DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 03-01

The Collective Household Model with Competing Pre-Marital Investments

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January 2003

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Abstract

We develop a "collective" model of the household in which spousal incomes are determined by pre-marital investments, the marriage market is characterized by assortative matching, and a sharing rule forms the basis of intra-household allocations. We identify the properties of the sharing rules that are maritally sustainable in this model. We find that the unconditionally efficient outcomes, in which both pre-marital investments and intra-household allocations are efficient, can be supported by intra-marital sharing rules that are consistent with the collective approach. In particular, when marriage does not generate a surplus, we show that only one sharing rule, which is purely a function of the gender wage gap, is sustainable in the marriage market. The outcome under this sharing rule is unconditionally Pareto efficient. When marriage generates a surplus and the numbers and distributions of men and women in the marriage market are identical, we demonstrate that there exists a continuum of maritally sustainable sharing rules. Associated with each of these sharing rules is a continuum of equilibria only one of which is unconditionally efficient. In contrast, when marriage involves a surplus and the numbers and distributions of men and women in the marriage market differ, the sharing rule which supports the unconditionally efficient equilibrium associated with the wives' threat point–at least for couples in the lowest assortative order–emerges as the maritally sustainable outcome.

Keywords: The Collective Model, Marriage, Bargaining, Household Labor Supply.
JEL Classification Numbers: C78, D61, D70.

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1. Introduction

Recent literature has shown that a treatment of the household as a single decision unit is not consistent with a growing body of empirical evidence on intra-household allocations.[1] Instead, the "collective" view, in which intra-household allocations are assumed to be efficient and individual members of the family are treated as the core decision-makers, has emerged as a compelling alternative.[2] Collective household models suggest–and the empirical evidence supports–the notion that relative spousal incomes influence household allocations.[3] Nonetheless, these spousal incomes are determined, at least in part, by decisions individuals make prior to marriage.[4] Thus, implicit in the construction of the collective framework is the idea that pre-marital investments influence wage earnings, and hence, intra-marital sharing rules.[5] Despite this premise, the existing collective models do not directly address how pre-marital investments can influence the efficiency of both pre-marital choices and intra-marital allocations. However, the emergence of the collective model as a fruitful approach to study household behavior and the inextricable links between pre-marital choices, marital matching, and intra-marital allocations suggest that it is imperative to do so.

In this paper, we incorporate the notion of pre-marital investments into a "collective" model where marital matching is assortative and a sharing rule forms the basis of intra-household allocations. Using this model, we examine the properties of intra-marital sharing rules that are sustainable in the markets for marriage. In particular, we study the conditions under which intra-marital sharing rules–which are according to the collective approach Pareto efficient conditional on pre-marital choices–yield unconditionally

[1] See, for example, Browning et al. (1994), Chiappori et al. (2002), and Udry (1996).

[2] The generalized version of this literature was spearheaded by Becker (1981) and developed further by Chiappori (1988, 1992).

[3] See, for instance, Browning et al. (1994) and Thomas (1990).

[4] In models of the household where spousal incomes are pure public goods, such decisions can lead to inefficient pre-marital choices and intra-household allocations, although the efficiency of pre-marital investments can be restored as a result of spousal competition in the markets for marriage. For instance, Bergstrom et al. (1986) demonstrate that economic agents under-invest prior to the beginning of a cooperative stage during which their potential partners' incomes are pure public goods. Recently, Peters and Siow (2002) have combined such a setup with assortative spousal matching to reveal how competition in large marriage markets restore Pareto efficiency.

[5] Moreover, to the extent that pre-marital choices of the offspring are made by their parents, they account for a share of intra-marital spending.

efficient and maritally sustainable outcomes.[6]

Our main finding is that unconditionally efficient outcomes, in which both pre-marital investments and intra-household allocations are Pareto efficient, can be supported by sharing rules that are consistent with the collective framework. When marriage does not generate a marital surplus, only the unconditionally efficient intra-marital sharing rule is sustainable in the marriage market. In this case, the uniquely determined efficient sharing rule is solely a function of the gender wage gap. When marriage generates a surplus and the numbers and distributions of men and women in the marriage market do not differ, we demonstrate that there exists a continuum of maritally sustainable sharing rules. Associated with each of these sharing rules is a continuum of equilibria only one of which is unconditionally efficient.

There are two corollaries to this finding: First, only when marriage involves a surplus, do distributional factors such as marriage and divorce legislation and sex ratios have the potential to influence the marital sharing rules. In the model, these distributional factors influence intra-marital allocations only in a corner solution where, for the lowest income marital matches, one of the spouses receives the reservation level of utility. These corner solutions emerge as the marital outcomes when the sex ratio is not equal to unity so that the numbers of men and women in the marriage market differ. For couples with the lowest household incomes, we argue that the sharing rule that emerges in such cases also supports the unconditionally efficient equilibrium. Second, reductions in the gender wage gap influence the set of inefficient but maritally sustainable sharing rules in two potentially offsetting ways: First, holding constant the level of household income, a smaller gender wage gap raises the reservation level of utility above which women choose to get married and shrinks the set of inefficient sharing rules that are maritally sustainable. Second, holding constant husbands' wages, a smaller gender wage gap also raises household income and expands the set of inefficient sharing rules that both spouses find acceptable.

[6] Hereafter, we refer to equilibria outcomes as *unconditionally Pareto efficient* if neither pre-marital investments nor intra-marital allocations can be altered to make one spouse strictly better off while leaving the other no worse off. We define outcomes as *conditionally Pareto efficient* if, given the choices spouses have made prior to marriage, intra-marital allocations cannot be altered to make one spouse better off while not affecting the other spouse.

2. Related Literature

This paper sits at the juncture of three strands in the economics literature. The first strand is on "collective" household models, and early- and late-generation marital bargaining models. These allow for differences between spouses to affect the choices households make by relying on a sharing-rule or an intra-household bargaining mechanism. The common analytical basis of this strand is that family members with potentially different preferences make Pareto-efficient household decisions. Among the earliest examples of the collective models are Becker (1981), Chiappori (1988, 1992), and Bourguignon and Chiappori (1994), and those of exogenous marital bargaining are Manser and Brown (1980), McElroy and Horney (1981), and Sen (1983). Each of these models assume that the sharing rule or the bargaining power of the two sexes are determined exogenously and that couples have different preferences over the choice sets. In two exceptions, Basu (2001) and Iyigun and Walsh (2002) suggest models that treat the bargaining power of the sexes as determined endogenously according to actual relative earnings. Neither of these models, however, examine how the existence of pre-marital investments might impact intra-marital allocations in a collective household setting.

The second strand of the literature to which this work is related includes papers that explore how matching influences investments. Earlier work in this line, such as Bergstrom, Blume, and Varian (1986), and MacLeod and Malcomson (1993), have shown that the equilibrium level of educational investments are below the Pareto efficient level when pre-marital investments are a public good in marriage. These papers do not take into account how endogenous matching might influence pre-marital investments. Peters and Siow (2002) argue that families make investments in education that are Pareto optimal once marital matching is endogenized. According to their results, large marriage markets, assortative matching and bilateral efficiency together guarantee that the equilibrium distribution of pre-marital investments is efficient. This is due to the fact that, when spousal wealth is a public good in marriage, the competitive marriage market and the assortative matching that occurs within it guide families to indirectly and reciprocally compensate each other for the investments that they make in their own children. Neither of the papers in this strand, including Peters and Siow, address how pre-marital investments might be influenced in a collective household setting.

Finally, our paper is related to work by Cole et al. (2001a, 2001b) which identifies that, in models with complementary investments by individuals, efficient outcomes as well as inefficient ones are sustainable in equilibrium. Our work is related to this strand because we study a modified version of their model to explore the marriage market and intra-household allocations. In particular, by examining the conditions under which the collective household models with pre-marital investments sustain efficient marital outcomes, we identify the properties of intra-marital sharing rules that are consistent with the collective approaches to household behavior.

The remainder of our paper is organized as follows: In section 3, we incorporate pre-marital investments into a collective model without a marital surplus and solve for the equilibrium in which intra-household allocations are determined by a sharing rule. In Section 4, we solve for the unconditionally Pareto efficient levels of pre-marital investments and intra-marital allocations. In Section 5, we compare the collective model outcomes, which are conditionally efficient, with the unconditionally Pareto efficient frontier and the maritally sustainable allocations. This comparison demonstrates that only the efficient allocation rule is sustainable. In Section 6, we extend the basic model to include a marital surplus and identify the conditions under which the unconditionally efficient and inefficient outcomes can be sustained in equilibrium. In Section 7, we conclude.

3. The Basic Model

The economy is made up of individuals who live for two periods. The total mass of women in the economy is equal to F and that of men is equal to M.[7] Let $G(N)$ and $H(N)$ respectively be measures of the sets of males and females whose endowments lie in the set N. All individuals are endowed with an initial wealth of y, where $y \in (0, Y]$.

When young, individuals allocate their wealth to consumption and a form of investment that augments their future incomes (i.e. education). When they get old, individuals

[7] Most of our notation and definitions regarding the assortative matching in the marriage market follows Peters and Siow.

either marry or stay single but they all work and consume. The wage rate per unit of education equals one for men and ψ, $\psi \leq 1$, for women.[8] The efficiency units of labor for each individual is linearly proportional to his or her pre-marital investment, ω_i. Thus, a married couple can generate $\psi\omega_f + \omega_m$ of household income (of which $\psi\omega_f$ is generated by the wife and ω_m by the husband).

3.1. Preferences and Budget Constraints

Individual preferences are defined over first- and second-period consumption. For an individual i, let c_i, $i = f, m$, denote second-period consumption. We assume that preferences of i are represented by the following inter-temporal utility function:

$$U_i \ = \ v(y_i - \omega_i) \ + \ u(c_i); \qquad i \ = \ f, \ m. \tag{1}$$

where the function U_i, $i = f, m$, satisfies the neo-classical Inada conditions. That is, $\forall$ ω_i, $c_i \in [0, y_i)$, u', $v' > 0$, u'', $v'' < 0$, $\lim_{(y_i - \omega_i)\to 0} v' = \lim_{c_i \to 0} u' = \infty$ and $\lim_{(y_i - \omega_i)\to \infty}$ $v' = \lim_{c_i \to \infty} u' = 0$.

The young augment their future incomes by maximizing (1) subject to the constraint $\omega_i \leq y_i$, $i = f, m$.[9] When they get older, individuals can either remain single or match in the marriage market assortatively according to their wage incomes ω_i. For a given income level y_i, let ω_i^s, $i = f, m$, denote the optimal investment level of an individual who remains single during adulthood. Thus, for a given level of income y_i,

$$\omega_i^s \ = \ \arg\max \ U_i \ = \ \begin{cases} v(y_f - \omega_f) \ + \ u(\psi\omega_f) & \text{if } i \ = \ f \ , \\ v(y_m - \omega_m) \ + \ u(\omega_m) & \text{if } i \ = \ m \ . \end{cases} \tag{2}$$

The optimal investment levels of single men and women respectively satisfy

[8] We allow the wage rates paid to men and women to differ in order to explore the effects of changes in the gender wage gap on household dynamics.

[9] For simplicity, we assume that there are only pecuniary costs of education, although extending the model to allow time costs would not alter our main results.

$$u'(\omega_m^s) = v'(y_m - \omega_m^s) \quad \text{and} \quad \psi u'(\psi\omega_f^s) = v'(y_f - \omega_f^s) \ . \tag{3}$$

Let U_i^s, $i = f, m$, denote the utility levels associated with optimal investment levels of singles,

$$U_i^s \ = \ \begin{cases} v(y_f - \omega_f^s) \ + \ u(\psi\omega_f^s) & \text{if } i \ = \ f \ , \\ v(y_m - \omega_m^s) \ + \ u(\omega_m^s) & \text{if } i \ = \ m \ . \end{cases} \tag{4}$$

It is straightforward to show that, $\forall \, \omega_m^s, \, \omega_f^s \in [0, \, y_i]$, $\partial U_f^s / \partial \psi > 0$ and that $\partial U_i^s / \partial y_i > 0$, $i = f, \, m$.

Married couples allocate their household income according to an intra-marital sharing rule. Denoting the wives' share in intra-household allocations by θ, $\theta \in [0,1]$, we have

$$c_f \ = \ \theta(\psi\omega_f + \omega_m) \qquad \text{and} \qquad c_m \ = \ (1-\theta)(\psi\omega_f + \omega_m) \tag{5}$$

Note that, by construction, the marital allocation rule satisfies the familial budget constraint.

$$c_f \ + \ c_m \ = \ \psi\omega_f + \omega_m \ . \tag{6}$$

3.2. Assortative Matching and Conditional Efficiency

Given the intra-marital allocation rule, a married individual's utility function can be re-written as follows:

$$U_i \ = \begin{cases} v(y_f - \omega_f) \ + \ u[\theta(\psi\omega_f + \omega_m)] & \text{if } i \ = \ f \\ v(y_m - \omega_m) \ + \ u[(1-\theta)(\psi\omega_f + \omega_m)] & \text{if } i \ = \ m \end{cases} \tag{7}$$

We consider the equilibrium outcome in marriage as follows. First, we follow Peters and Siow and demonstrate that, in an assortative marriage market, individuals' pre-marital investments are efficient conditional on the intra-marital sharing rule. Next, we establish the properties of the marital contract curve (the combinations of spousal pre-marital investments that are efficient conditional on the given intra-marital sharing rule). Finally, we identify the points on the marital contract curve that are sustainable in the marriage market.

Let $g(\omega_m)$ represent the pre-marital investment of the wife that each husband in M expects to match with as a result of a pre-marital investment of ω_m. If individuals' expectations are realized, then $g^{-1}(\omega_f)$ will represent the pre-marital investment of the husband that each wife in F expects to match with due to an investment of ω_f. Under assortative matching, higher pre-marital investment attracts a higher income spouse. Hence, in such a market, $g(\omega_m)$ is non-decreasing in ω_m.

Now let us formally define the *rational expectations equilibrium.* The return function $g(\omega_m)$ is a rational expectations equilibrium if there exist pre-marital investment strategies $\sigma_f(y_m)$ and $\sigma_m(y_m)$ for individuals in F and M, respectively, such that

1. $\forall\ y_m \in (0, Y]$, $\sigma_m(y_m) = \arg\max \{v(y_m - x) + u[(1-\theta)(\psi g(x) + x)]\}$ and $\sigma_f(y_m)$ $= \arg\max \{v(y_f - x) + u[\theta(\psi x + g^{-1}(x))]\}$;

2. $G\{y_m : \sigma_m(y_m) \geq \omega_m\} = H\{y_f : \sigma_f(y_m) \geq g(\omega_m)\}$.

Part 1 of the definition indicates that all individuals choose their pre-marital investments optimally given the return function $g(\omega_m)$ and conditional on a given intra-marital sharing rule θ. Part 2 of the definition is the marriage market-clearing condition. It guarantees that, by assortative matching, each husband that invests ω_m or more will be able to find a spouse who invests $g(\omega_m)$ or more.

In Figure 1, we depict two possible rational expectations equilibria. The pre-marital investment levels of the women are shown on the horizontal axis and those of the men are on the vertical axis. The two upward-sloping dashed lines represent two different equilibrium matching functions $g^{-1}(\omega_m)$. The upward convex curves are the indifference curves of the wives and those that are convex downward are the indifference curves of the husbands.[10] Due to the assortative matching equilibrium, couples for whom the husband has higher initial endowment, y_m, invest more than those for whom the husband has a lower initial endowment. If distributional factors favor men more than they do women, then the equilibrium matching function will tend to shift to the right leading to more investment by the wives and less by the husbands.

[Figure 1 about here.]

In what follows, we derive the optimal investment levels and explore how they compare with the unconditionally efficient outcomes. Consider a couple for whom the husband's initial endowment equals y. Due to assortative matching, a husband with an income of y will match with a wife whose income is $\alpha(y)$. Rational expectations equilibrium requires that it be optimal for a husband with an endowment of $\overline{y}_m$ to invest $\sigma_m(\overline{y}_m)$ rather than $\sigma_m(y'_m)$ as he would if his income were y'_m instead of $\overline{y}_m$. And similarly, the wife who matches with this husband and who has an endowment of $\overline{y}_f$ chooses to invest $\sigma_f(\overline{y}_f)$ rather than $\sigma_f(y'_f)$. In equilibrium with assortative matching, each spouse's level of utility can then be written as a function of the income type whose investment level they choose (y_i):

$$U_i = \begin{cases} v[\overline{y}_f - \sigma_f(y_f)] \ + \ u\{\theta[\psi\sigma_f(y_f) + \sigma_m[\alpha^{-1}(y_f)]\} & \text{if } i = f \\ v[\overline{y}_m - \sigma_m(y_m)] \ + \ u\{(1-\theta)[\psi\sigma_f[\alpha(y_m)] + \sigma_m(y_m)]\} & \text{if } i = m \end{cases} \tag{8}$$

[10] The assumptions we impose on the utility function U_i help to ensure that the indifference curves are single crossing and strictly convex. Together these properties guarantee the existence and uniqueness of the assortative matching equilibrium. More precisely, using the implicit function theorem, we can show that along an indifference curve for the wives, where $v(y-\omega_f) + u[\theta(\psi\omega_f + \omega_m)] = \bar{U}_f$, $\partial\omega_m/\partial\omega_f = (v'_f/\theta u'_f) - \psi$ and $\partial^2\omega_m/\partial\omega_f^2 = (-1/\theta)\ \{(v''_f u'_f + v'_f u''_f)\ /\ (u'_f)^2\}$. The second partial derivative is strictly positive which verifies that the indifference curves are strictly convex. The first partial derivative is also positive in the vicinity of pre-marital investments that are given by the second equality in (3). Moreover, $\forall\ y' > y$, it follows that $\partial\omega_m/\partial\omega_f = \{v'_f(y'-\omega_f)\ /\ \theta u'_f[\theta\ (\psi\omega_f + \omega_m)]\} - \psi < \partial\omega_m/\partial\omega_f = \{v'_f(y-\omega_f)\ /\ \theta u'_f[\theta\ (\psi\omega_f + \omega_m)]\} - \psi$. Thus, the indifference curves of two women with different levels of income y and y' are single crossing. Following the same procedure will establish the same properties for the indifference curves of husbands as well. And together with the strict convexity of the indifference curves, this ensures the existence and uniqueness of the assortative marital equilibrium for a given reservation utility.

In equilibrium, the level of y_i that maximizes the utility levels given by (8) must be equal to the initial endowment $\overline{y}_i$. Let u_i' and v_i', $i = f$, m, respectively denote the marginal utilities of consumption in the first and second periods of life. Differentiating (8) with respect to y_i, and then setting $\overline{y}_i = y_i$ yields the following set of first order conditions which together identify the set of potential rational expectations equilibria:

$$-v_f'\sigma_f' \ + \ \psi\theta u_f'\sigma_f' \ = \ -\frac{\theta u_f'\sigma_m'}{\alpha'} \tag{9}$$

and,

$$-\psi(1-\theta)u_m'\sigma_f'\alpha' \ = \ -v_m'\sigma_m' \ + \ (1-\theta)u_m'\sigma_m' \ . \tag{10}$$

Dividing (9) through by (10) and cross-multiplying, we find that the sum of the spouses' marginal utilities from pre-marital investments relative to their cost equals one. This demonstrates efficiency of the investments conditional on the intra-marital sharing rule.

$$\frac{\psi\theta u_f'}{v_f'} \ + \ \frac{(1-\theta)u_m'}{v_m'} \ = \ 1 \tag{11}$$

The differential equation in (11) implicitly defines the set of allowable equilibrium matching functions $g(w_m)$. Different initial conditions (threat points) map into different unique equilibria. Put another way, for a given income match, equation (11) traces out the marital contract curve (set of potential equilibrium investment levels). Figure 2 depicts two marital contract curves drawn for two different given intra-marital sharing rules at a given income match. As shown, a higher level of θ raises the intra-allocations of the wife and rotates the contract curve clockwise.

[Figure 2 about here.]

We now establish the properties of pre-marital investments and intra-household allocations that are unconditionally efficient.

4. Pareto Efficient Pre-Marital Investments and Intra-Household Allocations

For the above couple, the unconditionally efficient pre-marital investments and intra-household allocations can be determined by solving the following maximization problem:

$$\max_{\{\omega f,\, \omega_m,\, c_f,\, c_m\}} \; v(y_m - \omega_m) \; + \; u(c_m) \tag{12}$$

subject to:

$$v[\alpha(y_m) - \omega_f] \; + \; u(c_f) \; \geq \; \bar{U}_f \tag{13}$$

$$c_f \; + \; c_m \; \leq \; \psi\omega_f \; + \; \omega_m \; , \tag{14}$$

and,

$$\omega_m \; \leq \; y_m \quad \text{and} \quad \omega_f \; \leq \; \alpha(y_m) \; . \tag{15}$$

The first-order conditions for this problem yield

$$u'_m \; = \; v'_m \quad \text{and} \quad \psi u'_f \; = \; v'_f \; . \tag{16}$$

When combined with the familial budget constraint, equation (14), and the wife's utility constraint, equation (13), these equations tie down the unique unconditionally efficient allocation associated with the wife attaining utility equal to $\bar{U}_f$. The solution to (12) also yields $u'_m/u'_f = \lambda$, where λ is the shadow price of the wife's reservation level of utility evaluated at $\bar{U}_f$. Further, dividing the first equation in (16) by the second one yields

$$\frac{u'_m}{v'_m} \; = \; \frac{\psi u'_f}{v'_f}. \tag{17}$$

Along the Pareto efficient frontier, equation (17) equates the relative marginal utility of pre-marital investments to its disutility.

5. Matching, Pre-Marital Investments, and Efficiency

We now examine the properties of intra-household sharing rules that are both *unconditionally efficient* and a *sustainable rational expectations equilibria* in the marriage market. Note that such a sharing rule:

1. must satisfy equation (4) in order to satisfy *marital sustainability*;

2. must satisfy equation (11) in order to be a *rational expectations equilibria*;

3. must satisfy equations (13), (14), (16) and (17) because it is *unconditionally efficient*;

4. must be *invariant* across endowment matches.

Consider a sharing rule which, for both spouses, yields the levels of pre-marital investment and consumption associated with being single. If such a sharing rule exists, it will yield an intra-marital share $\check{\theta}$. By construction, $c_f = \psi\omega_f^s = \check{\theta}(\psi\omega_f^s + \omega_m^s)$ and $c_m = \omega_m^s = (1 - \check{\theta})\ (\psi\omega_f^s + \omega_m^s)$. Thus, $\check{\theta}$ must equal $\psi\omega_f^s/(\psi\omega_f^s + \omega_m^s)$. This sharing rule and the associated investment levels (ω_m^s and ω_f^s) satisfy equation (11) and are therefore a rational expectations equilibrium in the marriage market. Further, these allocations satisfy equations (13), (14), (16) and (17) and the associated allocation is therefore unconditionally efficient. To establish point 4, we need to impose additional structure on preferences.[11] If U_i, $i = f, m$, is homothetic, the ratio $u'_m/u'_f = u'[(1 - \check{\theta})(\psi\omega_f^s + \omega_m^s)]\ /\ u'[\check{\theta}(\psi\omega_f^s + \omega_m^s)]$ is H^0 in its arguments.[12] Thus, $\forall\ y_f,\ y_m \in (0, Y]$, $\check{\theta}$ is such that $u'_m/u'_f = \lambda$ and point 4 is satisfied.[13]

[11] Note that in the general case where marriage generates a surplus, this assumption is not necessary.

[12] For example, suppose that equation (1) is given by the specific functional form $(y - \omega_i)^\eta + (c_i)^\eta$. Then U_i is H^η and u'_i is $H^{\eta-1}$. Thus, $\forall\ \eta \geq 1$, the relative marginal utilities are constant and invariant in the endowments y.

[13] Note that, if $F \neq M$, there may exist assortative marital matching equilibria with different levels of relative incomes across the assortartive order. Because the unique and sustainable sharing rule is a function of relative spousal incomes, in such equilibria a constant sharing rule that is invariant to the relative spousal incomes cannot be sustained. However, for $F \neq M$, there always exists a maritally sustainable assortative matching equilibrium in which, $\forall\ y_f,\ y_m \in (0, Y]$, the number of individuals who belong to the excess-supply sex remains single and the relative spousal incomes are held constant. Hence, such an equlibrium will be consistent with a constant sharing rule and it will satisfy point 4.

It is also the case that no other sharing rule $\theta \neq \check{\theta}$ exists that is sustainable in the assortative marriage markets. The strictly monotonic nature of the utility functions in (1) suggest that, $\forall\ \theta \neq \check{\theta}$, one of the spouses is worse-off in marriage than he or she is when single and therefore the marital equilibrium cannot be sustained. In particular, it is straightforward to verify that, $\forall\ \theta < \check{\theta}$, the wife is worse off in marriage than she is single, and $\forall\ \theta > \check{\theta}$, the husband is worse off in marriage than he is single. Thus, $\forall\ \theta \neq \check{\theta}$, either the women or the men would rather remain single, and hence, point 1 cannot be satisfied.

In Figure 3, we super-impose the loci of the Pareto efficient frontier and the reservation utilities on the marital contract curve, the latter which was originally depicted in Figure 2. Given $\theta = \theta_1$, only point A, which lies on the marital contract curve associated with sharing rule θ_1, is unconditionally efficient and sustainable in the marriage market. Any other point like B on the marital contract curve is conditionally efficient but not maritally sustainable. However, associated with sharing rule θ_2 there exists a point C that is also unconditionally efficient and yields a higher intra-marital share, $\theta_2 > \theta_1$, for the wives. This point can be sustainable in the marriage market only with a lower gender wage gap.

[Figure 3 about here.]

6. The Collective Household with a Marital Surplus

So far, we have shown that only the efficient sharing rule, which is solely a function of the gender wage gap, is sustainable as a marriage market equilibrium. This is due to the fact that marriage does not generate a surplus that can enable transfers between the two spouses. In this section, we augment the collective household model to include a marital surplus and revisit the issue of marital sustainability of conditionally efficient and unconditionally efficient sharing rules.

For simplicity, suppose that the state of being married increases individuals' utility during marriage by an amount k, $k > 1$.[14] Individual utility originally given by equation (7) is now given by

$$U_i = \begin{cases} v(y_f - \omega_f) \ + \ u[\theta(\psi\omega_f + \omega_m)] \ + \ k & \text{if } i \ = \ f \ , \\ v(y_m - \omega_m) \ + \ u[(1-\theta)(\psi\omega_f + \omega_m)] \ + \ k & \text{if } i \ = \ m \ . \end{cases} \tag{18}$$

Since the marital surplus accrues to spouses only in marriage, the utility of single individuals are still given by equation (1). Thus, the optimal investment levels of single men and women remain unchanged and continue to satisfy equation (3).

Following once again the couple discussed above and the methodology we employed in Section 3.2, we can re-write this couple's utility in marriage as:

$$U_i = \begin{cases} v[\overline{y}_f - \sigma_f(y_f)] \ + \ u\{\theta[\psi\sigma_f(y_f) + \sigma_m[\alpha^{-1}(y_f)]\} + k & \text{if } i \ = \ f \\ v[\overline{y}_m - \sigma_m(y_m)] \ + \ u\{(1-\theta)[\psi\sigma_f[\alpha(y_m)] + \sigma_m(y_m)]\} + k & \text{if } i \ = \ m \end{cases} \tag{19}$$

Given equation (19), the first-order conditions originally given by (9) and (10) and the locus of marital outcomes given by (11) still apply.

Turning our attention to the unconditionally efficient pre-marital investments and intra-household allocations, we modify the maximand in (12) and the subsequent constraint in (13) to read:

$$\max_{\{\omega f,\, \omega_m,\, c_f,\, c_m\}} \ v(y_m - \omega_m) \ + \ u(c_m) \ + \ k \tag{20}$$

and,

$$v[\alpha(y_m) - \omega_f] \ + \ u(c_f) \ + \ k \geq \ \bar{U}_f \ \geq \ U_f^s \tag{21}$$

[14] Even though the exposition gets more complicated, the main results presented here go through unaltered if marriage does not generate a direct surplus but involves the production of marital public goods.

As before, we get four first-order conditions which together yield equations (16) and (17).

The difference now arises from the fact that marriage generates a surplus that in effect widens the set of spousal pre-marital investments that are maritally sustainable. That is, the presence of a marital surplus leads to a set of pre-marital investments and allocations that were not sustainable under the model of equation (7) to yield utility above the reservation levels.[15] For all potential spouses, distributional factors such as the sex ratios in the markets for marriage and the degree to which legislature pertaining to marriage and divorce are favorable to either sex, will help determine the set of sustainable allocations.

In Figure 4, we depict how the set of sustainable outcomes is affected when marriage produces a surplus. As shown, compared to the case without a marital surplus, husbands' iso-utility locus, $\bar{U}_m$, shifts upward and to the left and that of the wives', $\bar{U}_f$, shifts downward and to the right. Consequently, all combinations of pre-marital investments and intra-household allocations that lie on the continuum $[B, C]$ are now sustainable. Thus, when $F = M$ and $G(N) = H(N) \ \forall \ N$, distributional factors other than the sex ratio will influence the location of the marital matching function, $g(\omega_m)$, and which of the continuum of equilibria on the segment $[B,\ C]$ is sustained as the matching equilibrium. When $F = M$ and $G(N) = H(N) \ \forall \ N$, the unique equilibrium sharing rule $\check{\theta}$ (derived under the no surplus model that is both unconditionally efficient and maritally sustainable) can lead to maritally sustainable and unconditionally efficient outcomes for all couples along the assortative order. By construction and location, such a point will satisfy points 1 through 3 listed above. Under the homotheticity of U_i, $i = f,\ m$, it will be invariant in couples' aggregate endowments $y_m + \alpha(y_m)$, and thus, will

[15] The same mechanism would be operative even if the production of the public good was not costless and it required the allocation of household resources. In that case, the family budget constraint given originally by equation (6) would be modified as $c_f + c_m + k = \psi\omega_f + \omega_m$. This would prompt marrried couples to allocate some of their joint resources to the production of k and lower their levels of private consumption c_f and c_m below those of single individuals who cannot by themselves produce the public good k. At the optimum, this would then imply that married individuals' marginal utility from private and public consumption would exceed singles' marginal utility from private consumption only. Hence, it would still be the case that the reservation utility levels under the collective model with public goods is strictly less than that under the model without public goods, i.e., $\bar{U}_i < U_i^s$, $i = f,\ m$.

also satisfy point 4.

Nonetheless, when $F = M$ and $G(N) = H(N)\ \forall\ N$, there exists a different continuum of outcomes for every different sharing rule, only one of which is unconditionally efficient. And, it is possible that the unique and efficient equilibrium associated with any particular sharing rule is not sustainable as a marital outcome. For instance, in Figure 4 we depict the marital contract curves for two different sharing rules θ_1 and θ_2. As shown, the efficient outcome associated with the sharing rule θ_1, which lies at point A, is maritally sustainable. So are the continuum of inefficient outcomes associated with θ_1 that lie in the segments $[B,\ C]$. In contrast, the efficient outcome for the sharing rule θ_2, that lies at point F, is not maritally sustainable but the continuum of inefficient allocations in the segment $[D,\ E]$ are.

[Figure 4 about here.]

As Becker originally identified, differential sex ratios in the marriage market where $F \neq M$ will lead to corner solutions in which $|F - M|$ measure of individuals with the lowest endowment levels y remain single. For example, if $F > M$, then the excess $F - M$ measure of women in the marriage markets will generate a competition among women that will leave all women with the lowest endowments y indifferent between getting married and remaining single, and enable men who marry them to extract all of the marital surplus to their wives (which equals k). Our model suggests that such a competition will not only transfer all of the marital surplus to the husbands at the lowest assortative rank, but also lead to the emeregence of the sharing rule that supports the unique and unconditionally efficient outcome for the lowest-income match. This outcome is depicted in Figure 5. When $F > M$, marriage market competition among women lowers the marital utility of all wives' with the lowest endowment to their reservation level of U_f^s. Moreover, we would generally not expect the dotted marital contract curve that yields point C to emerge because the sharing rule associated with the solid concave line that yields point D as the outcome generates strictly higher utility for the husbands without lowering that of the wives.

In general, when $F > M$, it is not possible to ascertain whether the unique sharing rule that emerges as the unconditionally efficient marital outcome for couples in the lowest assortative rank is also the unconditionally efficient and maritally sustainable sharing rule for all couples higher up in the assortative order. Nonetheless, we can make the following observation. Under the homotheticity of U_i, $i = f, m$, it is straightforward to establish that in restrictive cases where all assortative matches lead to constant relative endowments, i.e. $\forall\ y_m \in (0,\ Y]$, $\alpha(y_m) = \chi y_m$, $\chi \lesseqqgtr 1$, the sharing rule that leads to unconditional efficiency for the lowest-endowment couples is sustainable and generates unconditional efficiency for all couples.[16]

In other cases, an excess supply of one sex over the other will lead to the spousal endowment gap to narrow as the household rank in the assortative order increases. For instance, as long as $F > M$ and $\exists\ N > 0$, such that $G(N) \neq H(N)$, the endowments of husbands will be less than those of their wives and the husbands' relative endowments will be lower among low-endowment couples. The narrowing of the endowment gap along the assortative order then introduces the possibility that the sharing rule that induces the unconditional efficiency of outcomes for the lowest-endowment couples may not do so–and may not even be sustainable–for other couples higher up in the assortative order. The reason is simply as follows: On the one hand, for couples higher up in the assortative order, the endowment gap between the husband and the wife is smaller than those below them in the ranks. Ceteris paribus, this would induce the efficient share of wives in total household consumption, θ, to be lower the higher is the assortative rank of a couple. On the other hand, while none of the wives in the lowest ranks extract any marital surplus, wives higher up in the assortative order capture a fraction of the marital surplus that is commensurate with their endowment ranks.[17] Ceteris paribus, this would induce the

[16] One specific example is when the endowment distributions of the two genders is identical, except for the inclusion of an additional number of women with the lowest possible endowment.

[17] When $F > M$ and all assortative matches do not involve constant relative endowments, there will exist husbands with identical endowments who marry wives with different levels of endowments. Thus, in the assortative matching equilibrium, it has to be the case that such husbands are indifferent between marrying either the high-endowment or the low-endowment wife. For heuristic purposes, take a husband with the lowest endowment of $\tilde{y}$ who can marry a wife with an endowment of $\tilde{y}$ or another wife with a slightly higher endowment of $\bar{y}$, $\bar{y} = \tilde{y} + \varepsilon$, $\varepsilon > 0$. Such a husband's utility should be invariant to which of the two types of wives he can marry. Together with the fact that, when $F > M$, women with the lowest endowments, $\tilde{y}$, receive their reservation utility level, $\bar{U}_f$, it has to be the case that women with

efficient share of wives in total household consumption, θ, to be higher the higher is the assortative rank of a couple. And, unless these two opposite forces fully offset each other for all couples in the assortative order, the sharing rule that yields the unconditionally efficient outcomes for the lowest-endowment couples, will not do so for other couples with higher endowments.

[Figure 5 about here.]

Modifying the collective model to incorporate a marital surplus generates another important implication. When $F = M$, the set of inefficient sharing rules that are sustainable in the marriage market are associated with the segments of the marital contract curve $[B, A)$ and $(A, C]$. Reductions in the gender wage gap–via increases in ψ–generate two potentially offsetting effects on the size of this set: First, the wives' reservation level of utility increase while the husbands' reservation level of utility does not change. Thus the set of maritally sustainable outcomes shrinks. Second, a closing of the gender wage gap via an increase in ψ also raises the level of household income. As a result, the set of inefficient sharing rules that are not efficient but that are acceptable to both spouses grows as the $\bar{U}_f$ and $\bar{U}_m$ loci move farther apart.

In Figure 6.a, we demonstrate how the reservation levels of utility adjust due to both these effects. The dashed arrows show that, holding constant the household level of income, wives' reservation level of utility rises and the set of maritally sustainable pre-marital investments shrinks. The solid arrows depict how a higher ψ generates a higher level of household income, lowers both spouses' reservation utility, and enlarges the set of maritally sustainable pre-marital investments. The marital contract curve is also influenced by the two above-mentioned effects, where a higher ψ induces higher pre-marital investments by the wives holding constant that of the husbands'. This suggests that the marital contract curve shifts to the right. In Figure 6.b, we show how the shift in the marital contract curve, holding constant the reservation levels of utility, can influence the set of maritally sustainable sharing rules. For a lower ψ this set is given

the higher endowment level, $\bar{y}$, get more than their resevation utility level, $\bar{U}_f$.

by the segment $[B,\ C]$ and for a higher ψ it becomes $[D,\ E]$. Finally, in Figure 6.c, we combine all these changes to depict how the set of maritally sustainable pre-marital investments changes from $[B,\ C]$ to $[D,\ E]$. As shown, the net effect is ambiguous.

[Figures 6.a - 6.c about here.]

7. Conclusion

In recent years the "collective" model of the household, in which individual members of the family are treated as the core decision-makers and a sharing rule generates efficient intra-household allocations, has emerged as the most promising framework for understanding household behavior. These models suggest that relative spousal incomes influence household allocations but they do not account for the fact that the household income can be determined at least in part by decisions individuals make prior to marriage. In models where spousal incomes are pure public goods, existing work has shown that such decisions can lead to inefficient pre-marital choices and intra-household allocations and further that the efficiency of pre-marital investments can be restored as a result of spousal competition in the markets for marriage. The collective household models rely on the efficiency of intra-household allocations but they do not address how pre-marital investments and marital matching can influence such allocations. However, given their rising prominence in analyzing household behavior, it is important to do so.

In this paper, we present the first attempt to extend to collective household model to cover pre-marital investments and matching in the marriage markets. Our main goal is to examine the implications of pre-marital investments in the collective marital setting. To that end, we present a microeconomic model of the household where marital matching is assortative and a sharing rule forms the basis of intra-household allocations.

We reach several important conclusions. First, we find that the unconditionally efficient outcomes, in which both pre-marital investments and intra-household allocations are Pareto efficient, can be supported by sharing rules that are consistent with the collective framework. Second, we find that when marriage does not involve a marital surplus, only a single outcome, which is unconditionally efficient, is sustainable. In that case, the uniquely determined efficient sharing rule is only a function of the gender wage gap. Distributional factors such as marriage and divorce legislation, and the sex ratios in the marriage market cannot influence the unique and efficient sharing rule that can sustain the equilibrium in the marriage market. Third, when marriage generates a surplus, we demonstrate that there exists a continuum of maritally sustainable sharing rules. Associated with each of these sharing rules is a continuum of equilibria only one of which is unconditionally efficient. When the numbers and distributions of men and women in the marriage market do not differ, the efficient equilibrium associated with any particular sharing rule may or may not be sustainable in the marriage market. When the numbers and distributions of men and women in the marriage market differ, however, a corner solution emerges where, for the lowest income marital matches, one of the spouses receives the reservation level of utility. At least for couples with lowest household incomes, we argue that the sharing rule one should expect to emerge in such cases also supports the unconditionally efficient equilibrium. Finally, our findings imply that reductions in the gender wage gap influence the set of inefficient and maritally sustainable sharing rules in two potentially offsetting ways: For a given level of household income, a smaller gender wage gap raises the reservation level of utility above which women choose to get married, but holding constant husbands' wages, a smaller gender wage gap also raises household income. Thus, the net effect of changes in the gender wage gap on the set of maritally sustainable sharing rules is ambiguous.

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Figure 1: The Rational Expectations Equilibrium

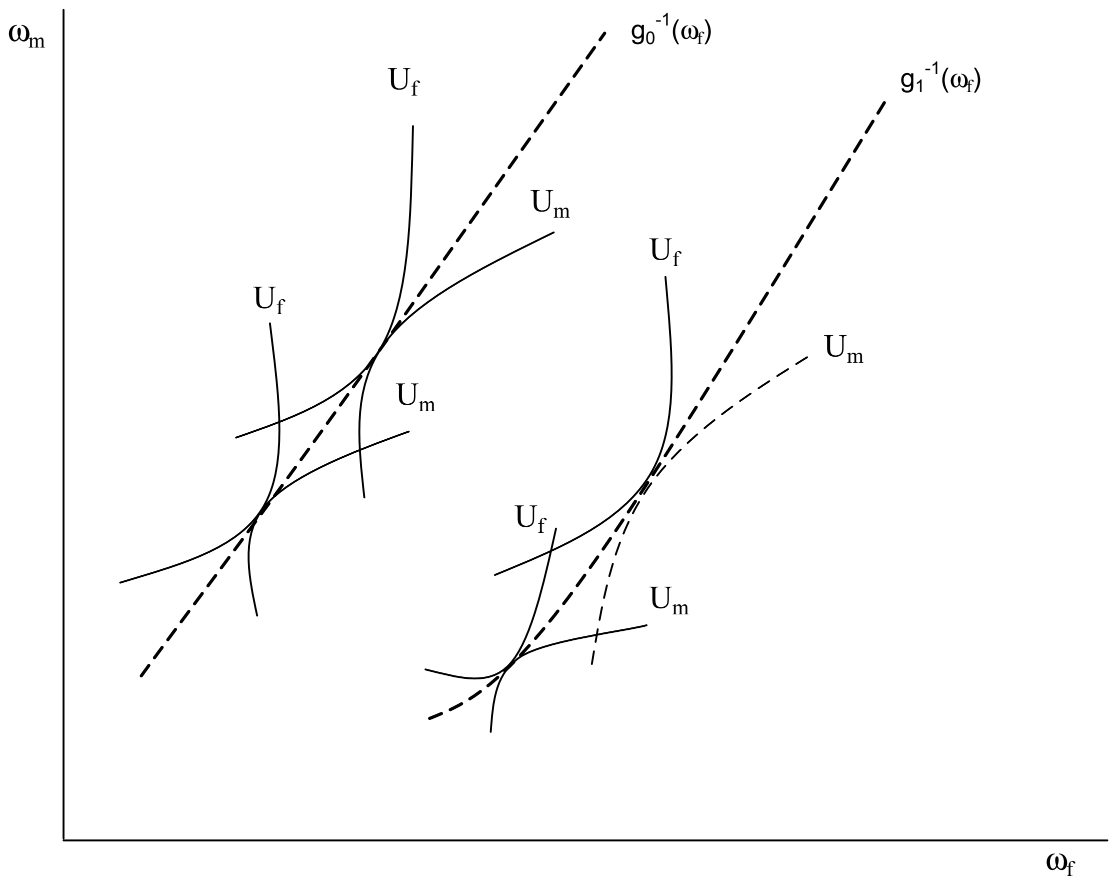

Figure 2: The Marital Contract Curve

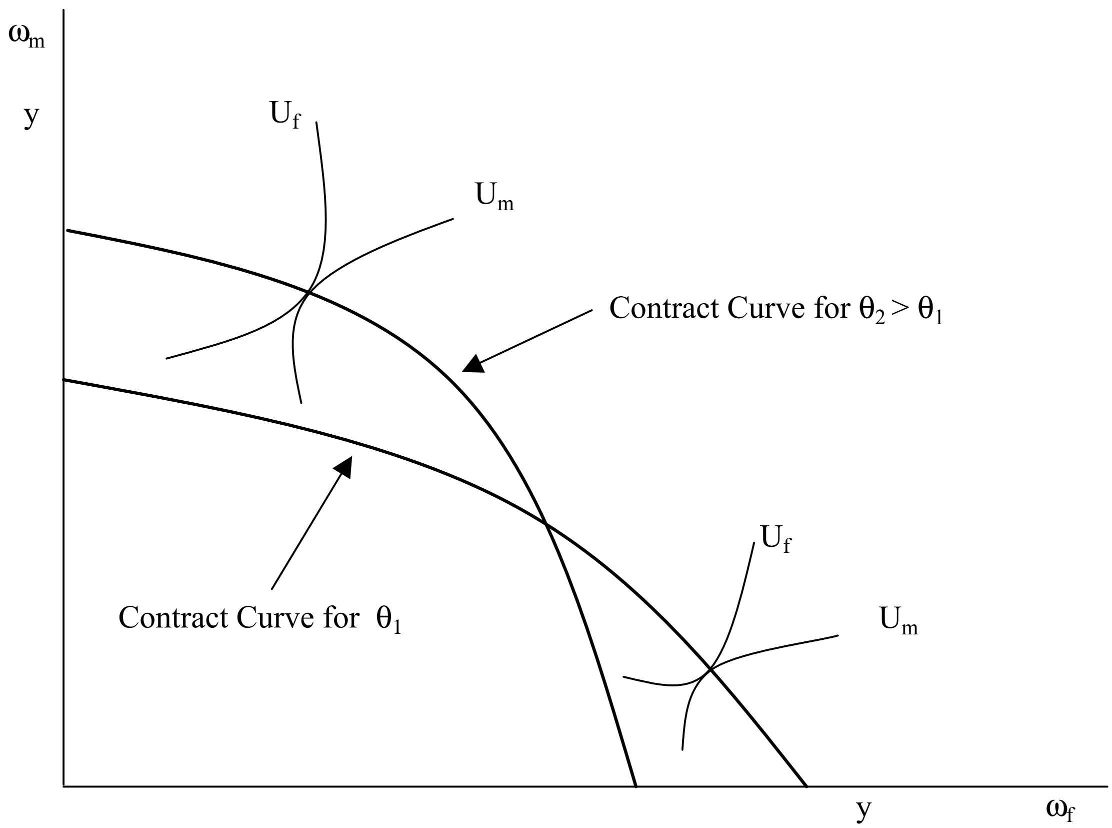

Figure 3: The Marital Contract Curve and the Efficient Frontier

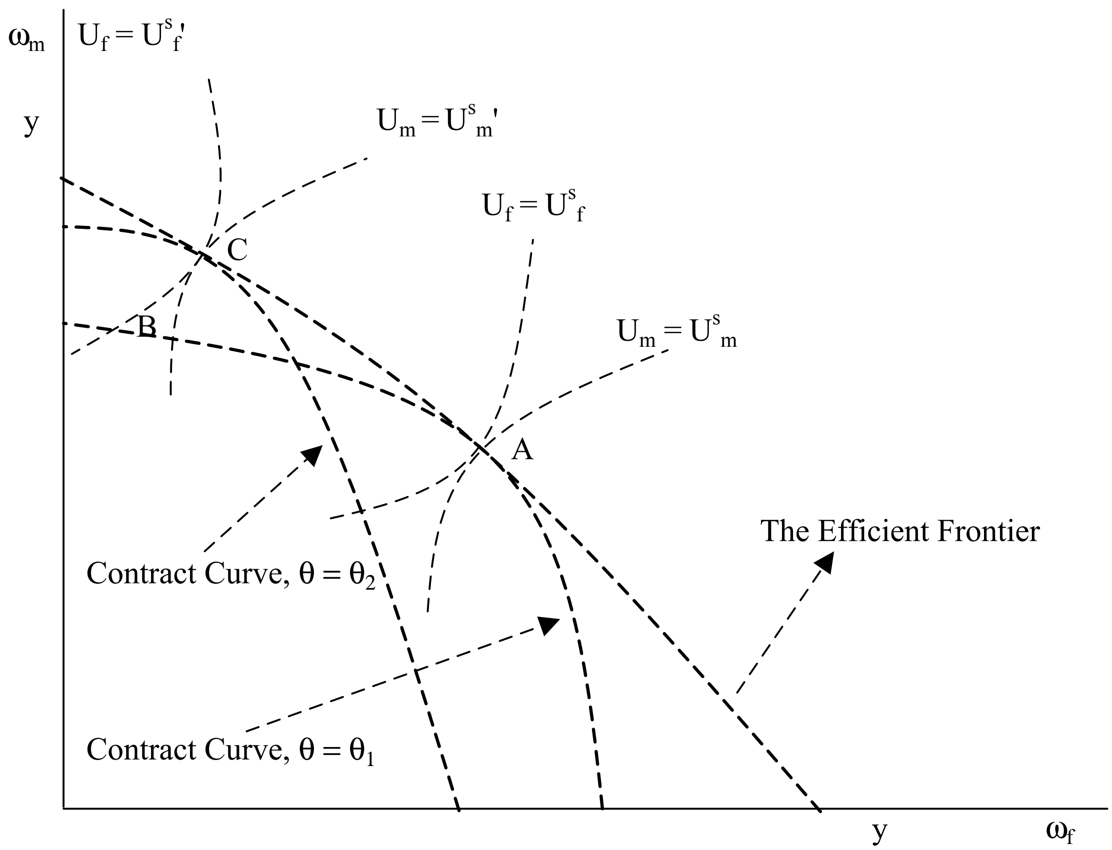

Figure 4: The Marital Contract Curve and the Efficient Frontier (with a Marital Surplus, k > 1)

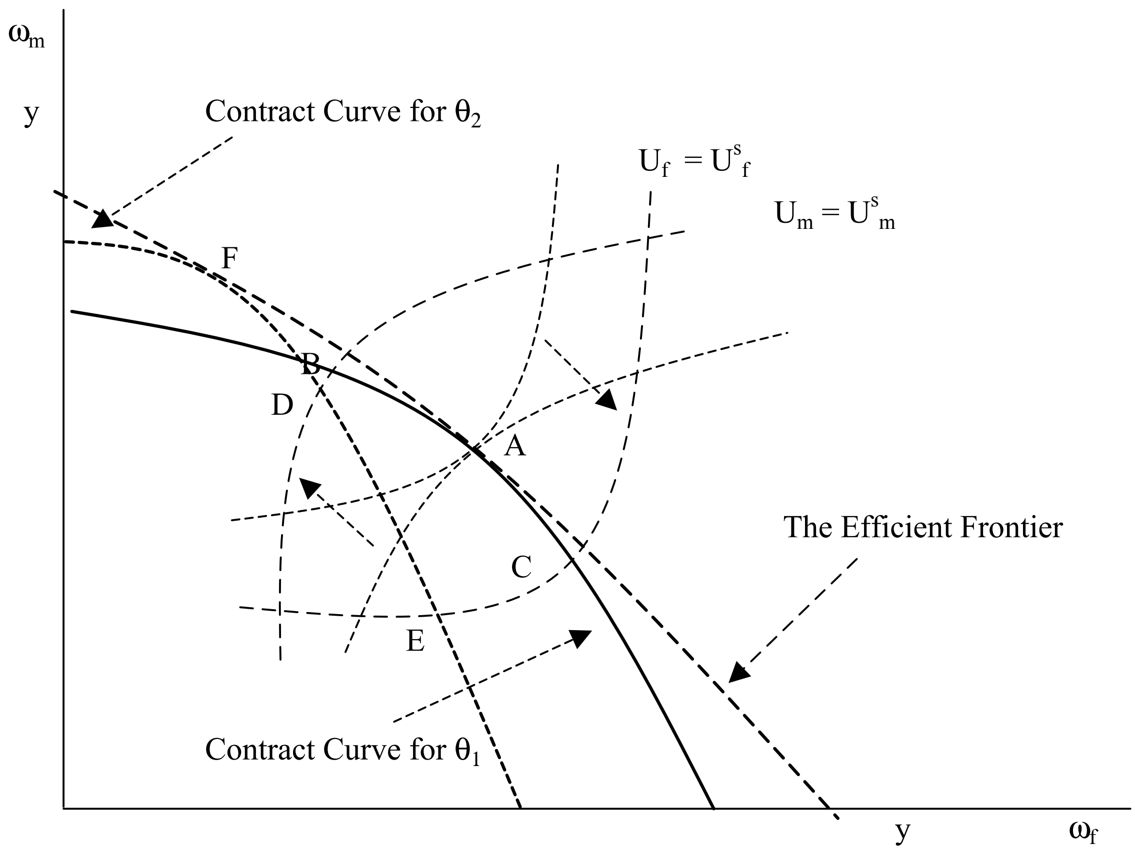

Figure 5: The Efficient Corner Solutions (with a Marital Surplus, k > 1)

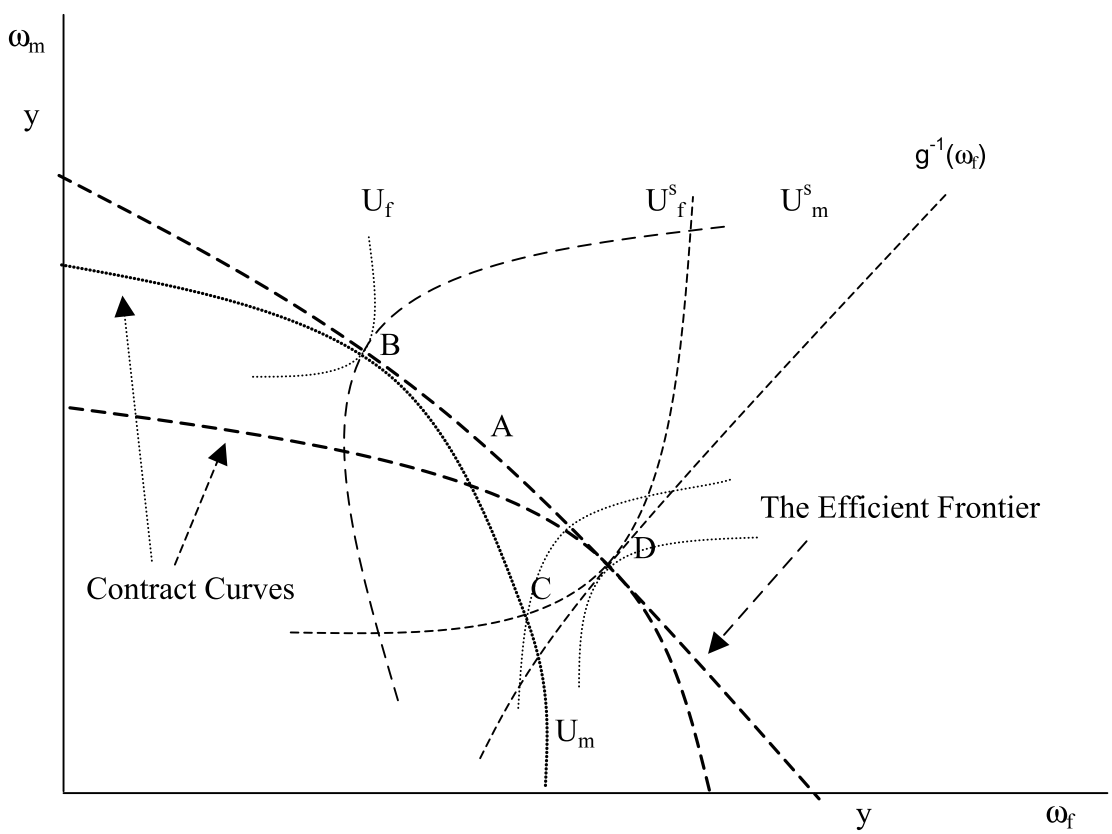

Figure 6.a: The Effect of a Smaller Gender Wage Gap (with a Marital Surplus, k > 1)

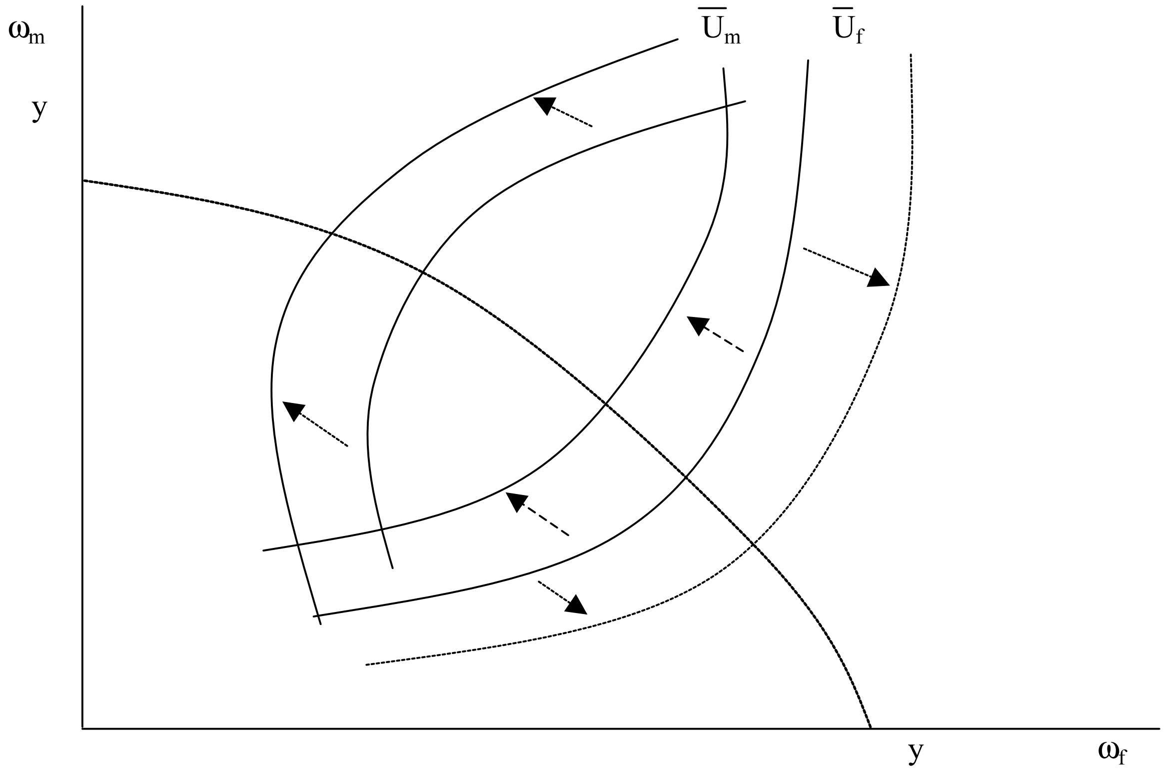

Figure 6.b:

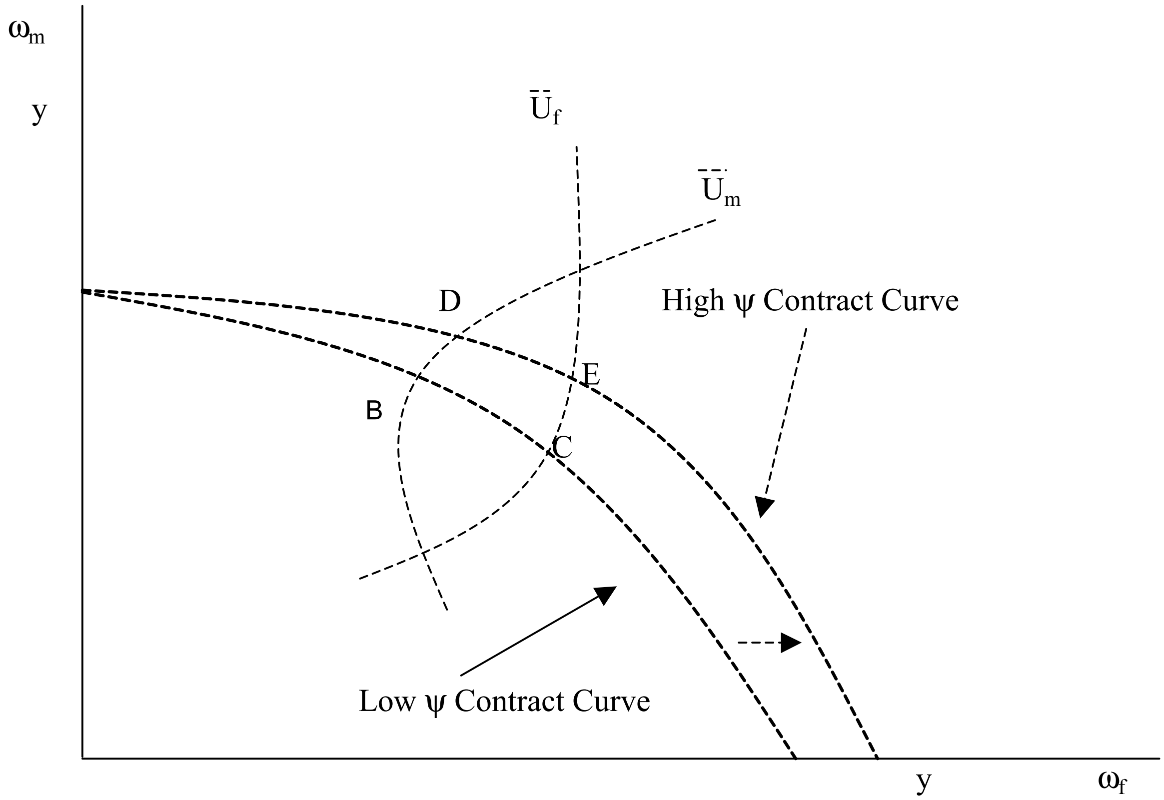

Figure 6.c: The Effect of a Change in Distributional Factors (with a Marital Surplus, k > 1)