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How much persistence should sticky-price models generate to match U.S. data?

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How much persistence should sticky-price models generate to match US data?

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Abstract

A number of recent papers find that sticky-price models fail to explain the persistence of output and inflation. We argue that this failure is partially attributable to low frequency fluctuations present in post-war US data, but absent from sticky-price models.

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1. Introduction

A number of recent papers argue that sticky-price monetary business cycle models with explicit microfoundations fail to explain the persistence of output and inflation in post-war US data. For example, Chari, Kehoe, and McGrattan (2000) conclude that monetary business cycle models based on Taylor's (1980) overlapping contracts require counterfactually long contracts to explain persistent output fluctuations. Nelson (1998) concludes that existing monetary business cycle models fail to explain persistent inflation changes.

Our objective is to show that the observed persistence of output and inflation at the business cycle frequency depends critically on the specification of their cyclical fluctuations. Output and inflation series exhibit low frequency fluctuations that add to their persistence. It is doubtful that these fluctuations are related to business cycle fluctuations. Moreover, they are absent from sticky-price models. These low frequency fluctuations must be removed to obtain meaningful business cycle fluctuations. Chari, Kehoe, and McGrattan (2000) remove a linear-quadratic trend from output. Nelson (1998) does not remove any low frequency fluctuations from inflation. We show that these choices are not innocuous, and that they partially account for the failure of sticky-price models to explain the persistence of output and inflation.

To achieve our objective, we compare the persistence of output and inflation (measured by their sample autocorrelations) from a sticky-price model to the persistence of cyclical fluctuations in post-war output and inflation. These cyclical variations are calculated using three different methods to remove low frequency fluctuations. Then, we use the model to quantify the extent of nominal rigidity required to explain the persistence of output and inflation for each of the three specifications of the low frequency fluctuations.

Section 2 presents our sticky-price monetary business cycle model. It consists of an economy populated by a consumer, a retailer, monopolistically competitive producers, and a monetary authority. As in Ireland (2001), producers find it costly to adjust nominal prices. This cost induces a nominal rigidity. We use this simple model because the extent of nominal rigidity is controlled by a single parameter. Also, as is standard, the model produces output and inflation series with business cycle fluctuations, but no lower frequency fluctuations.

Section 3 presents the post-war and model predicted autocorrelations of output and inflation. For these computations, the low frequency fluctuations are removed using a linear trend, a linear-quadratic (LQ) trend, or a Hodrick-Prescott (HP) filter trend. We use these low frequency specifications for their simplicity and wide usage. The autocorrelations computed under the three low-frequency filters differ greatly. Importantly, the model generates autocorrelations that match those from the post-war US data. To do this, the model simply requires different degrees of nominal rigidity.

Finally, Section 4 discusses our results. To highlight the amount of rigidity required to explain the post-war persistence, we translate the degrees of nominal rigidity in terms of production lost. We find that the instantaneous loss of production of a 10 point increase in the inflation rate is 2.5 percent if the low frequency fluctuations are specified as a linear trend. It drops to 0.15 percent when the low frequency fluctuations are specified as a LQ trend. It further drops to 0.01 percent when the low frequency fluctuations are specified as a HP trend.

2. A Sticky-Price Model

2.1 The Economic Environment

The consumer chooses consumption C, investment I, hours worked N, money holdings M, and contingent one-period nominal bond holdings B to solve

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\eta} \right) \left(\left[\omega C_t^{\frac{\hat{A}-1}{\hat{A}}} + (1-\omega)\,(M_t/P_t)^{\frac{\hat{A}-1}{\hat{A}}} \right]^{\frac{\hat{A}}{\hat{A}-1}} (1-N_t)^{\psi} \right)^{1-\eta} \right\} \tag{1}$$

subject to

$$P_t C_t + P_t I_t + M_t + \sum_{Z_{t+1}} q(Z_{t+1}, Z_t) B(Z_{t+1}) \leq$$
$$P_t w_t N_t + P_t r_t^k K_t + M_{t-1} + B(Z_t) + T_t + \Pi_t \tag{2}$$

and

$$K_{t+1} = I_t + (1-\delta)K_t - \frac{\nu}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t, \tag{3}$$

where P is the aggregate price level, K is the capital stock, T is nominal transfers, w is the real wage rate, r^k is the rental rate of capital, Π is the aggregate of all profits, q is the price of a bond, and Z is the state of the world. Equation (1) shows that the consumer maximizes expected lifetime utility. Equation (2) displays the consumer's budget constraint. Equation (3) describes capital accumulation, where the last term denotes investment costs.

The competitive retailer chooses purchases of individual goods s_i to solve

$$\max P_t G_t - \int p_{it}\, s_{it}\, di \tag{4}$$

subject to

$$G_t = \left[\int s_{it}^{\frac{\mu \text{¡} 1}{\mu}} di\right]^{\frac{\mu}{\mu \text{¡} 1}}, \tag{5}$$

where G is the quantity of aggregate goods sold to the consumer and p_i is the price of individual good i. Equation (4) shows that the retailer maximizes profits and equation (5) describes how individual goods are aggregated. The retailer's problem yields

$$s_{it} = \left[\frac{P_t}{p_{it}}\right]^{\theta} G_t \tag{6}$$

and

$$P_t = \left[\int p_{it}^{1-\theta} di\right]^{\frac{1}{1 \text{¡} \mu}}. \tag{7}$$

Equation (6) displays the retailer's demand for individual good i and equation (7) depicts the aggregate price index.

Monopolistic producer i chooses labor n_i, capital k_i, and prices p_i to solve

$$\max E_0 \sum_{t=0}^{\infty} \Lambda_t \left[p_{it} s_{it}^d - P_t w_t n_{it} - P_t r_t^k k_{it}\right] \tag{8}$$

subject to

$$y_{it} = \Gamma n_{it}^{\alpha} k_{it}^{1-\alpha}, \tag{9}$$

$$s_{it} = y_{it} - \frac{\varphi}{2}\left(\frac{p_{it}}{\pi p_{it-1}} - 1\right)^2 y_{it}, \tag{10}$$

and the demand function displayed in equation (6), where $\Lambda_t = \Pi_{j=0}^{t} R_j^{-1}$, R^{-1} is the market discount factor, and y_i is the quantity produced of good i. Equation (8) shows that the producer maximizes expected discounted profits. Equation (9) displays the production technology. Equation (10) shows that price adjustment costs drive a gap between production and sales, where the last term guarantees nominal price rigidity. The extent of this rigidity is controlled by φ.

The monetary authority provides nominal transfers according to $T_t = M_t - M_{t-1}$. The growth rate of the stock of money $\mu_t = \ln(M_t/M_{t-1})$ evolves as

$$\mu_t = (1-\rho)\ln(\pi) + \rho\mu_{t-1} + \epsilon_t, \tag{11}$$

where ϵ is a mean zero random variable with variance σ^2.

Clearing for the bond, labor, and capital markets requires $B(Z_t) = 0$, $N_t = \int n_{it}\, di$, and $K_t = \int k_{it}\, di$. Clearing of the goods market requires $C_t + I_t = G_t = S_t = Y_t - (\varphi/2)\left(P_t/(\pi P_{t-1}) - 1\right)^2 Y_t$, where the aggregate quantities are given by $Y_t = \int y_{it}\, di$ and $S_t = \int s_{it}\, di$. Also, note that $n_{it} = N_t$, $k_{it} = K_t$, $y_{it} = Y_t$, $s_{it} = S_t$, $g_{it} = G_t$, and $p_{it} = P_t$ because all producers are identical. Finally, the aggregate production function is

$$Y_t = \Gamma N_t^{\alpha} K_t^{1-\alpha}. \tag{12}$$

2.2 Calibration

We approximate a calibrated version of the model using the method described in King, Plosser, and Rebelo (1987). For the calibration, we follow Kydland and Prescott (1982) and set $\beta = 0.99$. We also follow Chari, Kehoe, and McGrattan (2000) and set $\eta = 1$, $\chi = 0.39$, $\omega = 0.94$, $\delta = 0.025$ $\alpha = 0.64$, $\Gamma = 1$, $\theta = 10$, $\pi = 1$, and $\rho = 0.57$. In addition, we set ψ and ν so that hours worked are 30 percent of the time endowment and that the standard deviation of investment is 2.9 times that of output. We follow Boileau and Letendre (2002) and set $\sigma = 0.006$, but our results are invariant to this parameter. Finally, we set φ to match the observed autocorrelations of output and inflation.

2.3 The Real Effects of Money Growth Shocks

Money growth shocks produce real effects because firms find it costly to change nominal prices. An unexpectedly higher money growth rate generates a larger transfer to the consumer. As long as prices are sticky, this raises the consumer's real balances and stimulates the demand for goods.

In reaction to an increase in demand, a producer raises its price and its production. The relative sizes of these changes depend on the marginal cost of production and the cost of changing nominal prices. The marginal cost of production $[1/\alpha]^{\alpha}[1/(1-\alpha)]^{1-\alpha}(1/\Gamma)w_t^{\alpha}r_t^{k(1-\alpha)}$ is increasing in production: raising production requires more inputs, which pushes wages and rental rates up. The cost of changing nominal prices depends on φ: the larger φ, the more costly it is to raise prices. If $\varphi = 0$, a producer only raises its price, and money is neutral. If φ is infinite, a producer only raises its production, and money is non-neutral. For reasonable values, $\varphi > 0$ and not too large, a producer raises both its price and production, and money is only neutral in the long run.

3. Empirical Results

3.1 Low Frequency Fluctuations of Output and Inflation

Figures 1 and 2 plot post-war US output and inflation, as well as their low frequency and business cycle fluctuations.[1] Output refers to the natural logarithm of per capita real gross domestic product and inflation to the first difference of the natural logarithm of the consumer price index.

Post-war output has a marked upward trend that must be removed to obtain the business cycle fluctuations of output. As a first attempt, we specify the output trend as the predicted values from a regression of output on a constant and a linear trend. The cycle is the residuals from this regression. Inflation, however, does not possess this upward trend. Accordingly, the trend is a series of zero and the cycle is inflation itself.

[1] The quarterly post-war sample covers the 1947:1 to 2000:1 period. It includes real gross domestic product (Bureau of Economic Analysis, NIPA Table 1.2), the consumer price index (Bureau of Economic Analysis, NIPA Table 7.1), and population (Citibase, P16).

Figures 1B and 2B suggests that the linear trend does not remove all low frequency fluctuations. For example, output grows on average less rapidly during the 1980s than during the 1960s and 1970s. Inflation is on average higher during the 1970s and early 1980s than during the 1960s and 1990s. Unfortunately, the linear trend does not entirely capture these low frequency movements. As a result, the output and inflation cycles inherit some of the persistence of the low frequency fluctuations.

To better capture these nonlinear fluctuations, we then specify the low frequency fluctuations as the predicted values of regressions of output and inflation on a constant, a linear trend, and a quadratic trend. Our third specification of the low frequency fluctuations consists of the predicted values from the HP filter with a smoothing parameter of 1600. For both specifications, the cycles are the residuals. The nonlinear part of the LQ and HP trends captures more fluctuations, but the effect is more dramatic for the HP trend.

3.2 Post-war and Predicted Autocorrelations of Output and Inflation

Figures 3 and 4 plot the post-war and predicted sample autocorrelations of the output and inflation cycles. The predicted autocorrelations are computed as averages over 1,000 simulations of 212 periods for different values of φ.

The post-war autocorrelations of output and inflation suggest that the persistence of these series depends importantly on the specification used to remove the low frequency fluctuations. When low frequency fluctuations are specified as a linear trend, the series are highly persistent. The autocorrelations of output slowly decline as the number of lags increases. The autocorrelations of inflation decline more rapidly at first, but then reach a plateau. To (informally) match these autocorrelations, the model requires a value of $\varphi = 50,000$.

When the low frequency fluctuations are specified as a LQ trend or as a HP trend, the post-war autocorrelations suggest that the series are less persistent. For the LQ trend, the autocorrelations decline rapidly, but still reach a plateau for inflation. The model requires a value of $\varphi = 3,000$ to match these autocorrelations. For the HP trend, the autocorrelations decline very rapidly, and the model only requires a value of $\varphi = 200$ to match the autocorrelations.

4. Discussion

The proportion of production lost by changing prices is $(\varphi/2)\left(P_t/(\pi P_{t-1})-1\right)^2$. For $\varphi = 50,000$, an instantaneous increase of the inflation rate from its steady state of 0 to 0.10 percent translates to an instantaneous 2.5 percent loss of production. For $\varphi = 3,000$, the increase translates to a 0.15 percent loss of production. For $\varphi = 200$, the increase translates to a small 0.01 percent loss of production.

Chari, Kehoe, and McGrattan (2000) remove a linear-quadratic trend from output before computing the persistence of output. Our results show that removing this trend may not fully remove low frequency fluctuations. Nelson (1998) does not remove any low frequency fluctuations. In both cases, however, the models studied do not possess low frequency fluctuations. This partially explains the failure of sticky-price models to replicate the post-war persistence of output and inflation.

Admittedly, the values of φ required to match the post-war persistence are larger than those estimated in Ireland (2001). He provides estimates that vary between 72.01 and 77.10. One reason for the discrepancy is that our model only has one source of shocks, while the model studied in Ireland (2001) has multiple sources of shocks. Another is that our sticky-price model is simple and only partially accounts for the post-war persistence. Recent work shows that reasonable extensions improve the model's predictions. For example, Fuhrer and Moore (1995) introduce relative wage contracting to raise the persistence of output and inflation. Their wage contracting framework, however, does not have explicit microfoundations. For models with explicit microfoundations, Boileau and Letendre (2002) introduce inventories, Johri (2002) exploits learning-by-doing, and Christiano, Eichenbaum, and Evans (2001) use habit formation, nominal wage rigidity, and capital utilization to raise the persistence of output and inflation.

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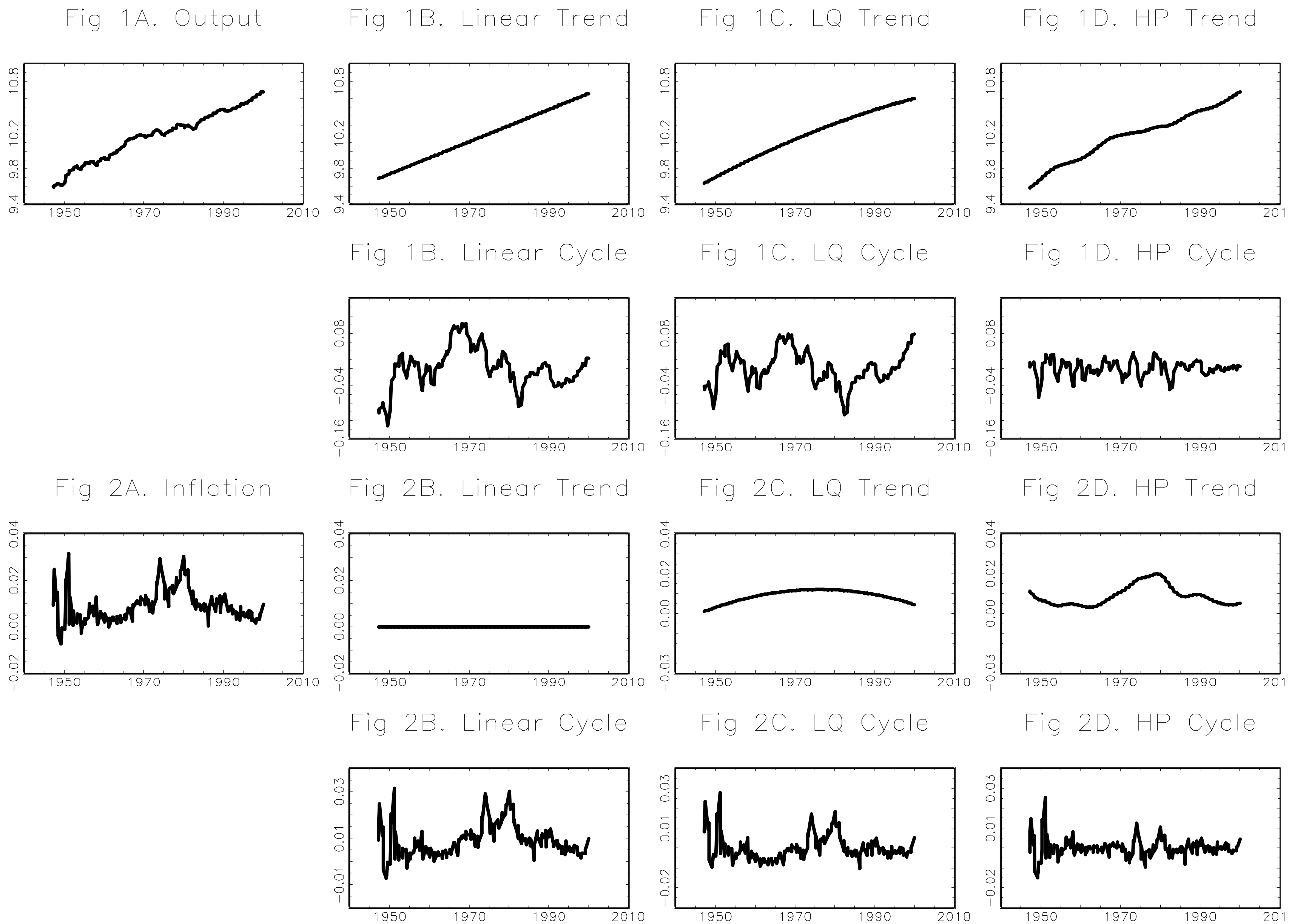
Fig 1A. Output
Fig 1B. Linear Trend
Fig 1C. LQ Trend
Fig 1D. HP Trend
Fig 1B. Linear Cycle
Fig 1C. LQ Cycle
Fig 1D. HP Cycle
Fig 2A. Inflation
Fig 2B. Linear Trend
Fig 2C. LQ Trend
Fig 2D. HP Trend
Fig 2B. Linear Cycle
Fig 2C. LQ Cycle
Fig 2D. HP Cycle

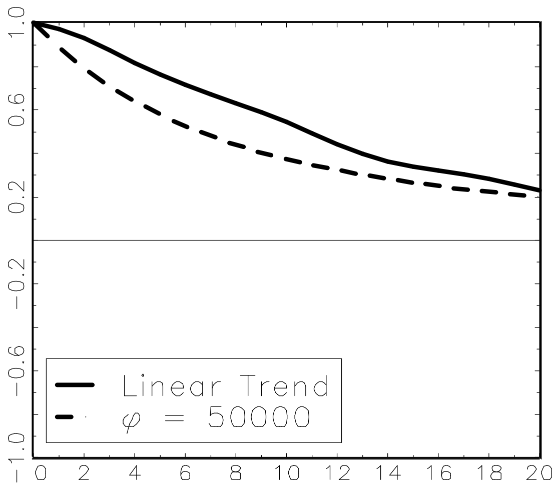

Fig 3A. Autocorrelations of Output Cycle

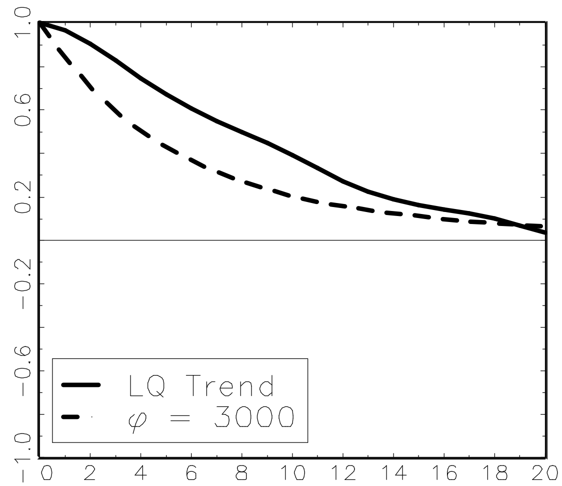

Fig 3B. Autocorrelations of Output Cycle

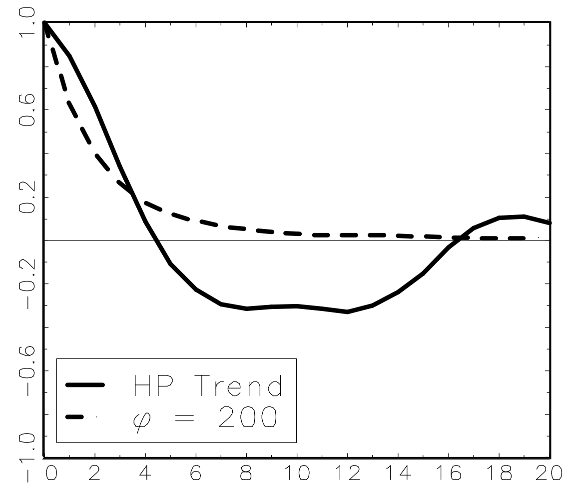

Fig 3C. Autocorrelations of Output Cycle

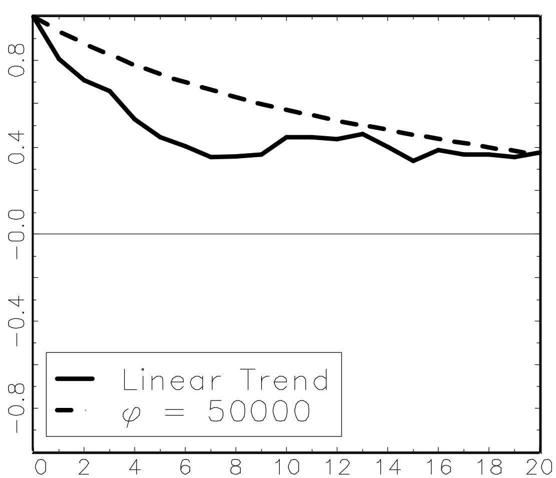

Fig 4A. Autocorrelations of Inflation Cycle

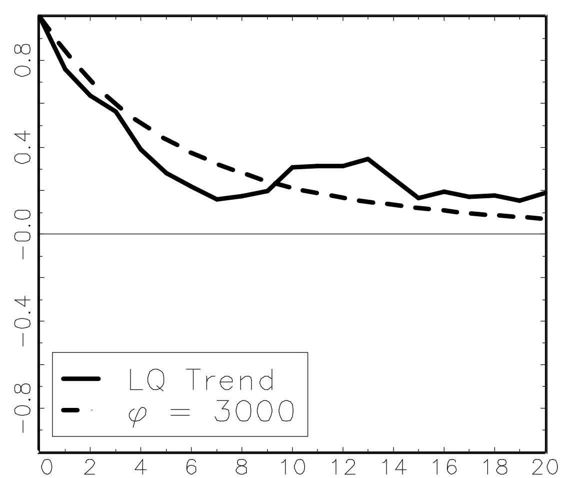

Fig 4B. Autocorrelations of Inflation Cycle

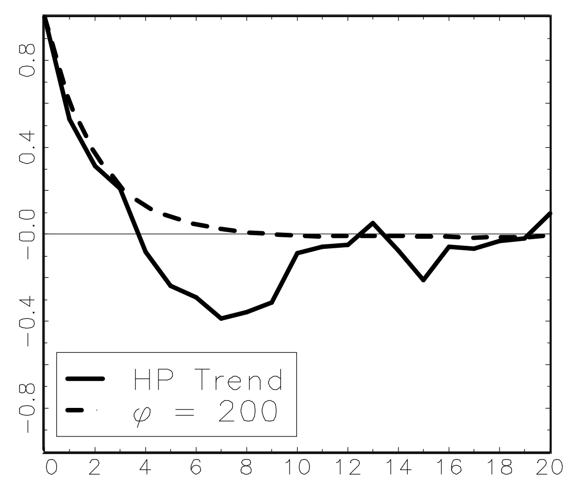

Fig 4C. Autocorrelations of Inflation Cycle