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Demand Growth and Strategically Useful Idle Capacity

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Abstract

This paper presents a capacity preemption game between an incumbent firm and a potential entrant. If entry occurs, then competition proceeds through Cournot quantity competition. My model, like those of (e.g.) Dixit and Ware, includes a strategic use of capacity prior to entry. However, it differs in that I consider a two period model, in which second period demand is larger than first period demand. I show that demand growth can result in a use for idle capacity. This result does not require the assumption of strategic complements, and therefore works with, e.g., linear demand.
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1 Introduction

The idea that a firm might create productive capacity for the purpose of preempting a (potential) rival is hardly novel. Further, there is no lack of empirical evidence of firms maintaining a persistent stock of idle capacity.[1] However, the current body of theoretical models concerning preemptive capacity has not directly addressed the issues in Justice Hand's decision on what has become the text book case on *preemptive idle capacity,* Alcoa Aluminum.[2] In his decision, Justice Hand suggests that Alcoa did "always anticipate increases in demand for ingot and be prepared to supply them." Further, he suggests that the rational behind Alcoa's behavior was that there was "no more effective exclusion than progressively to embrace each new opportunity as it opened, and to face every newcomer with new capacity..."[3] This paper investigates Justice Hand's assertion that the maintenance of idle capacity is an effective method of entry deterrence when demand growth is anticipated.

While Dixit (1980) and Bulow et. al. (1985) demonstrate that capacity can be an entry deterrent, they have tied the strategic use of *idle* capacity to cases in which the output of the two firms are strategic complements.[4] Similarly, Allen et. al. (2000) con-

[1]For example: Esposito and Esposito (1974), Cossuta and Grillo (1986), Rosembaum (1989). More to the point, Mathis and Koscianski (1995), Shaanon (1997) and Hall (1990) find evidence that firms use idle capacity to prevent entry. However, Ghemawat (1984) and Gilbert and Lieberman (1987) find otherwise.

[2]Covered in, for example, Martin (1993, pg. 98.)

[3]Hand (1941,1947).

[4]See also: Dixit (1979), Spence (1977), and Fudenberg and Tirole (1984). For elaborations see: Barnham and Ware (1993), Eaton and Lipsey (1980, 1981), Eaton and Ware(1987), Fudenberg and

sider capacity setting prior to a Betrand-Edgeworth price setting game. Here, prices are strategic complements, and conditions can be found under which idle capacity occurs. Basu and Singh (1990) have shown that idle capacity might arise if capacity is only one of the entry deterrence instruments available to the incumbent.[5] However, these models work with only a single period, and so assume away the possibility of demand growth. Consequently, the relationship between demand growth and strategically useful idle capacity suggested by Justice Hand can not be present. In this paper, I show that in the face of growing demand, entry deterrence may necessitate the maintenance of idle capacity. This result requires neither strategic complements, nor the presence of additional deterrence instruments. Rather, it follows from an entrant's willingness to take early losses in order to gain a foothold in a market and make profits in later stages. Knowing the value of a foothold, the incumbent firm recognizes that deterrence requires sufficient capacity to make both the current and future periods unprofitable for the potential entrant. If demand is growing, then this might make idle capacity necessary.

Beyond the Alcoa Case, these arguments illuminate Dupont's alleged attempts to achieve and maintain dominance in the titanium dioxide market. Dupont's advantage stemmed, not from incumbency, but rather from lower costs from learning by doing (see

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Tirole (1983), Salop (1979), Schmalensee (1981), and Spence (1979).

[5]Basu and Singh (1990) use a *Stackelberg perfect* equilibrium to capture the commitment value of the incumbent's other instruments.

e.g. Gilbert and Harris (1981).) However, part of the accusation leveled at Dupont involved the preemption of their rivals' capacity investment. In particular, Dupont built a plant in DeLisle Mississippi "despite the acknowledgment that the completed facility might have to be held in readiness for operation ... until market conditions had sufficiently improved."[6]

The formal model is a two period game of Cournot quantity competition with an incumbent and a potential entrant. Capacity is used as a commitment device through which the incumbent gains a first mover advantage. In the first period, the incumbent firm sets capacity before the potential entrant may do so. However, the incumbent maintains this advantage in the second period only if there is no entry in the first period. Otherwise, in the second period, the two firms set output simultaneously, without making any change to their capacity. That is, the value of a foothold is the negation of the incumbent's first mover advantage. This is modeled by removing the capacity choice from the post entry game. I find that a two period model behaves in many ways the same as a one period model. However, it is possible to establish that, given sufficient growth in demand, entry deterrence requires the presence of idle capacity. With linear demand, one can demonstrate the existence of cases in which entry deterrence with idle capacity is the subgame perfect equilibrium.

There have been previous temporal models with capacity choice. For example,

[6] Dobsons et. al. (1994, pg. 166).

Spulber (1981) also examines a two period model. However, Spulber does not distinguish between first and second period capacity, and does not allow entry to occur in the first period. Hence, even if Spulber's model did include demand growth, it would not allow the type of behavior studied here. Gilbert and Harris (1984), Eaton and Lipsey (1980) and Reynolds (1987) all examine dynamic capacity games, but assume away the possibility of idle capacity. Eaton and Lipsey (1979) consider a growing spatial market, and show that an incumbent will expand into new markets before entry occurs.[7] Reynolds (1986) performs simulations of the American aluminum industry after the Alcoa decision, and finds that a dominant firm model (Kydland, 1977) does the best job of replicating the persistent idle capacity in that market.[8]

The remainder of the paper is organized as follows: the model is presented in Section 2, and analyzed in Section 3. Section 4 provides discussion. Many proofs are contained in the appendix.

2 Model

The model presumes that an incumbent firm has a first mover advantage only until the entrant establishes a foothold in the industry. The timing of the model in period one is: 1) the Incumbent (I) sets capacity. 2) The entrant (E) makes his entry decision, and

[7]Eaton and Lipsey suggest that there is *excess* capacity in this model. However, there is no *idle* capacity.

[8]Following the decision Kaiser and Reynolds both became significant players in the Aluminum market.

sets capacity (if he enters,) and 3) firms in the market set output simultaneously at the intersection of their reaction functions.[9] If there is still only a single firm in the market at the beginning of the second period, then the timing in the second period mimics that in the first. However, if there was entry in period one, then in the second period firms are unable to change their capacity, and simply compete in quantities.

Throughout, subscripts will denote time periods, $t = 1, 2$, and superscripts will denote either players $i = I, E$. For example, q_t^i refers to firm i's output in period t. In period t, for an aggregate output Q, prices are determined by a inverse demand $P_t(Q)$. Demand in both periods is assumed to satisfy: $P_t' + P_t'' \cdot Q > 0$. Demand growth is formalized by requiring that $P_2(Q) > P_1(Q)$ and that $|P_2'| \leq |P_1'|$ for all aggregate outputs Q. This is satisfied, for example, with linear demands: $P_t = a_t - bQ$ with $a_2 > a_1$. These assumptions guarantee that reaction function are downward sloping in rival output, and shift outward between periods. Capacity, K_t^i is modeled as a commitment on marginal costs. Total operating costs for firm i in period t are $F + cK_t^i + c(\max\{q_t^i - K_t^i, 0\})$. Firms may not decrease their capacity so that $K_1^i \leq K_2^i$. In addition to the costs of operating within each period, there is a sunk cost of $\bar{F}$ which must be paid upon entry into the market. It will be convenient to denote by $\pi_t^i(q_t^I, q_t^E) = (P_t - c)q_t^i$, the firm's 'variable' profits.

Let $R_t(\cdot, K)$ be the reaction function in period t for a firm in the market with

[9]Of course the reaction functions are determined by capacity choices. It is presumed that output has no consequence for later periods, and is set at the single period Nash equilibrium.

capacity K. Let $\underline{R}_t(\cdot) = R_t(\cdot, 0)$ and $\bar{R}_t(\cdot) = R_t(\cdot, \infty)$. A superscript i on any of these functions indicates that it is firm i's reaction function.

My results depend upon Ware's (1984) analysis of a single period capacity setting game, so let us suppress the time subscripts for the moment. See Figure 1 for an illustration. Denote the (zero capacity) Cournot Nash equilibrium as the point $CN = (CN^I, CN^E)$ (throughout a superscript $i = I, E$ denotes the projection onto q^i) and denote the point where $\bar{R}^I$ and $\underline{R}^E$ intersect as V.[10] In the Dixit (1980) model, the Incumbent sets capacity so as to make his preferred point on $\underline{R}^E$ between CN and V the Nash equilibrium of the post entry output game. Presuming that both points are feasible, he chooses between accommodating entry at the Stackelberg point S and deterring entry by committing to the limit output. Ware (1984) modifies Dixit's model by allowing the (potential) Entrant to set capacity as well. At this point, the Entrant has the commitment opportunity, and sets his capacity to choose a point on $R^I(\cdot, K^I)$ between the intersections with $\underline{R}^E$ (point U) and $\bar{R}^E$ as the equilibrium of the quantity setting game. In equilibrium, the Entrant never uses this ability, but its presence constrains the Incumbent's capacity choice. Specifically, consider the point $\tilde{S}$, the Entrant's preferred point on $\bar{R}^I$ between U and V.[11] If the Incumbent sets her capacity too high, then the Entrant will prefer this point to the intersection of $R^I(\cdot, K^I)$ and

[10] Since the model is symmetric, $CN^I = CN^E$. The distinction between firms is made only to keep notation standard.

[11] This should perhaps be spoken of as the constrained preferred point, since it incorporates the constraint that the Entrant can not commit to more output than he produces at the point U.

$\underline{R}^E$. Let W be the point on $\underline{R}^E$ such that $\pi^E(W) = \pi^E(\tilde{S})$. Clearly the Incumbent has higher profits at W, than at $\tilde{S}$. Hence in the Ware (1984) one period model, the Incumbent firm does not set capacity higher than W^I. Deterrence is only possible if the limit output is less than W^I, and a Stackelberg leader would set her capacity to choose her preferred point on $\underline{R}^E$ between CN and W. Call this the generalized Stackelberg point, and denote it $\hat{S}$.

3 Analysis

Subgame Perfect equilibria can be placed within three classes based upon when entry takes place. There are: *entry equilibria*, with entry in the first period; *delayed entry equilibria* with entry in the second period; and *deterrence equilibria*, with no entry what so ever. The analysis of a delayed entry equilibrium adds little to Ware (1984). Further, as we shall see below, there is never idle capacity in a delayed entry equilibrium. Consequently, I focus on *entry equilibria*, and *deterrence equilibria.*

Second period output is set by the intersection of the reaction functions $R_2^i(\cdot, K_2^i)$. Since the game ends after the second period, neither firm will build idle capacity in the second period.[12] Furthermore, since the only purpose for idle capacity in the first period is to influence the second period, no firm would build capacity in the first period beyond what is used in the second period. Hence in equilibrium, there is never idle

[12] This follows from exactly the same logic which lies behind Dixit's (1980) results.

second period capacity.

Let us recall that, in an entry equilibrium, there is no second period capacity choice, leaving firms with $K_2^i = K_1^i$. Consequently, if both firms have $K_1^i \leq CN_2^i$, then CN_2 is the second period output. If one firm has $K_1^i > CN_2^i$, then that firm's first period capacity determines second period output. Of course, if entry does not take place in the first period, then the Incumbent maintains her advantage, and capacity is set in the second period.

Since adding another period to the game has not changed the fundamental role of capacity, some aspects of equilibria should remain qualitatively unchanged. Capacity should be built only if it has commitment value. The Incumbent's first mover advantage should, in equilibrium, leave the Entrant without a desire to use his capacity for commitment. That is, the Entrant, should he enter in the first period, should build only capacity he will use in the first period. And finally, the Incumbent should, at a minimum be able to guarantee himself the modified Stackelberg outcome, $\hat{S}_1$, in the first period.

Proposition 1 *In an entry equilibrium, the Entrant's first period capacity is no greater than his first period output, and the Incumbent's first period capacity is equal to her output in either the first or second period.*

In a deterrence equilibrium, if the Incumbent's first period capacity is greater than her first period output, then her capacity is greater than her second period Cournot Nash *output.*

In any equilibrium, the Incumbent's first period output is greater than or equal to the minimum of her first period monopoly and first period generalized Stackelberg output.

Proposition 1 implies that idle capacity can not occur in a *delayed entry equilibrium.* Let $\underline{M}_t = (\underline{R}_t^I(0), 0)$. The first term, $\underline{M}_t^I$, is monopoly output. Let $\bar{M}_t = (\bar{R}_t^I(0), 0)$. $\bar{M}_t^I$ would be monopoly output if a firm had no marginal costs. Consider an outcome with delayed entry and idle first period capacity. If capacity is left idle in the first period, then $K_1^I > \bar{M}_1^I$. Since the Entrant wishes to enter in the second period, but not in the first, it must be the case that $\pi_1^E(\tilde{S}_1) \leq F$. Hence $\pi_1^E(W_1) \equiv \pi_1^E(\tilde{S}_1) \leq F$. Now consider an outcome which is the same, except that $K_1^I = \max\{W_1^I, \underline{M}_1^I\} < \bar{M}_1^I$. The Entrant's incentive to enter in the second period is unchanged, and so we may expect the same second period profits for both firms. However, the Incumbent no longer has to pay the first period costs of maintaining idle capacity, and is producing closer to the first period profit maximizing output. Clearly the Incumbent's profits are higher, and the original outcome was not an equilibrium.

Proposition 2 *There is never idle capacity in a delayed entry equilibrium.*

In cases where idle first period capacity is possible, that possibility must depend upon the amount of demand growth.

Assumption G *There is sufficient growth in demand that commitment in the second period requires idle capacity in the first:* $CN_2^I > \bar{M}_1^I$.

By the definition of $\bar{M}_1^I$, idle capacity in the first period of a deterrence equilibrium occurs when $K_1^I > \bar{M}_1^I$. On the other hand, by Proposition 1, we know that for the Incumbent's first period capacity to have a consequence in the second period, it must be greater than CN_2^I. Hence Assumption G is the requirement that commitment in the second period requires idle capacity in the first period. This simplifies the analysis of both deterrence equilibria, and Stackelberg leadership.

Because the Incumbent's first period capacity determines the outcome in both periods, when acting as a Stackelberg leader, she trades off more profitable commitment today with less profitable commitment tomorrow and vice versa. Assumption G reduces Stackelberg leadership to two extreme cases. A type 1 Stackelberg leader chooses capacity to commit in the first period, but gives up the ability to commit in the second. That is, she chooses $K_1^I = q_1^I \leq W_1^I$. This leads to an equilibrium with outputs of $\hat{S}_1$ in the first period and CN_2 in the second. A type 2 Stackelberg leader commits in the second period, and 'over commits' in the first. That is she chooses $K_1^I = q_2^I > CN_2^I$.[13] In this case, the Incumbent has idle first period capacity and the first period output is $\tilde{S}_1$. In the second period the output is set at $(K_1^I, \underline{R}_2^E(K_1^I))$. If there is demand growth, but Assumption G does not hold, then there might be an intermediate type of Stackelberg leader, who both uses all of her capacity in the first period, and commits to an output in the second. In this case, the Incumbent chooses K_1^I such that

[13] Under Assumption G, $W_1^I < CN_2^I$. There is no reason to set capacity between these two values, as this would result in the same output pairs as setting capacity to W_1^I, but at a higher cost.

$\hat{S}_1^I \leq q_1^I = K_1^I = q_2^I \leq \hat{S}_2^I$.[14] Entrant outputs are at $\underline{R}_t^E(K_1^I)$.

We can now investigate the conditions under which idle capacity occurs in a deterrence equilibrium. Throughout what follows, Assumptions G is maintained. The following five conditions must be satisfied: (1) It is possible to deter first period entry, but only if the Incumbent maintains idle capacity. (2) It is possible to deter entry in the second period. (3) The Incumbent prefers entry deterrence to being a Stackelberg leader, and (4) given that entry has not occurred in the first period, the Incumbent prefers to deter it in the second period as well. The first two of these conditions are statements about the Entrant's payoffs in different situations. They might be restated as (1') $\pi_1^E(\tilde{S}_1) + \pi_2^E(W_2) \leq \bar{F} + 2F \leq \pi_1^E(W_1) + \pi_2^E(CN_2)$, and and (2') $\pi_2^E(W_2) \leq \bar{F} + 2F$.[15] Using the second inequality from (1'), one can transform (2') into $F \leq [\pi_2^E(CN_2) - \pi_2^E(W_2)] + \pi_1^E(W_1)$. Since $\pi_1^E(W_1) \equiv \pi_1^E(\tilde{S}_1)$, $\pi_2^E(W_2) < \pi_2^E(CN_2)$ and $0 < [\pi_2^E(CN_2) - \pi_2^E(W_2)] + \pi_1^E(W_1)$, there are $\bar{F}$, and F such that conditions 1 and 2 hold. This yields:

Proposition 3 *Under Assumption G, one can find levels of fixed and sunk costs (i.e. F and $\bar{F}$) such that a deterrence equilibrium requires idle first period capacity.*

Observe that Proposition 3 is merely a statement that there are circumstances under which, if the Incumbent wishes to deter entry, then he must maintain idle capacity.

[14] If $S_t^I < W_t^I$ in both periods, then it follows that $S_1^I < q_1^I = K_1^I = q_2^I < S_2^I$.

[15] For the sake of clarity, condition (1') is sufficient, but stronger than necessary.

To demonstrate that such equilibria actually exist, one must show that the Incumbent prefers deterrence through idle capacity to being a Stackelberg leader. Because it is not so easy to compare the Incumbent's payoffs in different circumstances, some further structure must be imposed. For the remainder of the paper, linear demand is assumed.

Assumption L Demand is linear: $P_t = a_t - b(q_t^E + q_t^I)$ with $a_2 > a_1 > c$.

There remains the problem that the payoffs for deterring entry depend upon the fixed and sunk costs. Hence in comparing payoffs, it is convenient to fix upon a particular case. Specifically, let us presume for now that $K_1^I = \underline{M}_2^I$ is sufficient to deter entry in both periods. The task of finding values for F and $\bar{F}$ which justify this presumption is addressed later. The first benefit from Assumption L is the ability to rule out type 2 Stackelberg leadership.

Proposition 4 *Let Assumption L hold. If there exists* $K_1^I \leq \min\{\underline{M}_2^I, W_2^I\}$ *which is sufficient to deter entry in both periods, then type 2 Stackelberg leadership never occurs in equilibrium.*

The intuition of Proposition 4 is that either the first period or the second period is in some sense more important. If the first period is more important, then the Incumbent prefers type 1 Stackelberg leadership to type 2. If the second period is more important, then the Incumbent prefers to deter entry, because by presumption, entry deterrence is not difficult. It now remains to show that there are cases in which the Incumbent prefers deterrence to type 1 Stackelberg leadership, for which the following assumption is useful.

Assumption D $(a_1)^2 + 2a_1c - (c)^2 \geq (a_2 - c)(5c - a_2)$.

Sufficient conditions for Assumption D to hold are: $\frac{a_1}{c} \geq 1.5$ or $\frac{a_2}{c} \geq 4.5$. Assumption D is an algebraic statement that the second period is more important than the first period, so that the Incumbent prefers deterrence, when it is relatively easy, to type 1 Stackelberg leadership.

Proposition 5 *With linear demand,* $\pi_1^I(\bar{M}_1) + \pi_2^I(\underline{M}_2) - c(\underline{M}_2 - \bar{M}_1) \geq \pi_1^I(\hat{S}_1) + \pi_2^I(CN_2)$ *if and only if Assumption D holds.*

Proposition 5 is a statement that the Incumbent would be willing to hold the second period monopoly capacity, $\underline{M}_2^I$, in the first period to deter entry. Hence, while Assumption D is 'tight' for Proposition 5, there are clearly cases in which Assumption D does not hold, but the Incumbent is nonetheless willing to hold idle capacity. Likewise, if Assumption D holds with a strict inequality, then the Incumbent would be willing to hold capacity greater than $\underline{M}_2^I$ to deter entry. However, this gives us an easy case to check for parameter values such that the Incumbent finds entry deterrence both possible and desirable.

Let us fix $\Omega = (K^L, \underline{R}_2^E(K^L))$ for come capacity level K^L. Our task is completed by finding F and $\bar{F}$ such that there exists K^L, with $CN_2^I < K^L \leq \hat{S}_2^I$, satisfying the following two conditions. Entry deterrence is possible in the first period, if and only if the Incumbent maintains at least capacity K^L; $\pi_1^E(W_1) + \pi_2^E(\Omega) = \bar{F} + 2F <$

$\pi_1^E(W_1) + \pi_2^E(CN_2)$ Since $K^L > CN_2^I$, and Assumption G holds, this involves idle capacity. The second condition is that deterrence in the second period is feasible, and never requires more than the monopoly level of capacity; $\pi_2^E(\hat{S}_2) \leq \bar{F} + F$. Hence, delayed entry equilibria are ruled out. Now observe that since $\pi_2^E(\Omega) < \pi_2^E(CN_2)$, it is always possible to assure these conditions.[16] By Assumption D, $\pi_1^I(\bar{M}_1) + \pi_2^I(\underline{M}_2) - c(K^L - \bar{M}_1) \geq \pi_1^I(\bar{M}_1) + \pi_2^I(\underline{M}_2) - c(\underline{M}_2 - \bar{M}_1) \geq \pi_1^I(\hat{S}_1) + \pi_2^I(CN_2)$. This guarantees that the Incumbent prefers bearing the cost of idle capacity to sharing the market.

Proposition 6 *Let Assumptions G, L, and D hold. One can choose fixed and sunk costs such that the unique equilibrium is a deterrence equilibrium in which the Incumbent maintains idle first period capacity.*

4 Discussion

We have found that for an incumbent firm to deter entry when there is demand growth can require the maintenance of idle capacity. The model was chosen for simplicity, but could be generalized. One simple extension would be the addition of physical depreciation. Since it is only the post depreciation capacity that has any commitment value, depreciation would, all other things being equal, increase the capacity requirements for entry deterrence. Consequently, physical depreciation should serve much the same role as demand growth in making idle capacity strategically meaningful.

[16] By essentially the same arguments as Proposition 3.

Less trivial would be to extend the model to a longer (possibly infinite) sequence of periods, allowing capacity setting in each period. Since a foothold is valuable in all future periods, an incumbent firm would have to consider demand in all future periods when choosing capacity. Reynolds (1987) has analyzed such a model for a duopoly market. He finds that concern over future periods increases the capacity which firms hold. This seems to indicate that results similar to those contained herein could be found in an infinite horizon model. However, Reynolds' analysis depends upon firm payoffs being quadratic in capacity, which disallows the possibility of idle capacity.[17]

5 Appendix

The following four lemmas are for the purpose of proving Proposition 1.

Lemma 5.1 *In an entry equilibrium, both firms set first period capacity less than or equal to first period output, or equal to second period output.*

In a deterrence equilibrium, the Incumbent sets her capacity less than or equal to her first period output, or strictly greater than the second period Cournot output.

Proof: We already know that capacity is not set above second period output, so it remain to rule out a choice of capacity greater than first period output, but less than second period output. In this case, the two firms' second period reaction functions must

[17] Reynolds analyses a differential game within a continuous time framework.

cross at a point where the Incumbent produces more than his capacity. Since a decrease in capacity will not altered this intersection, but will lower the Incumbent's first period costs, this can not be an equilibrium. ♣

Lemma 5.2 *In an entry equilibrium, if* $CN_2^I \leq K_1^I$*, then the Entrant sets his capacity less than or equal to his first period output.*

Proof: If $CN_2^I \leq K_1^I \leq W_2^I$ then we know that the Entrant gets no benefit from capacity in the second period, because his optimal second period output is (holding $K_2^I = K_1^I$) $\underline{R}_2(K_1^I)$ which he will receive even if $K_1^E = 0$. If $K_1^I \geq W_2^I$, then the Entrant might wish to carry extra capacity in the first period in order to improve his second period profits. However, just as the Incumbent never wants to set his capacity higher than W^I in a one period model, in a two period model the Incumbent will never set his capacity high enough to trigger this sort of behavior. Hence the Entrant sets $K_1^E \leq q_1^E$.♣

The point of the following two Lemmas is to rule out the case where the dominant firm wishes to act like a Stackelberg follower in the second period, and hence chooses a low capacity to induce the Entrant to chooses a 'leader' capacity. Hence in these proofs there is a possibility that the dominant firm sets his capacity at say $\tilde{S}_1^E$.

Lemma 5.3 *Presume that there is an entry equilibrium in which the Incumbent chooses* $K_1^I < CN_2^I$*, but the Entrant chooses* $K_1^E > CN_2^I$*.*

1) If $K_1^E \leq W_1^I$ *then* $K_1^I \leq \underline{R}_1(K_1^E)$ *or* $K_1^I = \underline{R}_2(K_1^E) > \underline{R}_1(K_1^E)$*.*

2) If $K_1^E > W_1^I$ then $K_1^I = \tilde{S}_1^E$ or $K_1^I = \underline{R}_2(K_1^E) \geq \tilde{S}_1^E$

Proof: Let $\bar{K}$ denote $\underline{R}_1(K_1^E)$ for case 1 and $\tilde{S}_1^E$ for case 2. Observe, that from Lemma 5.2 we know that $\bar{K}$ would be an optimal response by the Incumbent if the Entrant moved first and chose the capacity suggested in one of the cases. If the Incumbent has to choose some $\underline{K} < \bar{K}$ in order to get the Entrant to choose K_1^E, then there is no equilibrium in which the Entrant chooses K_1^E, because $\bar{K}$ and an Entrant capacity lower than K_1^E is preferred (by the Incumbent) to $\bar{K}$ and K_1^E which is in turn (weakly) preferred to $\underline{K}$ and K_1^E. Now consider the possible choice of $K > \bar{K}$ in period 1 by the Incumbent. The only reason to make such a choice would be to take away the Entrant's incentive to set an even higher capacity, and this is done as cheaply as possible at $K_1^I = \underline{R}_2(K_1^E)$. ♣

Lemma 5.4 *If the dominant firm chooses $K_1^I \leq CN_2^I$ in an entry equilibrium, then the Entrant chooses $K_1^E \leq CN_2^I$.*

Proof: I assume that $K_1^I \leq CN_2^I < K_1^E$, and derive a contradiction. By Lemma 5.3 it is evident that if the Incumbent chose the Entrant's equilibrium capacity, then a reversal of outputs would result. Therefore the Incumbent must be making higher net profits than the Entrant. In addition, the Entrant could simply choose to mimic the Incumbent's capacity choice, so that this must yield lower profits than the Entrant's equilibrium choice. However, by choosing this lower capacity, the Entrant would make at least as much profits in the first period, and strictly more profits in the second period than the Incumbent is making in equilibrium. Therefore the Entrant is making higher net profits than the Incumbent, by which contradiction the Lemma is proven. ♣

Proof of Proposition 1.

Given Lemma 5.1, there remain two things to show. By putting Lemmas 5.1 and 5.4 together, we see that a first period Entrant sets $K_1^E \leq q_1^E$. It remains now to show only that $q_1^I \geq \min\{\hat{S}_1^I, \underline{M}_1^I\}$. This is now immediate, since if there is no first period entry, then the Incumbent will wish to produce at least his monopoly output, $\underline{M}_1^I$, and if there is entry, then he will wish to produce at least $\hat{S}_1^I$ in both periods, even if she could commit period by period. ♣

Proof of Proposition 4

The first step is to quantify the profits from Type 2 Stackelberg leadership. Observe, that capacity is set purely on it's value in the second period, but has to be carried for two periods. Hence it is set to maximize: $(a_2 - 2c - b(K + \frac{a_2 - c - bK}{2b}))K$ with solution $K^* = \frac{a_2 - 3c}{2b}$. Clearly for type two behavior to be valid it must be the case that $K^* > q_2^{CN}$ which requires $a_2 > 7c$. Presuming that $a_2 > 7c$, and recognizing that the first period outcome must be $\tilde{S}_1$, then net profits from type 2 Stackelberg leadership are no more than $\frac{(a_1+2c)^2}{16b} - \frac{a_2 c - 3c^2}{2b} + \frac{(a_2+c)(a_2-3c)}{8b}$. We need now to compare these net profits with $\pi_1^I(\bar{M}_1) + \pi_2^I(\underline{M}_2) - c(\underline{M}_2 - \bar{M}_1) = \frac{a_1(a_1-2c)}{4b} + \frac{(a_2-c)^2}{4b} - \frac{(a_2-a_1-c)c}{2b}$.

Algebra reveals that type 2 Stackelberg are the smaller of the two if $a_1(3a_1 - 4c) >$

$-2((a_2)^2 - 4a_2c - 5c^2)$ which itself follows with $a_1 > c$ and $a_2 > 7c$. ♣

Proof of Proposition 5

$\pi_1^I(\bar{M}_1) + \pi_2^I(\underline{M}_2) - c(\underline{M}_2 - \bar{M}_1) \geq \pi_1^I(\hat{S}_1) + \pi_2^I(CN_2) \Leftrightarrow \frac{a_1(a_1-2c)}{4b} + \frac{(a_2-c)^2}{4b} - \frac{(a_2-a_1-c)c}{2b} \geq \frac{(a_1-c)^2}{8b} + \frac{(a_2-c)^2}{9b} \Leftrightarrow (a_1)^2 + 2a_1c - (c)^2 \geq (a_2-c)(5c-a_2)$. ♣

6 Linear Appendix

For the linear model, I provide here algebraic expressions for the values defined in section 2. Since these are based upon the one period model, I drop the time subscript. When I mention points in q^I, q^E space, the q^I value is listed first.

$$CN = (\frac{a-c}{3b} \frac{a-c}{3b}) \tag{1}$$

$$V = (\frac{a+c}{3b}, \frac{a-2c}{3b}) \tag{2}$$

$$U = (\frac{a}{3b}, \frac{a}{3b}) \tag{3}$$

$$S = (\frac{a-c}{2b}, \frac{a-c}{4b}) \tag{4}$$

$$\tilde{S} = \begin{cases} (\frac{a+2c}{4b}, \frac{a-2c}{2b}) & \text{if } a \leq 6c \\ U & \text{if } a \geq 6c \end{cases} \tag{5}$$

$$W^I = \begin{cases} \frac{a-c}{b} - \frac{a-2c}{b\sqrt{2}}) & \text{if } a \leq 6c \\ \frac{a-c}{b} - \frac{2\sqrt{a(a-3c)}}{3b} & \text{if } a \geq 6c \end{cases} \tag{6}$$

$$W^E = \begin{cases} \frac{a-2c}{2b\sqrt{2}} & \text{if } a \leq 6c \\ \frac{\sqrt{a(a-3c)}}{3b} & \text{if } a \geq 6c \end{cases} \tag{7}$$

$$\hat{S} = (\min\{S^I, W^I\}, \max\{S^E, W^E\}) \tag{8}$$

$$\pi^E(\tilde{S}) = \pi^E(W) = \begin{cases} \frac{(a-2c)^2}{8b} & \text{if } a \leq 6c \\ \frac{a(a-3c)}{9b} & \text{if } a \geq 6c \end{cases} \tag{9}$$

$$\pi^I(\tilde{S}) = = \begin{cases} \frac{a^2-4c^2}{16b} & \text{if } a \leq 6c \\ \frac{a(a-3c)}{9b} & \text{if } a \geq 6c \end{cases} \tag{10}$$

$$\pi^I(W) = = \begin{cases} \frac{(a-2c)(a+c\sqrt{2})}{8b} \cdot (2\sqrt{2}-2) & \text{if } a \leq 6c \\ \frac{(a-c)\sqrt{a(a-3c}}{3b} - \frac{2a(a-3c)}{9b} & \text{if } a \geq 6c \end{cases} \tag{11}$$

In the linear model, $R_t(\cdot, K)$ is defined as follows.

$$R_t(q, K) = \begin{cases} \frac{a_t - bq}{2b} & \text{if } q < \frac{a_t - 2bK}{b} \\ \frac{a_t - bq - c}{2b} & \text{if } q > \frac{a_t - 2bK - c}{b} \\ K & \text{otherwise} \end{cases} \tag{12}$$

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