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Market Structure and Product Innovation

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Abstract

This paper considers a model where current producers of a product and an entrant compete in developing a new patentable product. An incumbent's win enables it to internalize the externalities between the new and existing products, but may either strengthen or weaken its competitive position in the existing product. Whether an incumbent or the new entrant will spend more on innovation depends crucially on strategic relations between the two products. The incumbent tends to spend more (less) when the products are strategic complements (substitutes). The analysis offers testable implications about whether an industry tends to become more or less concentrated as new market opportunities arise.

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I. Introduction

An intriguing issue in industrial organization and antitrust is whether market dominance persists. In their pioneering study of patent races between an incumbent and an entrant, Gilbert and Newbery(1982) argued that monopoly tends to persists since by winning the new product the incumbent avoids dissipation of rents through competition. Subsequent research has considered other possibilities. Reinganum (1983) shows that a monopoly may not be able to preempt an entrant in acquiring a new technology if the discovery process is uncertain. On the other hand, within the framework of deterministic discovery process, Kamien and Zang (1990), Krishna (1993), and Lewis (1983), have shown that monopoly need not persist it there are innovative opportunities. More recently, Chen (2000) shows that, when the incumbent faces entry threat to its existing business, who wins the bidding for a new product depends importantly on the strategic relationship between the new and existing products.

In this paper, I pose a more general question: As new market opportunities arise, will an industry remain as concentrated or becomes less so? Thus, in contrast to all the existing studies, where the incumbent is a monopolist, I allow the incumbents to be oligopolists; and instead of the study of innovative incentives of a monopolist and an entrant, I investigate more generally the relationship between market structure and product innovation. In one respect, this exercise will add more realism to the existing analysis, since some kind of competition between incumbents is present in most industries. But more importantly, this will provide a unified framework to address the issue of market structure and innovation. The existing models in the literature will be shown as special cases of the model that I will develop in this paper.

I consider a model where n firms are current producers of an existing product. They, together with an entrant, can invest in R&D in developing a new product. The existing and the new products are related: They can be strategic substitutes or strategic complements, as in Chen (2000). The discovery process for the new product is stochastic. Winning the new product by an incumbent enables it to internalize the externalities between the two products, but may either strengthen or weaken its competitive position in its current product where it competes with other incumbents. When the two products are strategic substitutes, the entrant tends to have more incentive in innovating the new product; and when the two products are complements the incumbents tend to have more incentives. Thus, my results suggest that if an industry's innovative opportunity is on products that are strategic complements of the existing product(s), the industry tends to maintain its existing concentration; but if the innovative opportunity is on products that are strategic substitutes of the existing product(s), the industry tends to become less concentrated overtime.

While more empirical work is needed to test our theoretical predictions, casual observations suggest some supporting examples. In the early 1980's, for instance, IBM was both the dominant producer of personal computers (PC) and mainframe computers. However, as rapid innovation took place in the PC industry, PC's became closer substitutes to the faster mainframe computers. Overtime IBM lagged behind the innovation frontier in PC and gave way to entrants such as Compaq. It is argued that IBM held back on PC development in order to protect its dominant position in the mainframe market (see Chen (2000) pp.164). Another example is Microsoft's dominant position in both applications software and operating systems software markets. Microsoft continues to be at the frontier in innovating new software both in the applications and the operating systems market. These two product categories are complementary and serve to strengthen Microsoft's dominant position.

My research is also related to Vickers (1986), who makes the point that when a sequence of innovative opportunities is present, strategic decisions to innovate must account for complicated reciprocal effects. Firms in a patent race will take account not only of the immediate effect of the patent race, but also of its indirect effect upon future patent races. Unlike Vickers, the simple model that I present in this paper only allows one innovative opportunity. This allows me to focus on the effect that the strategic relationship between new and already existing products have on the strategic decision to innovate without confounding the analysis.

Also related is Sutton (2000), who aimed at uncovering mechanisms at work that influence the relation between industry concentration and innovation. He found that whether a R&D intensive industry becomes more or less concentrated depends crucially on the ability of the new product to capture market share of existing products. His argument is intuitively appealing. He argues that if a firm has the option of pursuing a number of research trajectories where each trajectory is expected to lead to discovery of a distinct product belonging to a distinct product group, and if product groups are close substitutes, then the firm will escalate its R&D spending along only one trajectory. This is because discovery along any one- research trajectory will capture enough market share from other product groups sufficient to cover high R&D spending. The eventual equilibrium configuration of such an industry is one of high R&D intensity and high industry concentration. On the other hand, if product groups belonging to the same industry are poor substitutes, then the R&D intensive firms will spread R&D spending across several research trajectories rather than escalate spending along any one trajectory. This is a rational research strategy for research-intensive firms since discovery along any one trajectory is not as profitable as if product groups are close substitutes. He therefore argues that the eventual equilibrium configuration of a research-intensive industry with product groups that are poor substitutes is high innovation and low industry concentration.

I present the model in the next section. The solution to the model is presented in section III, and section IV concludes.

II. The Model

The single industry in this partial equilibrium model will eventually comprise two related markets, Q and Y. Q is an already existing market while Y is a new market to be operated by a monopolist subsequent to patenting of this product. The discovery, patenting and subsequent production of good Y can be described as product innovation. There are n Cournot competitors in market Q. A potential entrant, E, along with the n existing firms in Q, enter a patent race for the right to be the sole producer of the new product Y. To be a part of the patent race, each firm and the potential entrant must spend real resources on R&D.

In order for a firm to enter the already existing market Q, it must pay an entry cost K. We can consider K to be a setup cost, which I assume is sufficiently large so that entry is blockaded to market Q. Note that entry is not blockaded to the industry in general since entry is possible through product innovation. Therefore, firm E can only compete indirectly with the other n firms in market Q through Y's strategic relation to Q by winning the patent race for good Y. The assumption that entry to market Q is blockaded allows me to focus on preemption via product innovation. The entire model is a dynamic game that can be decomposed into two major sub-games, the production stage sub-game and the R&D sub-game. However, we can further decompose the production stage sub-game into three sub-games. The solution concept used is sub-game perfect Nash equilibrium. I will first describe the production stage sub-game and then the R&D sub-game.

Production stage Sub-game

Marginal cost for each of the n firms in Q is normalized to zero while, marginal cost for any firm producing in Y market is constant and equal to c. This normalization has absolutely no impact on the results but has the benefit of simplifying notation. There is no uncertainty about demand or cost conditions. Uncertainty will only occur in the R&D sub-game. Inverse demand functions for market Q and Y are given by $P_Q = f(Q, Y)$ and $P_y = g(Y, Q)$ respectively. These demand functions have the usual properties, that is, $f_1(Q, Y) < 0$ and $g_1(Y, Q) < 0$, where subscripts denote partial derivatives and the value of the numerical subscript corresponds to the argument to which the derivative is taken. In terms of restrictions on the second derivatives of these demand functions, all we need to assume is that the second derivatives are not sufficiently large to overturn the concavity of the profit functions. This only implies a restriction on how convex the demand function can be since linear and concave demand functions cannot overturn the concavity of the profit function[1]. The signs of $f_2(Q, Y)$ and $g_2(Y, Q)$ capture the relationship between good Q and Y. If $f_2(Q, Y) < 0$ and $g_2(Y, Q) < 0$, the two goods are substitutes in the usual sense while, if $f_2(Q, Y) > 0$ and $g_2(Y, Q) > 0$, the two goods are complements in the usual sense. I am careful here to say substitutes and complements in the usual sense because shortly I will define the cases where goods Q and Y are strategic substitutes and strategic complements.

[1] For example, if the profit function is given by, $p = [g(Y, Q) - c]Y$, concavity of p with respect to Y requires that, $g_{11}(\bullet)Y + 2g_1(\bullet) < 0$. Therefore, we allow for $g_{11}(Y, Q) \leq 0$ and $g_{11}(Y, Q) \geq 0$ as long as when $g_{11}(Y, Q) \geq 0$ it is not too large such that $g_{11}(\bullet)Y + 2g_1(\bullet) > 0$. Of course, there is an equivalent restriction on $f_{11}(Q, Y)$.

There are three possible sub-games within the production stage sub-game. The first sub-game is the pre-innovation production sub-game. This game is played by the n Cournot competitors in market Q each solving the following problem:

$$(1) \qquad \underset{q_i}{Max}\ p_i^N = f(Q, Y)q_i$$

where p_i^N and q_i denote incumbent i's profit and output respectively in the pre-innovation industry.

In the post-innovation industry there are two possible production sub-games. First, we have the case where one of the incumbents wins the patent race for Y. The winner solves the following problem:

$$(2) \qquad \underset{q_i, Y}{Max}\ p_i^w = f(Q, Y)q_i + [g(Y, Q) - c]Y$$

where p_i^w, q_i and Y denote the winning incumbent's profit, output in market Q, and output in market Y respectively. The other n-1 incumbents therefore solve:

$$(3) \qquad \underset{q_k}{Max}\ p_k^{LI} = f(Q, Y)q_k$$

where p_k^{LI} and q_k denote each unsuccessful incumbent's profit and output respectively.

Having introduced one of the production sub-games in the post-innovation industry, I think this is an appropriate time to define what it means when goods are strategic substitutes and strategic complements. The definition of strategic substitutes and complements used in this paper follows that used by Bulow, Greanakoplos, and Klemperer (1985). When two goods are strategic substitutes, an increase in the output of good Y lowers marginal profits in market Q, that is, $\frac{\partial^2 p_i}{\partial q_i \partial Y} < 0$. Conversely, when goods are strategic complements, an increase in the output of good Y increases marginal profits in market Q, that is, $\frac{\partial^2 p_i}{\partial q_i \partial Y} > 0$. It does not have to be the case that when goods are substitutes in the usual sense they are also strategic substitutes or that when goods are complements in the usual sense they are strategic complements[2]. However, throughout this paper I impose the restriction that whenever goods are substitutes in the usual sense they are also strategic substitutes and whenever goods are complements in the usual sense they are also strategic complements. This restriction is always true for linear demands.

The other production sub-game in the post-innovation industry occurs when the entrant wins the patent race and competes indirectly with the other n incumbents via the strategic relationship between Q and Y. The entrant therefore solves the following problem:

(4) $$\underset{Y}{Max}\ p_E = [g(Y, Q) - c]Y$$

where p_E and Y denote the entrant's profit and production of good Y respectively. The other n incumbents solve the following:

(5) $$\underset{q_i}{Max}\ p_i^{LE} = f(Q, Y)q_i$$

where p_i^{LE} and q_i denote each incumbents' profit and output respectively when the entrant wins the patent race. This completes the description of the production sub-games. Let me now describe the R&D sub-game.

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[2] For example, if the profit function of the winning incumbent is given by $p_i^w = f(Q, Y)q_i + [g(Y, Q) - c]Y$, then strategic complements require $f_{12}(\bullet)q_i + f_2(\bullet) + g_{21}(\bullet)Y + g_2(\bullet) > 0$ and strategic substitutes require $f_{12}(\bullet)q_i + f_2(\bullet) + g_{21}(\bullet)Y + g_2(\bullet) < 0$. Clearly if the size of the cross partials are small in absolute value, then the sign of the expression depends on the signs of $f_2(\bullet)$ and $g_2(\bullet)$. Thus the restriction that when goods are substitutes (complements) in the usual sense, they should also be strategic substitutes (complements), implies a restriction the absolute sizes of the cross partials. The restriction always holds for linear demand since cross partials are zero in this case.

R&D Sub-game

As mentioned before, n incumbents and an entrant are simultaneously attempting to invent and patent a new product. The stochastic patent race developed here is much like that in Reinganum (1983), Wilde and Lee (1980) and Delbono and Denicolo (1991). The main difference between the patent race here and those developed in the other papers just mentioned, is that the patent race here is for development of the new product while, in the other models the race was to develop a cost reducing production process for an already existing product.

Technological uncertainty takes the form of a stochastic relationship between the rate of investment and the eventual date of successful completion of the new technology. If x_i represents the rate of R&D investment of an incumbent, and $t_i(x_i)$ the random success date of the incumbent, then $\Pr(t_i(x_i) \leq t) = 1 - e^{-h(x_i)t}$ for $t \in [0, \infty)$. Similarly, if z and $t(z)$ represent the investment rate and the random success date for the entrant, then $\Pr(t(z) \leq t) = 1 - e^{-h(z)t}$. The expected date of success in each case is given by $\frac{1}{h(\bullet)}$. The hazard function, $h(\bullet)$, is twice continuously differentiable, with $h'(\bullet) > 0$ and $h''(\bullet) < 0$ for all $x, z \in [0, \infty)$. Furthermore, $h(0) = 0 = \lim_{x, z \to \infty} h'(\bullet)$.

Since the new product is patentable, the race ends with the first success. The expected profit to an incumbent, i, for the profile of investment rates $(z, x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$ is given by:

$$(6) \qquad V_i = \int_0^\infty e^{-rt} e^{-\left[h(x_i) + \sum_{k \neq i}^{n} h(x_k) + h(z)\right]t} \left[p_i^N - x_i + h(x_i)\frac{p_i^w}{r} + h(z)\frac{p_i^{LE}}{r} + \sum_{k \neq i}^{n} h(x_k)\frac{p_i^{LI}}{r}\right] dt$$

For a more detailed and intuitive derivation of equations (6) and (7), see Tirole (1988) pp.395. As defined in the production stage game, p_i^N is incumbent i profit before any innovation takes place, p_i^w is incumbent i profit if she wins the patent race, p_i^{LE} is incumbent i profit if she loses patent race to the entrant, and p_i^{LI} is incumbent i profit if she loses patent race to another incumbent. r represents market interest rate.

The expected profit of the entrant is given by:

$$(7) \qquad V_E = \int_0^\infty e^{-rt} e^{-\left[h(x_i)+\sum_{k\neq i}^{n} h(x_k)+h(z)\right]t} \left[h(z)\frac{p_E}{r} - z\right] dt$$

Again as described in the production stage game, p_E denotes the entrant's profit if she wins the patent race. Equation (6) and (7) can be rewritten as:

$$(6') \qquad V_i = \frac{p_i^N - x_i + h(x_i)\frac{p_i^w}{r} + h(z)\frac{p_i^{LE}}{r} + \sum_{k\neq i}^{n} h(x_k)\frac{p_i^{LI}}{r}}{r + h(x_i) + \sum_{k\neq i}^{n} h(x_k) + h(z)}$$

$$(7') \qquad V_E = \frac{h(z)\frac{p_E}{r} - z}{r + h(x_i) + \sum_{k\neq i}^{n} h(x_k) + h(z)}$$

respectively. This completes the description of the R&D subgame.

III. Solving the Model

To get an idea how to solve this dynamic model, let me summarize the optimizing behavior of a typical firm. At time zero each firm solves an infinite-horizon dynamic optimization problem. That is, each firm will maximize lifetime expected profits by choosing optimal R&D investment rate conditional on her profits from each possible production sub-game and rivals' best response R&D spending. To do this optimization, each firm must know the Nash equilibrium outputs and thus profits from each possible production sub-game along with rivals' best response R&D spending. Firms in this model can accurately predict these equilibrium profits for each possible production sub-game since there is no uncertainty in demand or cost. The only uncertainty the firm faces is, knowing which production sub-game it will eventually end up playing. Of course, it is the stochastic innovation process that drives the uncertainty.

Let me now turn to the task of solving the model. The first partials of $(6')$ and $(7')$ are given by:

$$(8)\qquad \frac{\partial V_i}{\partial x_i}=\frac{\left[r+h(x_i)+\sum_{k\neq i}^{n}h(x_k)+h(z)\right]\left[h'(x_i)\frac{p_i^w}{r}-1\right]-\left[p_i^N-x_i+h(x_i)\frac{p_i^w}{r}+h(z)\frac{p_i^{LE}}{r}+\sum_{k\neq i}^{n}h(x_k)\frac{p_i^{LI}}{r}\right]h'(x_i)}{\left[r+h(x_i)+\sum_{k\neq i}^{n}h(x_k)+h(z)\right]^2}$$

$$(9)\qquad \frac{\partial V_E}{\partial z}=\frac{\left[r+h(x_i)+\sum_{k\neq i}^{n}h(x_k)+h(z)\right]\left[h'(z)\frac{p_E}{r}-1\right]-\left[h(z)\frac{p_E}{r}-z\right]h'(z)}{\left[r+h(x_i)+\sum_{k\neq i}^{n}h(x_k)+h(z)\right]^2}$$

A Nash equilibrium profile of R&D spending, $\left(z^*,\, x_1^*,\, x_2^*,\ldots,\, x_n^*\right)$, where * denotes optimal levels, must satisfy:

$$(8')\qquad \left[r+h(x_i)+\sum_{k\neq i}^{n}h(x_k)+h(z)\right]\left[h'(x_i)\frac{p_i^w}{r}-1\right]-\left[p_i^N-x_i+h(x_i)\frac{p_i^w}{r}+h(z)\frac{p_i^{LE}}{r}+\sum_{k\neq i}^{n}h(x_k)\frac{p_i^{LI}}{r}\right]h'(x_i)=0$$

$$(9')\qquad \left[r+h(x_i)+\sum_{k\neq i}^{n}h(x_k)+h(z)\right]\left[h'(z)\frac{p_E}{r}-1\right]-\left[h(z)\frac{p_E}{r}-z\right]h'(z)=0$$

Equation $(8')$ represents incumbent i reaction function. For example, if we rearrange equation $(8')$ in a way that has incumbent i's R&D spending on the left hand side and all his rivals R&D spending on the right hand side, then the resulting equation gives incumbent i's best response spending given the spending of his rivals. An incumbent's rivals comprise all the other n-1 incumbents plus the entrant. Of course, each incumbent has his own reaction function. Similarly, equation $(9')$ represents the reaction function of the entrant. In total, there are n+1 reaction functions. Thus a firm's reaction function in this model is in a n+1 dimensional space. Exploiting the symmetry of the model can reduce the dimensions.

Due to the symmetry of the n incumbents, a symmetric equilibrium requires that $x_i = x \forall i$. As such, we can drop the subscript on all x. Equations $(8')$ and $(9')$ can then be written as:

$$(10) \quad h'(x)\left[p_i^w - p_i^N\right] + \frac{1}{r} h(z)h'(x)\left[p_i^w - p_i^{LE}\right] + \frac{1}{r}(n-1)h(x)h'(x)\left[p_i^W - p_i^{LI}\right] - r - \left[h(z) + nh(x)\right] + xh'(x) = 0$$

$$(11) \quad \left[h'(z) + \frac{1}{r} nh(x)h'(z)\right]\left[p_E - 0\right] - r - \left[h(z) + nh(x)\right] + zh'(z) = 0$$

For an incumbent, $\left[p_i^w - p_i^{LE}\right]$ and $\left[p_i^w - p_i^{LI}\right]$ in equation (10) represent the presence of rivalry from entrant and other incumbents respectively. In a somewhat similar model setup to this model, Delbono and Denicolo (1991) call these two terms "competitive threats". $\left[p_i^w - p_i^N\right]$ is an incumbent's incentive to invest in the absence of rivalry and is called the "profit incentive". In equation (11) we can see that the entrant only faces a profit incentive given by $\left[p_E - 0\right]$. This results from the assumption that the entrant is not producing any product prior to innovation.

Equation (10) can be written in the form, $x = R_i(z)$, where $R_i(\bullet)$ is an incumbents' reaction function. Note that in writing $R_i(\bullet)$, for exposition, I have suppressed all parameters that appear in equation (10). Thus, for a given R&D spending of the entrant, z, assuming all other parameters held constant, $R_i(z)$ gives the incumbent's best response R&D spending. Similarly, equation (11) can be written in the form, $z = R_E(x)$, where $R_E(\bullet)$ is the entrant's reaction function and again parameters are suppressed for notational convenience. Note I have exploited the symmetry of the model in writing down the reaction functions.

By examining equations (10) and (11), it is difficult to conclude whether the equilibrium R&D spending of an incumbent is greater or less than that of the entrant, that is, whether $x^* \geq z^*$ or $x^* \leq z^*$. One way to proceed is to impose restrictions on equation (10) in a manner that allows us to predict the rank in the level of equilibrium spending. Assumptions can then be systematically relaxed and its effect on the previous rank in level of spending determined. If we assume that, (1) there exist only one incumbent, that is n=1, and (2) the new product displaces the old product, that is $p_i^{LE} = 0$, then we can rewrite equations (10) and (11) as:

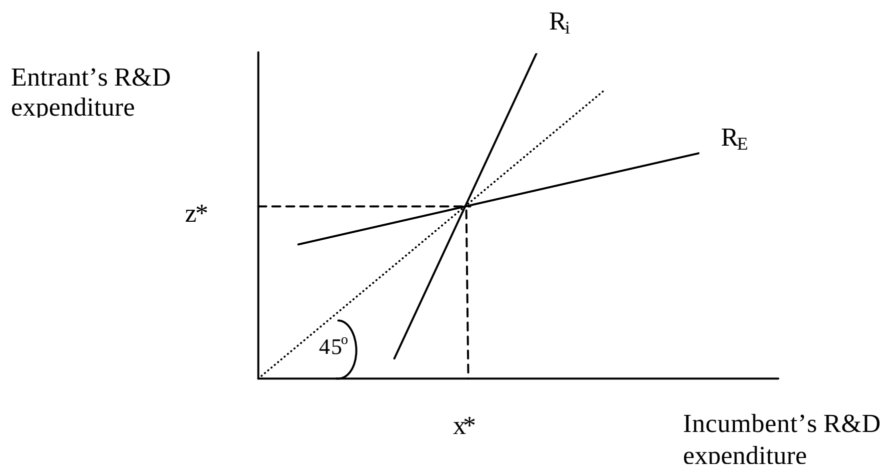
(12) $$\left[h'(x) + \frac{1}{r} h(z)h'(x)\right]p_i^w - h'(x)p_i^N - r - [h(z) + h(x)] + xh'(x) = 0$$

(13) $$\left[h'(z) + \frac{1}{r} h(x)h'(z)\right]p_E - r - [h(z) + h(x)] + zh'(z) = 0$$

For n=1, $p_i^{LE} = 0$ and also $p_i^N = 0$, equations (12) and (13) represent two symmetric upward sloping reaction functions, one for the incumbent and the other for the entrant. The fact that they are upward sloping follows from, $\frac{dx}{dz} > 0$ for the reaction function of the incumbent and $\frac{dz}{dx} > 0$ for the reaction function of the entrant. The proof of the sign of these derivatives is in the appendix. I impose the condition that the incumbent's reaction function must be steeper than the entrant's reaction function at a Nash equilibrium[3]. Without loss of generality I can depict the reaction functions as seen in figure 1.

[3] This is the usual stability condition that is satisfied as long as $\left|\frac{\partial^2 V_i}{\partial x^2}\right| > \left|\frac{\partial^2 V_i}{\partial x \partial z}\right|$ and $\left|\frac{\partial^2 V_E}{\partial z^2}\right| > \left|\frac{\partial^2 V_E}{\partial z \partial x}\right|$. Also with this condition I am assured that the qualitative results illustrated with linear reaction functions will not change with nonlinear reaction functions.

Figure 1



In figure 1, R_i and R_E denote the reaction functions of an incumbent and an entrant respectively. We can see that the Nash equilibrium occurs on the 45° line. This implies that $x^* = z^*$. This result follows from the symmetry of the reaction functions when n=1, $p_i^{LE} = 0$ and $p_i^N = 0$. If we should change any of the parameter values we have set, this would cause a shift in one or both reaction functions depending on the parameter that is perturbed. I will start by allowing $p_i^N > 0$ while maintaining that n=1, and $p_i^{LE} = 0$. We should only observe a shift in the incumbent's reaction function since p_i^N only appears in equation (12) and not (13). For $p_i^N > 0$, all other things held constant, the reaction function of the incumbent must shift to the left, which follows directly from lemma 1.

Lemma 1. For n=1 and $p_i^{LE} = 0$, $\dfrac{dx}{dp_i^N} < 0$.

Proof:

Define equation (12) as:

$$G = \left[h'(x) + \frac{1}{r}\,h(z)h'(x)\right]p_i^w - h'(x)p_i^N - r - [h(z) + h(x)] + xh'(x)$$

By the implicit function theorem:

$$\frac{dx}{dp_i^N} = -\frac{\partial G / \partial p_i^N}{\partial G / \partial x} = \frac{h'(x)}{h''(x)\left(p_i^w - p_i^N\right) + \frac{1}{r} h(z) h''(x) p_i^w + x h''(x)}$$

The concavity of $h(\bullet)$ gets us the result.

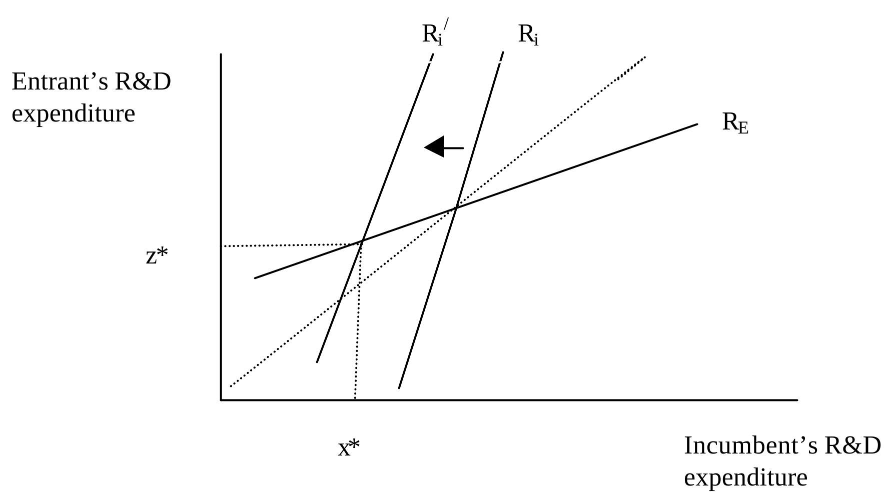
QED.

Using figure 1 as the base case, a leftward shift of the incumbent's reaction function must yield a new Nash equilibrium in which $x^* < z^*$. This is illustrated in figure 2 and leads us to proposition 1.

Proposition 1. For n=1 and $p_i^{LE} = 0$, any positive pre-innovation profit for the incumbent requires that equilibrium R&D spending of the incumbent be less than that of the entrant, that is, $x^* < z^*$.

Formal proof of proposition 1 is in the appendix.

Figure 2



Proposition 1 is not surprising since it resembles the result from Reinganum's (1983) process innovation model. Reinganum's (1983) result can thus be considered as a special case of the more general model presented in this paper. Reinganum (1983) investigated whether an incumbent monopolist is more likely to win a patent race for a cost reducing technology when the only challenger in the patent race is a potential entrant. Thus in her model there is only one homogeneous product. She proved that in the case of drastic innovation, as long as the incumbent monopolist has positive profits in the pre-innovation industry, the potential entrant is more likely to win the patent race and become the new monopolist in the post- innovation industry. Drastic innovation means that the new technology reduces cost to the extent that the owner of the new technology becomes the new monopolist. The intuition behind her result is that positive profits in the pre-innovation industry reduces the incumbent's incentive to spend on R&D because higher R&D spending stochastically reduces the length of time over which the incumbent earns these positive profits. In a sense the incumbent is faced with a dilemma because higher R&D spending increases the probability of winning by stochastically bringing forward the discovery date, but an earlier discovery date reduce the time over which the incumbent earns his pre-innovation profits. The potential entrant is not faced with this dilemma because in the pre-innovation industry the potential entrant has zero profits.

My model is a model of product innovation, therefore my analogous case of Reinganum's drastic innovation is the case where the new product completely displaces the old product ($p_i^{LE} = 0$). Since in proposition 1 I also set n=1 the pre-innovation industry is monopoly like Reinganum's (1983) model. The intuition for proposition 1 is therefore just like Reinganum, the incumbent monopolist with positive pre-innovation profits spends less on R&D compare to the entrant in equilibrium because the incumbent does not want to hasten his own replacement. Since proposition 1 is just an intermediate result let me press on to more interesting analysis.

If we now let $p_i^{LE} > 0$ but still maintain that n=1, we can re-write equation (10) as:

$$(14) \qquad \left[h'(x) + \frac{1}{r}h(z)h'(x)\right]p_i^{w} - h'(x)p_i^{N} - \frac{1}{r}h(z)h'(x)p_i^{LE} - r - [h(z) + h(x)] + xh'(x) = 0$$

Given that $p_i^{LE} > 0$, we now have the co-existence of the already existing product and the new product in the post-innovation industry. Thus I am relaxing the assumption analogous to drastic innovation made in proposition 1. This implies that the profit the incumbent gets if she wins, p_i^w, is not equal to the profit the entrant gets if she wins, p_E. If the entrant wins there will be two single product firms in the industry while, if the incumbent wins, there is only one multi-product firm in the industry. It is therefore natural to think that $p_i^w > p_E$. Even though p_i^{LE} only appears in the incumbent's reaction function, it would be inaccurate to conclude that changing this parameter has no effect on the entrant's reaction function. Changing p_i^{LE} affects the entrant's reaction function via p_E. Assuming all other things held constant, a higher p_i^{LE} should imply lower p_E. It is also the case that if we let n>1, both reaction functions are affected because n affects all profit levels. Since both reaction functions are shifting with a change in p_i^{LE} and n, the relevant concern now is: What can we conclude about the net effect on the equilibrium R&D spending?

To proceed with the analysis I parameterize the model and continue with numerical analysis. Assume the demand functions described earlier in the production sub-games are linear and take the following form:

(15) $$P_q = 1 - \sum_{i=1}^{n} q_i + by \quad ; \quad P_y = 1 - y + b\sum_{i=1}^{n} q_i$$

Total production in the already existing industry is given by $Q = \sum_{i=1}^{n} q_i$. Given these demand functions in (15), it turns out that when β>0 goods Q and Y are both complements in the usual sense and strategic complements. Conversely, whenever β<0 goods are both substitutes in the usual sense and strategic substitutes. A summary of the reduced-form equations is presented in table 1.

Table 1

Description	**Reduced-form functions**
Profit to each incumbent in the pre-innovation industry	$p_i^N = \frac{1}{(n+1)^2}$
Profit to the winning incumbent in the post-innovation industry	$p_i^w = \frac{1}{(n+1)^2} - \frac{(c - b - 1)^2}{4(b^2 - 1)}$
Given that an incumbent wins the patent race, the profit to each of the other incumbents	$p_i^{LI} = \frac{1}{(n+1)^2}$
Profit to the entrant if she wins patent race	$p_E = \frac{\left[(c-1)(n+1) - nb\right]^2}{\left[n(b^2-2)-2\right]^2}$
Profit to the other n incumbents given that the entrant wins the patent race	$p_i^{LE} = \frac{\left[b(c-1) - 2\right]^2}{\left[n(b^2-2) - 2\right]^2}$
Hazard function for an incumbent	$h(x) = 2m_i x^{\frac{1}{2}}$
Hazard function for the entrant	$h(z) = 2m_E z^{\frac{1}{2}}$

μ_i and μ_E are the parameters in the hazard functions that represent R&D efficiency of firms (see hazard functions in table1). The probability of winning the patent race increases with μ for all given levels of R&D spending. Recall that the probability that incumbent i makes a discovery before time t is given by $\Pr(t_i(x_i) \le t) = 1 - e^{-h(x_i)t}$. If we substitute the explicit functional form assumed for $h(x_i)$ in the probability expression, we can easily verify that $\frac{\partial \Pr(t_i(x_i) \le t)}{\partial m_i} = 2x_i^{\frac{1}{2}} t e^{-(2m_i x_i^{\frac{1}{2}})t} > 0$. Except for figure 7, all simulations were done assuming that all firms are equally efficient in R&D ($\mu_i = \mu_E = \mu = 0.7\ \forall i$). Since in reality it is likely that incumbents are more efficient in R&D compared to potential entrants, in figure 7 we analyze the impact of assuming that $\mu_i > \mu_E \; \forall i$.

In figure 3, I present simulations of the two reaction functions assuming n=1 and $\beta = 0.3$. Since β is positive, goods are complements. In figure 3, we can see that the Nash equilibrium R&D spending is to the right of the 45^0 line. This implies that the incumbent is willing to spend more than the entrant in equilibrium. In figure 4, the simulation is redone with the same parameter values except that β is now -0.3. Thus products are now substitutes. The interesting result here is that the equilibrium R&D spending is still to the right of the 45^0 line. Even though the Nash equilibrium R&D spending for both the incumbent and the entrant is lower when products are substitutes, the incumbent outspends the entrant both when products are substitutes and complements. This result is reminiscent of Chen (2000). Chen found that once there is no threat of entry to the incumbent monopolists existing business the incumbent monopolist will always outspend the entrant to acquire the new product irrespective of the strategic relation between the new and already existing product.

Figure 3

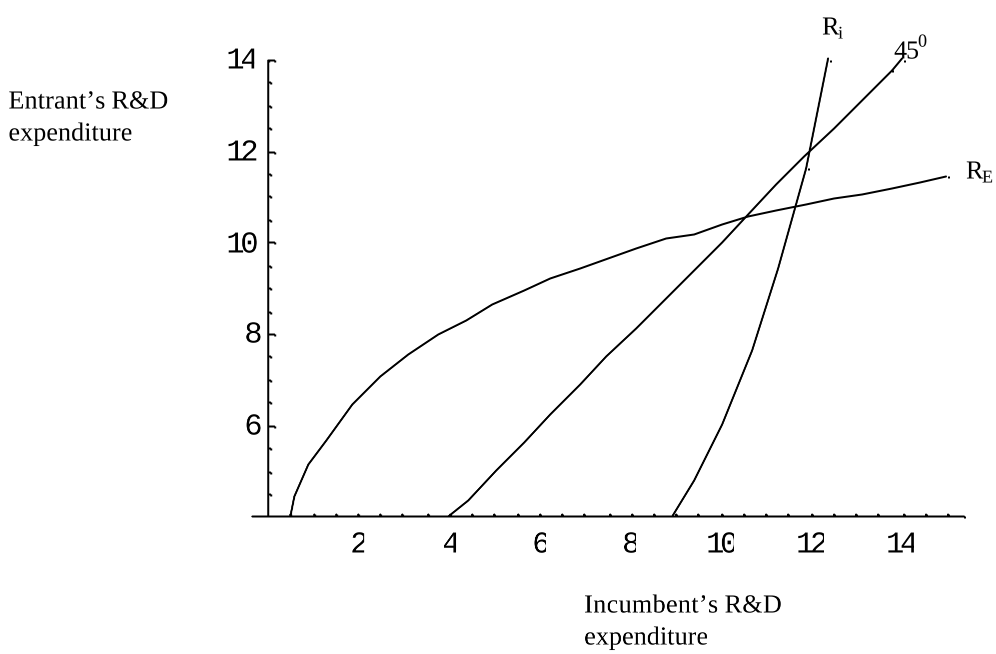
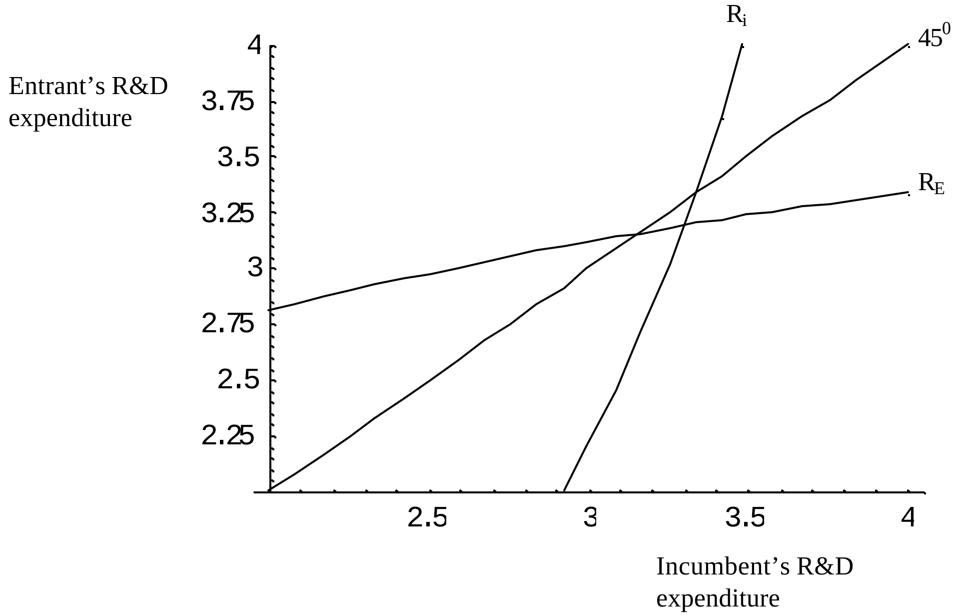


Figure 4



Let me give some intuition on Chen's result. The idea is that as long as entry to the incumbent monopolist existing market is blockaded, the monopolist has a higher valuation, relative to the entrant, for the new product because only the incumbent can become a multi-product firm. By becoming a multi-product firm, the incumbent monopolist can fully internalize reciprocal external pricing effects across related products. For example, if products are substitutes, a reduction in the price of one also reduces the demand and profitability of the other. If a firm controls the supply of both products, then it can control the level of competition between both products and is better off relative to the case where it is a single product firm. On the other hand, when products are complements, a reduction in the price of one good, for example, increases demand and profitability of the other good. A multi-product firm could internalize these profitable decisions while a single product firm cannot.

One difference between Chen's model and the model in this paper is that Chen's model is completely deterministic while the model in this paper introduces uncertainty in the innovation process. This implies that outspending a rival in my model does not necessarily guarantee victory in the patent race. Outspending a rival implies that the higher spender has a larger probability of winning the patent race. Let us now allow for more than one competitor in the already existing market and see how this affects the result.

Figure 5

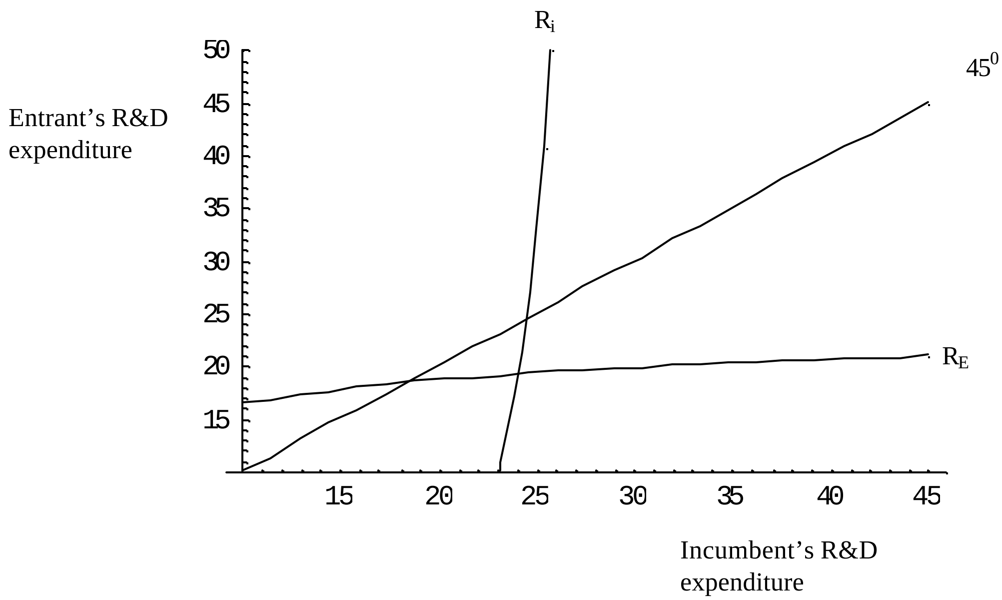
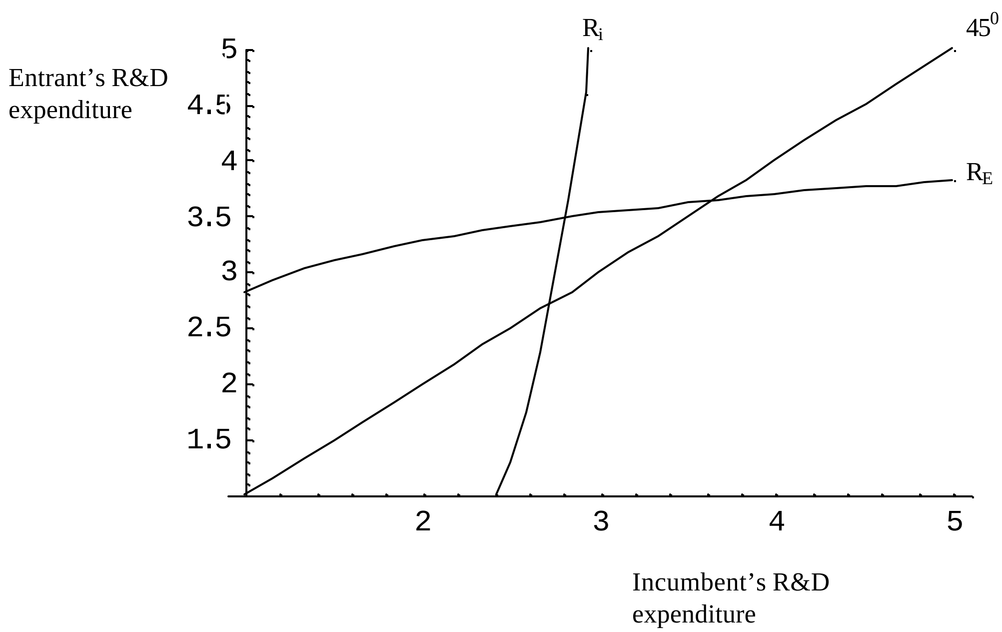


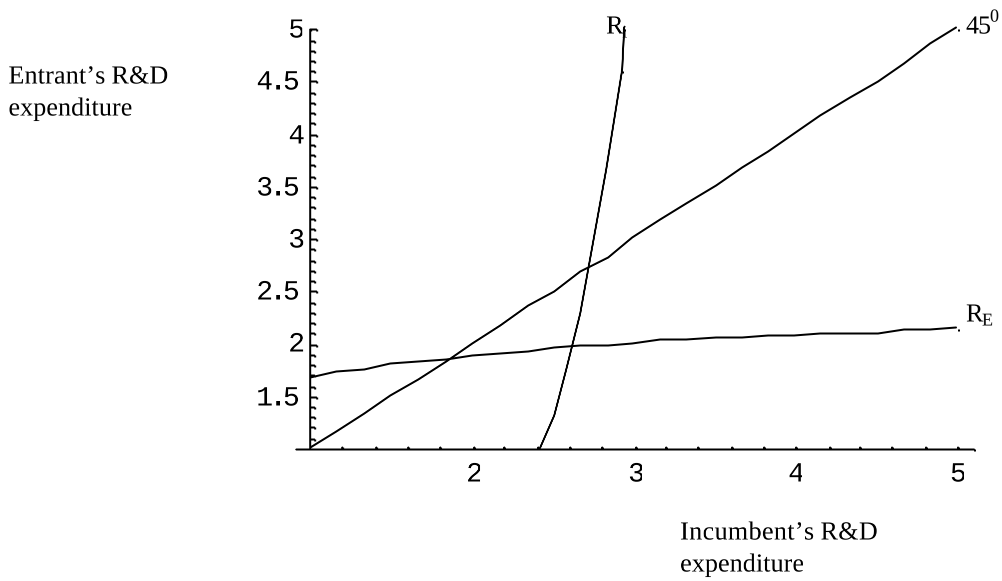
Figure 6



In figures 5 and 6 I made n=2, that is, there are now two Cournot competitors in the already existing market. Now that there are two incumbents, each incumbent must assess how acquiring the new product affects her strategic position in her already existing business. The difference between figure 5 and 6 is that in figure 5 goods are strategic complements (β=0.3) while in 6 goods are strategic substitutes (β= -0.3). The interesting result we observe is that when goods are strategic substitutes the Nash equilibrium R&D spending is to the left of the 45^0 line. This implies that a typical incumbent spends less on R&D compare to an entrant when goods are strategic substitutes. This is because, when goods are strategic substitutes, a multi-product incumbent's strategic position is weakened in her already existing business relative to the other single-product incumbents. If the multi-product firm takes an aggressive posture in her already existing market, (increase output in market Q), this lowers her profit in both markets when goods are strategic substitutes. However, when goods are strategic complements, a multi-product incumbent's strategic position in her already existing market is strengthened. This is because an aggressive posture by the multi-product firm in the already existing market increases her profit in the complementary market. Thus we observe in figure 5 that a typical incumbent will outspend an entrant.

Again the result here is consistent with Chen(2000), but the more general oligopoly model in this paper that makes the innovative process stochastic, seems to lend itself to sharper empirical hypotheses. One empirical hypothesis is that the probability that an entrant wins a patent race is greater than the probability that an incumbent wins when new and already existing products are strategic substitutes. However, this theoretical model does not rule out the equilibrium where the incumbent wins the patent race when new and already existing goods are strategic substitutes since the incumbent has positive probability of winning. Secondly, the oligopoly structure of this model allows for a more conducive environment to study industry concentration. The model predicts that there is a greater probability that the industry becomes more concentrated when goods are complements. A third interesting feature of the more general model presented in this paper is that we can impose asymmetry in the R&D efficiency of incumbents relative to entrants and thus analyze the impact of this asymmetry. This sort of analysis is not possible in Chen's(2000) bidding model. The impact of imposing this sort of asymmetry in R&D efficiency is analyzed in figure 7.

Figure 7



In figure 7 we assume that n=2, B=-0.3, u_i=0.7, and u_E=0.5. Thus the relevant comparison is figure 6 and 7. The only difference between figures 6 and 7 is that in figure 6 u_E=0.7 while in figure 7 u_E=0.5. In other words, in figure 7 the entrant is made less efficient at R&D or put differently, incumbents are more efficient at R&D relative to the entrant. The interesting result here is that the Nash equilibrium is on the right hand side of the 45^o line even though goods are strategic substitutes. Thus even though the winning incumbent compromise his strategic position in his already existing market when goods are strategic substitutes, because incumbents are more efficient at doing R&D, we get the result that they spend more on R&D. It is therefore more likely that market dominance will persist and entry deterred when incumbents are more efficient at doing R&D irrespective of the strategic relation between new and already existing products.

IV. Conclusion

This research has found that when discovery time of product innovation is uncertain in a Cournot oligopoly model, the entrant is willing to spend more on R&D when products are strategic substitutes, but incumbents outspend the entrant when products are strategic complements. The model also posits a testable hypothesis: an entrant has a higher probability to win a patent race than an incumbent when new and existing products are strategic substitutes.

Appendix

Proof that non-negativity of expected profits requires that $\left[h'(z)\frac{p_E}{r} - 1\right] \geq 0$.

Using the entrant's reaction function (equation (11)) we can get the following:

$$\frac{p_E}{r} = \frac{h(z) - zh'(z) + r + nh(x)}{h'(z)[r + nh(x)]}$$

Substitute for $\frac{p_E}{r}$ in equation (7/) and rearrange terms yields:

$$V_E = \frac{h(z) - zh'(z)}{h'(z)[r + nh(x)]}$$

Since $V_E \geq 0$ by the restriction that expected profits are non-negative, it must be the case that $h(z) - zh'(z) \geq 0$. But from the entrant's reaction function we have:

$$[r + nh(x)]\left[h'(z)\frac{p_E}{r} - 1\right] - [h(z) - zh'(z)] = 0$$

Given that $h(z) - zh'(z) \geq 0$, from the reaction function we must have $\left[h'(z)\frac{p_E}{r} - 1\right] \geq 0$. It also follows by an identical argument that when the incumbent's reaction function is symmetric to the entrant's reaction function (n=1, $p_i^{LE} = 0$ and $p_i^N = 0$) we must also have $\left[h'(x)\frac{p^w}{r} - 1\right] \geq 0$.

QED.

Proof that the symmetric reaction functions are upward sloping:

Recall symmetry occurs when n=1, $p_i^{LE} = 0$ and $p_i^N = 0$. In the case of the reaction function of the incumbent, let $G = \left[h'(x) + \frac{1}{r}h(z)h'(x)\right]p_i^w - h'(x)p_i^N - r - [h(z) + h(x)] + xh'(x)$. By the implicit function theorem:

$$\frac{dx}{dz} = -\frac{\partial G / \partial z}{\partial G / \partial x} = -\frac{h'(z)\left[h'(x)\frac{p_i^w}{r} - 1\right]}{h''(x)\left[1 + \frac{h(z)}{r}\right]p_i^w + xh''(x)}$$

The ratio is negative but the minus sign in front makes $\frac{dx}{dz} > 0$. The numerator is non-negative due to the requirement that expected profits be non-negative, while the denominator is negative by the concavity of $h(\bullet)$. In the case of the reaction function of the entrant, let $F = \left[h'(z) + \frac{1}{r}h(x)h'(z)\right]p_E - r - [h(z) + h(x)] + zh'(z)$. By the implicit function theorem:

$$\frac{dz}{dx} = -\frac{\partial F / \partial x}{\partial F / \partial z} = -\frac{h'(x)\left[h'(z)\frac{p_E}{r} - 1\right]}{h''(z)\left[1 + \frac{h(x)}{r}\right]p_E + zh''(z)}$$

The non-negativity of the numerator and the concavity of $h(\bullet)$ gets us the result just as in the case of the incumbent.

QED.

Proof of proposition 1:

Suppose there exist a Nash equilibrium where $x^* \geq z^*$. By definition of Nash equilibrium, equation (13) must be satisfied at this proposed equilibrium, that is:

$$\left[h'(z^*) + \frac{1}{r}h(x^*)h'(z^*)\right]p_E - r - [h(z^*) + h(x^*)] + z^*h'(z^*) = 0$$

If we substitute z^* wherever we see x^*, then by the fact that $h'(\bullet) > 0$ and $\left[h'(z)\frac{p_E}{r} - 1\right] \geq 0$ we have:

$$0 \geq \left[h'(z^*) + \frac{1}{r}h(z^*)h'(z^*)\right]p_E - r - [h(z^*) + h(z^*)] + z^*h'(z^*)$$

Further, if we subtract $h'(z^*)p^N$ from the right hand side we have:

$$0 > \left[h'(z^*) + \frac{1}{r}h(z^*)h'(z^*)\right]p_E - h'(z^*)p^N - r - [h(z^*) + h(z^*)] + z^*h'(z^*)$$

If we now replace z^* with x^* wherever x^* appear in the incumbent's reaction function (equation 12), then by the concavity of the incumbent's expected profit function (equation 6), we must have

$$0 > \left[h'(z^*) + \frac{1}{r} h(z^*) h'(z^*) \right] p_E - h'(z^*) p^N - r - \left[h(z^*) + h(z^*) \right] + z^* h'(z^*) \geq$$

$$\left[h'(x^*) + \frac{1}{r} h(z^*) h'(x^*) \right] p_E - h'(x^*) p^N - r - \left[h(z^*) + h(x^*) \right] + x^* h'(x^*)$$

In this case, $p^w = p_E$, since we assume that the new product displaces the old. Thus I have shown that the incumbent's reaction function is not satisfied by the proposed Nash equilibrium, which contradicts that it is Nash.

QED.

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