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Mergers and Acquisitions in Network Industries

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Abstract

Information goods characterized by strong positive network externalities and effects are playing an increasingly prominent economic role. In this paper, a merger simulation is performed using a logit model of oligopolistic competition incorporating network externalities and effects.

While the model possesses features common to models detailed in the network literature, it provides a variety of new insights into producer and consumer behavior in such markets. Oligopoly producers are found to respond to higher price elasticities with lower prices and markups. Strategic behaviors arise that do not exist in the absence of network externalities. Network externalities are found to dramatically impact post-merger prices and market concentration.

JEL classification: D11; D43; L13; L40

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1 Introduction

Products exhibiting positive network effects and externalities are playing an increasingly important economic role. eBay, perhaps the most successful pure play internet business to date, benefits from strong network externalities in which the web site is more useful as more people use it. The “applications barrier to entry,” mentioned in the Microsoft antitrust litigation,[1] describes a network effect in which a disproportionate number of applications are produced for the dominant operating system, which in turn makes the operating system more desirable.

Although a number of stylized models have been advanced, no framework has been established suitable for ready applied examination of information goods that exhibit network externalities or effects. To that end, a discrete choice random utility model of positive network externalities and effects is presented with a focus on analyzing mergers and acquisitions in network industries.[2]

The model, referred to throughout as the Network MNL (multinomial logit) introduces two innovations in a traditional logit setting: A utility function in which a consumer explicitly values the consumption of a product and compatible products by others and a production function with a compatibility decision and an associated

[1] [1, p. 28]

[2] Network externalities are mentioned briefly in the context of logit demand in [3]; however, only the symmetric case is considered. Notably, they comment that in the asymmetric case, “The major difficulty with these equilibria is to characterize them and prove existence.” In this letter, it is shown that numerical treatment of the asymmetric case is both feasible and computationally tractable. An existence proof can be found in [4] appendix C.

cost of compatibility. Although the model is formulated in terms of network externalities, a simple reformulation of the network term will admit network effects and the results are roughly equivalent in either case.

2 The Model

N consumers make a discrete choice over a set of I products in which each consumer first chooses a single product from among the available choices, then purchases a continuous quantity of the product. Utility is modeled by a random utility function in which a consumer derives utility from an aggregation of consumption of a good, network externalities or effects associated with others' consumption of that good or compatible goods, and an idiosyncratic valuation that is independently and identically distributed according to a type 1 extreme value distribution with a zero location parameter

$$u_{i,n} = y + q_i - \gamma p_i + v(z_{i,n}) + \sigma \epsilon_{i,n} \tag{1}$$

where $i \in \{1, \ldots, I\}$ denotes product i, n denotes consumer n where $n \in \{1, \ldots, N\}$, $u_{i,n}$ is the utility of product i for consumer n, y is consumer income, q_i is the perceived quality of product i, p_i is the price of good i with γ its elasticity parameter, σ is a scaling parameter corresponding to the degree of heterogeneity across products, and $\epsilon_{i,n}$ is the consumer's idiosyncratic valuation of product i.

Consumer n's perception of the value of the network of product i, v, is taken to be a continuous and strictly increasing function of consumer n's perceived network size of product i $z_{i,n}$, that is, others' consumption of product i and compatible products. It is also given that $v(0) = 0$. Compatibility between products i and j is given by $\rho_{i,j}$, where $\rho_{i,j} \in [0,1]$ and $\rho_{i,j}\frac{\partial v}{\partial z_{i,m}}\frac{\partial z_{i,m}}{\partial x_{i,n}} = \frac{\partial v}{\partial z_{i,m}}\frac{\partial z_{i,m}}{\partial x_{j,n}}$ $\forall\ i \neq j,\ m \neq n$. $\rho_{i,j} = 0$ represents complete incompatibility whereas $\rho_{i,j} = 1$ implies that products i and j are fully compatible.

Associated with each consumer n and product i is a probability $\mathcal{P}_n(i)$ where $\mathcal{P}_n(i) = P(u_{i,n} = \max_{j=1,\ldots,I} u_{j,n})$. A symmetry assumption, $\mathcal{P}_m(i) = \mathcal{P}_n(i)\ \forall\ m,n \in \{1,\ldots,N\}$, is imposed on $\mathcal{P}_n(i)$ to provide both analytic and computational tractability and allows us to abbreviate $z_{i,n}$ as z_i and $\mathcal{P}_n(i)$ as $\mathcal{P}(i)$.

Integration ([4] appendix A) will show that a closed-form solution for $\mathcal{P}(i)$ is given by

$$\mathcal{P}(i) = \Psi_i(x;p,q,\rho) = \frac{e^{q_i - \gamma p_i + v(z_i)}}{\sum_{j=1}^{I} e^{q_j - \gamma p_j + v(z_j)}} \tag{2}$$

Production of good i involves a cost associated with the level of product quality, a constant marginal cost, and a compatibility cost associated with making a product compatible with other competing products. Producers are oligopolists; however, their compatibility decisions enable them to draw on the size of the consumer base of other producers' products. Formally,

$$\max_{p_i,\phi_i}(p_i - b_i)y_i(p,\phi) - a_i(q_i) - \sum_{j\neq i} c_{i,j}\phi_{i,j} \tag{3}$$

where y_i is the production of good i, b_i is the marginal cost of producing good i, a_i is a strictly convex, increasing function representing the cost of producing quality q_i, and $c_{i,j}$ is the cost of making product i compatible with product j, $i \neq j$, and $\phi_{i,j}$ represents the level of spending on compatibility.

The level of spending on compatibility impacts compatibility through the parameter $\rho_{i,j}$, determined by the continuous function ρ where $\rho_{i,j}$ denotes $\rho(\phi_{i,j}, \phi_{j,i})$. ρ is strictly increasing in $\phi_{i,j}$, nondecreasing in $\phi_{j,i}$ and concave in its arguments. If product i is compatible with product j, it does not imply that product j is equally compatible with product i. In this sense, a compatibility decision can involve construction of either a one-way or two-way adapter or something in between. With no spending on compatibility, products are fully incompatible and producers experience diminishing marginal compatibility. That is, it is assumed that $\rho(0,0) = 0$ and $\lim_{\phi_{i,j}\to\infty} \frac{\partial \rho_{i,j}}{\partial \phi_{i,j}} = \lim_{\phi_{j,i}\to\infty} \frac{\partial \rho_{i,j}}{\partial \phi_{j,i}} = 0$.

Equilibrium results from a simultaneous move Bertrand-Nash game. Producers and consumers form expectations regarding consumers' choices with complete information about the consumers' response functions. Producers simultaneously choose price and compatibility to maximize profit with complete information about the consumers' response functions. Consumers simultaneously maximize utility by choosing consumption taking prices, product quality, and compatibility as given. In equilibrium, both producers' and consumers' expectations of network size are realized; that is, expectations are rational.

First-order conditions for profit maximization are given by

$$x_i + (p_i - b_i)\frac{dx_i}{dp_i} \leq 0 \quad \perp \quad p_i \geq 0 \tag{4}$$

$$(p_i - b_i)\frac{dx_i}{dq_i} - \frac{da_i}{dq_i} \leq 0 \quad \perp \quad q_i \geq 0 \tag{5}$$

$$(p_i - b_i)\,\frac{dx_i}{d\phi_{i,j}} - c_{i,j} \leq 0 \quad \perp \quad \phi_{i,j} \geq 0 \tag{6}$$

where, from equation 2, firms face demand derivatives of

$$\frac{dx_i}{dp_i} = e_i\left(\sum_{n=0}^{\infty}\mathcal{J}^n\right)\mathcal{D}_p e_i^T \tag{7}$$

$$\frac{dx_i}{dq_i} = e_i\left(\sum_{n=0}^{\infty}\mathcal{J}^n\right)\mathcal{D}_q e_i^T \tag{8}$$

$$\frac{dx_i}{d\phi_{i,j}} = e_i\left(\sum_{n=0}^{\infty}\mathcal{J}^n\right)\mathcal{D}_{\phi_i} e_j^T \tag{9}$$

where $\mathcal{J} = \left(\frac{N-1}{N}\right)\left[\sum_{i=1}^{I}\frac{\partial\Psi_m}{\partial v_i}\frac{\partial v_i}{\partial x_n}\right]$, $\mathcal{D}_p = \left[\frac{\partial\Psi_m}{\partial p_n}\right]$, $\mathcal{D}_q = \left[\frac{\partial\Psi_m}{\partial q_n}\right]$, and $\mathcal{D}_{\phi_i} = \left[\frac{\partial\Psi_m}{\partial v_i}\frac{\partial v_i}{\partial\rho_{i,n}}\frac{\partial\rho_{i,n}}{\partial\phi_{i,n}} + \frac{\partial\Psi_m}{\partial v_n}\frac{\partial v_n}{\partial\rho_{n,i}}\frac{\partial\rho_{n,i}}{\partial\phi_{i,n}}\right]$. Although second-order regularity cannot be assured, numerical techniques have been found to be generally robust to perturbation of the calibration set.

The MNL has practical appeal as a "rough and ready" model [2] for the ease with which existing market data can be calibrated against the demand specification and counterfactuals introduced to analyze relevant policy decisions. The Network MNL is no different in this regard, but involves additional steps to calibrate the scale of the network externalities or effects and incorporate costs of compatibility. While preferences are estimable by well-established econometric techniques and prices and market shares are typically readily observable, compatibility levels may not be. The means by which compatibility levels would be determined would likely be product-specific. It is not atypical that some products may be wholly incompatible, in which case a cost of compatibility must be extrapolated from reasonable assumptions and observed calibration costs with respect to similar products.

3 Application

Unlike the traditional logit demand system, due to the increasing returns inherent in positive network externalities, multiple equilibria are quite common in the Network MNL; indeed, they are to be expected as a fundamental characteristic of the system when the value of network externalities is sufficiently large and convex in perceived network size. However, even in the presence of convex positive network externalities, multiple equilibria are not guaranteed. With weak network externalities and sufficient differentiation between products in terms of core attributes and/or pricing, a single stable equilibria will be found.

In general, positive network externalities exacerbate consumers' price responses, often quite dramatically. With preferences convex in network size, producers anticipate that consumers are more responsive to changing prices or product attributes than they would otherwise be when considering products that do not exhibit network externalities. On the other hand, in the limit as the number of consumers becomes large, with full compatibility, the elasticity approaches that found in a market without network externalities. Intuitively, any loss in network size is made up by a corresponding gain in the network size of compatible products, resulting in no net impact to the effective network size.

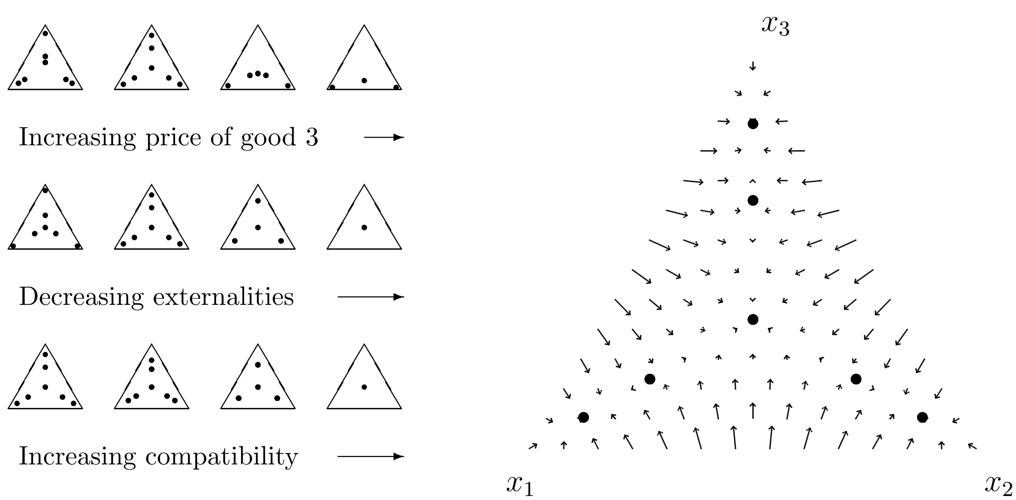

Figure 1: Demand equilibria and consumer tatônnement over the simplex

The positive feedback inherent to positive network externalities naturally gives rise to markets in which there is significant concentration on a single product. When there are sufficiently strong, convex network externalities, the relative importance of the differentiating features of a product are subsumed by the need to standardize on one of the available choices.

In figure 1, equilibria are plotted on a probability simplex representing a system with one consumer type and three producers. [3] Each of the three vertices represents the consumers' choice probability with respect to the associated good. As the

[3]Appendix A contains details of the model specification. Simulation tools are available from the author on request.

strength of network externalities decrease and consumers progressively favor features over standardization, market dominance becomes less exaggerated and the extreme equilibria draw progressively farther away from the vertices and toward the center of the simplex. [4] It is easy to see in the phase diagram, which describes a consumer tatônnement process in which consumers update their choice probabilities in each time period based on the previous choice probabilities of their peers, that both the extreme equilibria and the equilibrium describing symmetric choice probabilities are stable while saddle points separate the stable equilibria.

Symmetric or asymmetric, even with relatively mild network externalities, the logit demand formulation can admit extreme equilibria with the dominant firm commanding a substantial portion of the market.

$\eta_{i,j}$	$j=1$	$j=2$	$j=3$	$\eta_{i,j}$	$j=1$	$j=2$	$j=3$
$i=1$	-1.98	0.99	0.99	$i=1$	-1.17	0.58	0.58
$i=2$	31.84	-25.99	-5.86	$i=2$	18.83	-19.42	0.58
$i=3$	31.84	-5.86	-25.99	$i=3$	18.83	0.58	-19.42

Table 1: Price elasticities with and without network externalities

Network externalities and effects can also fundamentally alter the strategic incentives in the marketplace. As shown in table 1, with producer 1 the dominant producer and producers 2 and 3 forming a competitive fringe, convex positive network externalities can turn what would otherwise be substitute goods into complements. When a fringe producer steals market share from a dominant producer, thereby eroding the value of a very attractive network, other fringe producers' networks become relatively

[4] Without network externalities, a single equilibrium exists. In the symmetric cases, the equilibrium is represented by a point in the center of the simplex.

more valuable as a result. This dynamic may give rise to counterintuitive strategic behavior. A fringe producer may actually be incented to encourage a competitor to drop its price or subsidize the improvement of the quality of a competitor's product. [5]

	Share-weighted percentage price increase by merging firms	Percentage price increase by non-merging firm	% Δ HHI
No externalities	3.30%	0.41%	4.69%
Externalities	2.33%	-0.63%	7.38%

Table 2: Merger simulation

In table 2, results are given for a simulated merger between the dominant and a fringe firm with all firms producing goods symmetric in product quality and the dominant firm initially capturing 84% of the market. With or without network externalities, the dominant firm is incented to drive the price of the fringe firm up to effectively exclude it from the market. With the traditional antitrust logit model, any merger results in prices unambiguously higher. Consistent with traditional logit merger simulation, the newly-consolidated firm raises its share-weighted average price; however, facing more elastic demand, it does not raise its average price as high as it would in the absence of network externalities. On the other hand, the fringe firm finds it optimal to drop its price as it faces a more entrenched competitor.

[5] The recent phenomenon of "open source" software, in which the licensing terms of the software ensure that any improvements made by one vendor are shared with others, may in part benefit from such complementarities. IBM was reported to have spent $1 billion in 2002 on Linux, subsidizing development of the Linux operating systems and many related "open source" applications. According to the terms of the Linux licensing arrangement, commonly known as the GPL (Gnu Public License), any improvements to Linux or derivative works must be returned to the Linux development team and freely licensed to any third party.

Network externalities can even fundamentally change oligopoly pricing behavior following from a merger. With weak network externalities, it can be shown that average price may fall across the board following a merger. The impact of network effects and externalities on a merger are a mixed blessing, however. Though price effects are mitigated by the presence of network externalities, market concentration is exacerbated.

The introduction of compatibility changes can dramatically alter the dynamics of a merger. Table 3 details three merger simulations, two in the presence of relatively mild network externalities in which at the benchmark calibration point the dominant producer's good is 30% compatible with each of the fringe firms, the fringe firms' goods are 60% compatible with the dominant good, and the fringe goods are fully incompatible with each other. Compatibility is assumed introduced via a one-way adapter.

	Share-weighted percentage price increase by merging firms	Percentage price increase by nonmerging firm	% Δ HHI
No externalities	3.65%	0.99%	12.37%
Externalities without compatibility adjustment	-1.27%	-1.23%	47.40%
Externalities with compatibility adjustment	8.48%	0.64%	53.75%

Table 3: Merger simulation with compatibility changes

The first simulation presents a traditional merger simulation without network externalities. The second simulation presents an equivalently calibrated merger simulation with network externalities in which product quality and compatibility are held constant. The third simulation is conducted with the same framework and calibration as the second, but firms are presumed able to adjust compatibility.

The differences in firm responses when compatibility adjustment is introduced are profound. Without compatibility changes, consistent with the analysis reflected in table 2, additional market consolidation accompanies a merger when network externalities are introduced and the remaining fringe firm drops its price in the face of a more entrenched competitor. The merged firm, as is consistent with the previous simulations, raises the price of the product of the smaller of the merging firms to effectively drive it from the market. As it consolidates its market position, the price elasticity of the product of the larger of the two merged firms increases slightly, resulting in a lower share-weighted price increase by the merged firm. In stark contrast with the typical results from a traditional merger simulation, lower average prices shift consumption toward the the lower-cost product of the merged firm. With the additional network benefits that accompany a consolidated market, consumer welfare may increases following a merger.

If firms are permitted to adjust compatibility, the dominant firm is given an additional strategic instrument. With convex network externalities, as the dominant firm consolidates its market share, compatibility becomes progressively less cost effective for the remaining fringe firm. In the scenario above, the fringe firm's compatibility with all other firms is effectively driven to zero, and in the process, it is further marginalized as the merged firm further consolidates its market position.

While consumers gain value from the additional network externalities accruing from standardization on the merged firm's dominant product, they lose the network benefit conferred by the fringe good from its compatibility with the dominant network. The merged firm also benefits from the consolidation of market power, allowing it to maintain a higher oligopoly price and increasing industry profits.

While without compatibility as a strategic choice, consumers may benefit from a merger, the combination of the loss of network benefit from compatibility with a dominant network and the increase in prices may very well qualitatively change the nature of the merger to consumers.

4 Conclusion

If we are to believe that market behavior in the presence of network externalities is sufficiently similar to the Network MNL, a market in which consumers explicitly value network size behaves quite differently from one which does not. Furthermore, common inferences about preferences no longer apply in the presence of network externalities. The dominant good may be of inferior quality. Strategic opportunities arise that do not exist in markets without network externalities.

The Network MNL should be of interest in many settings in which the traditional MNL demand formulation is used, including merger and acquisition simulation analysis, theoretical analysis of competition in differentiated products industries, and marketing, and is particularly relevant for the analysis of many information goods and technology products which naturally exhibit network externalities and effects.

A Model Formulation

100 identical consumers were assumed to maximize utility over the function $u_{i,n} = q_i - \gamma p_i + (z_{i,n}/K)^2$ where i indexes products and n indexes consumers. q_i was assigned a value of 21 for all i and p_i was assigned a value of 20 for all i with the exception of figure the series representing increasing price in which p_1 assumed the values 19.94, 20.00, 20.15, and 20.58. K was assigned a value of 54 with the exception of figure the series representing decreasing strength of externalities in which it assumed the values of 52.00, 54.00, 55.10, and 56.00. $z_{i,n}$ was given by $(N-1)[P(i) - \sum_{j \neq i} \rho_{i,j} P(j)]$. Complete incompatibility was assumed with the exception of the series representing increasing compatibility in which $\rho_{i,j}$ assumed values of 0, 0.29, 0.31, and 0.98 for all $i \neq j$.

Marginal costs were calibrated to $b_i = p_i + x_i / \frac{dx_i}{dp_i}$ where $x_i = NP(i)$. Quality costs can be recovered from the expression $(p_i - b_i)\frac{dx_i}{dq_i} = \frac{da_i}{dq_i}$ and compatibility costs from $c_{i,j} = (p_i - b_i)\,\frac{dx_i}{d\phi_{i,j}}$.

The merger analysis of table 2 assumed the merger of firms 1 (the dominant firm) and 2, resulting in the merged firm maximizing combined profits. Product quality in the case without network externalities was calibrated to given prices, compatibilities, and market shares. Product quality and compatibility were assumed to be exogenous and products were taken to be fully incompatible. The merger analysis of table 3 likewise assumed the merger of firms 1 (the dominant firm) and 2.

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