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Inventories, Sticky Prices, and the Propagation of Nominal Shock

Martin Boileau
Department of Economics, University of Colorado at Boulder
Boulder, Colorado

Marc-André Letendre
Department of Economics, McMaster University
Hamilton, Ontario

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Center for Economic Analysis
Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

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Martin Boileau and Marc-André Letendre

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Abstract

Post-war business cycle fluctuations of output and inflation are remarkably persistent. Recent sticky-price monetary business cycle models, however, grossly underpredict this persistence. We assess whether adding inventories to a standard sticky-price model raises the persistence of output and inflation. For this addition, we consider three different frameworks: a linear-quadratic inventory model, a factor of production model, and a transaction costs model. We find that adding inventories increases the persistence of output and inflation, but that the increase is smaller for inflation. Overall, the transaction costs model explains more the persistence of output and inflation than the other models.

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Martin Boileau
Department of Economics
University of Colorado
256 UCB
Boulder, Colorado 80309
United States
Tel.: (303) 492-2108
Fax: (303) 492-8960
E-mail: martin.boileau@colorado.edu

Marc-André Letendre
Department of Economics
McMaster University
1280 Main Street West
Hamilton, Ontario L8S 4M4
Canada
Tel.: (905) 525-9140 x23817
Fax: (905) 521-8232
E-mail: letendre@mcmaster.ca

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1. Introduction

Post-war US business cycle fluctuations of output and inflation are remarkably persistent. A number of recent papers study the ability of sticky-price monetary business cycle models to explain this persistence (e.g. Andersen 1998, Ascari 2000, Chari, Kehoe, and McGrattan 2000, Christiano, Eichenbaum, and Evans 2001, Fuhrer and Moore 1995, Ireland 2001, Kiley 1996, and Nelson 1998). With the exception of Christiano, Eichenbaum, and Evans (2001), these papers show that existing monetary business cycle models with explicit microfoundations fail to explain the observed persistence of output and inflation. For example, Chari, Kehoe, and McGrattan (2000) show that monetary business cycle models based on the Taylor (1980) overlapping contracts require counterfactually long contracts to explain persistent output fluctuations. Also, Nelson (1998) documents that several existing monetary business cycle models fail to explain persistent inflation changes.

Our objective is to determine whether adding inventories to a standard sticky-price monetary business cycle model raises the persistence of output and inflation. There is a large body of literature on inventories. Two good surveys are found in Blinder and Maccini (1991) and Ramey and West (1999). In this literature, some studies examine the relation between inventories and the business cycle (Bils and Kahn 2000, Blinder and Fischer 1981, Fisher and Hornstein 2000, West 1990), between inventories and sticky prices (Blinder 1982, Borenstein and Shepard 1996, Hornstein and Sarte 1998), and between inventories and costly price changes (Aguirregabiria 1999). We follow these studies and focus on the impact of inventories for several reasons. First, Ramey and West (1999) document that, although changes in inventories form on average less than one percent of gross domestic product, reductions in inventories arithmetically account for about 49 percent of the fall in gross domestic product during post-war US recessions. Second, Blinder and Fisher (1981) argue that the gradual adjustment of the stock of inventories is responsible for lasting real effects of changes in the stock of money. Finally, Blinder (1982) argues that inventories generate (real) price stickiness.

The last two reasons suggest that the gradual adjustment of inventory stocks is an explanation for the sluggishness of both output and inflation changes. In the terminology of Ball and Romer (1990), inventories create a real rigidity. They write "Researchers have presented a wide range of explanations for wage and price rigidities: examples include implicit contracts, customer markets, social customs, efficiency wages, *inventory* models, and counter-cyclical markups" (page 183). In other words, the effects of money growth shocks on the real economy created by nominal rigidities become quantitatively important and persistent with inventories.

To achieve our objective, we compare the persistence of output and inflation in monetary business cycle models with and without inventories. We evaluate whether adding inventories raises the persistence by directly comparing the sample autocorrelations of output and inflation produced by the different models. In addition, we verify whether the models with inventories reproduce two features of the data: sales are less volatile than output and changes in inventories are procyclical.

Section 2 presents our baseline sticky-price monetary business cycle model without inventories. It consists of an artifical economy populated by an infinitely-lived representative consumer, a representative competitive retailer, monopolistically competitive producers, and a monetary authority. The consumer purchases an aggregate good from the retailer. The retailer purchases individual goods from producers and aggregates them. As both the consumer and the retailer are price-takers, our economy is equivalent to one where the consumer purchases individual goods directly from producers. We nevertheless introduce the retailer because this modeling choice simplifies the exposition. Individual goods are produced by monopolistically competitive producers using labor and capital. As in Ireland (2001), producers find it costly to adjust nominal prices. We find that the nominal rigidity explains lasting effects of money growth shocks on output and inflation. The persistence of these effects, however, is much smaller than that found in post-war US data. In particular, we find that the sample autocorrelations of output and inflation approach zero much more rapidly than those of a post-war US sample. The autocorrelations of output predicted by the baseline model are positive for the first 16 lags (a period is a quarter), but are virtually zero after the first nine lags. The autocorrelations of inflation predicted by the baseline model are positive only for the first six lags. In contrast, the autocorrelations of output computed from quarterly post-war US data are positive for the first 18 lags. Those of inflation are positive for the first 11 lags.

Sections 3, 4, and 5 verify whether adding inventories to the baseline model enhances the lasting effects of money growth shocks. In each of these inventories models, only producers hold inventories, while the retailer does not. Ramey and West (1999) document that, for 1995, about 37 percent of inventories were held in manufacturing and 52 percent were held in either retail or wholesale trade. We abstract from inventories in the retail sector for two reasons. First, we introduce the retailer only to simplify the exposition. Our approach is equivalent to one where the consumer purchases goods directly from producers. Second, we are interested in the interaction between inventories and pricing decisions of monopolistic producers. In doing so, we follow Blinder and Fischer (1981) and Hornstein and Sarte (1998).

Section 3 discusses the persistence of output and inflation in a monetary business cycle model with inventories that share several features with the linear-quadratic model of West (1990). In this model, producers manage an inventory stock of goods, but face costs of changing the level of production and costs of deviating from a ratio of sales to inventories. The first cost provides a production smoothing motive and the second represents stockout costs. Overall, we find that adding inventories raises the persistence of output, but not that of inflation. The linear-quadratic model generates autocorrelations of output that are positive for the first nine lags and that are larger than those produced by the baseline model. It also generates autocorrelations of inflation that are positive for the first four lags only. The increase in the persistence of output results only partially from the smoothing motive. That is, without the costs of changing the level of production, the autocorrelations of output are still larger than those produced by the baseline model. The linear-quadratic model, however, counterfactually predicts that sales are more volatile than output and that changes in inventories are countercyclical.

Section 4 discusses the persistence of output and inflation in a factor of production model that embodies a feature found in the model of Kydland and Prescott (1982). In this model, producers manage an inventory stock of goods that is a direct input in production. The inventory stock is a production input because it helps economize on the cost of restocking and the cost of shifting production from one type of good to another. We again find that adding inventories increases the persistence of output, but has little effect on the persistence of inflation. The autocorrelations of output predicted by the factor of production model are positive for the first ten lags, while those of inflation are positive for the first six lags. As for the linear-quadratic model, the factor of production model counterfactually predicts that sales are more volatile than output and that changes in inventories are countercyclical.

Section 5 discusses the persistence of output and inflation in a transaction costs model that shares features with the model of Bils and Kahn (2000). In this model, the consumer finds shopping activities costly. A larger stock of inventory increases the stock of available goods, which makes it easier to shop. We find that adding inventories using the transaction costs model significantly raises the persistence of output and inflation. The autocorrelations of output predicted by the transaction costs model are large and positive for the first 19 lags. Moreover, these autocorrelations replicate those computed from the post-war US sample. The autocorrelations of inflation predicted by the transaction costs model are positive for the first nine lags. These autocorrelations are reasonably close to those computed from the post-war US sample. In addition, the transaction costs model predicts procyclical inventories, but this result is sensitive to the degree of risk aversion. Finally, as in our previous models, sales are counterfactually more volatile than output.

2. The Baseline Model

The baseline model does not include inventories. It depicts a stochastic economy populated by an infinitely lived representative consumer, monopolistically competitive producers, a representative retailer, and a passive monetary authority. Production of individual goods requires both labor and capital. Producers find it costly to change nominal prices. For convenience, the retailer aggregates individual goods, and sells the aggregate to the consumer. The monetary authority supplies money according to a stochastic rule.

2.1 The Consumer

The representative consumer's expected lifetime utility is

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t U\left(C_t, M_t/P_t, N_t\right)\right\}, \tag{1}$$

where C denotes consumption, M is nominal money balances, P is the aggregate price level, N is hours worked, and

$$U(C,M/P,N)=\frac{1}{1-\sigma}\left(\left[\omega C^{\frac{\chi-1}{\chi}}+(1-\omega)\left(\frac{M}{P}\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}(1-N)^{\psi}\right)^{1-\sigma}.$$

The consumer faces the budget constraint

$$P_tC_t+P_tI_t+M_t+\sum_{Z_{t+1}}q(Z_{t+1},Z_t)B(Z_{t+1})\leq P_tw_tN_t+P_tr_t^kK_t+M_{t-1}+B(Z_t)+T_t+\Pi_t, \tag{2}$$

where I is investment, K is the capital stock, T is nominal transfers, w is the real wage rate, r^k is the rental rate of capital, and Π is the aggregate of all profits. Also, the consumer purchases contingent one-period nominal bonds B at price q, but face the borrowing constraint $B \geq \bar{B}$ for some large negative number $\bar{B}$. Finally, the state of the world Z follows a process with transition probability density $f(Z_{t+1}, Z_t)$.

The capital stock evolves according to

$$K_{t+1}=I_t+(1-\delta)K_t-\frac{\nu}{2}\left(\frac{I_t}{K_t}-\delta\right)^2K_t, \tag{3}$$

where the last term of equation (3) denotes investment costs.

The consumer chooses consumption, investment, hours worked, money holdings, and bond holdings to maximize expected lifetime utility subject to his budget. The first-order necessary conditions of this problem are

$$w_t=-\frac{U_n(Z_t)}{U_c(Z_t)}, \tag{4.1}$$

$$\frac{U_c(Z_t)}{P_t}-\frac{U_m(Z_t)}{P_t}=\beta E_t\left\{\frac{U_c(Z_{t+1})}{P_{t+1}}\right\}, \tag{4.2}$$

$$\frac{U_c(Z_t)}{\left[1-\nu\left(\frac{I_t}{K_t}-\delta\right)\right]} = \beta E_t\left\{\frac{U_c(Z_{t+1})}{\left[1-\nu\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\right]}\left(r^k_{t+1}\left[1-\nu\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\right]\right.\right.$$

$$\left.\left.+(1-\delta)-\frac{\nu}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)^2+\nu\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\frac{I_{t+1}}{K_{t+1}}\right)\right\}, \tag{4.3}$$

$$q(Z_{t+1},Z_t) = \beta f(Z_{t+1},Z_t)\frac{U_c(Z_{t+1})}{U_c(Z_t)}\frac{P_t}{P_{t+1}}, \tag{4.4}$$

$$1 = \beta\frac{U_c(Z_{t+1})}{U_c(Z_t)}\frac{P_t}{P_{t+1}}R_{t+1}, \tag{4.5}$$

where R^{-1} is the one-period market discount factor ($R_0 = 1$).

2.2 Producers

Monopolistic producer i's expected discounted profits are

$$E_0\sum_{t=0}^{\infty}\Lambda_t\left[p_{it}s^d_{it} - P_t w_t n_{it} - P_t r^k_t k_{it}\right], \tag{5}$$

where $\Lambda_t = \Pi^t_{j=0}R^{-1}_j$, p_i is the sales price for good i, s_i is sales, n_i is labor, and k_i is capital. Good i is produced using the technology

$$y_{it} = \Gamma n^{\alpha}_{it}k^{1-\alpha}_{it}, \tag{6}$$

where y_i is output. Price adjustment costs drive a gap between output and sales:

$$s_{it} = y_{it} - \frac{\phi_p}{2}\left(\frac{p_{it}}{\pi p_{it-1}}-1\right)^2 y_{it}, \tag{7}$$

where π denotes the level of inflation in steady state. The last term of equation (7) guarantees nominal price rigidity. The extent of this rigidity is controlled by ϕ_p. Finally, the demand for good i is

$$s^d_{it} = \left[\frac{P_t}{p_{it}}\right]^{\theta} G_t, \tag{8}$$

where G denotes the quantity of aggregate goods sold to the consumer.

Producer i chooses labor, capital, and prices to maximize expected discounted profits. The first-order necessary conditions of this problem are

$$P_t w_t = \lambda_{it}\left[1-\frac{\phi_p}{2}\left(\frac{p_{it}}{\pi p_{it-1}}-1\right)^2\right]\alpha\frac{y_{it}}{n_{it}}, \tag{9.1}$$

$$\alpha\, r_t^k k_{it} = (1-\alpha) w_t n_{it}, \tag{9.2}$$

$$(\theta-1)p_{it}s_{it} - \lambda_{it}\left[\theta s_{it} - \phi_p\left(\frac{p_{it}}{\pi p_{it-1}} - 1\right)\frac{p_{it}}{\pi p_{it-1}} y_{it}\right] = \\ E_t\left\{R_{t+1}^{-1}\lambda_{t+1}\phi_p\left(\frac{p_{it+1}}{\pi p_{it}} - 1\right)\frac{p_{it+1}}{\pi p_{it}} y_{it+1}\right\}, \tag{9.3}$$

where λ_i is the multiplier associated with constraint (7).

2.3 The Retailer

The competitive retailer's profits are

$$P_t G_t - \int p_{it}\, s_{it}\, di. \tag{10}$$

The retailer aggregates individual goods using the technology

$$G_t = \left[\int g_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{11}$$

where $g_{it} = s_{it}$.

The retailer chooses inputs and output to maximize profits. The first-order necessary conditions of this problem imply the goods demand function displayed in equation (8). The demand functions for all goods and the retailer's zero-profit condition yield the aggregate price index

$$P_t = \left[\int p_{it}^{1-\theta} di\right]^{\frac{1}{1-\theta}}. \tag{12}$$

2.4 The Monetary Authority

The monetary authority is passive and provides nominal transfers according to

$$T_t = M_t - M_{t-1}. \tag{13}$$

The growth rate of money, $\mu_t = \ln(M_t/M_{t-1})$, evolves as

$$\mu_t = (1-\rho)\ln(\pi) + \rho\mu_{t-1} + \epsilon_t, \tag{14}$$

where ϵ is a mean zero random variable with variance σ_μ^2.

2.5 Market Clearing and Aggregation

Clearing of the bond, labor, capital, and goods markets requires

$$B(Z_t) = 0, \tag{15.1}$$

$$N_t = \int n_{it}\, di, \tag{15.2}$$

$$K_t = \int k_{it}\, di, \tag{15.3}$$

$$S_t = Y_t - \frac{\phi_p}{2}\left(\frac{P_t}{\pi P_{t-1}} - 1\right)^2 Y_t, \tag{15.4}$$

$$G_t = S_t, \tag{15.5}$$

$$C_t + I_t = G_t, \tag{15.6}$$

where aggregate quantities are given by $Y_t = \int y_{it}\, di$ and $S_t = \int s_{it}\, di$. Also, note that $n_{it} = N_t$, $k_{it} = K_t$, $y_{it} = Y_t$, $s_{it} = S_t$, $g_{it} = G_t$, and $p_{it} = P_t$ because all producers are identical. Finally, the aggregate production function is

$$Y_t = \Gamma N_t^{\alpha} K_t^{1-\alpha}. \tag{16}$$

2.6 Calibration

The baseline model does not have an analytical solution for general values of the underlying parameters. Instead, we find an approximate solution using the method described in King, Plosser, and Rebelo (1987). This method requires that values be assigned to all parameters.

Table 1 displays parameter values for the different models. For the baseline model, we set several parameters to the values used in Chari, Kehoe, and McGrattan (2000): $\sigma = 1$, $\chi = 0.39$, $\omega = 0.94$, $\delta = 0.025$ $\alpha = 0.36$, $\Gamma = 1$, $\theta = 10$, $\pi = 1$. In addition, we follow their guidelines and set $\psi = 1.7119$ and $\nu = 5.71$ to ensure that hours worked are 30 percent of the time endowment and that the standard deviation of investment is 2.9 times the standard deviation of output, as in our post-war US sample.

The source of nominal rigidity in our baseline model differs from that used in Chari, Kehoe, and McGrattan (2000). This influences the values of both β and ϕ_p. We follow

Kydland and Prescott (1982) and set $\beta = 0.99$. We also follow Ireland (2001) and use his estimated value of ϕ_p for the pre-1979 period: $\phi_p = 72.01$ (See Ireland Table 1). Our empirical results, however, are qualitatively similar if we use the estimate for the post-1979 period ($\phi_p = 77.10$). Finally, we use quarterly data on $M2$ to estimate ρ and σ_μ. The results are $\rho = 0.72$ and $\mu = 0.006$.

2.7 Empirical Results

Figure 1 displays autocorrelations of output and inflation for up to 20 lags. The autocorrelations in the post-war US sample are computed as the sample autocorrelations from the logarithm of output and the inflation rate over the 1959:1 to 2000:1 period (see Data Appendix). Output corresponds to per capita gross domestic product and the inflation rate to the first difference of the logarithm of the consumer price index. In the data, the logarithm of output displays an upward trend. For our computations, we remove the trend by regressing the logarithm of output on a constant, a linear trend, and a quadratic trend. The residual of this ordinary least squares regression is our definition for the cycle of the logarithm of output. Although the inflation rate does not possess a trend, we nevertheless remove one similarly to that of output. We do so for two reasons. First, we wish to treat inflation and the logarithm of output similarly. Second, we wish to account for the fact that post-war US inflation is on average much higher during the 1970s and early 1980s than during the 1960s and 1990s. This feature alone would suggest that inflation fluctuations are extremely persistent. It is doubtful, however, that it reflects a business cycle fluctuation of inflation. Our detrending method may not completely eliminate the influence of this period, but it is a step in the right direction. Overall, it is important to note that the sample autocorrelations obtained for both output and inflation are sensitive to the detrending method. For this reason, the Results Appendix shows autocorrelations computed from data where the Hodrick-Prescott filter is applied to the logarithm of output and the inflation rate. In the model, neither the logarithm of output nor the inflation rate possess a trend. We nevertheless detrend both to ensure that the artificial data is treated similarly to the post-war US data. Finally, the autocorrelations predicted by the baseline model are computed as the average autocorrelations over 1000 simulations of 164 quarters (the number of quarters of the post-war US sample).

A comparison between the autocorrelations computed from the post-war US sample and those predicted by the baseline model shows that the model grossly underpredicts the persistence of output and inflation. The autocorrelations of output and inflation predicted by the baseline model decline much more rapidly than those computed on the post-war US sample. The autocorrelations of output computed from the post-war US sample are positive for the first 18 lags, and negative after. Those of inflation are positive for the first 11 lags. The autocorrelations of output predicted by the baseline model are positive for the first 16 lags, but are very close to zero after the first nine lags. The autocorrelations of inflation are only positive for the first six lags.

Figure 2 displays the dynamic responses of output, inflation, and money growth in percent deviations from their steady-state levels computed from the baseline model. The responses show that the baseline model generates real effects to the money growth shock. This occurs because firms find it costly to change nominal prices. The mechanism works as follows. The higher money growth generates a larger transfer from the monetary authority to the consumer. As long as prices are sticky, the larger transfer raises the consumer's real balances. The increase in real balances stimulates the consumer's demand for the aggregate good, because it raises his wealth and because real balances and consumption are complements. The increase in the demand for the aggregate good raises the demand for all individual goods.

In reaction to the increase in the demand for its good, a monopolistic producer can change its price and output levels. The larger the change in price, the smaller the change in output required to meet the new demand. The relative sizes of the price and output changes depend on the cost of changing nominal prices and the marginal cost of production. The cost of changing nominal prices depends on ϕ_p: the larger ϕ_p, the more costly it is to raise prices. The marginal cost of production is $[1/\alpha]^{\alpha}[1/(1-\alpha)]^{1-\alpha}(1/\Gamma)w_t^{\alpha}r_t^{k(1-\alpha)}$. In equilibrium, the marginal cost is increasing in output. That is, raising output requires an increase in the demand for inputs, which pushes wages and rental rates up and increases the marginal cost.

If prices are not costly to change ($\phi_p = 0$), a producer meets the new demand by increasing its price, and no output response is necessary. If prices are costly to change ($\phi_p > 0$) while output is not (the marginal cost is constant), a producer meets the new demand by raising output, and no price response is necessary. As shown in Figure 2, a producer trades off the two costs and raises both its price and its output to meet the new demand.

The persistence of the changes in price and output also depends on the cost of changing nominal prices. Fluctuations in inflation and output become more persistent with larger values for ϕ_p. However, Figure 1 shows that, even with our calibrated large value for ϕ_p, a monetary shock does not have long-lasting effects on output and inflation in the baseline model.

3. The Linear-Quadratic Model

The linear-quadratic model adds inventories to the baseline model. For this addition, we borrow several features from West (1990). In particular, producers face quadratic costs of changing the level of production and of deviating from a target ratio of sales to inventories. Our version of the linear-quadratic model, however, differs from that of West. Our producers are monopolistic competitors that produce goods with both labor and capital, while his producer is a monopolist that produces goods with labor only. Also, our demand shocks are money growth shocks, while his are taste shocks.

Our version of the linear-quadratic model uses the consumer, the retailer, and the monetary authority of the baseline model.

3.1 Producers

Producer i's expected discounted profits are

$$E_0 \sum_{t=0}^{\infty} \Lambda_t \left[p_{it} s_{it}^d - P_t w_t l_{it} - P_t r_t^K k_{it} \right], \tag{17}$$

where l_i denotes labor usage. Similarly to West (1990), labor usage is

$$l_{it} = n_{it} + \frac{\zeta_1}{2} \left(\Delta y_{it} \right)^2 + \frac{\zeta_2}{2} \left(x_{it} - \eta s_{it} \right)^2, \tag{18}$$

where x_i is the stock of inventories. Labor is used in three activities. The first term on the right side of equation (18) represents the time allocated to production. The second term reflects the labor used to change the level of production. Finally, the last term is a labor cost due to deviations of inventories from a fraction of sales. This term represents the labor cost associated with stockouts and is often called the convenience yield.

Inventories evolve as

$$x_{it+1} = x_{it} + y_{it} - s_{it} - \frac{\phi_p}{2}\left(\frac{p_{it}}{\pi p_{it-1}} - 1\right)^2 y_{it}. \tag{19}$$

Also, producer i faces a demand given by equation (8).

Producer i chooses labor, capital, inventories, and prices to maximize expected discounted profits. This problem yields the following first-order necessary conditions:

$$\frac{1}{\alpha}P_t w_t \frac{n_{it}}{y_{it}} + \zeta_1 P_t w_t \Delta y_{it} - \lambda_{it}\left[1 - \frac{\phi_p}{2}\left(\frac{p_{it}}{\pi p_{it-1}} - 1\right)^2\right] = E_t\Big\{R_{t+1}^{-1}\zeta_1 P_{t+1} w_{t+1} \Delta y_{it+1}\Big\}, \tag{20.1}$$

$$\alpha\, r_t^k k_{it} = (1-\alpha) w_t n_{it}, \tag{20.2}$$

$$(\theta-1)p_{it}s_{it} + \theta\eta\zeta_2 P_t w_t (x_{it} - \eta s_{it}) s_{it} - \lambda_{it}\left[\theta s_{it} - \phi_p\left(\frac{p_{it}}{\pi p_{it-1}} - 1\right)\right]\frac{p_{it}}{\pi p_{it-1}} y_{it} = E_t\left\{R_{t+1}^{-1}\lambda_{t+1}\phi_p\left(\frac{p_{it+1}}{\pi p_{it}} - 1\right)\frac{p_{it+1}}{\pi p_{it}} y_{it+1}\right\}, \tag{20.3}$$

$$\lambda_{it} = E_t\Big\{R_{t+1}^{-1}\left[\lambda_{it+1} - P_{t+1} w_{t+1} \zeta_2 (x_{it+1} - \eta s_{it+1})\right]\Big\}, \tag{20.4}$$

where λ_i is the shadow price of inventories or the multiplier associated with equation (19).

3.2 Market Clearing and Aggregation

Clearing of the bond and capital markets are still described by equations (15.1) and (15.3). Clearing of the labor market is given by

$$N_t = n_t + \frac{\zeta_1}{2}(Y_t - Y_{t-1})^2 + \frac{\zeta_2}{2}(X_t - \eta S_t)^2. \tag{21.1}$$

Clearing of the goods market requires

$$S_t + X_{t+1} - X_t = Y_t - \frac{\phi_p}{2}\left(\frac{P_t}{\pi P_{t-1}} - 1\right)^2 Y_t, \tag{21.2}$$

as well as equations (15.5) and (15.6). Aggregate quantities are as before, except for $n_{it} = n_t$ and $x_{it} = X_t$. Finally, the aggregate production function is

$$Y_t = \Gamma n_t^{\alpha} K_t^{1-\alpha}. \tag{22}$$

3.3 Calibration

Table 1 also reports the calibration of the linear-quadratic model. The calibration is similar to that of the baseline model. That is, we set β, σ, ω, χ, δ, Γ, α, ϕ_p, θ, ρ, and σ_μ to the same values. We also set ψ and ν so that hours worked are 30 percent of the time endowment and that the standard deviation of investment is 2.9 times that of output.

The linear-quadratic model has three additional parameters: ζ_1, ζ_2, and η. West (1990) estimates a cost function similar to that in equation (18). Although the exact specification differs, West's estimate offer a good benchmark (see West Table III). Estimates for ζ_1 range from 2×0.344 to 2×0.366 and estimates for ζ_2 range from 2×0.111 to 2×0.145. Accordingly, we set $\zeta_1 = 0.7$ and $\zeta_2 = 0.25$. West also provides estimates for η that range between -0.040 and -0.057, but argues that a value between 0.4 and 0.7 reflects the general concensus. We set $\eta = 0.68$ so that steady-state sales are 60 percent of available goods (output plus inventories) as in the post-war US sample.

3.4 Empirical Results

Figure 3 shows sample autocorrelations of output and inflation for up to 20 lags. The autocorrelations are computed as before. Table 2 reports the relative volatility of sales to output, the relative volatility of changes in inventories to output, and the correlation between changes in inventories and output. As for the autocorrelations, these moments are computed from the post-war US sample and the model. In the post-war US sample, the relative volatility of sales is the ratio of the standard deviation of the logarithm of per capita sales to the standard deviation of the logarithm of per capita gross domestic product, where sales are computed from the data using equation (21.2). The relative volatility of inventories is the ratio of the standard deviation of changes in inventories to the standard deviation of the logarithm of output. Changes in inventories corresponds to the ratio of changes in private per capita inventories to per capita gross domestic product. The moments predicted by the model are averages over 1000 simulations of 164 periods. As for the autocorrelations, the different moments are computed from detrended variables.

The results for our benchmark calibration of the linear-quadratic model appear as Benchmark in Figure 3 and Table 2. As for the baseline model, the linear-quadratic model underpredicts the persistence of output and inflation. The autocorrelations predicted by the benchmark calibration, however, suggests that adding inventories raises the persistence of output fluctuations. The autocorrelations of output produced by the benchmark linear-quadratic model are positive for the first nine lags, and slightly larger than those produced by the baseline model. Unfortunately, adding inventories does not increase the persistence of inflation. The autocorrelations of inflation produced by the benchmark model are positive only for the first four lags.

The benchmark linear-quadratic model also fails to replicate some standard inventories facts. Even though the model is consistent with the fact that changes in inventories are less volatile than output, sales are counterfactually more volatile than output and changes in inventories are counterfactually countercyclical. Note that these failures are not independent. An increase in sales can trigger changes in both output and inventories. If inventories are countercyclical, output is raised and inventories are depleted to meet an increase in sales. The result is that sales are more volatile than output. If inventories are procyclical, output is raised more than the increase in sales, such that sales are less volatile than output.

Figure 4 displays the dynamic responses of output, inflation, and money growth produced by the benchmark linear-quadratic model. The responses of output predicted by the benchmark linear-quadratic model are more persistent than those predicted by the baseline model. The responses of inflation, however, are not. The higher persistence of output predicted by the benchmark linear-quadratic model is attributable to the fact that producers can vary inventories to meet the new demand. In the baseline model, a producer meets a larger demand by increasing price and output. In making his decisions, he accounts for the cost of adjusting prices and for the (increasing) marginal cost of production. In the linear-quadratic model, a producer meets a larger demand by increasing price and output, and by depleting inventories. He must account for the cost of adjusting prices and the marginal cost of production, as well as for the cost of changing output and the cost of having inventories deviate from a fraction of sales. With the benchmark calibration, a producer adjusts price, output, and inventories to trade off all these costs. The reduction in inventories ensures that output does not increase as much as in the baseline model. It also ensures that the change in output is lasting to gradually replenish inventories.

We wish to verify the robustness of these results to the values of the additional parameters ζ_1, ζ_2, and η. To that end, we perform three experiments on the linear-quadratic model.

Our first experiment investigates the effects of the cost of changing production. This cost offers a production smoothing motive that may explain the increase in the persistence of output. For this experiment, we reduce this cost by lowering ζ_1 from 0.7 to 0.01. The results of this experiment appear as Low Smoothing in Figure 3 and Table 2. Diminishing the cost of changing output reduces the predicted autocorrelations of output, but these autocorrelations are still larger than those predicted by the baseline model. Clearly, the gradual adjustment of inventories adds to the persistence of output fluctuations. Otherwise, diminishing the cost of changing output has little effects. The autocorrelations of inflation are still small, sales are still more volatile than output, and changes in inventories are still countercyclical.

Our second experiment investigates the effects of the cost of having inventories deviate from a fraction of sales (the convenience yield cost). It might be possible to make changes in inventories procyclical by increasing this cost and forcing inventories to track sales more closely. For this experiment, we make the deviations more costly by raising ζ_2 from 0.25 to 4, while keeping $\zeta_1 = 0.01$. The results appear as & High Yield Costs. Raising this cost makes inventories procyclical and sales less volatile than output. It also severely reduces the predicted autocorrelations of output and inflation. By making inventories procyclical, a higher cost eliminates the need for the lasting increase in output required to replenish inventories.

Our last experiment investigates the effects of the steady-state level of the ratio of sales to all available goods. A large steady-state level of this ratio is associated with a low convenience of having inventories and a low steady-state level of inventories. For this experiment, we raise the steady-state ratio of sales to available goods from 0.6 to 0.82 by reducing η from 0.68 to 0.24. This value for the ratio is similar to that obtained in Bils and Kahn (2000). The results of this experiment appear as Low Convenience. The increase in the steady-state ratio of sales to available goods raises the autocorrelations of output, but has very little impact on the autocorrelations of inflation as well as on moments of sales and changes in inventories.

4. The Factor of Production Model

The factor of production model adds inventories to the baseline model by following Kydland and Prescott (1982). In particular, inventories are an input in production, because they reduce down time and help economize on labor. Our version of the factor of production model is different from that of Kydland and Prescott. Importantly, our producers are monopolistic competitors, while theirs are perfect competitors. Also, we consider only monetary growth shocks, while they consider only real technology shocks.

Our version of the factor of production model retains the consumer, the retailer, and the monetary authority of the baseline model.

4.1 Producers

Monopolistic producer i's expected discounted profits are given by equation (5). Good i is produced using the technology

$$y_{it} = \Gamma n_{it}^{\alpha} \Big(\big[(1-\ell) k_{it}^{-\varepsilon} + \ell\, x_{it}^{-\varepsilon} \big]^{-1/\varepsilon} \Big)^{1-\alpha}. \tag{23}$$

The elasticity of substitution between capital and inventories is $1/(1+\varepsilon)$. Inventories evolve as in equation (19) and the demand for good i is displayed in equation (8).

The producer chooses labor, capital, inventories, and prices to maximize expected discounted profits. The problem yields the following first-order necessary conditions:

$$P_t w_t = \lambda_{it} \left[1 - \frac{\phi_p}{2} \left(\frac{p_{it}}{\pi p_{it-1}} - 1 \right)^2 \right] \alpha \frac{y_{it}}{n_{it}}, \tag{24.1}$$

$$\alpha r_t^k k_{it} = (1-\alpha) w_t n_{it} (1-\ell) \left[\frac{k_{it}^{-\varepsilon}}{(1-\ell) k_{it}^{-\varepsilon} + \ell\, x_{it}^{-\varepsilon}} \right], \tag{24.2}$$

$$(\theta - 1)\, p_{it} s_{it} - \theta \lambda_{it} s_{it} + \lambda_{it} \phi_p \left(\frac{p_{it}}{\pi p_{it-1}} - 1 \right) \frac{p_{it}}{\pi p_{it-1}} y_{it} = E_t \left\{ R_{t+1}^{-1} \lambda_{it+1} \phi_p \left(\frac{p_{it+1}}{\pi p_{it}} - 1 \right) \frac{p_{it+1}}{\pi p_{it}} y_{it+1} \right\}, \tag{24.3}$$

$$\lambda_{it} = E_t \left\{ R_{t+1}^{-1} \left(\lambda_{it+1} + \lambda_{it+1} \left[1 - \frac{\phi_p}{2} \left(\frac{p_{it+1}}{\pi p_{it}} - 1 \right)^2 \right] (1-\alpha) \frac{y_{it+1}}{x_{it+1}} \ell \left[\frac{x_{it+1}^{-\varepsilon}}{(1-\ell) k_{it+1}^{-\varepsilon} + \ell\, x_{it+1}^{-\varepsilon}} \right] \right) \right\}, \tag{24.4}$$

where λ_i is the shadow price of inventories or the multiplier associated with equation (19).

4.2 Market Clearing and Aggregation

Clearing of the bond, labor, capital, and goods markets are as in equations (15.1), (15.2), (15.3), (21.2), (15.5), and (15.6). Aggregate quantities are as in the linear-quadratic model, except for $n_{it} = N_t$. Finally, the aggregate production function is

$$Y_t = \Gamma N_t^{\alpha} \left(\left[(1-\ell) K_t^{-\varepsilon} + \ell\, X_t^{-\varepsilon} \right]^{-1/\varepsilon} \right)^{1-\alpha}. \tag{25}$$

4.3 Calibration

Table 1 reports the calibration. As for the linear-quadratic model, we set β, σ, ω, χ, δ, Γ, α, ϕ_p, θ, ρ, and σ_μ to the values used in the baseline model, and set ψ and ν so that hours worked are 30 percent of the time endowment and that the standard deviation of investment is 2.9 times that of output.

The factor of production model has two new parameters: ε and ℓ. Kydland and Prescott (1982) set $\varepsilon = 4$ and $\ell = 0.28 \times 10^{-5}$ to ensure that the elasticity of substitution between capital and inventories is low, that inventories represent about one-fourth of output, and that the capital stock is 10 times the output. Following these guidelines, we set $\varepsilon = 5$ and $\ell = 0.3 \times 10^{-7}$ so that the elasticity is low, that steady-state sales are 60 percent of available goods, and that the steady-state capital stock is 9.2 times the output (as in the baseline model).

4.4 Empirical Results

The results for the benchmark calibration appear in Figure 5 and Table 2. The sample autocorrelations shown in Figure 5 suggest that having inventories as an input raises the persistence of output beyond that predicted by both the baseline and linear-quadratic model. It also slightly increases the persistence of inflation. The autocorrelations of output predicted by the benchmark factor of production model are positive for the first ten lags, and larger than those predicted by the benchmark linear-quadratic model. The autocorrelations of inflation are positive for the first six lags, and also larger than those predicted by the benchmark linear-quadratic model. Otherwise, the factor of production model behaves similarly to the linear-quadratic model: changes in inventories are less volatile than output, sales are much more volatile than output, and changes in inventories are countercyclical.

Figure 6 displays the dynamic responses of output, inflation, and money growth produced by the benchmark factor of production model. As for the linear-quadratic model, the dynamic responses of the factor of production model differ from that of the baseline model because producers can vary inventories to respond to changes in demand. In the factor of production model, as in the linear-quadratic model, a producer meets a larger demand by increasing price and output, and by depleting inventories. In making his decisions, he accounts for the cost of adjusting prices and the increasing marginal cost of production. In this case, the short-run marginal cost of production depends on inventories. A reduction of inventories, however, is not very costly in terms of lost output, because inventories play only a minor role in production. As in the linear-quadratic model, the depletion of inventories requires lasting output increases to gradually replenish inventories.

We wish to verify the robustness of these results to the values of the additional parameters ε and ℓ. For this, we perform two experiments on the factor of production model.

Our first experiment investigates the effects of the elasticity of substitution between capital and inventories. A reduction of the elasticity forces capital and inventories to be less substitutable which might generate procyclical changes in inventories. For our experiment, we reduce the elasticity by raising ε from 5 to 100. The results of this experiment appear as Low Elasticity in Figure 5 and Table 2. Reducing the elasticity makes inventories somewhat less countercyclical and reduces the relative volatility of sales, but these effects are small. The lower elasticity also reduces the autocorrelations of output and inflation, but that are still larger than those produced by the baseline model.

Our second experiment investigates the effects of the steady-state level of the ratio of sales to all available goods. As in the linear-quadratic model, a large steady-state level of this ratio is associated with a low convenience of having inventories and a low steady-state level of inventories. For our experiment, we raise the steady-state ratio of sales to available goods from 0.60 to 0.82 by lowering ℓ from 0.3×10^{-7} to 0.5×10^{-10}. The results appear as Low Convenience. Reducing the steady-state level of inventories diminishes the autocorrelations of output, but has very little impact on the autocorrelations of inflation. Changes in inventories become marginally more countercyclical, and the relative volatilities of sales and changes in inventories are reduced.

5. The Transaction Costs Model

The transaction costs model adds inventories to the baseline model by adopting some elements of Bils and Kahn (2000). In particular, producers face a demand that depends on the available stock of goods. That is, consumers, via retailers, find it costly to engage in shopping activities. A larger stock of available goods help economize on the resources expanded while shopping. Our transaction costs model, however, differs from that of Bils and Kahn. Our producers are monopolistic competitors, while theirs are perfect competitors. Also, our demand for goods is derived from the consumer's problem, while their demand for goods is a reduced form. Finally, our demand shocks are money growth shocks, while theirs are real demand shocks.

Our version of the transaction costs model uses the consumer and the monetary authority of the baseline model.

5.1 Producers

Producer i's expected discounted profits are described in equation (5). The production technology is given in equation (6) and the stock of inventories evolves as in equation (19). Producer i faces the demand

$$s_{it}^d = \left[\frac{P_t}{p_{it}}\right]^\theta G_t \gamma^{\theta-1} a_{it}^{\xi(\theta-1)}, \tag{26}$$

where $a_{it} = y_{it} + x_{it}$ is the stock of good i available.

The producer chooses labor, capital, inventories, and prices to maximize expected discounted profits. This problem yields the following necessary first-order conditions:

$$P_t w_t = (p_{it} - \lambda_{it})\,\xi(\theta-1)\frac{s_{it}}{a_{it}}\alpha\,\frac{y_{it}}{n_{it}} + \lambda_{it}\left[1 - \frac{\phi_p}{2}\left(\frac{p_{it}}{\pi p_{it-1}} - 1\right)^2\right]\alpha\,\frac{y_{it}}{n_{it}}, \tag{27.1}$$

$$\alpha\, r_t^k k_{it} = (1-\alpha) w_t n_{it}, \tag{27.2}$$

$$\begin{aligned}(\theta-1)p_{it}s_{it} - \lambda_{it}\left[\theta s_{it} - \phi_p\left(\frac{p_{it}}{\pi p_{it-1}} - 1\right)\frac{p_{it}}{\pi p_{it-1}} y_{it}\right] = \\ E_t\left\{R_{t+1}^{-1}\lambda_{it+1}\phi_p\left(\frac{p_{it+1}}{\pi p_{it}} - 1\right)\frac{p_{it+1}}{\pi p_{it}} y_{it+1}\right\},\end{aligned} \tag{27.3}$$

$$\lambda_{it} = E_t\left\{R_{t+1}^{-1}\left(\lambda_{it+1} + (p_{it+1} - \lambda_{it+1})\xi(\theta-1)\frac{s_{it+1}}{a_{it+1}}\right)\right\}, \tag{27.4}$$

where λ_i is the shadow price of inventories or the multiplier associated with equation (19).

5.2 The Retailer

The retailer's profits are depicted in equation (10). The retailer aggregates goods using the technology displayed in equation (11). The retailer finds it costly to purchase goods. The cost of purchasing s_{it} goods is $(1 - \gamma a_{it}^\xi)s_{it}$, such that

$$g_{it} = \gamma a_{it}^\xi s_{it}. \tag{28}$$

The representative retailer chooses inputs and output to maximize profits. For convenience, we split the representative retailer in two retailers. The first retailer purchases s_{it} at price p_{it} and sells g_{it} at price $\hat{p}_{it}$ to the second retailer. The zero profit condition of the first retailer is $\hat{p}_{it}g_{it} = p_{it}s_{it}$. Because of equation (28), this zero profit condition implies $\hat{p}_{it}\gamma a_{it}^{\xi} = p_{it}$. The second retailer purchases g_{it} at price $\hat{p}_{it}$ and sells the aggregate G_t to the consumer at price P_t. The zero profit condition of the second retailer is $P_t G_t = \int \hat{p}_{it} g_{it}\, di$. The demand function of the second retailer is $g_{it}^d = [P_t/\hat{p}_{it}]^{\theta} G_t$. Substituting (28) and $\hat{p}_{it}$ in the demand of the second retailer yields the demand given by equation (26). Finally, the demand for all goods combined with the zero-profit conditions of both retailers yield the price index

$$P_t = \left[\int \hat{p}_{it}^{1-\theta} di\right]^{\frac{1}{1-\theta}}. \tag{29}$$

where $p_{it} = \hat{p}_{it}\gamma a_{it}^{\xi}$.

5.3 Market Clearing and Aggregation

Clearing of the bond, labor, and capital markets are as in equations (15.1), (15.2), and (15.3). Clearing of the goods market requires

$$G_t = S_t \gamma A_t^{\xi}, \tag{30.1}$$

$$S_t + X_{t+1} - X_t = Y_t - \frac{\phi_p}{2}\left(\frac{p_t}{\pi p_{t-1}} - 1\right)^2 Y_t, \tag{30.2}$$

and (15.6). Aggregate quantities are as in the factor of production model, with the addition of $A_t = \int a_{it} di$. Also, $a_{it} = A_t$, $p_{it} = p_t$, and $p_t = P_t \gamma A_t^{\xi}$. Finally, the aggregate production function is given by equation (16).

5.4 Calibration

Table 1 reports the calibration of the transaction costs model. The calibration is similar to that of the baseline model. We set β, σ, ω, χ, δ, Γ, α, ϕ_p, θ, ρ, and σ_μ to the same values. We also set ψ and ν so that hours worked are 30 percent of the time endowment and that the standard deviation of investment is 2.9 times that of output.

The transaction costs model has two new parameters: γ and ξ. Although the models differ, the parameter estimates of Bils and Kahn (2000) offer a good benchmark. They provide estimates for $\xi(\theta - 1)$ (See Bils and Kahn Table 6). The constrained estimates range from 0.023 to 0.486. As in our previous models, we set $\xi = 0.0168$ so that steady-state sales are 60 percent of available goods. Given our value of $\theta = 10$, the implied value of $\xi(\theta - 1)$ is 0.151, which is well within Bils and Kahn's range of estimates. Finally, we set $\gamma = 0.9906$ to remove steady-state transaction costs.

5.5 Empirical Results

The results for the benchmark calibration appear as Benchmark in Figure 7 and Table 2. The predictions of the transaction costs model differ from those of the previous models in a number of ways. The autocorrelations of output predicted by the benchmark transaction costs model are much larger than those predicted by our previous inventories models, and replicate the autocorrelations computed from the post-war US sample. The predicted autocorrelations of output are large and positive for the first 19 lags. The autocorrelations of inflation predicted by the benchmark transaction costs model are also larger than those predicted by our previous models, and are closer to the autocorrelations computed from the post-war US sample. The predicted autocorrelations of inflation are positive for the first nine lags. Also, Table 2 shows that the relative volatility of sales and changes in inventories predicted by the benchmark transaction costs model are larger than those predicted by the previous models. Changes in inventories are procyclical in the benchmark transaction costs model as in the post-war US data, while countercyclical in the previous models.

Figure 8 displays the dynamic responses of output, inflation, and money growth produced by the benchmark transaction cost model. As for the previous inventories models, the dynamic responses of the transaction costs model differ from that of the baseline model because producers can vary inventories to respond to changes in demand. In the transaction costs model a producer meets a larger demand by increasing price, reducing output, and depleting inventories. In making his decisions, he accounts for the cost of adjusting prices and the increasing marginal cost of production, as well as the impact of his output and inventories decisions on sales (*e.g.* a reduction of output reduces the stock of available goods and makes shopping more difficult). The producer changes price, output, and inventories to save on production costs. This behavior has two implications. First, changes in inventories are procyclical because both output and inventories are reduced. Second, sales are much more volatile than output. This occurs because the volatility of both sales and output declines, but the reduction is larger for output (given the impact of both available goods and prices on sales). Over time, the producer gradually replenish its inventories and manages the demand by smoothly increasing output.

We wish to verify the robustness of these results to the values of the coefficient of risk aversion σ and the additional parameter ξ. We are interested in the coefficient of risk aversion because the benchmark transaction costs model predicts that sales (and consumption) are much more volatile than output. We are not interested, however, by the other additional parameter γ, because it is set to remove steady-state transaction costs.

Our first experiment investigates the effects of the coefficient of risk aversion. An increase in this coefficient will raise the willingness of the consumer to smooth consumption, which may reduce the counterfactually large relative volatility of sales. For our experiment, we increase risk aversion by raising σ from 1 to 2. The results of this experiment appear as High Risk Aversion in Figure 7 and Table 2. A higher risk aversion coefficient reduces the relative volatility of sales, but also makes inventories countercyclical. It also raises the autocorrelations of output, but has no effect on the autocorrelations of inflation.

Finally, our second experiment investigates the effects of the steady-state level of the ratio of sales to all available goods. As before, a large steady-state level of this ratio is associated with a low convenience of having inventories and a low steady-state level of inventories. For our experiment, we raise the steady-state ratio of sales to available goods from 0.60 to 0.82 by reducing ξ from 0.0168 to 0.0123, while keeping $\sigma = 2$. The results of this experiment appear under & Low Convenience. Reducing the steady-state level of inventories has little impact on the autocorrelations of output and inflation. Changes in inventories become somewhat less countercyclical and the relative volatility of sales to output is reduced.

6. Conclusion

Postwar US business cycle fluctuations of output and inflation are remarkably persistent. Standard sticky-price monetary business cycle models with explicit microfoundations, however, fail to explain this persistence. Our objective is to determine whether adding inventories to a standard sticky-price monetary business cycle model raises the predicted persistence of output and inflation.

To fulfill this objective, we compare the persistence of output and inflation computed from three different models with inventories to the persistence computed in a model without inventories. Our three models with inventories are a linear-quadratic model, a factor of production model, and a transaction costs model. These models emphasize different roles for inventories. In the linear-quadratic model, producers manage inventories to avoid the costs associated with changing output and with having inventories deviate from a target fraction of sales. In the factor of production model, producers manage a stock of inventories that is an input in production. Finally, in the transaction costs model, producers manage inventories that affect the demand for its goods by making it easier for consumers to shop.

We find that the propagation properties of inventories depend on the role played by inventories. Adding inventories as in the linear-quadratic model or as in the factor of production model raises the persistence of output, but not sufficiently to replicate the persistence of output in post-war US data. Adding inventories as in these models has little effect on the persistence of inflation. Adding inventories as in the transaction costs model, however, significantly raises the persistence of both output and inflation. In fact, it raises the persistence of output to that observed in post-war US data. It also raises the persistence of inflation fairly close to that observed in post-war US data. Finally, we find that all three models counterfactually predict that sales are more volatile than output and that changes in inventories are countercyclical.

Data Appendix

Our quarterly post-war US sample covers the 1959:1 to 2000:1 period. It comprises the following: *Gross Domestic Product:* Bureau of Economic Analysis, NIPA Table 1.2; *Change in Private Inventories:* Bureau of Economic Analysis, NIPA Tables 1.2, 5.11A, 5.11B; *Private Inventories:* Bureau of Economic Analysis, NIPA Tables 5.13A, 5.13B; *Consumer Price Index:* Bureau of Economic Analysis, NIPA Table 7.1; *Investment:* fixed investment, Citibase, mnemonic GIFQF; *Population:* Citibase, mnemonic P16; and *M2 Money Stock:* FRED.

We construct per capita output Y_t and per capita inventories X_t by dividing Gross Domestic Product and Private Inventories by Population. Our measure of the price index P_t is the Consumer Price Index. Finally, we construct sales as $S_t = Y_t + X_t - X_{t+1} - (\phi_p/2)\left(P_t/(\pi^* P_{t-1}) - 1\right)^2 Y_t$ with $\phi_p = 72.01$ and $\pi^* = 1.04$, where the value of π^* is the average inflation over the sample. Finally, we construct quarterly M2 data by averaging the monthly data.

Results Appendix

Figures A1, A3, A5, and A7 display the autocorrelations of output and inflation, while Table A2 displays the relative volatility of sales, the relative volatility of changes in inventories, and the correlation between changes in inventories and output.

There are two main computational differences between these statistics and those shown in Figures 1, 3, 5, and 7, and in Table 2. The first difference is that, for both the post-war US sample and the models, all variables are detrended by the Hodrick-Prescott filter with a smoothness parameter of 1600. The second difference is that, for the models, the calibration is slightly different. The difference relates to the parameter ν that controls investment costs. This parameter is set to ensure that the ratio of the standard deviation of investment is 3.6 times that of output (as in our post-war US sample detrended with the Hodrick-Prescott filter).

Overall, these figures and the table suggest very similar conclusions. First, the baseline model fails to reproduce the persistence of output and inflation. Second, introducing inventories significantly raises the persistence of output, but marginally raises the persistence of inflation. Third, the models with inventories counterfactually predict that sales are more volatile than output and that changes in inventories are countercyclical. Finally, only the transaction costs model reproduces the post-war US persistence of output.

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Table 1: Benchmark Parameter Values

The Baseline Model	
Consumers	$\beta = 0.99$, $\sigma = 1$, $\omega = 0.94$, $\chi = 0.39$, $\psi = 1.7119$, $\delta = 0.025$, $\nu = 5.71$
Producers	$\Gamma = 1$, $\alpha = 0.64$, $\phi_p = 72.01$
Retailers	$\theta = 10$
Monetary Authority	$\rho = 0.72$, $\sigma_\mu = 0.006$
The Linear-Quadratic Model	
Consumers	$\beta = 0.99$, $\sigma = 1$, $\omega = 0.94$, $\chi = 0.39$, $\psi = 1.6967$, $\delta = 0.025$, $\nu = 14.6$
Producers	$\Gamma = 1$, $\alpha = 0.64$, $\phi_p = 72.01$, $\zeta_1 = 0.7$, $\zeta_2 = 0.25$, $\eta = 0.68$
Retailers	$\theta = 10$
Monetary Authority	$\rho = 0.72$, $\sigma_\mu = 0.006$
The Factor of Production Model	
Consumers	$\beta = 0.99$, $\sigma = 1$, $\omega = 0.94$, $\chi = 0.39$, $\psi = 1.7028$, $\delta = 0.025$, $\nu = 19.4$
Producers	$\Gamma = 1$, $\alpha = 0.64$, $\phi_p = 72.01$, $\ell = 0.3 \times 10^{-7}$, $\varepsilon = 5$
Retailers	$\theta = 10$
Monetary Authority	$\rho = 0.72$, $\sigma_\mu = 0.006$
The Transaction Costs Model	
Consumers	$\beta = 0.99$, $\sigma = 1$, $\omega = 0.94$, $\chi = 0.39$, $\psi = 1.7345$, $\delta = 0.025$, $\nu = 54.52$
Producers	$\Gamma = 1$, $\alpha = 0.64$, $\phi_p = 72.01$
Retailers	$\theta = 10$, $\gamma = 0.9906$, $\xi = 0.0168$
Monetary Authority	$\rho = 0.72$, $\sigma_\mu = 0.006$

Note: Several parameters are set endogenously. The values for ψ and ν ensure that hours worked are 30 percent of the time endowment in the steady state and that the ratio of the standard deviations of the logarithm of investment and the logarithm of output is 2.9. The values for η, ℓ, and ξ are set so that sales are 60 percent of all available goods (output and inventories) in the steady state. Finally, the value for γ is set to eliminate steady state transaction costs.

Table 2. Empirical Results: Sales, Inventories, and Output

	Volatility Relative to Output		*Correlation with Output*
	Sales	Inventories	Inventories
Post-war US	0.92	0.13	0.50
Baseline	1.00	—	—
Linear-Quadratic			
Benchmark	1.41	0.77	-0.24
Low Smoothing	1.23	0.35	-0.54
& High Yield Costs	0.92	0.10	0.79
Low Convenience	1.37	0.75	-0.21
Factor of Production			
Benchmark	1.66	0.96	-0.43
Low Elasticity	1.45	0.76	-0.35
Low Convenience	1.38	0.60	-0.45
Transaction Costs			
Benchmark	3.37	3.72	0.47
High Risk Aversion	2.63	1.88	-0.64
& Low Convenience	2.51	1.77	-0.61

Note: The variables are as follows: output refers to the logarithm of per capita real gross domestic product, inflation to the difference in logarithm of the consumer price index, sales to the logarithm of per capita real sales, and inventories to the ratio of changes in per capita inventories to per capita output. Variables are detrended by removing a linear trend and a quadratic trend before computations. Entries under *Volatility Relative to Output* are the ratio of the standard deviation of the variable to the standard deviation of output. Entries under *Correlation with Output* show the correlation coefficient between the variable and output. For each model, Benchmark refers to the calibration in Table 1. The alternative calibrations change the benchmark calibrations as follows. For the linear-quadratic model, the alternative calibrations are: Low Smoothing (ζ_1=0.01), & High Yields Costs (ζ_1=0.01 and ζ_2=4), and Low Convenience (η=0.24). For the factor of production model, the alternative calibrations are: Low Elasticity (ε=100) and Low Convenience (ℓ=0.5×10^{-10}). For the transaction costs model, the alternative calibrations are: High Risk Aversion (σ=2) and & Low Convenience (σ=2 and ξ=0.0123). The volatility and correlation are computed as the average from 1000 simulations of 164 periods.

Figure 1. Baseline
Autocorrelations of Output

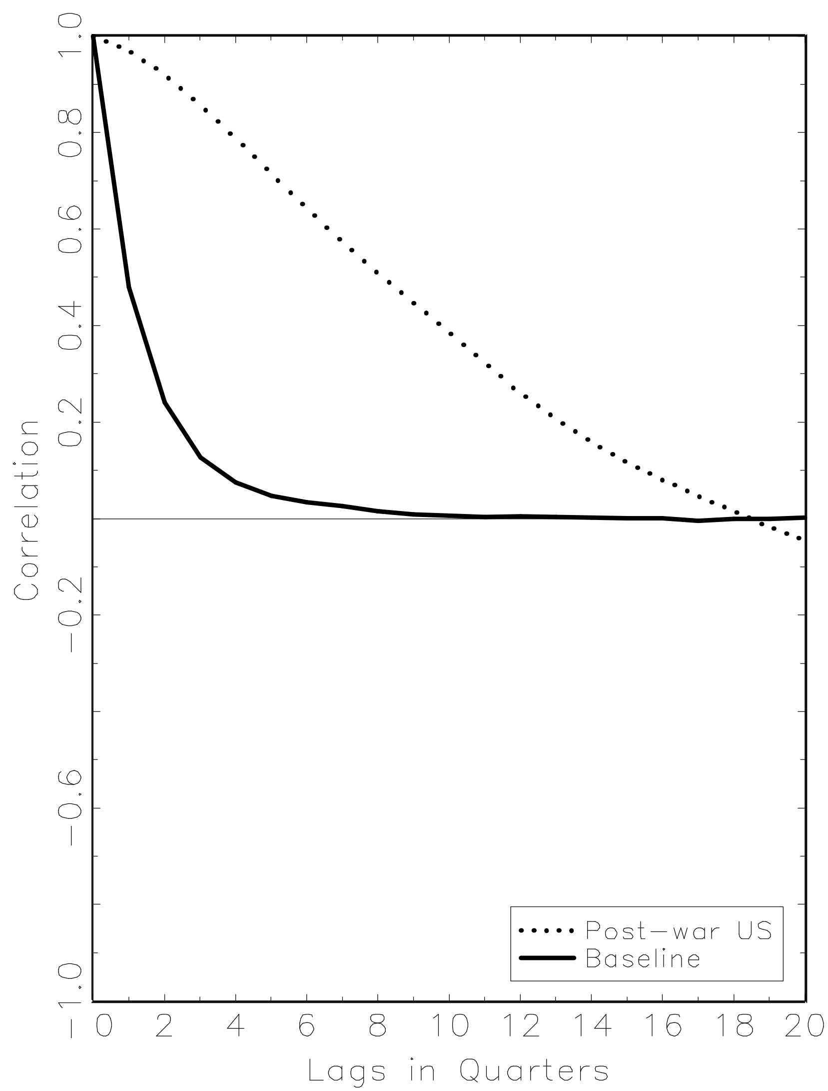

Figure 1. Baseline
Autocorrelations of Inflation

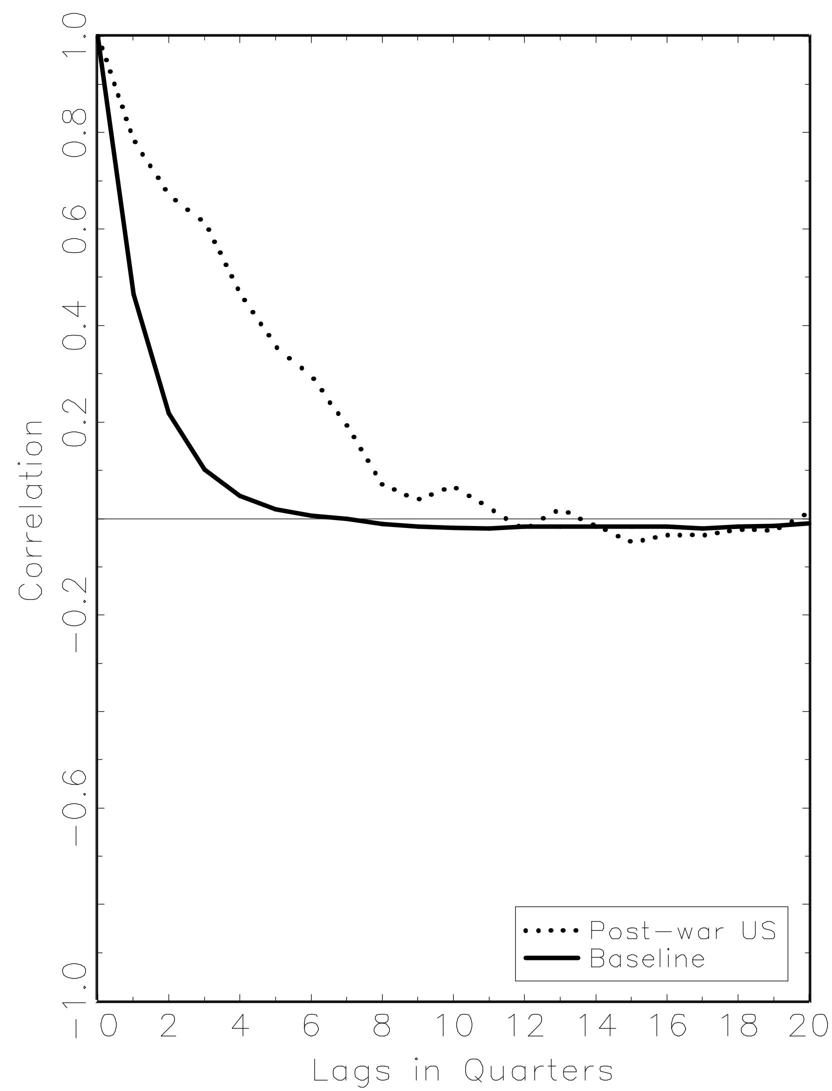

Figure 2. Baseline
Impulse Response Functions

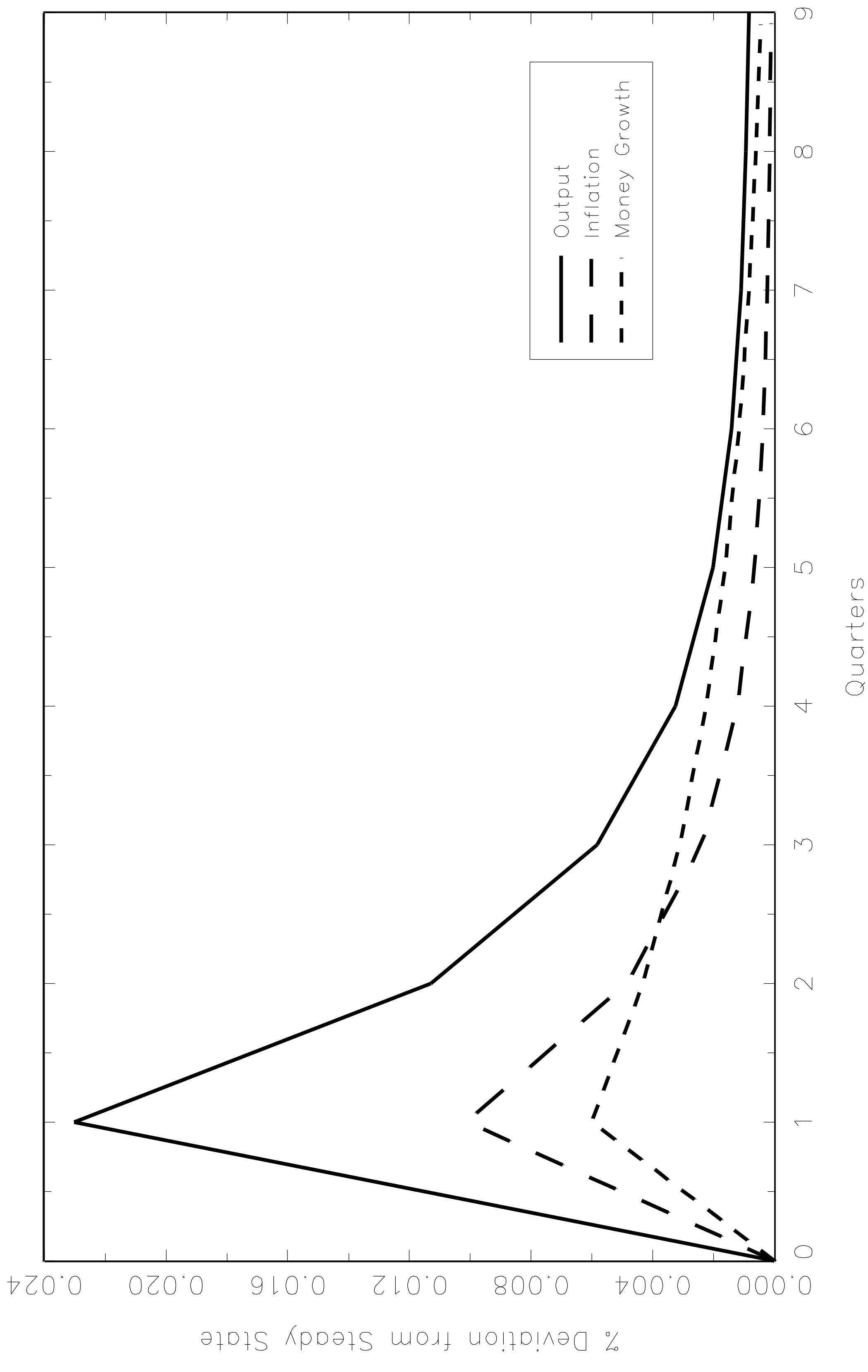

Figure 3. Linear−Quadratic
Autocorrelations of Output

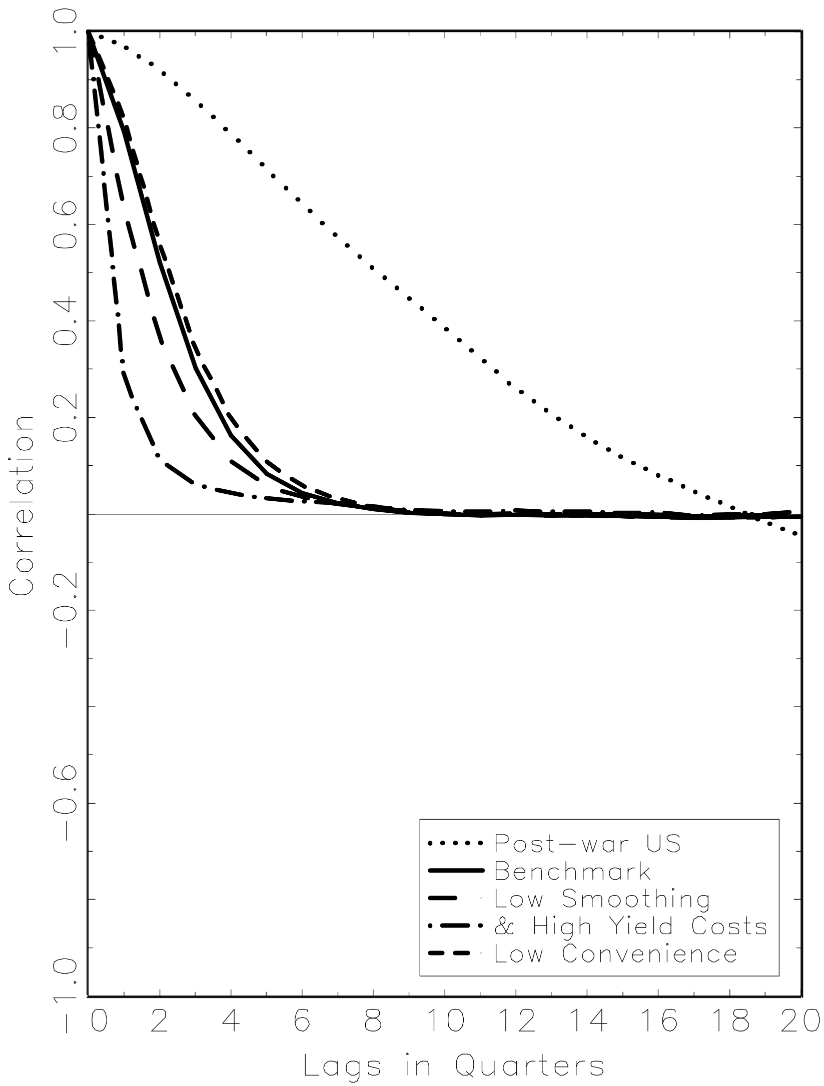

Figure 3. Linear−Quadratic
Autocorrelations of Inflation

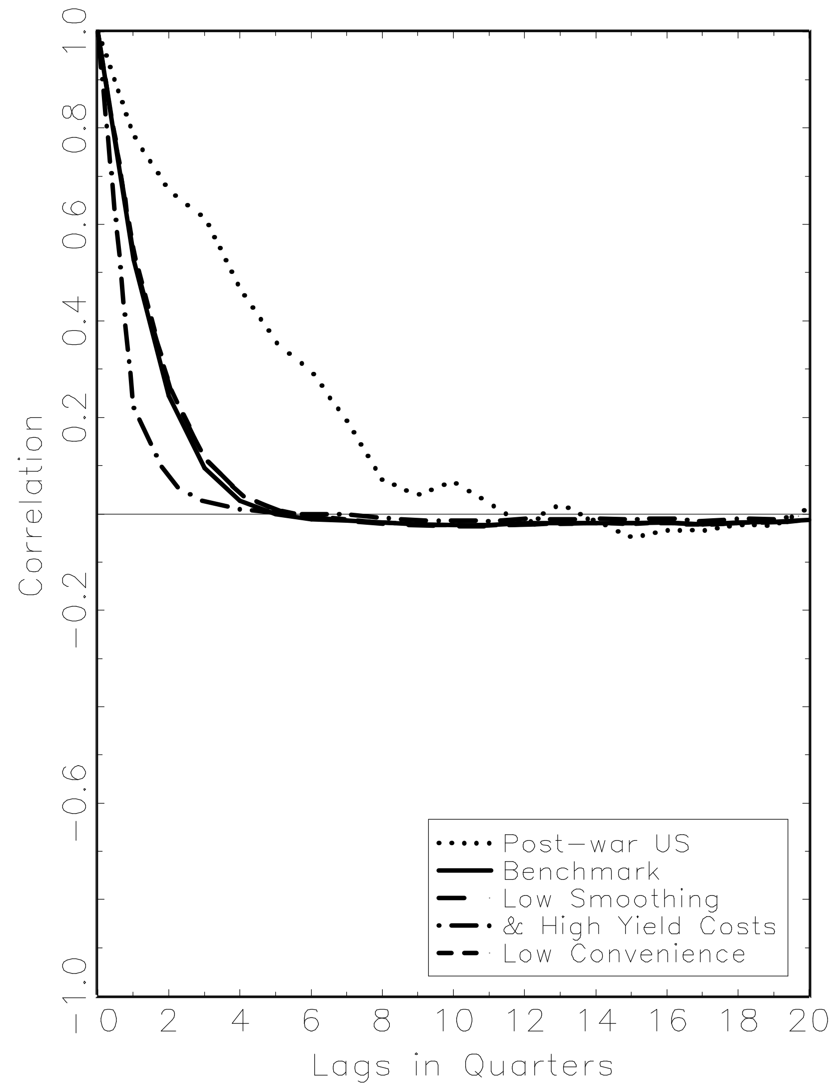

Figure 4. Linear–Quadratic
Impulse Response Functions

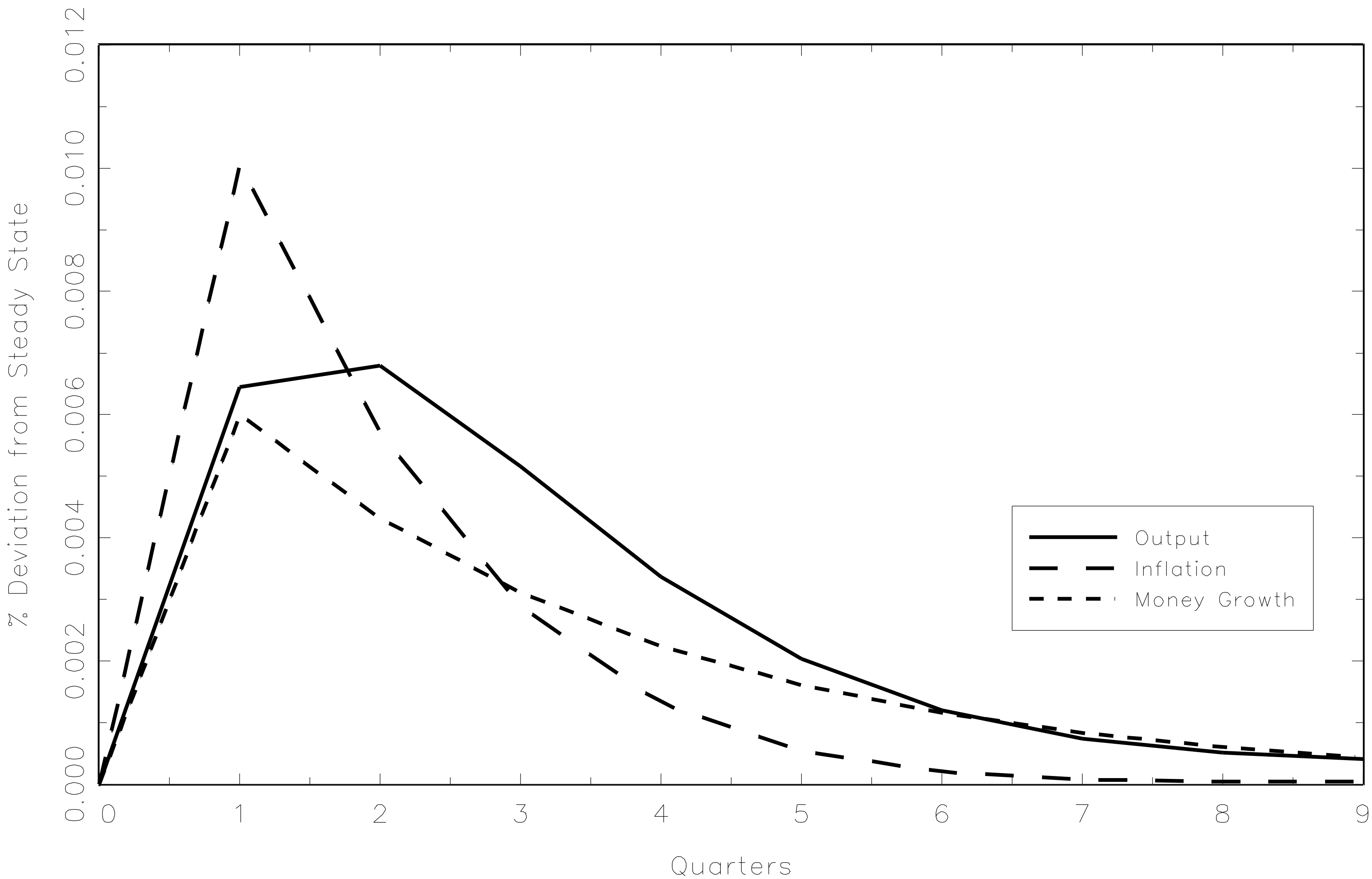

Figure 5. Factor of Production
Autocorrelations of Output

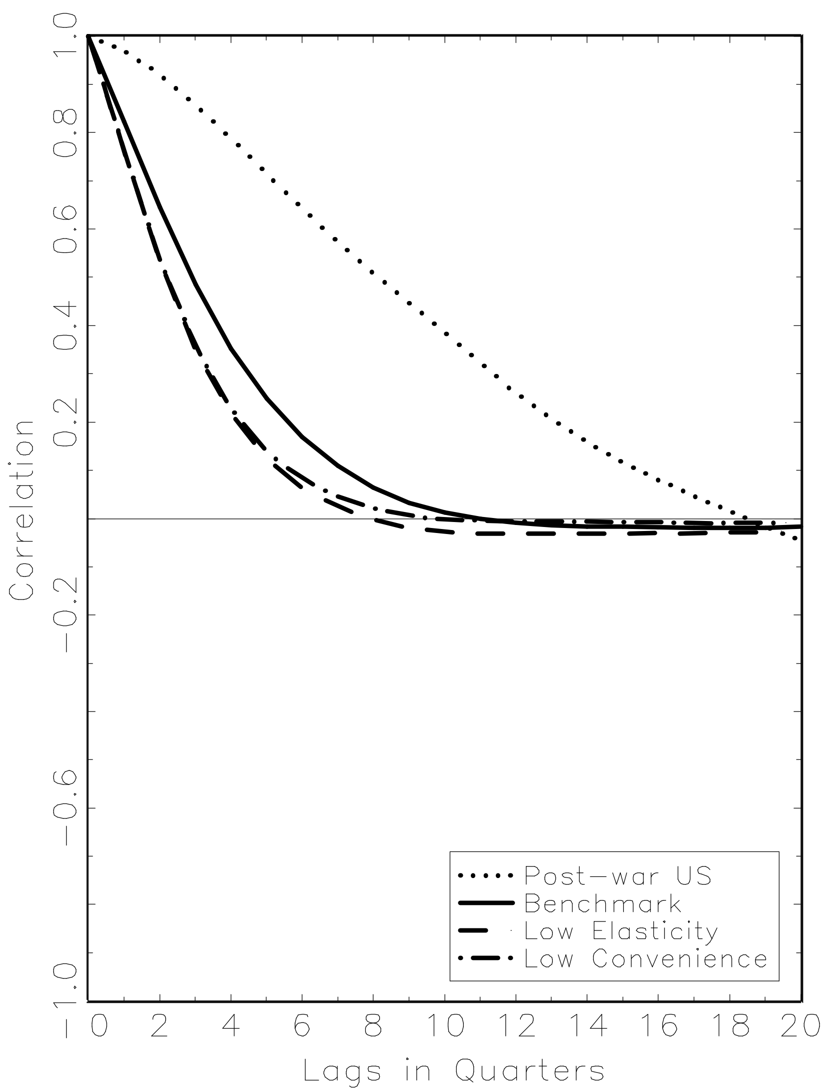

Figure 5. Factor of Production
Autocorrelations of Inflation

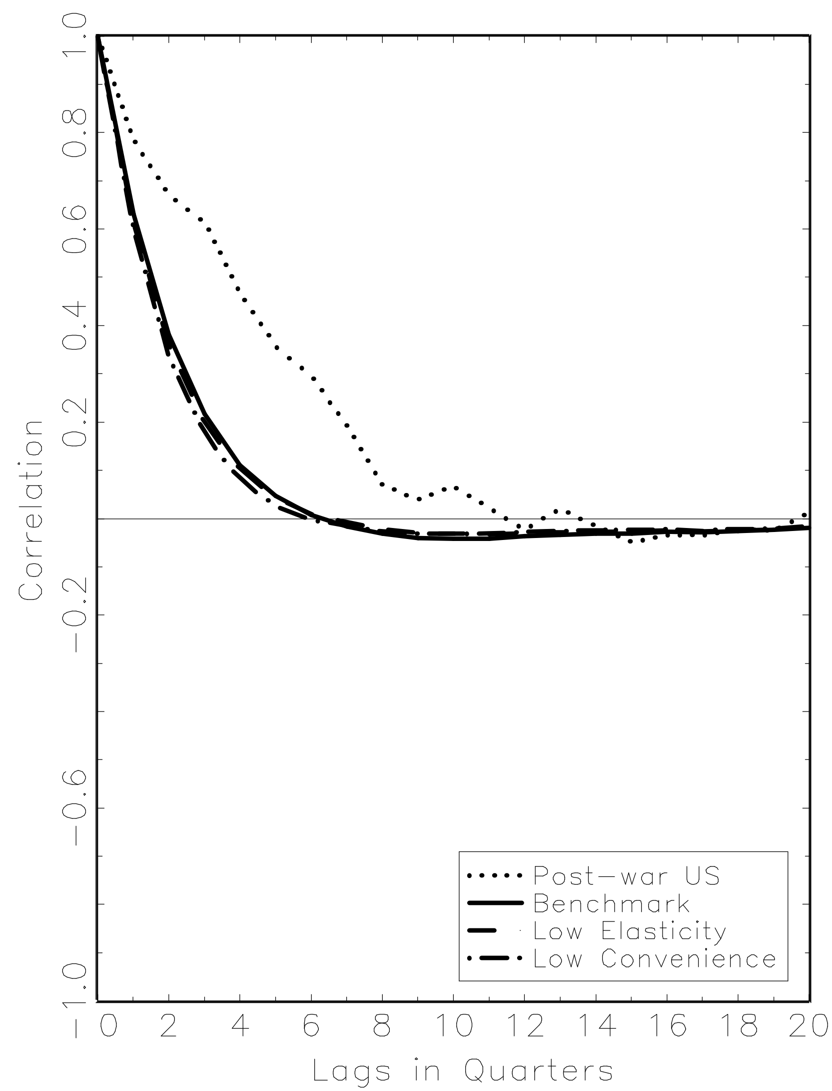

Figure 6. Factor of Production
Impulse Response Functions

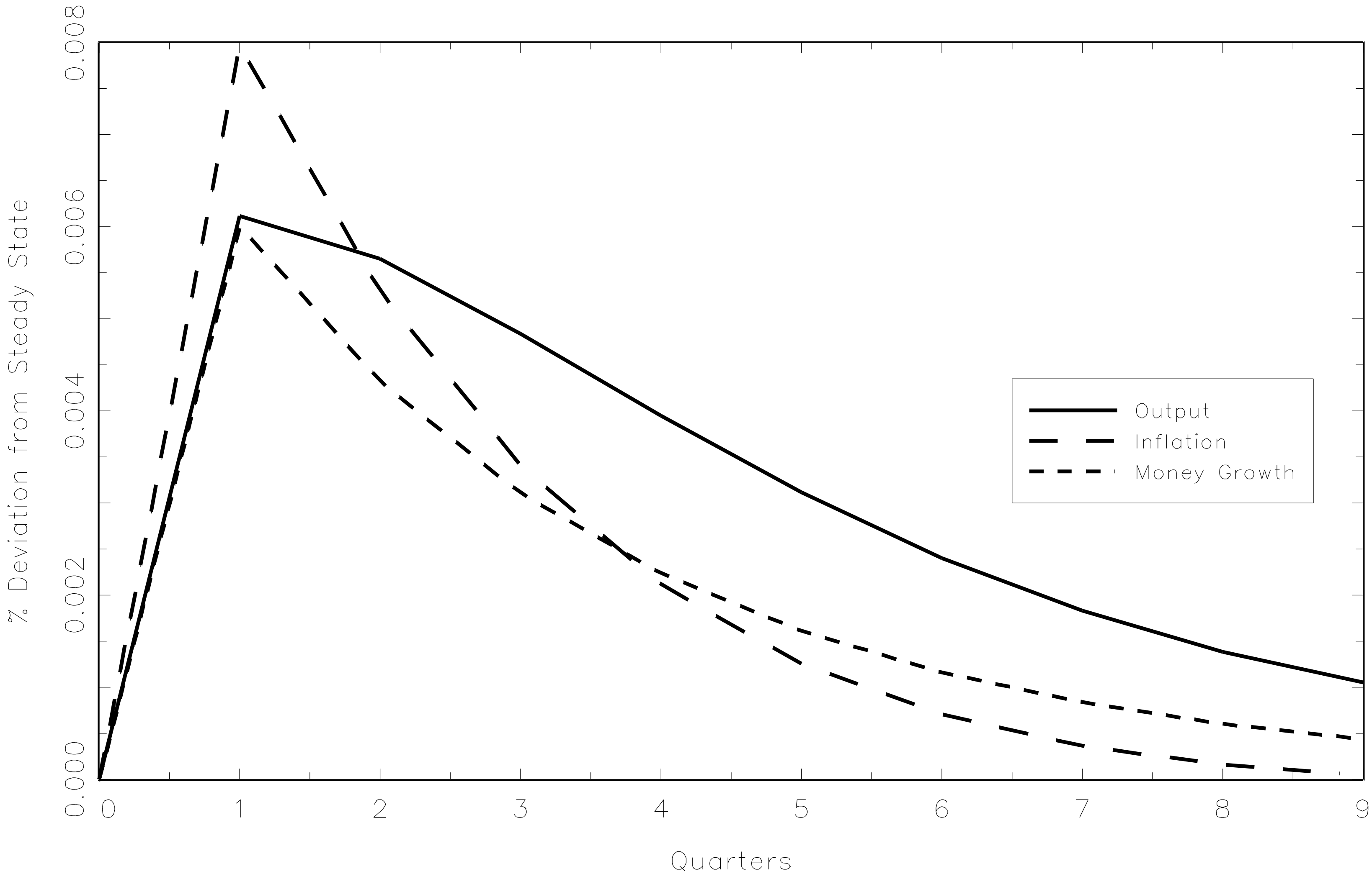

Figure 7. Transaction Costs
Autocorrelations of Output

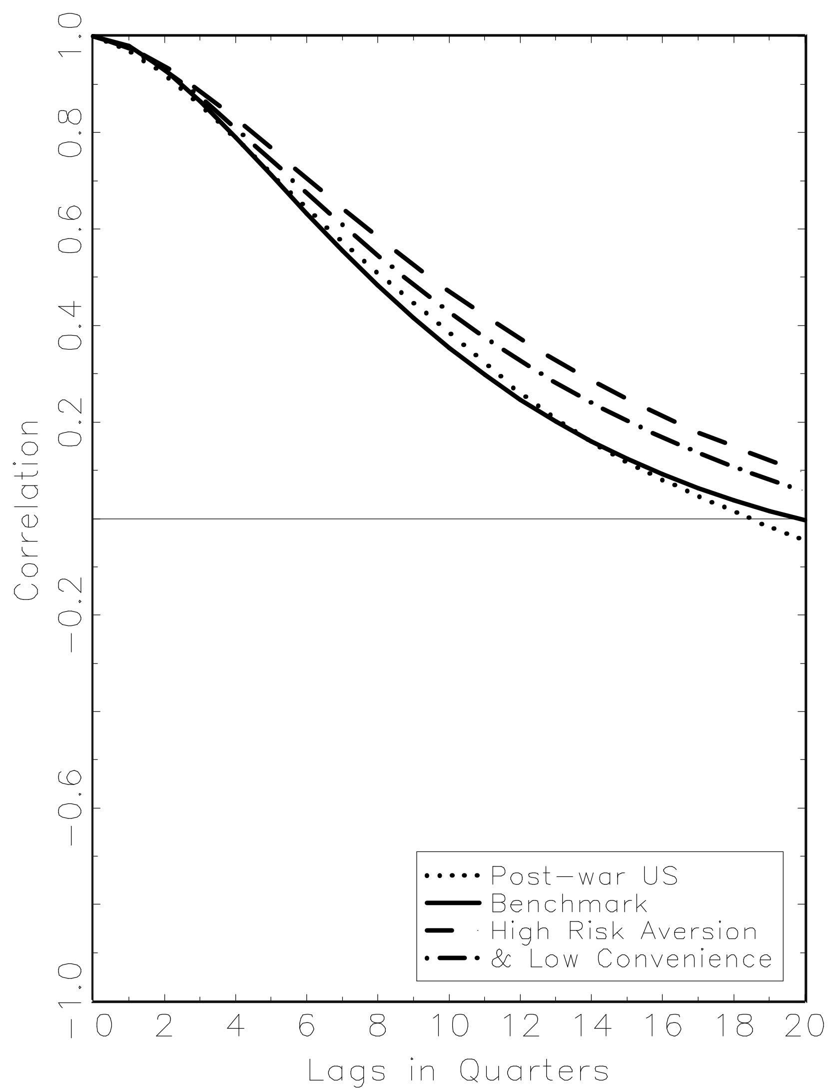

Figure 7. Transaction Costs
Autocorrelations of Inflation

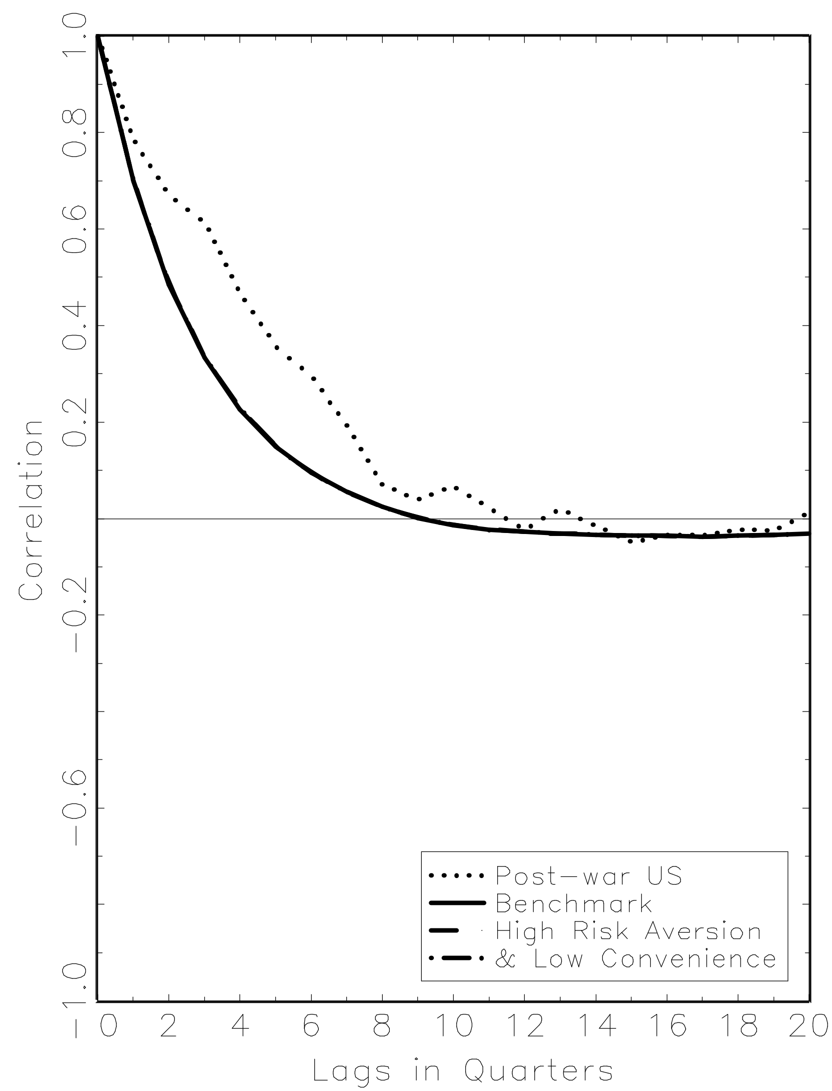

Figure 8. Transaction Costs
Impulse Response Functions

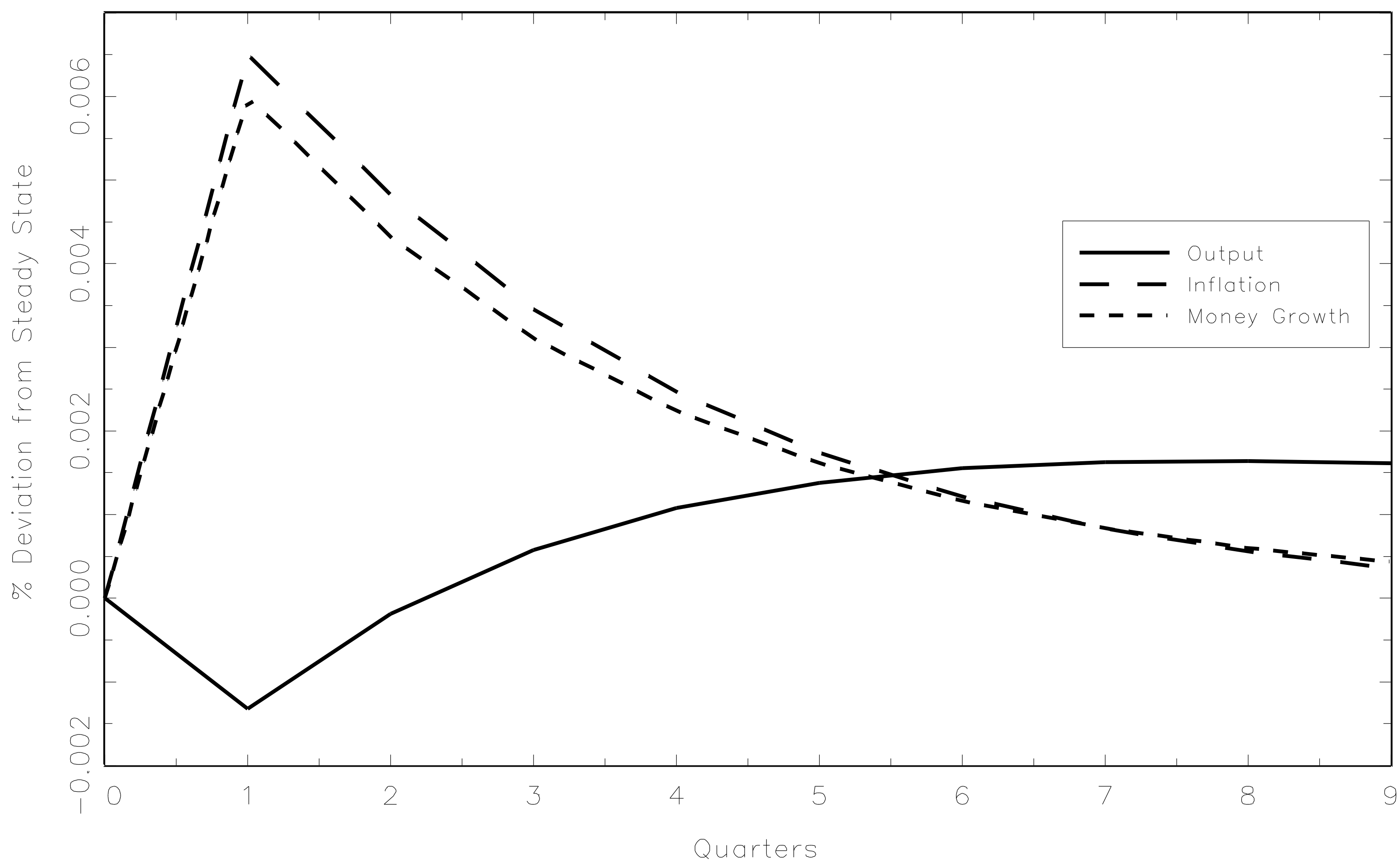

Table A2. Empirical Results of Hodrick-Prescott Filtered Data

	Volatility Relative to Output		*Correlation with Output*
	Sales	Inventories	Inventories
Post-war US	0.85	0.28	0.60
Baseline	1.00	—	—
Linear-Quadratic			
Benchmark	1.60	0.98	-0.31
Low Smoothing	1.34	0.45	-0.63
& High Yield Costs	0.99	0.02	0.61
Low Convenience	1.57	0.96	-0.28
Factor of Production			
Benchmark	2.00	1.25	-0.58
Low Elasticity	1.67	0.93	-0.48
Low Convenience	1.56	0.76	-0.56
Transaction Costs			
Benchmark	5.31	6.03	0.76
High Risk Aversion	3.46	2.53	-0.91
& Low Convenience	3.33	2.41	-0.89

Note: The variables are as follows: output refers to the logarithm of per capita real gross domestic product, inflation to the difference in logarithm of the consumer price index, sales to the logarithm of per capita real sales, and inventories to the ratio of changes in per capita inventories to per capita output. All variables are detrended using the Hodrick-Prescott filter (smoothing parameter set to 1600) before computations. Entries under *Volatility Relative to Output* are the ratio of the standard deviation of the variable to the standard deviation of output. Entries under *Correlation with Output* show the correlation coefficient between the variable and output. For each model, Benchmark refers to the calibration in Table 1. The alternative calibrations change the benchmark calibrations as follows. For the linear-quadratic model, the alternative calibrations are: Low Smoothing (ζ_1=0.01), & High Yields Costs (ζ_1=0.01 and ζ_2=4), and Low Convenience (η=0.24). For the factor of production model, the alternative calibrations are: Low Elasticity (ε=100) and Low Convenience ($\ell=0.5\times10^{-10}$). For the transaction costs model, the alternative calibrations are: High Risk Aversion (σ=2) and & Low Convenience (σ=2 and ξ=0.0123). The volatility and correlation are computed as the average from 1000 simulations of 164 periods.

Figure A1. Baseline
Autocorrelations of Output

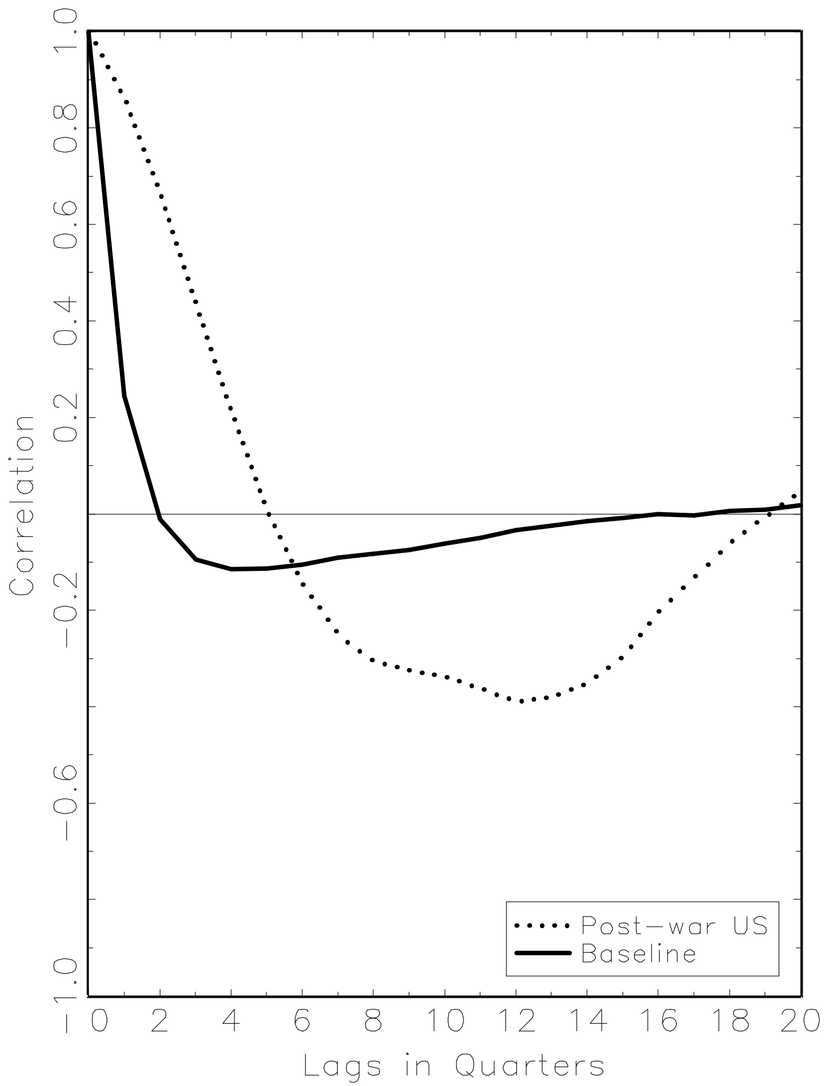

Figure A1. Baseline
Autocorrelations of Inflation

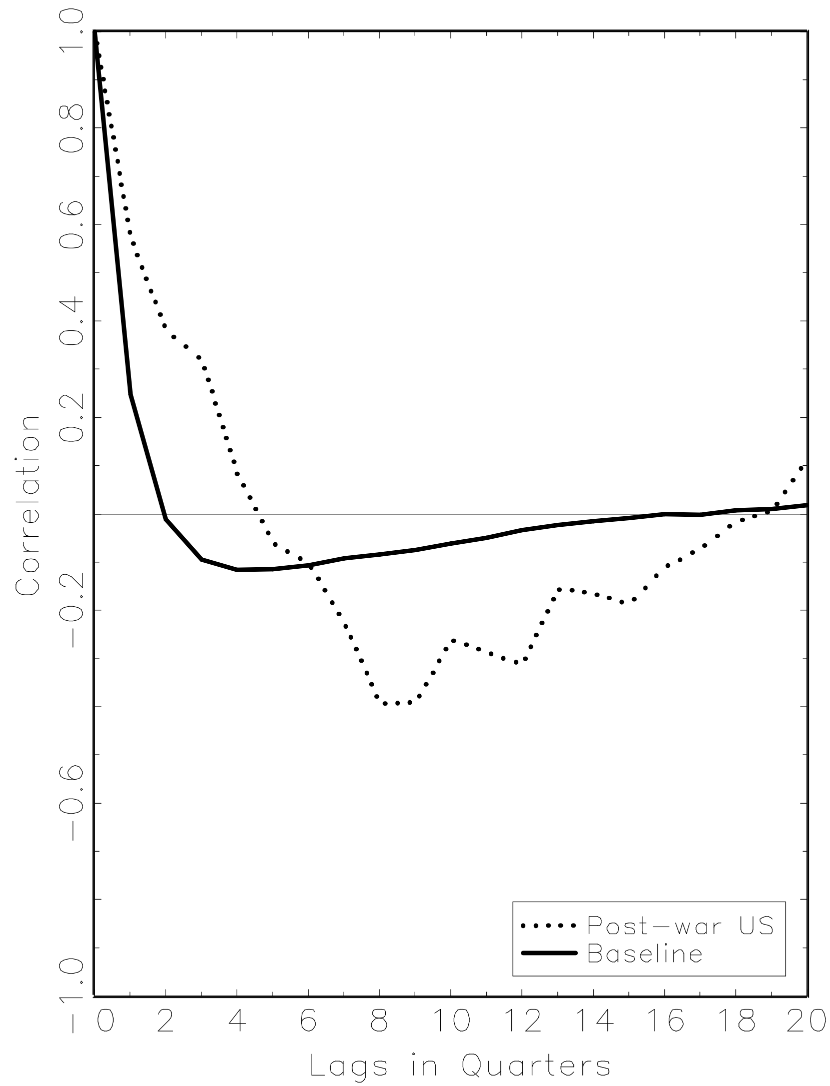

Figure A3. Linear–Quadratic Autocorrelations of Output

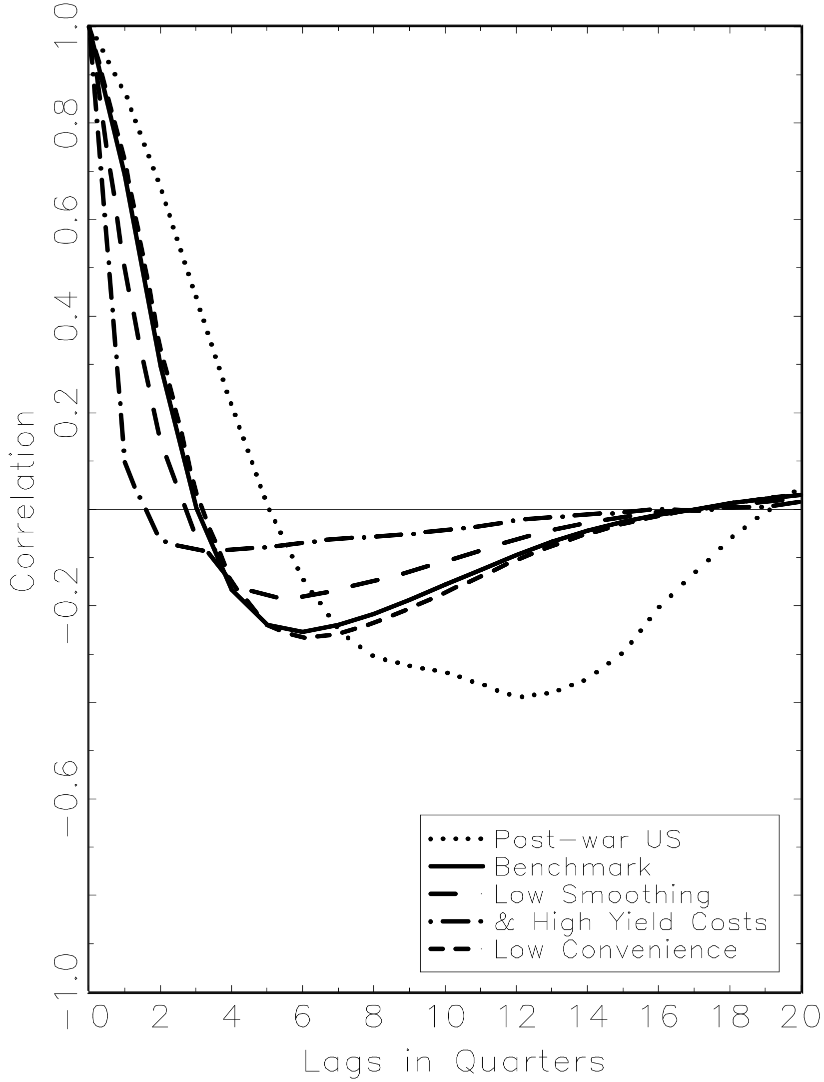

Figure A3. Linear–Quadratic Autocorrelations of Inflation

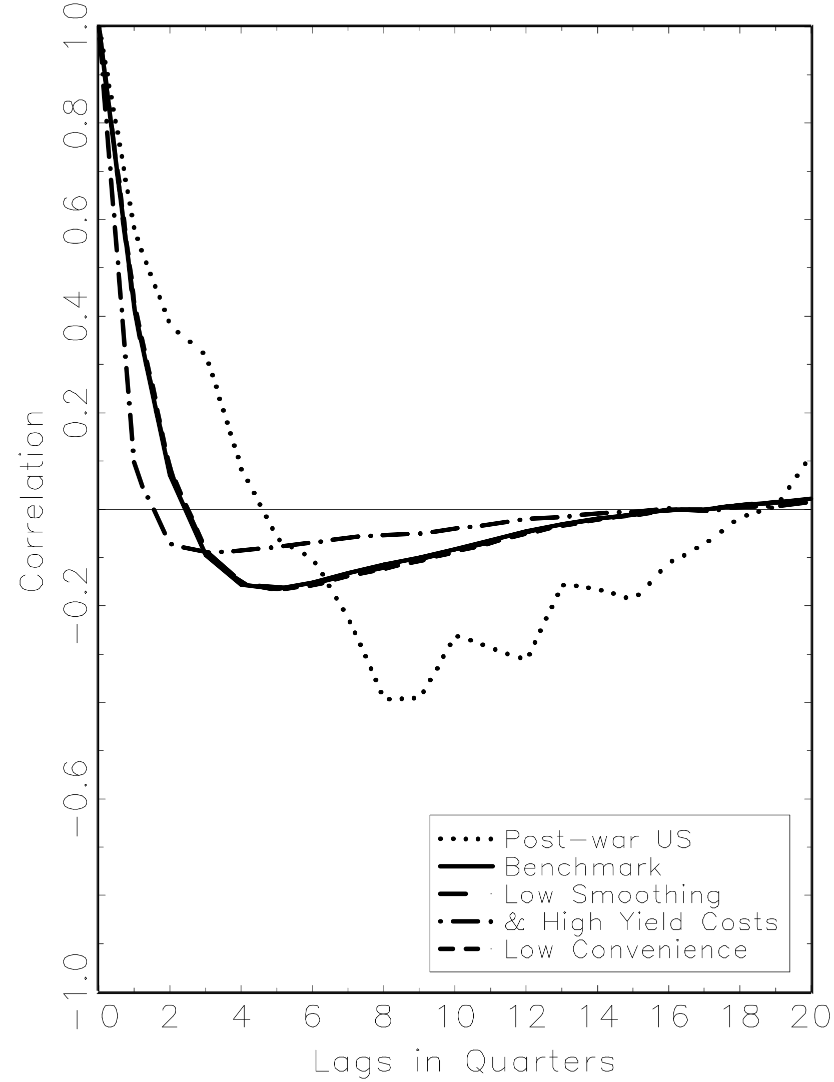

Figure A5. Factor of Production
Autocorrelations of Output

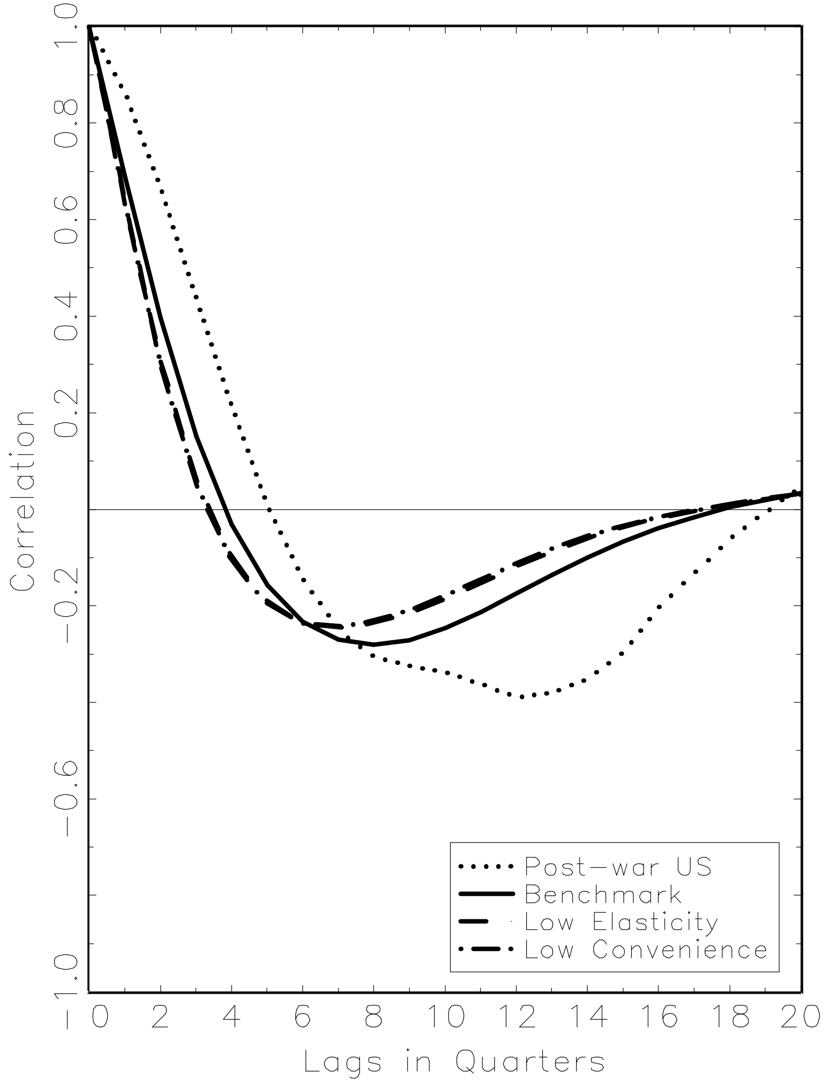

Figure A5. Factor of Production
Autocorrelations of Inflation

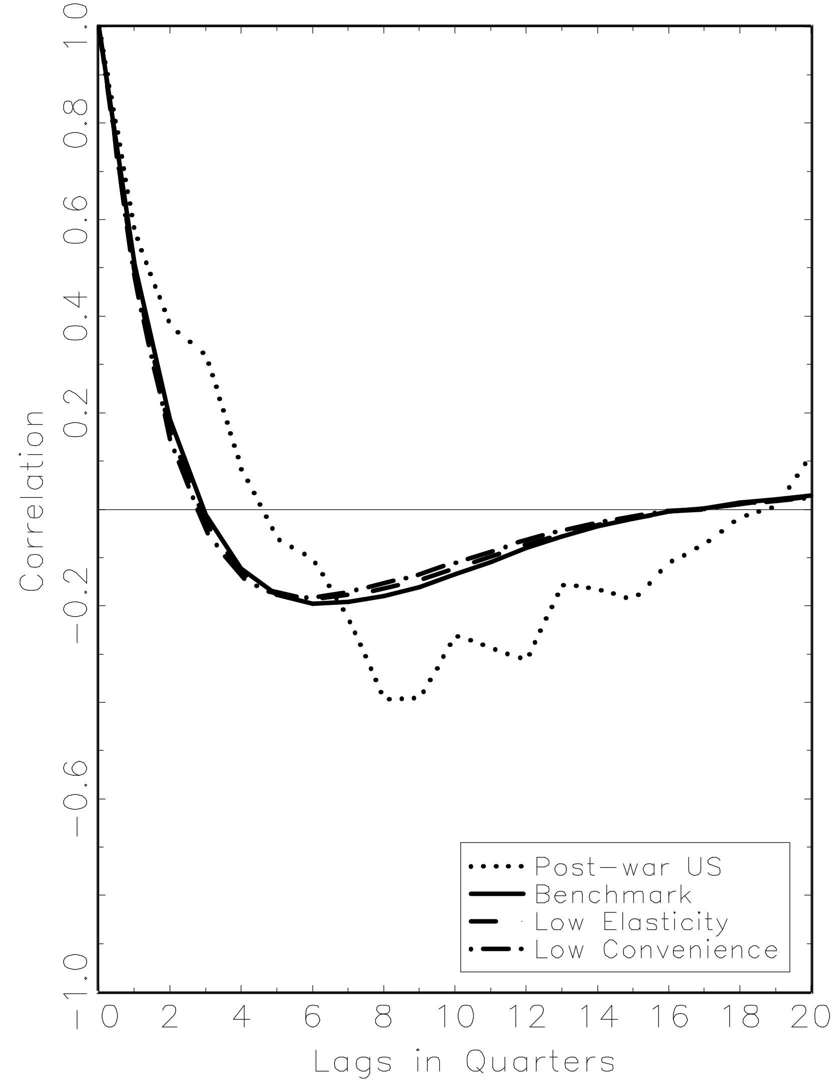

Figure A7. Transaction Costs
Autocorrelations of Output

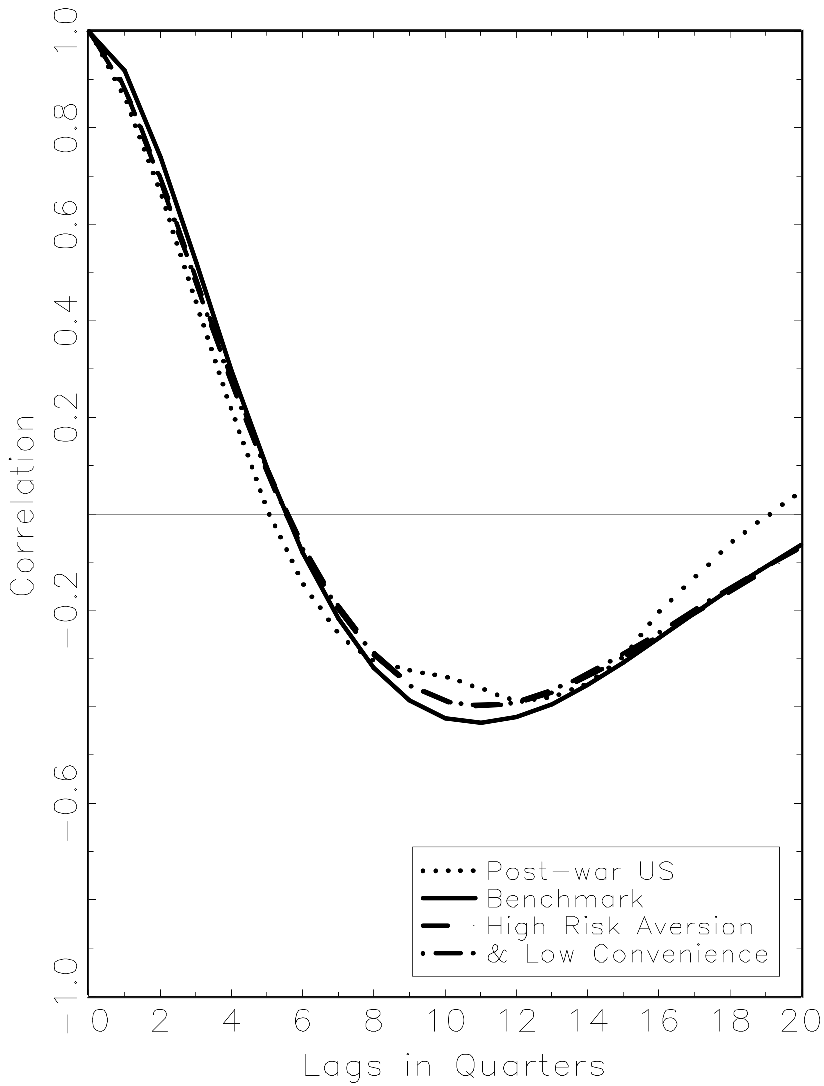

Figure A7. Transaction Costs
Autocorrelations of Inflation