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The Valuation of Public Goods: Why Do We Work?

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Abstract

Conventional analysis of public goods provision aggregates individual willingness to pay while treating income as exogenous. This process ignores the fact that we generate income to allow us to purchase utility-generating goods. From the individual perspective, there exists an optimal level of income generated that would be associated with an optimal level of public goods. To the extent that there is an output market failure in the provision of public goods, there will also exist an accompanying input failure in labor supplied/leisure demanded. We explore the implications of this input market failure by explicitly considering leisure demand in a model of public goods provision. We consider two areas of public goods analysis, benefit analysis and optimal provision. Based on our analysis, we conclude that by misinterpreting the basic nature of income, conventional analysis generally leads to an incorrect provision of public goods.

1. Introduction

The public goods provision problem is undoubtedly one of the most celebrated economic problems and also one of the most difficult to solve. While Samuelson (1954) presented us with a characterization of socially optimal provision, he ended on a pessimistic note, suggesting that no decentralized pricing mechanism will yield the optimal levels of public goods, because "it is in the selfish interest of each person to give *false* signals, to pretend to have less interest in a given collective consumption activity than he really has, etc." While demand revealing mechanisms have been proposed (*e.g.* Clarke (1971) and Groves and Ledyard (1977)), the bulk of public goods analysis is still carried out by welfare economists attempting to weigh the benefits of additional provision against the costs or in exceptional cases, attempting to find the optimal level of provision by applying Samuelson's results.

Absent from Samuelson's analysis and almost all welfare analysis of public goods provision is the explicit consideration of the demand for leisure or its mirror image, the supply of labor. While a closely related issue, the relationship between the optimal allocation of public goods and distribution of wealth, has been analyzed,[1] these papers fail to explicitly consider leisure demand/labor supply and its implications for the analysis of public goods. With the exception of the appendix in Just et al. (1982), the leisure/labor tradeoff is absent from textbook treatments of welfare economics which is one of the best indicators of conventional thinking. The general equilibrium literature, including papers that analyze the so called double dividend is one area of exception where leisure/labor tradeoffs are explicitly modeled.[2] However, as pointed

[1]Musgrave (1969) raises the problem that one cannot separate issues of optimal public goods provision and optimal distribution of wealth. Bergstrom and Cornes (1983) provide a general class of preferences that avoid Musgrave's problem. Flores (forthcoming) explores the relationship between income distribution and willingness to pay in the context of non-paternalistic altruism.

[2]The double divided is the potential welfare gain that may occur when revenues from pollution taxes are used to reduce distortionary labor taxes. For an overview of this literature see Parry and Oates (2000).

out by Schwartz and Repetto (2000), most of these analyses suffer from the assumption of separability of public goods and other goods, including leisure. Though separability is now being addressed by general equilibrium modelers, estimates of the marginal utility of public goods used in calibrating these models are likely to come from partial equilibrium welfare analyses that ignore leisure/labor tradeoffs. Hence overall, public goods provision analysis is mired in a conceptual regime that, to varying degrees, ignores leisure/labor tradeoffs.[3]

In this paper we explicitly consider leisure/labor tradeoffs in a model of public goods provision and address what we believe are several important questions for the analysis of public goods. First, how does the individual's ability to choose or not choose the level of public goods affect the generation of income? Second, what are the implications of treating income as exogenous in benefits estimation? Finally, what are the implications of treating income as exogenous for the determination of optimal levels of public goods under a Samuelson type rule?

2. Public Goods and Input Market Failure

Public goods provision is problematic because everyone faces the same level of public goods (common provision) and benefits are non-exclusive. While together these two characteristics give rise to the free-rider problem in the output market, common provision in particular implies that people do not get to purchase public goods up to their most preferred level.[4] The inability to *individually* buy public goods means that individuals make different leisure/labor choices than would be made if they were able to individually choose the level of

[3]Another area where researchers have considered labor supply decisions is with regard to recreational demand literature. The emphasis in this literature has been on inferring a more precise value of time that can then be used in calculating travel costs. A rigorous treatment is provided in Bockstael et al. (1987).

[4]Rationed goods generally result in this problem, even if private. Many pundits argue that rationing of consumption created the disincentive for work that ultimately led to the collapse of the Soviet state. The input market failure we identify is closely related, though distinct from the incentive problems that occurred in the Soviet Republics under communism.

public goods.[5] In a very real sense, there is a heretofore unrecognized input market failure that corresponds to the output market failure: inability to *individually* buy the public good will result in a reduced desire to generate income.

Note that our observation is distinct from the well-recognized point made by Musgrave (1969) that the optimal level of public goods cannot be determined independently of the "optimal" distribution of income where the optimal distribution is judged by a particular social welfare function. It is generally true, even when allowing for leisure/labor choice, that if the income distribution changes through transfers, then individuals' relative values for the public good will change and subsequently the optimal level of the public good. Our point, however, is that the overall amount of income generated is too low as a result of an "input market failure." People work to acquire goods. If a certain class of goods, public goods in this case, cannot be acquired by parting with income, rational individuals will not incur the effort to generate that income.

In order to more establish the implications for income generation, we begin with the simplest of examples. Suppose that a consumer has preferences for a vector of market goods, X, a public good, q, and leisure, L that are represented by a simple Cobb-Douglas utility function, $U(X, q, L) = \alpha_X \log(X) + \alpha_q \log(q) + \alpha_L \log(L)$.[6] The consumer faces a constraint on the amount of hours worked, H, and chooses to spend time in leisure, L, or working. The hourly wage earned working is w and $H - L$ is the hours of labor supplied. The budget constraint is given by $pX + \tau q + wL = wH$ where τ is the consumer's unit tax rate for q. The consumer is

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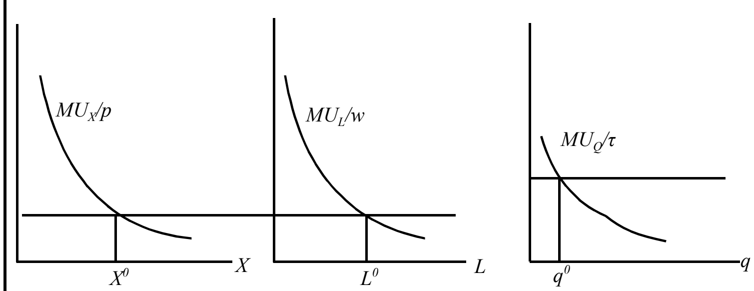
[5]Samuelson assumes this interconnectedness away: "Provided economic quantities can be divided into two groups, (1) *outputs* or goods which everyone always wants to maximize and (2) inputs or factors which everyone always wants to minimize, we are free to change the algebraic signs of the latter category and from then on to work only with 'goods,' knowing that the case of factor inputs is covered as well."

[6]Here we are not claiming that *all* individuals have these same Cobb-Douglas preferences or even Cobb-Douglas preferences at all. Rather we are simply using these preferences to explain the relationship between public goods and income generated..

able to choose the levels of the market goods and the amount of leisure, but not q. Now due to the revelation problem identified by Samuelson, q is under-provided given the consumer's preferences. The marginal utility of q relative to its price exceeds the marginal utility of any of the market goods relative to price or the marginal utility of leisure relative to the wage as shown in Figure 1. Without loss of generality, in this section we define X as a scalar in order to avoid subscripts that distinguish among the different market goods.

Figure 1

Price-Scaled Marginal Utility with Public Goods Under-Provision

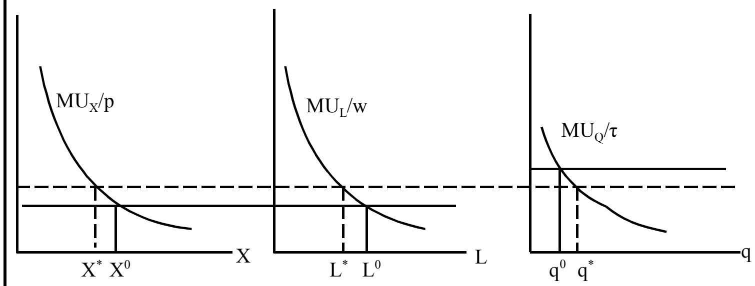


Because of the separable form of utility, the curves for each good in Figure 1 do not shift with changes in the levels of public goods, the reason for providing price scaled marginal utility as opposed to demand.[7] In looking at the graphs, it is easy to see that qualitatively, this individual would like to increase the purchase of the public good at the expense of market good expenditures and leisure. The optimal choice of q for this individual is accompanied by less leisure, *i.e.* more work and earned income, and less consumption in market goods as shown in Figure 2.

[7] The curves are similar in that they are given by the relevant α utility parameter divided by the price multiplied by the level of the good. For example, the price scaled marginal utility of market goods is $\alpha_X/(p\ X)$.

Figure 2

Price-Scaled Marginal Utility
Optimal versus Under-Provision



From the graphs we can see that earned income is greater at the most preferred level of the public good, but by how much? To this end we contrast earned income between the extreme cases of free riding, $q^0 = 0$ and optimal provision for a handful of parameter values. By considering the ratio of observed hours worked to the hours worked under optimal provision, $(H - L^0)/(H - L^*)$, we only need to consider the utility parameter values.[8]

Table 1
Ratio of Earned Income: Free-Riding versus Optimal Provision

α_X	α_q	α_L	Proportion of Earned Income $(H - L^0)/(H - L^*)$
0.45	0	0.55	1
0.40	0.10	0.50	0.89
0.35	0.20	0.45	0.80

For these parameter values we see that free-riding results in a 0 to 20% reduction in earned income relative to optimal provision for an individual, depending on preferences for the

[8] In this case, $(H - L^0)/(H - L^*) = [H - (H\alpha_L)/(\alpha_X + \alpha_L)]/[H - (H\alpha_L)/(\alpha_X + \alpha_L + \alpha_q)] = [\alpha_X/(\alpha_X + \alpha_L)]/[(\alpha_X + \alpha_q)/(\alpha_X + \alpha_L + \alpha_q)]$.

public good. The proportion of earned income depends on all of the parameter values. Optimal leisure demanded relative to leisure demanded under free-riding depends only on α_q in the case when $\alpha_X + \alpha_q + \alpha_L = 1$, $L^*/L^0 = 1 - \alpha_q$. The stronger the preference for q, the less leisure demanded at the optimum relative to free riding.

The intuition for our simple model above is most readily grasped in comparing individuals of two distinct types. First consider Wanda Lott, who has very strong preferences for market goods and cares little for public goods (*e.g.* environmental quality). In her pursuit of market goods consumption, Wanda will want to generate a high income to finance her desired high consumption levels. She will work long hours at her job to acquire the goods she desires in such abundance. She will do this because she knows that, if she gets the income to do it, she can have the goods she wants.

In stark contrast to Wanda Lott is Sten (for strong environmentalist) who has strong preferences for environmental quality and only modest desires for ordinary goods. If environmental goods (say, species preservation, air quality and CO_2 abatement, for concreteness) *could* be bought like an ordinary good, Sten would generate the income to buy it. He would be observed buying it, until the marginal values per dollar spent were equated across each argument of his utility function (exhibiting a large total valuation for the environment and a high marginal valuation for any quantity of the environment substantially smaller than at his optimal bundle). However, Sten knows he cannot, in fact, affect environmental quality by his individual demands; he is too small to make a difference, say, in saving a species or in CO_2 abatement. So, Sten, and others like him, both consume their small demands for ordinary goods and *reduce their income* by increasing their leisure, variables they can affect. Indeed, Sten is also likely to invest less in education than Wanda Lott and those like her. Sten will do this because he knows that, even if he gets the income, he cannot have the goods he wants.

Sten's true willingness to pay, *if he could purchase public goods in the same way he can private goods,* would be much larger than his apparent willingness to pay at the low income he generates in a world in which he cannot so purchase public goods. After all, Sten is poor because he optimally chose not to work. In a very real sense, the *same* market failure that causes non-optimal provision of public goods in the market setting also creates a market failure in input markets: the failure to generate income when that income cannot buy what we would like.

3. Benefit Analysis and Input Market Failure

Standard welfare analysis for valuing a change in a quantity rationed public good typically treats income as being fixed. This implicitly assumes that leisure demanded/labor supplied, is constrained to the status quo level. It generally is true that if we constrain a person to the same level of leisure, then the gain in utility for an increase in the public good will be less than if we allow for adjustment in leisure. In turn, the compensating variation/willingness to pay for the increase under constrained leisure choices will be less than willingness to pay under a regime of free choice of leisure. Similarly for the same reduction, equivalent variation/willingness to accept compensation will also be less than if we had allowed for leisure adjustment.

Our general model is very similar to standard models of public goods provision. A representative consumer's preferences are defined over a vector of market goods, X, a public good, q, but now we add leisure, L. Though we use a representative agent in developing our analytical points, it is not critical that everyone have identical preferences. We focus on a representative consumer because our observations need only be discussed in the context of a single individual; the main points carry directly over to multiple consumers. While our model is similar in spirit to the typical public goods analysis, the addition of leisure differs from the standard analysis. Some, but not necessarily all, market goods and leisure are assumed necessary goods.[9] The primal problem is to maximize utility by choosing levels of market goods and leisure in a single period subject to the same budget constraint as in the earlier example, $pX + \tau q + wL = wH$, and a rationed level of the public good $q = q^0$. The formal problem is stated as follows.

$$\max_{\{X,\ L\}} \; U(X,q,L) \quad s.t. \quad pX + Lw + \tau q \leq Hw,\ q = q^0 \tag{1}$$

The ordinary demands for market goods will depend on prices, the wage rate, the tax rate for the public good, and the level of the public good, $X^m = X^m(p,w,\tau,q)$, $L^m(p,w,\tau,q)$.[10] While these ordinary demands dictate the observable behavior, they do not provide the proper insights into the monetary measures used in welfare analysis. In order to develop proper welfare measures, we consider the dual problem of minimizing expenditures on market goods and leisure subject to the level of q and the level of utility.

$$\min_{\{X,\ L\}} \; pX + Lw + \tau q \quad s.t. \quad U(X,q,L) = U^0,\ q = q^0 \tag{2}$$

The solutions to the dual problem depend on prices, the wage rate, the tax rate on the public good, the level of the public good, and the level of utility, $X^h = X^h(p,w,\tau,q,U)$,

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[9]The marginal utility of necessary goods tends to infinity as the level tends to zero.

[10]The *m* superscript refers to ordinary, Marshallian solutions.

$L^h = L^h(p, w, \tau, q, U)$.[11] Using the demands, we can easily represent the amount of wealth adjustment that would leave our consumer indifferent between obtaining an increase in the public good from an initial level q^0 to a new, higher level q^1. As in standard welfare analysis that treats income as exogenous, the wealth adjustment that makes the consumer indifferent is referred to as compensating variation and is equal to the difference in the minimized expenditures.

$$\begin{aligned} CV &= p \cdot [X^h(p,w,q^0,U^0) - X^h(p,w,q^1,U^0)] \\ &\quad + w \cdot [L^h(p,w,q^0,U^0) - L^h(p,w,q^1,U^0)] + \tau[q^0 - q^1] \end{aligned} \tag{3}$$

In order to contrast (3) with a standard welfare analysis that treats leisure demand as fixed, we constrain the choice of our consumer while providing the same increase in the public good. The minimization problem will be identical to the problem in (2) with the exception that leisure is constrained to the status quo level of leisure which we will refer to as L^0. Now the only choice variables in the expenditure minimization problem are the levels of the market goods, *X*. Given the constraint we alter our notation to reflect this constraint, $X_S^h = X_S^h(p,w,L^0,\tau,q,U^0)$, letting the *S* subscript refer to the "standard" notion of compensated demand where leisure is fixed. With our new notation, we can express the standard notion of compensating variation found in the welfare economics literature.

$$\begin{aligned} CV_S &= p \cdot [X_S^h(p,w,L^0,\tau,q^0,U^0) - X_S^h(p,w,L^0,\tau,q^1,U^0)] \\ &\quad + w \cdot [L^0 - L^0] + \tau[q^0 - q^1] \end{aligned} \tag{4}$$

As our intuition suggests, the relationship between the standard compensating variation (4) and compensating variation with flexible leisure demand (3) is easy to establish.

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[11] The *h* superscript refers to the solution to the dual problem, the compensated or Hicksian demands.

Proposition 1: For an increase in the public good from q^0 to a new higher level q^1, it will generally be true that $CV_S \leq CV$. Furthermore, in most cases $CV_S < CV$.

Proof: *Part 1.* The first statement in proposition 1 is very easy to establish. First note that by definition $p \cdot X^h(p,w,\tau,q^0,U^0) + w \cdot L^h(p,w,\tau,q^0,U^0) + \tau q^0 = p \cdot X_S^h(p,w,L^0,\tau,q^0,U^0) + w \cdot L^0 + \tau q^0$; that is initial expenditures are equal for both problems. Given the respective minimization problems that provide the demands in (3) and (4), we know that it must be the case that $p \cdot X^h(p,w,\tau,q^1,U^0) + w \cdot L^h(p,w,\tau,q^1,U^0) + \tau q^1 \leq p \cdot X_S^h(p,w,L^0,\tau,q^1,U^0) + w \cdot L^0 + \tau q^1$. The facts that initial expenditures are equal and $-p \cdot X_S^h(p,w,L^0,\tau,q^1,U^0) - w \cdot L^0 - \tau q^0 \leq - p \cdot X^h(p,w,\tau,q^1,U^0) - w \cdot L^h(p,w,\tau,q^1,U^0) - \tau q^1$ leads to the consequent $CV_S \leq CV$.

Part 2. Part 1 of the proof establishes the weak inequality. To prove the second statement we need only establish the conditions under which $CV_S = CV$. $CV_S = CV$ when $L^h(p,w,\tau,q^0,U^0) = L^h(p,w,\tau,q^1,U^0)$, that is when compensated leisure demand does not respond to the change in *q*. Compensated leisure demand from minimization problem (4) will differ from ordinary leisure demand in problem (1) once *q* is changed from the initial level. The two diverge since as *q* is increased, wealth must be decreased in the compensated case. There will be an income effect if leisure responds to changes in wealth, regardless of whether leisure is a normal good, *i.e.* increases with wealth, or an inferior good in the same sense. In the case of wealth sensitive leisure demand of either form (normal or inferior), $L^h(p,w,\tau,q^0,U^0) = L^h(p,w,\tau,q^1,U^0)$ only when the substitution effect exactly cancels out the effect of the reduction in income for the compensated demands. In the case where leisure demand is independent of wealth, then $L^h(p,w,\tau,q^0,U^0) = L^h(p,w,\tau,q^1,U^0)$ only when there is no substitution effect. Both of these cases are the razor's edge, particularly the case where leisure demand is sensitive to wealth effects.

Willingness to accept compensation is a special case of compensating variation that differs from the discussion above in that $q^0 > q^1$. It will still be the case that $CV_S \leq CV$, but now both numbers will be negative. Thus for reductions, ignoring income results in inferring that *more* compensation is required than is actually necessary. The general results on compensating variation make intuitive sense. When leisure is flexible, a person will obtain maximum value for increases in the public good, relative to leisure fixed, and be willing to pay more. For decreases in the public good, flexibility dampens the blow and so less compensation is required. In either case, treating income as fixed results in measurement errors. In the U.S., the analysis of natural resource damages and Federal environmental policy relies on a willingness to pay approach, even in cases in which willingness to accept is more the appropriate concept. The justification for this policy is that it results in a more conservative approach to benefits assessment. Thus according to our analysis, the bulk of environmental policy analysis will result in a tendency to generally under provide these public goods.

Analyzing the impact of assuming a fixed leisure demand/labor supply creates greater difficulties for equivalent variation. In the case of equivalent variation, a fixed income approach assumes that earned income after the change in the public good equals earned income before the change. When this assumption is violated, a fixed income approach infers the wrong earned income after the change takes place and infers the wrong income necessary to maintain the post-change utility level. Under a fixed income approach, equivalent variation is essentially the difference between two incorrect money measures.

Before moving onto our analysis of optimal provision, a discussion of further implications for welfare analysis is warranted. Our analysis directly extends to price changes. A fixed income approach will understate the welfare benefits of lower prices and overstate the welfare losses of higher prices when using compensating variation as the welfare measure. Thus our results really apply to welfare economics in general. It may be the case that changes in public goods may be implemented through pollution taxes instead of lump sum taxes as suggested in our model. Given the immediate extension of our results to price changes, our model readily handles this general class of problems. Simply set the direct tax rate to zero and simultaneously implement price changes with changes in public goods and we arrive at the analysis of pollution taxes.

4. Optimal Provision of Pubic Goods and Input Market Failure

In the previous section we used a utility-constant approach which is standard for welfare economic analysis. In this section we move away from a utility-constant analysis and consider optimal provision which must account for utility change. We show that when income is treated as exogenous, under provision will generally result even when using a Samuelson price scheme.

4.1. Efficient Pricing with Leisure Demand

The basic model for this section is the same as in section 3. Restating, the consumer's problem is to maximize utility in the choice of market goods, X, and leisure, L. The public good, q, is quantity rationed. The marginal tax rate charged to the consumer for q is τ. The maximization problem is as follows.

$$\max_{X,\, L} \; U(X,\, q,\, L) \quad s.t. \quad pX + wL + \tau q = wH,\; q = q^0 \tag{5}$$

In order to discuss efficient pricing of the public good, it is useful to write out the Lagrangean for this problem and the ensuing first order conditions.

$$\mathcal{L} = U(X,\, q,\, L) + \lambda(wH - pX - wL - \tau q) + \gamma(q^0 - q) \tag{6}$$

An efficient price consistent with Samuelson's characterization requires that τ is such that the marginal value of relaxing the rationing constraint, γ, exactly equals zero. If we were to determine the efficient price schedule, $\tau(q)$, that would induce our individual to want to demand that specified level of q for all q, we would recover the inverse demand curve. Optimal provision would require the sum of these inverse demand curves to exactly equal the marginal cost of provision which is a form of modified Samuelson rule that explicitly recognizes that labor supply is an integral part of the problem.

4.2. Marginal Valuation with Leisure Demand

A problem with an inverse demand approach is that when q is not matched with the efficient price, we are essentially observing demand for X and L along demands that are conditioned on the level of q. Furthermore, the rationed level of q is not along the demand curve since the rationing constraint is binding. The companion to the inverse demand is the marginal valuation function. For a given rationing level, the marginal value of more q is simply the marginal utility of q scaled by the marginal utility of income.[12]

$$p^{q}(q \mid p, w, \tau) = \frac{\partial U(X,\ q,\ L)}{\partial q}\frac{1}{\lambda} \tag{7}$$

The values for X, L, and λ are the solutions to the rationed problem for the specified level of q. Of course increasing q comes at a marginal cost of τ. Thus the net marginal value which equals γ/λ is simply the marginal value minus τ.

$$NMV = p^{q}(q \mid p, \tau, w) - \tau \tag{8}$$

At an efficient price τ, the net marginal value exactly equals zero. In this case, the marginal valuation curve exactly equals the inverse demand. However when the net marginal value is greater than zero, the marginal valuation curve diverges from the inverse demand at that particular value of q.

4.3. Marginal Valuation with Exogenous Income

As noted above, the standard analysis treats income as exogenous, modeling preferences only over market goods and the public good. The problem is presented as maximizing utility in

[12] The marginal valuation curve will also depend on the prices of market goods and the tax rate for q. We are suppressing these arguments to economize on notation.

the choices of X by allocating money among the X goods subject to a rationed level of q.

$$\max_{X} \; U(X,\ q) \quad s.t. \quad pX + \tau q = y,\ q = q^0 \tag{9}$$

Translating back to our model where preferences also include leisure, the problem in (9) is obtained by rationing leisure which results in the same initial level of income, $y^0 = wH - wL^0$, where L^0 is leisure demanded at rationed level q^0. As above, treating income as exogenous when it is not is equivalent to assuming the consumer solves the following problem.[13]

$$\max_{X} \; U(X,\ q,\ L) \quad s.t. \quad pX + wL + \tau q = wH,\ q = q^0,\ L = L^0 \tag{10}$$

In this case, the marginal valuation function and net marginal valuation function are similar in form to the problem where leisure is allowed to vary with the exception that the value is conditional on the rationed level of leisure. Similarly, there will exist a net marginal valuation function that an analyst would use to determine optimal provision while treating income as exogenous.

$$p^s(q \mid p, w, L, \tau) = \frac{\partial U(X,\ q,\ L)}{\partial q} \frac{1}{\lambda^s} \tag{11}$$

[13]Hanemann and Morey (1992) consider the consequences of estimating an incomplete demand system on calculating welfare measures. While our work is related in that some choice variables are being ignored in the analysis, the implication is more fundamental for our work because income is not just ignored, but rather the wrong income is used in the analysis.

$$NMV^s = p^s(q|p,w,L,\tau) - \tau \tag{12}$$

At q^0, the marginal valuation curves from the leisure choice approach and the income exogenous approaches are equal as are the net marginal valuation curves. However when q is increased, the two will diverge due to the constraint implicitly imposed on leisure when treating income as exogenous. The general result we seek to prove is that for $q > q^0$, $p^s(q|p,w,L^0,\tau) < p^q(q|p,w,\tau)$. To this end we must introduce compensated valuation curves.

4.4. Compensated Marginal Valuation

Recall the dual utility-constant formulation from above.

$$\min_{X,\ L} \ pX + wL + \tau q \quad s.t.\ U(X,\ q,\ L) = U^0,\ q = q^0 \tag{13}$$

The Lagrangean for this problem is given as follows.

$$\mathfrak{L} = pX + wL + \tau q + \lambda_c(U^0 - U(X,\ q,\ L)) + \gamma_c(q - q^0) \tag{14}$$

The respective marginal value and net marginal value functions for this problem are given below.

$$p^v(q|p,w,\tau,U) = \frac{\partial U(X,\ q,\ L)}{\partial q}\lambda_c \tag{15}$$

$$NMV = p^q(q|p,w,\tau,U) - \tau \tag{16}$$

At q^0, all the marginal valuation functions are equal at the initial reference utility level.

$$p^{q}(q^{0}|p,w,\tau) = p^{s}(q^{0}|p,w,L^{0},\tau) = p^{v}(q^{0}|p,w,\tau,U^{0}) \qquad \textbf{(17)}$$

At an increased level of q, say q^{1}, we know that moving along both uncompensated marginal valuation curves, utility is increasing when the good is under-provided. Given the constraint on leisure when treating income as exogenous, we know that the utility gain in the leisure restricted case is bounded above by the utility gain obtained when leisure is not restricted. Let U^{1} represent the utility level obtained when leisure is not restricted and $U^{\prime}$ the utility gain when leisure is restricted. Further assume that q is a normal good in the sense that if we provided additional non-labor income, demand for q would increase if the consumer did not face the ration constraint on q. Under strict convexity of preferences, $U^{0} < U^{\prime} < U^{1}$. The compensated marginal valuation function can be used to match the marginal valuation curves that are uncompensated. For the case of moving along the curve where leisure is not restricted, we know the following.

$$p^{q}(q^{1}|p,w,\tau) = p^{v}(q^{1}|p,w,\tau,U^{1}) \qquad \textbf{(18)}$$

Given the assumption of normal goods, $p^{v}(q|p,w,\tau,U^{0}) < p^{v}(q|p,w,\tau,U^{1})$ for all q which implies that $p^{q}(q^{0}|p,w,\tau) < p^{q}(q^{1}|p,w,\tau)$. Now consider $p^{s}(q^{1}|p,w,L^{0},\tau)$. In terms of the compensated marginal value, there does exist a compensated problem that would exactly support the choice of market goods and leisure that is realized under the fixed leisure problem. In order to keep the choice of leisure fixed at L^{0}, a different wage would be required to keep leisure at the initial level. We refer to that wage as $w^{\prime}$ and note that $p^{s}(q^{1}|p,w,\tau) = p^{v}(q^{1}|p,w^{\prime},\tau,U^{\prime})$. From the assumption of normal goods, $p^{v}(q^{1}|p,w,\tau,U^{\prime}) < p^{v}(q|p,w,\tau,U^{1})$. The question is one of what is the effect of the wage change. We need to consider two cases.

Case 1: Compensated Substitution Between q and L

Compensated substitution can be expressed in several equivalent ways according to the rationing classifications of Madden (1991). One way of defining substitution is as the case when the compensated demand for *L* goes down with increasing levels of the public good . In this case, the wage must decrease in order to keep leisure fixed at L^0. In the case of substitution, it must be the case that $w > w'$. Thus for the case of substitutes, there will exist a lower wage that will satisfy the condition $p^s(q^1|p,w,\tau) = p^v(q^1|p,w',\tau,U')$. For normal goods, we already know that $p^v(q^1|p,w,\tau,U') < p^v(q|p,w,\tau,U^1)$. Following Madden, an equivalent way of expressing substitution between *q* and *L* is that the change in the marginal value for rationed *q* increases as the wage increases or more importantly in this case, decreases as the wage decreases. In the case of substitution between *q* and *L* we have the following inequalities satisfied:

$$\begin{gathered} p^s(q^1|,p,w,L^0,\tau) = \\ p^v(q^1|p,w',\tau,U') < p^v(q^1|p,w,\tau,U') < p^v(q|p,w,\tau,U^1) \\ = p^q(q^1|p,w,\tau) \end{gathered} \qquad \textbf{(19)}$$

Proposition 2: Suppose that preferences are strictly convex, *q* is a normal good, and that *q* and *L* are compensated substitutes. Then for all $q > q^0$, $p^s(q|p,w,L^0,\tau) < p^q(q|p,w,\tau)$.

Case 2: Compensated Complementarity Between q and L

In the case of complements, compensated *L* increases with an increase in *q*. In order to keep leisure fixed at the lower level, the price of leisure must be increased and so $w < w'$. It is still the case that $p^v(q^1|p,w,\tau,U') < p^v(q|p,w,\tau,U^1)$. Under complementarity, it is the case that the marginal value for *q* decreases as the wage increases. Under complementarity we have the same set of inequalities.

$$p^{s}(q^{1}|,p,w,L^{0},\tau) =$$

$$p^{v}(q^{1}|p,w^{\prime},\tau,U^{\prime}) < p^{v}(q^{1}|p,w,\tau,U^{\prime}) < p^{v}(q|p,w,\tau,U^{1}) \tag{20}$$

$$= p^{q}(q^{1}|p,w,\tau)$$

Proposition 3: Suppose that preferences are strictly convex, q is a normal good, and that q and L are compensated complements. Then for all $q > q^{0}$, $p^{s}(q|p,w,L^{0},\tau) < p^{q}(q|p,w,\tau)$.

From propositions 2 and 3, we find that the marginal valuation curve treating income as exogenous lies below the true marginal valuation curve. Thus using the income exogenous valuation curve to determine optimal provision will generally result in under provision relative to the true optimum.

5. Discussion of Implications and Extensions

If we cannot acquire the goods we desire, we lose the incentive to work. In the case of public goods, the degree of this effect is determined by the relative strength of individual preferences for private versus public goods. Collectively, we will under-supply work effort, generate too little income, and under-supply the public good.

These work-leisure choices extend to the realm of human capital accumulation. Individuals, caring in varying degrees about the environment, make human capital decisions over their life cycles, as well as making the leisure/labor decisions in each period emphasized here. Each will choose to accumulate different levels of human capital, depending on expected levels of public goods provision. For some, private demands for the public good will be only modestly greater than the social provision level, hence only minor adjustments in human capital decisions and leisure/goods consumption might occur. This may well be a typical situation among those not characterizing themselves as "environmentalists."

Others, like Sten, will have demands for environmental quality far in excess of the level chosen collectively. Unable to unilaterally choose the level of environmental quality and not caring much about consumption goods, these individuals may choose not to accumulate much at all in the way of human capital. They may effectively "drop-out" in the jargon of the '60s, perhaps acquiring a farm to grow organic food.

Still others, under these conditions, may invest in human capital that is complementary with influencing society's environmental quality choices (e.g. working for environmental advocacy groups in a variety of positions). Pay within such advocacy organizations is typically relatively low, implying substantial non-monetary benefits for rational workers.[14]

While it is true that the direction of the bias in the individual valuation for public goods is unambiguous (true values are always greater than apparent willingness-to-pay, unless increments to the public good have zero or negative marginal value[15]), this does not mean that socially-optimal increases in public goods levels would necessarily win a popular vote. Incorporating willingness to pay after allowing for alterations in the labor/leisure and human capital investment decisions, as we do here, does not eliminate the possibility that a majority might have increases in costs that exceed their incremental benefits, though the likelihood of such an outcome would be lower than at present.

Moving from current under provision of public goods to the larger socially optimal quantity that would exist with proper valuation would have potentially pronounced impacts on private goods markets. The under provision of public goods implies that there will be an under

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[14]While our examples and analyses are specific to environmental issues, the results and observations carry over to a much larger class of examples. Advocates for other social causes that involve publicly rationed goods can also be described by our analysis.

[15]Negative marginal values might be possibilities for, say, national defense (depending on one's "theory of peace," fears of global destruction, and so on) but in the environmental context this would be unusual. Ranchers wishing to eliminate wolves, in the face of wolf reintroduction plans is not really a counter-example, but rather a case where the costs of the policy might be inappropriately distributed. That is, the rancher could have a positive wolf preservation demand that might be greatly exceeded by the costs he must pay to receive that benefit; full compensation for lost cattle might reveal his positive preservation value.

provision of private good complements and an over provision of private good substitutes.[16]

Of importance is the implication of the analysis for the selection of the appropriate social rate of discount to be used in evaluating long-term projects. Those placing high values on public goods currently do not earn the large incomes they would have earned if they could have directly purchased the goods they care about. They also will not have saved as much as they would have...because they would *like* to be able to save environmental resources, not merely financial goods which they don't care much about (and about which their children, raised as they will be, won't care much about). Their inability to save for a better environment *individually* results in both smaller incomes and reduced savings. With proper valuation, there might be much more total saving, which means that the appropriate social discount rate could be much smaller than currently presumed.[17]

How important, quantitatively, are the qualitative points made here likely to be? We believe that there is substantial undervaluation of public goods, particularly public goods without “special interest support.” Consider the case of environmental quality. Assume that roughly four percent of GDP is being spent to obtain current environmental quality levels ($400 billion out of about a $10 trillion economy). This would seem to suggest, that environmental quality is a very small component of both utility and expenditure.

It is not, however, implausible to argue that there might be a ten to twenty percent larger income, if people could in fact buy environmental quality like they can buy ordinary goods. Presume that income would be only ten percent ($1,000 billion) larger. Under independence, a

[16]For example, failure to build enough lighthouses may result in reduced demand for boats. A substitute for cleaning up urban air quality might be moving to a large lot in a distant suburb, hence failure to clean up urban air will (non-optimally) exacerbate the exodus to the suburbs and to ex-urban areas.

[17]In the present Second Best world, in which the analytical extensions discussed here are not incorporated in the valuation of public goods, a case can be made for applying a lower discount rate to projects involving public goods such as environmental quality vis-a-vis projects involving private goods. This could compensate for the current undervaluation of environmental goods.

rough partial equilibrium guess at the "proper" amount of environmental goods to be produced is $1,400 billion, two and a half times the current provision levels.[18] Large optimal percentage increases in public goods result from even small percentage increases in income, because of the relatively small base of the former. An extension of the present effort could fruitfully examine the quantitative significance of the observations made here in greater depth.

Economists and environmentalists have pronounced disagreements about proper levels of environmental goods. The extent of that disagreement would likely be far smaller were environmental public goods provided at the levels suggested by the analysis here, levels that both economists and environmentalists should recognize as improvements on the status quo.

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[18]The text numbers would be reduced some in general equilibrium as depicted in Figure 2.

6. References

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