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A Collective Household Model with Choice-Dependent Sharing Rules

Murat Iyigun
Department of Economics, University of Colorado at Boulder
Boulder, Colorado

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Center for Economic Analysis
Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

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Murat Iyigun

University of Colorado

Abstract

This paper presents a collective household model in which there are marital gains to assortative spousal matching, individuals face a labor-leisure choice and intra-marital allocations are determined by an endogenous sharing rule that is driven by actual wage earnings. The latter two features of the model introduce the potential for inefficiently high levels of labor supply because spouses recognize that changes in their labor supply would influence not only total household income but also their respective shares in intra-household allocations. Nonetheless, when sex ratios are imbalanced or external distribution factors are not gender neutral, competition among potential spouses in the large marriage markets helps to generate maritally sustainable Pareto efficient levels of labor supply and intra-household allocations. In such cases, the sharing rule that supports the maritally sustainable and Pareto efficient equilibrium outcome is also unique for each couple along the assortative order.

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1. Introduction

The traditional approach to analyze household choices takes the family as the relevant decision-making unit.[1] The collective household model provides an alternative to this approach by treating the individual members of the family–not the family as a whole–as the core decision-makers.[2] Starting in the early 1990s, the empirical literature began to provide strong support for the notion that relative spousal incomes matter for family decisions and intra-household allocations.[3] Consequently, the collective approach to household decision-making has emerged as the compelling theoretical tool for analyzing the economics of the family.

The collective model is based on the premise that external distribution factors such as the sex ratios in the markets for marriage and the distributions of income within the households determine the intra-marital sharing rules. It requires that the latter *do not* depend on variables that enter spousal choice sets. But what if sharing rules, to some extent, *do* depend on spousal choices made during the marriage? Then, there are two seemingly fundamental obstacles. First, it is not clear how one would model, for example, the household labor supply choices in a framework in which individuals value leisure and the marital decision-making power of the spouses depends on their relative *actual* labor incomes. In that case and in the absence of binding commitments prior to the formation of marriage, the spousal levels of labor supply and leisure could be determined via a decision-making process that is non-cooperative and competitive in nature. Such a solution method could make it less likely that there is specialization within the household. Then would modeling the household labor supply as the outcome of a non-cooperative process be reasonable and empirically consistent?

Second, a vital building block of the collective model is Pareto efficiency. As demonstrated by Chiappori (1988, 1992), Pareto optimality enables one to recover the

[1]The theoretical foundations of this literature is provided by two seminal papers. In Samuelson (1956), a consensus approach is emphasized as the rationale for treating the household allocation problem as that of maximizing a single household utility function. In Becker (1981), the existence of an altruistic household member is shown to generate outcomes that maximize total family income even in the presence of family members with divergent preferences.

[2]The generalized underpinning of this model is provided by Becker (1981) and Chiappori (1988, 1992).

[3]See, for example, Browning et al. (1994), Chiappori et al. (2002), and Udry (1996).

underlying preference structure of the individuals within the household as well as the implicit sharing rule that influences the intra-household allocations among different family members.[4] For existing households, efficiency is a robust assumption as long as the sharing rules consistent with the collective model are primarily driven by external factors, such as the sex ratios in the markets for marriage, divorce legislation, and *potential* (not actual) spousal incomes. Pareto efficiency could become suspect, however, in models where the marital decision-making power of spouses depends on their actual labor incomes relative to that of their partners. Then, it is quite possible that the household labor supply would be inefficiently high as spouses would recognize that their labor supply choices influence not only total household income but also their decision-making power within the marriage.

The conventional models of the collective household typically avoid these complications by either ruling out leisure from individual preferences or assuming that the incomes relevant for intra-marital allocations are those that the spouses *could earn* entering a marriage–not those that the husband and the wife *actually do earn* once all labor supply, household production and leisure choices are made.[5] For instance, if two stay-home wives have different levels of education, they either value leisure and are compensated differently in their marriages ceteris paribus, or have no preference for leisure and are compensated roughly similarly.

Since some empirical studies that find support for the collective model focus on the observed levels of total household earnings and how those are distributed within the household, they suggest that *actual* spousal earnings do matter for intra-marital allocations.[6] Hence, it is important to address whether sharing rules that depend on

[4]In fact, the critical feature of the collective approach is *conditional efficiency.* The latter defines intra-marital allocations that are Pareto efficient *conditional* on the choices spouses have made prior to marriage or on the choices that spouses have committed to make during the marriage.

[5]In almost all versions of the collective household model it is implicit either that (i) spousal incomes that matter for intra-household allocations are those given prior to the determination of endogenous and relevant household choices (such as spousal labor supply, leisure or specialization in home production); or that (ii) even if intra-marital sharing rules are influenced by spousal incomes that reflect the endogenous choices made within the households, individuals do not take this fact into account.

[6]Of course, the spousal levels of labor supply manifest themselves in both the observed levels of total household earnings and their distribution between the spouses.

spousal choices and the possibility for strategic spousal behavior during marriage alters–or even worse invalidates–the collective household approach.

In what follows, I present a collective household model in which there are marital gains to assortative spousal matching, individuals face a labor-leisure choice and intra-marital allocations are determined by an endogenous sharing rule that is driven by actual wage earnings. My main findings are that, even in the presence of competitive behavior and externalities in marriage, the process of marital matching in the large marriage markets goes a long way in (a) pinning down the levels of spousal labor supply and (b) maintaining the efficiency of intra-household decisions. In particular, I find that, when the sex ratios in the marriage markets are not equal to unity or external distribution factors (such as marriage and divorce legislation) are not gender neutral, marriage market competition among potential spouses helps to generate maritally sustainable and Pareto efficient levels of labor supply and spousal consumption. In such cases, the sharing rule that supports the efficient, maritally sustainable equilibrium for each couple along the assortative order is also unique.

These findings are fairly important because they suggest that neither strategic, non-cooperative interactions between the spouses nor the endogeneity of intra-marital sharing rules with respect to spousal choices made during the marriage need to be accounted for if the marriage markets are large and the external distribution factors are asymmetric. The reason is that, when the marriage markets are large and the external distribution factors are asymmetric, the efficiency of household choices are generally restored because marriage market competition helps to ensure that each spouse is compensated according to his or her marginal contribution to the marriage.

2. Related Literature

There are various strands in the economics of the family literature to which this paper is related. Of course, the main one is the "collective" household model, which encompasses the early- and late-generation marital bargaining theories. In general, the collective household model allows for differences between spousal preferences to affect households choices by relying on an intra-household sharing rule. Its special case the marital bargaining model generates the same feature via spousal Nash-bargaining weights. Among the earliest examples of the collective models are Becker (1981), Chiappori (1988, 1992), and Bourguignon and Chiappori (1994), and those of marital bargaining are Manser and Brown (1980), McElroy and Horney (1981), and Sen (1983). All of these models assume that and rely on the fact that the sharing rule or the bargaining power of spouses are determined exogenously. As a consequence, they all yield Pareto efficient intra-household allocations.

At least on a theoretical basis, it is not clear that spousal bargaining power (or the shares spouses extract from marital output) should be a function of *potential* relative spousal earnings and not *actual* relative labor income. Taking this distinction seriously, Basu (2001) and Iyigun-Walsh (2002) suggest models that treat the bargaining power of the spouses as determined endogenously according to actual relative earnings. Due to the fact that neither of these models consider and endogenize spousal matching, however, they both yield inefficient household choices and allocations.

Even in models where spousal wealth is a public good in marriage, inefficient allocations and choices can result. But as Peters and Siow (2002) have shown convincingly, families make investments in education that are Pareto optimal once marital matching is endogenized. According to their results, assortative matching and bilateral efficiency together guarantee that, in the large marriage markets, the equilibrium distribution of pre-marital investments are efficient. This is due to the fact that, when spousal wealth is a public good in marriage, the competitive marriage market and the assortative matching that occurs within it guide families to indirectly and reciprocally compensate each other for the investments that they make in their own children.

In the model below, I take the potential impact of marital matching on household decisions seriously and incorporate it into a collective household framework. In addition, I let the spousal labor supply decisions influence intra-marital sharing rules (and as a consequence, the intra-household allocations). What I present here differs Peters and Siow on three accounts: (a) spousal endowments are not public goods; (b) household choices are made based on the collective approach; and (c) spouses recognize that their labor supply choices influence intra-marital allocations.

This paper is most similar to Becker-Murphy (2000), Browning-Chiappori-Weiss (2003) and Iyigun-Walsh (2004). All three represent the early attempts to broaden the collective approach to cover aspects of household formation that precede marriage.[7] Becker-Murphy and Browning-Chiappori-Weiss share similarities in that they both merge the collective household model with marital sorting to explore the implications of spousal matching. In both contributions, however, the endowment each spouse brings to the marriage is taken as given. Iyigun and Walsh extend the collective model to cover pre-marital investments and marital sorting. They find that matching in the marriage markets helps to generate unique sharing rules that support *unconditionally Pareto efficient* outcomes (where both intra-household allocations and pre-marital choices are Pareto efficient).

The remainder of this paper is organized as follows: In section 3, I present some basics and discuss the choices of single men and women. The results derived in that section helps to establish the reservation levels of utility in marriage. In Section 4, I derive the Nash equilibrium labor supply choices of a given couple. In Section 5, I describe the marriage market outcomes and how the expected intra-household allocations influence these outcomes. In Section 6, I establish the properties of the efficiency frontier. In Section 7, I examine how the maritally sustainable intra-household allocations are related to the Pareto efficient frontier. In Section 8, I present an analytical example to highlight some of the main findings. In Section 9, I conclude.

3. The Basic Model

The total mass of women in the economy is equal to F and that of men is equal to M. Let $G(N)$ and $H(N)$ respectively be measures of the sets of males and females whose endowments lie in the continuum $[0, N]$ and let r, $r \lesseqgtr 1$, denote the measure of women relative to men.[8] Individuals are endowed with y units of total labor endowment, where $y \in [0, Y]$ and $Y > 0$.

[7]For an integrated collective household model with both spousal matching and the possibility of divorce, see Chiappori, Iyigun and Weiss (2004).

[8]Hence, r equals one if there are equal measures of men and women and it is less (greater) than one if there are less (more) women than men.

Preferences are defined over the consumption of a single good and leisure, c_i and $y_i - l_i$ respectively, where l_i denotes individual i's endogenously-determined labor supply. For males and females, preferences are represented by the following inter-temporal utility functions respectively:

$$U \ = \ u(y_m - l_m) \ + \ u(c_m) \ , \tag{1}$$

and

$$V \ = v(y_f - l_f) \ + \ v(c_f) \ , \tag{2}$$

where the functions U and V satisfy u', $v' > 0$, u'', $v'' < 0$, and the other neo-classical Inada restrictions.

The marital production technology is given by $h(l_m,\, l_f)$. If a man with a labor supply of l_m remains single, his intra-temporal output is given by $h(l_m,\, 0)$ and if a woman with an income of l_f remains single, her intra-temporal output is given by $h(0,\, l_f)$. I assume that the function $h(l_m,\, l_f)$ is increasing in l_m and l_f and that $h(0,\, 0) = 0$. The essential feature of the problem is the interaction in the traits that a couple brings to its marriage. As Becker-Murphy and Browning-Chiappori-Weiss note, when income is the only important trait and the couple shares a public good, spousal incomes are complements in the marital production function. Hereafter, I shall focus on such cases so that the function $h(l_m, l_f)$ is super modular which implies that $h_{12}(l_m, l_f) > 0$. The positive interaction creates gains from marriage and the marital surplus will be positive unless one (or both) of the spouses has a labor supply of zero.

Let l_i^s, $i = f,\, m$, denote the optimal labor supply of an individual who remains single. Then the output of a single male is given by $h(l_m^s,\, 0)$ and that of a single female is given by $h(0,\, l_f^s)$.[9] Thus, for single men and women we have $c_m^s = h(l_m^s,\, 0)$, $c_f^s = h(0,\, l_f^s)$ and utility from consumption respectively equals

[9] The supermodularity of the marital production function imples that, $\forall l_i > 0$, $i = f,\, m$, $h(0,\, l_f)$, $h(l_m,\, 0) < h(l_m,\, l_f)$, $h_{l_f}(0,\, l_f) < h_{l_f}(l_m,\, l_f)$ and $h_{l_m}(l_m,\, 0) < h_{l_m}(l_m,\, l_f)$.

$$u[h(l_m^s, 0)] \qquad \text{and} \qquad v[h(0, l_f^s)]. \tag{3}$$

For those individuals who remain single, the optimal levels of labor supply, l_i^s, $i = f,\ m$, are

$$l_i^s \;=\; \begin{cases} \arg\max\ U \ = u(y_m - l_m^s) \ + \ u[h(l_m^s, 0)] & \text{if } i \ = \ m\ , \\ \arg\max\ V \ = v(y_f - l_f^s) \ + \ v[h(0, l_f^s)] & \text{if } i \ = \ f\ . \end{cases} \tag{4}$$

The optimal labor supply of single men and women respectively satisfy the following first-order conditions:

$$u'(y_m - l_m^s) = u'[h(l_m^s, 0)]h_1(l_m^s, 0) \quad \text{and} \quad v'(y_f - l_f^s) = v'[h(0, l_f^s)]h_2(0, l_f^s)\ . \tag{5}$$

4. The Equilibrium Household Labor Supply

I now discuss the household equilibrium of a given spousal match. Let the pair (y_m^*, y_f^*) denote a couple whose husband and wife possess the endowments of y_m^* and y_f^* respectively, where $y_i^* \in [0,\ Y]$, $i = f,\ m$. As noted above, spouses choose their labor supply in order to maximize their own utility in marriage. This implies that they recognize the impact of their choices on not only total household income but also their own intra-household allocations.

For the couple (y_m^*, y_f^*), there exists a sharing arrangement that divides marital output, $h(l_m^*, l_f^*)$, between the spouses. That is

$$c_m(l_m^*) \ + \ c_f(l_f^*) \ = \ h(l_m^*, l_f^*) \ + \ g\ ; \qquad g \ \geq \ 0 \tag{6}$$

where g represents the common gain from marriage that is unrelated to spousal incomes.[10] Note that equation (6) holds only for couples that match with each other in the marriage market (and not for those who have chosen not to match with each other).[11] Due to the super modularity of the marital output function, also keep in mind that, $\forall\ (l_m^*,\, l_f^*) >> 0$, $h(0,\, l_f^*) + h(l_m^*,\, 0) < h(l_m^*,\, l_f^*)$. Therefore, the function $h(l_m,\, l_f)$ explicitly incorporates the "gains" from marriage.

The couple $(y_m^*,\, y_f^*)$ plays a non-cooperative Nash game in which each spouse takes as given the other's actions. Let the labor supply response function of a husband be defined as:

$$\begin{aligned} l_m(\bar{l}_f) &= \arg\max\ U(l_m \,|\bar{l}_f\,) \\ &= \arg\max\ \{u(y_m^* - l_m) \,+\, u[c_m(h(l_m,\, \bar{l}_f))]\}\ . \end{aligned} \tag{7}$$

In similar fashion, let the labor supply of a wife as a function of that of her husband be defined as:

$$\begin{aligned} l_f(\bar{l}_m) &= \arg\max\ V(l_f \,|\bar{l}_m\,) \\ &= \arg\max\ \{v(y_f^* - l_f) \,+\, v[c_f(h(\bar{l}_m,\, l_f))]\}\ . \end{aligned} \tag{8}$$

The related first-order conditions are

$$u'(c_m)c_m' \;=\; u'(y_m^* - l_m) \tag{9}$$

[10]In an alternative specification, g can represent the utility gain associated with the status of being married. In that case, g would not be part of the consumption levels, c_m and c_f, but it would appear as an additive term directly in the utility functions U and V. The main qualitative conclusions of the paper should remain intact under such an alternative.

[11]For marital matches not formed $c_m(l_m^*) + c_f(l_f^*) > h(l_m^*,\, l_f^*) + g$ holds and the demand of both spouses exceed what the potential marriage could produce.

and,

$$v'(c_f)c_f' \;=\; v'(y_f^* - l_f)\ . \tag{10}$$

Since the terms $c_m$and c_m' in (9) depend on a given level of the wife's labor supply through the total household output level, $h(l_m, \bar{l}_f) + g$, and c_f and c_f' in (10) depend on a given level of the husband's labor supply through the total household output level, equations (9) and (10) implicitly define two labor response functions.[12] Letting $l_m = \lambda(l_f)$ and $l_f = \mu(l_m)$ denote the response functions associated with equations (7) and (8) respectively, we can define a household equilibrium as $l_m^* = \lambda(l_f^*)$ and $l_f^* = \mu(l_m^*)$, where l_m^* and l_f^* respectively represent the solutions to (7) and (8) taking as given l_f^* and l_m^* respectively.

By applying the implicit function theorem to (9) and (10), some properties of the response functions can be determined. In particular, in the $(l_m,\, l_f)$ map, the slopes of the response functions equal $\partial l_f / \partial l_m$.[13] For the husband's response function, this turns out to be $[-u''(c_m)\ (c_m')^2\ h_1(l_m,\, l_f) - u'(c_m)\ c_m'' h_1(l_m, l_f) - u''(y_m^* - l_m)]\ /\ [u'(c_m)\ c_m''\ h_2(l_m, l_f) + u''(c_m)\ c_m'\ h_2(l_m, l_f)]$. For the wife's response function, the slope equals $[v'(c_f)\ c_f''\ h_1(l_m, l_f) + v''(c_f) c_f' h_1(l_m, l_f)]\ /\ [-v''(c_f)\ (c_f')^2\ h_2(l_m,\, l_f) - v'(c_f)\ c_f'' h_2(l_m, l_f) - v''(y_f^* - l_f)]$. Given that $h_1(l_m,\, l_f)$, $h_2(l_m,\, l_f)$, u', $v' > 0$, u'', $v'' < 0$, both response functions are downward sloping if, $\forall\ l_m,\, l_f \geq 0$, c_i'', $i = m,\, f$, is non-positive. Note that even if $\forall\ l_m,\, l_f \geq 0$, c_i'', $i = m,\, f$, is strictly positive, the response functions can still be downward sloping. In Figure 1, I depict two such labor response functions as well as the equilibrium levels of labor supply that emerge based on these functions under the assumption that c_i'', $i = m,\, f$, is non-positive.

[Figure 1 about here.]

[12] Of course, those response functions are also influenced by the sharing rule that generates, for each spouse, the level of consumption, c_i, and the marginal increase in consumption due to an increase in the labor supply, c_i', $i = m,\, f$.

[13] Given the notation introduced above, the slopes of the labor response functions of the wife and the husband respectively correspond to $\mu'(l_m^*)$ and $\lambda'^{-1}(l_m^*)$ on such a map.

Two observations are now in order. First, the structure of the problem by itself ensures neither the existence of a spousal labor supply equilibrium nor its uniqueness. Nonetheless, if household sharing rules are such that, $\forall\ (y_m^*,\ y_f^*) \in [0,\ Y]$, either $\lim_{l_i \to 0} c_i(l_i) \neq 0$ or $\lim_{l_i \to 0} c_i'(l_i) = c_i''(l_i) = 0$, then there will be at least one spousal labor supply equilibrium.[14] It is also possible that, for each couple, there exists multiple spousal labor supply equilibria. However, as I shall demonstrate below, marital sorting and external distribution factors may help to pin down a unique spousal labor supply equilibrium as the maritally sustainable one for each couple. Second, the slope of the response functions reflect how severe the competition between the spouses are: ceteris paribus, when sharing rules are such that the marginal effect of an increase in the labor supply of one spouse sufficiently benefits (harms) the consumption of his/her companion, then in equilibrium, the latter's labor supply declines (rises). As shown above, a sufficient but not necessary condition for this to be the case is, $\forall\ l_m,\ l_f \geq 0,\ c_i'' \leq 0,\ i = m,\ f$. Of course, the sharing rule that generates the marginal increase in each spouse's consumption, $c_i',\ i = m,\ f$, ought to be maritally sustainable and consistent with the external marriage market conditions (like the sex ratio, r, the marriage and divorce legislation, etc.). I will identify the properties of such sharing rules in due course. But first I shall elaborate on the marital matching process that paired up the couple with endowments of $(y_m^*,\ y_f^*)$.

5. Stable Marital Matchings

Under assortative matching, the allocations in marriage, $c_m(l_m^*)$ and $c_f(l_f^*)$, define a *rational expectations marriage market equilibrium* if there exist labor response functions, $l_m = \lambda(l_f)$ and $l_f = \mu(l_m)$, for all pairs $(y_m^*,\ y_f^*)$ in the set of married couples such that the following hold:

1. $1 - G(y_m^*) = r[1 - H(y_f^*)]$;

[14] With $c_i''(l_i) = 0,\ i = f,\ m$, both response functions would be downward sloping. Moreover, the additional restrictions would yield, $\forall\ l_f > 0$, $\lim_{l_m \to 0} [\partial l_f / \partial l_m\ |\lambda(l_f)] > \lim_{l_m \to y_m} [\partial l_f / \partial l_m \mid \lambda(l_f)] = -\infty$ and, $\forall\ l_m > 0$, $\lim_{l_f \to 0} [\partial l_f / \partial l_m\ |\mu(l_m)] > \lim_{l_f \to y_f} [\partial l_f / \partial l_m \mid \mu(l_m)] = -\infty$. These charateristics would then help to ensure that there is at least one non-trivial spousal labor supply equilibrium.

2. $\forall\ y_m^*,\ y_f^* \in [0,\, Y]$, $l_m^* = \lambda(l_f^*)$ and $l_f^* = \mu(l_m^*)$;

3. $\forall\ y_m^* \in [0,\, Y]$, $y_f^* = \arg\max\{u(y_m^* - l_m^*) + u[h(l_m^*, l_f^*) + g - c_f(l_f^*)]\}$;

4. $\forall\ y_f^* \in [0,\, Y]$, $y_m^* = \arg\max\{v(y_f^* - l_f^*) + v[h(l_m^*, l_f^*) + g - c_m(l_m^*)]\}$.

Part 1 of the definition is the marriage market-clearing condition which guarantees that, by assortative matching, each husband that is endowed with y_m^* or more will be able to match with a spouse who is endowed with at least y_f^*. It generates the following spousal matching functions:

$$y_m^* \;=\; \Phi\{1 \;- r(1 \;-\; H(y_f^*)]\} \;\equiv\; \phi(y_f^*) \tag{11}$$

and,

$$y_f^* \;=\; \Psi\left\{1 \;- \frac{1}{r}(1 \;-\; G(y_m^*)]\right\} \;\equiv\; \psi(y_m^*) \tag{12}$$

where $\Phi \equiv G^{-1}$ and $\Psi \equiv H^{-1}$. Note that either of the functions $\phi(y_f^*)$ and $\psi(y_m^*)$ fully describe the nature of spousal matching.

Part 2 of the definition reflects the fact that, once married, couples play a Nash-equilibrium game to determine their labor supply choices. Parts 3 and 4 of the definition indicate that all individuals choose their spouses optimally in order to maximize their gains from marriage recognizing that, once they are married, they play the Nash-equilibrium game. Accordingly, Parts 3 and 4 yield the following first-order conditions:[15]

$$u'(y_m^* - l_m^*)\frac{\partial l_m^*}{\partial l_f^*}\frac{\partial l_f^*}{\partial y_f^*} \;=\; u'[h(l_m^*, l_f^*) + g - c_f(l_f^*)]\left\{h_1\frac{\partial l_m^*}{\partial l_f^*}\frac{\partial l_f^*}{\partial y_f^*} + [h_2 - c_f'(l_f^*)]\frac{\partial l_f^*}{\partial y_f^*}\right\}\ , \tag{13}$$

and,

[15]Note that the expressions below represent the first-order conditions after the the envelope theorem is applied.

$$v'(y_f^* - l_f^*)\frac{\partial l_f^*}{\partial l_m^*}\frac{\partial l_m^*}{\partial y_m^*} \quad = \quad v'[h(l_m^*, l_f^*) + g - c_m(l_m^*)]\left\{h_2\frac{\partial l_f^*}{\partial l_m^*}\frac{\partial l_m^*}{\partial y_m^*} + [h_1 - c_m'(l_m^*)]\frac{\partial l_m^*}{\partial y_m^*}\right\} \ . \tag{14}$$

Equation (13) implies that there are both direct and indirect effects of a husband with an endowment of y_m^* marrying a wife with y_f^*. The direct effect is captured by the last term on the right hand side of (13) and it represents the impact of the best-response labor supply of the wife on the marital gain of her husband. If the wife receives less (more) than her marginal contribution to the marriage, then the direct effect of a marginal increase in her labor supply on her husband is positive (negative). There are two indirect effects of a husband with y_m^* marrying the wife with y_f^*. The best-response labor supply of this husband influences his leisure, captured by the term on the left hand side of (13), as well as his marital gain, denoted by the first term on the right hand side of equation (13). The interpretation of equation (14) is, of course, similar to that of (13).

Note that, $\forall$ (y_m^*, y_f^*), the rational expectations equilibrium implicitly defines two distributions functions $\hat{G}(l_m^*)$ and $\hat{H}(l_f^*)$ such that $1 - \hat{G}(l_m^*) = r[1 - \hat{H}(l_f^*)]$. On that basis and consistent with the notation above, we can re-define the spousal matching functions as $l_m^* = \hat{\phi}(l_f^*)$ and $l_f^* = \hat{\psi}(l_m^*)$. In Figure 2, I rely on these labor supply distributions and depict two possible rational expectations equilibria that could emerge in the marriage market.[16] The labor supply of the men are drawn on the horizontal axis and those of the women are on the vertical axis. The two upward-sloping dashed lines represent two different equilibrium matching functions $\hat{\psi}(l_m^*)$ for a given rule of intra-marital sharing of spousal consumption. The upward convex curves are the indifference curves of the husbands and those that are convex downward are the indifference curves of the wives. Both types of indifference curves incorporate the sharing rules associated with each potential spousal match. Due to the assortative matching equilibrium, couples for which the wife has a higher initial endowment, y_f, work more than those for which

[16] Both marriage market equilibria shown in the figure are conditional on the labor supply equilibrium that emerges for each couple along the assortative matching order.

the wife has a lower initial endowment. If distributional factors favor women more than they do men then, for a given sharing arrangement of spousal consumption within the households, the equilibrium matching function will tend to shift to the right leading to a higher labor supply by the husbands and less by the wives. For each matched couple, the tangency point of the indifference curves of husbands and wives also correspond to the intersection point of the labor supply response functions $l_m = \lambda^{-1}(l_f)$ and $l_f = \mu(l_m)$ (which were originally depicted in Figure 1). One such point is identified as the point A in Figure 2.

[Figure 2 about here.]

6. The Pareto Efficient Frontier

For the couple $(y_m^*,\ y_f^*)$, the unconditionally efficient levels of labor supply and intra-household allocations of consumption can be determined by solving the following maximization problem:

$$\max_{\{lf,\ l_m, c_f, c_m\}} U \ = \ u(y_m^* - l_m) \ + \ u(c_m) \tag{15}$$

subject to:

$$V \ = \ v(y_f^* - l_f) \ + \ v(c_f) \ \geq \ \bar{V} \ , \tag{16}$$

$$c_m \ + \ c_f \ \leq \ h(l_m,\ l_f) \ + \ g \tag{17}$$

and,

$$l_m \ \leq \ y_m^* \quad \text{and} \quad l_f \ \leq \ y_f^* \ . \tag{18}$$

The four first-order conditions to this problem yield

$$u'(y_m^* - l_m^*) \;=\; u'(c_m)\; h_1(l_m^*,\, l_f^*)\;, \tag{19}$$

and,

$$v'(y_f^* - l_f^*) \;=\; v'(c_f)\; h_2(l_m^*,\, l_f^*)\;. \tag{20}$$

Utilizing the restrictions imposed on this problem, these conditions can be re-written as

$$\frac{u'(y_m^* - l_m^*)}{u'[c_m(l_m^*)]h_1[l_m^*,\, \psi(l_m^*)]} = \frac{v'(y_f^* - l_f^*)}{v'[c_f(l_f^*)]h_2[\phi(l_f^*),\, l_f^*]} \quad . \tag{21}$$

Along the Pareto efficient frontier, equation (21) equates spouses' ratios of marginal utility of leisure to marginal utility of consumption. When combined with the endowment constraint, equation (17), the first order conditions of equations (19) and (20) determine the Pareto efficient frontier. Along this frontier, the wife's utility constraint, equation (16), ties down the allocation associated with the wife attaining utility equal to $\bar{V}$.

7. Equilibrium Sharing Rules and Marital Stability

We are now in position to address whether the marital matching process and the subsequent allocations of intra-marital consumption and leisure satisfy Pareto efficiency. The sharing rules that hold in equilibrium and that are therefore maritally sustainable need to be compatible with equations (9), (10), (13) and (14), all of which need to be satisfied for all married couples along the assortative order.

Combining these four equations and rearranging a bit, we get

$$\frac{u'(y_m^* - l_m^*)}{u'[c_m(l_m^*)]c_m'(l_m^*)} = \frac{v'(y_f^* - l_f^*)}{v'[c_f(l_f^*)]c_f'(l_f^*)} \;=\; 1\;, \tag{22}$$

$$\frac{u'(y_m^* - l_m^*)}{u'[c_m(l_m^*)]} \;=\; \frac{1}{\frac{\partial l_m^*}{\partial l_f^*}\frac{\partial l_f^*}{\partial y_f^*}} \left\{ h_1(l_m^*,\, l_f^*)\frac{\partial l_m^*}{\partial l_f^*}\frac{\partial l_f^*}{\partial y_f^*} + \left[h_2(l_m^*,\, l_f^*) - c_f'(l_f^*)\right]\frac{\partial l_f^*}{\partial y_f^*} \right\} \tag{23}$$

and,

$$\frac{v'(y_f^* - l_f^*)}{v'[c_f(l_f^*)]} = \frac{1}{\frac{\partial l_f^*}{\partial l_m^*}\frac{\partial l_m^*}{\partial y_m^*}} \left\{ h_2(l_m^*,\, l_f^*) \frac{\partial l_f^*}{\partial l_m^*} \frac{\partial l_m^*}{\partial y_m^*} + [h_1 - c_m'(l_m^*)] \frac{\partial l_m^*}{\partial y_m^*} \right\} \tag{24}$$

If equations (22) through (24) are satisfied simultaneously, then spousal choices of labor supply and intra-household allocations are maritally sustainable. If, in addition, equation (21) is satisfied, they are also Pareto efficient. An examination of equations (21) through (24) reveals that maritally sustainable outcomes are Pareto efficient if and only if intra-marital sharing rules yield $h_1(l_m^*,\, l_f^*) = c_m'(l_m^*)$ and $h_2(l_m^*,\, l_f^*) = c_f'(l_f^*)$.

Does spousal matching yield such intra-marital sharing rules? Combining equations (22)-(24), we derive

$$h_1 \; - \; c_m'(l_m^*) \; = -[h_2 - c_f'(l_f^*)] \frac{\partial l_f^*}{\partial l_m^*} \; . \tag{25}$$

With (25), and in the absence of further restrictions, we find there exists a continuum of maritally sustainable intra-household sharing rules. As we verified above, only one unique sharing rule in that continuum (the one that yields $h_1 - c_m'(l_m^*) = h_2 - c_f'(l_f^*)$ $= 0$) is Pareto efficient. Figure 3 depicts the continuum of maritally sustainable equilibria and identifies the unique, Pareto efficient one. In the diagram, I super-impose the loci of the Pareto efficient frontier and the reservation utilities on the curve that shows the equilibrium combinations of labor supply, the latter which was originally depicted in Figure 2. All sharing rules that satisfy equation (25) lie on the marital contract curve but only some of them–those that generate the segment $[B,\, C]$–yield intra-marital shares that are acceptable to both spouses.[17] And while the whole continuum that lies on the marital contract curve segment $[B,\, C]$ is maritally sustainable, only the point A on that segment is associated with the Pareto efficient sharing rule, which yields $h_1 - c_m'(l_m^*) = h_2 - c_f'(l_f^*) = 0$.

[17]In other words, only the allocations in the line segment $[B,\, C]$ yield spousal utility that equal or exceed the singles utility levels consistent with equations (3)-(5).

[Figure 3 about here.]

When the sex ratio, r, is equal to unity and the external distribution factors are neutral, all individuals marry and every husband and wife with a strictly positive endowment exceeds his or her reservation utility level.[18] Then, we cannot move beyond equation (25) and all we can conclude is that there exists a continuum of maritally sustainable intra-household sharing rules–only one of which is Pareto efficient.[19] [20]

In contrast, consider a case in which $r > 1$ or external distributions heavily favor men so that, among couples in the lowest assortative rank (when $r > 1$, those with $y_f^0 > y_m^0 = 0$), wives receive their reservation utility, which equals $v(y_f - l_f^s) + v[h(0, l_f^s)]$. In that case, we establish that equation (5) holds for married women in the lowest assortative rank. That is $v'(y_f^0 - l_f^s) = v'[h(0, l_f^s)]h_2(0, l_f^s)$. But since equations (9) and (10) also hold for all lowest-ranked couples, we can establish that $h_1(0,\, l_f^s) = c'_m(0)$ and $h_2(0,\, l_f^s) = c'_f(l_f^s)$.[21]

What about couples in the higher ranks? Based on the continuity of endowments over the support $(0, Y]$, we can show that either (13) or (14) would be violated if, $\exists\ (y_m^*,\, y_f^*)$, y_f^*, $y_m^* \in [0, Y]$, for which equilibrium intra-marital allocations are characterized by $h_1(l_m^*,\, l_f^*) \neq c'_m(l_m^*)$ and $h_2(l_m^*,\, l_f^*) \neq c'_f(l_f^*)$. In particular, define a couple $(\tilde{y}_m,\, \tilde{y}_f)$ such that $\tilde{y}_m = \varepsilon$ and $\tilde{y}_f > y_f^0$, where $\varepsilon > 0$. Recall that $r > 1$ and that $y_f^0 > 0$ and $y_m^0 = 0$ are the endowments of the wife and the husband in the lowest assortative rank. For

[18] Recall that, as shown in equation (4), the reservation levels of utility are $u(y_m - l_m^s) + u[h(l_m^s, 0)]$ and $v(y_f - l_f^s) + v[h(0, l_f^s)]$ for men and women respectively.

[19] However if, in addition, the underlying preference structure of men and women are identical so that $u(.) = v(.)$, we can make more progress. We can determine that it is likely–but not guaranteed–the unique sharing rule that supports the Pareto efficient intra-marital allocations would emerge as the only maritally sustainable outcome even when the sex ratio, r, is equal to unity and the external distribution factors are neutral. To see this, note that, $\forall\ (y_m^*,\, y_f^*)$, $y_m^* = y_f^*$, $h_1 = h_2$, and $c'_m = c'_f$ are more likely to hold in this case. Together with the fact that $u(.) = v(.)$, we would then have, $\forall\ (y_m^*,\, y_f^*)$, $l_m^* = l_f^*$. Hence, $\partial l_f^* / \partial l_m^* = 1$. As a result, equation (25) yields $\forall\ (y_m^*,\, y_f^*)$, $h_1 = c'_m$ and $h_2 = c'_f$.

[20] Implicit in this discussion is the assumption that the spousal labor response functions given by (9) and (10) behave well enough to generate a unique labor supply equilibrium for each couple $(l_m^*,\, l_f^*)$. In fact, in cases where the spousal labor response functions do not yield a unique marital labor supply equilibrium (and therefore, for each couple, there exists multiple labor supply equilibria), even the equilibrium spousal labor supply levels, $(l_m^*,\, l_f^*)$, could be indeterminate.

[21] Note that a total differentiation of equation (6) yields equation (25).

this couple, let $h_1(\tilde{l}_m, \tilde{l}_f) \neq c'_m(\tilde{l}_m)$ and $h_2(\tilde{l}_m, \tilde{l}_f) \neq c'_f(\tilde{l}_f)$. Now consider the analog of equation (14) for the wife with the endowment of $\tilde{y}_f$. If she marries a husband with $\tilde{y}_m$ and gets a share in marriage associated with $h_2(\tilde{l}_m, \tilde{l}_f) \neq c'_f(\tilde{l}_f)$ and $h_1(\tilde{l}_m, \tilde{l}_f) \neq c'_m(\tilde{l}_m)$, we have

$$\frac{v'(\tilde{y}_f - \tilde{l}_f)}{v'[c_f(\tilde{l}_f)]} = \frac{1}{\frac{\partial \tilde{l}_f}{\partial \tilde{l}_m}\frac{\partial \tilde{l}_m}{\partial \tilde{y}_m}} \left\{ h_2(\tilde{l}_m, \tilde{l}_f)\frac{\partial \tilde{l}_f}{\partial \tilde{l}_m}\frac{\partial \tilde{l}_m}{\partial \tilde{y}_m} + [h_1 - c'_m(\tilde{l}_m)]\frac{\partial \tilde{l}_m}{\partial \tilde{y}_m} \right\} \tag{26}$$

In contrast, if the same wife marries a lower ranked husband with $y_m^0 = 0$ and she earns the marital share consistent with $h_2(0, \tilde{l}_f) = c'_f(\tilde{l}_f)$ and $h_1(0, \tilde{l}_f) = c'_m(0)$,[22] we have

$$\frac{v'(\tilde{y}_f - \tilde{l}_f)}{v'[c_f(\tilde{l}_f)]} = \frac{1}{\frac{\partial \tilde{l}_f}{\partial l_m^0}\frac{\partial l_m^0}{\partial y_m^0}} \left\{ h_2(0, \tilde{l}_f)\frac{\partial \tilde{l}_f}{\partial l_m^0}\frac{\partial l_m^0}{\partial y_m^0} \right\} \tag{27}$$

Equation (26) evaluated at $\lim_{\tilde{y}_m \to (y_m^0)^+} = \lim_{\tilde{y}_m \to 0^+}$ yields[23]

$$\frac{v'(\tilde{y}_f - \tilde{l}_f)}{v'[c_f(\tilde{l}_f)]} = \frac{1}{\frac{\partial \tilde{l}_f}{\partial l_m^0}\frac{\partial l_m^0}{\partial y_m^0}} \left\{ h_2(0, \tilde{l}_f)\frac{\partial \tilde{l}_f}{\partial l_m^0}\frac{\partial l_m^0}{\partial y_m^0} + [h_1 - c'_m(0)]\frac{\partial l_m^0}{\partial y_m^0} \right\} \tag{28}$$

For the husband with $\tilde{y}_m$ to be the optimal spouse for the wife with $\tilde{y}_f$, equations (27) and (28) need to hold simultaneously. Because if (27) and (28) do not hold simultaneously, either the wife with $\tilde{y}_f$ or the husband with $\tilde{y}_m$ could be better off marrying a lower-ranked spouse and getting a share in marriage that is Pareto efficient (instead of marrying the higher-ranked $\tilde{y}_i$ spouse and getting an intra-marital share that is not Pareto efficient). However, it is obvious that, as long as $h_2(\tilde{l}_m, \tilde{l}_f) \neq c'_f(\tilde{l}_f)$ and $h_1(\tilde{l}_m, \tilde{l}_f) \neq c'_m(\tilde{l}_m)$, (27) and (28) cannot hold together. This contradicts that the pairing ($\tilde{y}_m$,

[22] As discussed above, this will have to be the intra-marital sharing arrangement due to the fact that, when $r > 1$, the husband with $y_m^0 = 0$ gets his reservation level of utility, which satisfies $h_1(0, l_f^s) = c'_m(0)$.

[23] The continuity of endowments over the support $[0, Y]$ guarantees the existence of a potential husband with an endowment of $\lim_{\varepsilon \to 0^+}(\tilde{y}_m)$.

$\tilde{y}_f$) is maritally sustainable if the intra-marital allocations for that pair are consistent with $h_1(\tilde{l}_m,\, \tilde{l}_f) \neq c'_m(\tilde{l}_m)$ and $h_2(\tilde{l}_m,\, \tilde{l}_f) \neq c'_f(\tilde{l}_f)$. Put differently, such a pairing would be maritally sustainable if and only if it yields Pareto efficient intra-marital allocations (i.e. the conditions $h_1(\tilde{l}_m,\, \tilde{l}_f) = c'_m(\tilde{l}_m)$ and $h_2(\tilde{l}_m,\, \tilde{l}_f) = c'_f(\tilde{l}_f)$ need to hold if $h_1(0,\, l_f^0)$ $= c'_m(0)$ and $h_2(0,\, l_f^0) = c'_f(l_f^0)$ hold).

Extending this logic further up the assortative order establishes that, for $r >$ 1 or external distributions that heavily favor men so that lowest-ranked wives receive their reservation utility, maritally sustainable intra-marital allocations would be Pareto efficient and they would satisfy, $\forall$ $(y_m^*,\, y_f^*)$, $h_1(l_m^*,\, l_f^*) = c'_m(l_m^*)$ and $h_2(l_m^*,\, l_f^*) = c'_f(l_f^*)$. Applying the same argument to a case in which $r < 1$ or external distributions heavily favor women such that, among couples in the lowest assortative rank, husbands get their reservation utility, maritally sustainable intra-marital allocations would again be Pareto efficient so that, $\forall$ $(y_m^*,\, y_f^*)$, $h_1(l_m^*,\, l_f^*) = c'_m(l_m^*)$ and $h_2(l_m^*,\, l_f^*) = c'_f(l_f^*)$ hold.[24]

Figure 4 illustrates the maritally sustainable outcome that attains when $r < 1$. In the figure, the marital outcome for couples at two different points in the assortative order are depicted; one couple is of the lowest rank and the other is of a higher rank. For both couples, the marriage market yields Pareto efficient and sustainable outcomes. However, the husband of the lowest-rank couple can only attain his reservation utility level, denoted as U^s, because there are fewer women in the marriage market. For this couple, the marriage market outcome is given by the point A on their marital contact curve. In contrast, the husband of any higher ranked couple (and in particular the one shown in the figure) attains a higher utility than his reservation level, U^s, because (a) the equilibrium sharing rule satisfies $h_1(l_m^*,\, l_f^*) = c'_m(l_m^*)$ and $h_2(l_m^*,\, l_f^*) = c'_f(l_f^*)$; (b) the wife has a strictly positive labor supply level; and (c) there are complementarities in marital production (i.e., $h_{12} > 0$). For this couple, the marriage market outcome is the point C on their marital contract curve.

[24] Of course, the size of the marriage markets play a vital role in generating such outcomes: without large markets with many potential spouses on both sides of the transaction–and the continuity of spousal endowments typically associated with such markets–inefficient intra-household allocations and labor supply could be sustained even if husbands or wives in the lowest assortative rank get their reservation utility in marriage.

[Figure 4 about here.]

What if the labor supply functions described by equations (9) and (10) generate multiple labor supply equilibria for each couple? When either wives or husbands in the lowest assortative order receive their reservation levels of utility (as would be the case when $r \neq 1$), it is clear that the above reasoning (which ensures that, $\forall$ (y_m^*, y_f^*), $h_1(l_m^*, l_f^*) = c_m'(l_m^*)$ and $h_2(l_m^*, l_f^*) = c_f'(l_f^*)$ hold), also pins down which one of the spousal labor supply equilibria would emerge as the maritally sustainable equilibrium.

In general, for $r \neq 1$ or external distribution factors that heavily favor one spouse over the other, we can derive the intra-marital allocations of each spouse along the assortative marital order by integrating the expressions $h_1(l_m^*, l_f^*) = c_m'(l_m^*)$ and $h_2(l_m^*, l_f^*) = c_f'(l_f)$:

$$c_m(l_m^*) \;=\; k \;+\; \int_{l_m^0}^{l_m^*} h_1[s,\, \hat{\psi}(s)]ds \;. \tag{29}$$

and,

$$c_f(l_f^*) \;=\; k' \;+\; \int_{l_f^0}^{l_f^*} h_2[\hat{\phi}(t),\, t]dt \;. \tag{30}$$

Note that, for a couple that is in the lowest assortative order, it has to be the case that $k = h(l_m^s, 0)$ if $r < 1$ and $k' = h(0, l_f^s)$ if $r > 1$. If $r = 1$ multiple equilibria are possible and all we can say is that $k + k' = g + h(0, 0) = g$. Essentially, these allocations ensure that, when the sex ratio is not balanced (i.e. $r \neq 1$), spouses in the lowest assortative order work as if they were single and the spouse from the overabundant group receives his or her reservation level of utility, which is represented by what is in equation (4).

The labor response functions $l_m = \lambda(l_f)$ and $l_f = \mu(l_m)$ define the marital equilibrium for each couple. In a stable marriage market equilibrium, the functions $l_m^* = \lambda(l_f^*)$ and $l_f^* = \mu(l_m^*)$ would be related to the matching functions defined by $l_m^* = \hat{\phi}(l_f^*)$ and $l_f^* = \hat{\psi}(l_m^*)$ because, in equilibrium, $l_m^* = \lambda(l_f^*) = \hat{\phi}(l_f^*)$ and $l_f^* = \mu(l_m^*) = \hat{\psi}(l_m^*)$. Put differently, the marital matching functions $\hat{\phi}(l_f)$ and $\hat{\psi}(l_m)$ are such that, $\forall$ (l_m^*, l_f^*), l_m^* $= \lambda(l_f^*) = \hat{\phi}(l_f^*)$ and $l_f^* = \mu(l_m^*) = \hat{\psi}(l_m^*)$.

8. An Example

For simplicity, let the marital gain, g, equal zero and the marital production function be given by

$$h(l_m, l_f) \;=\; l_m + l_f + l_m\; l_f\ . \tag{31}$$

Suppose that the preferences of males and females are represented by the following inter-temporal utility functions respectively:

$$U \;=\; \alpha \ln(y_m - l_m) \;+\; (1-\alpha)\ln(c_m)\ , \tag{32}$$

and

$$V \;=\; \beta \ln(y_f - l_f) \;+\; (1-\beta)\ln(c_f)\ , \tag{33}$$

where α, $\beta \in (0,\, 1)$ and the consumption levels of men and women are given by

$$c_m + c_f \;\geq\; l_m + l_f + l_m l_f\ . \tag{34}$$

.

We can now explore the outcomes under three different cases:

1. If $r = 1$ so that the measures of men and women in the marriage market are identical, all individuals marry. As a result, we can establish that, $\forall$ (y_m^*, y_f^*), y_m^* $= y_f^*$. The analogs of equations (9) and (10) correspond to the following:

$$\frac{\alpha c_m(l_m)}{y_m^* - l_m} \;=\; (1-\alpha)\; c_m'(l_m)\;, \tag{35}$$

$$\frac{\beta c_f(l_f)}{y_f^* - l_f} \;=\; (1-\beta)\; c_f'(l_f). \tag{36}$$

And the analog of (25) is

$$1 \;+\; l_f^* \;-\; c_m'(l_m^*) \;=\; -\left[1 \;+\; l_m^* \;-\; c_f'(l_f^*)\right] \frac{\partial l_f^*}{\partial l_m^*}\;. \tag{37}$$

In the general case in which $\alpha \neq \beta$, the labor supply response functions would not be symmetric, and hence, $\forall\ (y_m^*,\, y_f^*)$, $\partial l_f^* / \partial l_m^* \neq 1$. Consequently, (37) would be satisfied for a continuum of sharing rules and there is no guarantee that the sharing rule consistent with the Pareto efficient intra-marital allocations (i.e. the one which yields, $\forall\ (y_m^*,\, y_f^*)$, $1 + l_f^* = c_m'(l_m^*)$ and $1 + l_m^* = c_f'(l_f^*)$) would emerge in equilibrium. However, if $\alpha = \beta$, then, as discussed in footnote 19, the underlying preference structure would be identical for men and women. In that special case, the unique sharing rule associated with the Pareto efficient intra-marital outcome would more likely emerge as the only sustainable solution if, in addition, external distribution factors are neutral. Then, $\forall\ (y_m^*,\, y_f^*)$, equations (35) and (36) would yield $l_m^* = l_f^*$, due to the fact that $c_m' = c_f'$, and $c_m = c_f$ (which would in turn suggest that $\partial l_f^* / \partial l_m^* = 1$). In that case, (37) would only be satisfied, $\forall\ (y_m^*,\, y_f^*)$, if and only if $1 + l_f^* = c_m'(l_m^*)$ and $1 + l_m^* = c_f'(l_f^*)$.

With these restrictions in place, we can derive the intra-marital allocations of each spouse along the assortative marital order by integrating $1 + l_f^* = c_m'(l_m^*)$ and $1 + l_m^* = c_f'(l_f^*)$:

$$c_m(l_m^*) \;=\; \int_0^{l_m^*} (1 \;+\; s)\; ds \;= l_m^* \;+\; \frac{(l_m^*)^2}{2}\;, \tag{38}$$

and,

$$c_f(l_f^*) \;=\; \int_0^{l_f^*} (1 \;+\; t)\; dt \;= l_f^* \;+\; \frac{(l_f^*)^2}{2}\;, \tag{39}$$

where, $\forall\ (y_m^*,\, y_f^*)$, $l_m^* = l_f^*$.

Using equations (35), (36), (38) and (39), we can then solve for the optimal levels of labor supply: $\forall\ (y_m^*,\, y_f^*)$,

$$l_i^* \;=\; \frac{y_i^* \;-\; 2 \;+\; \sqrt{(2 - y_i^*)^2 + 6}}{3}\;; \qquad\qquad i \;=\; f,\; m\;. \tag{40}$$

When $\alpha = \beta$ and $r = 1$, all individuals marry and the endowments of both spouses in all marriages along the assortative order are identical. As a result, the labor supply response functions are more likely to be symmetric if also all other external distribution factors are gender neutral. For all couples, this generates identical amounts of equilibrium labor supply and, in all marriages along the assortative order, both spouses get equal shares of the marital output and surplus. Due to the fact that $r = 1$ and the underlying preference structure of men and women are the same, a unique sharing rule supports these Pareto efficient intramarital allocations in all marriages.

2. If $r < 1$ so that there are fewer women than men in the marriage market, there will be some unmarried men in equilibrium. Our starting point in this case is the men in the lowest assortative rank who will have to marry women with endowments of $y_f^0 = 0$. We know that such men will receive their reservation levels of utility in marriage. The optimal behavior of these men is fully characterized by equations (4) and (5). Hence, $\forall\ (y_m^0,\, 0)$, we have $u'(y_m^0 - l_m) = 1/(y_m^0 - l_m) = u'[h(l_m, 0)]h_1(l_m, 0) = 1/l_m$. Consequently, it has to be the case that, $\forall\ (y_m^0,\, 0)$, $l_m^* = l_m^s = y_m^0/2$ and

$l_f^* = l_f^s \; = 0$. Together with the analogs of (9) and (10), we find that, for these couples, the intra-marital allocations and the spousal supplies of labor ought to be Pareto efficient: $\forall\ (y_m^0,\, 0)$, $h_1(l_m, 0) = 1 = c_m'$, $h_2(0,\, l_f) = 0 = c_f'$, $c_m = y_m^0/2$ and $c_f = 0$.

Now take a couple $(\tilde{y}_m,\, \tilde{y}_f)$ that is slightly higher ranked. Let ε, $\varepsilon > 0$, denote the endowment of the wife of such a couple so that $\tilde{y}_f = \varepsilon$ and $\tilde{y}_m > y_m^0$. Suppose that for this couple the sharing rule does not yield the Pareto efficient allocations and labor supply. That is let $h_1(\tilde{l}_m, \tilde{l}_f) = 1 + \tilde{l}_f \neq c_m'$ and $h_2(\tilde{l}_m,\, \tilde{l}_f) = 1 + \tilde{l}_m \neq c_f'$. According to Parts 3 and 4 of the definition of rational expectations marriage market equilibrium, this couple is together because equations (23) and (24) are satisfied:

$$\frac{c_m(\tilde{l}_m)}{\tilde{y}_m - \tilde{l}_m} = \frac{1}{\frac{\partial \tilde{l}_m}{\partial \tilde{l}_f}\frac{\partial \tilde{l}_f}{\partial \tilde{y}_f}} \left\{ (1+\tilde{l}_f)\frac{\partial \tilde{l}_m}{\partial \tilde{l}_f}\frac{\partial \tilde{l}_f}{\partial \tilde{y}_f} + [1 + \tilde{l}_m - c_f'(\tilde{l}_f)]\frac{\partial \tilde{l}_f}{\partial \tilde{y}_f} \right\} \tag{41}$$

and,

$$\frac{c_f(\tilde{l}_f)}{\tilde{y}_f - \tilde{l}_f} = \frac{1}{\frac{\partial \tilde{l}_f}{\partial \tilde{l}_m}\frac{\partial \tilde{l}_m}{\partial \tilde{y}_m}} \left\{ (1+\tilde{l}_m)\frac{\partial \tilde{l}_f}{\partial \tilde{l}_m}\frac{\partial \tilde{l}_m}{\partial \tilde{y}_m} + [1 + \tilde{l}_f - c_m'(\tilde{l}_m)]\frac{\partial \tilde{l}_m}{\partial \tilde{y}_m} \right\} . \tag{42}$$

The important claim here is that such a marriage market equilibrium cannot be stable. Why? Because if $r < 1$, the endowment distribution is continuous, and the intra-marital allocations and spousal labor supply levels of the higher-ranked couple is not Pareto efficient, then either higher-ranked spouse would be better off marrying someone of a lower assortative rank but one who is willing to accept Pareto efficient intra-marital outcomes. To illustrate, consider the case in which $\varepsilon \to 0^+$. Suppose that the higher-ranked husband is getting less than his marginal contribution to marital output (i.e., $h_1(\tilde{l}_m, \tilde{l}_f) = 1 + \tilde{l}_f(\varepsilon) > c_m'$). If instead he marries a single woman with no endowment, he could receive his marginal contribution (i.e., $h_1(\tilde{l}_m, 0) = 1 = c_m'$) because such a woman would be indifferent between marrying him and remaining single. Hence, for $\varepsilon \to 0^+ \Rightarrow \tilde{l}_f(\varepsilon) \to 0^+$, this new spousal match would be dominating for the husband with the endowment of $\tilde{y}_m$, in contradiction of the fact that the existing marriage market equilibrium is stable. Only if the intra-marital sharing rule yields the Pareto efficient outcomes so that, $\forall\ (\tilde{y}_m,\, \tilde{y}_f)$, $h_1(\tilde{l}_m, \tilde{l}_f) = 1 + \tilde{l}_f = c'_m$ and $h_2(\tilde{l}_m,\, \tilde{l}_f) = 1 + \tilde{l}_m = c'_f$, would the existing assortative marriage market equilibrium be stable. Moreover, given the continuity of the endowment distributions over the support $[0,\, Y]$, the process just described would yield the unique sharing rule that supports the Pareto efficient intra-marital allocations and levels of spousal labor supply for all marriages along the assortative order. Then, using equations (11), (12), (29) and (30), we can derive that, for $r < 1$,

$$c_m(l_m^*) \;=\; \frac{1}{r}\int_0^{l_m^*} (2r \;-\; 1 \;+\; s)\; ds \;= 2l_m^* \;-\; \frac{l_m^*}{r} \;+\; \frac{(l_m^*)^2}{2r}\;, \tag{43}$$

and,

$$\begin{aligned} c_f(l_f^*) \;&=\; \int_{l_f^0=(r-1)/r}^{l_f^*} (2 \;-\; r \;+\; rt)\; dt \qquad\qquad\qquad\qquad (44) \\ &=\; (2 \;-\; r)\left(l_f^* \;-\; \frac{r-1}{r}\right) \;+\; \frac{r}{2}\left[(l_f^*)^2 \;-\; \left(\frac{r-1}{r}\right)^2\right]. \end{aligned}$$

Although it is not possible to derive closed form solutions for the optimal levels of spousal labor supply in this case, they could then be derived–in implicit form–as in case 1.

3. If $r > 1$ so that there are more women than men in the marriage market, there will be some unmarried women in equilibrium. As in case 2, only if the intra-marital sharing rule yields the Pareto efficient outcomes so that, $\forall\ (y_m^*,\, y_f^*)$, $h_1(l_m^*, l_f^*) = 1 + l_f^* = c'_m$ and $h_2(l_m^*,\, l_f^*) = 1 + l_m^* = c'_f$, would the existing assortative marriage market equilibrium be stable. Using equations (11), (12), (29) and (30), we can derive that, for $r < 1$,

$$c_m(l_m^*) \;=\; \frac{1}{r}\int_{l_m^0}^{l_m^*}(2r \;-\; 1 \;+\; s)\; ds \qquad (45)$$

$$= \; 2l_m^* \;-\; \frac{l_m^*}{r} \;+\; \frac{(l_m^*)^2}{2r} \;+\; \frac{3r}{2} \;+\; \frac{1}{2r} \;-\; 2 \;,$$

and,

$$c_f(l_f^*) \;=\; \int_0^{l_f^*}(2 \;-\; r \;+\; rt)\; dt \;=\; (2 \;-\; r)l_f^* \;+\; \frac{r}{2}(l_f^*)^2 \;. \qquad (46)$$

Again, the optimal spousal levels of labor supply could be derived as in case 1.

9. Conclusion

In analyzing intra-marital family decisions, the collective household model treats each individual family member–as opposed to the whole family–as the relevant decision making unit. Empirical studies carried out in the last decade or so have provided consistent support for the idea that relative spousal incomes matter for family decisions and intra-household allocations. Hence, the collective approach to household decision-making has emerged as the compelling theoretical tool for analyzing the economics of the family.

The collective model relies on the assumption that external distribution factors such as the sex ratios in the markets for marriage and the distributions of income within the households determine the intra-marital sharing rules. Conventionally, it requires that the intra-marital sharing rules do not depend on internal distribution factors; that is, variables that enter spousal choice sets. As a consequence, either leisure is ruled out from individual preferences or the incomes relevant for intra-marital allocations are assumed to be those that the spouses *could earn* entering a marriage (and not those that the husband and the wife *actually do earn* once all labor supply, household production and leisure choices are made). But what if sharing rules depend on choices individuals make during the marriage? To take an example, how should we treat cases in which leisure enters individual preferences and intra-marital sharing rules are influenced by the household distribution of actual wage earnings? Then, there are at least two important issues. First, it is not clear how one would model the household labor supply choices. In the absence of binding commitments prior to the formation of marriage, the household labor supply could to be derived via a decision-making process that is non-cooperative in nature. Such a process would render household specialization less likely if not impossible. Then it is also important to ask if modeling the household labor supply as the outcome of a non-cooperative process is reasonable and empirically valid.

Moreover, the collective model relies on the Pareto efficiency of choices made within the households. In many versions of the collective household model, Pareto efficiency is a robust assumption as long as the sharing rules consistent with the collective model are primarily driven by external factors, such as the sex ratios in the markets for marriage, divorce legislation, and the *potential* (not actual) spousal incomes. However, in many other plausible extensions of the collective model where the marital decision-making power of the spouses depends on their actual labor incomes relative to that of their partners, Pareto efficiency becomes suspect. As I emphasized in the introduction, in such extensions and in the absence of binding commitments prior to the formation of marriage, the household decision-making process would be non-cooperative and competitive in nature. This raises the possibility of inefficiently high levels of labor supply as spouses would recognize that their choices not only influence total household income but also their decision-making power within the marriage.

In this paper, I have presented a collective household model in which there are marital gains to assortative spousal matching, individuals face a labor-leisure choice and intra-marital allocations are determined by an endogenous sharing rule that is driven by actual wage earnings. What I have found is that, even in the presence of competitive behavior and externalities in marriage, the process of spousal matching in the large marriage markets can help to (a) establish the levels of spousal labor supply and (b) maintain the efficiency of intra-household decisions. In particular, when the sex ratios in the marriage markets are not equal to unity or external distribution factors (such as marriage and divorce legislation) are not gender neutral, marriage market competition among potential spouses helps to generate maritally sustainable and Pareto efficient levels of labor supply and spousal consumption. In such cases, the sharing rule that supports the efficient, maritally sustainable equilibrium is also unique for each couple along the assortative order.

In sum, I have identified that neither strategic spousal interactions nor the endogeneity of intra-marital sharing rules with respect to spousal choices made during the marriage need to be accounted for if the marriage markets are large and the external distribution factors are asymmetric. Then, the efficiency of household choices are generally restored because marriage market competition helps to ensure that each spouse is compensated according to his or her marginal contribution to the marriage.

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Figure 1: The Spousal Labor Response Functions and the Equilibrium

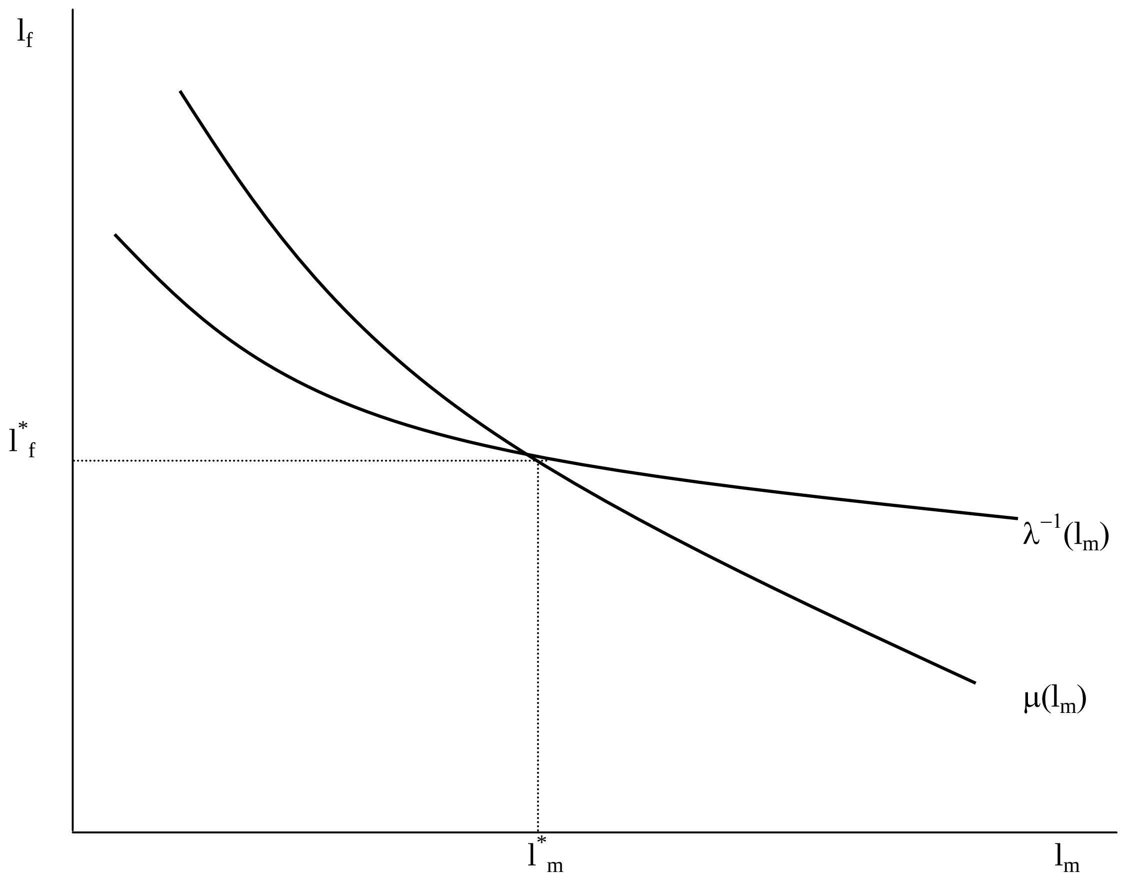

Figure 2: The Marital Matching Function

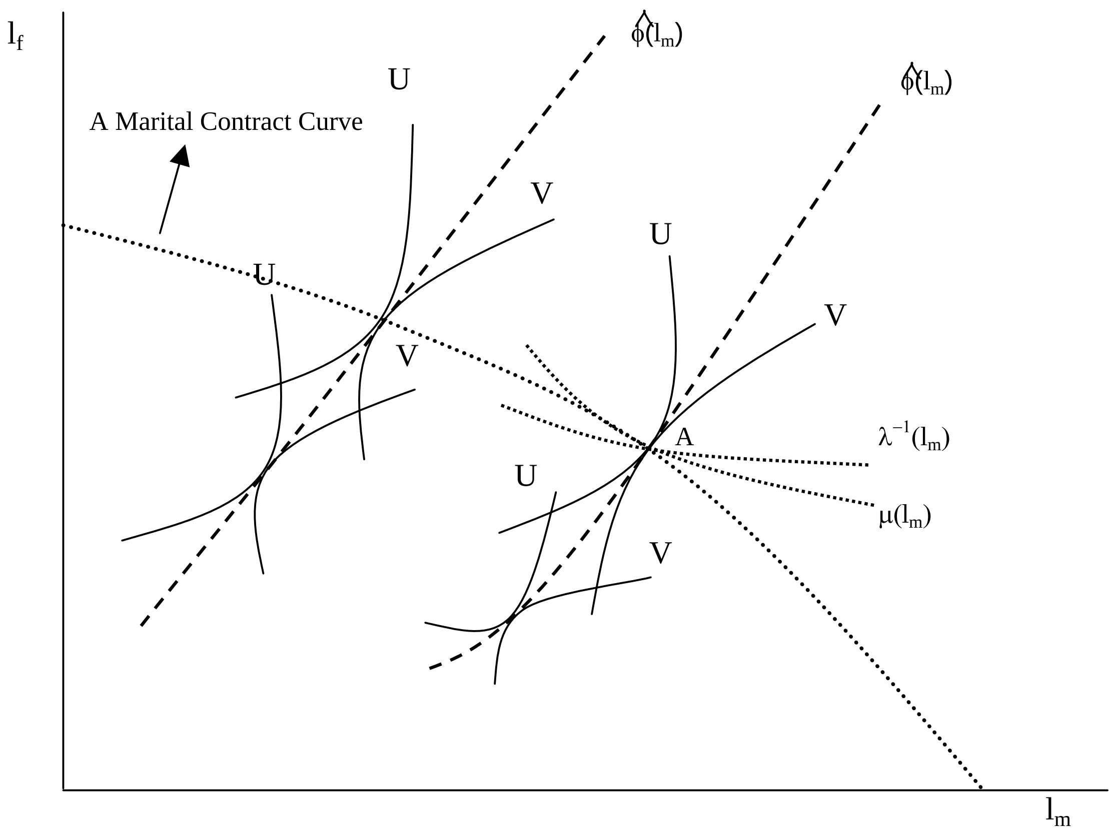

Figure 3: The Marital Contract Curve and the Efficient Frontier

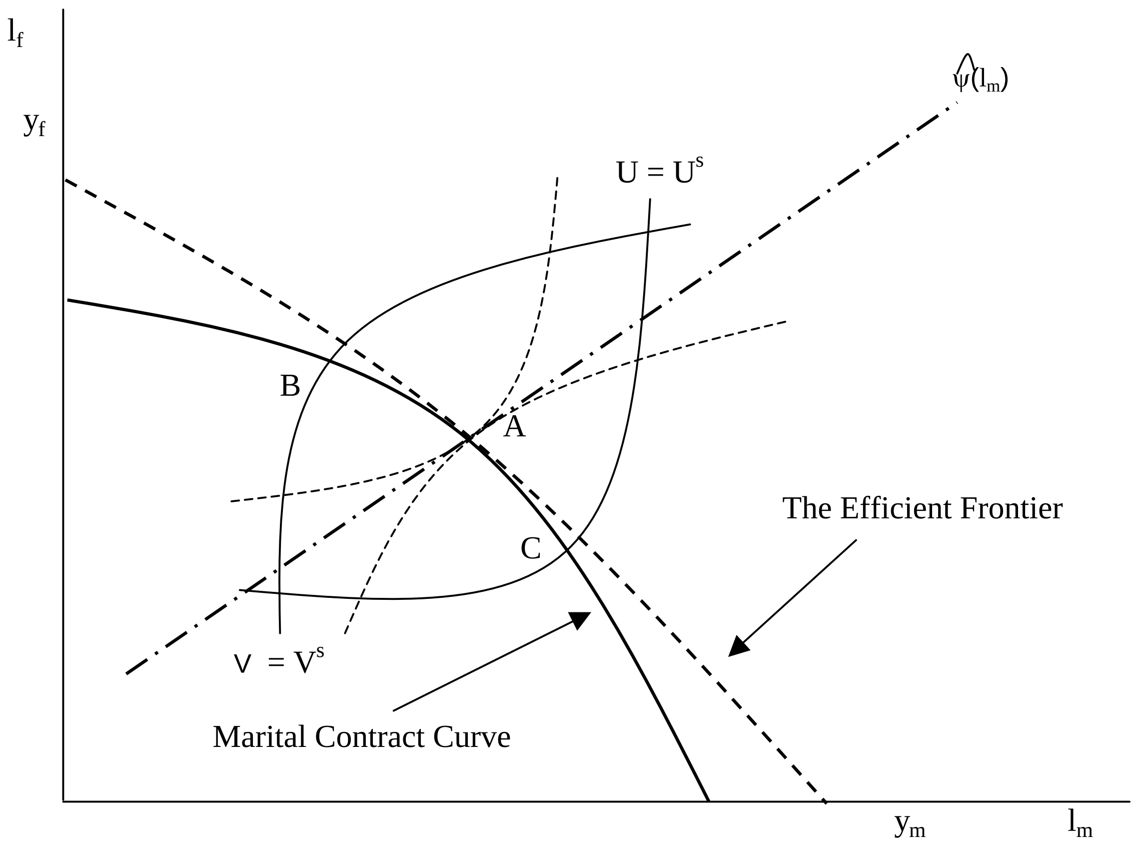

Figure 4: The Marital Contract Curve and the Efficient Frontier