tax revenue from the higher tax rate on pre-existing exports of kits and the fourth term is the loss in tax revenue due to the lower quantity of exports.

Now consider the case $t_k^1 > t_k^2$. Differentiating Equation (2) and collecting terms:

$$t_k^1 > t_k^2 \ : \quad \frac{dR^1}{dt_k^1} = N^1 \int_{\frac{t_k^1 - t_k^2}{\delta}}^{\frac{t_a^1 - t_k^1}{T}} Q\,f(w)\,dw \ + \ N^1 (t_a^1 \ - \ t_k^1)\, Q\,f\left(\frac{t_a^1 \ - \ t_k^1}{T} \right) \frac{1}{T} - N^1 \, t_k^1 \, Q\,f\left(\frac{t_k^1 \ - \ t_k^2}{\delta} \right) \frac{1}{\delta} \,. \tag{4}$$

The first term is the gain in tax revenue from the higher tax rate on pre-existing sales of own kits, the second term is the gain in tax revenue from the sales shifted from low-tax own kits to high-tax own assembled product and the third term is the loss in tax revenue due to sales shifted from own kits to imported kits.

Comparing Equations (3) and (4) in the limit as $t_k^1 \rightarrow t_k^2$, note that dR^1 / dt_k^1 is continuous at $t_k^1 = t_k^2$ only if $N^1 = N^2$. In this case, there is continuity in the rate at which export sales are gained as t_k^1 is lowered below t_k^2 and in the rate at which own sales are lost if t_k^1

is raised above t_k^2. Otherwise, if $N^1 \neq N^2$, dR^1/dt_k^1 is discontinuous at $t_k^1 = t_k^2$.

Finally, for small deviations of t_a^1 around t_a^2, the only effect is to shift smokers between own kits and own assembled product. The tax revenue is:

$$R^1 = N^1 \int_0^{\frac{t_a^1 - t_k^1}{T}} t_k^1\, Q f(w)\, dw \; + \; N^1 \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1\, Q f(w)\, dw .$$

Differentiating with respect to t_a^1 and collecting terms,

$$\frac{dR^1}{dt_a^1} = N^1 \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} Q f(w)\, dw \; - \; N^1 (t_a^1 - t_k^1)\, Q f\!\left(\frac{t_a^1 - t_k^1}{T} \right) \frac{1}{T} . \tag{5}$$

The first term is the tax revenue gain on the preexisting sales of assembled product and the second term is the tax revenue loss associated with marginal smokers shifting from assembled product to kits.

At the symmetric Nash Equilibrium, $N^1 = N^2 = N$, $t_k^1 = t_k^2 = t_k$ and $t_a^1 = t_a^2 = t_a$. t_k and t_a are

set so that no region gains tax revenue by a marginal deviation in its tax rate. Knowing dR^1/dt_k^1 is continuous through $t_k^1 = t_k$ implies that t_k occurs when

$$\frac{1}{N} \frac{dR^1}{dt_k^1} \Big|_{t_k^1 = t_k^2 = t_k} = \int_0^{\frac{t_a - t_k}{T}} Q f(w)\, dw \; + \; (t_a - t_k) Q f\!\left(\frac{t_a - t_k}{T} \right) \frac{1}{T} \; - \; t_k Q f(0) \frac{1}{\delta} = 0; \tag{6}$$

similarly t_a occurs when

$$\frac{1}{N}\frac{dR^1}{dt_a^1}\Big|_{t_a^1 = t_a^2 = t_a} = \int_{\frac{t_a - t_k}{T}}^{\bar{w}} Qf(w)\, dw - (t_a - t_k)\, Qf\left(\frac{t_a - t_k}{T}\right)\frac{1}{T} = 0 . \tag{7}$$

Solving,

$$t_k = \frac{\delta}{f(0)} .$$

A region's tax on kits depends on *f(0)* because a region's external margin is the individual with $w = 0$. As *f(0)* increases, there is more gain by making illegal exports to the other region and each region is more aggressive in setting its tax rate on kits. Hence the Nash Equilibrium value of t_k falls. Conversely, as δ increases, each region generates less exports if it lowers the tax rate on kits: it is therefore less aggressive and the equilibrium value of t_k rises.

If wages are uniformly distributed, $f(.) = 1/\bar{w}$ and

$$t_k = \delta\bar{w} .$$

In this case Equation (7) yields:

$$t_a = (\delta + \frac{T}{2})\bar{w} .$$

As T increases, fewer individuals at the internal margin shift from assembled product to kits as a region raises its tax rate on assembled product; regions respond by raising the "tax premium" on assembled product.

The above discussion is heuristic and focuses on small deviations from the equilibrium strategy. The derived conditions are necessary conditions. Proposition 2 below confirms that, for the uniform distribution, they are sufficient conditions provided $\delta \geq 2T$.

PROPOSITION 2: *If wages are distributed uniformly on* $[0, \bar{w}]$, *the strategies* $\left(t_k^1 = \delta\,\bar{w},\ \ t_a^1 = \left(\delta + \frac{T}{2}\right)\bar{w}\right)$ *and* $\left(t_k^2 = \delta\,\bar{w},\ \ t_a^2 = \left(\delta + \frac{T}{2}\right)\bar{w}\right)$ *are a symmetric Nash Equilibrium provided* $\delta \geq 2T$.

PROOF: See Appendix A.

If $\delta < 2T$, the risk premium is sufficiently low that Region 1 can generate a large quanity of exports by slightly lowering its tax rates. Its best response is to lower its tax rate on assembled product to sell cigarettes to *all* individuals in Region 2 who would otherwise buy kits taxed in Region 2.

Propositions 1 and 2 are the central results of this paper. In the absence of tax competition (i.e., if the residents in each region could only buy own cigarettes), a Leviathan region would tax kits at the same rate as assembled product and in consequence kits would not be sold (i.e., they would be a "redundant" product). Tax competition creates the incentive for regions to seek to lower taxes in order to gain tax revenue on sales in foreign regions. Because of the risk penalty, the marginal individual who buys the imported product has a low-wage. But low-wage individuals are particularly suited to the kit and hence, in order to gain foreign sales while maintaining high taxes on its "base" of assembled product, a region differentially lowers the tax rate on the kit. By so doing, it ensures he production of the “redundant” product.

4. ASYMMETRIC REGIONS

I now consider the asymmetric cases of unequal populations and different wage distributions. First, suppose that Region 1 is the smaller region, $N^1 < N^2$, and that the two regions continue to have identical wage distributions. The larger market size of Region 2 means that there is a greater advantage to Region 1 than to Region 2 in lowering its tax rate on kits and having kits exported. Conversely, having a larger domestic tax base, Region 2 is more reluctant to lower its tax rate to defend its tax base from marginal imports. Therefore Region 1 is more aggressive or $t_k^1 < t_k^2$. This is now shown formally.

PROPOSITION 3: *for a small perturbation from the symmetric equilibrium,*[6] $N^1 < N^2$ *implies* $t_k^1 < t_k^2$.

PROOF: see Appendix B

Second, suppose the regions have different wage distributions (but equal populations) and Region 2 has the greater density of individuals with $w = 0$. Suppose the regions apply the same tax rate on kits. If Region 1 lowers slightly its tax rate on kits, it generates more exports than if Region 2 lowers its tax rate on kits by an equal amount. Hence Region 1 has a greater incentive to undercut, or equal tax rates on kits cannot be an equilibrium.

If Region 1 sets a lower tax rate on kits than Region 2, it exports kits to Region 2 so that Region 2 has a smaller tax base than Region 1. As a thought-experiment, suppose Region 1 increases its tax rate on kits and on assembled product; it gains additional tax revenue from applying the higher tax rate to its pre-existing sales but loses some export sales and the

associated tax revenue. If Region 1 is setting its tax rates as the best response to the tax rates applied by Region 2, these effects must offset so that no additional revenue is generated. Compare these results with those of an alternative thought-experiment in which Region 2 lowers its tax rate on kits and on assembled product by the same amount as Region 1 raised them. Region 2's smaller tax base means that it loses less tax revenue on its pre-existing sales than the tax revenue that was gained by Region 1 on its pre-existing sales. Kit imports decrease so that Region 2 gains the same quantity of own kit sales as Region 1 lost in the earlier thought experiment. However, it is charging a higher tax rate so that this gain of kit sales translates into a larger tax revenue gain than the tax revenue loss of Region 1 in the earlier thought-experiment. Overall, if Region 1 experiences no change in tax revenue from the tax rate increase, Region 2 must gain tax revenue from the tax rate decrease. Or Region 2's tax rates cannot be a best response. Summarizing, if Region 2 has a higher density of individuals with
$w = 0$ (and equal population), Region 1 has an incentive to have a lower tax rate on kits than Region 2, but this cannot be an equilibrium. Put differently, if equilibrium exists, it cannot be a small perturbation from the symmetric case.

PROPOSITION 4: if wage distributions differ so that the density of individuals with $w = 0$ differs in the two regions (but populations are the same), there is no Nash Equilibrium which is a small perturbation from the symmetric case.

PROOF: see Appendix B.

Propositions 3 and 4 apply to a general distribution and are not restricted to the uniform distribution. They are similar to results already in the literature. For example, in Kanbur and

Keen (1993), at equilibrium the small country strictly undercuts the large country. In their model, if both countries have the same size but differ even slightly in their transport costs per mile, equilibrium in pure strategies ceases to exist.

5. WELFARE IMPLICATIONS

If each region was an autarky, the inelastic demand for cigarettes implies that a government could collect almost unlimited tax revenue by raising the tax rate on cigarettes. As noted in the Introduction, tax competition limits the ability of the government to collect tax revenue. But this limitation comes at a welfare cost - individuals buy kits instead of assembled product although kits are of higher inclusive cost. The welfare framework adopted in this paper deviates from the standard paradigm. That literature searches for welfare changes when taxes are perturbed from their equilibrium values in a competitive Nash game (e.g. Mintz and Tulkens (1986), de Crombrugghe and Tulkens (1990), Haufler (1998) and the review by Wilson (1999)). Instead of analyzing the welfare impact of differing strategies I pursue a traditional excess burden calculation.

With cigarette demand being inelastic at Q, changes in the tax rate do not change the total quantity of cigarettes smoked. If the only cigarette form is the assembled cigarette and there is no smuggling (as in the symmetric case), a cigarette tax is an “as if” lump-sum tax levied on smokers and there is no excess burden. However, this paper stresses that tax competition creates a new product - the kit with an additional resource cost wT. The excess burden calculation holds the level of tax revenue constant and calculates the additional resources which could be extracted from the system if the proportional tax is replaced by a lump-sum tax and individual utilities are unchanged. In this case, it translates to the opportunity cost of the time spent by individuals assembling the kits and the risk cost of individuals smoking illegal cigarettes. In the symmetric case there is no smuggling and the excess burden is:

$$2NQ\int_0^{\frac{t_a - t_k}{T}} wTf(w)\,dw$$

where the factor 2 is introduced as there are two regions. With a uniform wage density, $f(w) = 1/\bar{w}$ and $(t_a - t_k)/T = \bar{w}/2$ and the excess burden becomes

$$2NQ\int_0^{\frac{\bar{w}}{2}} wT\frac{1}{\bar{w}}\,dw = NQ\,\frac{T}{4}\bar{w}$$

6. CONCLUSION

This paper presents a model in which tax competition creates new products which are identical to other products except that they require additional time for their assembly. The motivating example is the cigarette market in Canada and the presence of low-tax kit cigarettes. However, as noted in the Introduction, the model can be interpreted as a traditional model of individuals in one jurisdiction being able to take advantage of lower taxes in a neighboring jurisdiction by traveling to that jurisdiction to shop: in this case the risk premium $w\delta$ associated with buying illegal product is reinterpreted as the opportunity cost of traveling to shop outside the own jurisdiction. The model predicts that, with cross-border shopping, jurisdictions set lower tax rates on kits and other products which require time to assemble than on the assembled product. By doing so, a welfare cost is created.

APPENDIX A: PROOF OF PROPOSITION 2

PROOF:I need to show that, if Region 2's strategy is $(t_k^2 = \delta \bar{w},\ \ t_a^2 = (\delta + T/2)\bar{w})$, the tax revenue collected by Region 1 using the proposed strategy $(t_k^1 = \delta \bar{w},\ \ t_a^1 = (\delta + T/2)\bar{w})$ is no less than the tax revenue collected by Region 1 using all possible alternative combinations of t_k^1 and t_a^1. Due to the large number of possible cases (hinted at below), this is laborious. Therefore in this appendix (1) I establish the tax revenue collected under the proposed strategy; (2) I show that a necessary condition for the proposed strategy to be a best response is $\delta > T$; (3) using $\delta > T$, I illustrate the method by considering a particular case and show that $\delta \geq 2T$. The full proof is available in de Bartolome (2005).

(1) Tax revenue of Region 1 using the strategy $(t_k^1 = \delta \bar{w},\ \ t_a^1 = (\delta + T/2)\bar{w})$. Individuals in Region 1 buy kits if $p + t_k^1 + wT \leq p + t_a^1$ or if $w \leq (t_a^1 - t_k^1)/T$. Region 1 makes no exports; its tax revenue is

$$R^{1*} = \int_0^{\frac{t_a^1 - t_k^1}{T}} t_k^1\, Q f(w)\, dw \ + \ \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}} t_a^1\, Q f(w)\, dw \ ;$$

Using the uniform distribution, $f(w) = 1/\bar{w}$, and integrating

$$R^{1*} = \frac{Q}{\bar{w}} \left[t_k^1 \frac{t_a^1 - t_k^1}{T} + t_a^1 \left(\bar{w} - \frac{t_a^1 - t_k^1}{T} \right) \right].$$

Setting $t_k^1 = \delta \bar{w},\ \ t_a^1 = (\delta + \frac{T}{2})\bar{w},$

$$R^{1*} = \left(\delta + \frac{T}{4} \right) Q \, \bar{w} \, .$$

(2) To show that a necessary condition for the proposed Nash Equilibrium is that $\delta > T$.

Suppose $\delta \le T$ and Region 2 sets tax rates as $t_k^2 = \delta\bar{w}$ and $t_a^2 = (\delta + T/2)\bar{w}$. I find a strategy for Region 1 which creates strictly more tax revenue than R^{1*}.

Consider the tax revenue which Region 1 could generate if it set tax rates as $t_k^1 = t_a^1 \le t_k^2 = \delta\bar{w} < t_a^2 = (\delta + T/2)\,\bar{w}$. Region 1 produces no kits and in Region 1

$$t_a^1 \le \min\,[t_k^2 + wT + w\delta\,,\, t_a^2 + w\delta]$$

or Region 1 does not import cigarettes. However, Region 1 may export to Region 2.

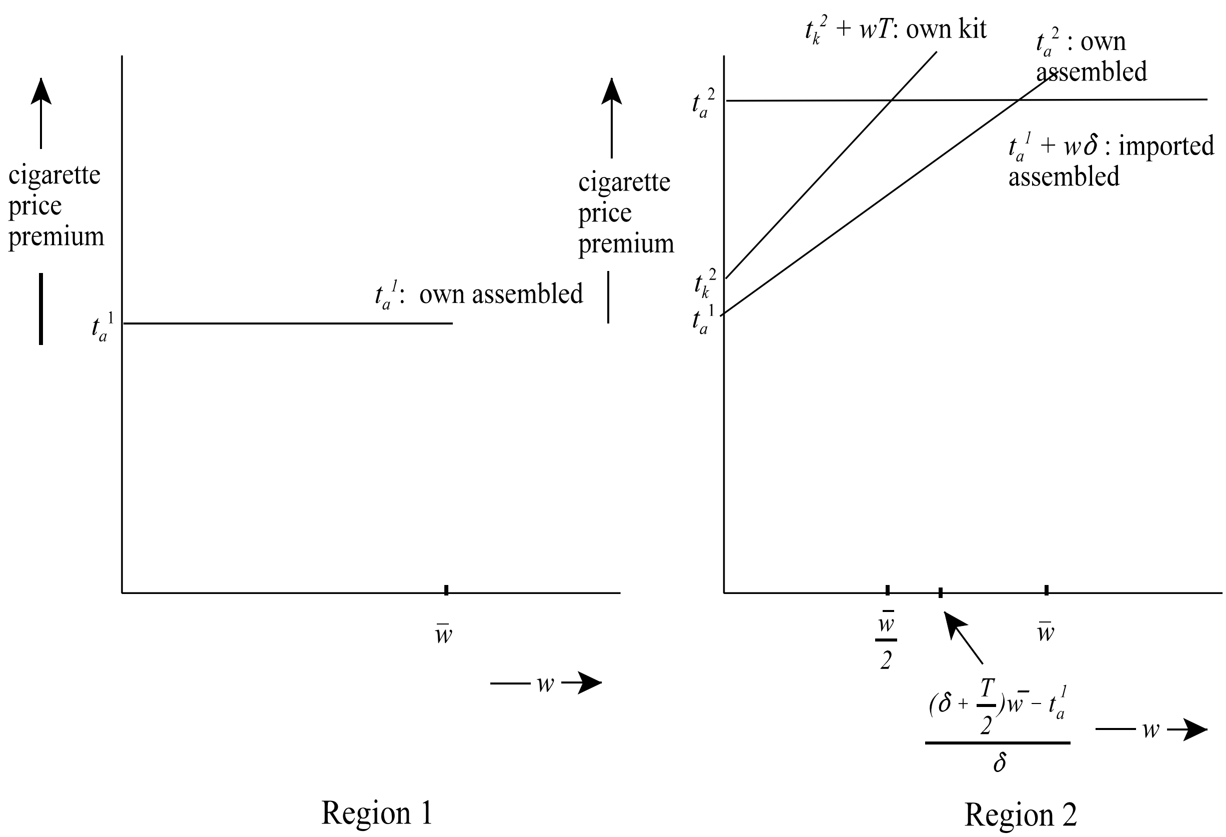


Figure A.1: inclusive prices with $t_a^1 \le t_k^2 \le t_a^2$

An individual in Region 2 faces the same inclusive price for an own assembled product and an imported assembled product if $p + t_a^2 = p + t_a^1 + w\delta$ or, setting $t_a^2 = (\delta + T/2)\bar{w}$, if

$w = ((\delta + T/2)\bar{w} - t_a^1) / \delta$. Region 1's problem is:

$$\max_{t_a^1} R^1 = \int_0^{\bar{w}} t_a^1 Q f(w)\, dw + \int_0^{\frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta}} t_a^1 Q f(w)\, dw$$

$$\text{s.t.} \qquad t_a^1 \le t_k^2 = \delta\bar{w}.$$

Setting $f(w) = 1 / \bar{w}$ and integrating, tax revenue is:

$$R^1 = t_a^1 \bar{w} + t_a^1 \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta}.$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}}\left[t_a^1 \bar{w} + t_a^1 \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta} \right] + A(\delta\bar{w} - t_a^1)\ .$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}}\left[\bar{w} - \frac{t_a^1}{\delta} + \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta} \right] - A = 0\ ; \qquad \text{(A.1)}$$

$$\frac{\partial \mathcal{L}}{\partial A} = \delta\bar{w} - t_a^1 \ge 0 \qquad\qquad CS \quad A \ge 0; \qquad \text{(A.2)}$$

where *CS* denotes “complementary slackness”. Try A > 0. From Equation (A.2):

$t_a^1 = \delta \bar{w}.$

From Equation (A.1):

$$A = \frac{Q}{\bar{w}}\left[\bar{w} - \bar{w} + \frac{(\delta + \frac{T}{2})\bar{w} - \delta\bar{w}}{\delta}\right] = \frac{T}{2\delta}Q > 0 \quad \text{as required .}$$

And tax revenue:

$$R^1 = \frac{Q}{\bar{w}}\left[\delta\bar{w}\bar{w} + \delta\bar{w}\frac{(\delta + \frac{T}{2})\bar{w} - \delta\bar{w}}{\delta}\right] = (\delta + \frac{T}{2})\,Q\,\bar{w} > R^{1*}$$

Summarizing, if $\delta \le T$ and $t_k^2 = \delta\bar{w}$ and $t_a^2 = (\delta + T/2)\bar{w}$, the best response of Region 1 is not to set $t_k^1 = \delta\bar{w}$ and $t_a^1 = (\delta + T/2)\bar{w}$. Therefore I restrict attention to $\delta > T$.

(3) Tax revenue under possible alternative strategies: To show that $t_k^1 = \delta\,\bar{w}$

and $t_a^1 = (\delta + T/2)\bar{w}$

are the best response, I must consider the tax revenue which can be generated by Region 1 under all possible alternative strategies. Given $t_k^1 \le t_a^1$ (see the proof of Proposition 1) and $t_k^2 < t_a^2$, there are 6 possible orderings between (t_k^1, t_a^1) and (t_k^2, t_a^2):

CASE A: $t_k^1 \le t_a^1 \le t_k^2 < t_a^2$;

CASE B: $t_k^1 \le t_k^2 \le t_a^1 \le t_a^2$;

CASE C: $t_k^1 \le t_k^2 < t_a^2 \le t_a^1$;

CASE D: $t_k^2 \le t_k^1 \le t_a^1 \le t_a^2$;

CASE E: $t_k^2 \le t_k^1 \le t_a^2 < t_a^1$;

CASE F: $t_k^2 < t_a^2 \le t_k^1 < t_a^1$;

The tax revenue function for Region 1 differs in each case (and possible subcases) and so each case must be considered separately. The full proof considers all possible cases and is available in de Bartolome (2005). I illustrate the methodology by initialing focusing on Case A and considering the particular subcase which leads to the restriction $\delta \ge 2T$.

$$\text{CASE A: } t_k^1 \le t_a^1 \le t_k^2 < t_a^2.$$

In Region 1, $\min[\, t_k^1 + wT,\, t_a^1\,] < \min[\, t_k^2 + wT + w\delta,\, t_a^2 + w\delta\,]$ or Region 1 does not import. The individual in Region 1 who faces the same inclusive price for an own kit and own preassembled cigarette has wage such that $p + t_k^1 + wT = p + t_a^1$, or $w = (t_a^1 - t_k^1)/T$.

An individual in Region 2 is indifferent between an own kit and an own assembled product if $p + t_k^2 + wT = p + t_a^2$ or, setting $t_k^2 = \delta \bar{w}$ and $t_a^2 = (\delta + T/2)\bar{w}$, if

$$w = \frac{\bar{w}}{2}.$$

The inclusive price for an own cigarette in Region 2 varies with an individual's income as the bold line ABC in Figure A.2.

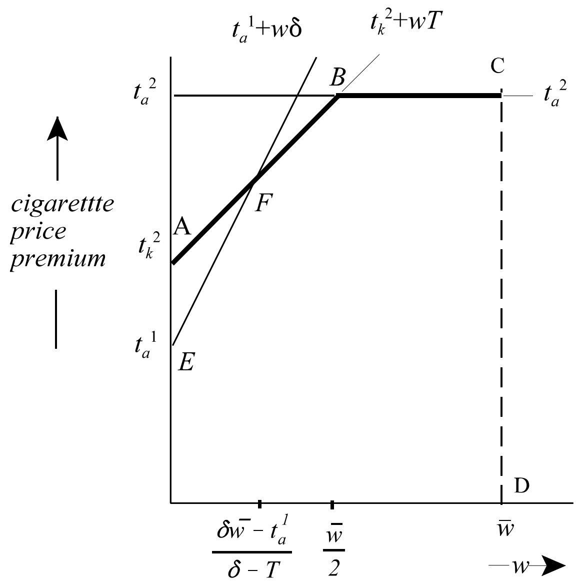


Figure A.2: inclusive price schedules in Region 2

An individual in Region 2 faces the same price for an imported assembled and an own kit if $p + t_a^1 + w\delta = p + t_k^2 + wT$ or, setting $t_k^2 = \delta\,\bar{w}$, if

$$w = \frac{\delta\bar{w} - t_a^1}{\delta - T}.$$

An individual in Region 2 faces the same price for an imported assembled and an own assembled if $p + t_a^1 + w\delta = p + t_a^2$ or, setting $t_a^2 = (\delta + T/2)\bar{w}$, if

$$w = \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta}$$

The inclusive price line of an imported assembled cigarette can intersect the envelope ABC

(1) on AB, or if $(\delta\bar{w} - t_a^1)/(\delta - T) \leq \bar{w}/2$. This case is illustrated in Figure A.2.

(2) on BC, or if $\bar{w}/2 \leq ((\delta + T/2)\bar{w} - t_a^1)/\delta \leq \bar{w}$.

(3) on $w = \bar{w}$ below C.

I focus on Case (1) (although the full proof considers all cases). In Case (1), there are three possible subcases which are characterized by the intersection of the inclusive price line of imported kits with the envelope EFBC:

(i) on EF

(ii) on FB

(iii) on BC

(iv) on $w = \bar{w}$ below C

I focus on Case (i) (although the full proof considers all subcases). The inclusive prices in the two regions are shown in the figure below:

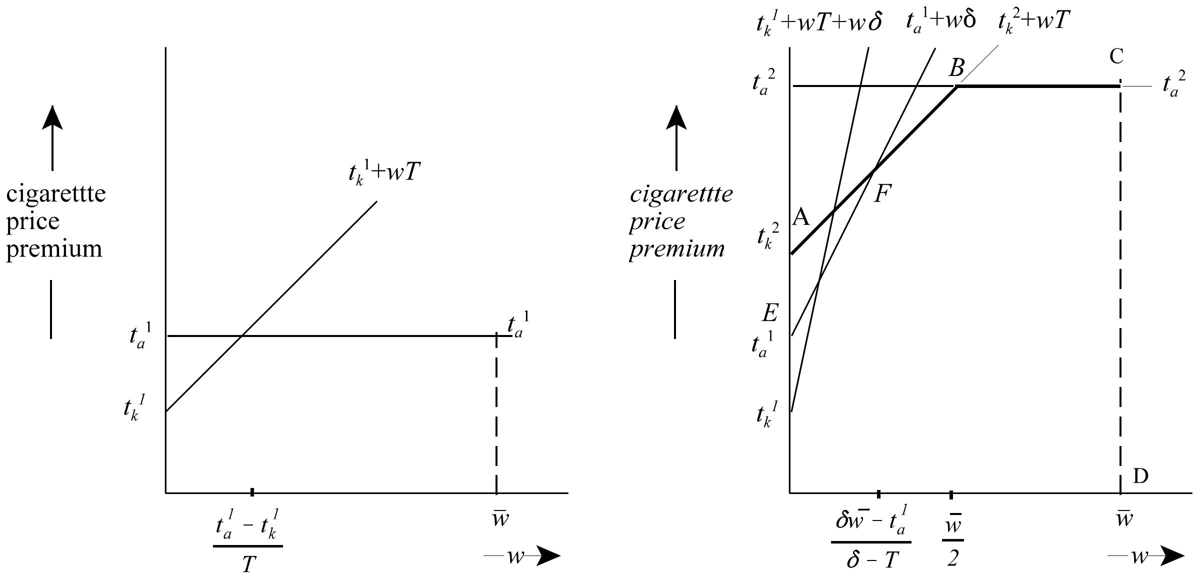


Figure A3: inclusive price lines for Case A.1.i

If $t_k^1 < t_a^1$ (as drawn in Figure A.3), by raising t_k^1 to t_a^1 Region 1 does not change its total cigarette sales in either region but it does cause some individuals in both regions to substitute out of its low-tax kits into its high-tax assembled product. Tax revenue increases. Therefore, for this case Region 1 maximizes its tax revenue by setting $t_k^1 = t_a^1$, or there is a single tax rate to be chosen. Region 1's problem becomes:

$$\max_{t_a^1} \; R^1 = \int_0^{\bar{w}} t_a^1 \, Q f(w) \, dw + \int_0^{\frac{\bar{w}\delta - t_a^1}{\delta - T}} t_a^1 \, Q f(w) \, dw$$

s.t. $\quad t_a^1 \le t_k^2 = \delta \bar{w};$

$$\frac{\delta \bar{w} - t_a^1}{\delta - T} \leq \frac{\bar{w}}{2}.$$

Using the uniform distribution, $f(w) = 1 / \bar{w}$ and integrating, R^1 becomes

$$R^1 = \frac{Q}{\bar{w}}\left[t_a^1 \bar{w} + t_a^1 \frac{\delta \bar{w} - t_a^1}{\delta - T} \right].$$

The Lagrangean is:

$$\mathcal{L} = \frac{Q}{\bar{w}}\left[t_a^1 \bar{w} + t_a^1 \frac{\delta \bar{w} - t_a^1}{\delta - T} \right] + A\,(\delta \bar{w} - t_a^1) + B\,(\frac{\bar{w}}{2} - \frac{\delta \bar{w} - t_a^1}{\delta - T}).$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{\bar{w}}\left[\bar{w} - \frac{t_a^1}{\delta - T} + \frac{\delta \bar{w} - t_a^1}{\delta - T} \right] - A + \frac{B}{\delta - T} = 0; \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - t_a^1 \geq 0 \qquad \text{CS} \quad A \geq 0; \tag{A.4}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\bar{w}}{2} - \frac{\delta \bar{w} - t_a^1}{\delta - T} \geq 0 \qquad \text{CS} \quad B \geq 0. \tag{A.5}$$

Try $A = B = 0$. From Equation (A.3):

$$t_a^1 = \frac{2\delta - T}{2}\bar{w}.$$

And

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - \frac{2\delta - T}{2} \bar{w} = \frac{T}{2} \bar{w} \geq 0 \qquad \text{as required ;}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\bar{w}}{2} - \frac{\delta \bar{w} - \frac{2\delta - T}{2} \bar{w}}{\delta - T} = \frac{(\delta - 2T)}{2(\delta - T)} \bar{w}.$$

If $\delta \geq 2T$, $\partial \mathcal{L}/\partial B \geq 0$ as required. In this case:

$$R^1 = \frac{Q}{\bar{\bar{w}}} \left[\frac{2\delta - T}{2} \bar{w}\, \bar{w} + \frac{2\delta - T}{2} \bar{w} \frac{\delta \bar{w} - \frac{2\delta - T}{2} \bar{w}}{\delta - T} \right] = \frac{2\delta - T}{2} \frac{2\delta - T}{2(\delta - T)} Q \bar{w} < R^{1*}.$$

If $T < \delta < 2T$: Try $A = 0$ and $B > 0$; From Equation (A.5):

$$t_a^1 = \frac{\delta + T}{2} \bar{w};$$

And

$$\frac{\partial \mathcal{L}}{\partial A} = \delta \bar{w} - \frac{\delta + T}{2} \bar{w} = \frac{\delta - T}{2} \bar{w} > 0 \ \text{ as required.}$$

From Equation (A.3):

$$B = Q(2T - \delta) > 0 \quad \text{as required}$$

In this case,

$$R^1 = \frac{Q}{\bar{\bar{w}}} \left[\frac{\delta + T}{2} \bar{w}\, \bar{w} + \frac{\delta + T}{2} \bar{w} \frac{\delta \bar{w} - \frac{\delta + T}{2} \bar{w}}{\delta - T} \right] = \frac{3}{4} (\delta + T) Q \bar{w} > R^{1*},$$

or $t_k^1 = \delta \bar{w}$, $t_a^1 = (\delta + T/2)\bar{w}$ is not the best response of Region 1.

APPENDIX B: PROOFS OF PROPOSITIONS 3 AND 4

The proofs of Propositions 3 and 4 are similar and so I combine the analysis. Denote the populations of Regions 1 and 2 as N^1 and N^2 respectively, and denote the wage densities in Regions 1 and 2 as $f^1(w)$ on $[0, \bar{w}^1]$ and $f^2(w)$ on $[0, \bar{w}^2]$ respectively. I will consider the two cases:

either Proposition 3: $N^1 < N^2$ and $f^1(0) = f^2(0)$;

or Proposition 4: $f^1(0) < f^2(0)$ and $N^1 = N^2$.

To establish the claims, I show (1) that $t_k^1 = t_k^2$ is not a Nash Equilibrium and (2) that $t_k^2 < t_k^1$ is not a Nash Equilibrium. It follows that at the Nash Equilibrium (if it exists) $t_k^1 < t_k^2$. I establish a necessary condition for the Nash Equilibrium with $t_k^1 < t_k^2$ and show that it is not satisfied if $f^1(0) < f^2(0)$ and $N^1 = N^2$.

(1) I suppose that, at the Nash Equilibrium, $t_k^1 = t_k^2$ and show a contradiction. If $t_k^1 = t_k^2$ is the best response for Region 1, then

$$\lim_{t_k^1 \to t_k^{2-}} \frac{dR^1}{dt_k^1} \geq 0 \geq \lim_{t_k^1 \to t_k^{2+}} \frac{dR^1}{dt_k^1}$$

Modifying Equations (3) and (4) to take account of the different density functions,

$$\lim_{t_k^1 \to t_k^{2-}} \frac{dR^1}{dt_k^1} = N^1 \int_0^{\frac{t_a^1 - t_k^2}{T}} Qf^1(w)\,dw + N^1(t_a^1 - t_k^2)\,Qf^1\left(\frac{t_a^1 - t_k^2}{T}\right)\frac{1}{T} - N^2 t_k^2\,Qf^2(0)\,\frac{1}{\delta}\,;$$

and

$$\lim_{t_r^1 \to t_k^{2+}} \frac{dR^1}{dt_k^1} = N^1 \int_0^{\frac{t_a^1 - t_k^2}{T}} Qf^1(w)\,dw + N^1(t_a^1 - t_k^2)\,Qf^1\left(\frac{t_a^1 - t_k^2}{T}\right)\frac{1}{T} - N^1 t_k^2\,Qf^1(0)\,\frac{1}{\delta}\,.$$

Hence if *either* $N^1 < N^2$ and $f^1(0) = f^2(0)$ *or* if $\mathrm{f}^1(0) < f^2(0)$ and $N^1 = N^2$,

$$\lim_{t_k^1 \to t_k^{2-}} \frac{dR^1}{dt_k^1} < \lim_{t_k^1 \to t_k^{2+}} \frac{dR^1}{dt_k^1}$$

which gives the contradiction.

(2) I suppose that, at the Nash Equilibrium, $t_k^2 < t_k^1$ and show a contradiction. I know that t_k^1 is the best response of Region 1 and that dR^1/dt_k^1 is continuous in the range for $t_k^1 > t_k^2$. Hence at the equilibrium value of t_k^1:

$$\frac{dR^1}{dt_k^1} = 0\,.$$

By assumption, $t_k^1 > t_k^2$. Modifying Equation (4) to take account of the different density functions:

$$\frac{dR^1}{dt_k^1} = N^1 \int_{\frac{t_k^1 - t_k^2}{\delta}}^{\frac{t_a^1 - t_k^1}{T}} Qf^1(w)\,dw + N^1 (t_a^1 - t_k^1)\, Qf^1\left(\frac{t_a^1 - t_k^1}{T}\right)\frac{1}{T} - N^1 t_k^1\, Qf^1\left(\frac{t_k^1 - t_k^2}{\delta}\right)\frac{1}{\delta}\,.$$

(B.1)

I know that t_k^2 is the best response for Region 2 or, modifying Equation (3) to take account of the different density functions and applying to Region 2,

$$\frac{dR^2}{dt_k^2} = N^2 \int_0^{\frac{t_a^2 - t_k^2}{T}} Qf^2(w)\,dw + N^2 (t_a^2 - t_k^2) Qf^2\left(\frac{t_a^2 - t_k^2}{T}\right)\frac{1}{T}$$

$$+ N^1 \int_0^{\frac{t_k^1 - t_k^2}{\delta}} Qf^1(w)\,dw - N^1 t_k^2 Qf^1 \left(\frac{t_k^1 - t_k^2}{\delta}\right)\frac{1}{\delta} = 0\,. \quad \text{(B.2)}$$

I also know that t_a^1 and t_a^2 are best responses for the two regions or, modifying Equation (5) to take account of the different density functions,

$$\frac{dR}{dt_a^1} = N^1 \int_{\frac{t_a^1 - t_k^1}{T}}^{\bar{w}^1} Qf^1(w)\,dw - N^1 (t_a^1 - t_k^1)\, Qf^1\left(\frac{t_a^1 - t_k^1}{T}\right)\frac{1}{T} = 0\,. \qquad \text{(B.3)}$$

$$\frac{dR^2}{dt_a^2} = N^2 \int_{\frac{t_a^2 - t_k^2}{T}}^{\bar{w}^2} Qf^2(w)\,dw - N^2 (t_a^2 - t_k^2)\, Qf^2\left(\frac{t_a^2 - t_k^2}{T}\right)\frac{1}{T} = 0$$

(B.4)

Adding Equations (B.1) and (B.3), I have

$$\frac{dR^1}{dt_k^1} = N^1 \int_{\frac{t_k^1 - t_k^2}{\delta}}^{\bar{w}^1} Qf^1(w)\,dw - N^1 t_k^1 Qf^1\left(\frac{t_k^1 - t_k^2}{\delta} \right) \frac{1}{\delta}\,; \tag{B.5}$$

Adding Equation (B.2) and (B.4), I have

$$N^2 \int_0^{\bar{w}^2} Qf^2(w)\,dw + N^1 \int_0^{\frac{t_k^1 - t_k^2}{\delta}} Qf^1(w)\,dw - N^1 t_k^2 Qf^1\left(\frac{t_k^1 - t_k^2}{\delta} \right) \frac{1}{\delta} = 0 \tag{B.6}$$

If $N^1 \leq N^2$:

$$N^1 \int_{\frac{t_k^1 - t_k^2}{\delta}}^{\bar{w}^1} Qf^1(w)\,dw < N^2 \int_0^{\bar{w}} Qf(w)\,dw + N^1 \int_0^{\frac{t_k^1 - t_k^2}{\delta}} Qf^1(w)\,dw \tag{B.7}$$

and $t_k^1 > t_k^2$ (by assumption) implies

$$N^1 t_k^1 Qf\left(\frac{t_k^1 - t_k^2}{\delta} \right) \frac{1}{\delta} > N^1 t_k^2 Qf\left(\frac{t_k^1 - t_k^2}{\delta} \right) \frac{1}{\delta}. \tag{B.8}$$

Inserting Equation (B.6) and Inequalities (B.7) and (B.8) into Equation (B.5) implies

$$\frac{1}{N^1}\frac{dR^1}{dt_k^1} < 0$$

which gives the contradiction.

(3) Repeat the analysis of Part (2) but with $t_k^1 < t_k^2$. The equation analogous to Equation (B.5) is:

$$\frac{dR^1}{dt_k^1} = N^1 \int_0^{\bar{w}^1} Qf^1(w)\,dw + N^2 \int_0^{\frac{t_k^2 - t_k^1}{\delta}} Qf^2(w)\,dw - N^2\, t_k^1 Qf^2\left(\frac{t_k^2 - t_k^1}{\delta}\right)\frac{1}{\delta}\,; \tag{B.9}$$

The analogue to Equation (B.6) is

$$N^2 \int_{\frac{t_k^2 - t_k^1}{\delta}}^{\bar{w}^2} Qf^2(w)\,dw - N^2\, t_k^2 Qf^2\left(\frac{t_k^2 - t_k^1}{\delta}\right)\frac{1}{\delta} = 0\,. \tag{B.10}$$

With $t_k^1 < t_k^2$, a necessary condition for $dR^1 / dt_k^1 = 0$ is that

$$N^2 \int_{\frac{t_k^2 - t_k^1}{\delta}}^{\bar{w}^2} Qf^2(w)\,dw > N^1 \int_0^{\bar{w}^1} Qf^1(w)\,dw + N^2 \int_0^{\frac{t_k^2 - t_k^1}{\delta}} Qf^2(w)\,dw$$

If $N^1 = N^2$, this cannot be satisfied. Hence there is no Nash Equilibrium of the assumed form in pure strategies.

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ENDNOTES

1. Note that the taxing authority is assumed to be a "Leviathan" which extracts tax revenue from the population without providing any public good. With this type of objective, keeping tax rates low is advantageous and this is achieved by tax competition. The welfare conclusion may be different if the tax revenue is spent on a public good.

2. The assumption that the firm's cost of the two products is the same understates my case. The roll-your-own-cigarette kit contains tobacco and a preformed paper cylinder; these are packaged separately so that the package of the kit is more bulky than the package of the traditional cigarette. Hence transport costs and retailing costs for the roll-your-own cigarette are likely to be higher than for the traditional cigarette.

3. Cigarettes - but not necessarily in the general case of kits - are probably better modeled as an oligopoly.

4. δ is the risk cost measured in time units. If the model is interpreted as a model of cross-jurisdiction shopping, δ would be the time cost of cross-jurisdiction shopping. Haufler (1998) uses a transportation cost which is strictly convex whereas my time cost is linear.

5. Individuals are assumed to always have sufficient income to purchase Q cigarettes. This means that a zero-wage individual is assumed to have an exogenous income source (e.g. government aid).

6. By "a small perturbation from the symmetric equilibrium," I mean an equilibrium in which $t_k^1, t_k^2 < t_a^1, t_a^2$ and in which only kits are exported.