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Anarchy and Demand for the State in a Trade Environment

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ANARCHY AND DEMAND FOR THE STATE IN A TRADE ENVIRONMENT*

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Abstract

We investigate the relationship between polity formation and the level of economic activity. We posit a dynamic search environment in which opportunities for mutually beneficial trade may be hampered by theft. Agents search for potential trading partners and, if matched, optimally choose whether to attempt to trade or to steal from each other. The excludability of goods – in the form of respected property rights – is endogenously determined as a result.

We compare the equilibria of this game under anarchy to those of an identical environment in which there is a "government" in the minimal sense of an agency that protects property rights. In exchange for protection, agents pay a certain amount to enter the market. We find that agents' willingness to pay for these services – i.e., to be taxed – is increasing in potential gains from trade, and this result is robust to corruption in government.

Keywords: property rights, theft, gains from trade, role for government

JEL Codes: H11, H41, K42, 017

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1 Introduction

Much has been written regarding the beneficial effects of government on the level of economic activity (see Shleifer (1998), Besley (1995) *inter alia*). There is a consensus that a necessary condition to modern growth is the existence of an incentive system that benefits those who incur the costs of productive endeavor. In particular, a precondition for the voluntary exchange of goods and services in a non-autarkic economy of non-altruistic agents is the existence of enforced property rights, such that agents may reap the benefits of costs that they incur. For example, if levels of theft are high, agents will not have an incentive to produce translatable goods, and will find it optimal to revert to autarky or subsistence.

The conventional rationale for government to arise in an autarkic anarchy is the presence of returns to scale in the protection technology. For instance, in Grossman (2002), a government centralizes protection activities with the aim of decreasing the probability that a good belonging to an individual is expropriated by another. Centralizing protection, therefore, can eliminate the over-investment of productive resources in an "arms race" that would otherwise obtain in a decentralized ("anarchic") society, as in Skaperdas (1992).

The focus of our paper is different. Instead, we examine the potential role of government as *arbiter*. Agents in our environment do not enjoy the good they produce and possess. Rather, the value that they attach to goods reflects the possibility of *exchanging them in the marketplace* for goods they *can* beneficially consume. In this context, the protection of property rights amounts to assuring that traders can leave the "negotiation table" possessing goods of the same or greater private value than those they began with. The focus is thus fully on the nature, and institutions, of exchange.

We begin with anarchy in the sense of Hirshleifer (1995):

> [...] anarchy is a social arrangement in which contenders struggle to conquer and defend durable resources, without effective regulation by either higher authorities or social pressures (Hirshleifer (1995), p. 27).

We then follow Nozick (1974) in defining a role for government in such an environment. In "Anarchy, State, and Utopia" (ASU), Nozick suggests that an agency that enforces property rights can arise through market-like mechanisms. In ASU, agents are willing to pay for the services of a "protection agency" (or a "minimal government") which adjudicates in disputes and protects property rights. We adopt a simple environment in which disputes are clear-cut in that they only arise in the case of overt theft, and derive demand for such an entity from economic primitives.[1] Unlike Nozick, we do not envision any notion of entitlements beyond institutions, and explicitly derive the conditions under which both property and, indeed, trade itself, may emerge.[2] We ignore the possibility that government may serve other functions in order to focus on its role protecting property rights.

The model addresses some fundamental issues regarding the nature of goods and exchange. The extent of rivalry and excludability in goods is at the basis of many lines of thinking in the economics of externalities, from public choice to technological change. Rivalry and excludability are traditionally taken as given in discussions of private goods. However, as Romer (1990) insightfully points out, rivalry is a technological (or "physical") property whereas *excludability* is both technological and *institutional* in nature. In the present framework, the extent of excludability is endogenously determined by the interaction between institutional and technological constraints *vis a vis* agents' equilibrium strategies.

To simplify the exposition, we assume an exchange economy with indivisible goods. Both goods and preferences are heterogeneous,[3] so that a meeting of two agents will not necessarily generate opportunities for trade. In the case of a Smithian "double coincidence of wants," potential traders may swap goods *quid pro quo*, which benefits them both symmetrically. And so they do, we assume, if both decide to "trade fairly" – or, equivalently, to *respect a notion of property rights*. However, they may independently decide not to, instead, choosing to obtain the desired good without "paying" for it with their own produce. The value of property, therefore, depends critically on the probability of meeting a robber in the market place.

It is worth underlining that agents do not differ in their attitudes towards "criminal

[1] Bös and Kolmar (2000) adopts a related approach, studying "rules of voluntary redistribution that Pareto-improve an anarchic initial situation".

[2] Institutions "transform possession into property." We thank Serge-Christophe Kolm for this concise formulation.

[3] The model admits an interpretation in terms of specialization and the division of labor. We thank Yang Yoon for this insight.

activity,"[4] so that the decision to trade or thieve is purely "rational." Institutions that decrease this probability enhance the value of tradeable goods, potentially generating a *willingness to pay* for access to a safer marketplace. The demand for a "minimal government" that can provide such a marketplace then arises naturally. Our main result is that the demand for protection increases with the "gains from trade," the immediate enjoyment from consumption of the good produced by the successfully matched trading partner.

The remainder of the paper is organized as follows. We describe the trade environment and its equilibria under anarchy in Section 2. The minimal government is introduced and its influence on equilibrium structure addressed in Section 3. The following section, 4, extends the model to allow for corruption in government. We find that the model is robust to this extension. In Section 5, we consider an environment in which agents return repeatedly to the market, and derive welfare implications. We then discuss possible extensions. Section 7 concludes.

2 Model Economy

2.1 Basic Setup

The model is a simple discrete-time matching setting similar to that found in, for example, Kiyotaki and Wright (1993).[5] There is a continuum of infinitely-lived households distributed evenly on a circumference, henceforth referred to as Farmland.

Agents are characterized by their "type" $i \in [0,1]$. In any given period, agent $i \in [0,1]$ may also produce a translatable good i. It does not yield her any utility; she is, however, willing to consume some of the translatables produced by other agents. In particular, she is equally happy to consume any item j from the interval

$$j \in \left[\left(i + \frac{1-\beta}{2}\right) \operatorname{mod} 1, \left(i + \frac{1+\beta}{2}\right) \operatorname{mod} 1\right] \tag{1}$$

[4] In this sense our approach departs from the literature on crime prevention that builds on the assumption that agents are *born* with different "tolerance[s] for engaging in illegal transactions" – see Boadway et al. (2000) among others.

[5] An interesting extension might involve a monetary environment such as theirs.

for $\beta \in (0,1)$. Thus, each agent seeks a proportion β of the goods produced by the others. Note that, if agent i likes the good produced by agent j, the converse also holds. Therefore, if two agents are anonymously matched pairwise, the conditional probability of a "double coincidence of wants" is β. This setup is represented in Figure 1.

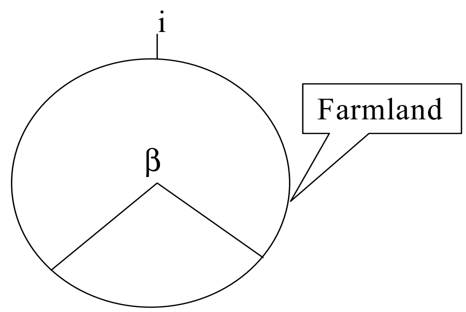

Figure 1: Preferred goods of agent i

Agents earn utility G from consuming a desired good. Goods are indivisible and durable; they do not depreciate until they are consumed, and agents can carry at most one unit of them. Farmers are risk-neutral, discounting the future using a common factor $\delta \in (0,1)$.

In order to find goods that they desire, agents must locate other farmers and obtain their goods – via trade or otherwise. We henceforth refer to the parameter G as "potential gains from trade".

There exists an additional location, which we denote "Market town." This is the locus of a matching technology that each period randomly pairs any agents who locate there. Once matched, agents can observe each other's good before they decide to interact. Having verified the coincidence of wants, they independently attempt to either trade or rob, where robbery is interpreted as the appropriation of another's good for nothing in exchange. This interaction will be referred to as the exchange-theft game.

In what follows we will assume that a positive fraction of agents is always present in the Market town.[6] The assumptions are such that agents who do not have a good

[6] No matter what institutions are in place, there is always a "Ghost Town" equilibrium, in which nobody trades, because there are no traders in the Market town in the first place.

to exchange have no purpose in staying in Market town since, lacking a good, nobody will approach them close enough for any interaction. Theft, being their only option, has no opportunity to materialize.

We also assume that agents' histories are unobservable, and that the probability of meeting the same agent twice is zero. We also focus on stationary equilibria, in which agents do not adopt strategies that are contingent upon their own histories.[7] As there is a continuum of agents in the Market Town, we restrict attention to strategies involving pure actions, in order to avoid "measurability" problems.[8]

Conditional on a match, an agent expects her counterpart to be fair trader (as opposed to a thief) with probability $\gamma \in [0, 1]$, which she takes as given. γ is an *equilibrium,* if it describes a fraction of fair traders in a stationary subgame perfect equilibrium of the exchange-theft game. Observe that equilibrium value of γ, as well as being a fraction of fair traders, may be interpreted as the (endogenous) extent of excludability of the goods that the agents produce in this economy. Agents' payoffs, and therefore the equilibria, will depend on the institutions in place. The exact nature of this relationship is examined in the following sections.

First, we consider an environment in which agents travel to Market town only once. They view the value of their possession as the expected stream of utility that they can exchange it for in the marketplace. We demonstrate that this value increases in the presence of the minimal state, and that the increase is rising with the "gains from trade", G. Section (5) closes the model by adding production and considering the "lifetime value of being a farmer" with and without government; similar results are found to hold. All proofs are either in the Appendix, or available upon request.

2.2 Anarchy

Let "anarchy" denote the basic environment devoid of institutions. Under anarchy, meetings are observed only by the affected parties. Assume that there is a double coincidence of wants among matched agents. Suppose an agent decides to trade fairly. In case the partner is also a fair trader, she obtains a payoff of $G > 0$ and leaves the

[7]These assumptions rule out decentralized punishments, which might support certain "norms of behavior" – say, trading fairly. See section 7.

[8]See Judd (1985), Hammond and Sun (2000) among others.

market. If the partner has chosen to rob, she looses her good and leaves empty handed. Thus, the instantaneous expected payoff conditional on a match is $u^a\left(\text{trade}\right) \equiv \gamma G$.If, instead, the agent decides to rob, she deprives her partner of his good and consumes it for a payoff of G. In this case she remains in the marketplace, maintaining possession of her own produce for a continuation payoff V^a in the following period. Denote by V^a the value attached to a tradeable good under anarchy. Thus, the encounter yields $W^a \equiv G + \delta V^a$.[9] If, however, the partners simultaneously attempt to rob, each succeeds with probability one half. In this case, the payoff conditional on the match is $u^a\left(\text{rob}\right) \equiv \gamma W^a + (1-\gamma)\frac{1}{2}W^a$. Hence, in equilibrium,

$$V^a = \beta \max\left\{u^a\left(\text{trade}\right), u^a\left(\text{rob}\right)\right\} + (1-\beta)\,\delta V^a. \tag{2}$$

It is straightforward to demonstrate that

Lemma 1 *The only equilibrium under anarchy is* $\gamma = 0$.

The Market town becomes a "Den of Thieves": farmers bring their produce to market only to be robbed or to steal from others. They use their good, like a "bait," to attract partners with desired goods. Having verified the "double coincidence of wants", they struggle over each other's possessions. Under anarchy, being a fair trader eliminates the chance of consuming the desired goods altogether.

3 Minimal State

Consider now an environment identical to that above, except that there is an agency (the minimal government) that covertly observes any given match with probability $\omega \in (0,1)$. If a robbery is observed, the agency is capable of inflicting a cost $c > 0$ upon the robber, and reinstating the good to the victim.[10] [11] This cost can be

[9] W is for "win": combat against fair traders is always successful.

[10] Although we do not restrict the severity of punishment, we recognize that most "civilized" punishments are bounded, as is the claim against future earnings.

[11] Assume for now that a detected robber and a government agent can not collude. We relax this assumption later.

thought of as a physical punishment, ignominy, or as a claim towards a stream of goods to be produced in the future.[12]

Now, when a the fair trader meets a robber with a desired good, with probability ω she can continue trading in the next period, which yields $\omega\delta V^g$, where V^g is the expected value of a good in the marketplace under the minimal government. If she meets a fair trader, her payoff is G as before. Thus, the expected value of an encounter with the owner of a desired good is $u^g\,(\text{trade}) = \gamma G + (1-\gamma)\,\omega\delta V^g$. In the presence of a protection agency, a robber who is matched with a fair trader may be observed and punished. If detected – which occurs with probability ω – she is afflicted with c, returns the stolen good to its "owner", and awaits another match.

On the other hand, if the robbery goes *un*detected, she consumes the good of her partner and remains in the marketplace, as under anarchy. Thus her payoff (denoted $W^g\,(\omega, c)$ for "win", as before) is $W^g\,(\omega, c) \equiv G(1-\omega) - c\omega + \delta V^g$. This value differs from that under anarchy, as it depends on the probability of being observed, ω, and the severity of the punishment, c.

If *both* agents attempt to rob and this is observed, only the successful robber is punished and the other's good is reinstated. The presumption is that it is impossible to verify an unsuccessful robbery attempt (or an "intent" to rob).[13] Thus, in case a robber meets another robber with the desired good she can either win the fight and get $W^g\,(\omega, c)$, or else fail, which, in the presence of government, may still yield a positive continuation value, for the interaction can be observed (with probability

[12] Punishment c is neither history-dependent nor modeled as a term of imprisonment. Immediate, history-independent punishments characterize most past cultures and judicial systems, except where slavery was used for purposes of retribution or redress. For example, in Europe, jurisprudence ignored individual characteristics (save for political power) until the 18th Century. Deeper in history, the punishment for theft in the Code of Hammurabi of the 20th Century BC is a fine – or death if payment is beyond the ability of the perpetrator. Again, punishment is not history-dependent – except in the trivial case in which the perpetrator is incapable of fulfilling the punishment. See Jastrow (1980). Jewish and Islamic criminal codes are similar to – and, indeed, have roots in – the Babylonian codes.

As for imprisonment, according to Foucault (1975) in Europe, a term of service and fines were the usual common forms of punishment through the early Middle Ages, being replaced by a system of corporeal and capital punishment later on. Imprisonment as punishment did not appear until the 17th Century, and was the lot of few until the early 19th Century, when an elaborate prison system developed. See also Kirchheimer and Rusche (1939).

[13] This assumption stacks the deck against the minimal state yielding any welfare improvements, as not all attempted robberies are detected.

ω) and her good re-instated, thus, enabling her to stay in the marketplace yielding $\omega\delta V^g$. The expected value of an appropriate match for the robber is then $u^g(\text{rob}) = \gamma W^g(\omega, c) + (1-\gamma)\left[\frac{1}{2}W^g(\omega, c) + \frac{1}{2}\omega\delta V^g\right]$.

The value traders attach to the good they bring to market is

$$V^g = \beta \max\{u^g(\text{trade}), u^g(\text{rob})\} + (1-\beta)\delta V^g \tag{3}$$

Recall that an equilibrium is a fraction of fair traders γ consistent with the optimal behavior of all potential traders. In a stationary equilibrium, an agent will choose either one action (to rob or to trade) "forever", or she will be indifferent between the two. We are interested in understanding what equilibria can be "induced" by a protection agency using punishment c and intensity ω of observing trades.

We start with two simple observations.[14] First, if c is high enough, it is an equilibrium for everybody to trade fairly. Second, in spite of the minimal state, the "Den of Thieves" equilibrium exists, if detection and retribution are lenient. Indeed, *both* "corner" equilibria exist for some range of punishments c.

In addition, there are 'interior' equilibria in which fair traders and robbers are present in the Market Town. To find these, we look at the difference $F(\gamma; c)$ between the value of tradeable goods for perpetual fair traders V_t^g and that of chronic robbers V_r^g.[15] [16]

Figure 2 plots $F(\gamma; c)$ for three different values of c, keeping other parameters fixed.[17] Any roots of F in $(0, 1)$ correspond to interior equilibria.

Figure 2 suggests that there are values of c that potentially generate *two* interior equilibria. Higher values of c are associated with only one interior equilibrium, while $\gamma = 1$ is also an equilibrium since $F(1, c) > 0$. On the other hand, punishments may also be so low that $\gamma = 0$ is the only equilibrium. According to the Figure 2, this "no trade" equilibrium is also present in the two cases mentioned above, since $F(0, c) < 0$ in all the cases depicted.

Finally, it is not surprising that, if c is high enough, $\gamma = 1$ is the unique equilib-

[14] See lemmata 14, 15 in Appendix $A.2$.

[15] See Appendix $A.2$ for the validity of this approach.

[16] More precisely, we use a function $F(\gamma; c)$ that has the same *sign* as this difference. See the detailed definitions in Appendix $A.1$.

[17] Parameter values in this example are $\omega = 0.5$, $G = 1$, $\beta = 0.5$, and $\delta = 0.9$.

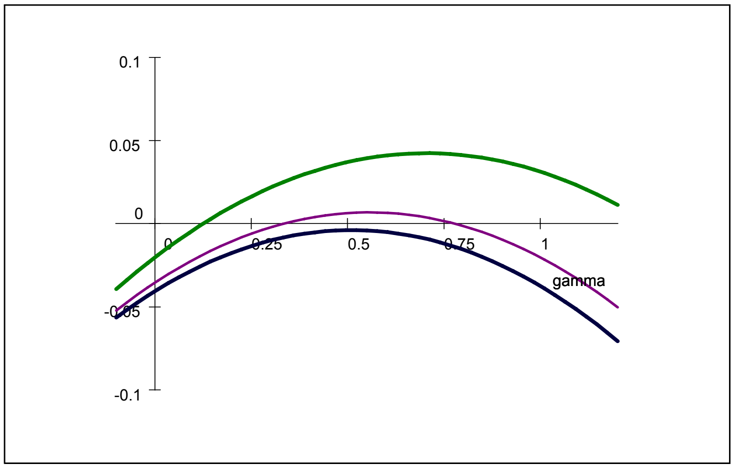

Figure 2: The lower line corresponds to $c = 1/4$, the middle one is for $c = 9/16$ and the upper line is for $c = 3/4$.

rium. Increasing punishment hurts thieves ,while leaving traders indifferent except via the equilibrium effect on γ, so that it is possible to completely deter robbery by imposing very severe punishments. Formally,

Proposition 2 *If $\beta\delta + \delta \leq 1$, then the set of equilibria $\Gamma(c)$ can be described as follows:*

$$\Gamma(c) = \begin{cases} \{0\} & \text{if } c < \underline{c}(\omega) \\ \{0, \gamma_L(c), 1\} & \text{if } c \in [\underline{c}(\omega), \bar{c}(\omega)] \\ \{1\} & \text{if } c > \bar{c}(\omega) \end{cases} \tag{4}$$

If, on the other hand, $\beta\delta + \delta > 1$, then the set of equilibria becomes:

$$\Gamma(c) = \begin{cases} \{0\} & \text{if } c < \underline{\underline{c}}(\omega) \\ \{0, \gamma_L(c), \gamma_H(c)\} & \text{if } c \in [\underline{\underline{c}}(\omega), \underline{c}(\omega)) \\ \{0, \gamma_L(c), 1\} & \text{if } c \in [\underline{c}(\omega), \bar{c}(\omega)] \\ \{1\} & \text{if } c > \bar{c}(\omega) \end{cases} \tag{5}$$

where $\gamma_L(c), \gamma_H(c) \in (0, 1)$ and $\gamma_L(c) < \gamma_H(c)$.

Moreover, $\gamma_L(c)$ is decreasing in the severity of punishment c, while $\gamma_H(c)$ is increasing in c.

Corollary 3 *If* $\omega \geq \frac{\beta\delta}{1-\delta+\beta\delta}$*, then* $\gamma = 1$ *(full trade) is an equilibrium even if* $c = 0$*.*

By corollary 3, if the detection rate is high, there is no need to inflict direct cost c on the observed robbers to prevent robbery altogether. The fact that the government reinstates the stolen item to the owner obliges the thieves to *wait* for a future opportunity before stealing or trading, which is a sufficient deterrent in itself. If agents are impatient or meetings are rare, this effect is exacerbated.

The relation between the equilibrium values of γ and punishment c can be conveniently represented graphically. Figure 3 depicts an example of an environment described in the second part of Proposition 2. Observe that $\gamma_L(c)$ approaches zero as c approaches $\overline{c}(\omega)$, and that $\gamma_L(c)$ and $\gamma_H(c)$ converge as c decreases towards $\underline{\underline{c}}(\omega)$.

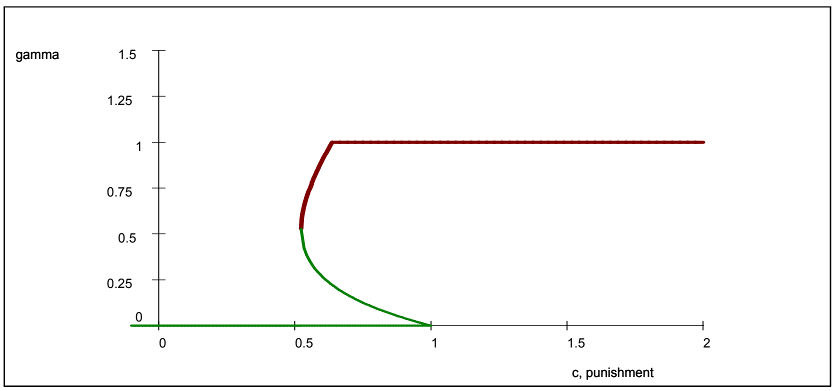

Figure 3: Here $\omega = .5$, $G = 1$, $\beta = .5$, $\delta = .9$, as in the previous example. Note that it implies $\beta\delta + \delta > 1$. In this case $\underline{\underline{c}} = .0.523$, $\underline{c} = 0.636$, $\bar{c} = 1$. Note also that γ_L (depicted as a thin line) is decreasing in c and γ_H (depicted as a thick line) is increasing in c.

Figure 4, in turn, illustrates the first part of Proposition 2. Here the "upper" equilibrium is always unity. This case also illustrates Corollary 3, in which theft is not a dominant strategy even without punishment ($c = 0$). Note the contrast with the anarchic Market Town, where theft leads to higher payoffs regardless of γ.

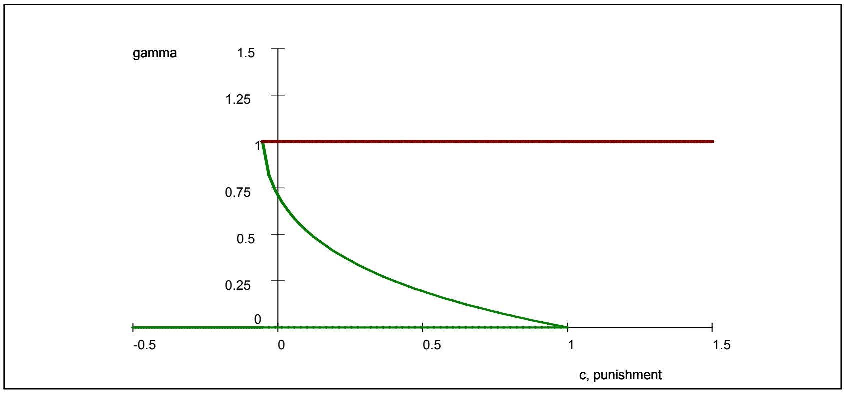

Figure 4: Here all the parameters are the same as before apart from $\beta = .1$. Note that it implies $\beta\delta + \delta < 1$. In this case $\underline{c} = -5.2632 \times 10^{-2} < 0$, $\bar{c} = 1$. Note also that γ_L (depicted as green line connecting the two corner equilibria) is decreasing in c .

3.1 Discussion

3.1.1 Crime and Punishment

Interestingly, there is a sense in which the severity of punishment and the rate of theft in the population may be *positively* related. Recall that $\gamma_L(c)$ and c are *negatively* related, so that harsher punishments are associated with higher robbery rates for this class of equilibria. However, it is also true that this type of equilibrium is *unstable*, in the sense that small perturbations to the punishment c lead agents to equilibria that are not in a neighborhood of $\gamma_L(c)$. This is not the case for equilibria in which $\gamma = 0$, $\gamma = \gamma_H(c)$ or $\gamma = 1$. See Claim 19 in Appendix $A.2$ for details.

Although it is reasonable to consider the intensity of observing trades, ω, to reflect a technological constraint, the severity of punishment c is, at least to some extent, under control of protection agencies (see footnote (12)). Such an argument suggests that it is appropriate to focus on the "good" stable equilibrium $\gamma_H(c) \leq 1$, which is positively related to punishment. If, by setting c at a certain level the outcome is γ_L, the agency could slightly increase punishment and switch to a "higher" equilibrium with $\gamma = \gamma_H(c)$. Moreover, as we will see in what follows, the demand for a protection agency is higher, if the marketplace it provides is safer. Hence, it will be in the interest of the protection agency to induce the equilibrium with larger fraction of fair traders, if it can do so.

In addition observe that more severe punishments are needed to discourage robbery in "patient" societies. The reason is related to Corollary 3. A high discount rate means that the component of the disincentive to trade that hinges on consumption deferral (due to $\omega > 0$) is less severe. This, on the margin, favors theft. Indeed, the lowest punishment ($\underline{c}$) that can support the all-trade equilibrium ($\gamma = 1$) is increasing in both the probability of a successful match (β) and the discount rate of an agent (δ). Thus, it is not surprising that "patient enough" societies with good enough probability of meeting a trading partner with the desired good ($\beta\delta + \delta > 1$) can deter some – but not all – theft, while the same punishment is sufficient to discourage robbery completely in less patient societies (at least in one of the equilibria).

3.2 Demand for Protection

Do agents benefit *vis-a-vis* an anarchic situation – and, if so, how much would they be willing to pay for an alternative?

To determine this quantity, suppose that agents can enforce anarchic payoffs if it is in their interest. This will occur, for example, if there is an alternative, anarchic locus for the matching technology, where farmers may enforce the "Den of Thieves" equilibrium derived in Lemma (1). An additional interpretation is that agents may overthrow the Minimal State.

Let us define the *demand for protection*, $D(\gamma, G)$, as the difference between the value of a tradeable good in a "protected" marketplace and that in an "anarchic" market. In other words, this demand corresponds to the willingness to pay of a trader, who has a tradeable good, to enter the protected marketplace, the amount he can be charged "at the gate" of the Market Town. As one would expect, this amount depends on the *safety of the marketplace*, or the fraction of the fair traders in it, γ. Moreover, it varies with the "gains from trade", G, or with the magnitude of his desire to consume the good of another trader. We will focus on stable trading equilibria.

Proposition 4 *The demand for protection arises whenever $\gamma > \underline{\gamma}$, where $\underline{\gamma} \in (0, 1)$. Moreover, in this case the demand increases in the gains from trade, G, provided that the safety of the marketplace, γ, is kept constant.*

Proposition (4) states that the inhabitants of Farmland are ready to pay for the access to the Market Town, if the induced proportion of fair traders is above a certain threshold, in other words, if the marketplace is safe enough. It is also easy to check that the demand grows with γ. What is striking though is that agent's willingness to pay for the government that protects *her* property rights is increasing in her valuation of the good produced and owned by *other* traders ("gains from trade", G). This creates an additional "niche" for the State quite distinct from those described in the literature, where the willingness to pay for protection depends on the utility from consumption of *one's own* goods.

Comparing across "stable" equilibria, so long as $c > \underline{c}(\omega)$, an increase in G implies that the willingness to pay increases also – at least locally – since $\gamma = 1$ will remain an equilibrium for sufficiently small perturbations of G. However, for $\gamma = 1$ to be an equilibrium, the severity of punishment must exceed $\underline{c}(\omega)$ and, for $\gamma = 1$ to be the *only* equilibrium, the condition is that $c > \bar{c}(\omega)$. Both thresholds increase in G,[18] so that it becomes harder to induce the equilibrium in which everybody trades fairly as the "stakes" become high.

More broadly, therefore, the protection agency may have to adjust punishments to keep the safety of the marketplace constant. Indeed, as the desirability of goods produced by other traders increases for a given agent, the temptation to thieve may become strong enough to lure them into robbing, if the punishment remains unchanged. This will be necessary, even locally, if G and c are such that the equilibrium prevailing in the Market Town is initially $0 < \gamma_H(c) < 1$.

Lemma 5 *Assume $\beta\delta + \delta > 1$ and $c \in \left(\underline{\underline{c}}(\omega), \underline{c}(\omega)\right]$. Then, as the gains from trade G increase, in order to preserve the safety of the marketplace (γ_H), the severity of punishment c must increase.*[19]

To summarize, as the gains from trade increase, so will the demand for the State

[18] The former does if $\omega < \frac{\beta\delta}{1-\delta+\beta\delta}$.

[19] An alternative formulation of lemma 5 is as follows. Define a correspondence $K(x)$ on the interval $[0, 1]$:

$$K(x) = \{(c, G) : \gamma_H(c; G) = x\} \tag{6}$$

$K(\gamma)$ is an "*isoklept*", the locus of punishment-gain combinations that lead to a constant level of theft. It can be shown that the isoklept curves are single-valued and positively sloped under the assumptions of the lemma.

that enforces property rights. In some cases the Minimal State, viewed as a unit, may be motivated to increase the severity of punishment as the gains from trade increase, if it is capable of capturing part of the additional willingness to pay for protection that arises as a result (so long as they are not constrained by the mores of the cultural environment, or by technology).[20]

In the next section we relax the assumption that prevented collusion between the observed robbers and protection agents. We, thus, extend the main result of the paper to the case of a corrupt government.

4 Should the observers be trusted?

The answer is: no, not always. However, the results of the previous section do not qualitatively change, as the following analysis shows.

Clearly, protection agents may be tempted to accept bribes from detected robbers. The highest bribe that a protection agent can expect is $G+c$, which is what the robber foregoes, when caught. Assume that if a bribe is accepted, the robber can keep her own good and continue operating in Market town in the following period.[21]

If the robbers know that the protection agency is corrupt, they will always be interested in bribing.[22] If all robbers bribe, the good is never re-instated to the owner – this reduces the gains from fair trading given γ. The instantaneous payoff from doing so becomes the same as under anarchy, apart from the fact that in equilibrium

[20]There is another sense in which increasing punishment may be necessary to preserve the "safety of the marketplace". Consider a non-stationary extension in which G increases at an exponential rate g. If c increases at the same rate, this is equivalent to the current framework (net of a change of variables) so long as $e^g\delta < 1$: there would be a "balanced growth path" with a constant level of theft. If c were not to increase at this rate, we conjecture that $\gamma_t \to 0$.

[21]Clearly, if the good is still re-installed to the owner and it is the punishment ($-c$) that can be avoided by bribing, the payoff structure for the fair traders will not change. In case the protection agent can extort the maximal bribe, the value function of a robber will not change either. Hence, the equilibrium should be the same as before (i.e., under non-corrupt minimal state). If the bribe is smaller, so that the robber gets some benefit from the deal with the protection agent, the effective punishment of the former is thus decreased. This, in turn, will reduce the equilibrium γ and, hence, the demand for government. It would be interesting to see the degree of corruption that would be fostered by a "rational" protection agency.

[22]We assume this is true, even if the protection agents take away all the surplus from the deal, i.e., leave them indifferent between paying to the agent and being punished. This "tie-breaking rule" simplifies our exposition.

γ may be different from zero. We denote this $u^{cg}\left(\text{trade}\right) = \gamma G$, where the superscript cg refers to "corrupt government".

As for robbers, the value of their good decreases as well. In case of an unsuccessful theft (which occurs with probability $\frac{1}{2}$ conditional on meeting another robber), they leave the marketplace empty handed. Thus,

$$u^{cg}\left(\text{rob}\right) = \gamma W^{g}\left(\omega, c\right) + \left(1 - \gamma\right) \frac{1}{2} W^{g}\left(\omega, c\right) \tag{7}$$

where $W^{g}\left(\omega, c\right) = G + \delta V^{gc} - \omega\left(c + G\right) = \left(1 - \omega\right) G - \omega c + \delta V^{gc}$.

4.1 Robustness

Assuming that government agents take bribes changes the equilibria of the model. However, their *structure* remains identical to those under a non-corrupt minimal state.

Proposition 6 *Assume protection agents can receive maximal bribe, $c + G$. If $\beta\delta + \delta \leq 1$, then the set of equilibria $\Gamma^{cg}\left(c\right)$ can be described as follows:*

$$\Gamma^{cg}\left(c\right) = \begin{cases} \{0\} & \text{if } c < \underline{c}\left(\omega\right) \\ \{0, \gamma_{L}^{cg}\left(c\right), 1\} & \text{if } c \in \left[\underline{c}\left(\omega\right), \bar{c}\left(\omega\right)\right] \\ \{1\} & \text{if } c > \bar{c}\left(\omega\right) \end{cases} \tag{8}$$

If $\beta\delta + \delta > 1$, then the set of equilibria $\Gamma^{cg}\left(c\right)$ can be described as follows:

$$\Gamma^{cg}\left(c\right) = \begin{cases} \{0\} & \text{if } c < \underline{\underline{c}}^{cg}\left(\omega\right) \\ \{0, \gamma_{L}^{cg}\left(c\right), \gamma_{H}^{cg}\left(c\right)\} & \text{if } c \in \left[\underline{\underline{c}}^{cg}\left(\omega\right), \underline{c}\left(\omega\right)\right) \\ \{0, \gamma_{L}^{cg}\left(c\right), 1\} & \text{if } c \in \left[\underline{c}\left(\omega\right), \bar{c}\left(\omega\right)\right] \\ \{1\} & \text{if } c > \bar{c}\left(\omega\right) \end{cases} \tag{9}$$

with $\gamma_{L}^{cg}\left(c\right), \gamma_{H}^{cg}\left(c\right) \in \left(0, 1\right)$, $\gamma_{L}^{cg}\left(c\right) < \gamma_{H}^{cg}\left(c\right)$.

The lower bound $\underline{c}\left(\omega\right)$ on $\gamma = 1$ equilibria does not vary, regardless of whether or not there is corruption, so nothing can be said in this range about the effects that corruption may have on equilibrium *existence*. However, when $\beta\delta + \delta > 1$, the ranges $\left[\underline{\underline{c}}\left(\omega\right), \underline{c}\left(\omega\right)\right)$ and $\left[\underline{\underline{c}}^{cg}\left(\omega\right), \underline{c}\left(\omega\right)\right)$ do provide such an opportunity. Interestingly, corruption introduces the possibility of a minimal state for punishments that were previously too low.

Proposition 7 *Assume* $\beta\delta + \delta > 1$*. Then* $\underline{\underline{c}}^{cg}(\omega) < \underline{\underline{c}}(\omega)$*. Moreover, if* $c \in \left[\underline{\underline{c}}(\omega), \underline{c}(\omega)\right)$*. Then,* $\gamma_L^{cg}(c) < \gamma_L(c)$*, and* $\gamma_H^{cg}(c) > \gamma_H(c)$*.*

Corollary 8 *Suppose that* $c \in \left[\underline{\underline{c}}^{cg}(\omega), \underline{\underline{c}}(\omega)\right)$*. Trade can be supported in equilibrium under corruption, whereas it cannot in its absence.*

That corrupt government leads to there being *less* theft in a stable equilibrium (i.e., that $\gamma_H^{cg}(c) > \gamma_H(c)$) may appear surprising. For a given γ, however, thieves are always better off when government is corrupt. Hence, for payoffs to be equal across strategies, γ must be higher, to "encourage" the fair traders.[23]

In spite of corruption, the government continues to play a role that agents are willing to pay for. Let $D^{cg}(\gamma, G)$ be the willingness to pay for a government in the case of the maximal bribe $c + G$. This demand for government is defined as the difference between the value of a tradeable good in the protected marketplace and that under anarchy, as in the previous section. This demand is still increasing in the gains from trade, provided the safety of the marketplace (γ) is kept constant.

Proposition 9 *Assume* $\beta\delta + \delta > 1$*;* $c \in \left[\underline{\underline{c}}(\omega), \underline{c}(\omega)\right)$ *and* $\gamma_H^{cg}(G, c) > \underline{\gamma}^{cg}$*, where* $1 > \underline{\gamma}^{cg} > \underline{\gamma} > 0$*. Then,*

1. $D^{cg}(\gamma, G) > 0$;
2. $D^{cg}(\gamma, G)$ *is increasing in* G *keeping* γ *constant;*
3. $\gamma_H^{cg}(G, c)$ *is decreasing in* G *and increasing in* c*.*

Here, again, in order for the government to capture the increasing willingness to pay for its services, the punishment may have to be more severe as the gains from trade increase, in the same sense as before.

[23] Clearly, robbers also prefer environments in which traders are numerous; it can be shown, however, that the increase in the payoff of a fair trader is higher than that of the robber when γ increases, provided γ is high enough to start with.

4.2 Payoffs from Corruption

An interesting question is whether protection agents will engage in corruption, if they are availed of the choice. We modify the model to address this question by simply assuming that the agency is a revenue-maximizing agent.[24] The agent is able to either charge a lump-sum tax τ, or to appropriate a fraction ϖ of demand D in return for its services. We focus on stable equilibria with trade.

First, if $c \in [\underline{c}(\omega), \bar{c}(\omega)]$, the only stable trading equilibrium is $\gamma = 1$, both in corrupt and in non-corrupt environments. Therefore payoffs, demand for government and government revenue are identical, since no bribes will be paid. A more interesting case is where $\beta\delta + \delta > 1$, $c \in \left[\underline{\underline{c}}(\omega), \underline{c}(\omega)\right)$ so that, in both environments, there are stable trading equilibria, in which theft is not completely deterred. On one hand, the proceeds from corruption amount for $(1 - \gamma^{cg})(c + G)\omega > 0$ of the agency's surplus. On the other hand, the demand for its services falls in the presence of corruption (see lemma 22.)

Proposition 10 *Suppose that $\beta\delta + \delta > 1$, $c \in \left[\underline{\underline{c}}(\omega), \underline{c}(\omega)\right)$ and $\gamma_H > \underline{\gamma}^{cg}$. Comparing across stable equilibria, the protection agency's return is higher in a corrupt environment regardless of ϖ or τ, provided $\varpi, \tau > 0$.*

If robbers can retain some gains from the deal with corrupt protection agents, the payoff from being a robber is greater than in the case analyzed above. We conjecture that the structure of equilibria will remain the same, but the decision about whether or not to accept the bribes will not be as clear cut as in case of maximal bribes. In the extreme, if robbers keep all the surplus, the analysis falls back to the case of anarchy – in which there is no advantage to being a fair trader – and so government is not sustainable: there is no demand for a protection agency that neither re-instates property nor punishes robbers. A precondition for there to be a demand for government, when this government is corrupt, is then that it be "sufficiently corrupt" as to allow indicted thieves little bargaining power, according to our conjecture.

[24] For a fixed intensity of observing trades, ω, (increasing which may be costly), maximizing revenue amounts to maximizing profits.

5 Welfare Implications

In this section, we close the model by extending it to an environment with repeated production. Although the value function does not have as clear an interpretation as before[25]; however, this environment is better suited to addressing the welfare implications of introducing the minimal state. In the extended "economy" we can determine the quantity produced and consumed in a steady state and thus, compare this quantity across equilibria with and without the government. As will be shown below, the Minimal State, by protecting property rights, induces more production and consumption (per a time period), thus increasing well-being of the farmers.

Additional goods produced with enforced property rights can be thought of as a "real" source of the willingness to pay for the government.

5.1 Production Economy

We now turn to a more detailed analysis. Instead of a one-off trading opportunity, farmers may travel between locations at their discretion. When in Farmland, they may choose to produce the translatable good and, if they leave Market town, they are free to return to Farmland to obtain more of the translatable good. Hence, in any trading equilibrium, value functions represent the expected value not only of holding one good but to being a farmer for the indefinite future, namely, producing a good in a Farmland and selling it in the Market town. Travel between locations takes one period, and is otherwise costless. If they choose to remain in Farmland, they earn utility ψ – which is set to zero for now.

Normalize the total mass of agents to unity. The fraction of people in Market town at time t is denoted by n_t. Let x_{in}^t be the fraction of people entering Market town, and x_{out}^t be the fraction of people leaving the market place at time t. The evolution of the population in Market town n_t is then

$$n_{t+1} = n_t + x_{in}^t - x_{out}^t \tag{10}$$

The exact values of these flows will depend on institutions, and on the actions of

[25]Recall that the value function introduced above has the interpretation of the discounted expected value of holding a particular unit of the translatable good.

agents.

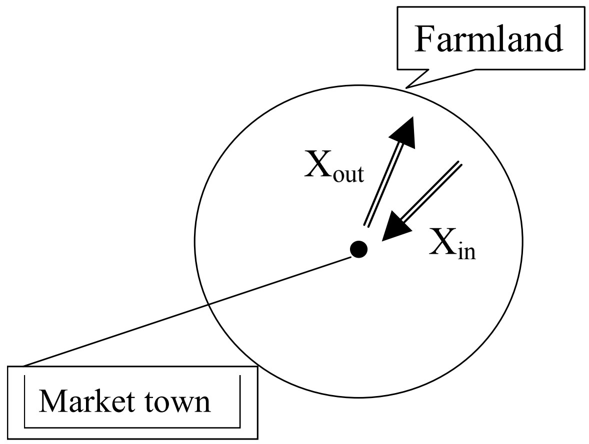

Figure 5: Geography of the Environment

The structure of equilibria in this closed model (economy with production) resembles that in the pure exchange economy.

Proposition 11 *In the "closed" environment, the set of equilibria* $\Gamma^{cm}(c)$ *is:*

$$\Gamma^{cm}(c) = \begin{cases} \{0\} & \text{if } c \leq \underline{c}^{cm}(\omega) \\ \{0, \gamma_L^{cm}(c), 1\} & \text{if } c \in (\underline{c}^{cm}(\omega), \bar{c}^{cm}(\omega)) \\ \{1\} & \text{if } c \geq \bar{c}(\omega) \end{cases} \tag{11}$$

where $\gamma_L^{cm}(c) \in [0,1]$ *and* $\underline{c}^{cm}(\omega) < \underline{c}(\omega)$.

There are no longer stable equilibria in which $\gamma_H(c) < 1$. Observe that $\underline{c}^{cm}(\omega) < \underline{c}(\omega)$. This suggests an explanation: the future value of returning to Market town increases the incentive to trade over robbing in perpetuity, as the delay in consumption that robbers experience when caught affects not only their current presence in Market town but also all future opportunities upon which they might return.

5.2 Aggregate Welfare

We now compare aggregate welfare across steady state equilibria.

Suppose that a protection agency is present. It stands at the gate, and charges a tax τ on all agents attempting to enter Market town. So long as the charge is below the willingness to pay, producers come to trade in the Market town. We assume for now that τ is a constant, and later discuss the factors that underlie tax determination.

So long as agents can profit from being there, the population of Market town n_t at time t evolves as

$$n_{t+1} = 1 - \frac{\beta}{2}\left[2\gamma\left(1-\gamma\right)\left(1-\omega\right) + 2\gamma^2 + \left(1-\gamma\right)^2\left(1-\omega\right)\right] n_t \tag{12}$$

In equation (12), the flow into Market town is composed of the agents who receive production opportunities $1 - n_t$; adding to these the agents that were already there (n_t) leaves $1\left(= n_t + x_{in}^t\right)$. The flow out is a more complicated object. If the number of agents in Market town is n_t, the mass of active matches is $\frac{1}{2}\beta n_t$. Conditional on such a match, agents who leave Market town are, first, unsuccessful robbers who were victims of an unobserved robbery; these matches are proportion $\left(1-\gamma\right)^2\left(1-\omega\right)$, and they each release one agent. Second are fair traders who met fair traders, (their fraction is γ^2), each such match releasing two agents. Finally, there are fair traders who met robbers and were victims of an unobserved robbery (fraction $2\gamma\left(1-\gamma\right)\left(1-\omega\right)$), a match that releases one agent.

In a steady state, $n_t = n^*\left(\gamma\right)$. The inflow is equal to the outflow at a steady state, so that the fraction of the people in the Market Town at any given period is

$$n^*\left(\gamma\right) = \frac{2}{\left(\beta - \beta\omega + \beta\gamma^2 + \beta\gamma^2\omega + 2\right)} \tag{13}$$

It is immediate that $n^*\left(\gamma\right)$ is decreasing in γ. Consequently, the fraction of people *producing* $\left(1 - n^*\left(\gamma\right)\right)$ in a stationary equilibrium is *increasing* with γ. As all the goods produced are also consumed, we conclude that aggregating over individuals consumption per period is increasing as well. Clearly, this implies that the agents are better off (on average) in the presence of government that protects property rights by inducing $\gamma > 0$ in the Market Town.

Potential punishment for being a robber reduces the value of time invested in staying in the marketplace in an attempt to rob, and makes the alternative, producing, more attractive in relative terms. The higher quantity of goods produced in the economy provides a "real" basis for the willingness to pay for the protection agency, on top of the redistributive effects of deterring theft.

One could argue that punishment – however interpreted – is a burden upon ex-ante identical agents. To evaluate improvements brought about by the government in this case, we can not avoid introducing an explicit form of a social welfare function. For demonstration purposes only we pick the traditional utilitarian form; sceptical readers are invited to skip the rest of this subsection.

The aggregate per-period value of consumption is $W(\gamma) = G(1 - n^*(\gamma))$. Incorporating punishment, the welfare measure is then

$$W^c(\gamma) = G(1 - n^*(\gamma)) - \beta c\omega(1 - \gamma) n^*(\gamma) \tag{14}$$

The welfare criterion is formulated in terms of the well-being of an "average citizen" who may decide to rob and can be caught and suffer punishment.

Proposition 12 *Assume* $c \in [\underline{c}^{cm}(\omega), \overline{c}^{cm}(\omega)]$. *Both* W *and* W^c *are increasing in* γ.

Proposition (12) shows that the Minimal state always improves upon anarchy, even if the cost of punishment is taken into account.

5.3 Taxation

Naturally, if the protection agency is a monopolist, it will set the highest possible entrance fees to the Market town. This will be the same *value* as in the "non-closed" model, $D(\gamma, G)$. The reason for this is that $D(\gamma, G)$ represents the expected value of a *particular* good, which is the maximum the agency could voluntarily extract at the gate; anything more would deter agents from leaving Farmland in the first place. If we interpret arrival at the Market town as an attempt to participate in the formal market, the most the government can extract voluntarily is the expected value of the goods that agents produce and bring to formality.

However, it could be that there exist other potential Market towns. If agents are fully informed, n^* represents the aggregate mass of traders across all competing Market towns in a steady state equilibrium, so long as γ is the same in all of them.

Depending on market structure[26], the portion of the surplus that farmers obtain should be in the $[0, D(\gamma, G)]$ interval, and farmers would be *ex-ante* strictly better off under the minimal state (except in the case of monopoly).

6 Possible Further Extensions

It is appropriate at this juncture to discuss the robustness of the model to further extensions. For example, suppose that there is a per-period utility flow $\psi > 0$ that accrues to agents that remain in Farmland. This may be interpreted as an autarkic payoff, or else as the payoff from less productive or limited trades that farmers may carry out among members of their household or village, so that they do not need a government to keep track of transactions but the quality or variety of goods is different. Then, Proposition (11) holds subject to the restriction that, in each case, the value of going to market town V^g exceeds $\frac{\psi}{1-\delta}$. The set of equilibria is censored by the opportunity cost of going to Market town. Moreover, introducing tastes for variety, allows to formulate the decision of a farmer about where to sell his good as a trade-off between exposure to a wider range of available goods on the "formal" market (with property rights enforced by the state) versus the taxes that are charged by the state for participating in such a market. The same is the case for a travel cost ξ, as they both represent opportunity costs of going to Market town. Interestingly, this suggests that a high opportunity cost of going to Market town may be another determinant of equilibrium selection, since agents will be unwilling to make the trip unless they expect a relatively benign trading environment. For simplicity we have concentrated on the case in which $\psi = \xi = 0$. Relaxing this assumption would also have an impact on the magnitude of taxes that the minimal state can raise, as higher values of ψ reduce the surplus agents can generate through the market.

More generally, ψ could be interpreted as a payoff an agent receives in an "informal" sector, in which any economic activity is not subject to the "third party" protection of property rights. The *extra* surplus generated by a well-functioning formal market $\left(V^g - \frac{\psi}{1-\delta}\right)$ is naturally smaller if the informal economy functions well.

[26] Bertrand competition suggests that a plurality of competitors is sufficient for farmers to avail themselves of all the surplus.

Hence, in environments with strong non-market systems of production and exchange or with strong social networks – each of which can be interpreted as high levels of ψ – the welfare-improving role of a Minimal State is diminished.

It is important to stress that the role of the "state" in this model does not necessarily have to be played by the official government. The only implication that we can draw from the analysis in this respect is that the emergence of trade (market economy) gives rise to the institution of property rights. If protection is not (adequately) provided by official organizations, it may emerge in other forms (as a part of a "shadow" economy, mafia, etc.), as the demand for this service is not met. This scenario can arise in countries in transition that move away from old institutions to a market economy. One of the factors that strengthens the role of a mafia in that case is the need to provide the missing "protection". For instance, this may partially explain Russia becoming a "criminal state".[27] Clearly, to better understand the workings of the shadow economy within our framework, one should introduce "competition" between the protection agencies that could result in the rise of a "dominant protection agency," – as suggested in Nozick (1974).

7 Conclusions

We develop an environment in which the excludability of rival goods is endogenously determined. Within such an environment, we show that the existence of potential gains from excludable trade can generate a "niche" for a protection agency, analogous to the Nozickian Minimal State. Individuals may be willing to be taxed in return for a safer trading environment.

In contrast to the previous work, governmentally induced welfare improvements are not due to the saved costs from centralizing protection, nor do they stem from the re-allocation of resources previously wasted in an arms race. We focus on the ability of the state to resolve a Prisoner's Dilemma that arises under anarchy: robbing is a dominant strategy, although everybody would prefer to be surrounded by fair traders. Government's role in this model is simple: it can observe some interactions,

[27]..organized crime presently controls about 40 percent of the Russian gross domestic product.. "Economic Crime and the Security of Citizens, Society, and the State." (1995) *Bisnes i bezopasnosti v Rossii*, No. 1.

punish detected robbers and re-instate property to its owners. By so doing it imposes direct or indirect costs on the robbers, thus encouraging more agents to trade fairly. Mutually beneficial trades occur more often, and more is produced and consumed. The Invisible Hand requires a Visible Fist.

We find that the willingness to pay grows with gains from trade, even if protection agents are corrupt. Hence, even if the "protection technology" is fixed, high enough gains from trade may generate sufficient demand for a protection agency to arise. We do not model the "supply" of protection in detail: whether it is provided by a benevolent or exploitative agency is not the focus of our analysis.[28]

It is true that introducing a "third party" (observer) is not the only solution to the Prisoner's Dilemma that lies at the core of the market transactions. Aside from various monitoring and punishment systems, certain rituals or social relations may be the mechanisms whereby a socioeconomic entity resolves this problem, and different systems will go some way towards improving aggregate welfare beyond that resulting simply from repetition of the Nash outcome at every meeting. Even in a repeated random matching environment, as Kandori (1995) shows, decentralized punishments may sustain cooperation in "large" economies. Their effectiveness is technologically limited, though. First, agents are required to display a summary of their past behavior before a new encounter, so that there is a need for a truthful "book-keeping" agency. Second, the discount factor restricts the severity of punishment that a community can impose on any deviators and, consequently, the level of cooperation that will be achieved. We show that "impatient societies" or those that can not afford or trust the "book-keepers" may yet benefit from a central "protection" agency. We leave the potentially interesting interaction between social pressures and a central authority for future work.

Although it is beyond the scope of this paper, it could also be interesting to investigate corruption considering an explicit model of government organization and the corresponding relationship between government composition and behavior.[29] Another interesting possibility is investigate how separate institutional structures over

[28]The latter interpretation may shed a light on formation of hierarchies in the underground economies.

[29]See, for example, Jean Hindriks and Muthoo (1999) for the analysis of corruption related to income tax reports.

which agents have an option may influence each other.

Finally, the framework is suitable for the analysis of other trade-related government functions, for example, the introduction of trade-related public goods.

A Appendix

Proof of lemma 1. Recall that a payoff to an agent is

$$V^a = \beta \max \{u^a (\text{trade}), u^a (\text{rob})\} + (1-\beta) \delta V^a. \tag{15}$$

But

$$\gamma G = u^a (\text{trade}) < u^a (\text{rob}) = \frac{1}{2} (\gamma + 1) (G + \delta V^a), \tag{16}$$

for any $\gamma \in [0,1]$, provided $V^a \geq 0$. The last inequality is true, since an agent can always opt for "trade" in every period, thus, ensuring a non-negative payoff

$$V_t^a = \beta \frac{\gamma G}{1 - \delta(1-\beta)} \geq 0$$

■

A.1 Notation

The value of a good held by a perpetual trader (trading forever) is

$$V_t^g (\gamma; c) \equiv \beta \frac{G\gamma}{1 - \delta + \beta\delta - \beta\delta (1-\gamma) \omega} \tag{17}$$

and that of a chronic robber (robbing in every period) is

$$V_r^g (\gamma; c) = \beta \frac{(\gamma + 1) (G (1-\omega) - c\omega)}{\beta\delta (1-\omega) (1-\gamma) + 2(1-\delta)}. \tag{18}$$

Let

$$\begin{aligned} F(\gamma;c) &= \kappa(\gamma)\left[V_t^g(\gamma;c) - V_r^g(\gamma;c)\right]; \\ \kappa(\gamma) &\equiv (\beta\delta(1-\omega)(1-\gamma) + 2(1-\delta))(1-\delta+\beta\delta(1-(1-\gamma)\omega)). \end{aligned} \tag{19}$$

Note that $\kappa > 0$. Therefore the sign of $F(\gamma;c)$ coincides with the sign of the difference $V_t^g(\gamma;c) - V_r^g(\gamma;c)$. Finding roots of F that lie in $[0,1]$ is equivalent to finding equilibria of in which agents are indifferent between actions. Note that F can be represented in the following way:

$$\begin{aligned} F(\gamma;c) &= \gamma^2 a_F(c) + \gamma b_F(c) + k_F(c), \\ k_F(c) &= (c\omega - G(1-\omega))(\beta\delta(1-\omega) + (1-\delta)); \\ b_F(c) &= G(1-\delta) + G\omega(1-\delta) + c\omega(1-\delta) + c\beta\delta\omega; \\ a_F(c) &= -\delta\beta\left(G(1-\omega^2) - c\omega^2\right) \end{aligned} \tag{20}$$

Note that $b_F(c)$ is positive as long as $c \geq \tilde{c} = (\beta\delta - \delta + 1)^{-1}\omega^{-1}(\omega+1)(\delta-1)G$, where $\tilde{c} < 0$.

A.2 Auxiliary Results

Lemma 13 *If $F(0;c) < 0$ then $\gamma = 0$ is an equilibrium. If $F(1;c) > 0$ then $\gamma = 1$ is an equilibrium.*

Proof of Lemma 13. Recall that the sign of $F(\gamma;c)$ is the same as the sign of $V_t^g(\gamma;c) - V_r^g(\gamma;c)$.

Assume the difference $V_t^g(0;c) - V_r^g(0;c)$ is negative, then everybody robbing in every period is a stationary subgame perfect Nash equilibrium. Indeed, the difference between the value of trading once and robbing thereafter and the value of robbing all the time is negative as well:

$$\begin{aligned} &\tilde{V}_r^g(0;c) - V_r^g(0;c) = \\ &= \beta(\gamma G + (1-\gamma)[\omega\delta V_r^g]) + (1-\beta)\delta V_r^g - V_r^g|_{\gamma=0} = \\ &= V_r^g(-\beta\delta(1-\omega) - (1-\delta)) < 0, \end{aligned} \tag{21}$$

provided $V_r^g > 0$. The last inequality stems from the fact that $V_t^g(0;c) - V_r^g(0;c) < 0$ (by assumption) and $V_t^g(0;c) = 0$ by definition.

Clearly, if $V_t^g(\gamma;c) - V_r^g(\gamma;c) < 0$ for a range of $\gamma : \gamma \neq 0$, then none of the values in the range is consistent with a stationary subgame perfect Nash equilibrium. Similarly $V_t^g(1;c) - V_r^g(1;c) > 0$ implies $\gamma = 1$ is an equilibrium, as the one shot deviation (rob and then trade) is unprofitable:

$$\tilde{V}_t^g(1;c) - V_t^g(1;c) < 0, \tag{22}$$

where

$$\tilde{V}_t^g(1;c) = \beta\left(G(1-\omega) - c\omega + \delta V_t^g(1;c)\right) + (1-\beta)\,\delta V_t^g(1;c). \tag{23}$$

Indeed,

$$\begin{aligned} & \tilde{V}_t^g(1;c) - V_t^g(1;c) = \\ = \;& G\beta - G\beta\omega - c\beta\omega - V_t^g(1;c)(1-\delta) < \\ < \;& G\beta - G\beta\omega - c\beta\omega - V_r^g(1;c)(1-\delta) = 0 \end{aligned} \tag{24}$$

■

Lemma 14 *If $c \geq \underline{c}(\omega)$, then $\gamma = 1$ is an equilibrium.*

Proof. Given $\gamma = 1$, the value of trading forever is

$$V_t^g(1;c) = \beta\frac{G}{1-\delta+\beta\delta}, \tag{25}$$

while the value of robbing forever is

$$V_r^g(1;c) = \beta\frac{(G(1-\omega) - c\omega)}{1-\delta}. \tag{26}$$

Clearly, if $F(1;c) = V_t^g(1;c) - V_r^g(1;c) = 0$, then $\gamma = 1$ is an equilibrium. If $F(1;c) > 0$ then $\gamma = 1$ is an equilibrium by lemma 13. But $F(1;c) > 0$ if and only if

$$c > \underline{c}(\omega) \equiv \frac{G\left(\beta\delta(1-\omega) - \omega(1-\delta)\right)}{\omega\left(\beta\delta + 1 - \delta\right)}. \tag{27}$$

■

Lemma 15 *If $c \leq \bar{c}(\omega)$, then $\gamma = 0$ is an equilibrium.*

Proof. The value of trading forever provided $\gamma = 0$ is zero. Thus, for $F(0; c) \leq 0$, it is sufficient to have the value of robbing forever to be positive positive, $V_r^g(0; c) \geq 0$, which is equivalent to requiring

$$G(1-\omega) - c\omega \geq 0, \tag{28}$$

so that

$$c \leq \bar{c}(\omega) \equiv \frac{G(1-\omega)}{\omega}. \tag{29}$$

The conclusion then follows, again, by lemma 13. ■

Lemma 16 *If $c > \bar{c}(\omega)$ there is a unique equilibrium $\gamma = 1$.*

Proof. If $c \geq \frac{G(1-\omega^2)}{\omega^2}$, then $F(\gamma; c) > 0$ for any $\gamma \geq 0$; in particular, $F(1; c) > 0$, which implies $\gamma = 1$ is an equilibrium by lemma 13. If $c < \frac{G(1-\omega^2)}{\omega^2}$, but $c > \bar{c}(\omega)$, then $k_F > 0, b_F > 0$, but $a_F < 0$ in representation (20). In this case the quadratic polynomial $F(\gamma; c)$ has two roots: one positive and one (*strictly*) negative. It is strictly positive in between these two roots. But as $c > \bar{c}(\omega) > \underline{c}(\omega) = \frac{\beta\delta(1-\omega)-\omega(1-\delta)}{(\beta\delta-\delta+1)\omega}G$, it must be the case that $F(1; c) > 0$ by lemma (14). Hence zero and one are strictly in between the two roots of the parabola, and F is above zero for any $\gamma \in [0, 1]$. Thus, the only equilibrium in this case is $\gamma = 1$. ■

Corollary 17 $\underline{c}(\omega) < \bar{c}(\omega)$: *both equilibria* $(\gamma = 0, \gamma = 1)$ *exist for an open set of punishments.*

Lemma 18 $\gamma_H(c)$ *is increasing in* c *and* $\gamma_L(c)$ *is decreasing in* c.

Proof. First note that $F(\gamma, c)$, is increasing in c. Second, $F(\gamma, c)$ is a quadratic polynomial in γ. For $c < \underline{c}(\omega)$ the coefficient of the quadratic term is negative (as in graph 2), so for any fixed c function $F(\gamma, c)$ has a unique maximum at some γ^*. Therefore F is decreasing in γ for $\gamma \geq \gamma^*$. Since $\gamma_H \geq \gamma^*$, γ_H and c should be positively related by the Implicit Function Theorem. Similarly $\gamma_L \leq \gamma^*$ is decreasing in c. ■

Claim 19 *$\gamma_L(c)$ is unstable and $\gamma_H(c)$ is stable.* [30]

Proof. Fix c_0 in the range in which γ_L is well defined. Assume that the fraction of fair traders is $\gamma_L(c_0)$. Let $c_1 = c_0 + \varepsilon$, $\varepsilon > 0$. Then $F(\gamma_L(c_0), c_1) > 0$, so that the fraction of the fair traders should increase.[31] It will continue to grow till it reaches $\gamma_H(c_1)$ as $\gamma_L(c_1) < \gamma_L(c_0) < \gamma_H(c_1)$ and $F(\gamma_H(c_1), c_1) = 0$. $F(\gamma, c_1) < 0$ for $\gamma > \gamma_H(c_1)$,[32] so agents should gravitate towards $\gamma_H(c_1)$ also. If the punishment is *reduced* even marginally at $\gamma_L(c_0)$, then $F(\gamma_L(c_0), c_1) < 0$, thus robbing becomes more attractive. As the last inequality is satisfied for all $\gamma < \gamma_L(c_0)$, the new equilibrium will be $\gamma = 0$. Thus, slightly more punishment will increase the proportion of fair traders "by a lot" and a small decrease in severity of punishment will eliminate fair trade. In no event will the perturbed equilibrium coincide with $\gamma_L(c_1)$. As for the equilibrium with $\gamma = 0$, it will not change as long as $c < \underline{c}(\omega)$, as in this case $F(0, c) < 0$, implying that being a robber is better than being a fair trader. Recall that the opposite is true if the punishment is high, $c \geq \underline{c}(\omega)$, moreover, in this case trading fair is a dominant strategy. This is independent of the fraction of fair traders in the population: $F(\gamma, c) > 0$, if $c > \underline{c}(\omega)$ for any γ. Finally, if in an equilibrium with $\gamma_H(c_0)$ the punishment is slightly increased to c_1, then as before $F(\gamma_L(c_0), c_1) > 0$, so it is more worthwhile to be a fair trader than a robber. As the fraction of fair traders increases to $\gamma_H(c_1)$,[33] the equilibrium is restored due to the fact that $F(\gamma_H(c_1), c_1) = 0$ and $F(\gamma, c_1) < 0$ for $\gamma > \gamma_H(c_1)$. Similar argument holds for a decrease in punishment starting from γ_H. ■

A.3 Main Results

Proof of Proposition 2. Recall, that the roots (γ) of polynomial $F(\gamma; c)$ defined in (19) correspond to the equilibria. Moreover, by lemma 13, $F(1; c) > 0$ indicates

[30] We could re-formulate this discussion in terms of small perturbations in γ, which leads to the same conclusions. The discussion is deliberately left informal. See DeMichelis and Germano (2000) for a formal approach that can be applied to this model.

[31] All that is required is a continuous dynamic that satisfies this monotonicity restriction and vanishes at γ such that $F(\gamma; c) = 0$.

[32] This argument is based on the assumption that $\gamma_H(c_1) < 1$. Clearly, if this is not the case, so that $\gamma_H(c_1) \geq 1$, then the all the population will trade fairly, thus not returning to $\gamma_L(c_1)$ either.

[33] Recall that γ_H and c are positively related, so that $\gamma_H(c_0) < \gamma_L(c_1)$, $c_1 > c_0$.

that $\gamma = 1$ is an equilibrium and $F(0;c) < 0$ implies $\gamma = 0$ is an equilibrium. Recall representation (20). Clearly, $k_F < 0$, if

$$c < \bar{c}(\omega) = G\frac{(1-\omega)}{\omega}, \tag{30}$$

while $b_F > 0$, and $a_F < 0$ if

$$c < \frac{G(1-\omega^2)}{\omega^2}. \tag{31}$$

The polynomial is maximized at $\gamma = \gamma^*$, where

$$\gamma^* = \frac{1}{2}\frac{(G(1-\delta) + G\omega(1-\delta) + c\omega(1-\delta+\beta\delta))}{(-G\omega^2 + G - c\omega^2)\beta\delta}. \tag{32}$$

If $\gamma^* > 1$, then $F(1;c) > 0$, as the upper root should be above unity. Evidently $\gamma^* > 1$ if and only if

$$c > c^* \equiv \frac{(2\beta\delta(1-\omega) - (1-\delta))}{(\beta\delta - \delta + 2\beta\delta\omega + 1)}\frac{(\omega+1)G}{\omega}. \tag{33}$$

To derive the lower bound, note that there are two possible cases that can lead the polynomial $F(\gamma;c)$ to be negative for all $\gamma \in [0,1]$. The first case occurs when γ^*, at which F is maximized, is above unity. In this case F hits zero at most once between zero and one. Thus, if $F(1;c) < 0$, then it is negative for any $\gamma \in [0,1]$. Secondly, if $\gamma^* < 1$ and $F(\gamma^*;c) < 0$, then, $F(\gamma;c) < 0$ for any γ. We will start with the first case, as it generates a higher lower bound on c, given that γ^* strictly increases in c (which can be verified directly from (32)).

Lemma 20 *If $\beta\delta + \delta \leq 1$ and $c < \underline{c}(\omega)$, then there is a unique equilibrium $\gamma = 0$.*

■

Proof. Note that if $c < \underline{c}(\omega)$, then $F(1;c) < 0$. If $c > c^*$, then $\gamma^* > 1$. Therefore if $c^* < \underline{c}(\omega)$, then $c \in [c^*, \underline{c}(\omega)]$ implies there is a unique equilibrium $\gamma = 0$ by the above argument. Assume $\beta\delta + \delta < 1$. Then

$$c^* - \underline{c}(\omega) = \tag{34}$$

$$= \frac{(\beta\delta(1-\omega) + (1-\delta))(\delta + \beta\delta - 1)}{(\beta\delta + 1 - \delta + 2\beta\delta\omega)(\beta\delta + 1 - \delta)}\frac{G}{\omega} < 0, \tag{35}$$

as required. It is left to show that in this case if $c < c^*$, then the equilibrium remains unique, $\gamma = 0$. Consider $c = c^*$. As $c^* < \underline{\underline{c}}(\omega)$, and $\gamma^* = 1$, it implies $F(\gamma^*(c^*); c^*) < 0$. As γ^* is the maximand of F, it follows that $F(\gamma; c^*) < 0$ for any γ. Now consider $c_0 < c^*$. It can be easily shown that F decreases in c for any γ. Therefore, $F(\gamma; c_0) < 0$. The case of equality $\beta\delta + \delta = 1$ is trivial. This completes the proof of proposition (2).

Lemma 21 *If $\beta\delta + \delta > 1$ and $c < \underline{\underline{c}}(\omega)$, then there is a unique equilibrium $\gamma = 0$.*

■

Proof. If $\beta\delta+\delta > 1$ then $c^* > \underline{\underline{c}}(\omega)$, therefore, for $c < \underline{\underline{c}}(\omega) < c^*$ first, $F(1; c) < 0$ and, second, $\gamma^* < 1$. Therefore, the parabola $F(\gamma; c)$ can cross zero twice if the discriminant

$$H(c; \beta, \delta, \omega) \equiv b_F^2(c) - 4a_F(c)\, k_F(c) \tag{36}$$

is positive. Whenever H is negative, $F(\gamma; c)$ lies below zero for any γ and, in this case, the only equilibrium is $\gamma = 0$. It remains to derive lower bound, $\underline{\underline{c}}(\omega)$, on the severity of punishment that assures that $H(c; \beta, \delta, \omega) < 0$. Clearly, $H(c; \beta, \delta, \omega)$ is quadratic in c :

$$H(c; \beta, \delta, \omega) = c^2 a_H(\beta) + c b_H(\beta) + k_H(\beta), \tag{37}$$

where

$$\begin{aligned}
k_H(\beta) &= (G - G\delta + G\omega - G\delta\omega)^2 - && (38)\\
&\quad -4G\beta\delta(1-\omega)\left(G\omega^2 - G\right)(\delta - \beta\delta + \beta\delta\omega - 1); && (39)\\
b_H(\beta) &= G\begin{pmatrix} 2(\omega - \delta\omega + \beta\delta\omega)(\omega - \delta\omega + 1 - \delta) + \\ +4\beta\delta(\beta\delta(1-\omega) + 1 - \delta)\,\omega(2\omega+1)(1-\omega)\end{pmatrix}; && (40)\\
a_H(\beta) &= \omega^2(\delta - \beta\delta + 2\beta\delta\omega - 1)^2 && (41)
\end{aligned}$$

Since $a_H(\beta) > 0, b_H(\beta) > 0$, in order to have a non-empty interval of values c that make the polynomial $H(c; \beta, \delta, \omega)$ negative, it has to be the case that $k_H(\beta) < 0$. In turn, $k_H(\beta)$ is quadratic in β :

$$\begin{aligned} k_H(\beta) &= a_k(\delta)\beta^2 + b_k(\delta)\beta + k_k(\delta), & (42)\\ k_k(\delta) &= G(1-\delta+\omega-\delta\omega)^2; & (43)\\ b_k(\delta) &= 4G^2\delta(1-\delta)(1-\omega)\left(\omega^2-1\right); & (44)\\ a_k(\delta) &= 4G^2\delta^2(1-\omega)^2\left(\omega^2-1\right), & (45)\end{aligned}$$

Observe that $k_k(\delta) > 0, b_k(\delta) < 0, a_k(\delta) < 0$. Thus, the polynomial $k_H(\beta)$ has two roots, as long as $k_k(\delta), b_k(\delta), a_k(\delta) \neq 0$, . The lower root is negative, while the upper one,

$$\beta_H = \frac{-b_k(\delta) - \sqrt{(b_k(\delta))^2 - 4a_k(\delta)k_k(\delta)}}{2a_k(\delta)} \quad (46)$$

$$\beta_H = \beta_1(\omega) \equiv \frac{(1-\delta)\left((\omega-1)^2 - \sqrt{2(1-\omega)^3}\right)}{2\delta(\omega-1)^3} \quad (47)$$

is positive. It is also below unity as long as

$$\delta > \delta_1(\omega) \equiv \frac{\sqrt{2(1-\omega)} - (1-\omega)}{\sqrt{2(1-\omega)} + (1-2\omega)(1-\omega)} \quad (48)$$

For $\delta > \delta_1(\omega)$ and $\beta > \beta_1(\omega)$, $k_H(\beta) < 0$. In this case polynomial $H(c;\beta,\delta,\omega)$ has two roots of opposing sign. Recall that $a_H(\beta) > 0$, so that $H(c;\beta,\delta,\omega)$ is negative for all the values of c in between the two roots, which means F does not have real roots (γ) and is always negative implying that the only equilibrium is $\gamma = 0$. If c is below the lower root of $H(c)$, quadratic polynomial $F(\gamma)$ has negative roots. Thus as long as c is below the upper (positive) root of $H(c;\beta,\delta,\omega)$, the only equilibrium is $\gamma = 0$. Denote this root by $\underline{\underline{c}}(\omega)$:

$$\underline{\underline{c}}(\omega) \equiv \frac{-b_H + \sqrt{b_H^2 - 4a_H k_H}}{2a_H}. \quad (49)$$

Finally, assume that $\beta\delta + \delta > 1$ and

$$\underline{\underline{c}}(\omega) < c < \underline{c}(\omega)$$

We have to show that in this case there are three equilibria: $\gamma = 0$, and a couple $\gamma_L < \gamma_H < 1$. The two roots of the polynomial $F(\gamma; c)$, are

$$\gamma_L(c) = \frac{-b_F(c) + \sqrt{H(c; \beta, \delta, \omega)}}{2a_F(c)}; \tag{50}$$

$$\gamma_H(c) = \frac{-b_F(c) - \sqrt{H(c; \beta, \delta, \omega)}}{2a_F(c)}, \tag{51}$$

where $H(c; \beta, \delta, \omega)$ is as defined in (36). Condition $c > \underline{\underline{c}}$ assures that $H(c; \beta, \delta, \omega)$ is strictly positive. Therefore $\gamma_L(c), \gamma_H(c)$ are real. As $a_F(c) < 0$ for $c < \bar{c}(\omega)$ and $k_F(c) < 0$, we have $0 < \gamma_L(c) < \gamma_H(c)$.

Since $\beta\delta + \delta > 1$, $c^* > \underline{c}$, thus $c < c^*$, which implies that the maximand of F, γ^*, is less than one. Moreover, as $c < \underline{c}$, $F(1, c) < 0$, this, along with the fact that $a_F(c) < 0$ and that the discriminant H is positive guarantees that $\gamma_H(c) < 1$. Finally, $F(0, c) < 0$, as $k_F(c) < 0$, which justifies the first equilibrium ($\gamma = 0$). Lastly, the uniqueness of $\gamma = 1$ for $c > \bar{c}$ is justified by lemma (16). ■

Proof of Propositions 4, 5 . Recall that there is no trade in the anarchy, $\gamma = 0$ so that $V^a = \frac{\beta G}{\beta\delta - 2\delta + 2}$. On the other hand, the agency has induced fair trade with some probability $\gamma > 0$, so that $V^g = \beta \frac{G\gamma}{1 - \delta + \beta\delta - \beta\delta(1-\gamma)\omega}$. We define the difference between the two as the willingness to pay for the protective agency, or demand $D(\gamma, G)$:

$$D(\gamma, G) = \frac{\delta - \beta\delta + \beta\delta\omega - 1 + \gamma(\beta\delta - \beta\delta\omega + 2 - 2\delta)}{(\beta\delta - \beta\delta\omega + \beta\gamma\delta\omega + 1 - \delta)(\beta\delta - 2\delta + 2)} G\beta \tag{52}$$

The demand is positive as long as

$$\gamma > \underline{\gamma} \equiv \frac{\beta\delta - \beta\delta\omega + 1 - \delta}{\beta\delta - \beta\delta\omega + 2 - 2\delta}. \tag{53}$$

Observe that $\underline{\gamma} < 1$. Moreover, provided $\gamma > \underline{\gamma}$, the demand $D(\gamma, G)$ is positively related to the gains from trade, G, keeping γ constant, $\frac{\partial D(\gamma, G)}{\partial G} > 0$. ■

Proof of lemma 5. It is enough to show that $\gamma_H(G)$ is decreasing in G and that $D(\gamma, G)$ is increasing in the proportion of fair traders, γ. The latter stems from the fact that

$$\frac{\partial D(\gamma, G)}{\partial \gamma} = \frac{G\beta(\beta\delta - \beta\delta\omega + 1 - \delta)}{(\beta\delta - \delta - \beta\delta\omega + \beta\gamma\delta\omega + 1)^2} > 0 \tag{54}$$

It is left to analyze the response of equilibrium value γ to an increase in the gains from trade, G.

Recall that condition $F(\gamma; c) = 0$ describes equilibria, provided that the values of γ satisfying this condition lie within $[0, 1]$ interval. We concentrate on the upper equilibrium, $\gamma = \gamma_H$ (see the discussion on page 23). It will be convenient to represent all the variables as functions of G, which we'll do (slightly abusing notation). By (51),

$$\gamma_H(G) = \gamma^*(G) + \frac{1}{2}\sqrt{K(G)}, \tag{55}$$

where

$$\begin{aligned} K(G) &\equiv \frac{H(G)}{|a_F(G)|^2} && (56) \\ H(G) &= b_F^2(G) - 4a_F(G)\,k_F(G), && (57) \\ \gamma^*(G) &= -\frac{b_F(G)}{2a_F(G)}, && (58) \end{aligned}$$

$$\begin{aligned} k_F(G) &= (c\omega - G(1-\omega))\,(\beta\delta(1-\omega) + (1-\delta)); && (59) \\ b_F(G) &= G(1-\delta) + G\omega(1-\delta) + c\omega(1-\delta) + c\beta\delta\omega; && (60) \\ a_F(G) &= -\delta\beta\left(G(1-\omega^2) - c\omega^2\right) && (61) \end{aligned}$$

It is easy to check that

$$\gamma^{*\prime}(G) = \frac{1}{2}\frac{c\omega(\omega+1)(\delta - \beta\delta + \beta\delta\omega - 1)}{\delta\beta\left(G(1-\omega^2) - c\omega^2\right)^2} < 0 \tag{62}$$

and

$$K'(G) = \frac{d}{dG}\left(\frac{b_F^2(G)}{|a_F(G)|^2} + \frac{4k_F(G)}{|a_F(G)|}\right) = \tag{63}$$

$$= 2T(G)\left(\frac{(\omega+1)N(G)}{\delta\beta\left(G(1-\omega^2)-c\omega^2\right)} + 2(1-\omega)\right) < 0, \tag{64}$$

where

$$N(G) = G(1-\delta) + G\omega(1-\delta) + c\omega(1-\delta) + c\beta\delta\omega, \tag{65}$$

$$T(G) = \frac{c\omega(\delta - \beta\delta + \beta\delta\omega - 1)}{\delta\beta\left(G(1-\omega^2)-c\omega^2\right)^2}. \tag{66}$$

Then

$$\gamma_H'(G) = \gamma^{*\prime}(G) + \frac{K'(G)}{\sqrt{K(G)}} < 0 \tag{67}$$

Then

$$\frac{\partial D(\gamma, G)}{\partial \gamma}\gamma_H'(G) < 0 \tag{68}$$

It has been shown that γ_H is increasing in c, thus the claim follows. ■

B Proofs of Additional Statements

Proof of Proposition 6. First, we have to assume that $c > 0$. The value of trading forever and robbing forever in this environment, correspondingly are

$$V_t^{cg}(\gamma) = \beta \frac{\gamma G}{1 - \delta(1-\beta)} \tag{69}$$

$$V_r^{cg}(\gamma; c) = \frac{\beta(\gamma+1)((1-\omega)G - \omega c)}{2(1-\delta) + \beta\delta(1-\gamma)}. \tag{70}$$

As before, if $c > \overline{c}(\omega)$, we have

$$V_r^{cg} < 0 \leq V_t^{cg}, \tag{71}$$

so the only equilibrium is $\gamma = 1$. Moreover, this is *an* equilibrium as long as $V_t^{cg}(1) - V_r^{cg}(1; c) \geq 0$, which is equivalent to setting, as before,

$$c > \underline{c}(\omega) = \frac{(\beta\delta(1-\omega) - \omega(1-\delta))G}{(\beta\delta - \delta + 1)\omega} \tag{72}$$

Now assume $c < \overline{c}(\omega) = G(1-\omega)/\omega$.

The difference between the two value from trading fairly, $V^{cg}(\text{trade})$, and that from robbing,$V^{cg}(\text{rob})$, should be equal to zero in the equilibrium. This is equivalent to requiring that $F^{cg}(\gamma, c) = 0$, where

$$F^{cg}(\gamma, c) \equiv (2(1-\delta) + \beta\delta(1-\gamma))\gamma G - (\gamma+1)((1-\omega)G - \omega c)(1 - \delta(1-\beta)) \tag{73}$$

Again, $F^{cg}(\gamma, c)$ is a quadratic polynomial in γ :

$$F^{cg}(\gamma, c) = k_1 + b_1\gamma + a_1\gamma^2, \tag{74}$$

where

$$k_1(c) = -(G(1-\omega) - c\omega)(1 - \delta(1-\beta)); \tag{75}$$

$$b_1(c) = G(\beta\delta - 2\delta + 2) - (G(1-\omega) - c\omega)(1 - \delta(1-\beta)) \tag{76}$$

$$a_1 = -G\beta\delta \tag{77}$$

$k_1(c) < 0, a_1(c) < 0$. If $b_1(c) < 0$, then $F^{cg}(\gamma; c) < 0$ for all $\gamma > 0$. Thus, equilibrium is $\gamma = 0$.

If $b_1(c) \geq 0$, in order to have roots, the polynomial has to satisfy $H^{cg}(\beta; \delta, c, \omega) \equiv b_1^2(c) - 4a_1(c) k_1(c) \geq 0$ also. This inequality is satisfied for all the relevant parameter values. But, similar to the proof of proposition 2, $H^{cg}(\beta; \delta, c, \omega)$ is quadratic in c:

$$
\begin{aligned}
H^{cg}(\beta; \delta, c, \omega) &= a_c c^2 + b_c c + k_c, && (78)\\
a_c &= \omega^2 (1 - \delta(1-\beta))^2 > 0; && (79)\\
b_c &= 2\omega G(\beta\delta - \delta + 1)(\omega - \delta\omega + 2\beta\delta + \beta\delta\omega + 1 - \delta) > 0; && (80)\\
k_c &= -4G^2 \beta\delta (1-\omega)(1 - \delta(1-\beta)) + && (81)\\
&\quad + (G(\beta\delta - 2\delta + 2) - G(1-\omega)(1 - \delta(1-\beta)))^2. && (82)
\end{aligned}
$$

Observe that if

$$
c = \tilde{c}(\omega) \equiv \frac{G(1-\omega)}{\omega} - G\frac{(\beta\delta + 2(1-\delta))}{\omega}(1 - \delta(1-\beta)), \tag{83}
$$

then $b_1(c) = 0$, therefore $F^{cg}(\gamma, c) < 0$ for all γ, thus its discriminant $H^{cg}(\beta; \delta, c, \omega)$ is strictly negative. Therefore, it always has real roots. If c is below the lower root, then $F^{cg}(\gamma; c)$ has negative (γ) roots. (Again, this means that the only equilibrium is $\gamma = 0$). If c is above the upper root of H, then $F^{cg}(\gamma; c)$ has positive (γ) roots. Moreover, if k_c is positive, H can have only negative roots. If k_c is negative, H has two roots (c) of the opposite sign. Denote the upper root of H by

$$
\underline{\underline{c}}^{cg}(\omega) \equiv \frac{-b_c + \sqrt{(b_c)^2 - 4a_c k_c}}{2a_c}. \tag{84}
$$

It is clear that $\underline{\underline{c}}^{cg}(\omega) > \tilde{c}(\omega)$, so that for $c > \underline{\underline{c}}^{cg}(\omega)$ coefficient $b_1(c)$ is strictly positive. Similarly to the parallel argument in the proof of proposition 2, $\underline{\underline{c}}^{cg}(\omega) < G(1-\omega)/\omega$. To summarize, if $c > \underline{\underline{c}}^{cg}(\omega)$, $F^{cg}(\gamma; c)$ has two positive roots $\gamma_L < \gamma_H$ (as $k_1(c) < 0, a_1(c) < 0$ and $b_1(c) > 0$). In addition, $F^{cg}(\gamma; c) < 0$ for $\gamma < \gamma_L$. To have a positive fraction of traders in equilibrium we need to assure that at least $\gamma_L < 1$. Similar to the case discussed in the proof of proposition (2) define γ^*_{cg}, the maximizer of $F^{cg}(\gamma; c)$,

$$\gamma^*_{cg} = \frac{1}{2\delta\beta G}\left(G - G\delta + G\omega + c\omega(1-\delta+\beta\delta) - G\delta\omega + G\beta\delta\omega\right) \tag{85}$$

It is easy to check that $\gamma^*_{cg} > 1$ iff $c > c^*_{cg}$, where

$$c^*_{cg} = \frac{G\left(2\beta\delta + \delta\omega - \beta\delta\omega + \delta - (1+\omega)\right)}{\omega(1-\delta+\beta\delta)} \tag{86}$$

Note that the difference

$$c^*_{cg} - \underline{c} = \frac{(\delta + \beta\delta - 1)\, G}{(\beta\delta - \delta + 1)\,\omega} \tag{87}$$

is negative whenever $\delta + \beta\delta \leq 1$. In this case there are three equilibria, $\{0, \gamma_L, 1\}$ with $\gamma_L \leq 1$ whenever $c \in [\underline{c}(\omega), \bar{c}(\omega)]$, with unique equilibrium $\gamma = 0$ for $c < \underline{c}(\omega)$ and unique equilibrium $\gamma = 1$ for the values $c > \bar{c}(\omega)$.

In the complementary case, $\delta + \beta\delta > 1$, there is a range of values, $c \in \left[\underline{\underline{c}}^{cg}(\omega), \underline{c}(\omega)\right]$, for which the three equilibria are $\{0, \gamma_L, \gamma_H\}$, where both γ_L and γ_H are below unity: $c < \underline{c}(\omega) < c^*_{cg}(\omega)$ implies $\gamma^*_{cg} < 1$ and $F^{cg}(1; c) < 1$, assuring that both roots (if real) of parabola $F^{cg}(\gamma; c)$ should be below unity. Condition $c > \underline{\underline{c}}^{cg}(\omega)$ assures that the roots are real. Finally, we have to check that $\underline{\underline{c}}^{cg}(\omega) < \underline{c}(\omega)$. Indeed, at $c = \underline{c}(\omega)$, expression $F^{cg}(1; c) = 0$ and $\gamma^*_{cg}(\underline{c}(\omega)) < 1$. Therefore, $F^{cg}\left(\gamma^*_{cg}(\underline{c}(\omega)); \underline{c}(\omega)\right) > 0$. Moreover, $F^{cg}(\gamma, c)$ is monotonically increasing in c. At $\underline{\underline{c}}^{cg}(\omega)$ the expression $F^{cg}(\gamma; c)$ is non-positive for any $\gamma \in [0,1]$, thus $F^{cg}\left(\gamma^*_{cg}(\underline{c}(\omega)); \underline{\underline{c}}^{cg}(\omega)\right) \leq 0$. Moreover, as by (85) $\gamma^*_{cg}(c)$ is strictly increasing in c, $F^{cg}\left(\gamma^*_{cg}(\underline{c}(\omega)); \underline{\underline{c}}^{cg}(\omega)\right) < 0$. It follows that $\underline{\underline{c}}^{cg}(\omega) < \underline{c}(\omega)$. ■

Proof of Proposition 7. Assume $c \in [\max\left\{\underline{\underline{c}}(\omega), \underline{\underline{c}}^{cg}(\omega)\right\}, \underline{c}(\omega))$. Let

$$F^x(\gamma, c) \equiv \frac{F^{cg}(\gamma, c) - F(\gamma, c)}{\omega\beta\delta} \tag{88}$$

$$= (1-\gamma)\left(G\omega - G + c\omega + G\gamma\omega + c\gamma\omega\right) \tag{89}$$

First, $F^x(1, c) = 0$ for any c, so that $F^{cg}(1, c) = F(1, c) = Z(c)$. Second, as was shown in lemma 14, $F(1, \underline{c}(\omega)) = 0$. Third, $F^{cg}(\gamma, c)$ and $F(\gamma, c)$ are both strictly increasing in c (for $\gamma \in [0,1]$), so that, $\forall c < \underline{c}(\omega)$, $Z(c) < 0$. Note also that $F^x(0, c) < 0$ (if $c < \bar{c}(\omega)$), so that $F^{cg}(0, c) < F(0, c)$. As for the second root of $F^x(\gamma, c)$, denote it by $\hat{\gamma}$,

$$\hat{\gamma} = \frac{G}{\omega\left(G+c\right)} - 1. \tag{90}$$

This root, $\hat{\gamma}$, is positive iff

$$c < \frac{G\left(1-\omega\right)}{\omega} = \bar{c}\left(\omega\right) \tag{91}$$

which is consistent with the assumptions in the statement of the proposition, as $c < \underline{c}\left(\omega\right) < \bar{c}\left(\omega\right)$. Observe that $F^{cg}(\widehat{\gamma}, c)$ is negative if $c < \underline{c}\left(\omega\right)$. Indeed,

$$F^{cg}(\widehat{\gamma}, c) = \frac{\left(G\omega + c\omega - G\beta\delta - G\delta\omega - c\delta\omega + G\beta\delta\omega + c\beta\delta\omega\right)}{\left(G+c\right)^{2}\omega^{2}}\left(G(1-\omega) - c\omega\right)G \tag{92}$$

Provided $\left(G(1-\omega) - c\omega\right) > 0$, so that $c < \bar{c}\left(\omega\right)$,

$$F^{cg}(\widehat{\gamma}, c) \leq 0 \tag{93}$$

if

$$G\omega + c\omega - G\beta\delta - G\delta\omega - c\delta\omega + G\beta\delta\omega + c\beta\delta\omega \leq 0 \tag{94}$$

The last inequality holds iff

$$c \leq \frac{\left(\beta\delta(1-\omega) - \omega(1-\delta)\right)G}{\left(\beta\delta - \delta + 1\right)\omega} = \underline{c}\left(\omega\right), \tag{95}$$

which is, again, consistent with the assumptions. Thus, $F^{cg}(\widehat{\gamma}, c)$ is indeed negative. To recapitulate, we know that $F^{x}\left(\gamma, c\right)$ has two positive roots (one equals 1) and that in both cases the roots are in a region where $F^{cg} = F < 0$.

There are now two possibilities. First, assume

$$c > \frac{G\left(1-2\omega\right)}{2\omega} = \hat{c}\left(\omega\right). \tag{96}$$

Note that $\hat{c}\left(\omega\right) < \underline{c}\left(\omega\right)$ whenever $\delta + \beta\delta > 1$, which is true by assumption. So, if $c > \hat{c}\left(\omega\right)$, then by definition (90), the two functions F and F^{cg} cross at $\hat{\gamma}$ less than unity. Therefore, as the difference F^{x} is positive between the roots, we have $F^{cg}\left(\gamma, c\right) > F\left(\gamma, c\right)$ for any $\gamma \in \left(\hat{\gamma}, 1\right)$. As, we assumed, $c \in [\max\left\{\underline{c}\left(\omega\right), \underline{\underline{c}}^{cg}\left(\omega\right)\right\}, \underline{c}\left(\omega\right))$, both quadratic polynomials F and F^{cg} have two real roots strictly between zero and one.

It follows that the lower root of $F^{cg}(\gamma, c)$ should be below the lower root of $F(\gamma, c)$ and the opposite is true for the upper root, i.e., $\gamma_H^{cg}(c) > \gamma_H(c)$, which corresponds to the claim in the proposition.

Second, consider the complementary case to (96), $c \leq \hat{c}(\omega)$. The goal is to show that neither F nor F^{cg} have no positive roots in this range, in other words, $\hat{c}(\omega) < \max\left\{\underline{\underline{c}}(\omega), \underline{\underline{c}}^{cg}(\omega)\right\}$. Indeed, take $c = \hat{c}(\omega)$, then the only intersection point of F and F^{cg} is $\hat{\gamma} = 1$:

$$F^{x}(\gamma, \hat{c}) = \left(-\frac{1}{2}\right) G(\gamma - 1)^2. \tag{97}$$

It follows that $F^{cg}(\gamma, \hat{c}) \leq F(\gamma, \hat{c})$ for any γ and the two are equal and tangent at $\gamma = 1$.

Moreover, the polynomial

$$F(\gamma, \hat{c}) = \frac{1}{2}G(\delta - \beta\delta + \beta\delta\omega - 1) + \frac{1}{2}G\gamma^2(\beta\delta\omega - 2\beta\delta) + \frac{1}{2}G\gamma(\beta\delta - 3\delta - 2\beta\delta\omega + 3) \tag{98}$$

has only negative roots, and therefore, is negative for $\gamma \in [0, 1]$. Indeed, its discriminant, $\left(\frac{1}{2}G\right)^2(\delta + \beta\delta - 1)(9\delta - 7\beta\delta + 8\beta\delta\omega - 9)$ is positive (for the relevant range of parameters, $\delta + \beta\delta > 1$) only if

$$\frac{7}{8} \leq \frac{9}{8\beta\delta} - \frac{9}{8\beta} + \frac{7}{8} \leq \omega. \tag{99}$$

But in this case all coefficients of $F(\gamma, \hat{c})$ are negative, so it can have only negative roots. To sum up, $F^{cg}(\gamma, \hat{c}) \leq F(\gamma, \hat{c}) < 0$ for $\gamma \in [0, 1]$. As both F and F^{cg} decrease in c for $\gamma \in [0, 1]$, we conclude that neither $F(\gamma; c)$ nor $F^{cg}(\gamma; c)$ have (γ) roots in the interval $[0, 1]$ if $c \leq \hat{c}$.

Finally, we have to show that $\max\left\{\underline{\underline{c}}(\omega), \underline{\underline{c}}^{cg}(\omega)\right\} = \underline{\underline{c}}(\omega)$. Thus, we need to show that there is a range of punishments c for which $F^{cg}(\gamma; c)$ has (γ) roots in the interval $[0, 1]$, but $F(\gamma; c)$ does not. For that consider function $\phi(c)$ defined as the first derivative of F at the point of intersection between F and F^{cg}, $\hat{\gamma}(c)$,

$$\phi(c) \equiv F'(\hat{\gamma}(c); c) \tag{100}$$

$$\phi(c) = 2\left(-\delta\beta\left(G(1-\omega^2) - c\omega^2\right)\right)\left(\frac{G}{\omega(G+c)} - 1\right) + \tag{101}$$

$$+G(1-\delta) + G\omega(1-\delta) + c\omega(1-\delta) + c\beta\delta\omega \tag{102}$$

Clearly, $\phi(c)$ is continuous in c for $c > -G$. But the range of punishments we are considering falls into this category, as $\hat{c} = \frac{G(1-2\omega)}{2\omega} > G$, clearly. (Besides, if $c+G \leq 0$, a thief will never pay a bribe.) Moreover,

$$\begin{aligned} \phi\left(\underline{c}(\omega)\right) &= \frac{(\beta\delta - \delta - \beta\delta\omega + 1)(\delta + \beta\delta - 1) G}{(\beta\delta - \delta + 1)} > 0 \\ \phi\left(\hat{c}(\omega)\right) &= \left(-\frac{3}{2}\right) G(\delta + \beta\delta - 1) < 0 \end{aligned}$$

Thus, there is a c_ϕ such that $\phi(c_\phi) = 0$. (It is easy to show that this value uniquely determined.) Similarly, define $\theta(c)$ as

$$\theta(c) \equiv F^{cg\prime}(\hat{\gamma}(c); c); \tag{103}$$

$$\theta(c) = 2(-G\beta\delta)\left(\frac{G}{\omega(G+c)} - 1\right) + \tag{104}$$

$$+G(\beta\delta - 2\delta + 2) - (G(1-\omega) - c\omega)(1 - \delta(1-\beta)) \tag{105}$$

Then,

$$\theta\left(\underline{c}(\omega)\right) = G(\delta + \beta\delta - 1) > 0; \tag{106}$$

$$\theta\left(\hat{c}(\omega)\right) = \phi\left(\hat{c}(\omega)\right) = \left(-\frac{3}{2}\right) G(\delta + \beta\delta - 1) < 0, \tag{107}$$

which, again, implies there is a unique c_θ such that $\phi(c_\theta) = 0$. Now,

$$\theta(c) - \phi(c) = \omega\delta\beta(2G\omega - G + 2c\omega) > 0 \tag{108}$$

if $c > \hat{c}(\omega)$. Therefore, $\theta(c_\phi) > 0$. But then $F(\hat{\gamma}(c_\phi), c_\phi) < 0$ (by (93) and definition of $\hat{\gamma}$). As $F'(\hat{\gamma}(c_\phi), c_\phi) = 0$, it follows that $F(\gamma, c_\phi) < 0$ for any γ. It follows that $c_\phi < \underline{\underline{c}}(\omega)$, at which F has only one (γ) root. Moreover, it follows that at $c = \underline{\underline{c}}(\omega)$ both $\theta\left(\underline{\underline{c}}(\omega)\right)$ and $\phi\left(\underline{\underline{c}}(\omega)\right)$ are positive, which means that the first intersection ($\hat{\gamma}$) between F and F^{cg} occurs when both are increasing and $F^{cg}\left(\gamma; \underline{\underline{c}}(\omega)\right) > F\left(\gamma; \underline{\underline{c}}(\omega)\right)$ for $\gamma \in (\hat{\gamma}, 1)$. Thus, $F^{cg}\left(\gamma; \underline{\underline{c}}(\omega)\right)$ has to have two distinct roots. In other words, $\underline{\underline{c}}(\omega) > \underline{\underline{c}}^{cg}(\omega)$, where, recall, $\underline{\underline{c}}^{cg}(\omega)$ is the value of punishment at which $F^{cg}(\gamma; c)$ has only one root between zero and unity (i.e., its discriminant is zero). ■

Proof of Proposition 9. In this case the equilibrium demand for government is

$$D^{cg}(\gamma, G) = \beta \frac{G\gamma}{1-\delta+\beta\delta} - \frac{\beta G}{\beta\delta - 2\delta + 2} = \tag{109}$$
$$= \frac{(2\gamma + \delta - \beta\delta - 2\gamma\delta + \beta\gamma\delta - 1)}{(\beta\delta - 2\delta + 2)(\beta\delta - \delta + 1)} G\beta,$$

which is, again proportional to the gains from trade and is positive (an increasing in G) iff

$$\gamma > \underline{\gamma}^{gc} \equiv \frac{1-\delta+\beta\delta}{2-2\delta+\beta\delta}. \tag{110}$$

Note that the lower bound on the equilibrium γ now is higher that under non-corrupt government (for strictly positive ω), $\underline{\gamma}^{gc} > \underline{\gamma}$.

It is also immediate from the definition of the demand (109) that it increases in γ.

As for the last claim, similar to lemma 5, it enough to show that both $\gamma_{cg}^{*}(G)$ and $H_{cg}(G)/(a_1(G))^2$ are decreasing in G, where $H^{cg}(G) \equiv b_1^2(G) - 4a_1(G)k_1(G)$ is the discriminant of the quadratic polynomial $F^{cg}(\gamma)$ with the coefficients

$$k_1(c) = -(G(1-\omega) - c\omega)(1 - \delta(1-\beta)); \tag{111}$$
$$b_1(c) = G(\beta\delta - 2\delta + 2) - (G(1-\omega) - c\omega)(1 - \delta(1-\beta)) \tag{112}$$
$$a_1 = -G\beta\delta. \tag{113}$$

In the view of the definition (85))

$$\frac{d}{dG}\left(\gamma_{cg}^{*}(G)\right) = \left(-\frac{1}{2}\right)\delta^{-1}\beta^{-1}G^{-2}(\beta\delta - \delta + 1)c\omega < 0$$

Next,

$$\frac{d}{dG}\left(H_{cg}(G)/(a_1(G))^2\right) = (-2)\frac{(\beta\delta - \delta + 1)c\omega}{\delta^2\beta^2G^3}K_1(G) < 0, \tag{114}$$

where

$$K_1(G) = (G(1-\delta)(1+\omega) + c\omega(1-\delta) + 2G\beta\delta + G\beta\delta\omega + c\beta\delta\omega) > 0. \tag{115}$$

Therefore, in the presence of corruption, again the upper equilibrium $\gamma_H^{cg}(G)$ is decreasing in G. ■

Lemma 22 *Assume $\beta\delta + \delta > 1$ and $c \in [\underline{\underline{c}}(\omega), \underline{c}(\omega))$, so that the protection agency can induce equilibrium $\gamma = \gamma_H < 1$. Assume also that protection agents extract all the surplus from farmers. Given γ, their return is higher in a corrupt environment.*

Proof. For any γ, in the willingness to pay for the government is lower under corruption:

$$D(\gamma, G) - D^{cg}(\gamma, G) = \frac{(1-\gamma)\, G\beta^2\gamma\delta\omega}{(\beta\delta(1-\omega) + \beta\gamma\delta\omega + 1 - \delta)\,(\beta\delta + 1 - \delta)} > 0 \qquad (116)$$

In this case the above loss in the willingness to pay has to be put against the proceeds from bribes: $(1-\gamma)(c+G)\omega$.The result holds so long as

$$(1-\gamma)(c+G)\,\omega > \frac{(1-\gamma)\, G\beta^2\gamma\delta\omega}{(\beta\delta(1-\omega) + \beta\gamma\delta\omega + 1 - \delta)\,(\beta\delta + 1 - \delta)} \qquad (117)$$

requiring

$$c > c^{\#} = G\left(\frac{\beta^2\gamma\delta}{(1-\delta+\beta\delta)\,(\beta\gamma\delta\omega + \beta\delta\,(1-\omega) + 1 - \delta)} - 1\right) \qquad (118)$$

but The sign of the derivative of $c^{\#}$ with respect to γ is the same as that of

$$\beta^2\delta\,(1-\delta+\beta\delta)\,(\beta\gamma\delta\omega + \beta\delta\,(1-\omega) + 1 - \delta) - \beta^2\gamma\delta\,(1-\delta+\beta\delta)\,\beta\delta\omega$$

which becomes

$$\beta\delta\,(1-\omega) + 1 - \delta \qquad (119)$$

which is positive. Hence, the expression for $c^{\#}$ is maximized where $\gamma = 1$. Of course, γ is endogenous, so that we plug in $\gamma = 1$ and get that

$$c^{\#} < G\left(\frac{\beta^2\delta}{(1-\delta+\beta\delta)^2} - 1\right) \qquad (120)$$

The derivative of the above expression with respect to δ is

$$\frac{\beta^2 (1-\delta+\beta\delta)^2 + 2\beta^2\delta (1-\beta)(1-\delta+\beta\delta)}{(1-\delta+\beta\delta)^4} > 0 \tag{121}$$

so it is maximized when $\delta = 1$. Hence,

$$c^{\#} < G\left(\frac{\beta^2}{\beta^2} - 1\right) = 0 \tag{122}$$

So $c^{\#}$ has to be negative.

This implies that the threshold $c^{\#} < 0$. As by assumption, $c \geq \underline{c} > 0$, we conclude that $D(\gamma, G) - D^{cg}(\gamma, G) > 0$. ■

Proof of Theorem 10**.** In this environment, the population of Market town is exogenous. Normalize the population of Market town to equal one, as it will merely operate as a multiplicative constant to the profits of the agency. Distinguish γ, the equilibrium proportion of traders in the regular environment, from γ^{cg}, the equilibrium proportion in the corrupt environment. Total bribes are $(1-\gamma^{cg})(c+G)\omega$. The relative payoff from corruption is

$$\Pi(\gamma^{cg}, \gamma) \equiv (1-\gamma^{cg})(c+G)\omega + \varpi\left[D^{cg}(\gamma^{cg}, c) - D(\gamma, c)\right] \tag{123}$$

where ϖ is the proportion of the demand that is captured by the protection agency. Clearly if $\varpi = 0$ the agency is better off under corruption, so the interesting case is $\varpi = 1$. We know $\Pi(\gamma^{cg}, \gamma) > 0$ when $\gamma^{cg} = \gamma$ from Lemma (22). However, we know that $\gamma_H^{cg}(c) > \gamma_H(c)$. So long as $\gamma_H > \overline{\gamma}_{cg} > \overline{\gamma}$,

$$\frac{\partial D(\gamma, c)}{\partial \gamma} > 0 \Rightarrow \frac{\partial \Pi(\gamma^{cg}, \gamma)}{\partial \gamma} < 0. \tag{124}$$

Hence, $\Pi(\gamma^{cg}, \gamma) > \Pi(\gamma^{cg}, \gamma^{cg}) > 0$. ■

Lemma 23 *Under anarchy, for all parametrizations, there exists a unique equilibrium in which agents go to Market town. This equilibrium is stationary and, in all periods, $\gamma = 0$: there is no trade.*

Lemma 24 *Suppose $c \geq \underline{c}^{cm}(\omega)$. Then, an equilibrium with only trade exists.*

Lemma 25 *Suppose $c < \overline{c}^{cm}(\omega)$. Then, an equilibrium exists in which there is no trade: $\gamma = 0$.*

Proof of Lemma 23. Let H be the value function of being in Farmland.

$$H = \delta \max\{H, V\} \Rightarrow H = \max\{0, \delta V\} \tag{125}$$

In the case of anarchy, in a steady state,

$$\begin{aligned} V^a &= \beta \max\left\{\gamma G + \delta H, \gamma W^a + (1-\gamma)\frac{1}{2}(W^a + \delta H)\right\} \quad (126) \\ &\quad + (1-\beta)\,\delta V^a \quad (127) \end{aligned}$$

If the agent chooses to trade this period, her value function is defined by

$$V^a\,(trade) = \beta\,(\gamma G + \delta H) + (1-\beta)\,\delta V^a \tag{128}$$

If there are any equilibria other than the 'Ghost Town' equilibrium, it must be that $V^a \geq 0$, so that this becomes

$$V^a\,(trade) = \beta\left(\gamma G + \delta^2 V^a\right) + (1-\beta)\,\delta V^a \tag{129}$$

On the other hand, if she decides to rob her value function becomes

$$V^a\,(rob) = \beta\left(\gamma W^a + (1-\gamma)\frac{1}{2}(W^a + \delta F)\right) + (1-\beta)\,\delta V^a \tag{130}$$

By comparing the contemporaneous terms of equations (129) and (130) ,for any internal values of the parameters, $\beta, \delta \in (0,1)$, and regardless of continuation values and strategies,

$$V^a\,(\text{rob}) > V^a\,(\text{trade}) \tag{131}$$

so that, in equilibrium, $V^a = V^a(rob)$. Finally,

$$V^a = \frac{\beta G}{(2+\beta\delta)(1-\delta)} > 0 \tag{132}$$

■

Proof of Lemma 24. Assuming agents are adopting an optimal strategy after the current date that yields V^g, the value functions are given by

$$V^g\left(\text{trade}\right) = \beta\left(\gamma G + \gamma\delta H + (1-\gamma)\left[(1-\omega)\delta H + \omega\delta V^g\right]\right) + \\ + (1-\beta)\delta V^g \tag{133}$$

$$W^g(\omega, c) \equiv (1-\omega)\left[G + \delta V^g\right] + \omega\left[-c + \delta V^g\right] \tag{134}$$

$$V^g\left(\text{rob}\right) = \beta\left(\gamma W^g(\omega, c) + \frac{(1-\gamma)}{2}\left[W^g(\omega, c) + \omega\delta V^g + (1-\omega)\delta H\right]\right) \\ + (1-\beta)\delta V^g \tag{135}$$

Suppose $\gamma = 1$. Since it must be that it is at least as good to trade as to rob, it must be that

$$V^g \leq \frac{\omega(G+c)}{\delta(1-\delta)} \tag{136}$$

Second, it must be that $V^g = V^g\left(\text{trade}\right)$:

$$V^g = \frac{\beta G}{1 - \delta(1 - \beta(1-\delta))} \tag{137}$$

$$V^g = \frac{\beta G}{(1-\delta)(1+\beta\delta)} \tag{138}$$

Thus, for $\gamma = 1$, it is necessary and sufficient that

$$\frac{\beta G}{(1-\delta)(1-\beta(1-\delta))} \leq \frac{\omega(G+c)}{\delta(1-\delta)} \tag{139}$$

$$\frac{\beta G}{(1+\beta\delta)} \leq \frac{\omega(G+c)}{\delta} \tag{140}$$

which becomes

$$c \geq \underline{c}^{cm}(\omega) = \frac{(\beta\delta(1-\omega) - \omega)G}{\omega(1+\beta\delta)}$$

■

Proof of Lemma 25. Assume $\gamma = 0$. For this to be optimal, it must be at least as good to steal as to trade.

$$V^g \geq \frac{[\omega c - (1-\omega)G]}{\delta(1-\delta)(1-\omega)} \tag{141}$$

The value function must equal the value of stealing, so that

$$V^g = \beta\frac{[(1-\omega)G - \omega c]}{(1-\delta)[2+\delta\beta(1-\omega)]} \tag{142}$$

Thus, the corresponding parameter restriction is

$$\beta[(1-\omega)G - \omega c]\,\delta(1-\delta)(1-\omega) \geq (1-\delta)[2+\delta\beta(1-\omega)][\omega c - (1-\omega)G] \tag{143}$$

Now there are three possibilities. First, suppose that

$$(1-\omega)G - \omega c = 0 \tag{144}$$

then the condition holds as is. Second, suppose that

$$(1-\omega)G - \omega c > 0 \tag{145}$$

Then we have

$$\beta\delta(1-\omega) \geq -1 \tag{146}$$

which always holds. Third, suppose that

$$(1-\omega)G - \omega c < 0 \tag{147}$$

in that case, the inequality becomes

$$-\beta\delta(1-\omega) \geq 1 \tag{148}$$

which never holds. Consequently, an equilibrium with $\gamma = 0$ occurs if and only if

$$c \leq \overline{c}^{cm}(\omega) = \frac{(1-\omega)G}{\omega} \tag{149}$$

■

Proof of Proposition 11. Suppose $\gamma \in (0,1)$. It must be that

$$V^g\left(\text{trade}\right) = V^g\left(\text{rob}\right) \tag{150}$$

regardless of the continuation value V_g. Hence

$$V^g = \frac{(\gamma - 1 + \omega + \gamma\omega)\, G + \omega\,(1+\gamma)\, c}{\delta\,(1-\delta)\,(1+\gamma+\gamma\omega-\omega)} \tag{151}$$

Second, it must be that

$$V^g = V^g\left(\text{trade}\right) \tag{152}$$

so that

$$V^g = \beta\left(\gamma G + \gamma\delta^2 V + (1-\gamma)\left[(1-\omega)\,\delta^2 V + \omega\delta V^g\right]\right) + (1-\beta)\,\delta V^g \tag{153}$$

$$V^g = \frac{\beta\gamma G}{\left[1+\beta\delta\,(1-\omega\,(1-\gamma))\right](1-\delta)} \tag{154}$$

Thus γ is given by the roots of to the following quadratic equation, that is given by equating (151) and (154):

$$\begin{aligned}
F^{cm}(\gamma;c) &= \gamma^2 a_{cm}(c) + \gamma b_{cm}(c) + k_{cm}(c) && (155)\\
a_{cm}(c) &= \beta\delta\left[\omega^2 c - \left(1-\omega^2\right) G\right] && (156)\\
b_{cm}(c) &= \left[(1+\omega)\, G + \omega c\right] + \beta\delta\omega c > 0 && (157)\\
k_{cm}(c) &= \left[1+\beta\delta\,(1-\omega)\right]\left[\omega c - (1-\omega)\, G\right] && (158)
\end{aligned}$$

Note for a start that F^{cm} is strictly increasing in c and without upper bound for a given γ. Consequently, if c is large enough real roots will exist. Note that

$$\begin{aligned}
\frac{\partial F^{cm}(\gamma;c)}{\partial\gamma} &= 2\gamma a_{cm}(c) + b_{cm}(c) && (159)\\
\left.\frac{\partial F^{cm}(\gamma;c)}{\partial\gamma}\right|_{\gamma=0} &= b_{cm}(c) > 0 && (160)
\end{aligned}$$

Consequently, F^{cm} must be upward-sloping when it crosses the vertical axis. This leaves several possible cases of F^{cm}. First, suppose $a_{cm}(\omega) > 0$. Then, it must be that

$$c > \frac{(1-\omega^2) G}{\omega^2} > \frac{(1-\omega) G}{\omega} = \overline{c}^{cm}(\omega) \tag{161}$$

so that necessarily an equilibrium with $\gamma = 0$ cannot exist in this range. Note that, since $\left.\frac{\partial F^{cm}(\gamma;c)}{\partial \gamma}\right|_{\gamma=0} > 0$, it must be that $F^{cm}(\gamma; c) = 0$ at most one positive value, and this only if $k_{cm} < 0$. Will such an intersection exist? If there is no $\gamma-$intercept, it must be that

$$k_{cm}(c) \quad > \quad 0 \tag{162}$$

$$\Rightarrow \quad c > \frac{(1-\omega) G}{\omega} \tag{163}$$

which is already implied by the fact that $a_{cm}(\omega) > 0$. Consequently, we know that if

$$c > \frac{(1-\omega^2) G}{\omega^2} \tag{164}$$

then there are no interior equilibria. Moreover, we know there is no theft-only equilibrium for these parameter values.

Next, suppose that $a_{cm}(c) < 0$. Then,

$$c < \frac{(1-\omega^2) G}{\omega^2} \tag{165}$$

There are two possibilities here. In the first case, $k_{cm}(c) > 0$. In the second case, $k_{cm}(c) < 0$. Suppose that $k_{cm}(c) > 0$, i.e.

$$c \in \left[\frac{(1-\omega)G}{\omega}, \frac{(1-\omega^2) G}{\omega^2}\right] \tag{166}$$

First of all, we know that an equilibrium with $\gamma = 0$ cannot exist in this parameter range. Second, here, we know that there is exactly one positive root of the polynomial (since $4a_{cm}(\omega) k_{cm}(\omega)$ is negative), which has to be less than one to represent an equilibrium. Call it $\gamma_L^{cm}(c)$ (where the subscript denotes "low"). $\gamma_L^{cm}(c) \leq 1$ iff

$F^{cm}(1;c) \leq 0$, which implies that

$$c \leq \frac{\beta\delta\left(1-\omega\right)-\omega}{\left(1+\beta\delta\right)\omega}G = \underline{c}^{cm}\left(\omega\right) \tag{167}$$

However, this contradicts the initial assumption that $c \geq \overline{c}\left(\omega\right) > \underline{c}^{cm}\left(\omega\right)$. Hence this type of equilibrium does not exist, so that if

$$c \in \left[\frac{(1-\omega)G}{\omega}, \frac{\left(1-\omega^2\right)G}{\omega^2}\right] \tag{168}$$

then there is a unique equilibrium in which $\gamma = 1$ also. Our results so far are that if $c > \overline{c}\left(\omega\right)$, there is a unique equilibrium in which $\gamma = 1$.

In the second case, $k_{cm}(c) < 0$:

$$c < \frac{(1-\omega)G}{\omega} \tag{169}$$

so that there does exist an equilibrium with $\gamma = 0$. In addition, we know that, generically, $F^{cm}\left(\gamma;c\right)$ will have two positive real roots or none in this region of parameter space, since $k_{cm}(c) < 0$. Let

$$\gamma^*_{cm} = \arg\max_{\gamma} F^{cm}\left(\gamma;c\right) \tag{170}$$

The equation for γ^*_{cm} is

$$\frac{\partial F^{cm}\left(\gamma;c\right)}{\partial\gamma} = 2\gamma^*_{cm}a_{cm}(c) + b_{cm}(c) = 0 \tag{171}$$

$$\gamma^*_{cm} = \frac{\left(1+\omega\right)G + \omega c + \beta\delta\omega c}{2\beta\delta\left[\left(1-\omega^2\right)G - c\omega^2\right]} \tag{172}$$

Suppose that

$$F^{cm}(1;c) > 0 \tag{173}$$

This is a sufficient condition for $F^{cm}(\gamma^*;c) > 0$. Then, since we know $k_{cm}\left(c\right) = F^{cm}(0;c) < 0$, there must be a unique intermediate equilibrium by the intermediate value theorem. If $c > \underline{c}^{cm}\left(\omega\right)$ then there exists $\gamma^{cm}_L\left(c\right) \in (0,1)$ such that $\gamma^{cm}_L\left(c\right)$ is an equilibrium. How to show this? First,

$$F^{cm}(1;c) = \beta\delta\omega^2 c + \omega c + \beta\delta\omega c + \omega c + \beta\delta(1-\omega)\omega c \tag{174}$$

$$-\beta\delta\left(1-\omega^2\right)G + 2\omega G - \beta\delta(1-\omega)(1-\omega)G \tag{175}$$

$$F^{cm}(1;c) = 2\left[1+\beta\delta(1-\delta)\right]\omega c - 2\beta\delta(1-\delta)(1-\omega)G + 2\omega G \tag{176}$$

$F^{cm}(1,c)$ is positive iff

$$c > \frac{\beta\delta(1-\omega)-\omega}{(1+\beta\delta)\omega}G = \underline{c}^{cm}(\omega) \tag{177}$$

Suppose $F^{cm}(1,c)=0$. Then it must be that

$$0 = \beta\delta\omega^2 c - \beta\delta\left(1-\omega^2\right)G + 2\omega G + 2\omega c \tag{178}$$

$$+\beta\delta\omega c + \beta\delta(1-\omega)\omega c - \beta\delta(1-\omega)(1-\omega)G \tag{179}$$

$$\omega G + \omega c + \beta\delta\omega c - \beta\delta G + \beta\delta G\omega = 0 \tag{180}$$

$$c = \frac{(\beta\delta(1-\omega)-\omega)}{\omega(1+\beta\delta)}G \tag{181}$$

which is exactly when $c = \underline{c}^{cm}(\omega)$. Since $\frac{\partial F^{cm}(\gamma,c)}{\partial c} > 0$, decreasing c from here (into the region in which $F^{cm}(1,c) < 0$) enters a region in which there are *no interior equilibria*: $F^{cm}(1,c)$ may have roots, but they are outside the unit interval. This, combined with the above results, proves the proposition. ■

Proof of Proposition 12. We ignore τ, instead concentrating on the surplus and assuming that taxation is low enough for farmers to get some of it. ■

Proof. If the appropriate measure is given by W, the proof is trivial: W is decreasing in n^* and $\frac{\partial n^*(\gamma)}{\partial \gamma} < 0$. Suppose instead that welfare is given by W^c. Then

$$\frac{\partial W^c(\gamma)}{\partial \gamma} = -G\frac{\partial n^*(\gamma)}{\partial \gamma} - \beta c\omega(1-\gamma)\frac{\partial n^*(\gamma)}{\partial \gamma} + \beta c\omega\gamma n^*(\gamma)$$

which is positive. ■

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